Stuttgart Fall School in CL
Class 2: Rewrite Grammars, CFGs

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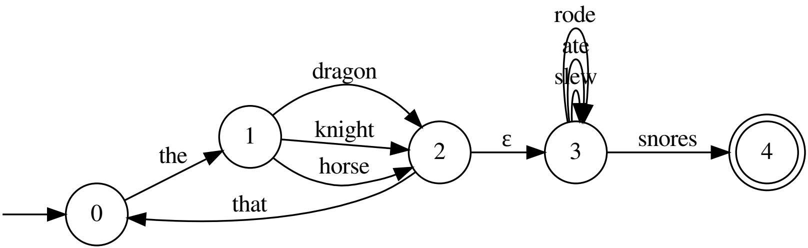
Review: Quiz

Which of these sentences is accepted by the following FSA?

1. the horse the knight rode snores
2. the dragon that the knight slew snores
3. that the dragon rode snores
4. the dragon ate
5. the horse that the knight that the dragon ate rode snores
6. the dragon rode ate slew ate snores



Rewrite Grammars

Let N be a set of *non-terminal symbols*, Σ a set of terminal symbols, and $N \cap \epsilon = \emptyset$. Choose a distinguished set of *start symbols* $S \subseteq N$.
Let $\alpha, \beta, \gamma \in (N \cup \Sigma)^*$ and $X \in N$. A *rewrite grammar* has a set of rules R of the form

$$\alpha X \beta \quad \rightarrow \quad \gamma$$

Example

0	$\rightarrow$	the 1
1	$\rightarrow$	dragon 2
1	$\rightarrow$	knight 2
1	$\rightarrow$	horse 2
2	$\rightarrow$	that 0
2	$\rightarrow$	ϵ 3
3	$\rightarrow$	rode 3
3	$\rightarrow$	ate 3
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Rewrite Grammars: Example

A *rewrite grammar* is a 4-tuple $\langle \Sigma, N, S, R \rangle$
(i.e. $\langle$terminals, nonterminals, start symbols, rules $\rangle$)

Example

$G_{\text{knight}} = \langle$\{the, that, dragon, knight, horse, that, rode, ate, slew, snores\},

$$\{0,1,2,3\},\ \{0\},\ \begin{array}{lcl} 0 & \to & \text{the } 1 \\ 1 & \to & \text{dragon } 2 \\ 1 & \to & \text{knight } 2 \\ 1 & \to & \text{horse } 2 \\ 2 & \to & \text{that } 0 \\ 2 & \to & \epsilon\ 3 \\ 3 & \to & \text{rode } 3 \\ 3 & \to & \text{ate } 3 \\ 3 & \to & \text{slew } 3 \\ 3 & \to & \text{snores} \end{array}$$

$\rangle$

The language of the grammar

A rewrite grammar G defines a language $L(G) \subseteq \Sigma^*$ as follows:

1. Take a start symbol s from S. This is your string w.
2. if $w \in \Sigma^*$ then $w \in L$.
3. If there is a substring x of w which matches the LHS of a rule, replace x with the RHS of the rule. Go back to step 2.

We say G *generates* L(G) (hence "generative grammar")

Language

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0 $\Rightarrow$ the 1 $\Rightarrow$ the knight 2 $\Rightarrow$ the knight that 0 $\Rightarrow$the knight that the 1 $\Rightarrow$ the knight that the horse 2 $\Rightarrow$ the knight that the horse 3 $\Rightarrow$ the knight that the horse snores $\in \Sigma^*$

Therefore, *the knight that the horse snores* $\in L(G_{\text{knight}})$

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Parse Trees

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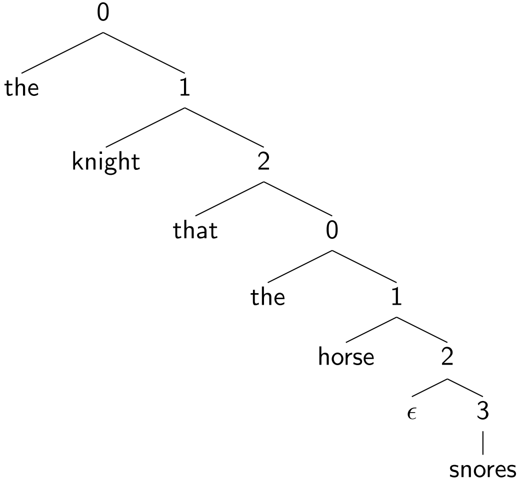
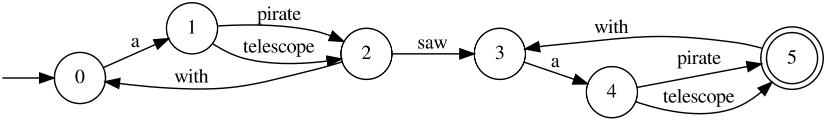


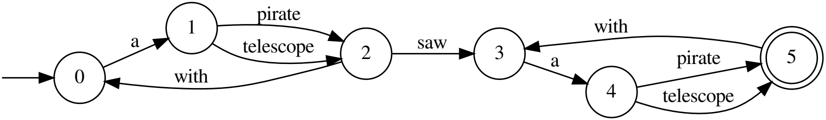
Exercise: parse trees

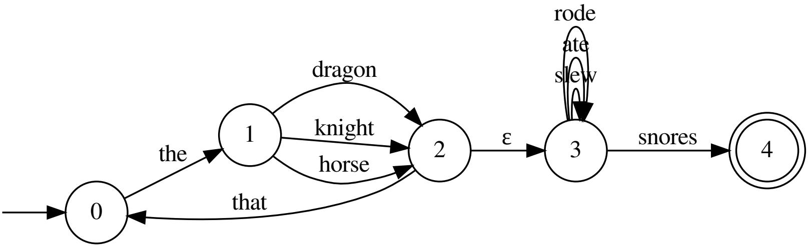
1. Rewrite G_{pirate} as a rewrite grammar
2. Draw parse trees for a few sentences in $L(G_{pirate})$.



Discussion

What are some conceptual and linguistic shortcomings of G_{pirate} and G_{knight}?





Rewrite Grammars

Recall rewrite grammar rule definition: Let $\alpha, \beta, \gamma \in (N \cup \Sigma)^*$ and $X \in N$.
A *rewrite grammar* has a set of rules R of the form

$\alpha X \beta \quad \rightarrow \quad \gamma$

But our rules all look like this:

$$\begin{array}{ccl} 1 & \rightarrow & \text{pirate } 2 \\ 5 & \rightarrow & \text{telescope} \\ \hline \text{NT}_1 & \rightarrow & \text{t } (\text{NT}_2) \end{array}$$

Even though possible rewrite rules include:

$$\begin{array}{rcl} 2\ 0 \text{ a with } 3 \text{ pirate} & \rightarrow & 5 \text{ saw with } 1 \\ 3 & \rightarrow & 1\ 2 \\ 2 \text{ telescope} & \rightarrow & \text{pirate} \end{array}$$

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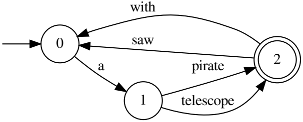
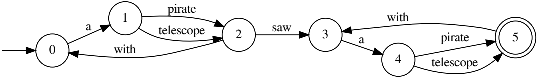
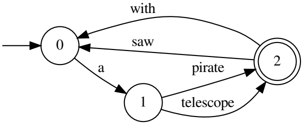
Context Free Grammars

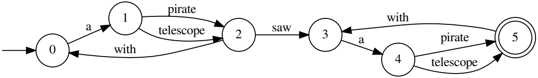




0	$\rightarrow$	a 1	DP	$\rightarrow$	D NP
3	$\rightarrow$	a 4			
			D	$\rightarrow$	a
1	$\rightarrow$	pirate 2	NP	$\rightarrow$	N (PP)
4	$\rightarrow$	pirate 5			
1	$\rightarrow$	telescope 2			
4	$\rightarrow$	telescope 5			
			N	$\rightarrow$	pirate | telescope
2	$\rightarrow$	with 0	PP	$\rightarrow$	P DP
5	$\rightarrow$	with 3			
			P	$\rightarrow$	with

Context Free Grammars



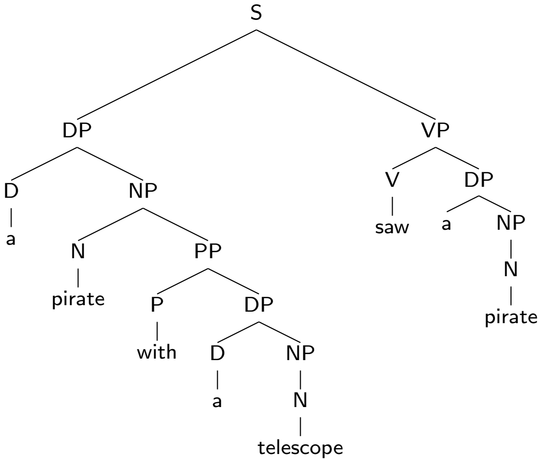


2	$\rightarrow$	saw 3	VP	$\rightarrow$	V DP
			S	$\rightarrow$	DP VP
			V	$\rightarrow$	saw

Context Free Grammars

$\mathbf{G}_{\textbf{pirate-CFG}}$

DP	$\rightarrow$	D NP
D	$\rightarrow$	a
NP	$\rightarrow$	N (PP)
N	$\rightarrow$	pirate | telescope
PP	$\rightarrow$	P DP
P	$\rightarrow$	with
VP	$\rightarrow$	V DP
S	$\rightarrow$	DP VP
V	$\rightarrow$	saw



Discussion: How is this parse tree different from the one for the same sentence using $\mathrm{G}_{\text{pirate-FSA}}$?

Chomksy Hierarchy

Recall rewrite grammar rule definition: Let $\alpha, \beta \in (N \cup \Sigma)^*$. A *rewrite grammar* has a set of rules R of the form

$$\alpha \quad \rightarrow \quad \beta$$

So far we've seen:

NT	$\rightarrow$	t (NT)	all rules	NT	$\rightarrow$	t NT
0	$\rightarrow$	a 1	OR all rules	NT	$\rightarrow$	NT t
4	$\rightarrow$	pirate				

NT	$\rightarrow$	NT NT	all rules	NT	$\rightarrow$	β
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So far we've seen:

Regular Grammar

NT	$\rightarrow$	t (NT)	all rules	NT	$\rightarrow$	t NT
0	$\rightarrow$	a 1	OR all rules	NT	$\rightarrow$	NT t
4	$\rightarrow$	pirate				

Context Free Grammar

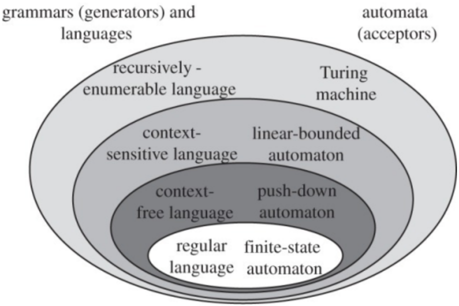
NT	$\rightarrow$	NT NT	all rules	NT	$\rightarrow$	β
S	$\rightarrow$	DP VP				
NT	$\rightarrow$	t				
N	$\rightarrow$	pirate				

Chomsky Hierarchy

Grammar	Language	Rewrite rules	Automaton
Type 0	Recursively Enumerable	$\alpha X \beta \rightarrow \gamma$	Turing machine
Type 1	Context Sensitive	$\alpha X \beta \rightarrow \alpha \gamma \beta$	Linear bounded non-deterministic Turing machine
Type 2	Context Free	$X \rightarrow \gamma$	Push-down Automaton
Type 3	Regular	$X \rightarrow t\ (X)$ or $X \rightarrow (X)\ t$	Finite State Automaton

$\alpha, \beta, \gamma \in (N \cup \Sigma)^*$, $X \in N$, $t \in \Sigma$

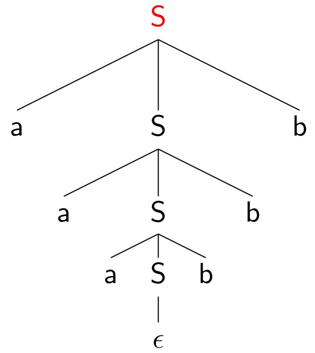
Chomsky Hierarchy



the traditional Chomsky hierarchy

Context-free grammars

aaabbb



Example: $a^n b^n$

1. S→a S b
2. S→ϵ

Question: *Can you generate $a^n b^n$ with a regular grammar?*

Weak Generative Capacity

Definition

The *weak generative capacity* of a (class of) grammar(s) is the (class of) string language(s) it can generate

Example: The language $a^n b^n$

- $a^n b^n = \{a^n b^n \mid n \in \mathbb{N}\} = \{\epsilon, ab, aabb, aaabbb, aaaabbbb, \dots\}$
- $a^n b^n$ is strictly context-free language (i.e. it's not regular).
- The weak generative capacity of context free grammars, but not of regular grammars, includes $a^n b^n$.

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Strong Generative Capacity

The *strong generative capacity* of a (class of) grammar(s) is the (class of) derivations/parses/structural descriptions of its language(s).

Exercise: Do G_1 and G_2 have the same weak and/or strong generative capacity?

G_1		
S	$\to$	A B
A	$\to$	a A
A	$\to$	ϵ
B	$\to$	b B
B	$\to$	ϵ

G_2		
S	$\to$	a S
S	$\to$	b B
B	$\to$	b B
S	$\to$	ϵ
B	$\to$	ϵ