

# HG2002 Semantics and Pragmatics

## Formal Semantics

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### Lecture 10

<https://bond-lab.github.io/Semantics-and-Pragmatics/>

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# Overview

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- Revision: Components
- Quantifiers and Higher Order Logic
- Modality
- (Dynamic Approaches to Discourse)
- Next Lecture: Chapter 11 — Cognitive Semantics

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# Revision: Componential Analysis

# Break word meaning into its components

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- components allow a compact description
- interact with morphology/syntax
- form part of our cognitive architecture
- For example:

<i>woman</i>	[FEMALE]	[ADULT]	[HUMAN]	
<i>spinster</i>	[FEMALE]	[ADULT]	[HUMAN]	[UNMARRIED]
<i>bachelor</i>	[MALE]	[ADULT]	[HUMAN]	[UNMARRIED]
<i>wife</i>	[FEMALE]	[ADULT]	[HUMAN]	[MARRIED]

- We can make things more economical (fewer components):

<i>woman</i>	[+FEMALE]	[+ADULT]	[+HUMAN]	
<i>spinster</i>	[+FEMALE]	[+ADULT]	[+HUMAN]	[−MARRIED]
<i>bachelor</i>	[−FEMALE]	[+ADULT]	[+HUMAN]	[−MARRIED]
<i>wife</i>	[+FEMALE]	[+ADULT]	[+HUMAN]	[+MARRIED]

# Defining Relations using Components

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- **hyponymy**: P is a hyponym of Q if all the components of Q are also in P.

*spinster*  $\subset$  *woman*; *wife*  $\subset$  *woman*

- **incompatibility**: P is incompatible with Q if they share some components but differ in one or more **contrasting** components

*spinster*  $\not\subset$  *wife*

- Redundancy Rules

[+HUMAN]	→	[+ANIMATE]	
[+ANIMATE]	→	[+CONCRETE]	
[+MARRIED]	→	[+ADULT]	
[+MARRIED]	→	[+HUMAN]	...

- Predicates with argument structure

*parent (of y)*(x,y) → [+PARENT](x,y)

# Katz's Semantic Theory

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- Semantic rules must be recursive to deal with infinite meaning
- Semantic rules interact with syntactic rule to build up meaning **compositionally**
- A **dictionary** pairs lexical items with semantic representations
  - \* (**semantic markers**) are the links that bind lexical items together in lexical relations
  - \* [**distinguishers**] serve to identify this particular lexical item

this information is not relevant to syntax
- **projection rules** show how meaning is built up
  - \* Information is passed up the tree and collected at the top.
  - \* **Selectional restrictions** help to reduce ambiguity and limit the possible readings

# Verb Classification

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- We can investigate the meaning of a verb by looking at its grammatical behavior
  - (1) Consider the following transitive verbs
    - a. *Margaret cut the bread*
    - b. *Janet broke the vase*
    - c. *Terry touched the cat*
    - d. *Carla hit the door*
- These do not all allow the same argument structure alternations

# Diathesis Alternations

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- **Causative/inchoative** alternation:  
*Kim broke the window* ↔ *The window broke*  
also *the window is broken* (state)
- **Middle construction** alternation:  
*Kim cut the bread* ↔ *The bread cut easily*
- **Conative** alternation:  
*Kim hit the door* ↔ *Kim hit at the door*
- **Body-part possessor ascension** alternation:  
*Kim cut Sandy's arm* ↔ *Kim cut Sandy on the arm*



# Diathesis Alternations and Verb Classes

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- A verb's (in)compatibility with different alternations is a strong predictor of its lexical semantics:

	<i>break</i>	<i>cut</i>	<i>hit</i>	<i>touch</i>
Causative	YES	NO	NO	NO
Middle	YES	YES	NO	NO
Conative	NO	YES	YES	NO
Body-part	NO	YES	YES	YES

*break* = {*break, chip, crack, crash, crush, ...*}

*cut* = {*chip, clip, cut, hack, hew, saw, ...*}

*hit* = {*bang, bash, batter, beat, bump, ...*}

*touch* = {*caress, graze, kiss, lick, nudge, ...*}

- *break* CAUSE, CHANGE  
*cut* CAUSE, CHANGE, CONTACT, MOTION  
*hit* CONTACT, MOTION  
*touch* CONTACT

# Cognitive Semantics

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- Major semantic components of Motion:
- \* **Figure**: object moving or located with respect to the **ground**
  - \* **Ground**: reference object
  - \* **Motion**: the presence of movement of location in the event
  - \* **Path**: the course followed or site occupied by the Figure
  - \* **Manner**: the type of motion

(2) *Kim swam away from the crocodile*  
Figure Manner Path Ground

(3) *The banana hung from the tree*  
Figure Manner Path Ground

- These are lexicalized differently in different languages.

Language (Family)	Verb Conflation Pattern
Romance, Semitic, Polynesian, ...	Path + fact-of-Motion
Indo-European (— Romance), Chinese	Manner/Cause + fact-of-Motion
Navajo, Atsuwegei, ...	Figure + fact-of-Motion

# Jackendoff's Lexical Conceptual Structure

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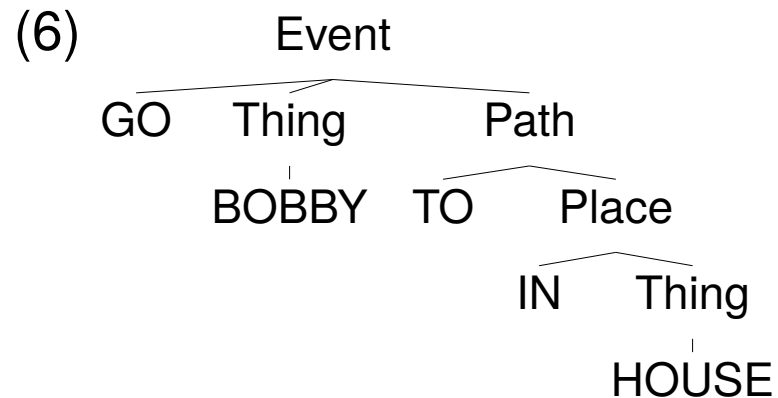
- An attempt to explain how we think
- **Mentalist Postulate**
  - Meaning in natural language is an information structure that is mentally encoded by human beings
- Universal Semantic Categories
  - \* **Event**
  - \* **State**
  - \* **Material Thing/Object**
  - \* **Path**
  - \* **Place**
  - \* **Property**

## Motion as a tree

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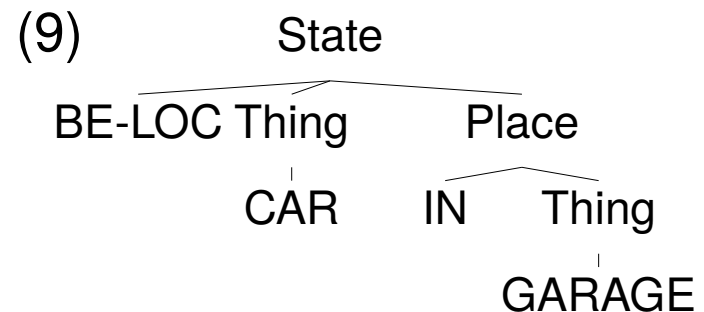
(4) *Bobby went into the house*

(5) “Bobby traverses a path that terminates at the interior of the house”



(7) *The car is in the garage*

(8) “The car is in the state located in the interior of the garage”



# Things: Boundedness and Internal Structure

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- Two components:

Boundedness	Internal Struct.	Type	Example
+b	−i	<b>individuals</b>	<i>a dog/two dogs</i>
+b	+i	<b>groups</b>	<i>a committee</i>
−b	−i	<b>substances</b>	<i>water</i>
−b	+i	<b>aggregates</b>	<i>buses, cattle</i>

- This can be extended to verb aspect (the verb event is also [ $\pm b$ ,  $\pm i$ ]).

*sleep* [−b], *cough* [+b], *eat* [ $\pm b$ ]

- (10) Bill ate two hot dogs in two hours.  
(11) \*Bill ate hot dogs in two hours.  
(12) #Bill ate two hot dogs for two hours.  
(13) Bill ate hot dogs for two hours.

# Conversion: Boundedness and Internal Structure

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## ➤ Including

**plural**

$[+b, -i] \rightarrow [-b, +i]$

*brick* → *bricks*

**composed of**

$[-b, +i] \rightarrow [+b, -i]$

*bricks* → *house of bricks*

**containing**

$[-b, -i] \rightarrow [+b, -i]$

*coffee* → *a cup of coffee/a coffee*

## ➤ Excluding

**element**

$[-b, +i] \rightarrow [+b, -i]$

*grain of rice*

**partitive**

$[-b, \pm i] \rightarrow [+b, -i]$

*top of the mountain, one of the*

**universal grinder**

$[+b, -i] \rightarrow [-b, -i]$

*There's dog all over the road*

# Pustejovsky's Generative Lexicon

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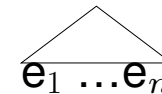
- Each lexical entry can have:
  - ARGUMENT STRUCTURE
  - EVENT STRUCTURE
  - LEXICAL INHERITANCE STRUCTURE
  - QUALIA STRUCTURE:
    - CONSTITUTIVE    constituent parts
    - FORMAL            relation to other things
    - TELIC             purpose
    - AGENTIVE        how it is made
- Interpretation is **generated** by combining word meanings
- Events have **complex** structure

## State

S  
|  
e

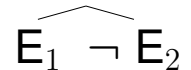
*understand, love, be tall*

## Process

P  
  
e<sub>1</sub> ... e<sub>n</sub>

*sing, walk, swim*

## Transition

T  
  
E<sub>1</sub>    E<sub>2</sub>

*open, close, build*





# Qualia Structure

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(17) *fast typist*

- a. a typist who is fast [at running]
- b. a typist who types fast

➤ typist  $\left[ \begin{array}{l} \text{ARGSTR} \left[ \text{ARG1 } x:\text{typist} \right] \\ \text{QUALIA} \left[ \begin{array}{l} \text{FORMAL } [x \subset \text{person}] \\ \text{TELIC } [\text{type}(e, x)] \end{array} \right] \end{array} \right]$

➤ (17a) *fast* modifies  $x$

➤ (17b) *fast* modifies  $e$

# Summary

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- Meaning can be broken up into units smaller than words: **components**
  - These can be combined to make larger meanings
  - At least some of them influence syntax
  - They may be psychologically real
- Problems with Components of Meaning
  - Primitives are no different from necessary and sufficient conditions
    - it is impossible to agree on the definitions
    - but they allow us to state generalizations better
  - Psycho-linguistic evidence is weak
  - It is just **markerese**
  - There is no **grounding**

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# Word Meaning: Meaning Postulates

# Defining Relations using Logic

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## ➤ hyponymy

➤  $\forall x(\text{DOG}(x) \rightarrow \text{ANIMAL}(x))$

## ➤ synonym

➤  $\forall x((\text{EGGPLANT}(x) \rightarrow \text{BRINJAL}(x)) \wedge (\text{BRINJAL}(x) \rightarrow \text{EGGPLANT}(x)))$

➤  $\forall x(\text{EGGPLANT}(x) \equiv \text{BRINJAL}(x))$

---

➤ **antonym**

- $\forall x(\text{DEAD}(x) \rightarrow \neg \text{ALIVE}(x));$   
+  $\forall x(\text{ALIVE}(x) \rightarrow \neg \text{DEAD}(x))$

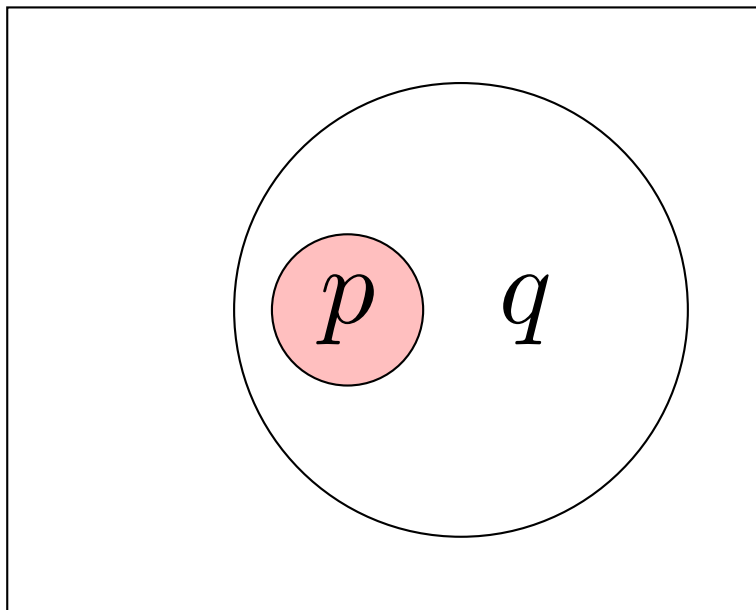
➤ **converse**

- $\forall x \forall y(\text{PARENT}(x,y) \rightarrow \text{CHILD}(y,x));$   
 $\forall x \forall y(\text{PARENT}(x,y) \rightarrow \neg \text{CHILD}(x,y))$
- $\forall x \forall y(\text{CHILD}(y,x) \rightarrow \text{PARENT}(x,y))$   
 $\forall x \forall y(\text{CHILD}(y,x) \rightarrow \neg \text{PARENT}(y,x))$

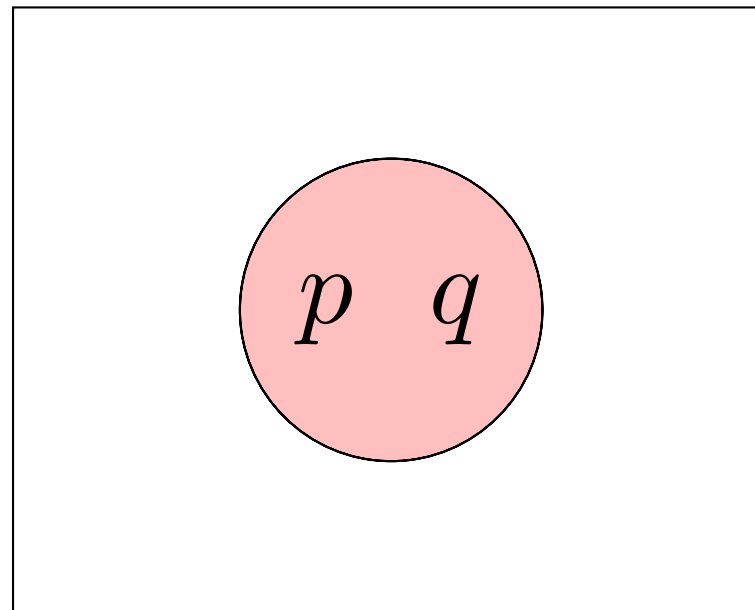
## Semantic Relations as Sets ( $p \subset q$ and $p \sim q$ )

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$p \subset q$  **hypernym**



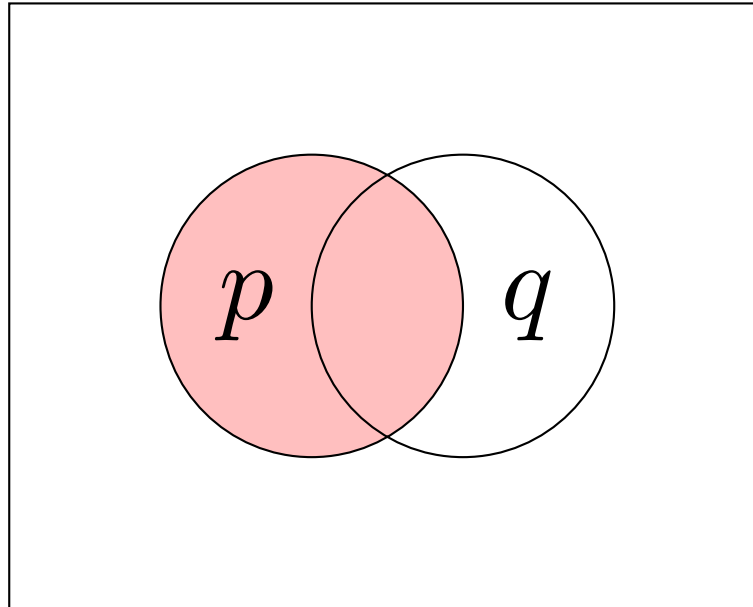
$p \sim q$  **synonym**



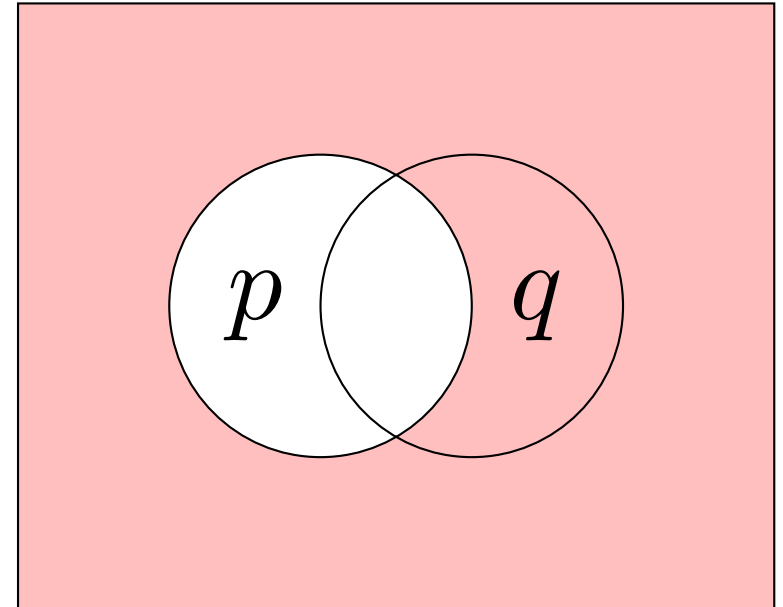
# Logical Connectives as Sets ( $p$ and $\neg p$ )

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$p$



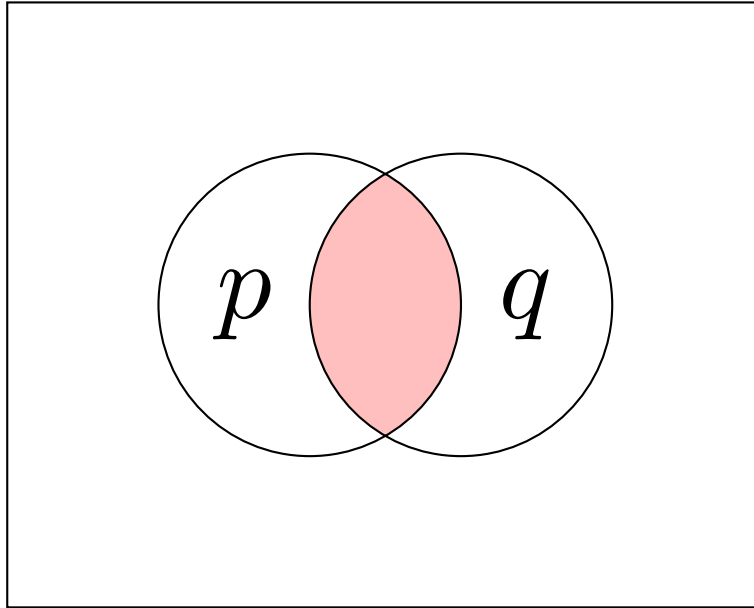
$\neg p$  “not”



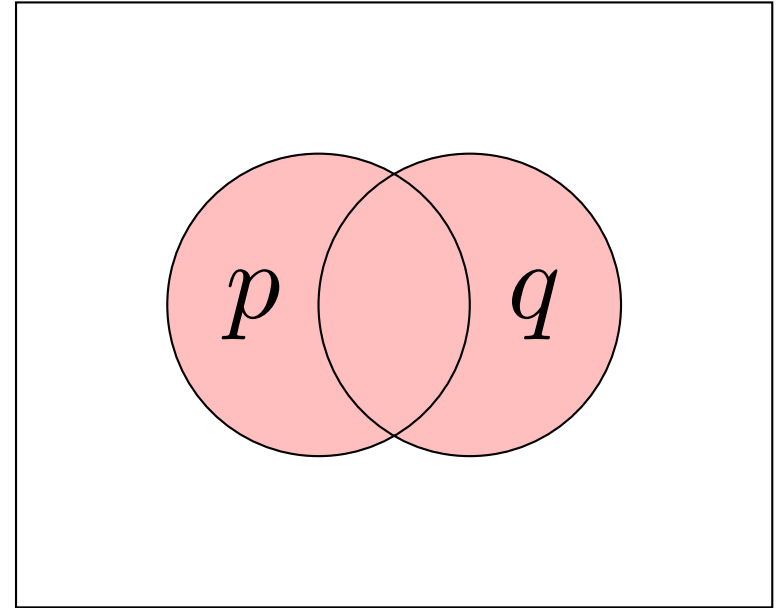
# Logical Connectives as Sets ( $p \wedge q$ and $p \vee q$ )

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$p \wedge q$  “and”



$p \vee q$  “or”

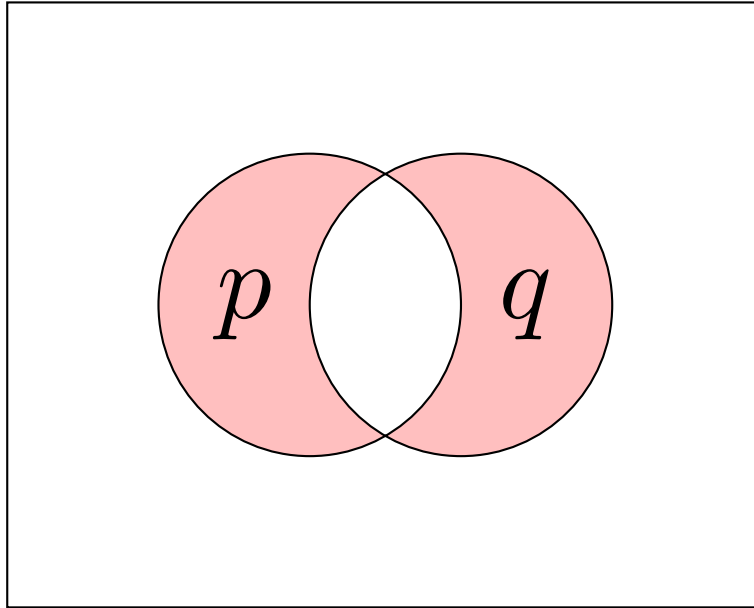




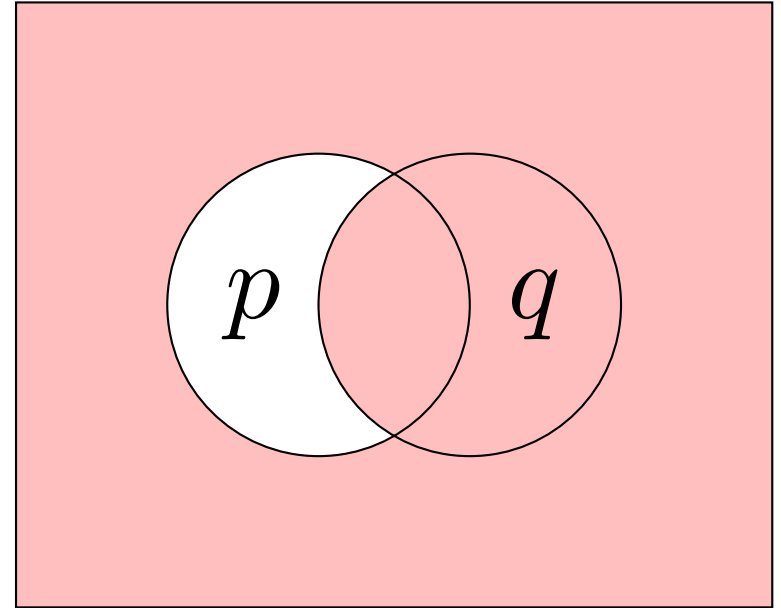
## Logical Connectives as Sets ( $p \oplus q$ and $p \rightarrow q$ )

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$p \oplus q$  “exclusive or”



$p \rightarrow q$  “if”



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# Natural Language Quantifiers and Higher Order Logic

# Restricted Quantifiers

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- *Most students read a book*
  - $\text{Most}(x)(S(x) \wedge R(x))$   
*most things are students and most things read books*
  - $\text{Most}(x)(S(x) \rightarrow R(x))$   
*most things are such that, if they are students, they read books*  
but also true for all things that are not students!
- We need to restrict the quantification
  - $(\text{Most } x: S(x)) R(x)$
- Sometimes we need to decompose
  - *everybody*  $(\forall x: P(x))$
  - *something*  $(\exists x: T(x))$

# Higher Order Logic

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- First-order logic over individuals
  - Second-order logic also quantifies over sets
  - Third-order logic also quantifies over sets of sets
  - Fourth-order logic also quantifies over sets of sets of sets
- ...

# Higher Order Logic

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➤ Recall *lan sings*

➤  $[S(i)]^{M_1} = 1$  iff  $[i]^{M_1} \in [S]^{M_1}$

The sentence is true if and only if the extension of *lan* is part of the set defined by *sings* in the model  $M_1$

➤ Remodel, with sing a property of lan:  $i(S)$

$[i(S)]^{M_1} = 1$  iff  $[S]^{M_1} \in [i]^{M_1}$

The sentence is true if and only if the denotation of the verb phrase *sings* is part of the extension of *lan* in the model  $M_1$

➤ *lan* is a set of sets of properties: **second-order logic**

# Generalized Quantifiers

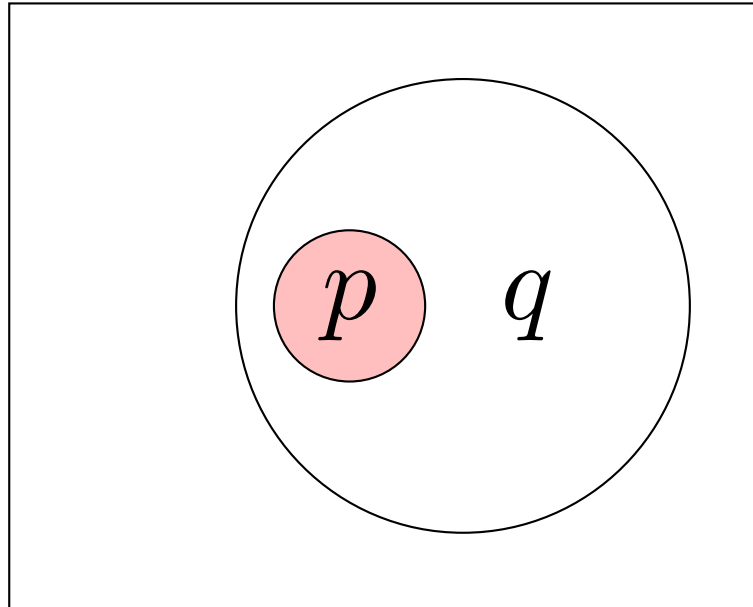
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- $Q(A,B)$ : *Q A are B*
- $\text{most}(A,B) = 1$  iff  $|A \cap B| > |A - B|$
- $\text{all}(A,B) = 1$  iff  $A \subseteq B$
- $\text{some}(A,B) = 1$  iff  $A \cap B \neq \emptyset$
- $\text{no}(A,B) = 1$  iff  $A \cap B = \emptyset$
- $\text{fewer than } x(A,B,X) = 1$  iff  $|A \cap B| < |X|$

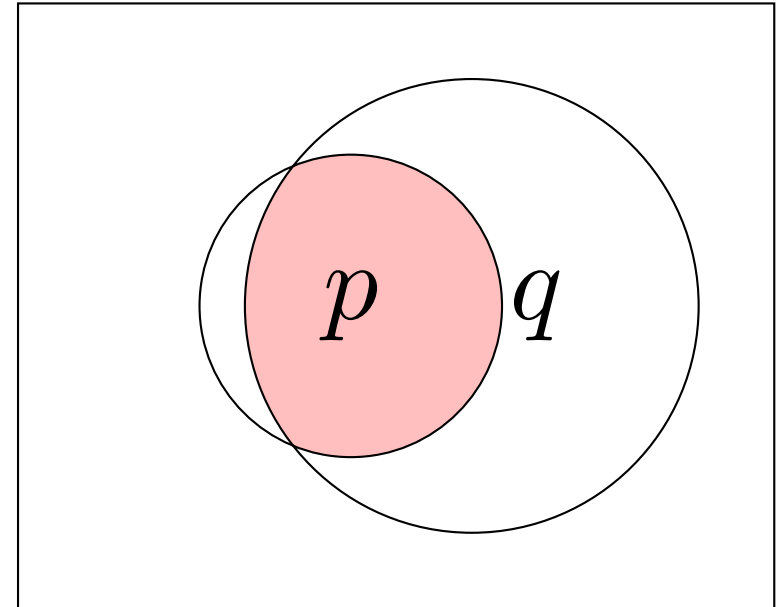
# Generalized Quantifiers: *all*, *most*

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*all p are q*



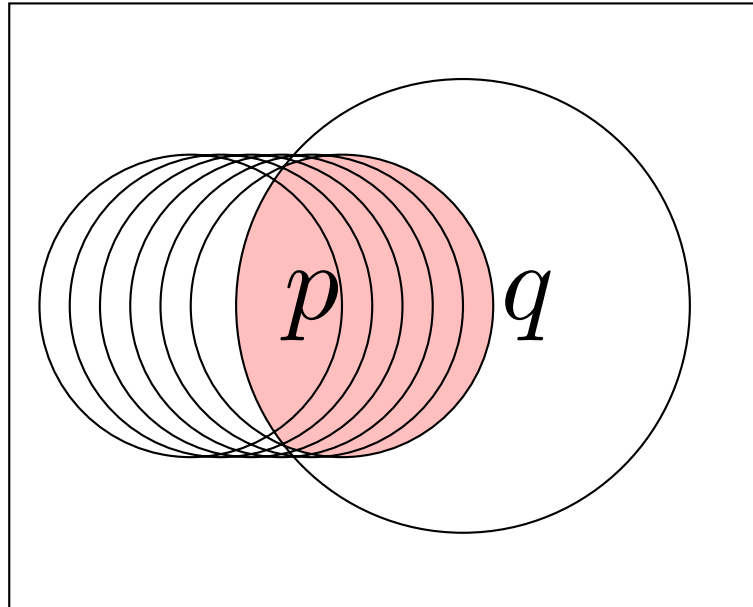
*most p are q*



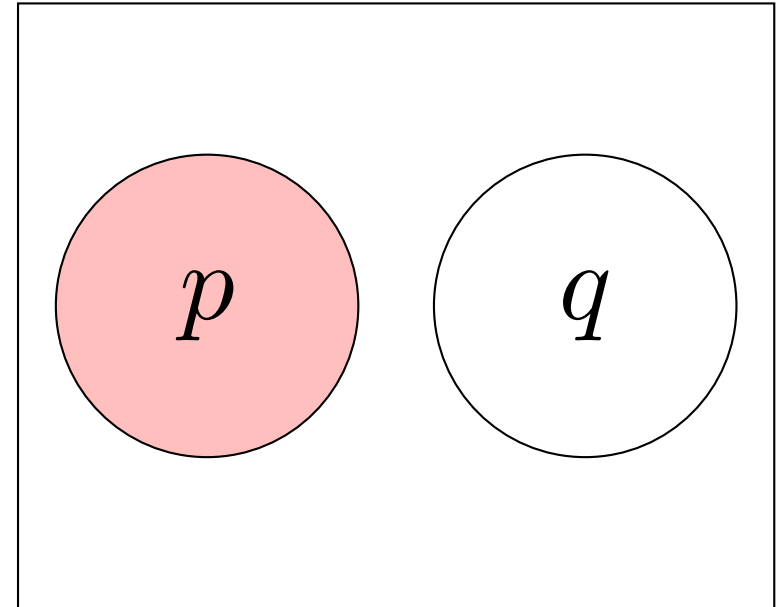
## Generalized Quantifiers: some, no

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some  $p$  are  $q$



no  $p$  are  $q$





## Strong/Weak Quantifiers

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(18) only **weak** quantifiers can occur in existential *there* sentences

- a. *There is a fox in the henhouse*
- b. *There are two foxes in the henhouse*
- c. *\*There is every fox in the henhouse*
- d. *\*There are both foxes in the henhouse*

➤ **symmetrical** (cardinal) quantifiers are **weak**  
 $\text{det}(A,B) = \text{det}(B,A)$

(19) *3 lecturers are Australian = 3 Australians are lecturers*

➤ **asymmetrical** (proportional) quantifiers are **strong**  
 $\text{det}(A,B) \neq \text{det}(B,A)$

(20) *most lecturers are Australian  $\neq$  most Australians are lecturers*

# Negative Polarity Items (NPI)

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- Some words in English mainly appear in negative environments

(21) a. *Kim doesn't ever eat dessert*

b. \**Kim does ever eat dessert*

(22) a. *Kim hasn't eaten dessert yet*

b. \**Kim has eaten dessert yet*

(23) a. *Few people have eaten dessert yet*

b. \**Many people have eaten dessert yet*

(24) a. *Rarely does Kim ever eat dessert*

b. \**Often does Kim ever eat dessert*

- Not just negation, but also some quantifiers

# Monotonicity

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- Some quantifiers control entailment between sets and subsets
  - **Upward entailment** goes from a subset to a set
  - **Downward entailment** goes from a set to a subset

- (25) a. *Kim doesn't eat dessert  $\Rightarrow$  Kim doesn't eat hot dessert*  
b. *Kim doesn't eat hot dessert  $\nRightarrow$  Kim doesn't eat dessert*

## Downward entailment

- (26) a. *Kim eats some desserts  $\nRightarrow$  Kim eats hot desserts*  
b. *Kim eats some hot desserts  $\Rightarrow$  Kim eats some desserts*

## Upward entailment

- **Negative Polarity Items** are licensed by **downward entailing expressions**

# Left and Right Monotonicity

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➤ The monotonicity may depend on the position

- (27) a. *Every student studies semantics*  $\nRightarrow$  *Every student studies formal semantics*  
b. *Every student studies formal semantics*  $\Rightarrow$  *Every student studies semantics*

**Upward entailment (right argument)**

- (28) a. *Every student studies semantics*  $\Rightarrow$  *Every linguistics student studies semantics*  
b. *Every linguistic student studies semantics*  $\nRightarrow$  *Every student studies semantics*

**Downward entailment (left argument)**

- 
- (29) a. *Every student who has ever studied semantics loves it*  
b. *\*Every student who has studied semantics ever loves it*  
c. *Few students who have ever studied semantics dislike it*  
d. *Few students who have studied semantics ever dislike it*

➤ Formal models of quantification can be used to make predictions about seemingly unrelated phenomena

## In other languages too!

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- (30) 我 没有 任何 朋友  
wǒ méi-yǒu rèn hé péngyǒu  
I NEG-have any friend  
“I don’t have any friends.”
- (31) \*我 有 任何 朋友  
wǒ yǒu rèn hé péngyǒu  
I have any friend  
\*“I have any friends.”

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# Modality

# Modality as a scale of Implicatures

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- (32) *I know that  $p$*
- (33) *I am absolutely certain that  $p$*
- (34) *I am almost certain that  $p$*
- (35) *I believe that  $p$*
- (36) *I am pretty certain that  $p$*
- ...
- (37) *Possibly  $p$*
- ...
- (38) *It is very unlikely that  $p$*
- (39) *It is almost impossible that  $p$*
- (40) *It is impossible that  $p$*
- (41) *It is not the case that  $p$*
- (42) *I am absolutely certain that not- $p$*



# Modal Logics

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➤ Add two modal operators for epistemic modality

➤  $\Diamond\phi = \textit{it is possible that } \phi$

➤  $\Box\phi = \textit{it is necessary that } \phi$

➤ Define them in terms of **possible worlds**

➤  $\Diamond\phi$ : true in at least one world

➤  $\Box\phi$ : true in all worlds

➤  $M = \{W, U, F\}$ : the model now has three parts

$W$  set of possible worlds

$U$  domain of individuals (universe)

$F$  denotation assignment function

# Deontic Modality

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- Add two modal operators for deontic modality
  - $P\phi = \textit{it is permitted that } \phi$
  - $O\phi = \textit{it is obligatorily } \phi$
- Define them in terms of **possible worlds**
  - $P\phi$ : true in at least one legal or morally ideal world
  - $O\phi$ : true in all legal or morally ideal worlds

---

# Dynamic Approaches to Discourse

# Anaphora

---

- (43) a. *R2D2<sub>i</sub> mistrusts itself<sub>i</sub>*  
b.  $M(r,r)$
- (44) a. *Every robot mistrusts itself*  
b.  $(\forall x: R(x)) M(x,x)$
- (45) a. *Luke bought a robot and it doesn't work*  
b.  $(\exists x: R(x)) B(l,x) \wedge \neg W(x)$
- (46) a. *Every robot went to Naboo. ?It met Jar Jar.*  
b.  $(\forall x: R(x)) W(x,n); M(x,j)$  unbound
- (47) a. *A robot went to Naboo. It met Jar Jar.*  
b.  $(\exists x: R(x)) W(x,n); M(x,j)$  ???
- indefinite nominals exist beyond the sentence: **discourse referents**
- (48) a. *Luke didn't buy a robot. ?It met Jar Jar.*  
indefinite nominals scope can still be limited

# Donkey Sentences

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- (49) a. *If R2D2<sub>i</sub> owns a ship it is rich*  
b.  $(\exists x (S(x) \wedge O(r,x))) \rightarrow R(x)$
- (50) a. *If a robot owns a ship it races it*  
b.  $*(\exists x \exists y (R(x) \wedge S(y) \wedge O(x,y))) \rightarrow R(x,y)$   
c.  $\forall x \forall y ((R(x) \wedge S(y) \wedge O(x,y)) \rightarrow R(x,y)$   
 $\exists$  needs to become  $\forall$
- (51) *Every farmer who owns a donkey beats it*

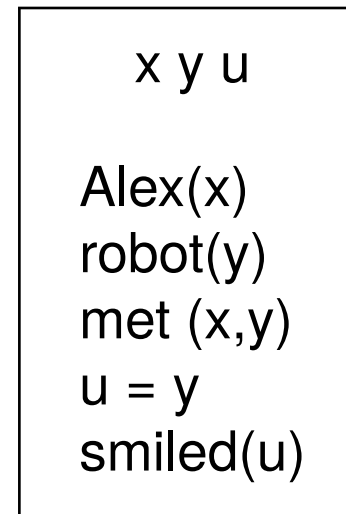
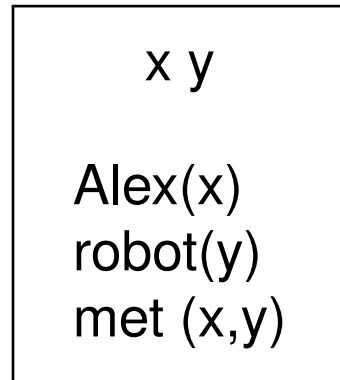
# Discourse Representation Theory

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➤ Build up Discourse Representation Structures

- (52) a. *Alex met a robot<sub>i</sub>*  
b. *It<sub>i</sub> smiled*

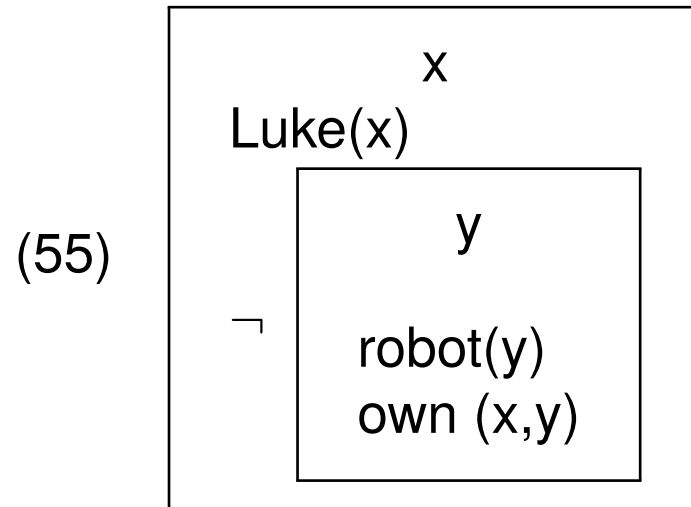
(53)



# Negative Contexts

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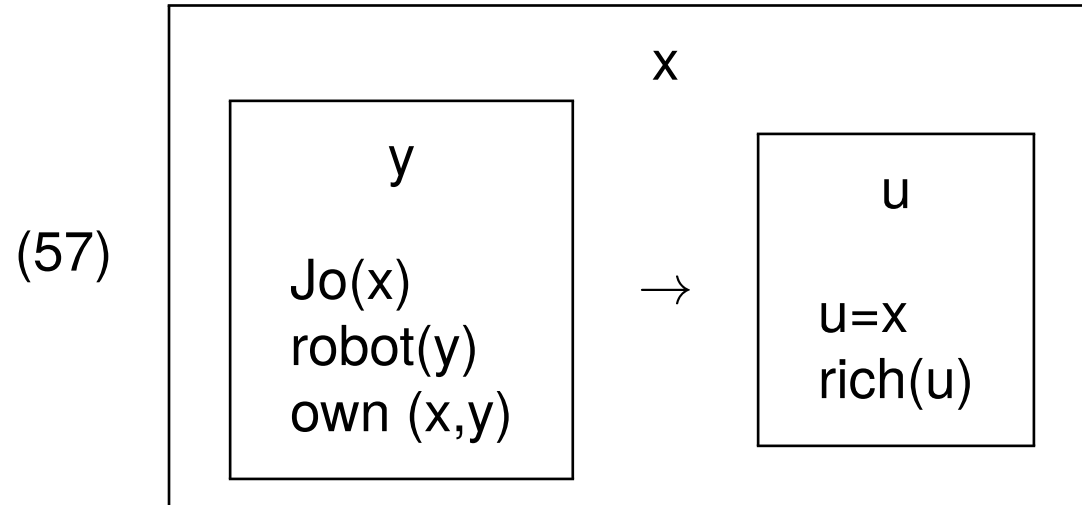
(54) a. *Luke does not own a robot*



- The contained DRS is **subordinate**
  - indefinite NPs in negated subordinate structures are inaccessible
  - names (constants) are always accessible

# Conditionals

(56) a. *If Jo owns a robot then they are rich*



➤ The contained DRS is **subordinate**

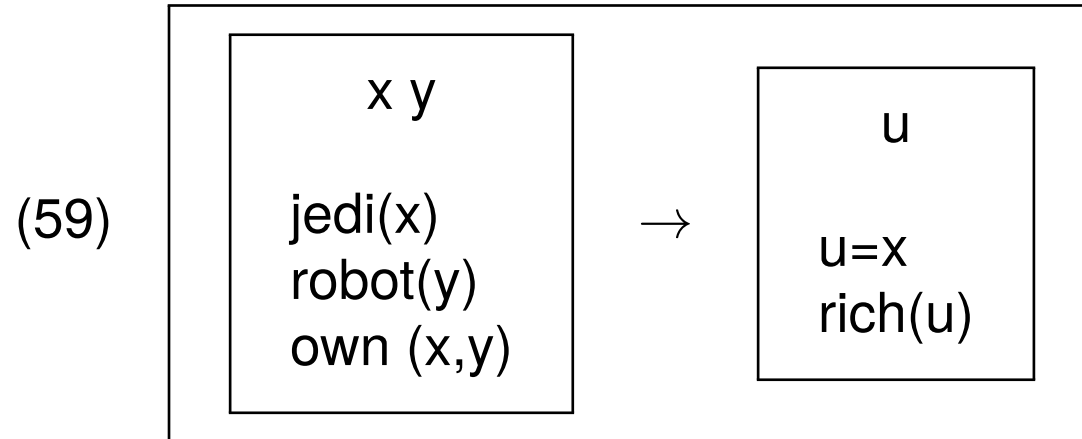
➤ indefinite NPs in the antecedent are accessible in the consequent



## More Conditionals

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(58) a. *If a Jedi owns a robot then they are rich*

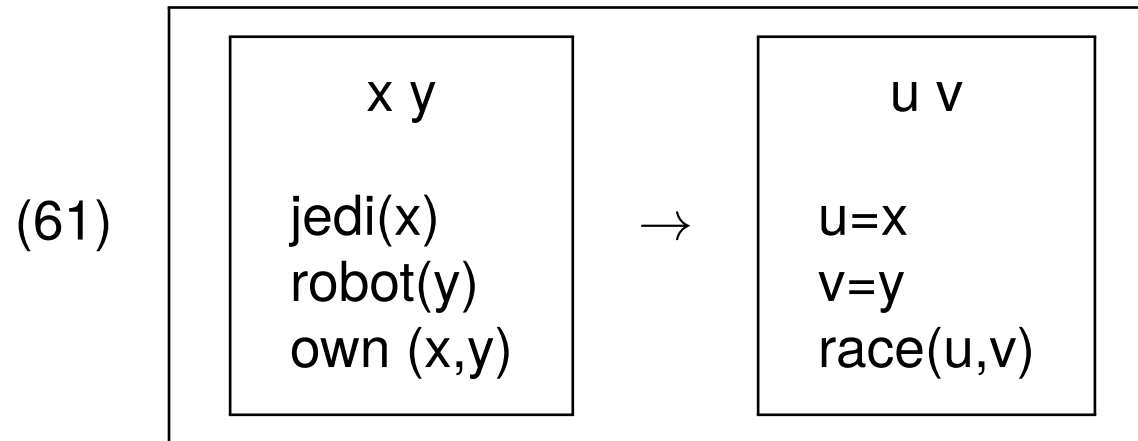


- The contained DRS is **subordinate**
- indefinite NPs in the antecedent are accessible in the consequent

## More Conditionals

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(60) a. *If a Jedi owns a robot then they race it*

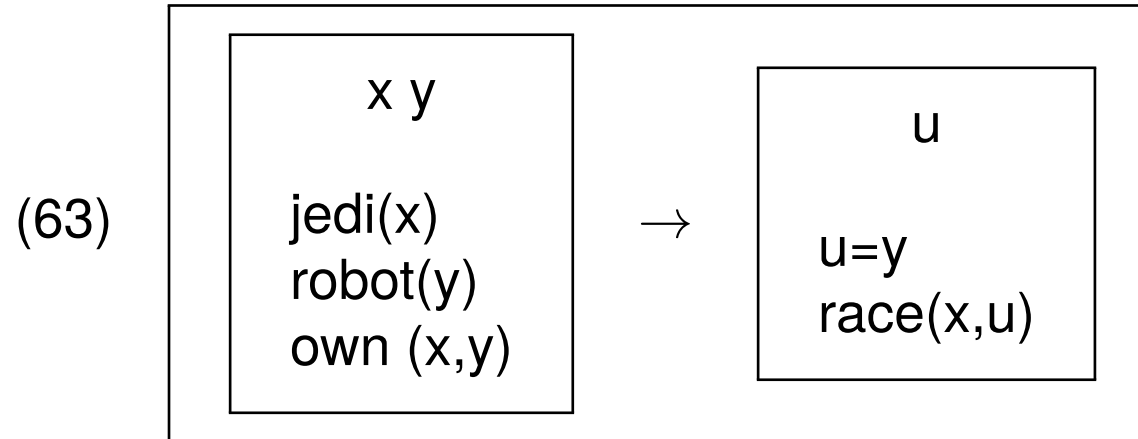


- The contained DRS is **subordinate**
- indefinite NPs in the antecedent are accessible in the consequent

## More Conditionals

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(62) a. *Every Jedi who owns a robot races it*



➤ The contained DRS is **subordinate**

➤ Universal Quantifiers copy the variable across the conditional

# Discourse Representation Theory

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- Explains how reference occurs across clauses and sentences
  - Distinguishes between names and indefinite NPS
  - Distinguishes between positive assertions, negative sentences, conditional sentences, universally quantified sentences
  - Is useful for modeling the incremental update of knowledge in a conversation

## Acknowledgments and References

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- Video *Regency Disco* from that Mitchel and Webb Look Episode 3.3, which was first broadcast on Thursday 25th June 2009.