Principles of Al Planning

7. Planning as search: relaxed planning tasks

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How to obtain a heuristic

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A simple heuristic for deterministic planning

STRIPS (Fikes & Nilsson, 1971) used the number of state variables that differ in current state s and a STRIPS goal $a_1 \wedge \cdots \wedge a_n$:

$$h(s) := |\{i \in \{1, \dots, n\} \mid s \not\models a_i\}|.$$

Intuition: more true goal literals \infty closer to the goal

→ STRIPS heuristic (a.k.a. goal-count heuristic) (properties?)

Note: From now on, for convenience we usually write heuristics as functions of states (as above), not nodes.

Node heuristic h' is defined from state heuristic h as $h'(\sigma) := h(\textit{state}(\sigma))$.

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Criticism of the STRIPS heuristic

What is wrong with the STRIPS heuristic?

- quite uninformative: the range of heuristic values in a given task is small; typically, most successors have the same estimate
- very sensitive to reformulation: can easily transform any planning task into an equivalent one where h(s)=1 for all non-goal states (how?)
- ignores almost all problem structure:
 heuristic value does not depend on the set of operators!
- → need a better, principled way of coming up with heuristics

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Coming up with heuristics in a principled way

General procedure for obtaining a heuristic

Solve an easier version of the problem.

Two common methods:

- relaxation: consider less constrained version of the problem
- abstraction: consider smaller version of real problem

Both have been very successfully applied in planning. We consider both in this course, beginning with relaxation.

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Relaxing a problem

How do we relax a problem?

Example (Route planning for a road network)

The road network is formalized as a weighted graph over points in the Euclidean plane. The weight of an edge is the road distance between two locations.

A relaxation drops constraints of the original problem.

Example (Relaxation for route planning)

Use the Euclidean distance $\sqrt{|x_1-x_2|^2+|y_1-y_2|^2}$ as a heuristic for the road distance between $\langle x_1,y_1\rangle$ and $\langle x_2,y_2\rangle$ This is a lower bound on the road distance (\leadsto admissible).

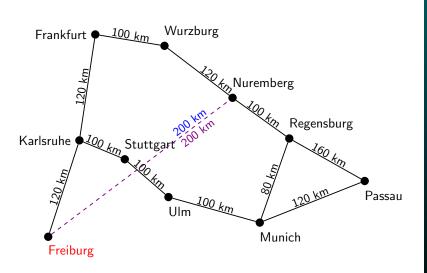
→ We drop the constraint of having to travel on roads.

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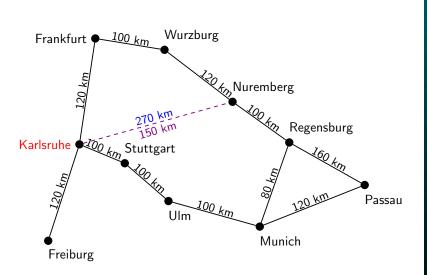


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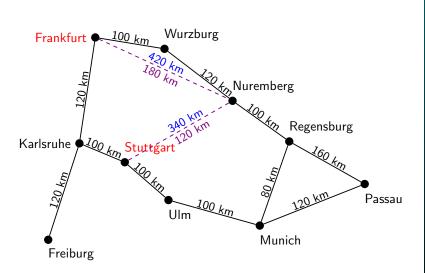


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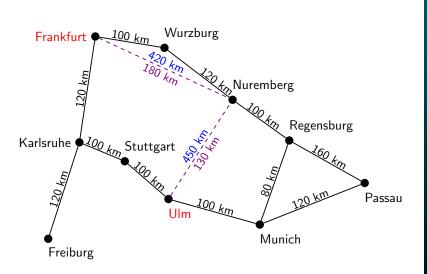


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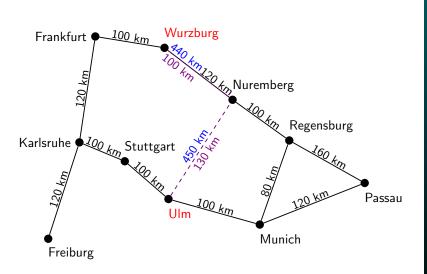


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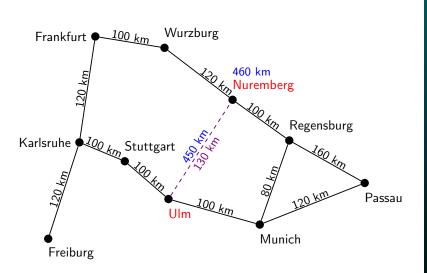


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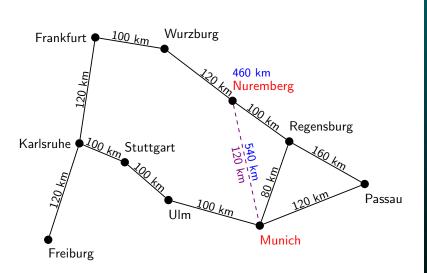


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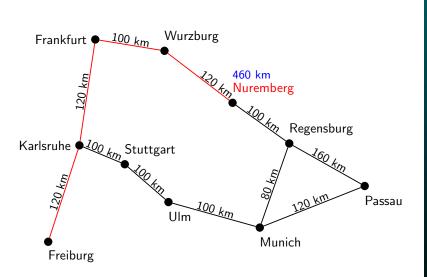


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Relaxed planning tasks: idea

In positive normal form (remember?), good and bad effects are easy to distinguish:

- Effects that make state variables true are good (add effects).
- Effects that make state variables false are bad (delete effects).

Idea for the heuristic: Ignore all delete effects.

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Definition (relaxation of operators)

The relaxation o^+ of an operator $o = \langle \chi, e \rangle$ in positive normal form is the operator which is obtained by replacing all negative effects $\neg a$ within e by the do-nothing effect \top .

Definition (relaxation of planning tasks)

The relaxation Π^+ of a planning task $\Pi = \langle A, I, O, \gamma \rangle$ in positive normal form is the planning task $\Pi^+ := \langle A, I, \{o^+ \mid o \in O\}, \gamma \rangle$.

Definition (relaxation of operator sequences)

The relaxation of an operator sequence $\pi = o_1 \dots o_n$ is the operator sequence $\pi^+ := o_1^+ \dots o_n^+$.

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Relaxed planning tasks: terminology

- Planning tasks in positive normal form without delete effects are called relaxed planning tasks.
- Plans for relaxed planning tasks are called relaxed plans.
- If Π is a planning task in positive normal form and π^+ is a plan for Π^+ , then π^+ is called a relaxed plan for Π .

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Dominating states

The on-set on(s) of a state s is the set of true state variables in s, i.e. $on(s) = s^{-1}(\{1\})$.

A state s' dominates another state s iff $on(s) \subseteq on(s')$.

Lemma (domination)

Let s and s' be valuations of a set of propositional variables A and let χ be a propositional formula over A which does not contain negation symbols.

If $s \models \chi$ and s' dominates s, then $s' \models \chi$.

Proof.

Proof by induction over the structure of χ .

- Base case $\chi = \top$: then $s' \models \top$.
- Base case $\chi = \bot$: then $s \not\models \bot$.

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Dominating states (ctd.)

Proof (ctd.)

- Base case $\chi = a \in A$: assume $s \models a$ and $on(s) \subseteq on(s')$. With $a \in on(s)$ we get $a \in on(s')$, hence $s' \models a$.
- Inductive case $\chi = \chi_1 \wedge \chi_2$: by induction hypothesis, our claim holds for the proper subformulas χ_1 and χ_2 of χ .

• Inductive case $\chi = \chi_1 \vee \chi_2$: Analogous.

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The relaxation lemma

For the rest of this chapter, we assume that all planning tasks are in positive normal form.

Lemma (relaxation)

Let s be a state, let s' be a state that dominates s, and let π be an operator sequence which is applicable in s. Then π^+ is applicable in s' and $\operatorname{app}_{\pi^+}(s')$ dominates $\operatorname{app}_{\pi}(s)$. Moreover, if π leads to a goal state from s, then π^+ leads to a goal state from s'.

Proof.

The "moreover" part follows from the rest by the domination lemma. Prove the rest by induction over the length of π .

```
Base case: \pi=\epsilon app_{\pi^+}(s')=s' dominates app_{\pi}(s)=s by assumption.
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Base case: $\pi = \epsilon$ $app_{\pi^+}(s') = s'$ dominates $app_{\pi}(s) = s$ by assumption.

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The relaxation

Proof (ctd.)

Inductive case: $\pi = o_1 \dots o_{n+1}$

By the induction hypothesis, $o_1^+ \dots o_n^+$ is applicable in s', and $t' = app_{o_1^+ \dots o_n^+}(s')$ dominates $t = app_{o_1 \dots o_n}(s)$.

Let $o:=o_{n+1}=\langle \chi,e\rangle$ and $o^+=\langle \chi,e^+\rangle$. By assumption, o is applicable in t, and thus $t\models \chi$. By the domination lemma, we get $t'\models \chi$ and hence o^+ is applicable in t'. Therefore, π^+ is applicable in s'.

Because o is in positive normal form, all effect conditions satisfied by t are also satisfied by t' (by the domination lemma). Therefore, $([e]_t \cap A) \subseteq [e^+]_{t'}$ (where A is the set of state variables, or positive literals).

We get $on(app_{\pi}(s)) \subseteq on(t) \cup ([e]_t \cap A) \subseteq on(t') \cup [e^+]_{t'} = on(app_{\pi^+}(s'))$, and thus $app_{\pi^+}(s')$ dominates $app_{\pi}(s)$.

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Consequences of the relaxation lemma

Corollary (relaxation leads to dominance and preserves plans)

Let π be an operator sequence which is applicable in state s. Then π^+ is applicable in s and $\operatorname{app}_{\pi^+}(s)$ dominates $\operatorname{app}_{\pi}(s)$. If π is a plan for Π , then π^+ is a plan for Π^+ .

Proof.

Apply relaxation lemma with s' = s.

- → Relaxations of plans are relaxed plans.
- → Relaxations are no harder to solve than the original task.
- Optimal relaxed plans are never longer than optimal plans for original tasks.

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Consequences of the relaxation lemma (ctd.)

Corollary (relaxation preserves dominance)

Let s be a state, let s' be a state that dominates s, and let π^+ be a relaxed operator sequence applicable in s. Then π^+ is applicable in s' and $\mathsf{app}_{\pi^+}(s')$ dominates $\mathsf{app}_{\pi^+}(s)$.

Proof.

Apply relaxation lemma with π^+ for π , noting that $(\pi^+)^+ = \pi^+$.

- \sim If there is a relaxed plan starting from state s, the same plan can be used starting from a dominating state s'.
- Making a transition to a dominating state never hurts in relaxed planning tasks.

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Monotonicity of relaxed planning tasks

We need one final property before we can provide an algorithm for solving relaxed planning tasks.

Lemma (monotonicity)

Let $o^+ = \langle \chi, e^+ \rangle$ be a relaxed operator and let s be a state in which o^+ is applicable.

Then $app_{o^+}(s)$ dominates s.

Proof.

Since relaxed operators only have positive effects, we have $on(s) \subseteq on(s) \cup [e^+]_s = on(app_{o^+}(s)).$

→ Together with our previous results, this means that making a transition in a relaxed planning task never hurts.

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Greedy algorithm for relaxed planning tasks

The relaxation and monotonicity lemmas suggest the following algorithm for solving relaxed planning tasks:

Greedy planning algorithm for $\langle A, I, O^+, \gamma \rangle$

```
s := I
\pi^+ := \epsilon
forever:
     if s \models \gamma:
           return \pi^+
     else if there is an operator o^+ \in O^+ applicable in s
              with app_{o^+}(s) \neq s:
           Append such an operator o^+ to \pi^+.
           s := app_{o^+}(s)
     else:
           return unsolvable
```

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Correctness of the greedy algorithm

The algorithm is sound:

- If it returns a plan, this is indeed a correct solution.
- If it returns "unsolvable", the task is indeed unsolvable
 - ullet Upon termination, there clearly is no relaxed plan from s.
 - ullet By iterated application of the monotonicity lemma, s dominates I.
 - ullet By the relaxation lemma, there is no solution from I.

What about completeness (termination) and runtime?

- ullet Each iteration of the loop adds at least one atom to $\mathit{on}(s)$.
- \bullet This guarantees termination after at most |A| iterations.
- Thus, the algorithm can clearly be implemented to run in polynomial time.
 - A good implementation runs in $O(\|\Pi\|)$.

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Using the greedy algorithm as a heuristic

We can apply the greedy algorithm within heuristic search:

- In a search node σ , solve the relaxation of the planning task with $state(\sigma)$ as the initial state.
- \bullet Set $h(\sigma)$ to the length of the generated relaxed plan.

Is this an admissible heuristic?

- Yes if the relaxed plans are optimal (due to the plan preservation corollary).
- However, usually they are not, because our greedy planning algorithm is very poor.

(What about safety? Goal-awareness? Consistency?)

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The set cover problem

To obtain an admissible heuristic, we need to generate optimal relaxed plans. Can we do this efficiently?

This question is related to the following problem:

Problem (set cover)

Given: a finite set U, a collection of subsets $C = \{C_1, \ldots, C_n\}$ with $C_i \subseteq U$ for all $i \in \{1, \ldots, n\}$, and a natural number K.

Question: Does there exist a set cover of size at most K, i. e., a subcollection $S = \{S_1, \ldots, S_m\} \subseteq C$ with $S_1 \cup \cdots \cup S_m = U$ and $m \leq K$?

The following is a classical result from complexity theory:

Theorem (Karp 1972)

The set cover problem is NP-complete.

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Hardness of optimal relaxed planning

Theorem (optimal relaxed planning is hard)

The problem of deciding whether a given relaxed planning task has a plan of length at most K is NP-complete.

Proof.

For membership in NP, guess a plan and verify. It is sufficient to check plans of length at most |A|, so this can be done in nondeterministic polynomial time.

For hardness, we reduce from the set cover problem.

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Hardness of optimal relaxed planning (ctd.)

Proof (ctd.)

Given a set cover instance $\langle U,C,K\rangle$, we generate the following relaxed planning task $\Pi^+=\langle A,I,O^+,\gamma\rangle$:

- \bullet A = U
- $\bullet \ I = \{a \mapsto 0 \mid a \in A\}$
- $O^+ = \{ \langle \top, \bigwedge_{a \in C_i} a \rangle \mid C_i \in C \}$
- $\gamma = \bigwedge_{a \in U} a$

If S is a set cover, the corresponding operators form a plan. Conversely, each plan induces a set cover by taking the subsets corresponding to the operators. Clearly, there exists a plan of length at most K iff there exists a set cover of size K.

Moreover, Π^+ can be generated from the set cover instance in polynomial time, so this is a polynomial reduction.

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Using relaxations in practice

How can we use relaxations for heuristic planning in practice? Different possibilities:

 Implement an optimal planner for relaxed planning tasks and use its solution lengths as an estimate, even though it is NP-hard.

```
\rightsquigarrow h^+ heuristic
```

- Do not actually solve the relaxed planning task, but compute an estimate of its difficulty in a different way.
 - $ightarrow h_{
 m max}$ heuristic, $h_{
 m add}$ heuristic, $h_{
 m LM-cut}$ heuristic
- Compute a solution for relaxed planning tasks which is not necessarily optimal, but "reasonable".

```
\rightsquigarrow h_{\mathsf{FF}} heuristic
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Summary

- Two general methods for coming up with heuristics:
 - relaxation: solve a less constrained problem
 - abstraction: solve a small problem
- Here, we consider the delete relaxation, which requires tasks in positive normal form and ignores delete effects.
- Delete-relaxed tasks have a domination property: it is always beneficial to make more fluents true.
- They also have a monotonicity property: applying operators leads to dominating states.
- Because of these two properties, finding some relaxed plan greedily is easy (polynomial).
- For an informative heuristic, we would ideally want to find optimal relaxed plans. This is NP-complete.

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