1 Untyped Syntax

value types	A	::=	Unit	
		|	Bool	
		|	Case A	
		|	OSum	
		|	$A \times A$	
		|	$A * A$	
		|	$U\underline{B}$	
computation types	B	::=	$A \to \underline{B}$	
		|	$A \mathrel{-\!\!*} \underline{B}$	
		|	$F\ A$	
values	V	::=	tt	
		|	true	
		|	false	
		|	x	variable
		|	$\mathrm{inj}_V\ V$	injection into the open sum
		|	(V, V)	
		|	$\pi_1\ V$	
		|	$\pi_2\ V$	
		|	$(V * V)$	separating product
		|	$\rho_1\ V$	
		|	$\rho_2\ V$	
		|	thunk M	
computations	M	::=	$\mho$	need monad for this
		|	force V	
		|	ret V	
		|	$x \leftarrow M;\ N$	
		|	$M\ N$	
		|	$M@N$	
		|	$\lambda x\colon A.M$	
		|	$\alpha x\colon A.M$	separating function
		|	$\mathrm{newcase}_A\ x;\ M$	"allocate" a new case in the open sum
		|	match V with V $\{\mathrm{inj}x.M \mid N\}$	match on open sum
		|	if V then M else N	
value typing context	Γ	::=	$\varnothing$	
		|	$x : A$	
		|	$\Gamma; \Gamma'$	cartesian product
		|	$\Gamma \fatsemi \Gamma'$	monoidal product
stoup	Θ	::=	$\cdot$	
		|	$\bullet : B$	

stoup forms?

2 Typed Syntax

This version of the syntax is a combination of the ν-calculus [**?**] [**?**] of Pitts and Stark and the affine $\alpha\lambda$-calculus of O'Hearn [**?**] expressed in CBPV and with the addition of an extensible sum type. for (rule airity > 1) non separating connectives, contexts don't need to be appended(Γ; Δ)?

2.1 Structural Rules

; admits weakening, contraction, and exchange (cartesian closed structure)
⨾ admits weakening and exchange (semicartisian symmetric monoidal structure)

2.2 Value Typing

2.2.1 Unit

$$\frac{}{\Gamma \vdash_v \mathsf{tt} : \mathsf{Unit}}\ \text{Intro}$$

2.2.2 Bool

$$\frac{}{\Gamma \vdash_v \mathsf{true} : \mathsf{Bool}}\ \text{Intro}_1$$

$$\frac{}{\Gamma \vdash_v \mathsf{false} : \mathsf{Bool}}\ \text{Intro}_2$$

$$\frac{\Gamma \vdash_v V : \mathsf{Bool} \qquad \Gamma \mid \cdot \vdash_c M_1 : B \qquad \Gamma \mid \cdot \vdash_C M_2 : B}{\Gamma \mid \cdot \vdash_c \mathsf{if}\ V\ \mathsf{then}\ M_1\ \mathsf{else}\ M_2 : B}\ \text{Elim}$$

2.2.3 Var

$$\frac{}{x : \mathrm{A} \vdash_v x : \mathrm{A}}\ \text{Intro}$$

2.2.4 OSum

$$\frac{\Gamma \vdash_v V_c : \mathsf{Case}\ \mathrm{A} \qquad \Gamma \vdash_v V : A}{\Gamma \vdash_v \mathsf{inj}_{V_c}\ V : \mathsf{OSum}}\ \text{Intro}$$

$$\frac{\Gamma \vdash_v V_T : \mathsf{Case}\ \mathrm{A} \qquad \Gamma \vdash_v V : \mathsf{OSum} \qquad \Gamma;\ (x : A) \mid \cdot \vdash_c M : B \qquad \Gamma \mid \cdot \vdash_c N : B}{\Gamma \mid \cdot \vdash_c \mathsf{match}\ V_T\ \mathsf{with}\ V\ \{\mathsf{inj}\ x.M \mid N\}}\ \text{Elim}$$

2.2.5 Value Product

$$\frac{\Gamma \vdash_v N : A_1 \qquad \Gamma \vdash_v M : A_2}{\Gamma \vdash_v (N, M) : A_1 \times A_2} \text{ Intro}$$

$$\frac{\Gamma \vdash_v P : A_1 \times A_2}{\Gamma \vdash_v \pi_1 P : A_1} \text{ Elim}_1$$

$$\frac{\Gamma \vdash_v P : A_1 \times A_2}{\Gamma \vdash_v \pi_2 P : A_2} \text{ Elim}_2$$

2.2.6 Separating Product

we have these because the monoidal product is semicartesian

$$\frac{\Gamma \vdash_v M : A_1 \qquad \Delta \vdash_v N : A_2}{\Gamma \fatsemi \Delta \vdash_v (M * N) : A_1 * A_2} \text{ Intro}$$

$$\frac{\Gamma \fatsemi \Delta \vdash_v P : A_1 * A_2}{\Gamma \vdash_v \rho_1 P : A_1} \text{ Elim}_1$$

$$\frac{\Gamma \fatsemi \Delta \vdash_v P : A_1 * A_2}{\Delta \vdash_v \rho_2 P : A_2} \text{ Elim}_2$$

2.2.7 Thunk

$$\frac{\Gamma \mid \cdot \vdash_c M : B}{\Gamma \vdash_v \text{thunk } M : UB} \text{ Intro}$$

$$\frac{\Gamma \vdash_v V : UB}{\Gamma \mid \cdot \vdash_c \text{force } V : B} \text{ Elim}$$

2.3 Computation Typing

2.3.1 Function

$$\frac{\Gamma;\, (x : A) \mid \cdot \vdash_c V : B}{\Gamma \mid \cdot \vdash_c (\lambda x : A.\, V) : A \to B} \text{ Intro}$$

$$\frac{\Gamma \mid \Theta \vdash_c M : A \to B \qquad \Delta \vdash_v N : A}{\Gamma;\, \Delta \mid \Theta \vdash_c MN : B} \text{ Elim}$$

2.3.2 Separating Function

$$\frac{\Gamma \fatsemi (x : A) \mid \cdot \vdash_c V : B}{\Gamma \mid \cdot \vdash_c (\alpha x : A.\, V) : A \mathrel{-\!\!*} B} \text{ Intro}$$

$$\frac{\Gamma \mid \Theta \vdash_c M : A \mathrel{-\!\!*} B \qquad \Delta \vdash_v N : A}{\Gamma \fatsemi \Delta \mid \Theta \vdash_c MN : B} \text{ Elim}$$

2.3.3 Return

$$\frac{\Gamma \vdash_v V : A}{\Gamma \mid \cdot \vdash_c \text{ret } V : FA}\ \text{Intro}$$

$$\frac{\Gamma \mid \Theta \vdash_c M : FA \qquad \Gamma; (x : A) \mid \cdot \vdash_c N : B}{\Gamma \mid \Theta \vdash_c x \leftarrow M;\ N : B}\ \text{Elim}$$

2.3.4 Error

$$\frac{}{\Gamma \mid \cdot \vdash_c \mho : B}\ \text{Intro}$$

2.4 Equations

for effects (but also computation rules)

2.4.1 Allocation

CITE The following are from Ian Stark's Categorial Models for Local Names

1. swap - allocation swap
2. drop - (our monad might not support this)
3. Fresh - (a kind of congruence)

2.4.2 Dynamic Type

the prism laws here

1.

3 Semantics

3.1 Worlds

Normally, the value types in a CBPV would be interpreted as Sets. However, in our case this is insufficient as the interpretation of the extensible sum type, $OSum$, is dependent on what $Cases$ have been "allocated" via newcase. Thus we interpret value types in a presheaf category on a category of worlds where objects are a map from a finite set to syntactic types. Let $SynTy$ be a set of syntactic types that can be used in the OSum Case store.

$$\begin{array}{rcl} SynTy & ::= & Unit \\ & \mid & Bool \\ & \mid & SynTy \times SynTy \end{array}$$

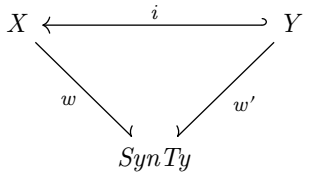
The denotation of these syntactic types is independent of what $Cases$ have been allocated. Limiting the extensible sum type to contain only base types and products of base types allows us to temporarily avoid two separate issues.

- The first issue has to do with circularity of definitions. If we say a world is a mapping from a fininte set to semantic types and that semantic types are presheaves on the category of worlds, then we have tied ourselves in a knot. This issue is pointed out in, among other places, [**?**].

$$World \cong X \rightharpoonup_{fin} SemTy \quad SemTy \cong [\mathbf{Worlds}, Set]$$

- The second issue is well-foundedness of the interpretation of $OSum$. Consider if we allow the extensible sum type to store the type $(OSum \to OSum)$ and try to recursively interpret $OSum$.

similar constructions: HeapV [**?**], Stirling univalent reference (or den), Variable binding, nominal sets book? Here we define the category **World**[1]. A $world : X \rightharpoonup_{fin} SynTy$ is a finite map of syntactic types. Morphisms $f : X \to Y$ in **World** are injective functions $i : Y \hookrightarrow X$ such that the following triangle commutes. The finite sets here represent a finite collection of case symbols and the injective map i represents "renaming". [2]



3.1.1 Monoidal Structure

We need to introduce additional structure on **World** to interpret the separating connectives $A * B$ and $A \mathrel{-\!\!*} B$. For this, we will pull from existing literature on categorical semantics of bunched typing, specifically [**?**] and section 3.1 of [**?**]. We use the Day convolution [**?**] to get a monoidal structure on $\widehat{\mathbf{Worlds}}$ from a monoidal structure on **World**. First, we define an operation $x \otimes_w y'$ on $worlds$ by taking the set coproduct of their domains[3] and using the recursor $X + Y \xrightarrow{rec\ w\ w'} SynTy$ to define a map into syntactic types. The action on morphisms is given by defining the top injection via $map : (A \to C) \to (B \to D) \to A+B \to C+D$, which is injective, and the commuting triangles still hold. The unit for this monoid structure is the empty map $\lambda()$ [4].
Using this operation, we define the monoidal product for $\widehat{\mathbf{Worlds}}$:

$$(A \otimes B)X = \int^{Y,Z} A(Y) \times B(Z) \times \mathbf{Worlds}[X, Y \otimes_w Z]$$

[1] Similar categories: $HeapV$ [**?**], $IFam_X$ [**?**]

[2] Formalized here as **World** := $(Inc \downarrow SynTy)^{op}$, where Inc is the inclusion functor from the category of fininte sets and injections into the category **Set** and $SynTy$ is a constant functor.

[3] This is not the coproduct in FinSet_mono [**?**]

[4] The empty map is also the terminal object in **Worlds**

The monoidal unit is given by:

$$I(X) = \mathbf{World}[X, \lambda()]$$

And the separating implication is defined by:

$$(A \twoheadrightarrow B)X = \int_Z \mathbf{Set}[A(Z), B(X *_w Z)] \cong \widehat{\mathbf{Worlds}}[A, B(X *_w \_)]$$

Additionally, we have that the tensor is left adjoint to magic wand $Hom(A \otimes B, C) \cong Hom(A, B \twoheadrightarrow C)$.

These definitions are standard in the literature on bunched implication with the exception that we specify that our "resources" are a specific kind of finite maps. The bicartesian stucture on $\widehat{\mathbf{Worlds}}$ along with the monoidal structure and its right adjoint bundled together has the structure of a bicartesian doubly closed category or BiDCC[5].

3.1.2 Affine BiDCC

The concrete model that we are working with has the property that $\mathbf{1} \cong I$[6][7]. That is, the cartesian unit is isomorphic to the monoidal unit. This property validates weakening for $\S$. Note that

$$\mathbf{1}(X) = \{*\}$$
$$I(X) = \mathbf{Worlds}[X, \lambda()]$$

$\lambda()$ is terminal in **Worlds** and the cardinality of $|\,\mathbf{Worlds}[\_, \lambda()]\,| = 1$.

3.2 Effect Monad

We need an appropriate monad on $\widehat{\mathbf{Worlds}}$ to interpret printing and error effects. First, we define an endofunctor T on $\widehat{\mathbf{Worlds}}$.

$$T_0(X) = Const_E + (X \times Const_{FooBar})$$

where $FooBar$ is a set containing all possible sequences of symbols "foo" and "bar" ($FooBar := \{s \mid s = (foo \mid bar)^+\}$) and $Const_{FooBar}$ is the constant presheaf mapping all $worlds$ to set $FooBar$. E is a singleton set representing the "error state".
The action on morphisms is define as:

$$T_1(f : X \to Y)(Z) := \begin{cases} Z & Z : E \\ (f(x), s) & Z = (x, s) \end{cases}$$

where f is a natural transformation between presheaves (overloading notation here).

[5] Terrible name, but I'm just following the literature here
[6] TODO, work this out in full detail
[7] See section 3.3 of [**?**]

- functor laws
- natural transformations (return and join)
- monad laws
- Show T is strong (define strength nt and show 4 diagrams commute https://ncatlab.org/nlab/show/strong+monad) strength for $\otimes$

[?] point to section 7, specifically the note about how the notion of finite support does not exist in the presheaf category, but does in the full subcategory of pullback preserving functors. Additionally that the shanuel topos models classical HOL rather than intuitionistic HOL

3.3 Computation Semantics

To interpret computation types, we use a free and forgetful adjunction between $\widehat{\textbf{Worlds}}$ and the Eilenberg Moore category of monad T denoted **TAlg**. **TAlg** has all limits which exist in $\widehat{\textbf{Worlds}}$[8]. Concrete constructions can be found here [**?**]. The singleton, exponential, product, and free algebra are "given", what remains is to construct the separating implication algebra. Assuming[9] T is a strong monad on $\otimes$, we have a map $T_{A\otimes B} : A \otimes T(B) \to T(A \otimes B)$, then the construction of the exponent algebra mirrors that of the separating algebra. I will reproduce the construction here.

Given an object $A : \widehat{\textbf{Worlds}}$ and algebra $\langle B, \theta_B : T(B) \to B\rangle$, we need to construct an algebra. Set the carrier of the algebra to be $A \multimap B$, we then need a map $\theta_{A\multimap B} : T(A \multimap B) \to (A \multimap B)$. Via the tensor wand adjunction, it suffices to construct a map $T(A \multimap B) \otimes A \to B$.

$$\begin{array}{ccc} T(A \multimap B) \otimes A & \xrightarrow{\theta_{A\multimap B}} & B \\ \downarrow{\scriptstyle T_{str}} & & \uparrow{\scriptstyle \theta_B} \\ T(A \otimes (A \multimap B)) & \xrightarrow{T_1(eval_{\multimap})} & T(B) \end{array}$$

What remains is to check the T-algebra laws. We call $\langle A \multimap B, \theta_{A\multimap B}\rangle$ the separating algebra.

[8] https://ncatlab.org/nlab/show/Eilenberg-Moore+category
[9] TODO: show

3.4 Interpreting Types

With all the categorical structure in place, we now give an interpretation of syntax. We first give an interpretation to syntactic types $SynTy$ via $[\![\_]\!]_{el} : SynTy \to \widehat{\mathbf{Worlds}}$. We recall that the interpretation of these types is not dependent on what cases have been allocated. $\mathbf{1}$ is the terminal presheaf in $\widehat{\mathbf{Worlds}}$ and $+, \times$ come from the bicartesian structure of $\widehat{\mathbf{Worlds}}$.

$$\begin{aligned}
[\![Unit]\!]_{el} &= \mathbf{1} \\
[\![Bool]\!]_{el} &= \mathbf{1} + \mathbf{1} \\
[\![A \times B]\!]_{el} &= [\![A]\!]_{el} \times [\![B]\!]_{el}
\end{aligned}$$

Next, we interpret the remaining value types in $\widehat{\mathbf{Worlds}}$. The separating product is interpreted as the monoidal product from the Day convolution and $|\, [\![B]\!]_c \,|$ is the forgetful functor which returns the carrier of the algebra $[\![B]\!]_c$. The main definitions of interest are the interpretation of $OSum$ and $Case$. $OSum$ is an extensible sum type where the current world determines the number and type of cases in the extensible sum. $Case\ A$ is interpreted to be the set of allocated case symbols for type A in the current world. Note that syntactic type equality is used in the set comprehension. This means that $\sigma : Case(A \times (B \times C))$ and $\sigma' : Case((A \times B) \times C)$ are distinct elements of two distinct sets. It may be nice to have a more flexible definition $\{\sigma \mid \sigma \in dom(\rho) \wedge [\![\rho(\sigma)]\!]_{el} \cong [\![A]\!]_{el}\}$. We would have to be more careful when constructing and destructing elements of $OSum$. Consider $\sigma : Case\ A$ and $\mathrm{inj}_\sigma(*, a) : OSum$. Should this be allowed with $[\![A]\!] \cong [\![Unit \times A]\!]$?

$$\begin{aligned}
[\![Unit]\!]_v &= [\![Unit]\!]_{el} \\
[\![Bool]\!]_v &= [\![Bool]\!]_{el} \\
[\![A \times B]\!]_v &= [\![A \times B]\!]_{el} \\
[\![OSum]\!]_v(\rho) &= \sum_{\sigma \in dom(\rho)} [\![\rho(\sigma)]\!]_{el}(\rho) \\
[\![Case\ A]\!]_v(\rho) &= \{\sigma \mid \sigma \in dom(\rho) \wedge \rho(\sigma) = A\} \\
[\![A * B]\!]_v &= [\![A]\!]_v \otimes [\![B]\!]_v \\
[\![U \underline{B}]\!]_v &= |\, [\![B]\!]_c \,|
\end{aligned}$$

Lastly, we interpret the computation types as their respective algebras in **TAlg**.

$$\begin{aligned}
[\![A \to B]\!]_c &= expAlg [\![A]\!]_v [\![B]\!]_c \\
[\![A \mathrel{-\!\!*} B]\!]_c &= sepAlg [\![A]\!]_v [\![B]\!]_c \\
[\![FA]\!]_c &= freeAlg [\![A]\!]_v
\end{aligned}$$

3.5 Interpreting Contexts

In *regular* type theory, contexts have the structure of a list and are interpreted as an n-ary product in the category used to interpret types. In the bunched type theory we are working with here, there are two connectives for constructing contexts (";" and "⨾") and the structure of contexts are "tree-like". We use ; to denote the usual context construction operation that is interpreted as a cartesian product and ⨾ to denote the new context construction operation that is interpreted as the tensor product.

$$
\begin{aligned}
[\![x : A]\!]_{ctx} &= [\![A]\!]_v \\
[\![\phi]\!]_{ctx} &= \mathbf{1} \\
[\![\Gamma \,;\, \Gamma]\!]_{ctx} &= [\![\Gamma]\!]_{ctx} \times [\![\Gamma]\!]_{ctx} \\
[\![\varphi]\!]_{ctx} &= I \\
[\![\Gamma \fatsemi \Gamma]\!]_{ctx} &= [\![\Gamma]\!]_{ctx} \otimes [\![\Gamma]\!]_{ctx}
\end{aligned}
$$

The cartesian fragment of the context supports weakening and contraction. The monoidal fragment supports weakening[10].

3.6 Interpreting Terms

Value terms $\Gamma \vdash M : A$ are interpreted as morphisms $[\![\Gamma]\!]_{ctx} \xrightarrow{M} [\![A]\!]_v$ in $\widehat{\mathbf{Worlds}}$ which are natural transformations from $\mathbf{World}^{\mathbf{op}}$ to $\mathbf{Set}$. So for any term $\Gamma \vdash M : A$, we need to provide a family of maps $\forall\, w,\ \alpha_w : [\![\Gamma]\!](w) \to [\![A]\!](w)$ such that given any morphism $f : w \to w'$ in $\mathbf{World}$ we have:

$$
\begin{array}{ccc}
[\![\Gamma]\!](w) & \xleftarrow[[\![\Gamma]\!](f)]{} & [\![\Gamma]\!](w') \\
\downarrow{\scriptstyle \alpha_w} & & \downarrow{\scriptstyle \alpha_{w'}} \\
[\![A]\!](w) & \xleftarrow[[\![A]\!](f)]{} & [\![A]\!](w')
\end{array}
$$

Here we define the terms as natural transformations by giving the components. check: do we get naturality via parametric polymorphism in this setting? w is the current world and $\gamma : [\![\Gamma]\!](w)$. The base terms are straightforward.

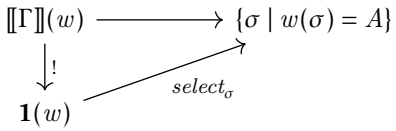
$$
\begin{aligned}
[\![\mathsf{tt}]\!]_{vtm}(w, \gamma) &= * \\
[\![\mathsf{true}]\!]_{vtm}(w, \gamma) &= inl\ * \\
[\![\mathsf{false}]\!]_{vtm}(w, \gamma) &= inr\ *
\end{aligned}
$$

[10] consequence of $\mathbf{1} \cong I$ in our model

Deleted whole page, becase our monoidal product is affine, we have projections [**?**](Definition 2.1.ii) We can use this process (call it *lookup*) to construct the proper denotation for variables.

$$[\![x]\!]_{vtm} = lookup(x)$$

For case symbols, we'd like a natural transformation $[\![\sigma]\!] : [\![\Gamma]\!] \to [\![Case\ A]\!]$ where components are defined as so:



which is definable as long as long as $\sigma \mapsto A \in w$. Consider the empty map $\lambda() \in Obj(\mathbf{Worlds}),\ \alpha_{\lambda()} : [\![\Gamma]\!](\lambda()) \to \varnothing$. This seems problematic..

$$[\![\sigma]\!]_{vtm}(w) = select_\sigma \circ !$$

For injection of values into the open sum type, we can assume we already have $[\![\Gamma]\!] \xrightarrow{\sigma} [\![Case\ A]\!]$ and $[\![\Gamma]\!] \xrightarrow{V} [\![A]\!]$. We then want to construct $[\![\text{inj}_\sigma\, V]\!] : [\![\Gamma]\!] \to [\![OSum]\!]$. First, lets consider the partially applied term $[\![\text{inj}_\sigma]\!] : [\![A]\!] \to [\![OSum]\!]$. Consider the subset $\{\alpha_w \mid w\ s.t.\ \sigma \mapsto A \in w\}$ of components of $[\![\text{inj}_\sigma]\!]$. These components will be of the form $\alpha_w = (inl \mid inr) \circ inr^*$ which will inject a value of type $[\![A]\!]$ into the open sum type at world w. $[\![\Gamma]\!] \xrightarrow{\sigma, V} [\![Case\ A]\!] \times [\![A]\!] \xrightarrow{\text{inj}_\sigma, id} [\![OSum]\!]^{[\![A]\!]} \times [\![A]\!] \xrightarrow{eval} [\![OSum]\!]$

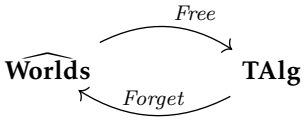
$$[\![\text{inj}_\sigma\, V]\!]_{vtm} = ?$$

The denotation of product and separating product are both akin to contructing bilinear maps.

$$[\![(V_1, V_2)]\!]_{vtm} = [\![V_1]\!], [\![V_2]\!]$$
$$[\![(V_1 * V_2)]\!]_{vtm} = [\![V_1]\!] * [\![V_2]\!]$$

And finally, the interpretation of thunk M is given by the forgetful functor which projects out the carrier of the algebra $[\![M]\!]_{ctm}$.

$$[\![\text{thunk}\ M]\!]_{vtm} = Forget([\![M]\!]_{ctm})$$



The interpretation of computation terms

$$
\begin{aligned}
[\![\mho]\!]_{ctm} &= \\
[\![\text{print } V]\!]_{ctm} &= \\
[\![\text{force } V]\!]_{ctm} &= [\![V]\!]_{vtm} \\
[\![\text{ret } V]\!]_{ctm} &= \mathit{Free}([\![V]\!]_{vtm}) \\
[\![x \leftarrow M;\ N]\!]_{ctm} &= \\
[\![M\ N]\!]_{ctm} &= \langle \mathit{eval}(|\,[\![M]\!]_{ctm}\,|)([\![N]\!]_{vtm}),\ \theta_{[\![M]\!]_{ctm}}[\![N]\!]_{vtm}\rangle \\
[\![M@N]\!]_{ctm} &= \langle \mathit{eval}^*(|\,[\![M]\!]_{ctm}\,|)([\![N]\!]_{vtm}),\ \theta_{[\![M]\!]_{ctm}}[\![N]\!]_{vtm}\rangle \\
[\![\lambda x : A.M]\!]_{ctm}(e : [\![\Gamma]\!]_v) &= \langle \lambda a : [\![A]\!]_v.\,|\,[\![M]\!]_{ctm}(e, a)\,|,\ \theta\rangle \\
[\![\alpha x : A.M]\!]_{ctm}(e : [\![\Gamma]\!]_v) &= \langle \alpha a : [\![A]\!]_v.\,|\,[\![M]\!]_{ctm}(e * a)\,|,\ \theta\rangle \\
[\![\text{newcase}_A\ \sigma;\ M]\!]_{ctm} &= \text{updated world and value environment?} \\
[\![\text{match } V_1 \text{ with } V_2\ \{x.M \mid N\}]\!]_{ctm} &= \\
[\![\text{let } (x, y) = V;\ M]\!]_{ctm} &= \\
[\![\text{let } (x * y) = V;\ M]\!]_{ctm} &= \\
[\![\text{if } V \text{ then } M \text{ else } N]\!]_{ctm} &=
\end{aligned}
$$

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