1 Judgements

The context is split into Local Value Types and Global Value Types.

$$\Gamma_G; \Gamma_L \text{ Ctx}$$

We have three *kinds* of type: Global Value Type, Local Value Type, and Computation Type. Global value types consist of the universes. remove global type context from local type judgement?(Do need global context for existential types)

$$\Gamma_G; \Gamma_L \vdash A \text{ Global}$$
$$\Gamma_G; \Gamma_L \vdash A \text{ Local}$$
$$\Gamma_G; \Gamma_L | \Delta \vdash \underline{B} \text{ Comp}$$

For the logic, we have Value Propositions and Computation Propositions.

$$\Gamma_G; \Gamma_L \vdash \phi \text{ Prop}_V$$
$$\Gamma_G; \Gamma_L | \Delta \vdash \underline{\phi} \text{ Prop}_C$$

Derivation judgements

$$\Gamma_G; \Gamma_L \vdash_d \phi$$
$$\Gamma_G; \Gamma_L | \Delta \vdash_d \underline{\phi}$$

2 Formation Rules

2.1 Contexts

$$\frac{}{\cdot; \cdot \text{ Ctx}}$$

$$\frac{\Gamma_G; \Gamma_L \text{ Ctx} \qquad \Gamma_G; \Gamma_L \vdash X \text{ Global}}{\Gamma_G, (x : \lfloor X \rfloor); \Gamma_L \text{ Ctx}}$$

$$\frac{\Gamma_G; \Gamma_L \text{ Ctx} \qquad \Gamma_G; \Gamma_L \vdash X \text{ Local}}{\Gamma_G; \Gamma_L, (x : X) \text{ Ctx}}$$

$$\frac{\Gamma_G; \Gamma_L \text{ Ctx} \qquad \Gamma_G; \Gamma'_L \text{ Ctx}}{\Gamma_G; \Gamma_L, \Gamma'_L \text{ Ctx}}$$

$$\frac{\Gamma_G; \Gamma_L \text{ Ctx} \qquad \Gamma_G; \Gamma'_L \text{ Ctx}}{\Gamma_G; \Gamma_L * \Gamma'_L \text{ Ctx}}$$

2.2 Types

2.2.1 Global Value Types

Global types consist of our *universe* types.

$$\frac{\Gamma_G;\Gamma_L \vdash A;\text{Local}}{\Gamma_G;\Gamma_L \vdash V A \text{ Global}}$$

The addition of $V A$ as a global type in conjunction with the context rules allows terms of Local Type to exist in the Global Type context. See the dependent linear non-linear paper

$$\frac{}{\Gamma_G;\Gamma_L \vdash U_V \text{ Global}}$$

$$\frac{}{\Gamma_G;\Gamma_L \vdash U_C \text{ Global}}$$

$$\frac{\Gamma_G;\Gamma_L \vdash A \text{ Local}}{\Gamma_G;\Gamma_L \vdash P_V\ A \text{ Global}}$$

$$\frac{\Gamma_G;\Gamma_L \vdash \underline{B} \text{ Comp}}{\Gamma_G;\Gamma_L \vdash P_C\ \underline{B} \text{ Global}}$$

$$\frac{\Gamma_G;\Gamma_L \vdash A \text{ Local} \qquad \Gamma_G;\Gamma_L \vdash A' \text{ Local}}{\Gamma_G;\Gamma_L \vdash R_V\ A\ A' \text{ Global}}$$

$$\frac{\Gamma_G;\Gamma_L \vdash \underline{B} \text{ Comp} \qquad \Gamma_G;\Gamma_L \vdash \underline{B}' \text{ Comp}}{\Gamma_G;\Gamma_L \vdash R_C\ \underline{B}\ \underline{B}' \text{ Global}}$$

2.2.2 Local Value Types

$$\frac{}{\Gamma_G;\Gamma_L \vdash \text{Unit Local}}$$

$$\frac{}{\Gamma_G;\Gamma_L \vdash \text{OSum Local}}$$

$$\frac{\Gamma_G;\Gamma_L \vdash A \text{ Local}}{\Gamma_G;\Gamma_L \vdash \text{Case } A \text{ Local}}$$

$$\frac{\Gamma_G;\Gamma_L \vdash A \text{ Local} \qquad \Gamma_G;\Gamma_L \vdash A' \text{ Local}}{\Gamma_G;\Gamma_L \vdash A \times A' \text{ Local}}$$

$$\frac{\Gamma_G;\Gamma_L \vdash A \text{ Local} \qquad \Gamma_G;\Gamma_L \vdash A' \text{ Local}}{\Gamma_G;\Gamma_L \vdash A * A' \text{ Local}}$$

$$\frac{\Gamma_G;\Gamma_L \vdash \underline{B} \text{ Comp}}{\Gamma_G;\Gamma_L \vdash U\underline{B} \text{ Local}}$$

existential type

2.2.3 Computation Types

$$\frac{\Gamma_G;\Gamma_L \vdash A \text{ Local} \qquad \Gamma_G;\Gamma_L \vdash \underline{B} \text{ Comp}}{\Gamma_G;\Gamma_L \vdash A \to \underline{B} \text{ Comp}}$$

$$\frac{\Gamma_G;\Gamma_L \vdash A \text{ Local} \qquad \Gamma_G;\Gamma_L \vdash \underline{B} \text{ Comp}}{\Gamma_G;\Gamma_L \vdash A \mathrel{-\!\!*} \underline{B} \text{ Comp}}$$

$$\frac{\Gamma_G;\Gamma_L \vdash \underline{B} \text{ Comp} \qquad \Gamma_G;\Gamma_L \vdash \underline{B'} \text{ Comp}}{\Gamma_G;\Gamma_L \vdash \underline{B} \mathbin{\underline{\times}} \underline{B'} \text{ Comp}}$$

$$\frac{\Gamma_G, X : U_L;\Gamma_L \vdash \underline{B} \text{ Comp}}{\Gamma_G;\Gamma_L \vdash \forall(X : U_V).\underline{B} \text{ Comp}}$$

$$\frac{\Gamma_G, \underline{X} : U_C;\Gamma_L \vdash \underline{B} \text{ Comp}}{\Gamma_G;\Gamma_L \vdash \forall(\underline{X} : U_C).\underline{B} \text{ Comp}}$$

$$\frac{\Gamma_G;\Gamma_L \vdash A \text{ Local}}{\Gamma_G;\Gamma_L \vdash F A \text{ Comp}}$$

2.3 Propositions

2.3.1 Value Propositions

$$\frac{}{\Gamma_G;\Gamma_L \vdash \top \text{ Prop}_V}$$

$$\frac{\Gamma_G;\Gamma_L \vdash \phi \text{ Prop}_V \qquad \Gamma_G;\Gamma_L \vdash \phi' \text{ Prop}_V}{\Gamma_G;\Gamma_L \vdash \phi \wedge \phi' \text{ Prop}_V}$$

$$\frac{\Gamma_G;\Gamma_L \vdash x : A \qquad \Gamma_G;\Gamma_L \vdash y : A}{\Gamma_G;\Gamma_L \vdash x =_A y \text{ Prop}_V}$$

$$\frac{\Gamma_G;\Gamma_L \vdash P : P_V\ A \qquad \Gamma_G;\Gamma_L \vdash x : A}{\Gamma_G;\Gamma_L \vdash P(x) \text{ Prop}_V}$$

$\sqcap \sqcup$?

$$\frac{\Gamma_G;\Gamma_L \vdash R : R_V\ A\ A' \qquad \Gamma_G;\Gamma_L \vdash x : A \qquad \Gamma_G;\Gamma_L \vdash y : A'}{\Gamma_G;\Gamma_L \vdash R(x, y) \text{ Prop}_V}$$

2.3.2 Computation Propositions

rethink the stoup component in these rules

$$\frac{}{\Gamma_G;\Gamma_L|\Delta \vdash \underline{\top}\ \mathrm{Prop}_C}$$

$$\frac{\Gamma_G;\Gamma_L|\Delta \vdash \underline{\phi}\ \mathrm{Prop}_C \qquad \Gamma_G;\Gamma_L|\Delta \vdash \underline{\phi'}\ \mathrm{Prop}_C}{\Gamma_G;\Gamma_L|\Delta \vdash \underline{\phi}\ \underline{\wedge}\ \underline{\phi'}\ \mathrm{Prop}_C}$$

$$\frac{\Gamma_G;\Gamma_L|\Delta \vdash x : \underline{B} \qquad \Gamma_G;\Gamma_L|\Delta \vdash y : \underline{B}}{\Gamma_G;\Gamma_L|\Delta \vdash x =_{\underline{B}} y\ \mathrm{Prop}_C}$$

$$\frac{\Gamma_G;\Gamma_L \vdash \phi\ \mathrm{Prop}_V \qquad \Gamma_G;\Gamma_L|\Delta \vdash \underline{\psi}\ \mathrm{Prop}_C}{\Gamma_G;\Gamma_L|\Delta \vdash \phi \implies \underline{\psi}\ \mathrm{Prop}_C}$$

$$\frac{\Gamma_G;\Gamma_L|\Delta \vdash \underline{P} : P_C\ \underline{B} \qquad \Gamma_G;\Gamma_L|\Delta \vdash x : \underline{B}}{\Gamma_G;\Gamma_L|\Delta \vdash \underline{P}(x)\ \mathrm{Prop}_C}$$

$\lceil \rceil \lfloor \rfloor$?

$$\frac{\Gamma_G;\Gamma_L|\Delta \vdash \underline{R} : R_C\ \underline{B}\ \underline{B'} \qquad \Gamma_G;\Gamma_L|\Delta \vdash x : \underline{B} \qquad \Gamma_G;\Gamma_L|\Delta \vdash y : \underline{B'}}{\Gamma_G;\Gamma_L|\Delta \vdash \underline{R}(x,y)\ \mathrm{Prop}_C}$$

Can we have one rule to capture quantification over all the global types? That is, one formation rule for value terms, value and computation types, value and computation predicates, value and computation relations. For type quantification, we need a side condition that for $x : U_V$ then $\lfloor x \rfloor \notin FTV(\Gamma_G;\Gamma_L|\Delta)$. TODO consider relevant free variable restrictions

$$\frac{\Gamma_G, x : G;\Gamma_L \Delta \vdash \phi\ \mathrm{Prop}_C}{\Gamma_G;\Gamma_L|\Delta \vdash \forall(x : G).\phi\ \mathrm{Prop}_C}$$

v.s. individual rules for each global type

$$\frac{\Gamma_G, (x : VA);\Gamma_L|\Delta \vdash \underline{\phi}\ \mathrm{Prop}_C}{\Gamma_G;\Gamma_L|\Delta \vdash \forall(x : VA).\underline{\phi}\ \mathrm{Prop}_C}$$

$$\frac{\Gamma_G, (x : U_V);\Gamma_L|\Delta \vdash \underline{\phi}\ \mathrm{Prop}_C}{\Gamma_G;\Gamma_L|\Delta \vdash \forall(x : U_V).\underline{\phi}\ \mathrm{Prop}_C}$$

...

2.4 Predicates & Relations

Try to cut out relations by taking them to be special predicates $P(A \times A')$. That seems to complicate the definition of relational substitution when performing two different type substitutions on the "input types" $(x : A[\overrightarrow{C}/\overrightarrow{X}], y : B[\overrightarrow{C'}/\overrightarrow{Y}])$

Are the following sufficient? What is the justification for the rules in figure 5 of the PE logic paper?

Why introduce new judgement forms when we can directly declare membership of the appropriate universe type? $(x : A).\phi \;\; \mathrm{Pred}_V \; A$ vs $(x : A).\phi : P_V A$. In the Plotkin Abadi logic, this corresponds to the only formation rule for relations

Check the usage of the bunch/hole syntax

$$\frac{\Gamma_G; \Gamma_L(x : X) \vdash \phi \; \mathrm{Prop}_V}{\Gamma_G; \Gamma_L \vdash (x : X).\phi \; \mathrm{Pred}_V \; X}$$

$$\frac{\Gamma_G; \Gamma_L(x : A, y : A') \vdash \phi \; \mathrm{Prop}_V}{\Gamma_G; \Gamma_L \vdash (x : A, y : A').\phi \; \mathrm{Rel}_V \; A \; A'}$$

$$\frac{\Gamma_G; \Gamma_L | \cdot \vdash \underline{\phi} \; \mathrm{Prop}_C}{\Gamma_G; \Gamma_L \vdash (x : F(Unit)).\underline{\phi} \; \mathrm{Pred}_C(F \; Unit)}$$

$$\frac{\Gamma_G; \Gamma_L | x : \underline{B} \vdash \underline{\phi} \; \mathrm{Prop}_C}{\Gamma_G; \Gamma_L \vdash (x : \underline{B}).\underline{\phi} \; \mathrm{Pred}_C \; \underline{B}}$$

$$\frac{\Gamma_G; \Gamma_L | (p : \underline{B} \; \underline{\times} \; \underline{B'}) \vdash \underline{\phi} \; \mathrm{Prop}_C}{\Gamma_G; \Gamma_L \vdash (x : \underline{B}, y : \underline{B'}).\underline{\phi} \; \mathrm{Rel}_C \; \underline{B} \; \underline{B'}}$$

Commented out formation rules akin to Figure 5 of PE logic paper.

3 Introduction & Elimination Rules

3.1 Types

3.1.1 Global Value Types

Check the rules for VA against the dependent linear non-linear paper.

$$\frac{\Gamma_G; \Gamma_L \vdash x : A}{\Gamma_G; \Gamma_L \vdash \lceil x \rceil : VA}$$

$$\frac{\Gamma_G; \Gamma_L \vdash x : VA}{\Gamma_G; \Gamma_L \vdash \lfloor x \rfloor : A}$$

$$\frac{\Gamma_G; \Gamma_L \vdash A \; \mathrm{Local}}{\Gamma_G; \Gamma_L \vdash \lceil A \rceil : U_V} \; U_V\text{-Intro}$$

$$\frac{\Gamma_G;\Gamma_L \vdash A : U_V}{\Gamma_G;\Gamma_L \vdash \lfloor A \rfloor \text{ Local}}\; U_V\text{-Elim}$$

$$\frac{\Gamma_G;\Gamma_L \vdash \underline{B} \text{ Comp}}{\Gamma_G;\Gamma_L \vdash \lceil \underline{B} \rceil : U_C}\; U_C\text{-Intro}$$

$$\frac{\Gamma_G;\Gamma_L \vdash \underline{B} : U_C}{\Gamma_G;\Gamma_L \vdash \lfloor \underline{B} \rfloor \text{ Comp}}\; U_C\text{-Elim}$$

$$\frac{\Gamma_G;\Gamma_L}{}$$

3.1.2 Local Value Types

$$\frac{}{\Gamma_G;\Gamma_L \vdash tt : \text{Unit}}\; \text{Unit-I}$$

$$\frac{\Gamma_G;\Gamma_L \vdash \sigma : \text{Case}A \qquad \Gamma_G;\Gamma_L \vdash V : A}{\Gamma_G;\Gamma_L \vdash \text{inj}_\sigma V : \text{OSum}}\; \text{OSum-I}$$

$$\frac{\Gamma_G;\Gamma_L \vdash V : \text{OSum} \qquad \Gamma_G;\Gamma_L \vdash \sigma : \text{Case } A \qquad \Gamma_G;\Gamma_L(x : A)|\Delta \vdash M : \underline{B} \qquad \Gamma_G;\Gamma_L|\Delta \vdash N : \underline{B}}{\Gamma_G;\Gamma_L|\Delta \vdash \text{match } V \text{ with } \sigma\{ \text{ inj } x.M \| N\} : \underline{B}}\; \text{OSum-E}$$

$$\frac{\Gamma_G;\Gamma_L \vdash V : A \qquad \Gamma_G;\Gamma_L \vdash V' : A'}{\Gamma_G;\Gamma_L \vdash (V, V') : A \times A'}\; \times\text{-I}$$

$$\frac{\Gamma_G;\Gamma_L \vdash p : A \times A'}{\Gamma_G;\Gamma_L \vdash \pi_1\, p : A}\; \times\text{-E1}$$

$$\frac{\Gamma_G;\Gamma_L \vdash p : A \times A'}{\Gamma_G;\Gamma_L \vdash \pi_2\, p : A'}\; \times\text{-E2}$$

$$\frac{\Gamma_G;\Gamma_{L_1} \vdash V_1 : A_1 \qquad \Gamma_G;\Gamma_{L_2} \vdash V_2 : A_2}{\Gamma_G;\Gamma_{L_1} * \Gamma_{L_2} \vdash (V_1 * V_2) : A_1 * A_2}\; *\text{-I}$$

$$\frac{\Gamma_G;\Gamma_L \vdash V : A * A' \qquad \Gamma_G;\Gamma_L(x : A * y : A')|\Delta \vdash M : \underline{B}}{\Gamma_G;\Gamma_L|\Delta \vdash \text{let } (x * y) = V; M : \underline{B}}\; *\text{-E}$$

$$\frac{\Gamma_G;\Gamma_L(x : A) \vdash M : A'}{\Gamma_G;\Gamma_L \vdash (\lambda(x : A).M) : A \to A'}\; \to\text{-I}$$

$$\frac{\Gamma_G;\Gamma_L \vdash M : A \to A' \qquad \Gamma_G;\Gamma_L \vdash N : A}{\Gamma_G;\Gamma_L \vdash MN : A'}\; \to\text{-E}$$

$$\frac{\text{TODO}}{\Gamma_G;\Gamma_L \vdash \text{pack}(A', V) \text{ as } \exists X.A : \exists X.A}\; \exists\text{-I}$$

$$\frac{\text{TODO}}{\Gamma_G;\Gamma_L \vdash : \text{unpack}(X, x) = V; M : \underline{B}}\; \exists\text{-E}$$

$$\frac{\Gamma_G;\Gamma_L|\Delta \vdash M : \underline{B}}{\Gamma_G;\Gamma_L \vdash \text{thunk } M : U\underline{B}}\; U\text{-I}$$

3.1.3 Computation Types

$$\frac{\Gamma_G;\Gamma_L(x:A)|\Delta \vdash M : \underline{B}}{\Gamma_G;\Gamma_L|\Delta \vdash \lambda x : A.M : A \to \underline{B}} \to_c\text{-I}$$

$$\frac{\Gamma_G;\Gamma_L|\Delta \vdash M : A \to \underline{B} \qquad \Gamma_G;\Gamma_L \vdash N : A}{\Gamma_G;\Gamma_L|\Delta \vdash MN : \underline{B}} \to_c\text{-E}$$

check bunch

$$\frac{\Gamma_G;\Gamma_L * x : A|\Delta \vdash M : \underline{B}}{\Gamma_G;\Gamma_L|\Delta \vdash \alpha x : A.M : A \mathrel{-\!\!*} \underline{B}} \mathrel{-\!\!*}\text{-I}$$

$$\frac{\Gamma_G;\Gamma_L|\Delta \vdash_M : A \mathrel{-\!\!*} \underline{B} \qquad \Gamma_G;\Gamma'_L \vdash N : A}{\Gamma_G;\Gamma_L * \Gamma'_L|\Delta \vdash M@N : \underline{B}} \mathrel{-\!\!*}\text{-E}$$

Commented rules to be converted.