# 1   Object Language

## 1.1   Raw Terms

The untyped syntax for CBPV OSum, add separating connectives, remove error.
Add $\forall \underline{X}.\underline{B}$ ?

| | | | |
|---|---|---|---|
| Value Types | $A$ | ::= | $X$ |
| | | $\mid$ | Unit |
| | | $\mid$ | Case $A$ |
| | | $\mid$ | OSum |
| | | $\mid$ | $A \times A$ |
| | | $\mid$ | $A * A$ |
| | | $\mid$ | $\exists X.A$ |
| | | $\mid$ | $U\underline{B}$ |
| Computation Types | $\underline{B}$ | ::= | $A \rightarrow \underline{B}$ |
| | | $\mid$ | $A \ast\!\!\rightarrow \underline{B}$ |
| | | $\mid$ | $\forall X.\underline{B}$ |
| | | $\mid$ | $FA$ |
| Values | $V$ | ::= | $x$ |
| | | $\mid$ | $tt$ |
| | | $\mid$ | $\sigma$ |
| | | $\mid$ | $\mathrm{inj}_V V$ |
| | | $\mid$ | $(V, V)$ |
| | | $\mid$ | $(V * V)$ |
| | | $\mid$ | pack $(A, V)$ as $\exists X.A$ |
| | | $\mid$ | thunk $M$ |
| Computations | $M$ | ::= | $\lambda x \colon A.M$ |
| | | $\mid$ | $MV$ |
| | | $\mid$ | $\alpha x \colon A.M$ |
| | | $\mid$ | $M@V$ |
| | | $\mid$ | $\Lambda X.M$ |
| | | $\mid$ | $M[A]$ |
| | | $\mid$ | ret $V$ |
| | | $\mid$ | $x \leftarrow M; N$ |
| | | $\mid$ | force $V$ |
| | | $\mid$ | $\mathrm{newcase}_A x; M$ |
| | | $\mid$ | match V with V $\{$ inj $x.M \| N \}$ |
| | | $\mid$ | let $(x, x) = V; M$ |
| | | $\mid$ | let $(x * x) = V; M$ |
| | | $\mid$ | unpack $(X, x) = V; M$ |
| Value Context | $\Gamma$ | ::= | $\cdot$ |
| | | $\mid$ | $\Gamma, x \colon A$ |
| | | $\mid$ | $\Gamma * x \colon A$ |
| Type Context | $\Xi$ | ::= | $\cdot$ |
| | | $\mid$ | $\Xi, X$ |

## 1.2 Typed Terms

$$\overline{\Xi; \Gamma, x : A \vdash_v x : A}$$

$$\overline{\Xi; \Gamma * x : A \vdash_v x : A}$$

$$\overline{\Xi; \Gamma \vdash_v \text{tt} : \text{Unit}}$$

$$\frac{\Xi; \Gamma \vdash_v \sigma : \text{Case} A \qquad \Xi; \Gamma \vdash_v V : A}{\Xi; \Gamma \vdash_v \text{inj}_\sigma V : \text{OSum}}$$

$$\frac{\Xi; \Gamma \vdash_v V_1 : A_1 \qquad \Xi; \Gamma \vdash_v V_2 : A_2}{\Xi; \Gamma \vdash_v (V_1, V_2) : A_1 \times A_2}$$

$$\frac{\Xi; \Gamma_1 \vdash_v V_1 : A_1 \qquad \Xi; \Gamma_2 \vdash_v V_2 : A_2}{\Xi; \Gamma_1 * \Gamma_2 \vdash_v (V_1 * V_2) : A_1 * A_2}$$

$$\frac{\Xi; \Gamma \vdash_v V : A[A'/X]}{\Xi; \Gamma \vdash_v \text{pack}(A', V) \text{ as } \exists X.A : \exists X.A}$$

$$\frac{\Xi; \Gamma \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_v \text{thunk } M : U\underline{B}}$$

$$\frac{\Xi; \Gamma, x : A \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_c \lambda x : A.M : A \to \underline{B}}$$

$$\frac{\Xi; \Gamma \vdash_c M : A \to \underline{B} \qquad \Xi; \Gamma \vdash_v N : A}{\Xi; \Gamma \vdash_c MN : \underline{B}}$$

$$\frac{\Xi; \Gamma * x : A \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_c \alpha x : A.M : A \multimap \underline{B}}$$

$$\frac{\Xi; \Gamma_1 \vdash_c M : A \multimap \underline{B} \qquad \Xi; \Gamma_2 \vdash_v N : A}{\Xi; \Gamma_1 * \Gamma_2 \vdash_c M@N : \underline{B}}$$

$$\frac{\Xi, X; \Gamma \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_c \Lambda X.M : \forall X.\underline{B}}$$

$$\frac{\Xi; \Gamma \vdash_c: M : \forall X.\underline{B} \qquad \Xi \vdash A}{\Xi; \Gamma \vdash_c M[A] : \underline{B}[A/X]}$$

$$\frac{\Xi; \Gamma \vdash_v V : A}{\Xi; \Gamma \vdash_c \text{ret } V : FA}$$

$$\frac{\Xi; \Gamma \vdash_c M : FA \qquad \Xi; \Gamma, x : A \vdash_c N : \underline{B}}{\Xi; \Gamma \vdash_c x \leftarrow M; N : \underline{B}}$$

$$\frac{\Xi; \Gamma \vdash_v V : U\underline{B}}{\Xi; \Gamma \vdash_c: \text{force } V : \underline{B}}$$

$$\frac{\Xi; \Gamma * (\sigma : \text{Case}A) \vdash_c M : \underline{B} \qquad \Xi \vdash A}{\Xi; \Gamma \vdash_c \text{newcase}_A x; M : \underline{B}}$$

$$\frac{\Xi; \Gamma \vdash_v V : \text{OSum} \qquad \Xi; \Gamma \vdash_v \sigma : \text{Case } A \qquad \Xi; \Gamma, x : A \vdash M : \underline{B} \qquad \Xi; \Gamma \vdash_c N : \underline{B}}{\Xi; \Gamma \vdash_c \text{match } V \text{ with } \sigma\{ \text{ inj } x.M \| N \} : \underline{B}}$$

$$\frac{\Xi; \Gamma \vdash_v V : A_1 \times A_2 \qquad \Xi; \Gamma, x : A_1, y : A_2 \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_c \text{let } (x, y) = V; M : \underline{B}}$$

$$\frac{\Xi; \Gamma \vdash_v V : A_1 * A_2 \qquad \Xi; \Gamma * x : A_1 * y : A_2 \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_c \text{let } (x * y) = V; M : \underline{B}}$$

$$\frac{\Xi \vdash \underline{B} \qquad \Xi; \Gamma \vdash_v V : \exists X.A \qquad \Xi, X; \Gamma, x : A \vdash_c M : \underline{B}}{\Xi; \Gamma \vdash_c: \text{unpack}(X, x) = V; M : \underline{B}}$$

# 2  Logic

## 2.1  Judgments

The relation environment, $\Theta$ in PE logic contains both value and computation relations. How does this work in the semantics when value relations are denoted as objects of $Sub_{\mathcal{V}}(A \times B)$ for $Rel_{\mathcal{V}}[A, B]$ and computation relations are denoted as objects of $Sub_{\mathcal{C}}(\underline{A} \times \underline{B})$ for $Rel_{\mathcal{V}}[\underline{A}, \underline{B}]$? Maybe we have separate relation environments?

$$\Xi; \Gamma; \Theta \vdash \phi \text{ VProp}$$
$$\Xi; \Gamma; \Theta \vdash (x : A, y : B).\phi : Rel_{\mathcal{V}}[A, B]$$
$$\Xi; \Gamma; \Theta | \Phi \vdash \phi$$
$$\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\phi} \text{ CProp}$$
$$\Xi; \Gamma; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}).\underline{\phi} : Rel_{\mathcal{C}}[\underline{A}, \underline{B}]$$
$$\Xi; \Gamma; \Delta; \Theta; \Omega | \Phi; \underline{\Psi} \vdash \underline{\phi}$$

for type environment $\Xi$, term environment $\Gamma$, stoup $\Delta$, value relation environment $\Theta$, computation relation environment $\Omega$, value proposition environment $\Phi$, and computation proposition environment $\underline{\Psi}$.

## 2.2 Formation Rules

### 2.2.1 Value Propositions

$$\phi := \top \mid \phi \wedge \phi \mid t =_A u \mid R(t,u)$$

$$\frac{}{\Xi; \Gamma; \Theta \vdash \top \text{ VProp}}$$

$$\frac{\Xi; \Gamma; \Theta \vdash \phi \text{ VProp} \qquad \Xi; \Gamma; \Theta \vdash \psi \text{ VProp}}{\Xi; \Gamma; \Theta \vdash \phi \wedge \psi \text{ VProp}}$$

$$\frac{\Xi; \Gamma \vdash_v t : A \qquad \Xi; \Gamma \vdash_v u : A}{\Xi; \Gamma; \Theta \vdash t =_A u \text{ VProp}}$$

$$\frac{\Xi; \Gamma \vdash_v t : A \qquad \Xi; \Gamma \vdash_v u : B \qquad R : Rel_{\mathcal{V}}[A, B] \in \Theta}{\Xi; \Gamma; \Theta \vdash R(t,u) \text{ VProp}}$$

### 2.2.2 Computation Propositions

$$\underline{\psi} := \underline{\top} \mid \underline{\psi} \wedge \underline{\psi} \mid t =_{\underline{B}} u \mid \phi \implies \underline{\psi} \mid \underline{R}(t,u) \mid \forall (x : A).\underline{\psi} \mid \forall X.\underline{\psi} \mid \forall \underline{X}.\underline{\psi} \mid \forall (R : Rel_{\mathcal{V}}[A, B]).\underline{\psi} \mid \forall (R : Rel_{\mathcal{C}}[\underline{A}, \underline{B}]).\underline{\psi}$$

$$\frac{}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\top} \text{ CProp}}$$

$$\frac{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\phi} \text{ CProp} \qquad \Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\psi} \text{ CProp}}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\phi} \wedge \underline{\psi} \text{ CProp}}$$

$$\frac{\Xi; \Gamma \vdash_c t : \underline{B} \qquad \Xi; \Gamma \vdash_c u : \underline{B}}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash t =_{\underline{B}} u \text{ CProp}}$$

$$\frac{\Xi; \Gamma; \Theta \vdash \phi \text{ VProp} \qquad \Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\psi} \text{ CProp}}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \phi \implies \underline{\psi} \text{ CProp}}$$

$$\frac{\Xi; \Gamma \vdash_c t : \underline{A} \qquad \Xi; \Gamma \vdash_c u : \underline{B} \qquad \underline{R} : Rel_{\mathcal{C}}[\underline{A}, \underline{B}] \in \Omega}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \underline{R}(t,u) \text{ CProp}}$$

$$\frac{\Xi; \Gamma, x : A; \Delta; \Theta; \Omega \vdash \underline{\psi} \text{ CProp}}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \forall (x : A).\underline{\psi} \text{ CProp}}$$

$$\frac{\Xi, X; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\psi} \text{ CProp}}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \forall X.\underline{\psi} \text{ CProp}}$$

$$\frac{\Xi, \underline{X}; \Gamma; \Delta; \Theta; \Omega \vdash \underline{\psi} \text{ CProp}}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \forall \underline{X}.\underline{\psi} \text{ CProp}}$$

$$\frac{\Xi; \Gamma; \Delta; \Theta, R; \Omega \vdash \underline{\psi} \text{ CProp}}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \forall R.\underline{\psi} \text{ CProp}}$$

$$\frac{\Xi; \Gamma; \Delta; \Theta; \Omega, \underline{R} \vdash \underline{\psi} \text{ CProp}}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash \forall \underline{R}.\underline{\psi} \text{ CProp}}$$

### 2.2.3   Value Relations

$$\frac{\Xi; \Gamma, x : A \vdash_v t : C \qquad \Xi; \Gamma, y : B \vdash_v u : C}{\Xi; \Gamma; \Theta \vdash (x : A, y : B).t =_C u : Rel_{\mathcal{V}}[A, B]}$$

$$\frac{\Xi; \Gamma, x : A \vdash_v t : C \qquad \Xi; \Gamma, y : B \vdash_v u : D}{\Xi; \Gamma; \Theta, R : Rel_{\mathcal{V}}[C, D] \vdash (x : A, y : B).R(t, u) : Rel_{\mathcal{V}}[A, B]}$$

### 2.2.4   Computation Relations

Something seems off including the stoup, $\Delta$, in the computation relation judgment..

$$\frac{\Xi; \Gamma | x : \underline{A} \vdash_c t : \underline{C} \qquad \Xi; \Gamma | y : \underline{B} \vdash_c u : \underline{C}}{\Xi; \Gamma; \Delta; \Theta; \Omega \vdash (x : \underline{A}, y : \underline{B}).t =_{\underline{C}} u : Rel_{\mathcal{C}}[\underline{A}, \underline{B}]}$$

$$\frac{\Xi; \Gamma | x : \underline{A} \vdash_c t : \underline{C} \qquad \Xi; \Gamma | y : \underline{B} \vdash_c u : \underline{D}}{\Xi; \Gamma; \Delta; \Theta; \Omega, \underline{R} : Rel_{\mathcal{C}}[\underline{C}, \underline{D}] \vdash (x : \underline{A}, y : \underline{B}).\underline{R}(t, u) : Rel_{\mathcal{C}}[\underline{A}, \underline{B}]}$$

## 2.3   Derivation Rules

### 2.3.1   Values

$$\frac{}{\Xi; \Gamma; \Theta | \Phi \vdash \top}$$

$$\frac{\Xi; \Gamma; \Theta | \Phi \vdash \phi \qquad \Xi; \Gamma; \Theta | \Phi \vdash \psi}{\Xi; \Gamma; \Theta | \Phi \vdash \phi \wedge \psi} \text{ I-}\wedge$$

$$\frac{\Xi; \Gamma; \Theta | \Phi \vdash \phi \wedge \psi}{\Xi; \Gamma; \Theta | \Phi \vdash \phi} \text{ E1-}\wedge$$

$$\frac{\Xi; \Gamma; \Theta | \Phi \vdash \phi \wedge \psi}{\Xi; \Gamma; \Theta | \Phi \vdash \psi} \text{ E2-}\wedge$$

$$\frac{\Xi; \Gamma \vdash_v t : A}{\Xi; \Gamma; \Theta | \Phi \vdash t =_A t}$$

$$\frac{\Xi; \Gamma; \Theta | \Phi \vdash t =_A u \qquad \Xi; \Gamma; \Theta | \Phi \vdash \phi[t/x]}{\Xi; \Gamma; \Theta | \Phi \vdash \phi[u/x]}$$

Rel

$$\frac{\Xi; \Gamma; \Theta | \Phi \vdash \qquad \Xi; \Gamma; \Theta | \Phi \vdash}{\Xi; \Gamma; \Theta | \Phi \vdash}$$

### 2.3.2 Computation

$$\overline{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\top}}$$

$$\frac{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\phi} \qquad \Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\psi}}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\phi} \wedge \underline{\psi}} \text{ I-}\wedge$$

$$\frac{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\phi} \wedge \underline{\psi}}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\phi}} \text{ E1-}\wedge$$

$$\frac{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\phi} \wedge \underline{\psi}}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\psi}} \text{ E2-}\wedge$$

$$\frac{\Xi;\Gamma \vdash_c t : \underline{B}}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash t =_{\underline{B}} t}$$

$$\frac{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash t =_{\underline{B}} u \qquad \Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\phi}[t/x]}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\phi}[u/x]}$$

$$\frac{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi,\phi;\Psi \vdash \underline{\psi}}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \phi \implies \underline{\psi}} \text{ I-} \implies$$

$$\frac{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \phi \implies \underline{\psi} \qquad \Xi;\Gamma;\Theta|\Phi \vdash \phi}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\psi}} \text{ E-} \implies$$

Rel?

$$\frac{\Xi;\Gamma,x:A;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\phi}}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \forall(x:A).\underline{\phi}} \text{ I-}\forall \text{ term , FV constraint?}$$

$$\frac{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \forall(x:A).\underline{\phi} \qquad \Xi;\Gamma \vdash_v t : A}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\phi}[t/x]} \text{ E-}\forall \text{ term}$$

$$\frac{\Xi,X;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\phi}}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \forall X.\underline{\phi}} \text{ I-}\forall \text{ vtype , FV constraint?}$$

$$\frac{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \forall X.\underline{\phi} \qquad \Xi \vdash A}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\phi}[A/X]} \text{ E-}\forall \text{ vtype}$$

$$\frac{\Xi,\underline{X};\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\phi}}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \forall\underline{X}.\underline{\phi}} \text{ I-}\forall \text{ ctype , FV constraint?}$$

$$\frac{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \forall\underline{X}.\phi \qquad \Xi \vdash \underline{A}}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \underline{\phi[\underline{A}/\underline{X}]}} \text{ E-}\forall \text{ ctype}$$

$$\frac{\Xi;\Gamma;\Delta;\Theta,(R:Rel_{\mathcal{V}}[A,B]);\Omega|\Phi;\Psi \vdash \phi}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \forall(R:Rel_{\mathcal{V}}[A,B]).\phi} \text{ I-}\forall \text{ vrel}$$

$$\frac{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \forall(R:Rel_{\mathcal{V}}[A,B]).\underline{\phi} \qquad \Xi;\Gamma;\Theta,\vdash (x:A,y:B).\psi:Rel_{\mathcal{V}}[A,B]}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \forall(R:Rel_{\mathcal{V}}[A,B]).\underline{\phi}[\psi[t/x,u/y]/R(t,u)]} \text{ E-}\forall \text{ vrel}$$

$$\frac{\Xi;\Gamma;\Delta;\Theta,(\underline{R}:Rel_{\mathcal{C}}[\underline{A},\underline{B}]);\Omega|\Phi;\Psi \vdash \phi}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \forall(\underline{R}:Rel_{\mathcal{C}}[\underline{A},\underline{B}]).\phi} \text{ I-}\forall \text{ crel}$$

$$\frac{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \forall(R:Rel_{\mathcal{C}}[\underline{A},\underline{B}]).\phi \qquad \Xi;\Gamma;\Delta;\Theta;\Omega,\vdash (x:\underline{A},y:\underline{B}).\psi:Rel_{\mathcal{C}}[\underline{A},\underline{B}]}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \forall(R:Rel_{\mathcal{C}}[\underline{A},\underline{B}]).\underline{\phi}[\underline{\psi}[t/x,u/y]/R(t,u)]} \text{ E-}\forall \text{ crel}$$

### 2.3.3 Congruences

$$\frac{\Xi;\Gamma \vdash_c t:\underline{B} \qquad \Xi;\Gamma \vdash_c u:\underline{B} \qquad \Xi;\Gamma,x:A;\Delta;\Theta;\Omega|\Phi;\Psi \vdash t=u}{\Xi;\Gamma;\Delta;\Theta;\Omega|\Phi;\Psi \vdash \lambda(x:A).t=\lambda(x:A).u} \text{ cong-}\lambda$$

## 2.4 Axioms

Beta/Eta/(parametricity schema?)

## 2.5 Logical Interpretation of Types

Let $\boldsymbol{X}$ and $\underline{\boldsymbol{X}}$ be vectors of value type and computation type variables of length $n$. Let $\boldsymbol{\rho}$ be a vector of value relations $\Xi;\Gamma;\Theta \vdash \rho_i:Rel_{\mathcal{V}}[C_i,C_i']$ for all $i \in 1..n$. Let $\underline{\boldsymbol{\rho}}$ be a vector of computation relations $\Xi;\Gamma;\Delta;\Theta;\Omega \vdash \underline{\rho_i}:Rel_{\mathcal{C}}[\underline{C_i},\underline{C_i'}]$ for all $i \in 1..n$.
Let $A$ be a **value type** with $FTV(A) \in \{\boldsymbol{X},\underline{\boldsymbol{X}}\}$. Define:

$$A[\boldsymbol{\rho}/\boldsymbol{X},\underline{\boldsymbol{\rho}}/\underline{\boldsymbol{X}}]:Rel_{\mathcal{V}}[A[\boldsymbol{C}/\boldsymbol{X},\underline{\boldsymbol{C}}/\underline{\boldsymbol{X}}],A[\boldsymbol{C'}/\boldsymbol{X},\underline{\boldsymbol{C'}}/\underline{\boldsymbol{X}}]]$$

by induction on $A$.

$$X_i[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}] = \rho_i$$

$$\text{Unit}[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}] = (x : Unit, y : Unit).x =_{Unit} y$$

$$\text{Case}A[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}] = (x : \text{Case } (A[\boldsymbol{C}/\boldsymbol{X}, \underline{\boldsymbol{C}}/\underline{\boldsymbol{X}}]), y : \text{Case } (A[\boldsymbol{C'}/\boldsymbol{X}, \underline{\boldsymbol{C'}}/\underline{\boldsymbol{X}}])).$$
$$\text{think exists?}$$

$$\text{OSum}[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}] = \text{think exists?}$$

$$A \times A'[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}] = ((x, y) : A \times A'[\boldsymbol{C}/\boldsymbol{X}, \underline{\boldsymbol{C}}/\underline{\boldsymbol{X}}], (x', y') : A \times A'[\boldsymbol{C'}/\boldsymbol{X}, \underline{\boldsymbol{C'}}/\underline{\boldsymbol{X}}]).$$
$$A[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}](x, x') \wedge A'[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}](y, y')$$

$$A * A'[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}] = \text{similar to product?}$$

$$\exists X.A[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}] = \text{standard}$$

$$U\underline{B}[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}] = \text{related thunks?}$$

Let $\underline{B}$ be a **computation type** with $FTV(\underline{B}) \in \{\boldsymbol{X}, \underline{\boldsymbol{X}}\}$. Define:

$$\underline{B}[\boldsymbol{\rho}/\boldsymbol{X}, \underline{\boldsymbol{\rho}}/\underline{\boldsymbol{X}}] : Rel_{\mathcal{C}}[\underline{B}[\boldsymbol{C}/\boldsymbol{X}, \underline{\boldsymbol{C}}/\underline{\boldsymbol{X}}], \underline{B}[\boldsymbol{C'}/\boldsymbol{X}, \underline{\boldsymbol{C'}}/\underline{\boldsymbol{X}}]]$$

by induction on $\underline{B}$.

$$A \to \underline{B}[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}] = \text{Add weakest precondition to the proposition syntax to interpret this?}$$

$$A \twoheadrightarrow \underline{B}[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}] =$$

$$\forall X.\underline{B}[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}] =$$

$$FA[\boldsymbol{\rho}, \underline{\boldsymbol{\rho}}] = \text{Could be encoded if we add } \forall \underline{X}.\underline{B}?$$

# 3   Goal

Try to prove

$$\vdash_d \forall A \ \underline{B}.$$
$$\quad \forall(y : \text{OSum}).$$
$$\quad \forall(f : U(\forall X.\text{Case } X \twoheadrightarrow \text{OSum} \to FX)).$$
$$\quad \forall(k \ k' : U(A \to \underline{B})).$$
$$\quad \text{newcase}_A \sigma; x \leftarrow (!f)[A]\sigma y; (!k)x$$
$$\quad \quad =$$
$$\quad \text{newcase}_A \sigma; x \leftarrow (!f)[A]\sigma y; (!k')x$$