

1 Object Language

1.1 Raw Terms

The untyped syntax for CBPV OSum, add separating connectives, remove error.

Add $\forall X. \underline{B}$?

Value Types	A	$::=$	X Unit $\text{Case } A$ OSum $A \times A$ $A * A$ $\exists X. A$ $U \underline{B}$
Computation Types	\underline{B}	$::=$	$A \rightarrow \underline{B}$ $A \multimap \underline{B}$ $\forall X. \underline{B}$ FA
Values	V	$::=$	x tt σ $\text{inj}_V V$ (V, V) $(V * V)$ $\text{pack } (A, V) \text{ as } \exists X. A$ $\text{thunk } M$
Computations	M	$::=$	$\lambda x: A. M$ MV $\alpha x: A. M$ $M @ V$ $\Lambda X. M$ $M[A]$ $\text{ret } V$ $x \leftarrow M; N$ $\text{force } V$ $\text{newcase}_A x; M$ $\text{match } V \text{ with } V \{ \text{inj } x. M \parallel N \}$ $\text{let } (x, x) = V; M$ $\text{let } (x * x) = V; M$ $\text{unpack } (X, x) = V; M$
Value Context	Γ	$::=$	\cdot $\Gamma, x: A$ $\Gamma * x: A$
Type Context	Ξ	$::=$	\cdot Ξ, X

1.2 Typed Terms

$$\begin{array}{c}
\frac{}{\Xi; \Gamma, x : A \vdash_v x : A} \\
\\
\frac{}{\Xi; \Gamma * x : A \vdash_v x : A} \\
\\
\frac{}{\Xi; \Gamma \vdash_v \text{tt} : \text{Unit}} \\
\\
\frac{\Xi; \Gamma \vdash_v \sigma : \text{Case}A \quad \Xi; \Gamma \vdash_v V : A}{\Xi; \Gamma \vdash_v \text{inj}_\sigma V : \text{OSum}} \\
\\
\frac{\Xi; \Gamma \vdash_v V_1 : A_1 \quad \Xi; \Gamma \vdash_v V_2 : A_2}{\Xi; \Gamma \vdash_v (V_1, V_2) : A_1 \times A_2} \\
\\
\frac{\Xi; \Gamma_1 \vdash_v V_1 : A_1 \quad \Xi; \Gamma_2 \vdash_v V_2 : A_2}{\Xi; \Gamma_1 * \Gamma_2 \vdash_v (V_1 * V_2) : A_1 * A_2} \\
\\
\frac{\Xi; \Gamma \vdash_v V : A[A'/X]}{\Xi; \Gamma \vdash_v \text{pack}(A', V) \text{ as } \exists X.A : \exists X.A} \\
\\
\frac{\Xi; \Gamma \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_v \text{thunk } M : U\underline{B}} \\
\\
\frac{\Xi; \Gamma, x : A \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_c \lambda x : A. M : A \rightarrow \underline{B}} \\
\\
\frac{\Xi; \Gamma \vdash_c M : A \rightarrow \underline{B} \quad \Xi; \Gamma \vdash_v N : A}{\Xi; \Gamma \vdash_c MN : \underline{B}} \\
\\
\frac{\Xi; \Gamma * x : A \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_c \alpha x : A. M : A \multimap \underline{B}} \\
\\
\frac{\Xi; \Gamma_1 \vdash_c M : A \multimap \underline{B} \quad \Xi; \Gamma_2 \vdash_v N : A}{\Xi; \Gamma_1 * \Gamma_2 \vdash_c M @ N : \underline{B}} \\
\\
\frac{\Xi, X; \Gamma \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_c \Lambda X. M : \forall X. \underline{B}} \\
\\
\frac{\Xi; \Gamma \vdash_c M : \forall X. \underline{B} \quad \Xi \vdash A}{\Xi; \Gamma \vdash_c M[A] : \underline{B}[A/X]} \\
\\
\frac{\Xi; \Gamma \vdash_v V : A}{\Xi; \Gamma \vdash_c \text{ret } V : FA}
\end{array}$$

$$\begin{array}{c}
\frac{\Xi; \Gamma \vdash_c M : FA \quad \Xi; \Gamma, x : A \vdash_c N : \underline{B}}{\Xi; \Gamma \vdash_c x \leftarrow M; N : \underline{B}} \\
\\
\frac{\Xi; \Gamma \vdash_v V : U\underline{B}}{\Xi; \Gamma \vdash_c \text{force } V : \underline{B}} \\
\\
\frac{\Xi; \Gamma * (\sigma : \text{Case}A) \vdash_c M : \underline{B} \quad \Xi \vdash A}{\Xi; \Gamma \vdash_c \text{newcase}_A x; M : \underline{B}} \\
\\
\frac{\Xi; \Gamma \vdash_v V : \text{OSum} \quad \Xi; \Gamma \vdash_v \sigma : \text{Case } A \quad \Xi; \Gamma, x : A \vdash M : \underline{B} \quad \Xi; \Gamma \vdash_c N : \underline{B}}{\Xi; \Gamma \vdash_c \text{match } V \text{ with } \sigma \{ \text{inj } x.M \parallel N \} : \underline{B}} \\
\\
\frac{\Xi; \Gamma \vdash_v V : A_1 \times A_2 \quad \Xi; \Gamma, x : A_1, y : A_2 \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_c \text{let } (x, y) = V; M : \underline{B}} \\
\\
\frac{\Xi; \Gamma \vdash_v V : A_1 * A_2 \quad \Xi; \Gamma * x : A_1 * y : A_2 \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_c \text{let } (x * y) = V; M : \underline{B}} \\
\\
\frac{\Xi \vdash \underline{B} \quad \Xi; \Gamma \vdash_v V : \exists X.A \quad \Xi, X; \Gamma, x : A \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_c \text{unpack}(X, x) = V; M : \underline{B}}
\end{array}$$

2 Logic

2.1 Judgments

The relation environment, Θ in PE logic contains both value and computation relations. How does this work in the semantics when value relations are denoted as objects of $Sub_V(A \times B)$ for $Rel_V[A, B]$ and computation relations are denoted as objects of $Sub_C(\underline{A} \times \underline{B})$ for $Rel_C[\underline{A}, \underline{B}]$? Maybe we have separate relation environments?

$$\begin{array}{l}
\Xi; \Gamma; \Theta \vdash \phi \text{ VProp} \\
\Xi; \Gamma; \Theta \vdash (x : A, y : B). \phi : Rel_V[A, B] \\
\Xi; \Gamma; \Theta \mid \Phi \vdash \phi \\
\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\phi} \text{ CProp} \\
\Xi; \Gamma; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}). \underline{\phi} : Rel_C[\underline{A}, \underline{B}] \\
\Xi; \Gamma; \Delta; \Theta; \Omega \mid \Phi; \underline{\Psi} \vdash \underline{\phi}
\end{array}$$

for type environment Ξ , term environment Γ , stoup Δ , value relation environment Θ , computation relation environment Ω , value proposition environment Φ , and computation proposition environment $\underline{\Psi}$.

2.2 Formation Rules

2.2.1 Value Propositions

$$\begin{array}{c}
\phi := \top | \phi \wedge \phi | t =_A u | R(t, u) \\
\hline
\Xi; \Gamma; \Theta \vdash \top \text{ VProp} \\
\hline
\Xi; \Gamma; \Theta \vdash \phi \text{ VProp} \quad \Xi; \Gamma; \Theta \vdash \psi \text{ VProp} \\
\hline
\Xi; \Gamma; \Theta \vdash \phi \wedge \psi \text{ VProp} \\
\hline
\Xi; \Gamma \vdash_v t : A \quad \Xi; \Gamma \vdash_v u : A \\
\hline
\Xi; \Gamma; \Theta \vdash t =_A u \text{ VProp} \\
\hline
\Xi; \Gamma \vdash_v t : A \quad \Xi; \Gamma \vdash_v u : B \quad R : \text{Rel}_V[A, B] \in \Theta \\
\hline
\Xi; \Gamma; \Theta \vdash R(t, u) \text{ VProp}
\end{array}$$

2.2.2 Computation Propositions

$$\underline{\psi} := \top | \underline{\psi} \wedge \underline{\psi} | t =_B u | \phi \implies \underline{\psi} | \underline{R}(t, u) | \forall (x : A). \underline{\psi} | \forall X. \underline{\psi} | \forall \underline{X}. \underline{\psi} | \forall (R : \text{Rel}_V[A, B]). \underline{\psi} | \forall (R : \text{Rel}_C[\underline{A}, \underline{B}]). \underline{\psi}$$

$$\begin{array}{c}
\hline
\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \top \text{ CProp} \\
\hline
\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\phi} \text{ CProp} \quad \Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\psi} \text{ CProp} \\
\hline
\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\phi} \wedge \underline{\psi} \text{ CProp} \\
\hline
\Xi; \Gamma \vdash_c t : \underline{B} \quad \Xi; \Gamma \vdash_c u : \underline{B} \\
\hline
\Xi; \Gamma; \Delta; \Theta; \Omega \vdash t =_{\underline{B}} u \text{ CProp} \\
\hline
\Xi; \Gamma; \Theta \vdash \phi \text{ VProp} \quad \Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\psi} \text{ CProp} \\
\hline
\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \phi \implies \underline{\psi} \text{ CProp} \\
\hline
\Xi; \Gamma \vdash_c t : \underline{A} \quad \Xi; \Gamma \vdash_c u : \underline{B} \quad \underline{R} : \text{Rel}_C[\underline{A}, \underline{B}] \in \Omega \\
\hline
\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{R}(t, u) \text{ CProp} \\
\hline
\Xi; \Gamma, x : A; \Delta; \Theta; \Omega \vdash \underline{\psi} \text{ CProp} \\
\hline
\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \forall (x : A). \underline{\psi} \text{ CProp} \\
\hline
\Xi, X; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\psi} \text{ CProp} \\
\hline
\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \forall X. \underline{\psi} \text{ CProp} \\
\hline
\Xi, \underline{X}; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\psi} \text{ CProp} \\
\hline
\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \forall \underline{X}. \underline{\psi} \text{ CProp} \\
\hline
\Xi; \Gamma; \Delta; \Theta, R; \Omega \vdash \underline{\psi} \text{ CProp} \\
\hline
\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \forall R. \underline{\psi} \text{ CProp} \\
\hline
\Xi; \Gamma; \Delta; \Theta; \Omega, \underline{R} \vdash \underline{\psi} \text{ CProp} \\
\hline
\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \forall \underline{R}. \underline{\psi} \text{ CProp}
\end{array}$$

2.2.3 Value Relations

$$\frac{\Xi; \Gamma, x : A \vdash_v t : C \quad \Xi; \Gamma, y : B \vdash_v u : C}{\Xi; \Gamma; \Theta \vdash (x : A, y : B).t =_C u : Rel_V[A, B]}$$

$$\frac{\Xi; \Gamma, x : A \vdash_v t : C \quad \Xi; \Gamma, y : B \vdash_v u : D}{\Xi; \Gamma; \Theta, R : Rel_V[C, D] \vdash (x : A, y : B).R(t, u) : Rel_V[A, B]}$$

Introduction Rule

$$\frac{\Xi; \Gamma, x : A, y : B; \Theta \vdash \phi}{\Xi; \Gamma; \Theta \vdash (x : A, y : B).\phi : Rel_V[A, B]}$$

2.2.4 Computation Relations

Something seems off including the stoup, Δ , in the computation relation judgment..

$$\frac{\Xi; \Gamma | x : \underline{A} \vdash_c t : \underline{C} \quad \Xi; \Gamma | y : \underline{B} \vdash_c u : \underline{C}}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}).t =_{\underline{C}} u : Rel_C[\underline{A}, \underline{B}]}$$

$$\frac{\Xi; \Gamma | x : \underline{A} \vdash_c t : \underline{C} \quad \Xi; \Gamma | y : \underline{B} \vdash_c u : \underline{D}}{\Xi; \Gamma; \Delta; \Theta; \Omega, \underline{R} : Rel_C[\underline{C}, \underline{D}] \vdash (x : \underline{A}, y : \underline{B}).\underline{R}(t, u) : Rel_C[\underline{A}, \underline{B}]}$$

$$\frac{\Xi; \Gamma; \Theta \vdash \phi \text{ VProp} \quad \Xi; \Gamma; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}).\underline{\psi} : Rel_C[\underline{A}, \underline{B}]}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}).\phi \implies \underline{\psi} : Rel_C[\underline{A}, \underline{B}]}$$

$$\frac{\Xi; \Gamma, z : C; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}).\underline{\phi} : Rel_C[\underline{A}, \underline{B}]}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}).\forall(z : C).\underline{\phi} : Rel_C[\underline{A}, \underline{B}]}$$

$$\frac{\Xi, X; \Gamma; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}).\underline{\phi} : Rel_C[\underline{A}, \underline{B}]}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}).\forall X.\underline{\phi} : Rel_C[\underline{A}, \underline{B}]}$$

$$\frac{\Xi, \underline{X}; \Gamma; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}).\underline{\phi} : Rel_C[\underline{A}, \underline{B}]}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}).\forall \underline{X}.\underline{\phi} : Rel_C[\underline{A}, \underline{B}]}$$

$$\frac{\Xi; \Gamma; \Delta; \Theta, R : Rel_V[A, B]; \Omega \vdash (x : \underline{A}, y : \underline{B}).\underline{\phi} : Rel_C[\underline{A}, \underline{B}]}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}).\forall(R : Rel_V[A, B]).\underline{\phi} : Rel_C[\underline{A}, \underline{B}]}$$

$$\frac{\Xi; \Gamma; \Delta; \Theta; \Omega, \underline{R} : Rel_C[\underline{A}, \underline{B}] \vdash (x : \underline{A}, y : \underline{B}).\underline{\phi} : Rel_C[\underline{A}, \underline{B}]}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}).\forall(\underline{R} : Rel_C[\underline{A}, \underline{B}]).\underline{\phi} : Rel_C[\underline{A}, \underline{B}]}$$

Introduction Rule. Compare this to the value relation introduction rule.. the stoup has at most one computation..

$$\frac{\Xi; \Gamma; x : \underline{B}; \Theta; \Omega \vdash \phi}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash ?? : Rel_C[?, ?]}$$

2.3 Derivation Rules

2.3.1 Values

$$\begin{array}{c}
\overline{\Xi; \Gamma; \Theta | \Phi \vdash \top} \\
\\
\frac{\Xi; \Gamma; \Theta | \Phi \vdash \phi \quad \Xi; \Gamma; \Theta | \Phi \vdash \psi}{\Xi; \Gamma; \Theta | \Phi \vdash \phi \wedge \psi} \text{I-}\wedge \\
\\
\frac{\Xi; \Gamma; \Theta | \Phi \vdash \phi \wedge \psi}{\Xi; \Gamma; \Theta | \Phi \vdash \phi} \text{E1-}\wedge \\
\\
\frac{\Xi; \Gamma; \Theta | \Phi \vdash \phi \wedge \psi}{\Xi; \Gamma; \Theta | \Phi \vdash \psi} \text{E2-}\wedge \\
\\
\frac{\Xi; \Gamma \vdash_v t : A}{\Xi; \Gamma; \Theta | \Phi \vdash t =_A t} \\
\\
\frac{\Xi; \Gamma; \Theta | \Phi \vdash t =_A u \quad \Xi; \Gamma; \Theta | \Phi \vdash \phi[t/x]}{\Xi; \Gamma; \Theta | \Phi \vdash \phi[u/x]}
\end{array}$$

Rel

$$\frac{\Xi; \Gamma; \Theta | \Phi \vdash \quad \Xi; \Gamma; \Theta | \Phi \vdash}{\Xi; \Gamma; \Theta | \Phi \vdash}$$

2.3.2 Computation

$$\begin{array}{c}
\overline{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \perp} \\
\\
\frac{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\phi} \quad \Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\psi}}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\phi} \wedge \underline{\psi}} \text{I-}\wedge \\
\\
\frac{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\phi} \wedge \underline{\psi}}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\phi}} \text{E1-}\wedge \\
\\
\frac{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\phi} \wedge \underline{\psi}}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\psi}} \text{E2-}\wedge \\
\\
\frac{\Xi; \Gamma \vdash_c t : B}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash t =_{\underline{B}} t} \\
\\
\frac{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash t =_{\underline{B}} u \quad \Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\phi}[t/x]}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\phi}[u/x]}
\end{array}$$

$$\frac{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi, \phi; \Psi \vdash \underline{\psi}}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \phi \implies \underline{\psi}} \text{I-} \implies$$

$$\frac{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \phi \implies \underline{\psi} \quad \Xi; \Gamma; \Theta | \Phi \vdash \phi}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\psi}} \text{E-} \implies$$

Rel?

$$\frac{\Xi; \Gamma, x : A; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\phi}}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \forall(x : A).\underline{\phi}} \text{I-}\forall \text{ term , FV constraint?}$$

$$\frac{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \forall(x : A).\underline{\phi} \quad \Xi; \Gamma \vdash_v t : A}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\phi}[t/x]} \text{E-}\forall \text{ term}$$

$$\frac{\Xi, X; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\phi}}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \forall X.\underline{\phi}} \text{I-}\forall \text{ vtype , FV constraint?}$$

$$\frac{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \forall X.\underline{\phi} \quad \Xi \vdash A}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\phi}[A/X]} \text{E-}\forall \text{ vtype}$$

$$\frac{\Xi, \underline{X}; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\phi}}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \forall \underline{X}.\underline{\phi}} \text{I-}\forall \text{ ctype , FV constraint?}$$

$$\frac{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \forall \underline{X}.\underline{\phi} \quad \Xi \vdash \underline{A}}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \underline{\phi}[\underline{A}/\underline{X}]} \text{E-}\forall \text{ ctype}$$

$$\frac{\Xi; \Gamma; \Delta; \Theta, (R : Rel_{\mathcal{V}}[A, B]); \Omega | \Phi; \Psi \vdash \underline{\phi}}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \forall(R : Rel_{\mathcal{V}}[A, B]).\underline{\phi}} \text{I-}\forall \text{ vrel}$$

$$\frac{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \forall(R : Rel_{\mathcal{V}}[A, B]).\underline{\phi} \quad \Xi; \Gamma; \Theta, \vdash (x : A, y : B).\psi : Rel_{\mathcal{V}}[A, B]}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \forall(R : Rel_{\mathcal{V}}[A, B]).\underline{\phi}[\psi[t/x, u/y]/R(t, u)]} \text{E-}\forall \text{ vrel}$$

$$\frac{\Xi; \Gamma; \Delta; \Theta, (\underline{R} : Rel_{\mathcal{C}}[\underline{A}, \underline{B}]); \Omega | \Phi; \Psi \vdash \underline{\phi}}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \forall(\underline{R} : Rel_{\mathcal{C}}[\underline{A}, \underline{B}]).\underline{\phi}} \text{I-}\forall \text{ crel}$$

$$\frac{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \forall(R : Rel_{\mathcal{C}}[\underline{A}, \underline{B}]).\underline{\phi} \quad \Xi; \Gamma; \Delta; \Theta; \Omega, \vdash (x : \underline{A}, y : \underline{B}).\underline{\psi} : Rel_{\mathcal{C}}[\underline{A}, \underline{B}]}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \forall(R : Rel_{\mathcal{C}}[\underline{A}, \underline{B}]).\underline{\phi}[\underline{\psi}[t/x, u/y]/R(t, u)]} \text{E-}\forall \text{ crel}$$

2.3.3 Congruences

$$\frac{\Xi; \Gamma \vdash_c t : \underline{B} \quad \Xi; \Gamma \vdash_c u : \underline{B} \quad \Xi; \Gamma, x : A; \Delta; \Theta; \Omega | \Phi; \Psi \vdash t = u}{\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \Psi \vdash \lambda(x : A).t = \lambda(x : A).u} \text{cong-}\lambda$$

2.4 Axioms

Beta/Eta/(parametricity schema?)

2.5 Logical Interpretation of Types

Let \mathbf{X} and $\underline{\mathbf{X}}$ be vectors of value type and computation type variables of length n . Let $\underline{\rho}$ be a vector of value relations $\Xi; \Gamma; \Theta \vdash \rho_i : Rel_V[C_i, C'_i]$ for all $i \in 1..n$. Let $\underline{\rho}$ be a vector of computation relations $\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\rho}_i : Rel_C[\underline{C}_i, \underline{C}'_i]$ for all $i \in 1..n$.

Let A be a **value type** with $FTV(A) \in \{\mathbf{X}, \underline{\mathbf{X}}\}$. Define:

$$A[\underline{\rho}/\mathbf{X}, \underline{\rho}/\underline{\mathbf{X}}] : Rel_V[A[C/\mathbf{X}, \underline{C}/\underline{\mathbf{X}}], A[C'/\mathbf{X}, \underline{C}'/\underline{\mathbf{X}}]]$$

by induction on A .

$$X_i[\underline{\rho}, \underline{\rho}] = \rho_i$$

$$Unit[\underline{\rho}, \underline{\rho}] = (x : Unit, y : Unit).x =_{Unit} y$$

$$CaseA[\underline{\rho}, \underline{\rho}] = (x : Case (A[C/\mathbf{X}, \underline{C}/\underline{\mathbf{X}}]), y : Case (A[C'/\mathbf{X}, \underline{C}'/\underline{\mathbf{X}}])).$$

think exists?

$$OSum[\underline{\rho}, \underline{\rho}] = \text{think exists?}$$

$$A \times A'[\underline{\rho}, \underline{\rho}] = ((x, y) : A \times A'[C/\mathbf{X}, \underline{C}/\underline{\mathbf{X}}], (x', y') : A \times A'[C'/\mathbf{X}, \underline{C}'/\underline{\mathbf{X}}]).$$

$$A[\underline{\rho}, \underline{\rho}](x, x') \wedge A'[\underline{\rho}, \underline{\rho}](y, y')$$

$$A * A'[\underline{\rho}, \underline{\rho}] = \text{similar to product?}$$

$$\exists X.A[\underline{\rho}, \underline{\rho}] = \text{standard}$$

$$UB[\underline{\rho}, \underline{\rho}] = \text{related thinks?}$$

Let \underline{B} be a **computation type** with $FTV(\underline{B}) \in \{\mathbf{X}, \underline{\mathbf{X}}\}$. Define:

$$\underline{B}[\underline{\rho}/\mathbf{X}, \underline{\rho}/\underline{\mathbf{X}}] : Rel_C[\underline{B}[C/\mathbf{X}, \underline{C}/\underline{\mathbf{X}}], \underline{B}[C'/\mathbf{X}, \underline{C}'/\underline{\mathbf{X}}]]$$

by induction on \underline{B} .

$$A \rightarrow \underline{B}[\underline{\rho}, \underline{\rho}] = \text{Add weakest precondition to the proposition syntax to interpret this?}$$

$$A * \underline{B}[\underline{\rho}, \underline{\rho}] =$$

$$\forall X.\underline{B}[\underline{\rho}, \underline{\rho}] =$$

$$FA[\underline{\rho}, \underline{\rho}] = \text{Could be encoded if we add } \forall \underline{X}.\underline{B}?$$

3 Goal

Try to prove

$$\begin{aligned}
& \vdash_d \forall A \underline{B}. \\
& \quad \forall (y : \text{OSum}). \\
& \quad \forall (f : U(\forall X. \text{Case } X \multimap \text{OSum} \rightarrow FX)). \\
& \quad \forall (k \ k' : U(A \rightarrow \underline{B})). \\
& \quad \text{newcase}_A \sigma; x \leftarrow (!f)[A]\sigma y; (!k)x \\
& \quad = \\
& \quad \text{newcase}_A \sigma; x \leftarrow (!f)[A]\sigma y; (!k')x
\end{aligned}$$

4 TODO/Questions

- Do we add the type $\forall \underline{X}, \underline{B}$ to the object lang and then encode FA ?
- Finalize the judgment forms, what contexts do they actually need to be displayed over? Do we need to split the relation and proposition contexts into distinct value/computation contexts?
- Check that the classification of logical connectives makes sense (value prop vs comp prop)
- Does the body of a value relation need to be value propositions? (same question for computation relation)
- What is the introduction rule for computation relations (definable computation relations)?
- Denotation of value/computation propositions
- Understand the operation \odot and its laws
- Denotation of value/computation derivations
- Define the operation \odot^* and find its laws
- Finish the known relational interpretation of types
- Attempt the relational interpretation of our new types
- Write up the beta and eta deduction rules
- Check the correctness of the relational interpretation of types. (By proving Reynold's Identity Extension Lemma?)
- PE Logic denotes the collection of computation relations, $Rel_C[\underline{A}, \underline{B}]$, by $Sub_C(\underline{A} \times \underline{B})$. However, they never define $\underline{A} \times \underline{B}$ or state that it is a derivable type.