1 Object Language

A simply typed CBPV language.

1.1 Raw Terms

Value Types	A	$::=$	Unit
		$\mid$	$A \times A$
		$\mid$	$U\underline{B}$
Computation Types	$\underline{B}$	$::=$	$A \to \underline{B}$
		$\mid$	FA
Values	V	$::=$	x
		$\mid$	tt
		$\mid$	(V, V)
		$\mid$	thunk M
Computations	M	$::=$	$\lambda x \colon A.M$
		$\mid$	MV
		$\mid$	ret V
		$\mid$	force V
		$\mid$	$x \leftarrow M; M'$
		$\mid$	let $x = V; M'$
		$\mid$	let $(x, y) = V; M$
Value Context	Γ	$::=$	$\cdot$
		$\mid$	$\Gamma, x \colon A$

1.2 Typed Terms

$$\frac{}{\Gamma, x \colon A \vdash_v x : A}\text{Var}$$

$$\frac{}{\Gamma \vdash_v tt : \text{Unit}}\text{I-Unit}$$

$$\frac{\Gamma \vdash_v t : A \qquad \Gamma \vdash_v t : A'}{\Gamma \vdash_v (t, u) : A \times A'}\text{I-}\times$$

$$\frac{\Gamma \vdash_c M : \underline{B}}{\Gamma \vdash_v \text{thunk } M : U\underline{B}}$$

$$\frac{\Gamma, x : A \vdash_c M : \underline{B}}{\Gamma \vdash_c (\lambda(x : A).M) : A \to \underline{B}}\text{I - }\to$$

$$\frac{\Gamma \vdash_c M : A \to \underline{B} \qquad \Gamma \vdash_v V : A}{\Gamma \vdash_c : MV : \underline{B}}\text{E-}\to$$

$$\frac{\Gamma \vdash_v V : A}{\Gamma \vdash_c \text{ret } V : FA}$$

$$\frac{\Gamma \vdash_v V : U\underline{B}}{\Gamma \vdash_c \text{force } V : \underline{B}}$$

$$\frac{\Gamma \vdash_c M : FA \qquad \Gamma, x : A \vdash_c N : \underline{B}}{\Gamma \vdash_c x \leftarrow M; N : \underline{B}}$$

$$\frac{\Gamma \vdash_v V : A \qquad \Gamma, x : A \vdash_c N : \underline{B}}{\Gamma \vdash_c \text{let x } = V; N : \underline{B}}$$

$$\frac{\Gamma \vdash_v V : A \times A' \qquad \Gamma, x : A, y : A' \vdash_c M : \underline{B}}{\Gamma \vdash_c \text{let (x,y) } = V; M : \underline{B}} \text{E-}\times$$

2 Simple Logic

The usual presentation of a logic except that we make a distinction between value and

2.1 Formation Rules

2.1.1 Value Fragment

Judgments: Value Propositions are over a value context.

$$\Gamma \vdash \phi \text{ VProp}$$

Connectives[1] [2]:

$$\phi := \top | \phi \wedge \phi | \phi \implies \phi$$

$$\frac{}{\Gamma \vdash \top \text{ VProp}}$$

$$\frac{\Gamma \vdash \phi \text{ VProp} \qquad \Gamma \vdash \psi \text{ VProp}}{\Gamma \vdash \phi \wedge \psi \text{ VProp}}$$

$$\frac{\Gamma \vdash \phi \text{ VProp} \qquad \Gamma \vdash \psi \text{ VProp}}{\Gamma \vdash \phi \implies \psi \text{ VProp}}$$

2.1.2 Computation Fragment

Judgments: Computation Propositions are over a value context and a stoup.

$$\Gamma; \Delta \vdash \underline{\phi} \text{ CProp}$$

In our intended semantics, this judgment should be interpreted as a subalgebra of $[\![\Gamma; \Delta]\!]$. We need some way of combining the value object $[\![\Gamma]\!]$ with computation object $[\![\Delta]\!]$. Maybe we can use some Γ fold copower[3] of Δ, $[\![\Gamma]\!] \oslash [\![\Delta]\!]$,

[1] Focusing on just a few connectives for the moment
[2] should there be a logical equivalent to $U\underline{B}$ ($U\underline{\phi}$) here?
[3] This copower connective is definable in PE

which is a connective in the Enriched Effect Calculus?
Connectives: The purpose of this document is to explore this new kind of connective, the computational implication. [4]

$$\underline{\phi} := \underline{\top} | \underline{\phi} \wedge \underline{\psi} | \phi \implies \underline{\psi}$$

$$\frac{}{\Gamma; \Delta \vdash \underline{\top} \text{ CProp}}$$

$$\frac{\Gamma; \Delta \vdash \underline{\phi} \text{ CProp} \qquad \Gamma; \Delta \vdash \underline{\psi} \text{ CProp}}{\Gamma; \Delta \vdash \underline{\phi} \wedge \underline{\psi} \text{ CProp}}$$

The computation implication formation rule, similar to the computation function type in CBPV, takes both a value proposition and a computation proposition.

$$\frac{\Gamma \vdash \phi \text{ VProp} \qquad \Gamma; \Delta \vdash \underline{\psi} \text{ CProp}}{\Gamma; \Delta \vdash \phi \implies \underline{\psi} \text{ CProp}}$$

2.2 Derivation Rules

2.2.1 Value Derivations

Value Derivation Judgement

$$\Gamma | \Phi \vdash \phi$$

where Φ is a conjunction of value propositions.

$$\frac{}{\Gamma | \Phi \vdash \top} \text{ I-}\top$$

$$\frac{\Gamma | \Phi \vdash \phi \qquad \Gamma | \Phi \vdash \psi}{\Gamma | \Phi \vdash \phi \wedge \psi} \text{ I-}\wedge$$

$$\frac{\Gamma | \Phi \vdash \phi \wedge \psi}{\Gamma | \Phi \vdash \phi} \text{ E1-}\wedge$$

$$\frac{\Gamma | \Phi \vdash \phi \wedge \psi}{\Gamma | \Phi \vdash \psi} \text{ E2-}\wedge$$

$$\frac{\Gamma | \Phi, \phi \vdash \psi}{\Gamma | \Phi \vdash \phi \implies \psi} \text{ I-} \implies$$

$$\frac{\Gamma | \Phi \vdash \phi \implies \psi \qquad \Gamma | \Phi \vdash \phi}{\Gamma | \Phi \vdash \psi} \text{ E-} \implies$$

[4] Need weakest precondition as part of the syntax for the logical interpretation of $A \to \underline{B}$?

2.2.2 Computation Derivations

Computation Derivation Judgment:

$$\Gamma; \Delta | \Phi; \underline{\Psi} \vdash \underline{\phi}$$

If computation derivations are allowed a context of computation propositions, we'll need a way to combine subobjects of the value category with subobjects of the computation category[5].

$$\oslash^*_{\Gamma;\Delta} : Sub_V(\llbracket \Gamma \rrbracket) \times Sub_C(\llbracket \Gamma \rrbracket \oslash \llbracket \Delta \rrbracket) \to Sub_C(\llbracket \Gamma \rrbracket \oslash \llbracket \Delta \rrbracket)$$
$$\llbracket \Gamma; \Delta | \Phi; \underline{\Psi} \vdash \underline{\phi} \rrbracket : Sub_C(\Gamma \oslash \Delta)[\oslash^*(\Phi, \underline{\Psi}), \underline{\phi}]$$

What is this operation? (should it be adjoint to computation implication?)

$$\frac{}{\Gamma; \Delta | \Phi; \underline{\Psi} \vdash \underline{\top}} \text{ I-}\top$$

$$\frac{\Gamma; \Delta | \Phi; \underline{\Psi} \vdash \underline{\phi} \qquad \Gamma; \Delta | \Phi; \underline{\Psi} \vdash \underline{\psi}}{\Gamma; \Delta | \Phi; \underline{\Psi} \vdash \underline{\phi} \wedge \underline{\psi}} \text{ I-}\wedge$$

$$\frac{\Gamma; \Delta | \Phi; \underline{\Psi} \vdash \underline{\phi} \wedge \underline{\psi}}{\Gamma; \Delta | \Phi; \underline{\Psi} \vdash \underline{\phi}} \text{ E1-}\wedge$$

$$\frac{\Gamma; \Delta | \Phi; \underline{\Psi} \vdash \underline{\phi} \wedge \underline{\psi}}{\Gamma; \Delta | \Phi; \underline{\Psi} \vdash \underline{\psi}} \text{ E2-}\wedge$$

Not clear what the semantics for this connective should be. Normally, the implication would be the exponential object in the category of subobjects over Γ. Here we might be able to say something about adjointness with $\oslash$.

$$\frac{\Gamma; \Delta | \Phi, \phi; \underline{\Psi} \vdash \underline{\psi}}{\Gamma; \Delta | \Phi; \underline{\Psi} \vdash \phi \implies \underline{\psi}} \text{ I-} \implies$$

$$\frac{\Gamma; \Delta | \Phi; \underline{\Psi} \vdash \phi \implies \underline{\psi} \qquad \Gamma | \Phi \vdash \phi}{\Gamma; \Delta | \Phi; \underline{\Psi} \vdash \underline{\psi}} \text{ E-} \implies$$

3 Model

3.1 Object Language Model

Assume we have a typical, set based, algebra model of a simply typed CBPV language consisting of a value category $\mathcal{V} = Set$, a monad T on $\mathcal{V}$, a computation category $\mathcal{C}$ which is the category of algebras for monad T, with the usual functors F, U.

[5]Similar to how value contexts are combined with the stop

3.2 Logic Model

Starting with the interpretation of the judgments

$$
\begin{aligned}
[\![\Gamma \vdash \phi]\!] &: \text{ object of } Sub_{\mathcal{V}}([\![\Gamma]\!]) \\
[\![\Gamma|\Phi \vdash \phi]\!] &: Sub_{\mathcal{V}}[[\![\Phi]\!], [\![\phi]\!]] \\
[\![\Gamma; \Delta \vdash \underline{\phi}]\!] &: \text{ object of } Sub_{\mathcal{C}}([\![\Gamma]\!] \oslash [\![\Delta]\!]) \\
[\![\Gamma; \Delta|\Phi; \underline{\Psi} \vdash \underline{\phi}]\!] &: Sub_{\mathcal{C}}([\![\Gamma]\!] \oslash [\![\Delta]\!])[\oslash^*_{\Gamma;\Delta}([\![\Phi]\!], [\![\underline{\Psi}]\!]), [\![\phi]\!]]
\end{aligned}
$$

What laws should we expect $\oslash$ and $\oslash^*_{\Gamma,\Delta}$ to obey?

3.2.1 Values

Denotation of the value propositions:

$$
\begin{aligned}
[\![\Gamma \vdash \top]\!] &= \mathbf{1} \\
[\![\Gamma \vdash \phi \wedge \psi]\!] &= [\![\Gamma \vdash \phi]\!] \times [\![\Gamma \vdash \psi]\!] \\
[\![\Gamma \vdash \phi \implies \psi]\!] &= [\![\Gamma \vdash \psi]\!]^{[\![\Gamma \vdash \phi]\!]}
\end{aligned}
$$

Denotation of the value derivations:
Unique map into the terminal object.

$$
\frac{}{[\![\Gamma|\Psi \vdash \top]\!] = !}
$$

3.2.2 Computations

Denotation of the computation propositions:

$$
\begin{aligned}
[\![\Gamma; \Delta \vdash \top]\!] &= \mathbf{1} \\
[\![\Gamma; \Delta \vdash \phi \wedge \psi]\!] &= [\![\Gamma; \Delta \vdash \phi]\!] \times [\![\Gamma; \Delta \vdash \psi]\!] \\
[\![\Gamma; \Delta \vdash \phi \implies \underline{\psi}]\!] &=?
\end{aligned}
$$

4 Unary Relational Interpretation

- Add predicates?
- Why do we need a unary relational interpretaiton of types? As a way to propogate predicates over types?
- Do we need to add weakest precondition as a piece of syntax to describe the unary relational interpretation of $A \to \underline{B}$?