

# Principles of AI Planning

## 5. Planning as search: progression and regression

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# Planning as (classical) search

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# What do we mean by search?

- **Search** is a very generic term.
- ⇒ Every algorithm that tries out various alternatives can be said to “search” in some way.
- Here, we mean **classical search** algorithms.
  - **Search nodes** are **expanded** to generate **successor nodes**.
  - **Examples:** breadth-first search, A\*, hill-climbing, ...
- To be brief, we just say **search** in the following (not “classical search”).

# Do you know this stuff already?

- We **assume prior knowledge** of basic search algorithms:
  - uninformed vs. informed
  - systematic vs. local
- There will be a small refresher in the next chapter.
- **Background:** Russell & Norvig, Artificial Intelligence – A Modern Approach, Ch. 3 (all of it), Ch. 4 (local search)

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# Search in planning

- **search:** one of the **big success stories** of AI
- many planning algorithms based on classical AI search (we'll see some other algorithms later, though)
- will be the focus of this and the following chapters (the majority of the course)

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# Satisficing or optimal planning?

Must carefully distinguish two different problems:

- **satisficing planning:** any solution is OK  
(although shorter solutions typically preferred)
- **optimal planning:** plans must have shortest possible length

Both are often solved by search, but:

- details are **very different**
- almost **no overlap** between good techniques for satisficing planning and good techniques for optimal planning
- many problems that are trivial for satisficing planners are impossibly hard for optimal planners

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# Planning by search

How to apply search to planning?  $\rightsquigarrow$  many choices to make!

## Choice 1: Search direction

- **progression**: forward from initial state to goal
- **regression**: backward from goal states to initial state
- **bidirectional search**

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# Planning by search

How to apply search to planning?  $\rightsquigarrow$  many choices to make!

## Choice 2: Search space representation

- search nodes are associated with **states**  
( $\rightsquigarrow$  **state-space search**)
- search nodes are associated with **sets of states**

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# Planning by search

How to apply search to planning?  $\rightsquigarrow$  many choices to make!

## Choice 3: Search algorithm

- **uninformed search:**  
depth-first, breadth-first, iterative depth-first, ...
- **heuristic search (systematic):**  
greedy best-first,  $A^*$ , Weighted  $A^*$ , IDA\*, ...
- **heuristic search (local):**  
hill-climbing, simulated annealing, beam search, ...

# Planning by search

How to apply search to planning?  $\rightsquigarrow$  many choices to make!

## Choice 4: Search control

- **heuristics** for informed search algorithms
- **pruning techniques**: invariants, symmetry elimination, partial-order reduction, helpful actions pruning, ...

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# Search-based satisficing planners

## FF (Hoffmann & Nebel, 2001)

- search direction: forward search
- search space representation: single states
- search algorithm: enforced hill-climbing (informed local)
- heuristic: FF heuristic (inadmissible)
- pruning technique: helpful actions (incomplete)

⇒ one of the best satisficing planners

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# Search-based optimal planners

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## Fast Downward Stone Soup (Helmert et al., 2011)

- search direction: forward search
- search space representation: single states
- search algorithm:  $A^*$  (informed systematic)
- heuristic: multiple admissible heuristics combined into a heuristic portfolio (LM-cut, M&S, blind, ...)
- pruning technique: none

↪ one of the best optimal planners

# Our plan for the next lectures

Choices to make:

- ➊ search direction: progression/regression/both  
~~> **this chapter**
- ➋ search space representation: states/sets of states  
~~> **this chapter**
- ➌ search algorithm: uninformed/heuristic; systematic/local  
~~> **next chapter**
- ➍ search control: heuristics, pruning techniques  
~~> **following chapters**

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# Progression

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# Planning by forward search: progression

**Progression:** Computing the successor state  $app_o(s)$  of a state  $s$  with respect to an operator  $o$ .

**Progression planners** find solutions by forward search:

- start from initial state
- iteratively pick a previously generated state and **progress it** through an operator, generating a new state
- solution found when a goal state generated

**pro:** very easy and efficient to implement

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# Search space representation in progression planners

Two alternative search spaces for progression planners:

① **search nodes correspond to states**

- when the same state is generated along different paths, it is not considered again (**duplicate detection**)
- **pro**: save time to consider same state again
- **con**: memory intensive (must maintain **closed list**)

② **search nodes correspond to operator sequences**

- different operator sequences may lead to identical states (**transpositions**); search does not notice this
- **pro**: can be very memory-efficient
- **con**: much wasted work (often exponentially slower)

↪ first alternative usually preferable in planning

(**unlike** many classical search benchmarks like 15-puzzle)

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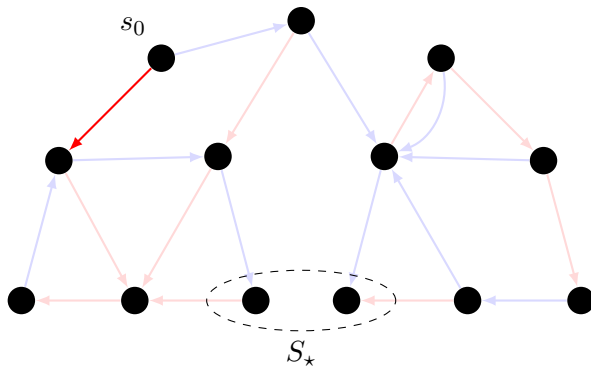
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# Progression planning example (depth-first search)

**Example** where search nodes correspond to operator sequences  
(no duplicate detection)



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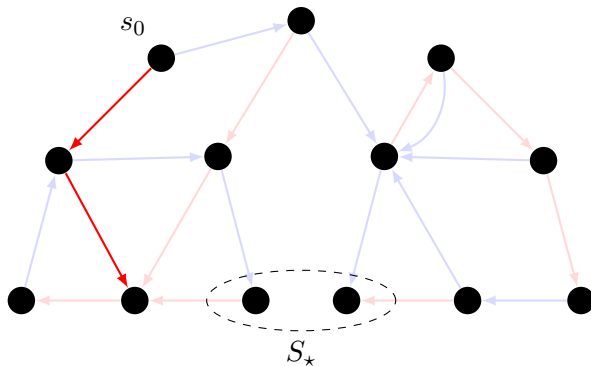
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**Example** where search nodes correspond to operator sequences (no duplicate detection)



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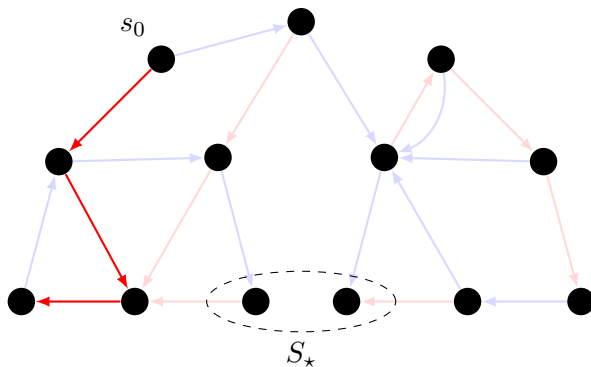
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**Example** where search nodes correspond to operator sequences (no duplicate detection)



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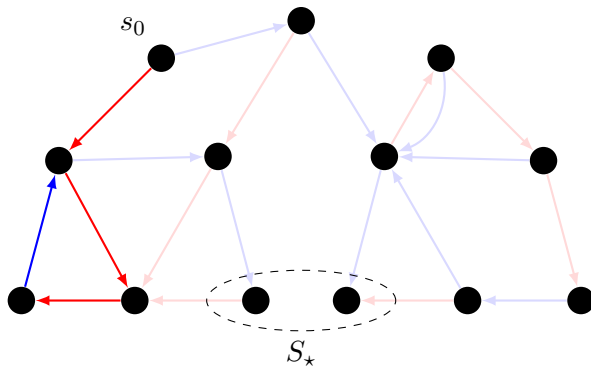
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# Progression planning example (depth-first search)

**Example** where search nodes correspond to operator sequences  
(no duplicate detection)



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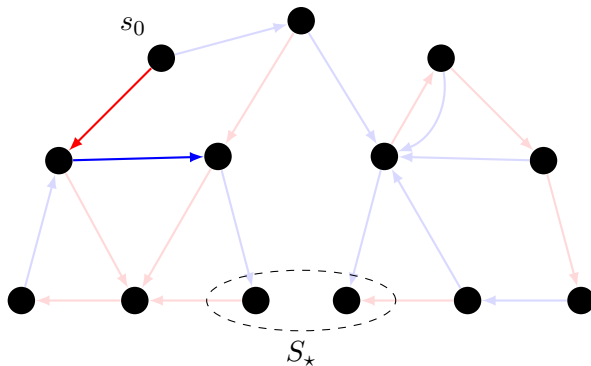
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**Example** where search nodes correspond to operator sequences (no duplicate detection)



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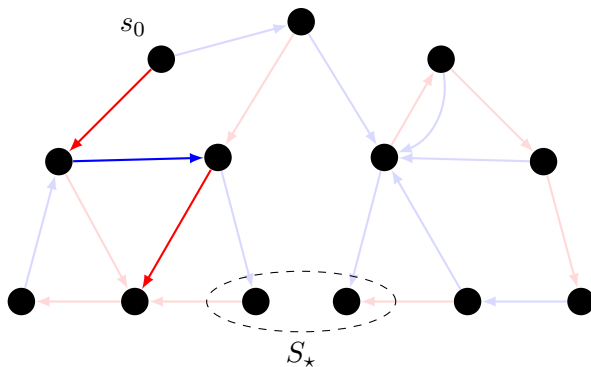
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**Example** where search nodes correspond to operator sequences (no duplicate detection)



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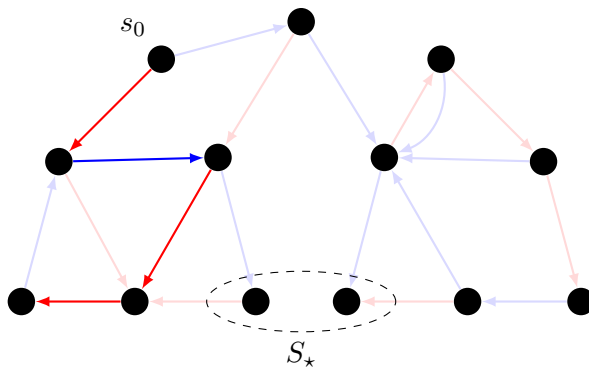
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**Example** where search nodes correspond to operator sequences (no duplicate detection)



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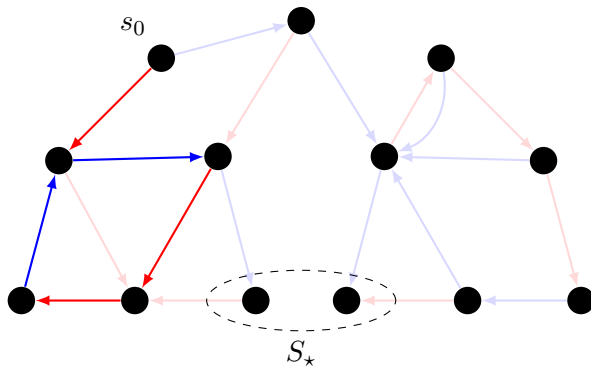
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**Example** where search nodes correspond to operator sequences  
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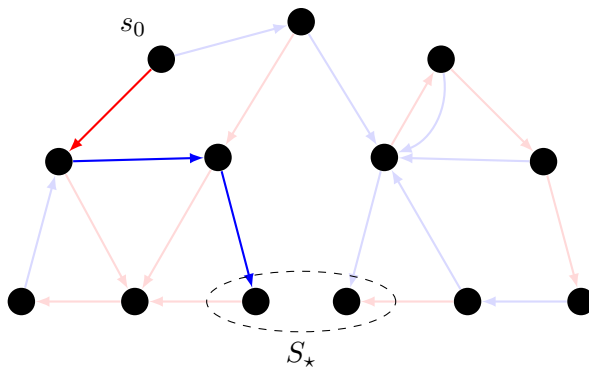
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# Progression planning example (depth-first search)

**Example** where search nodes correspond to operator sequences (no duplicate detection)



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# Forward search vs. backward search

Going through a transition graph in forward and backward directions is **not symmetric**:

- forward search starts from a **single** initial state; backward search starts from a **set** of goal states
- when applying an operator  $o$  in a state  $s$  in forward direction, there is a **unique successor state**  $s'$ ; if we applied operator  $o$  to end up in state  $s'$ , there can be **several possible predecessor states**  $s$

⇒ most natural representation for backward search in planning associates **sets of states** with search nodes

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# Planning by backward search: regression

**Regression:** Computing the possible predecessor states  $regr_o(G)$  of a set of states  $G$  with respect to the last operator  $o$  that was applied.

**Regression planners** find solutions by backward search:

- start from set of goal states
- iteratively pick a previously generated state set and **regress it** through an operator, generating a new state set
- solution found when a generated state set includes the initial state

**Pro:** can handle many states simultaneously

**Con:** basic operations complicated and expensive

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# Search space representation in regression planners

identify state sets with **logical formulae** (again):

- **search nodes correspond to state sets**
- each state set is represented by a **logical formula**:  
 $\varphi$  represents  $\{s \in S \mid s \models \varphi\}$
- many basic search operations like detecting duplicates are NP-hard or coNP-hard

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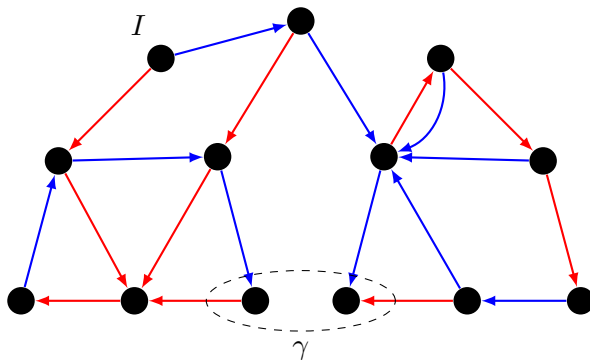
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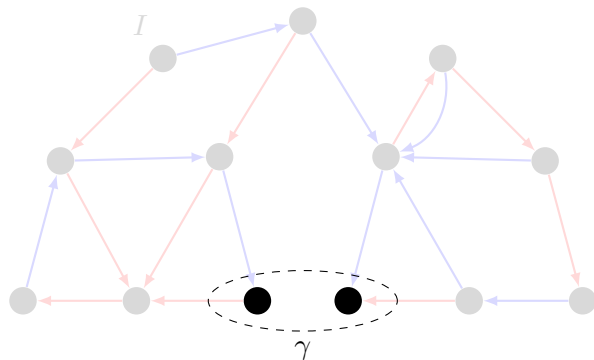
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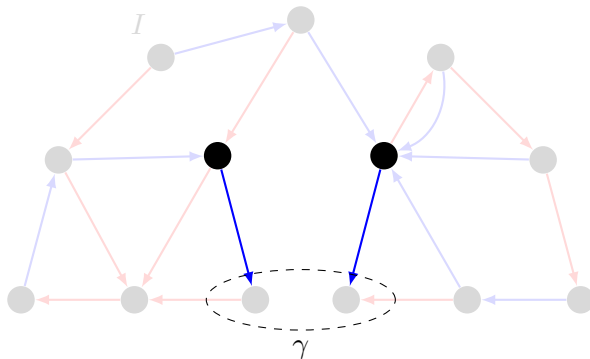
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# Regression planning example (depth-first search)

$$\varphi_1 = \text{regr}_{\rightarrow}(\gamma)$$

$$\varphi_1 \longrightarrow \gamma$$



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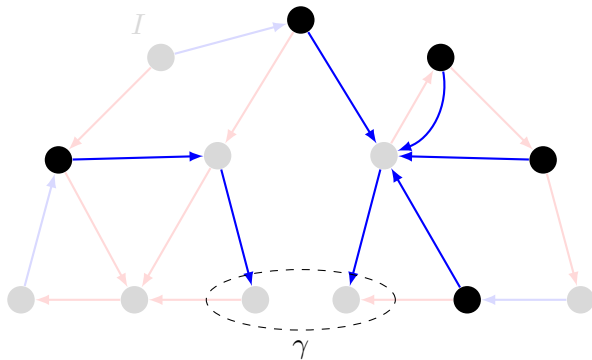
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# Regression planning example (depth-first search)

$$\varphi_1 = \text{regr}_{\rightarrow}(\gamma)$$

$$\varphi_2 = \text{regr}_{\rightarrow}(\varphi_1)$$

$$\varphi_2 \longrightarrow \varphi_1 \longrightarrow \gamma$$



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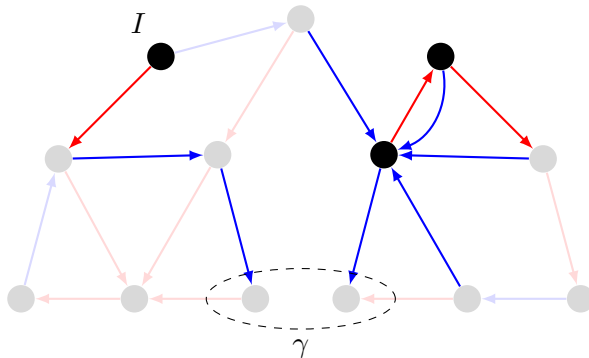
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# Regression planning example (depth-first search)

$$\begin{aligned}\varphi_1 &= \text{regr}_{\rightarrow}(\gamma) \\ \varphi_2 &= \text{regr}_{\rightarrow}(\varphi_1) \\ \varphi_3 &= \text{regr}_{\rightarrow}(\varphi_2), I \models \varphi_3\end{aligned}$$



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# Regression for STRIPS planning tasks

## Definition (STRIPS planning task)

A planning task is a **STRIPS planning task** if all operators are STRIPS operators and the goal is a conjunction of atoms.

Regression **for STRIPS planning tasks** is very simple:

- Goals are conjunctions of atoms  $a_1 \wedge \dots \wedge a_n$ .
  - **First step**: Choose an operator that makes none of  $a_1, \dots, a_n$  false.
  - **Second step**: Remove goal atoms achieved by the operator (if any) and add its preconditions.
- ↪ Outcome of regression is again conjunction of atoms.

**Optimization**: only consider operators making some  $a_i$  true

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# STRIPS regression

## Definition (STRIPS regression)

Let  $\varphi = \varphi_1 \wedge \dots \wedge \varphi_n$  be a conjunction of atoms, and let  $o = \langle \chi, e \rangle$  be a STRIPS operator which adds the atoms  $a_1, \dots, a_k$  and deletes the atoms  $d_1, \dots, d_l$ .

The **STRIPS regression** of  $\varphi$  with respect to  $o$  is

$$\text{sreg}_o(\varphi) := \begin{cases} \perp & \text{if } a_i = d_j \text{ for some } i, j \\ \perp & \text{if } \varphi_i = d_j \text{ for some } i, j \\ \chi \wedge \bigwedge (\{\varphi_1, \dots, \varphi_n\} \setminus \{a_1, \dots, a_k\}) & \text{otherwise} \end{cases}$$

**Note:**  $\text{sreg}_o(\varphi)$  is again a conjunction of atoms, or  $\perp$ .

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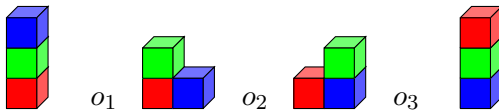
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# STRIPS regression example



**Note:** Predecessor states are in general not unique.  
This picture is just for illustration purposes.

$$\begin{aligned}
 o_1 &= \langle \text{blue on green} \wedge \text{blue clr}, & \neg \text{blue on green} \wedge \text{blue on T} \wedge \text{green clr} \rangle \\
 o_2 &= \langle \text{green on red} \wedge \text{green clr} \wedge \text{blue clr}, & \neg \text{blue clr} \wedge \neg \text{green on red} \wedge \text{green on blue} \wedge \text{red clr} \rangle \\
 o_3 &= \langle \text{red on T} \wedge \text{red clr} \wedge \text{green clr}, & \neg \text{green clr} \wedge \neg \text{red on T} \wedge \text{red on green} \rangle
 \end{aligned}$$

$$\gamma = \text{red on green} \wedge \text{green on blue}$$

$$\varphi_1 = \text{sregr}_{o_3}(\gamma) = \text{red on T} \wedge \text{red clr} \wedge \text{green clr} \wedge \text{green on blue}$$

$$\varphi_2 = \text{sregr}_{o_2}(\varphi_1) = \text{green on red} \wedge \text{green clr} \wedge \text{blue clr} \wedge \text{red on T}$$

$$\varphi_3 = \text{sregr}_{o_1}(\varphi_2) = \text{blue on green} \wedge \text{blue clr} \wedge \text{green on red} \wedge \text{red on T}$$

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# Regression for general planning tasks

- With disjunctions and conditional effects, things become more tricky. How to regress  $a \vee (b \wedge c)$  with respect to  $\langle q, d \triangleright b \rangle$ ?
- The story about goals and subgoals and fulfilling subgoals, as in the STRIPS case, is no longer useful.
- We present a general method for doing regression for any formula and any operator.
- Now we extensively use the idea of representing sets of states as formulae.

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# Effect preconditions

## Definition (effect precondition)

The **effect precondition**  $EPC_l(e)$  for literal  $l$  and effect  $e$  is defined as follows:

$$\begin{aligned}EPC_l(l) &= \top \\EPC_l(l') &= \perp \text{ if } l \neq l' \quad (\text{for literals } l') \\EPC_l(e_1 \wedge \dots \wedge e_n) &= EPC_l(e_1) \vee \dots \vee EPC_l(e_n) \\EPC_l(\chi \triangleright e) &= EPC_l(e) \wedge \chi\end{aligned}$$

**Intuition:**  $EPC_l(e)$  describes the situations in which effect  $e$  causes literal  $l$  to become true.

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# Effect precondition examples

## Example

$$\begin{aligned}EPC_a(b \wedge c) &= \perp \vee \perp \equiv \perp \\EPC_a(a \wedge (b \triangleright a)) &= \top \vee (\top \wedge b) \equiv \top \\EPC_a((c \triangleright a) \wedge (b \triangleright a)) &= (\top \wedge c) \vee (\top \wedge b) \equiv c \vee b\end{aligned}$$

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# Effect preconditions: connection to change sets

## Lemma (A)

*Let  $s$  be a state,  $l$  a literal and  $e$  an effect.  
Then  $l \in [e]_s$  if and only if  $s \models EPC_l(e)$ .*

## Proof.

Induction on the structure of the effect  $e$ .

Base case 1,  $e = l$ :  $l \in [l]_s = \{l\}$  by definition, and  $s \models EPC_l(l) = \top$  by definition. Both sides of the equivalence are true.

Base case 2,  $e = l'$  for some literal  $l' \neq l$ :  $l \notin [l']_s = \{l'\}$  by definition, and  $s \not\models EPC_l(l') = \perp$  by definition. Both sides are false.

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# Effect preconditions: connection to change sets

## Lemma (A)

*Let  $s$  be a state,  $l$  a literal and  $e$  an effect.*

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# Effect preconditions: connection to change sets

## Proof (ctd.)

Inductive case 1,  $e = e_1 \wedge \dots \wedge e_n$ :

$l \in [e]_s$  iff  $l \in [e_1]_s \cup \dots \cup [e_n]_s$  (Def  $[e_1 \wedge \dots \wedge e_n]_s$ )

iff  $l \in [e']_s$  for some  $e' \in \{e_1, \dots, e_n\}$

iff  $s \models EPC_l(e')$  for some  $e' \in \{e_1, \dots, e_n\}$  (IH)

iff  $s \models EPC_l(e_1) \vee \dots \vee EPC_l(e_n)$

iff  $s \models EPC_l(e_1 \wedge \dots \wedge e_n)$ . (Def  $EPC$ )

Inductive case 2,  $e = \chi \triangleright e'$ :

$l \in [\chi \triangleright e']_s$  iff  $l \in [e']_s$  and  $s \models \chi$  (Def  $[\chi \triangleright e']_s$ )

iff  $s \models EPC_l(e')$  and  $s \models \chi$  (IH)

iff  $s \models EPC_l(e') \wedge \chi$

iff  $s \models EPC_l(\chi \triangleright e')$ . (Def  $EPC$ )



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# Effect preconditions: connection to change sets

## Proof (ctd.)

Inductive case 1,  $e = e_1 \wedge \dots \wedge e_n$ :

$$\begin{aligned} l \in [e]_s &\text{ iff } l \in [e_1]_s \cup \dots \cup [e_n]_s && (\text{Def } [e_1 \wedge \dots \wedge e_n]_s) \\ &\text{ iff } l \in [e']_s \text{ for some } e' \in \{e_1, \dots, e_n\} \\ &\text{ iff } s \models EPC_l(e') \text{ for some } e' \in \{e_1, \dots, e_n\} && (\text{IH}) \\ &\text{ iff } s \models EPC_l(e_1) \vee \dots \vee EPC_l(e_n) \\ &\text{ iff } s \models EPC_l(e_1 \wedge \dots \wedge e_n). && (\text{Def } EPC) \end{aligned}$$

Inductive case 2,  $e = \chi \triangleright e'$ :

$$\begin{aligned} l \in [\chi \triangleright e']_s &\text{ iff } l \in [e']_s \text{ and } s \models \chi && (\text{Def } [\chi \triangleright e']_s) \\ &\text{ iff } s \models EPC_l(e') \text{ and } s \models \chi && (\text{IH}) \\ &\text{ iff } s \models EPC_l(e') \wedge \chi \\ &\text{ iff } s \models EPC_l(\chi \triangleright e'). && (\text{Def } EPC) \end{aligned}$$



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# Effect preconditions: connection to normal form

## Remark: *EPC* vs. effect normal form

Notice that in terms of  $EPC_a(e)$ , any operator  $\langle \chi, e \rangle$  can be expressed in effect normal form as

$$\left\langle \chi, \bigwedge_{a \in A} ((EPC_a(e) \triangleright a) \wedge (EPC_{\neg a}(e) \triangleright \neg a)) \right\rangle,$$

where  $A$  is the set of all state variables.

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# Regressing state variables

The formula  $EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$  expresses the **value of state variable  $a \in A$  after applying  $o$**  in terms of **values of state variables before applying  $o$** .

Either:

- $a$  became true, or
- $a$  was true before and it did not become false.

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# Regressing state variables: examples

## Example

Let  $e = (b \triangleright a) \wedge (c \triangleright \neg a) \wedge b \wedge \neg d$ .

variable $x$	$EPC_x(e) \vee (x \wedge \neg EPC_{\neg x}(e))$
$a$	$b \vee (a \wedge \neg c)$
$b$	$\top \vee (b \wedge \neg \perp) \equiv \top$
$c$	$\perp \vee (c \wedge \neg \perp) \equiv c$
$d$	$\perp \vee (d \wedge \neg \top) \equiv \perp$

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# Regressing state variables: correctness

## Lemma (B)

*Let  $a$  be a state variable,  $o = \langle \chi, e \rangle$  an operator,  $s$  a state, and  $s' = \text{app}_o(s)$ .*

*Then  $s \models \text{EPC}_a(e) \vee (a \wedge \neg \text{EPC}_{\neg a}(e))$  if and only if  $s' \models a$ .*

## Proof.

$(\Rightarrow)$ : Assume  $s \models \text{EPC}_a(e) \vee (a \wedge \neg \text{EPC}_{\neg a}(e))$ .

Do a case analysis on the two disjuncts.

- ① Assume that  $s \models \text{EPC}_a(e)$ . By Lemma A, we have  $a \in [e]_s$  and hence  $s' \models a$ .
- ② Assume that  $s \models a \wedge \neg \text{EPC}_{\neg a}(e)$ . By Lemma A, we have  $\neg a \notin [e]_s$ . Hence  $a$  remains true in  $s'$ .

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( $\Leftarrow$ ): We showed that if the formula is **true** in  $s$ , then  $a$  is **true** in  $s'$ . For the second part, we show that if the formula is **false** in  $s$ , then  $a$  is **false** in  $s'$ .

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# Regression: general definition

We base the definition of regression on formulae  $EPC_l(e)$ .

## Definition (general regression)

Let  $\varphi$  be a propositional formula and  $o = \langle \chi, e \rangle$  an operator. The **regression of  $\varphi$  with respect to  $o$**  is

$$\text{regr}_o(\varphi) = \chi \wedge \varphi_r \wedge \kappa$$

where

- ①  $\varphi_r$  is obtained from  $\varphi$  by replacing each  $a \in A$  by  $EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$ , and
- ②  $\kappa = \bigwedge_{a \in A} \neg(EPC_a(e) \wedge EPC_{\neg a}(e))$ .

The formula  $\kappa$  expresses that operators are only applicable in states where their change sets are consistent.

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# Regression examples

- $\text{regr}_{\langle a, b \rangle}(b) \equiv a \wedge (\top \vee (b \wedge \neg \perp)) \wedge \top \equiv a$
- $\text{regr}_{\langle a, b \rangle}(b \wedge c \wedge d)$   
 $\equiv a \wedge (\top \vee (b \wedge \neg \perp)) \wedge (\perp \vee (c \wedge \neg \perp)) \wedge (\perp \vee (d \wedge \neg \perp)) \wedge \top$   
 $\equiv a \wedge c \wedge d$
- $\text{regr}_{\langle a, c \triangleright b \rangle}(b) \equiv a \wedge (c \vee (b \wedge \neg \perp)) \wedge \top \equiv a \wedge (c \vee b)$
- $\text{regr}_{\langle a, (c \triangleright b) \wedge (b \triangleright \neg b) \rangle}(b) \equiv a \wedge (c \vee (b \wedge \neg b)) \wedge \neg(c \wedge b)$   
 $\equiv a \wedge c \wedge \neg b$
- $\text{regr}_{\langle a, (c \triangleright b) \wedge (d \triangleright \neg b) \rangle}(b) \equiv a \wedge (c \vee (b \wedge \neg d)) \wedge \neg(c \wedge d)$   
 $\equiv a \wedge (c \vee b) \wedge (c \vee \neg d) \wedge (\neg c \vee \neg d)$   
 $\equiv a \wedge (c \vee b) \wedge \neg d$

# Regression example: binary counter

$$\begin{aligned} & (\neg b_0 \triangleright b_0) \wedge \\ & ((\neg b_1 \wedge b_0) \triangleright (b_1 \wedge \neg b_0)) \wedge \\ & ((\neg b_2 \wedge b_1 \wedge b_0) \triangleright (b_2 \wedge \neg b_1 \wedge \neg b_0)) \end{aligned}$$

$$EPC_{b_2}(e) = \neg b_2 \wedge b_1 \wedge b_0$$

$$EPC_{b_1}(e) = \neg b_1 \wedge b_0$$

$$EPC_{b_0}(e) = \neg b_0$$

$$EPC_{\neg b_2}(e) = \perp$$

$$EPC_{\neg b_1}(e) = \neg b_2 \wedge b_1 \wedge b_0$$

$$EPC_{\neg b_0}(e) = (\neg b_1 \wedge b_0) \vee (\neg b_2 \wedge b_1 \wedge b_0) \equiv (\neg b_1 \vee \neg b_2) \wedge b_0$$

Regression replaces state variables as follows:

$$b_2 \quad \text{by} \quad (\neg b_2 \wedge b_1 \wedge b_0) \vee (b_2 \wedge \neg \perp) \equiv (b_1 \wedge b_0) \vee b_2$$

$$\begin{aligned} b_1 \quad \text{by} \quad & (\neg b_1 \wedge b_0) \vee (b_1 \wedge \neg(\neg b_2 \wedge b_1 \wedge b_0)) \\ & \equiv (\neg b_1 \wedge b_0) \vee (b_1 \wedge (b_2 \vee \neg b_0)) \end{aligned}$$

$$b_0 \quad \text{by} \quad \neg b_0 \vee (b_0 \wedge \neg((\neg b_1 \vee \neg b_2) \wedge b_0)) \equiv \neg b_0 \vee (b_1 \wedge b_2)$$

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# General regression: correctness

## Theorem (correctness of $\text{regr}_o(\varphi)$ )

*Let  $\varphi$  be a formula,  $o$  an operator and  $s$  a state.*

*Then  $s \models \text{regr}_o(\varphi)$  iff  $o$  is applicable in  $s$  and  $\text{app}_o(s) \models \varphi$ .*

## Proof.

Let  $o = \langle \chi, e \rangle$ . Recall that  $\text{regr}_o(\varphi) = \chi \wedge \varphi_r \wedge \kappa$ , where  $\varphi_r$  and  $\kappa$  are as defined previously.

If  $o$  is inapplicable in  $s$ , then  $s \not\models \chi \wedge \kappa$ , both sides of the “iff” condition are false, and we are done. Hence, we only further consider states  $s$  where  $o$  is applicable. Let  $s' := \text{app}_o(s)$ .

We know that  $s \models \chi \wedge \kappa$  (because  $o$  is applicable), so the “iff” condition we need to prove simplifies to:

$$s \models \varphi_r \text{ iff } s' \models \varphi.$$

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## Proof (ctd.)

To show:  $s \models \varphi_r$  iff  $s' \models \varphi$ .

We show that for all formulae  $\psi$ ,  $s \models \psi_r$  iff  $s' \models \psi$ , where  $\psi_r$  is  $\psi$  with every  $a \in A$  replaced by  $EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$ .

The proof is by structural induction on  $\psi$ .

Induction hypothesis  $s \models \psi_r$  if and only if  $s' \models \psi$ .

Base cases 1 & 2  $\psi = \top$  or  $\psi = \perp$ : trivial, as  $\psi_r = \psi$ .

Base case 3  $\psi = a$  for some  $a \in A$ :

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# General regression: correctness

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$$\begin{aligned} s \models \psi_r &\text{ iff } s \models (\neg\psi')_r \text{ iff } s \models \neg(\psi'_r) \text{ iff } s \not\models \psi'_r \\ &\text{ iff (IH) } s' \not\models \psi' \text{ iff } s' \models \neg\psi' \text{ iff } s' \models \psi \end{aligned}$$

Inductive case 2  $\psi = \psi' \vee \psi''$ :

$$\begin{aligned} s \models \psi_r &\text{ iff } s \models (\psi' \vee \psi'')_r \text{ iff } s \models \psi'_r \vee \psi''_r \\ &\text{ iff } s \models \psi'_r \text{ or } s \models \psi''_r \\ &\text{ iff (IH, twice) } s' \models \psi' \text{ or } s' \models \psi'' \\ &\text{ iff } s' \models \psi' \vee \psi'' \text{ iff } s' \models \psi \end{aligned}$$

Inductive case 3  $\psi = \psi' \wedge \psi''$ : Very similar to inductive case 2, just with  $\wedge$  instead of  $\vee$  and “and” instead of “or”.

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# Emptiness and subsumption testing

The following two tests are useful when performing regression searches to avoid exploring unpromising branches:

- Test that  $regr_o(\varphi)$  does not represent the empty set (which would mean that search is in a dead end).  
For example,  $regr_{\langle a, \neg p \rangle}(p) \equiv a \wedge \perp \equiv \perp$ .
- Test that  $regr_o(\varphi)$  does not represent a subset of  $\varphi$  (which would make the problem harder than before).  
For example,  $regr_{\langle b, c \rangle}(a) \equiv a \wedge b$ .

Both of these problems are **NP-hard**.

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# Formula growth

The formula  $\text{regr}_{o_1}(\text{regr}_{o_2}(\dots \text{regr}_{o_{n-1}}(\text{regr}_{o_n}(\varphi))))$  may have size  $O(|\varphi||o_1||o_2|\dots|o_{n-1}||o_n|)$ , i. e., the product of the sizes of  $\varphi$  and the operators.

$\rightsquigarrow$  worst-case **exponential** size  $O(m^n)$

## Logical simplifications

- $\perp \wedge \varphi \equiv \perp, \top \wedge \varphi \equiv \varphi, \perp \vee \varphi \equiv \varphi, \top \vee \varphi \equiv \top$
- $a \vee \varphi \equiv a \vee \varphi[\perp/a], \neg a \vee \varphi \equiv \neg a \vee \varphi[\top/a],$   
 $a \wedge \varphi \equiv a \wedge \varphi[\top/a], \neg a \wedge \varphi \equiv \neg a \wedge \varphi[\perp/a]$
- idempotency, absorption, commutativity, associativity, ...

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# Restricting formula growth in search trees

**Problem** very big formulae obtained by regression

**Cause** **disjunctivity** in the (NNF) formulae  
(formulae **without disjunctions** easily convertible to  
small formulae  $l_1 \wedge \dots \wedge l_n$  where  $l_i$  are literals and  $n$   
is at most the number of state variables.)

**Idea** handle disjunctivity when generating search trees

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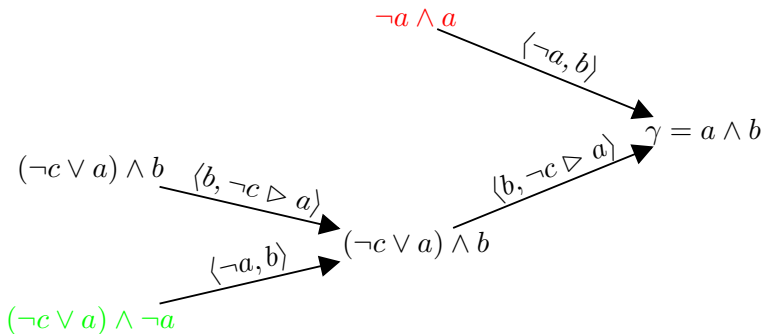
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# Unrestricted regression: search tree example

**Unrestricted regression:** do not treat disjunctions specially

Goal  $\gamma = a \wedge b$ , initial state  $I = \{a \mapsto 0, b \mapsto 0, c \mapsto 0\}$ .



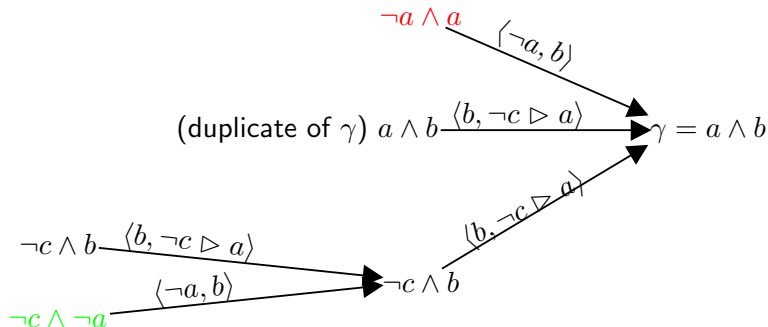
# Full splitting: search tree example

**Full splitting:** always remove all disjunctivity

Goal  $\gamma = a \wedge b$ , initial state  $I = \{a \mapsto 0, b \mapsto 0, c \mapsto 0\}$ .

$(\neg c \vee a) \wedge b$  in DNF:  $(\neg c \wedge b) \vee (a \wedge b)$

$\rightsquigarrow$  split into  $\neg c \wedge b$  and  $a \wedge b$



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# General splitting strategies

Alternatives:

- 1 Do nothing (**unrestricted regression**).
- 2 Always eliminate all disjunctivity (**full splitting**).
- 3 Reduce disjunctivity if formula becomes too big.

Discussion:

- With **unrestricted regression** the formulae may have **size that is exponential** in the number of state variables.
- With **full splitting** search tree can be **exponentially bigger** than without splitting.
- The third option lies between these two extremes.

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# Summary

- (Classical) **search** is a very important planning approach.
- Search-based planning algorithms differ along many dimensions, including
  - **search direction** (forward, backward)
  - **what each search node represents**  
(a state, a set of states, an operator sequence)
- **Progression search** proceeds forwards from the initial state.
  - If we use duplicate detection, each search node corresponds to a unique **state**.
  - If we do not use duplicate detection, each search node corresponds to a unique **operator sequence**.

# Summary (ctd.)

- **Regression search** proceeds backwards from the goal.
  - Each search node corresponds to a **set of states** represented by a **formula**.
  - Regression is simple for **STRIPS** operators.
  - The theory for **general regression** is more complex.
  - When applying regression in practice, additional considerations such as when and how to perform **splitting** come into play.

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