

# C++ Level 9 Group F

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**Problem b.** The results are shown as follows.

Table 1: Approximation of Put price for Batch 1 using FDM

J	N	$P_f$	error
325	1000	-4E45	4E45
325	2000	5.842167	0.004113
325	5000	5.842093	0.004187
325	10000	5.842068	0.004212
325	100000	5.842046	0.004234
325	500000	5.842044	0.004236
650	10000	5.845248	0.001032
650	100000	5.845226	0.001054
1300	10000	65535	65529.15372
1300	1000000	5.84602	0.000260

Table 2: Approximation of Put price for Batch 2 using FDM

J	N	$P_f$	error
500	10000	7.963211	0.0023590
500	1000000	7.963121	0.0024490
1000	10000	65535	65527.03443
1000	100000	7.964963	0.0006070
1000	200000	7.964958	0.0006120

Table 3: Approximation of Put price for Batch 3 using FDM

J	N	$P_f$	error
50	1000	4.071127	0.0021330
50	10000	4.071285	0.0019750
100	1000	65535	65530.92674
100	10000	4.072733	0.0005270
100	20000	4.072742	0.0005180

Table 4: Approximation of Put price for Batch 4 using FDM

J	N	$P_f$	error
100	10000	65535	65533.7525
100	100000	1.193278	0.054222
100	200000	1.1933	0.05420
200	100000	65535	65533.7525
200	500000	1.1953	0.05220
200	600000	1.195301	0.052199

As we can see from the above experiments, the accuracy of FDM to approximate option price is dependent on both  $J$  and  $N$  and the ratio between  $N$  and  $J^2$ . More specifically, when  $J$  is set,  $N$  can not be choose too close to  $J$ . Otherwise, there is a serious oscillation of approximate option price for the given stock price, which is the approximation either extremely large or extremely small (negative). Then we extend  $N$  and once  $N$  is around  $J^2$ , the approximate option price will become a constant (65535). Then we keep increasing  $N$  to become much larger than  $J^2$ , we will finally obtain the accurate approximation. We can see that for Batch 1 to Batch 3, we can obtain at least 2 decimal places of accuracy compared to the exact solution. For Batch 4, we can obtain 1 decimal place of accuracy. We also see that under condition  $N \gg J^2$ , once  $J$  is fixed, even we keep increasing  $N$ , the accuracy will not increase stable around a certain point. What's more, if we increase  $J$  and set  $N \gg J^2$ , we will obtain more accurate approximation which will converge to the exact solution.