Principles of AI Planning

4. Normal forms

Bernhard Nebel and Robert Mattmüller

Albert-Ludwigs-Universität Freiburg

November 4th, 2011

Motivation

Similarly to normal forms in propositional logic (DNF, CNF, NNF, . . .) we can define normal forms for effects, operators and planning tasks.

This is useful because algorithms (and proofs) then only need to deal with effects (resp. operators or tasks) in normal form.

Effect normal form

Equivalence of operators and effects

Definition (equivalent effects)

Two effects e and e' over state variables A are equivalent, written $e \equiv e'$, if for all states s over A, $[e]_s = [e']_s$.

Definition (equivalent operators)

Two operators o and o' over state variables A are equivalent, written $o \equiv o'$, if they are applicable in the same states, and for all states s where they are applicable, $\mathit{app}_o(s) = \mathit{app}_{o'}(s)$.

Theorem

Let $o = \langle \chi, e \rangle$ and $o' = \langle \chi', e' \rangle$ be operators with $\chi \equiv \chi'$ and $e \equiv e'$. Then $o \equiv o'$.

Note: The converse is not true. (Why not?)

Equivalence transformations for effects

$$e_1 \wedge e_2 \equiv e_2 \wedge e_1 \tag{1}$$

$$(e_1 \wedge e_2) \wedge e_3 \equiv e_1 \wedge (e_2 \wedge e_3) \tag{2}$$

$$\top \wedge e \equiv e \tag{3}$$

$$\chi \rhd e \equiv \chi' \rhd e \qquad \text{if } \chi \equiv \chi' \tag{4}$$

$$\top \rhd e \equiv e \tag{5}$$

$$\bot \rhd e \equiv \top \tag{6}$$

$$\chi_1 \rhd (\chi_2 \rhd e) \equiv (\chi_1 \wedge \chi_2) \rhd e \tag{7}$$

$$\chi \rhd (e_1 \wedge \cdots \wedge e_n) \equiv (\chi \rhd e_1) \wedge \cdots \wedge (\chi \rhd e_n) \tag{8}$$

$$(\chi_1 \rhd e) \wedge (\chi_2 \rhd e) \equiv (\chi_1 \vee \chi_2) \rhd e \tag{9}$$

Normal form for effects

We can define a normal form for effects:

- Nesting of conditionals, as in $a \triangleright (b \triangleright c)$, can be eliminated.
- Effects e within a conditional effect $\varphi \triangleright e$ can be restricted to atomic effects (a or $\neg a$).

Transformation to this effect normal form only gives a small polynomial size increase.

Compare: transformation to CNF or DNF may increase formula size exponentially.

Normal form for operators and effects

Definition

An operator $\langle \chi, e \rangle$ is in effect normal form (ENF) if for all occurrences of $\chi' \rhd e'$ in e the effect e' is either a or $\neg a$ for some $a \in A$, and there is at most one occurrence of any atomic effect in e.

Theorem

For every operator there is an equivalent one in effect normal form.

Proof is constructive: we can transform any operator into effect normal form using the equivalence transformations for effects.

Effect normal form example

Example

$$\begin{aligned}(a \rhd (b \wedge\\ &(c \rhd (\neg d \wedge e)))) \wedge\\ (\neg b \rhd e)\end{aligned}$$

transformed to effect normal form is

$$\begin{aligned}(a \rhd b) \wedge\\ ((a \wedge c) \rhd \neg d) \wedge\\ ((\neg b \vee (a \wedge c)) \rhd e)\end{aligned}$$

Positive normal form

Example: Freecell

Example (good and bad effects)

If we move a card c to a free tableau position, the good effect is that the card formerly below c is now available.
The bad effect is that we lose one free tableau position.

What is a good or bad effect?

Question: Which operator effects are good, and which are bad?

Difficult to answer in general, because it depends on context:

- Locking the entrance door is good if we want to keep burglars out.
- Locking the entrance door is bad if we want to enter.

We will now consider a reformulation of planning tasks that makes the distinction between good and bad effects obvious.

Positive normal form

Definition (operators in positive normal form)

An operator $o = \langle \chi, e \rangle$ is in positive normal form if it is in effect normal form, no negation symbols appear in χ, and no negation symbols appear in any effect condition in e.

Definition (planning tasks in positive normal form)

A planning task $\langle A, I, O, \gamma \rangle$ is in positive normal form if all operators in O are in positive normal form and no negation symbols occur in the goal γ.

Positive normal form: existence

Theorem (positive normal form)

Every planning task Π has an equivalent planning task Π' in positive normal form.
Moreover, Π' can be computed from Π in polynomial time.

Note: Equivalence here means that the represented transition systems of Π and Π', limited to the states that can be reached from the initial state, are isomorphic.

We prove the theorem by describing a suitable algorithm. (However, we do not prove its correctness or complexity.)

Positive normal form: algorithm

Transformation of $\langle A, I, O, \gamma\rangle$ to positive normal form

```
Convert all operators $o \in O$ to effect normal form.
Convert all conditions to negation normal form (NNF).
while any condition contains a negative literal $\neg a$:
    Let $a$ be a variable which occurs negatively in a condition.
    $A := A \cup \{\hat{a}\}$ for some new state variable $\hat{a}$
    $I(\hat{a}) := 1 - I(a)$
    Replace the effect $a$ by $(a \wedge \neg\hat{a})$ in all operators $o \in O$.
    Replace the effect $\neg a$ by $(\neg a \wedge \hat{a})$ in all operators $o \in O$.
    Replace $\neg a$ by $\hat{a}$ in all conditions.
Convert all operators $o \in O$ to effect normal form (again).
```

Here, *all conditions* refers to all operator preconditions, operator effect conditions and the goal.

Positive normal form: example

Example (transformation to positive normal form)

$$
\begin{aligned}
A &= \{\textit{home}, \textit{uni}, \textit{lecture}, \textit{bike}, \textit{bike-locked}\} \\
I &= \{\textit{home} \mapsto 1, \textit{bike} \mapsto 1, \textit{bike-locked} \mapsto 1, \\
&\qquad \textit{uni} \mapsto 0, \textit{lecture} \mapsto 0\} \\
O &= \{\langle \textit{home} \land \textit{bike} \land \neg\textit{bike-locked}, \neg\textit{home} \land \textit{uni}\rangle, \\
&\qquad \langle \textit{bike} \land \textit{bike-locked}, \neg\textit{bike-locked}\rangle, \\
&\qquad \langle \textit{bike} \land \neg\textit{bike-locked}, \textit{bike-locked}\rangle, \\
&\qquad \langle \textit{uni}, \textit{lecture} \land ((\textit{bike} \land \neg\textit{bike-locked}) \rhd \neg\textit{bike})\rangle\} \\
\gamma &= \textit{lecture} \land \textit{bike}
\end{aligned}
$$

Positive normal form: example

Example (transformation to positive normal form)

$$
\begin{aligned}
A &= \{\textit{home}, \textit{uni}, \textit{lecture}, \textit{bike}, \textit{bike-locked}\} \\
I &= \{\textit{home} \mapsto 1, \textit{bike} \mapsto 1, \textit{bike-locked} \mapsto 1, \\
&\qquad \textit{uni} \mapsto 0, \textit{lecture} \mapsto 0\} \\
O &= \{\langle \textit{home} \land \textit{bike} \land \neg\textit{bike-locked}, \neg\textit{home} \land \textit{uni}\rangle, \\
&\qquad \langle \textit{bike} \land \textit{bike-locked}, \neg\textit{bike-locked}\rangle, \\
&\qquad \langle \textit{bike} \land \neg\textit{bike-locked}, \textit{bike-locked}\rangle, \\
&\qquad \langle \textit{uni}, \textit{lecture} \land ((\textit{bike} \land \neg\textit{bike-locked}) \rhd \neg\textit{bike})\rangle\} \\
\gamma &= \textit{lecture} \land \textit{bike}
\end{aligned}
$$

Identify state variable a occurring negatively in conditions.

Positive normal form: example

Example (transformation to positive normal form)

$$
\begin{aligned}
A &= \{\textit{home}, \textit{uni}, \textit{lecture}, \textit{bike}, \textit{bike-locked}, \textit{bike-unlocked}\}\\
I &= \{\textit{home} \mapsto 1, \textit{bike} \mapsto 1, \textit{bike-locked} \mapsto 1,\\
&\qquad \textit{uni} \mapsto 0, \textit{lecture} \mapsto 0, \textit{bike-unlocked} \mapsto 0\}\\
O &= \{\langle \textit{home} \land \textit{bike} \land \neg\textit{bike-locked}, \neg\textit{home} \land \textit{uni}\rangle,\\
&\qquad \langle \textit{bike} \land \textit{bike-locked}, \neg\textit{bike-locked}\rangle,\\
&\qquad \langle \textit{bike} \land \neg\textit{bike-locked}, \textit{bike-locked}\rangle,\\
&\qquad \langle \textit{uni}, \textit{lecture} \land ((\textit{bike} \land \neg\textit{bike-locked}) \rhd \neg\textit{bike})\rangle\}\\
\gamma &= \textit{lecture} \land \textit{bike}
\end{aligned}
$$

Introduce new variable $\hat{a}$ with complementary initial value.

Positive normal form: example

Example (transformation to positive normal form)

$$
\begin{aligned}
A &= \{\mathit{home}, \mathit{uni}, \mathit{lecture}, \mathit{bike}, \mathit{bike\text{-}locked}, \mathit{bike\text{-}unlocked}\} \\
I &= \{\mathit{home} \mapsto 1, \mathit{bike} \mapsto 1, \mathit{bike\text{-}locked} \mapsto 1, \\
&\qquad \mathit{uni} \mapsto 0, \mathit{lecture} \mapsto 0, \mathit{bike\text{-}unlocked} \mapsto 0\} \\
O &= \{\langle \mathit{home} \land \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \lnot \mathit{home} \land \mathit{uni}\rangle, \\
&\qquad \langle \mathit{bike} \land \mathit{bike\text{-}locked}, \lnot \mathit{bike\text{-}locked}\rangle, \\
&\qquad \langle \mathit{bike} \land \lnot \mathit{bike\text{-}locked}, \mathit{bike\text{-}locked}\rangle, \\
&\qquad \langle \mathit{uni}, \mathit{lecture} \land ((\mathit{bike} \land \lnot \mathit{bike\text{-}locked}) \rhd \lnot \mathit{bike})\rangle\} \\
\gamma &= \mathit{lecture} \land \mathit{bike}
\end{aligned}
$$

Identify effects on variable a.

Positive normal form: example

Example (transformation to positive normal form)

$$
\begin{aligned}
A &= \{\textit{home}, \textit{uni}, \textit{lecture}, \textit{bike}, \textit{bike-locked}, \textit{bike-unlocked}\}\\
I &= \{\textit{home} \mapsto 1, \textit{bike} \mapsto 1, \textit{bike-locked} \mapsto 1,\\
&\qquad \textit{uni} \mapsto 0, \textit{lecture} \mapsto 0, \textit{bike-unlocked} \mapsto 0\}\\
O &= \{\langle \textit{home} \wedge \textit{bike} \wedge \neg \textit{bike-locked}, \neg \textit{home} \wedge \textit{uni}\rangle,\\
&\qquad \langle \textit{bike} \wedge \textit{bike-locked}, \neg \textit{bike-locked} \wedge \textit{bike-unlocked}\rangle,\\
&\qquad \langle \textit{bike} \wedge \neg \textit{bike-locked}, \textit{bike-locked} \wedge \neg \textit{bike-unlocked}\rangle,\\
&\qquad \langle \textit{uni}, \textit{lecture} \wedge ((\textit{bike} \wedge \neg \textit{bike-locked}) \rhd \neg \textit{bike})\rangle\}\\
\gamma &= \textit{lecture} \wedge \textit{bike}
\end{aligned}
$$

Introduce complementary effects for $\hat{a}$.

Positive normal form: example

Example (transformation to positive normal form)

$$A = \{\textit{home}, \textit{uni}, \textit{lecture}, \textit{bike}, \textit{bike-locked}, \textit{bike-unlocked}\}$$

$$I = \{\textit{home} \mapsto 1, \textit{bike} \mapsto 1, \textit{bike-locked} \mapsto 1,$$
$$\textit{uni} \mapsto 0, \textit{lecture} \mapsto 0, \textit{bike-unlocked} \mapsto 0\}$$

$$O = \{\langle \textit{home} \land \textit{bike} \land \neg\textit{bike-locked}, \neg\textit{home} \land \textit{uni}\rangle,$$
$$\langle \textit{bike} \land \textit{bike-locked}, \neg\textit{bike-locked} \land \textit{bike-unlocked}\rangle,$$
$$\langle \textit{bike} \land \neg\textit{bike-locked}, \textit{bike-locked} \land \neg\textit{bike-unlocked}\rangle,$$
$$\langle \textit{uni}, \textit{lecture} \land ((\textit{bike} \land \neg\textit{bike-locked}) \rhd \neg\textit{bike})\rangle\}$$

$$\gamma = \textit{lecture} \land \textit{bike}$$

Identify negative conditions for a.

Positive normal form: example

Example (transformation to positive normal form)

$$
\begin{aligned}
A &= \{\textit{home}, \textit{uni}, \textit{lecture}, \textit{bike}, \textit{bike-locked}, \textit{bike-unlocked}\}\\
I &= \{\textit{home} \mapsto 1, \textit{bike} \mapsto 1, \textit{bike-locked} \mapsto 1,\\
&\qquad \textit{uni} \mapsto 0, \textit{lecture} \mapsto 0, \textit{bike-unlocked} \mapsto 0\}\\
O &= \{\langle \textit{home} \wedge \textit{bike} \wedge \textit{bike-unlocked}, \neg\textit{home} \wedge \textit{uni}\rangle,\\
&\qquad \langle \textit{bike} \wedge \textit{bike-locked}, \neg\textit{bike-locked} \wedge \textit{bike-unlocked}\rangle,\\
&\qquad \langle \textit{bike} \wedge \textit{bike-unlocked}, \textit{bike-locked} \wedge \neg\textit{bike-unlocked}\rangle,\\
&\qquad \langle \textit{uni}, \textit{lecture} \wedge ((\textit{bike} \wedge \textit{bike-unlocked}) \rhd \neg\textit{bike})\rangle\}\\
\gamma &= \textit{lecture} \wedge \textit{bike}
\end{aligned}
$$

Replace by positive condition $\hat{a}$.

Positive normal form: example

Example (transformation to positive normal form)

$$
\begin{aligned}
A &= \{\mathit{home}, \mathit{uni}, \mathit{lecture}, \mathit{bike}, \mathit{bike\text{-}locked}, \mathit{bike\text{-}unlocked}\} \\
I &= \{\mathit{home} \mapsto 1, \mathit{bike} \mapsto 1, \mathit{bike\text{-}locked} \mapsto 1, \\
&\qquad \mathit{uni} \mapsto 0, \mathit{lecture} \mapsto 0, \mathit{bike\text{-}unlocked} \mapsto 0\} \\
O &= \{\langle \mathit{home} \wedge \mathit{bike} \wedge \mathit{bike\text{-}unlocked}, \neg \mathit{home} \wedge \mathit{uni}\rangle, \\
&\qquad \langle \mathit{bike} \wedge \mathit{bike\text{-}locked}, \neg \mathit{bike\text{-}locked} \wedge \mathit{bike\text{-}unlocked}\rangle, \\
&\qquad \langle \mathit{bike} \wedge \mathit{bike\text{-}unlocked}, \mathit{bike\text{-}locked} \wedge \neg \mathit{bike\text{-}unlocked}\rangle, \\
&\qquad \langle \mathit{uni}, \mathit{lecture} \wedge ((\mathit{bike} \wedge \mathit{bike\text{-}unlocked}) \rhd \neg \mathit{bike})\rangle\} \\
\gamma &= \mathit{lecture} \wedge \mathit{bike}
\end{aligned}
$$

Why positive normal form is interesting

In positive normal form, good and bad effects are easy to distinguish:

- Effects that make state variables true are good (add effects).
- Effects that make state variables false are bad (delete effects).

This is of high relevance for some planning techniques that we will see later in this course.

STRIPS operators

STRIPS operators

Definition

An operator $\langle \chi, e \rangle$ is a STRIPS operator if

- χ is a conjunction of atoms, and
- e is a conjunction of atomic effects.

Hence every STRIPS operator is of the form

$$\langle a_1 \wedge \cdots \wedge a_n, \quad l_1 \wedge \cdots \wedge l_m \rangle$$

where a_i are atoms and l_j are atomic effects.

Note: Sometimes we allow conjunctions of literals as preconditions. We denote this as STRIPS with negative preconditions.

Why STRIPS is interesting

- STRIPS operators are particularly simple, yet expressive enough to capture general planning problems.
- In particular, STRIPS planning is no easier than general planning problems.
- Most algorithms in the planning literature are only presented for STRIPS operators (generalization is often, but not always, obvious).

STRIPS

STanford Research Institute Planning System
(Fikes & Nilsson, 1971)

Transformation to STRIPS

- Not every operator is equivalent to a STRIPS operator.
- However, each operator can be transformed into a set of STRIPS operators whose "combination" is equivalent to the original operator. (How?)
- However, this transformation may exponentially increase the number of required operators. There are planning tasks for which such a blow-up is unavoidable.
- There are polynomial transformations of planning tasks to STRIPS, but these do not preserve the structure of the transition system (e. g., length of shortest plans may change).

Summary

- Effect normal form simplifies the structure of the operator effects: conditional effects contain only atomic effects, and there is at most one occurrence of any atomic effect.
- Positive normal form allows to distinguish good and bad effects.
- The form of STRIPS operators is even more restrictive than effect normal form, forbidding complex preconditions and conditional effects.
- All three forms are expressive enough to capture general planning problems.
- Transformation to effect normal form and positive normal form can be done with a small polynomial size increase.
- Structure preserving transformations of planning tasks to STRIPS can increase the number of operators exponentially.