

# Scout and NegaScout

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# Introduction

- It looks like alpha-beta pruning is the best we can do for a **generic searching procedure**.
  - What else can be done generically?
  - Alpha-beta pruning follows basically the “intelligent” searching behaviors used by human when domain knowledge is not involved.
  - Can we find some other “intelligent” behaviors used by human during searching?
- Intuition: One a MAX node
  - Suppose we know currently we have a way to gain at least 300 points at the first branch.
  - If there is an efficient way to know the second branch is at most gaining 300 points, then there is no need to search the second branch in detail.
    - ▷ *Is there a way to search a tree approximately?*
    - ▷ *Is searching approximately faster than searching exactly?*
- Similar intuition holds for a MIN node.

# SCOUT procedure

- Invented by Judea Pearl in 1980.
- It may be possible to verify whether the value of a branch is greater than a value  $v$  or not in a way that is faster than knowing its exact value.
- High level idea:
  - While searching a branch  $T_b$  of a MAX node, if we have already obtained a lower bound  $v_\ell$ .
    - ▷ *First TEST whether it is possible for  $T_b$  to return something greater than  $v_\ell$ .*
    - ▷ *If FALSE, then there is no need to search  $T_b$ . This is called **fails the test**.*
    - ▷ *If TRUE, then search  $T_b$ . This is called **passes the test**.*
  - While searching a branch  $T_c$  of a MIN node, if we have already obtained an upper bound  $v_u$ 
    - ▷ *First TEST whether it is possible for  $T_c$  to return something smaller than  $v_u$ .*
    - ▷ *If FALSE, then there is no need to search  $T_c$ . This is called **fails the test**.*
    - ▷ *If TRUE, then search  $T_c$ . This is called **passes the test**.*

# How to TEST

- procedure TEST(position  $p$ , value  $v$ , condition  $>$ )  
// test whether the value of the branch at  $p$  is  $> v$ 
  - determine the successor positions  $p_1, \dots, p_d$  of  $p$
  - if  $d = 0$ , then // terminal
    - ▷ return TRUE if  $f(p) > v$  //  $f()$ : evaluating function
    - ▷ return FALSE otherwise
  - for  $i := 1$  to  $d$  do
    - ▷ if  $p$  is a MAX node and TEST( $p_i, v, >$ ) is TRUE, then return TRUE
    - ▷ if  $p$  is a MIN node and TEST( $p_i, v, >$ ) is FALSE, then return FALSE
  - if  $p$  is a MAX node, then return FALSE
  - if  $p$  is a MIN node, then return TRUE
- Condition can be stated as  $\geq$  by properly revising the algorithm.
  - For the condition to be  $<$  or  $\leq$ , we need to switch conditions for the MAX and MIN nodes.
- Practical consideration:
  - Set a depth limit and evaluate the position's value when the limit is reached.

# Main SCOUT procedure

## ■ Algorithm SCOUT(position $p$ )

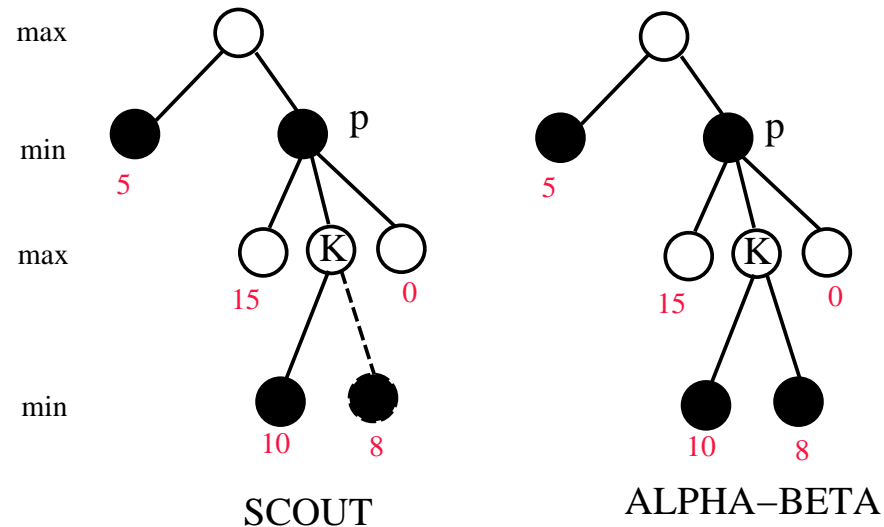
- determine the successor positions  $p_1, \dots, p_d$
- if  $d = 0$ , then return  $f(p)$
- else  $v = \text{SCOUT}(p_1)$  // SCOUT the first branch
- for  $i := 2$  to  $d$  do  
// TEST first for the rest of the branches
  - ▷ if  $p$  is a MAX node and  $\text{TEST}(p_i, v, >)$  is TRUE,  
then  $v = \text{SCOUT}(p_i)$  // find the value of this branch
  - ▷ if  $p$  is a MIN node and  $\text{TEST}(p_i, v, \geq)$  is FALSE,  
then  $v = \text{SCOUT}(p_i)$  // find the value of this branch
- return  $v$

## ■ Note that $v$ is the current best value at any moment.

- for a MAX node,  $p$ ,
  - ▷ For any  $i > 1$ , if  $\text{TEST}(p_i, v, >)$  is TRUE, then the value returned by  $\text{SCOUT}(p_i)$  must be greater than  $v$  for a MAX node.
  - ▷ We say the  $p_i$  **passes the test** if  $\text{TEST}(p_i, v, >)$  is TRUE.
- for a MIN node,  $p$ ,
  - ▷ For any  $i > 1$ , if  $\text{TEST}(p_i, v, \geq)$  is FALSE, then the value returned by  $\text{SCOUT}(p_i)$  must be smaller than  $v$ .
  - ▷ We say the  $p_i$  **passes the test** if  $\text{TEST}(p_i, v, \geq)$  is FALSE.

# Discussions for SCOUT (1/2)

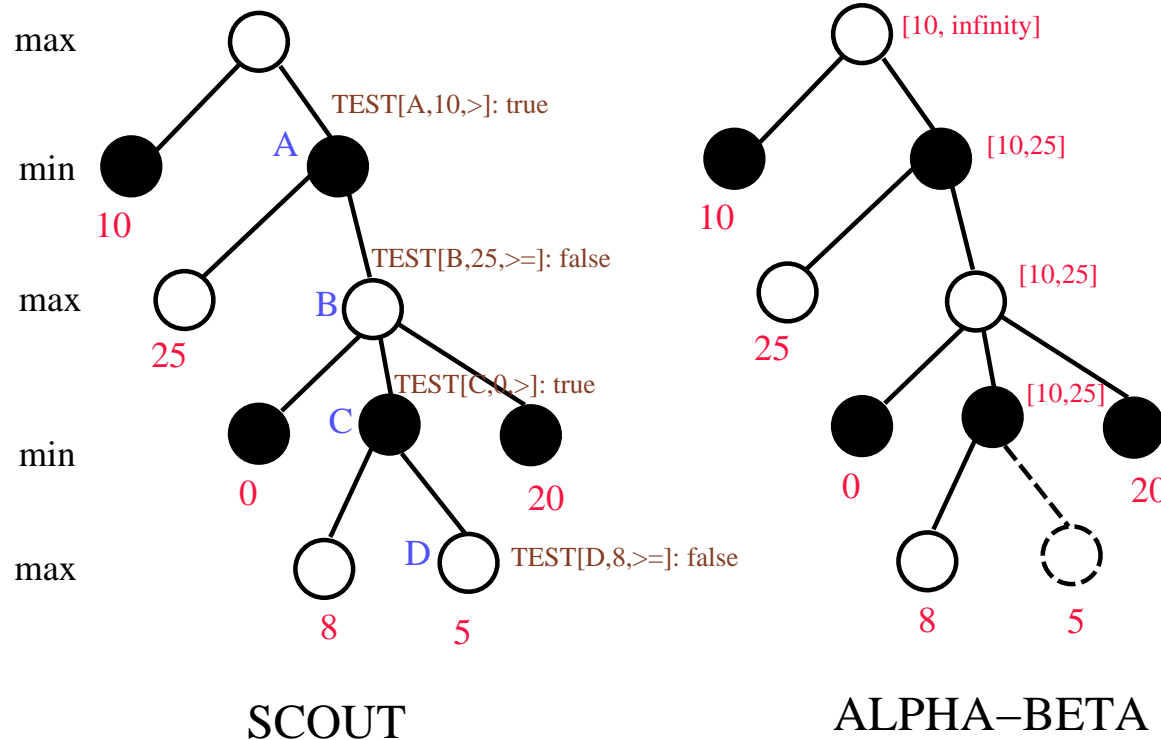
- TEST may visit less nodes than alpha-beta.



- Assume  $TEST(p, 5, >)$  is called by the root after the first branch is evaluated.
  - ▷ It calls  $TEST(K, 5, >)$  which skips  $K$ 's second branch.
  - ▷  $TEST(p, 5, >)$  is **FALSE**, i.e., fails the test, after returning from the 3rd branch.
  - ▷ No need to do SCOUT for the branch  $p$ .
- Alpha-beta needs to visit  $K$ 's second branch.

# Discussions for SCOUT (2/2)

- SCOUT may visit a node that is cut off by alpha-beta.



# Number of nodes visited (1/3)

- For **TEST** to return **TRUE** for a subtree  $T$ , it needs to evaluate at least
  - ▷ *one child for a MAX node in  $T$ , and*
  - ▷ *and all of the children for a MIN node in  $T$ .*
  - ▷ *If  $T$  has a fixed branching factor  $b$  and uniform depth  $d$ , the number of nodes evaluated is  $\Omega(b^{d/2})$ .*
- For **TEST** to return **FALSE** for a subtree  $T$ , it needs to evaluate at least
  - ▷ *one child for a MIN node in  $T$ , and*
  - ▷ *and all of the children for a MAX node in  $T$ .*
  - ▷ *If  $T$  has a fixed branching factor  $b$  and uniform depth  $d$ , the number of nodes evaluated is  $\Omega(b^{d/2})$ .*



# Number of nodes visited (2/3)

## ■ Assumptions:

- Assume a full complete  $d$ -ary tree with depth  $\ell$ .
- Assume  $\ell$  is even.
- The depth of the root, which is a MAX node, is 0.

■ The total number of nodes in the tree is  $\frac{d^{\ell+1}-1}{d-1}$ .

■ The minimum number of nodes visited by TEST when it returns TRUE.

▷ It is  $1 + 1 + d + d + d^2 + d^2 + d^3 + d^3 + \dots + d^{\ell/2-1} + d^{\ell/2-1} + d^{\ell/2}$ .

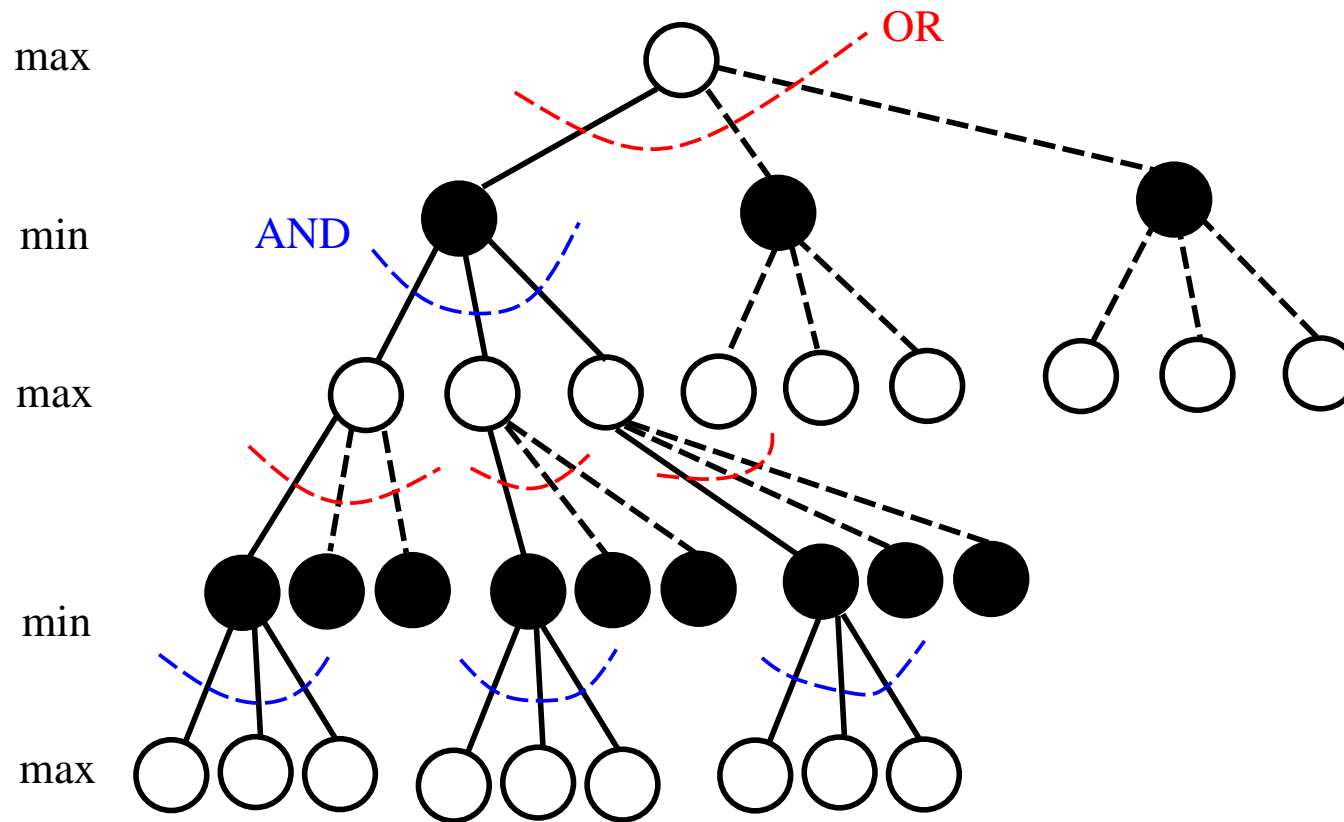
▷ It is  $2 \cdot (d^0 + d^1 + \dots + d^{\ell/2}) - d^{\ell/2} = 2 \cdot \frac{d^{\ell/2+1}-1}{d-1} - d^{\ell/2}$ .

■ The minimum number of nodes visited by alpha-beta.

▷ It is  $\sum_{i=0}^{\ell} d^{\lceil i/2 \rceil} + d^{\lfloor i/2 \rfloor} - 1$ .

▷ It is  $1 + d + (2d - 1) + (d^2 + d - 1) + \dots + (d^{\ell/2} + d^{\ell/2-1} - 1) + (2 \cdot d^{\ell/2} - 1)$ .

# Number of nodes visited (3/3)



# Comparisons

- When the first branch of a node has the best value, then TEST scans the tree fast.
  - The best value of the first  $i - 1$  branches is used to test whether the  $i$ th branch needs to be searched exactly.
  - If the value of the first  $i - 1$  branches of the root is better than the value of  $i$ th branch, then we do not have to evaluate exactly for the  $i$ th branch.
- Compared to alpha-beta pruning whose cut off comes from bounds of search windows.
  - It is possible to have some cut-off for alpha-beta as long as there are some relative move orderings are “good.”
    - ▷ *The moving orders of your children and the children of your ancestor who is odd level up decide a cut-off.*
  - The search bound is updated during the searching.
    - ▷ *Sometimes, a deep alpha-beta cut-off occurs because a bound found from your ancestor a distance away.*

# Performance of SCOUT (1/2)

- A node may be visited more than once.
  - First visit is to TEST.
  - Second visit is to SCOUT.
    - ▷ *During a SCOUT, it may be TESTed with a different value.*
  - Q: Can information obtained in the first search be used in the second search?
- SCOUT is a recursive procedure.
  - A node in a branch that is not the first child of a node with a depth of  $\ell$ .
    - ▷ *Every ancestor of you may initiate a TEST to visit you.*
    - ▷ *It can be visited  $\ell$  times by TEST.*
    - ▷ *Every ancestor of you may pass the TEST and decides to SCOUT you.*
    - ▷ *It can be visited  $\ell$  times by SCOUT.*

# Performance of SCOUT (2/2)

- Show great improvements on  $depth > 3$  for games with small branching factors.
  - It traverses most of the nodes without evaluating them preciously.
  - Few subtrees remained to be revisited to compute their exact mini-max values.
- Experimental data show [Pearl 1980]:
  - SCOUT favors “skinny” games, that are games with high depth-to-width ratios.
  - On depth = 5, it saves over 40% of time.
  - Maybe bad for games with a large branching factor.
  - Move ordering is very important.
    - ▷ *The first branch, if is good, offers a great chance of pruning further branches.*

# Alpha-beta revisited

- In an alpha-beta search with a window  $[alpha, beta]$ :
  - **Failed-high** means it returns a value that is larger than its upper bound  $beta$ .
  - **Failed-low** means it returns a value that is smaller than its lower bound  $alpha$ .
- **Null or Zero window search:**
  - Using alpha-beta search with the window  $[m, m + 1]$ .
  - The result can be either failed-high or failed-low.
  - Failed-high means the return value is at least  $m + 1$ .
    - ▷ *Equivalent to  $TEST(p, m, >)$  is true.*
  - Failed-low means the return value is at most  $m$ .
    - ▷ *Equivalent to  $TEST(p, m, >)$  is false.*

# Alpha-Beta + Scout

## ■ Intuition:

- Try to incooperate SCOUT and alpha-beta together.
- The searching window of alpha-beta if properly set can be used as TEST in SCOUT.
- Using a searching window is better than using a single bound as in SCOUT.
- Can also apply alpha-beta cut if it applies.

## ■ Modifications to the SCOUT algorithm:

- Traverse the tree with two bounds as the alpha-beta procedure does.
  - ▷ *A searching window.*
  - ▷ *Use the current best bound to guide the TEST value.*
- Use a fail soft version to get a better result when the returned value is out of the window.

# The NegaScout Algorithm – MiniMax (1/2)

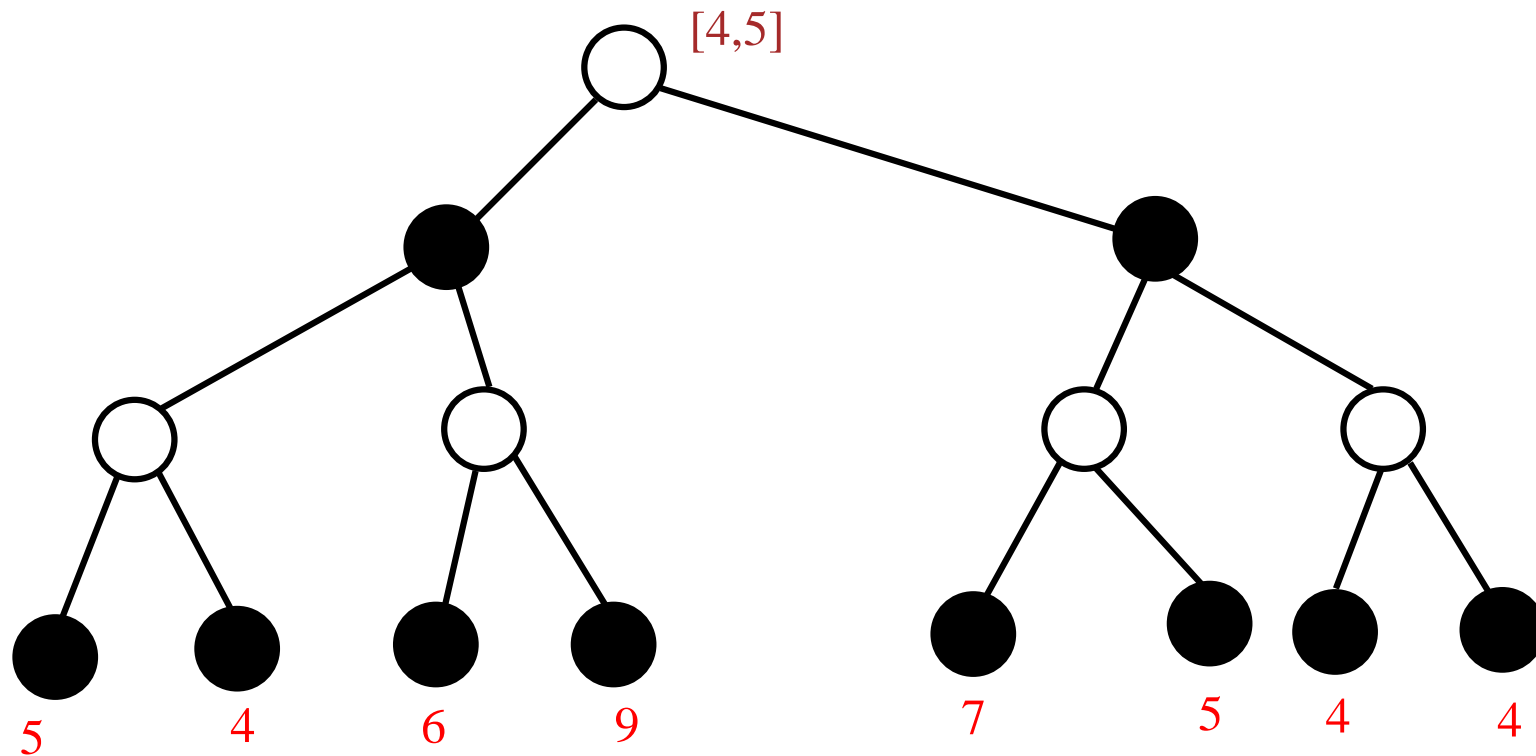
- Algorithm  $F4'$ (position  $p$ , value  $alpha$ , value  $beta$ , integer  $depth$ )
  - determine the successor positions  $p_1, \dots, p_d$
  - if  $d = 0$  // a terminal node  
or  $depth = 0$  //  $depth$  is the remaining depth to search  
or time is running up // from timing control  
or some other constraints are met // apply heuristic here
  - then return  $f(p)$  else  
begin
    - ▷  $m := -\infty$  //  $m$  is the current best lower bound; fail soft  
 $m := \max\{m, G4'(p_1, alpha, beta, depth - 1)\}$  // the first branch  
if  $m \geq beta$  then return( $m$ ) // beta cut off
    - ▷ for  $i := 2$  to  $d$  do
    - ▷ 9:  $t := G4'(p_i, m, m + 1, depth - 1)$  // null window search
    - ▷ 10: if  $t > m$  then // failed-high
    - 11: if ( $depth < 3$  or  $t \geq beta$ )
    - 12: then  $m := t$
    - 13: else  $m := G4'(p_i, t, beta, depth - 1)$  // re-search
    - ▷ 14: if  $m \geq beta$  then return( $m$ ) // beta cut off
  - end
  - return  $m$



# The NegaScout Algorithm – MiniMax (2/2)

- Algorithm  $G4'$ (position  $p$ , value  $alpha$ , value  $beta$ , integer  $depth$ )
  - determine the successor positions  $p_1, \dots, p_d$
  - if  $d = 0$  // a terminal node  
or  $depth = 0$  //  $depth$  is the remaining depth to search  
or time is running up // from timing control  
or some other constraints are met // apply heuristic here
  - then return  $g(p)$  else  
begin
    - ▷  $m = \infty$  //  $m$  is the current best upper bound; fail soft  
 $m := \min\{m, F4'(p_1, alpha, beta, depth - 1)\}$  // the first branch  
if  $m \leq alpha$  then return( $m$ ) // alpha cut off
    - ▷ for  $i := 2$  to  $d$  do
    - ▷ 9:  $t := F4'(p_i, m, m + 1, depth - 1)$  // null window search
    - ▷ 10: if  $t \leq m$  then // failed-low
    - 11: if ( $depth < 3$  or  $t \leq alpha$ )
    - 12: then  $m := t$
    - 13: else  $m := G4'(p_i, alpha, t, depth - 1)$  // re-search
    - ▷ 14: if  $m \leq alpha$  then return( $m$ ) // alpha cut off
  - end
  - return  $m$

# Example for NegaScout – MiniMax version



# The NegaScout Algorithm

- Use Nega-MAX format.
- Algorithm  $F4(\text{position } p, \text{value } \alpha, \text{value } \beta, \text{integer } \text{depth})$ 
  - determine the successor positions  $p_1, \dots, p_d$
  - if  $d = 0$  // a terminal node  
or  $\text{depth} = 0$  //  $\text{depth}$  is the remaining depth to search  
or time is running up // from timing control  
or some other constraints are met // apply heuristic here
  - then return  $f(p)$  else
    - ▷  $m := -\infty$  // the current lower bound; fail soft
    - ▷  $n := \beta$  // the current upper bound
    - ▷ for  $i := 1$  to  $d$  do
    - ▷ 9:  $t := -F4(p_i, -n, -\max\{\alpha, m\}, \text{depth} - 1)$
    - ▷ 10: if  $t > m$  then
    - ▷ 11:     if  $(n = \beta \text{ or } \text{depth} < 3 \text{ or } t \geq \beta)$
    - ▷ 12:     then  $m := t$
    - ▷ 13:     else  $m := -F4(p_i, -\beta, -t, \text{depth} - 1)$  // re-search
    - ▷ 14: if  $m \geq \beta$  then return( $m$ ) // cut off
    - ▷ 15:  $n := \max\{\alpha, m\} + 1$  // set up a null window
  - return  $m$

# Search behaviors (1/3)

- If the depth is enough or it is a terminal position, then stop searching further.
  - Return  $f(p)$  as the value computed by an evaluation function.
- Fail soft version.
- For the first child  $p_1$ , a normal alpha beta searching window is used.
  - line 9: normal alpha-beta search for the first child
  - the initial value of  $m$  is  $-\infty$ , hence  $-\max\{\alpha, m\} = -\alpha$ 
    - ▷  *$m$  is current best value*
  - that is, searching with the normal window  $[\alpha, \beta]$

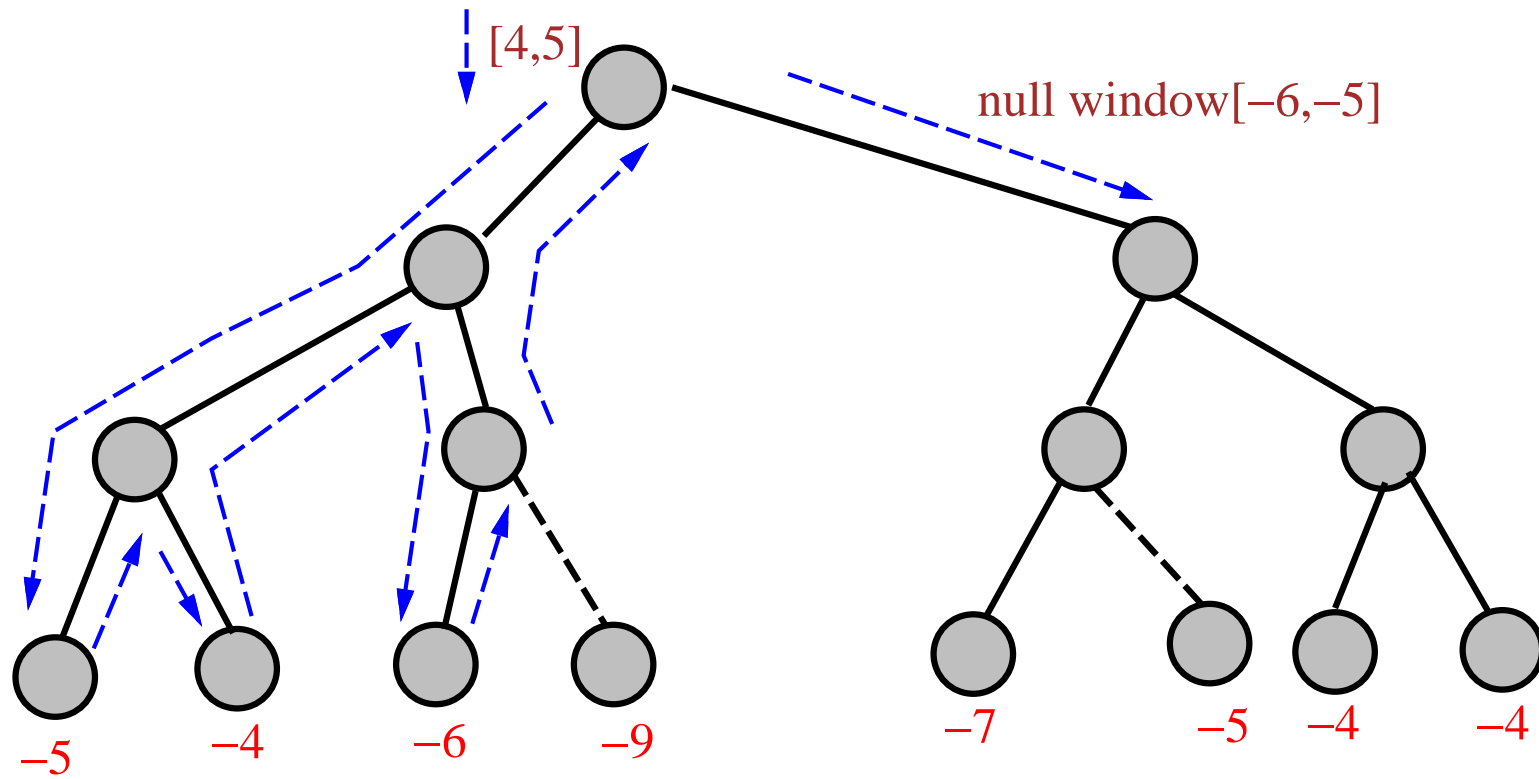
# Search behaviors (2/3)

- For the second child and beyond  $p_i$ ,  $i > 1$ , first perform a null window search for testing whether  $m$  is the answer.
  - line 9: a null-window of  $[m, m + 1]$  searches for the second child and beyond.
    - ▷  $m$  is best value obtained so far
    - ▷  $m$ 's value will be first set at line 12 because  $n = \text{beta}$
    - ▷ The null window is set at line 15.
  - line 11:
    - ▷  $n = \text{beta}$ : we are at first iteration.
    - ▷  $\text{depth} < 3$ : on a smaller depth subtree, i.e., depth at most 2, NegaScout always returns the best value.
    - ▷  $t \geq \text{beta}$ : we have obtained a good enough value from the failed-soft version to guarantee a beta cut.

# Search behaviors (3/3)

- For the second child and beyond  $p_i$ ,  $i > 1$ , first perform a null window search for testing whether  $m$  is the answer.
  - line 11: on a smaller depth subtree, i.e., depth at most 2, NegaScout always returns the best value.
    - ▷ *Normally, no need to do alpha-beta or any enhancement on very small subtrees.*
    - ▷ *The overhead is too large on small subtrees.*
  - line 13: **re-search** when the null window search fails high.
    - ▷ *The value of this subtree is at least  $t$ .*
    - ▷ *This means the best value in this subtree is more than  $m$ , the current best value.*
    - ▷ *This subtree must be re-searched with the the window  $[t, \text{beta}]$ .*
  - line 14: the normal pruning from alpha-beta.

# Example for NegaScout



# Refinements

- When a subtree is re-searched, it is best to use information on the previous search to speed up the current search.
  - Restart from the position that the value  $t$  is returned.
- Maybe want to re-search using the normal alpha-beta procedure.
- $F4$  runs much better with a good move ordering and transposition tables.
  - Order the moves in a best-first list.
  - Reduce the number of re-searches.



# Performances

- Experiments done on a uniform random game tree [Reinefeld 1983].
  - Normally superior to alpha-beta when searching game tree with branching factors from 20 to 60.
  - Shows about 10 to 20% of improvement.

# Comments

- Incooperating both SCOUT and alpha-beta.
- Used in state-of-the-art game search engines.
- The first search, though maybe unsuccessful, can provide useful information in the second search.
  - Information can be stored and then be reused.

# References and further readings

- \* A. Reinefeld. An improvement of the scout tree search algorithm. *ICCA Journal*, 6(4):4–14, 1983.
- \* J. Pearl. Asymptotic properties of minimax trees and game-searching procedures. *Artificial Intelligence*, 14(2):113–138, 1980.