Bounded Branching and Modalities in Non-Deterministic Planning

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Introduction

- We consider two extensions for the task of deciding the existence of solutions for non-deterministic planning problems:
    - Bounds in the number of branch points in a plan (solution)
    - Extending the description language with modal formulae
- The first applies to the cases of non-deterministic planning with complete and partial information
- The second applies only to the case of non-deterministic planning with partial information

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Outline

- Planning with Complete Information
- Planning with Partial Information
- Summary

Planning with Complete Information

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- Another possibility is to apply a sequence of actions without observing the system and then to observe the state of the system at the end of the sequence and planning thereafter
- Therefore, we can think of bounding the number of branch points in a plan and to question when such a plan exists

Deterministic Models

Understood in terms of:

- a discrete and finite state space S
- an initial state $s_0 \in S$
- a non-empty set of goal states $G \subseteq S$
- actions $A(s) \subseteq A$ applicable in each state s
- a function that maps states and actions into states $f(a,s) \in S$

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Solutions: sequences $(a_0, \ldots, a_n)$ of actions that “transform” s_0 into a goal state

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  - Actions are pairs $\langle prec, effect \rangle$
  - $prec$ is a propositional formula used to define $A(s)$
  - Effects are defined inductively as:
    - $\{l\}$ is an effect for literal l (atomic)
    - $(e_1 \wedge \ldots \wedge e_n)$ for effects $e_1, \ldots, e_n$ (parallel)
    - $(c \triangleright e)$ for effect e and formula c (conditional)

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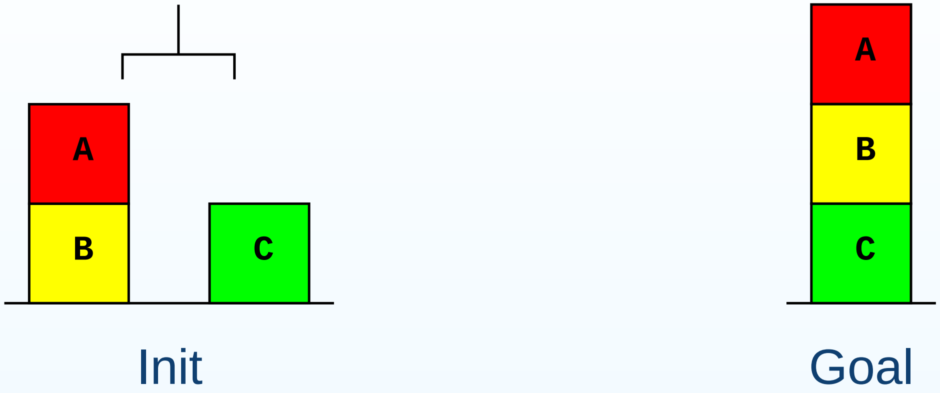
- The goal states defined by a propositional formula Φ_G

Example – Blocksworld (Deterministic)



- Propositions:
  - Blocks' positions: $\{$`on-table(B)`, `on(A,B)`, `on-table(C)`$\}$
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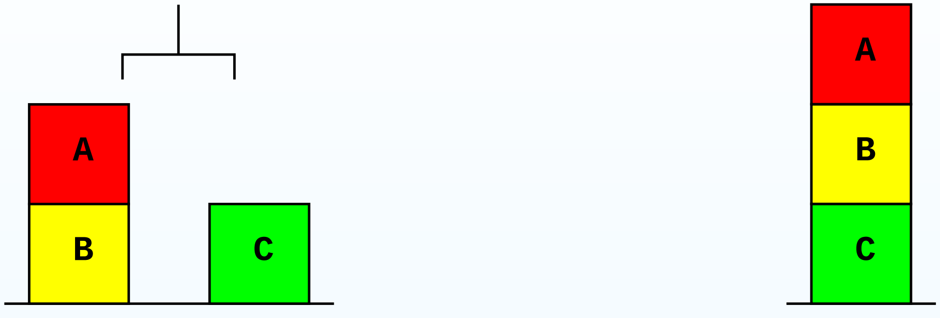

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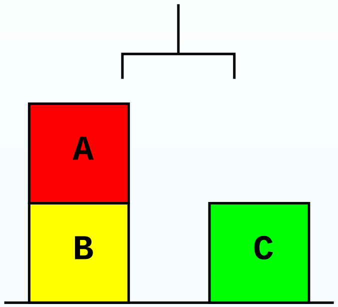

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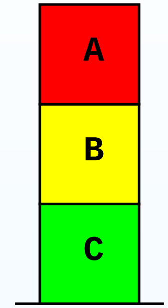

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- Plan: $(\texttt{unstack(A,B)}, \texttt{drop(A)}, \texttt{pick(B)}, \texttt{stack(B,C)}, \texttt{pick(A)}, \texttt{stack(A,B)})$

Complexity of Deterministic Planning

- The existence of a plan can be decided with the non-deterministic program:

  1. Let $counter := 0$
  2. Let $state := I$
  3. If $state \models \Phi_G$, then ACCEPT
  4. Choose applicable action a in $state$
  5. Let $state := f(a, state)$
  6. Let $counter := counter + 1$
  7. If $counter = 2^{|P|}$, then REJECT
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- The fact than PLAN-DET is PSPACE-hard was shown in [Bylander, 1994] with a direct simulation of DTMs with polynomial space bound

Non-Deterministic Models

- As deterministic models but the transition function maps states and actions into **sets of states** $F(a,s) \subseteq S$
- There can be more than one initial state described by formula Φ_I
- The description language is extended with non-deterministic effects:
  - $(e_1 \oplus \cdots \oplus e_n)$ for effects $e_1,\ldots,e_n$ (non-deterministic effect),
- Solutions can't be sequences of actions, but tree-like structures called **contingent plans**

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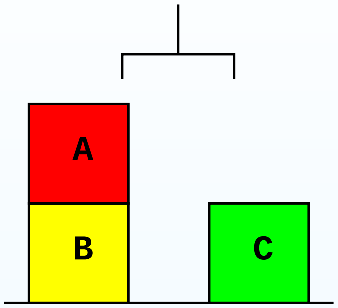

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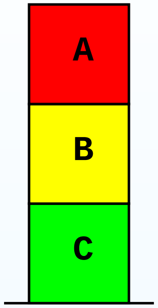

Goal

- New Action:
  - **unstack(A,B)**:

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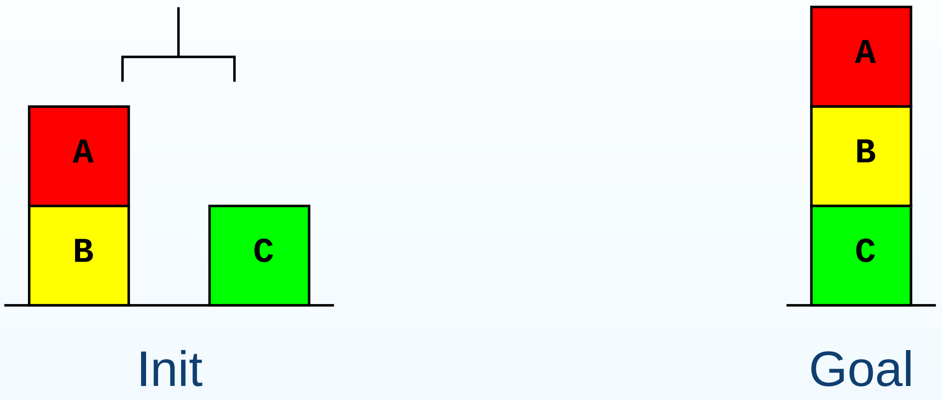

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- Contingent Plan:

$$\texttt{unstack(A,B)} \begin{cases} \texttt{drop(A)}, \texttt{pick(B)}, \texttt{stack(B,C)}, \texttt{pick(A)}, \texttt{stack(A,B)} \\ \texttt{pick(B)}, \texttt{stack(B,C)}, \texttt{pick(A)}, \texttt{stack(A,B)} \end{cases}$$

Complexity of Non-Deterministic Planning

- Plans are like policies for Markov Decisions Processes (MDPs)
- Deciding existence of solution for a contingent planning problem with full observability (i.e. PLAN-FO-CONT) is EXPTIME-complete
- Shown by [Rintanen, 2004] using Alternating TMs with polynomial space bound
- Interestingly, [Littman, 1997] showed that deciding the existence of acyclic policies, of bounded depth N, that reach the goal with probability $\geq T$ for MDPs is also EXPTIME-complete

Conformant Planning

- Consider now the following actions:
  - **unstack(A,B)**:
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- Since there are no preconditions, the actions are **always applicable**

Conformant Planning (Cont'd) and Complexity

- Therefore, the plan:

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- A conformant plan is a **no-branch** plan for a non-deterministic planning problem with full observability

Complexity of Conformant Planning

- Checking the existence of a conformant plan (i.e. PLAN-FO-CONF) is EXPSPACE-complete
- The inclusion is shown with a similar program that works with subsets of states instead of states
- Hardness shown by [Haslum & Jonsson, 1999] using Regular Expressions with Exponentiation and Non-deterministic Finite Automata with Counters

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- Example: Σ^n set of words of length n over alphabet Σ
- An REE α can be recognized with an NFA with Counters (NFAC)
- The NFAC can be "simulated" as a non-deterministic planning problem P such that $\alpha \neq \Sigma^*$ iff P has a conformant plan
- Therefore, since checking whether $\alpha = \Sigma^*$ is EXPSPACE-hard, checking if P has a conformant plan is co-EXPSPACE-hard
- Finish with the fact that EXPSPACE is closed under complementation

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- Checking the existence of a contingent plan with at most k branches (i.e. PLAN-FO-CONT-k) is EXPSPACE-complete
- Similar proof to that of [Haslum & Jonsson, 1999] for conformant planning

Summary

Problem	Complete for	Reference
PLAN-DET	PSPACE	[Bylander, 1994]
PLAN-FO-CONT	EXPTIME	[Rintanen, 2004]
PLAN-FO-CONF	EXPSPACE	[Haslum & Jonsson, 1999]
PLAN-FO-CONT-k	EXPSPACE	**New**

Remember:

- P $\subseteq$ NP $\subseteq$ PSPACE $\subseteq$ EXPTIME $\subseteq$ EXPSPACE
- PSPACE $= \cup_{k\geq 0}$DSPACE(n^k)
- EXPTIME $= \cup_{k\geq 0}$DTIME(2^{n^k})
- EXPSPACE $= \cup_{k\geq 0}$DSPACE(2^{n^k})

Planning with Partial Information

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- In general, each action depends on the **history** of actions/observations

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- Plans are tree-like structures that map belief states into actions

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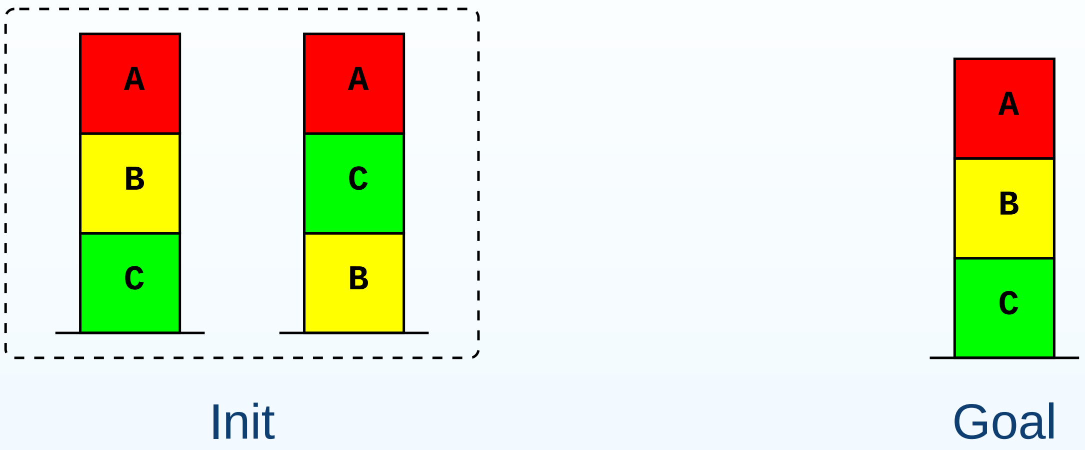

- Observables: $Z = \{$clear(A), clear(B), clear(C)$\}$
- Contingent Plan:

$$\texttt{pick(A)} \begin{cases} \texttt{stack(A,B)} \\ \texttt{drop(A)}, \texttt{pick(C)}, \texttt{drop(C)}, \texttt{pick(B)}, \texttt{stack(B,C)}, \texttt{pick(A)}, \texttt{stack(A,B)} \end{cases}$$

Complexity of Planning With Partial Information

- Plans are like policies for Partially Observable MDPs (POMDPs)
- Deciding existence of solution for a contingent planning problem with partial observability (i.e. PLAN-PO-CONT) is 2EXPTIME-complete
- 2EXPTIME $= \cup_{k\geq 0}$DTIME$(2^{2^{n^k}})$
- Shown by [Rintanen, 2004] using Alternating TMs with exponential space bound

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  - the number of "near" matches in the guess

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Another Example – Game of Mastermind

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- A modal formula is needed to represent such a goal

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  - $(s, b, \sigma) \models \Box\varphi$ iff $(s', b, s\sigma) \models \varphi$ for all $s' \in b$,
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  - $(s, b, s'\sigma) \models \varphi^*$ iff $(s', b, \sigma) \models \varphi$, and
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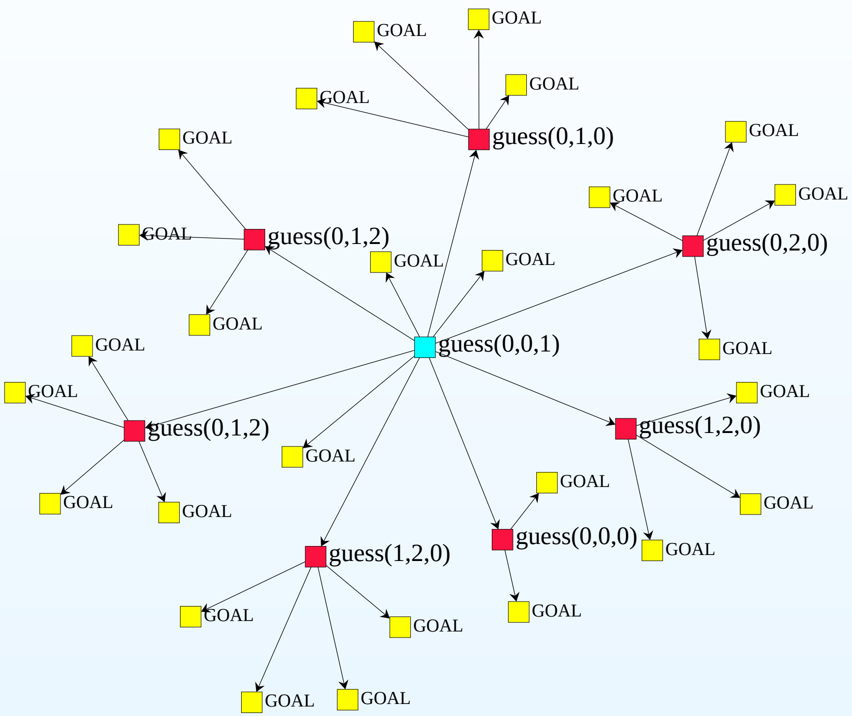
- Examples:
  - 'know φ' equivalent to $\Box\varphi \vee \Box\neg\varphi$
  - 'possibly φ' equivalent to $\Diamond\varphi$
  - $\Box\Diamond(p^* \leftrightarrow q)$ true in belief b iff for all $s \in b$ there is $s' \in b$ such that the value of p in s coincides with the value of q in s'

Mastermind Revisited: 3 colors, 3 pegs

- Contingent Plan:



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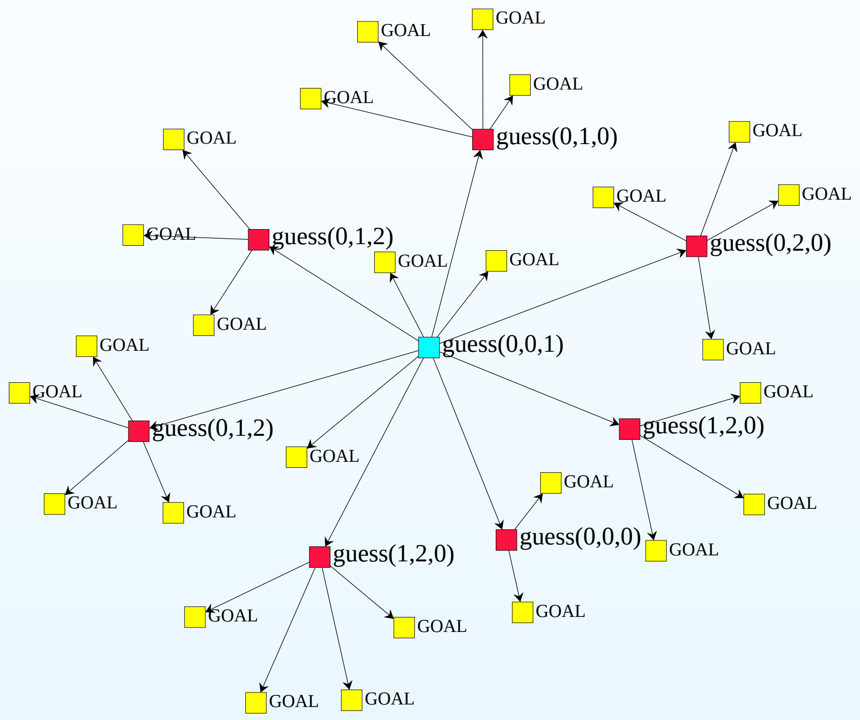

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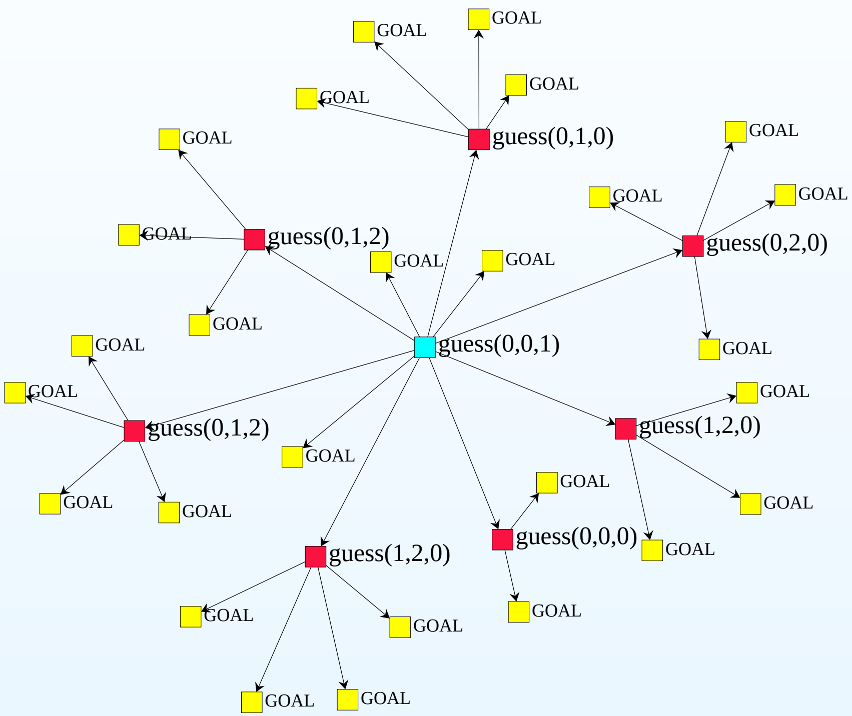

- As before, we can also compute conformant plans for this task
- The following plan discover the secret code no matter what's its value

$$\text{guess}(2,0,0)\ ,\ \text{guess}(2,1,0)\ ,\ \text{guess}(2,2,1)\,.$$

Complexity of Conformant Planning with Partial Information

- Deciding the existence of solution for a conformant planning problem with partial information (i.e. PLAN-PO-CONF) is 2EXPSPACE-complete

- Deciding the existence of solutions for a contingent planning problem with partial information and with at most k branch points (i.e. PLAN-PO-CONT-k) is also 2EXPSPACE-complete

- If there are no modal formulae in the planning problem, PLAN-PO-CONF and PLAN-PO-CONT-k become EXPSPACE-complete

Summary

Problem	Complete for	Reference
PLAN-PO-CONT	2EXPTIME	[Rintanen, 2004]
PLAN-PO-CONF	2EXPSPACE	**New**
PLAN-PO-CONT-k	2EXPSPACE	**New**

Remember:

- P $\subseteq$ NP $\subseteq$ PSPACE $\subseteq$ EXPTIME $\subseteq$ EXPSPACE $\subseteq$ 2EXPTIME $\subseteq$ 2EXPSPACE

Grand Summary

- New planning tasks considered: solutions of bounded branching and conformant tasks for partially observable problems
- Derived tight bounds on complexity for the new tasks
- Special classes also considered:
  - Problems with partial information and no modal formulae
  - Checking the existence of plans of polynomial length following [Turner, 2002]

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