# 1 Object Language

A simply typed CBPV language.

## 1.1 Raw Terms

$$
\begin{array}{llll}
\text{Value Types} & A & ::= & \text{Unit} \\
 & & \mid & A \times A \\
 & & \mid & U\underline{B} \\
\text{Computation Types} & \underline{B} & ::= & A \to \underline{B} \\
 & & \mid & FA \\
\text{Values} & V & ::= & x \\
 & & \mid & tt \\
 & & \mid & (V, V) \\
 & & \mid & \text{thunk } M \\
\text{Computations} & M & ::= & \lambda x \colon A.M \\
 & & \mid & MV \\
 & & \mid & \text{ret } V \\
 & & \mid & \text{force } V \\
 & & \mid & M \text{ to } x \text{ in } M' \\
 & & \mid & \text{let } x = V; M' \\
 & & \mid & \text{let } (x, y) = V; M \\
\text{Value Context} & \Gamma & ::= & \cdot \\
 & & \mid & \Gamma, x \colon A
\end{array}
$$

## 1.2 Typed Terms

$$\frac{}{\Gamma, x \colon A \vdash_v x : A}\,\text{Var}$$

$$\frac{}{\Gamma \vdash_v tt : \text{Unit}}\,\text{I-Unit}$$

$$\frac{\Gamma \vdash_v t : A \qquad \Gamma \vdash_v t : A'}{\Gamma \vdash_v (t, u) : A \times A'}\,\text{I-}\times$$

$$\frac{\Gamma \vdash_c M : \underline{B}}{\Gamma \vdash_v \text{thunk } M : U\underline{B}}$$

$$\frac{\Gamma, x : A \vdash_c M : \underline{B}}{\Gamma \vdash_c (\lambda(x : A).M) : A \to \underline{B}}\,\text{I - }\to$$

$$\frac{\Gamma \vdash_c M : A \to \underline{B} \qquad \Gamma \vdash_v V : A}{\Gamma \vdash_c : MV : \underline{B}}\,\text{E-}\to$$

$$\frac{\Gamma \vdash_v V : A}{\Gamma \vdash_c \text{ret } V : FA}$$

$$\frac{\Gamma \vdash_v V : U\underline{B}}{\Gamma \vdash_c \text{force } V : \underline{B}}$$

$$\frac{\Gamma \vdash_c M : FA \qquad \Gamma, x : A \vdash_c N : \underline{B}}{\Gamma \vdash_c M \text{ to } x \text{ in } N : \underline{B}}$$

$$\frac{\Gamma \vdash_v V : A \qquad \Gamma, x : A \vdash_c N : \underline{B}}{\Gamma \vdash_c \text{let x } = V; N : \underline{B}}$$

$$\frac{\Gamma \vdash_v V : A \times A' \qquad \Gamma, x : A, y : A' \vdash_c M : \underline{B}}{\Gamma \vdash_c \text{let (x,y) } = V; M : \underline{B}} \text{ E-}\times$$

# 2 Logic

## 2.1 Formation Rules

### 2.1.1 Value Fragment

Judgments: Value Propositions

$$\Gamma \vdash \phi \text{ VProp}$$

connectives:

$$\phi := \top | \phi \wedge \phi | \phi \implies \phi$$

$$\frac{}{\Gamma \vdash \top \text{ VProp}}$$

$$\frac{\Gamma \vdash \phi \text{ VProp} \qquad \Gamma \vdash \psi \text{ VProp}}{\Gamma \vdash \phi \wedge \psi \text{ VProp}}$$

$$\frac{\Gamma \vdash \phi \text{ VProp} \qquad \Gamma \vdash \psi \text{ VProp}}{\Gamma \vdash \phi \implies \psi \text{ VProp}}$$

### 2.1.2 Computation Fragment

Judgments: Computation Propositions

$$\Gamma; \Delta \vdash \underline{\phi} \text{ CProp}$$

connectives:

$$\underline{\phi} := \underline{\top} | \underline{\phi} \wedge \underline{\psi} | \phi \implies \underline{\psi}$$

$$\frac{}{\Gamma; \Delta \vdash \underline{\top} \text{ CProp}}$$

$$\frac{\Gamma; \Delta \vdash \underline{\phi} \text{ CProp} \qquad \Gamma; \Delta \vdash \underline{\psi} \text{ CProp}}{\Gamma; \Delta \vdash \underline{\phi} \wedge \underline{\psi} \text{ CProp}}$$

$$\frac{\Gamma \vdash \phi \text{ VProp} \qquad \Gamma; \Delta \vdash \underline{\psi} \text{ CProp}}{\Gamma; \Delta \vdash \phi \implies \underline{\psi} \text{ CProp}}$$

## 2.2 Derivation Rules

### 2.2.1 Value Derivations

Value Derivation Judgement

$$\Gamma | \Phi \vdash \phi$$

where $\Phi$ is a conjunction of value propositions.

$$\frac{}{\Gamma | \Phi \vdash \top} \text{ I-}\top$$

$$\frac{\Gamma | \Phi \vdash \phi \qquad \Gamma | \Phi \vdash \psi}{\Gamma | \Phi \vdash \phi \wedge \psi} \text{ I-}\wedge$$

$$\frac{\Gamma | \Phi \vdash \phi \wedge \psi}{\Gamma | \Phi \vdash \phi} \text{ E1-}\wedge$$

$$\frac{\Gamma | \Phi \vdash \phi \wedge \psi}{\Gamma | \Phi \vdash \psi} \text{ E2-}\wedge$$

$$\frac{\Gamma | \Phi, \phi \vdash \psi}{\Gamma | \Phi \vdash \phi \implies \psi} \text{ I-} \implies$$

$$\frac{\Gamma | \Phi \vdash \phi \implies \psi \qquad \Gamma | \Phi \vdash \phi}{\Gamma | \Phi \vdash \psi} \text{ E-} \implies$$

### 2.2.2 Computation Derivations

Computation Derivation Judgement:

$$\Gamma ; \Delta | \Phi ; \underline{\Psi} \vdash \underline{\phi}$$

context for computation propositions?

$$\oslash^* : Sub_V(\Gamma) \times Sub_C(\Gamma \oslash \Delta) \to Sub_C(\Gamma \oslash \Delta)$$

$$[\![\Gamma ; \Delta | \Phi ; \underline{\Psi} \vdash \underline{\phi}]\!] : Sub_C(\Gamma \oslash \Delta)[\oslash^*(\Phi, \underline{\Psi}), \underline{\phi}]$$

$$\frac{}{\Gamma ; \Delta | \Phi ; \underline{\Psi} \vdash \underline{\top}} \text{ I-}\top$$

$$\frac{\Gamma ; \Delta | \Phi ; \underline{\Psi} \vdash \underline{\phi} \qquad \Gamma ; \Delta | \Phi ; \underline{\Psi} \vdash \underline{\psi}}{\Gamma ; \Delta | \Phi ; \underline{\Psi} \vdash \underline{\phi} \wedge \underline{\psi}} \text{ I-}\wedge$$

$$\frac{\Gamma ; \Delta | \Phi ; \underline{\Psi} \vdash \underline{\phi} \wedge \underline{\psi}}{\Gamma ; \Delta | \Phi ; \underline{\Psi} \vdash \underline{\phi}} \text{ E1-}\wedge$$

$$\frac{\Gamma ; \Delta | \Phi ; \underline{\Psi} \vdash \underline{\phi} \wedge \underline{\psi}}{\Gamma ; \Delta | \Phi ; \underline{\Psi} \vdash \underline{\psi}} \text{ E2-}\wedge$$

$$\frac{}{\Gamma ; \Delta | \Phi ; \underline{\Psi} \vdash \phi \implies \underline{\psi}} \text{ I-} \implies$$

$$- \text{ E-} \implies$$