Intelligent Robot Studio
From Theory to Practice

Homework 2

Robot Kinematics and Motor Encoder

I. Theory

Question 1: Measure the radius of the driven wheel and the distance between two driven wheels. Based on your measurements, calculate the velocities of the left and right wheels so that the robot will go along a circle with the diameter of 2 m?

Question 2: Given your N20 geared encoder motor, calculate the maximum number of pulses you will get when the motor spins one revolution/round? Show details of your calculations.

Question 3: If your robot travels half of a circle with the diameter of 2 m, how many pulses will you get for the left and right wheels in the ideal condition (for example, no slippage, perfect mechanics, etc.).

Question 4*: As discussed in the lecture, the kinematics of a differential drive robot in the continuous time domain is given by:

$$\boxed{\begin{aligned} \dot{x} &= \frac{1}{2}(V_r + V_l)\cos\theta \\ \dot{y} &= \frac{1}{2}(V_r + V_l)\sin\theta \\ \dot{\theta} &= \frac{1}{l}(V_r - V_l) \end{aligned}} \tag{1}$$

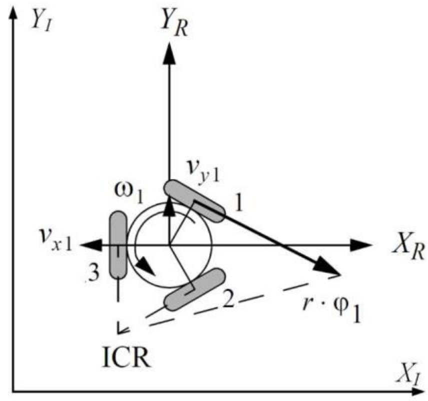
In practice, the robot is often controlled by a digital computer which works in the discrete time domain. Therefore, it is necessary to convert the kinematic equations to that domain. It can be done by approximating the derivative as:

$$f'(t) = \frac{f_t - f_{t-1}}{\Delta T} \tag{2}$$

where ΔT is the sampling period.

a) From equations (1) and (2), derive the kinematic equations of the robot in the discrete time domain.
b) Assume that at time $t_0 = 0$, the robot's pose is (0,0,0). Find its pose at time t_1 given that the velocities of the left and right wheels are $V_l = 1$ m/s, $V_r = 2$ m/s, respectively; the sampling period is 500 ms; and the distance l between the driven wheels is 30 cm.

Question 5: Given an omni-directional drive robot as below:



Its forward and inverse kinematics are given by:

- Forward kinematics:

$$\begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & 0 \\ \frac{-1}{3} & \frac{-1}{3} & \frac{2}{3} \\ \frac{-1}{3l} & \frac{-1}{3l} & \frac{-1}{3l} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

- Inverse kinematics:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} & -l \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} & -l \\ 0 & 1 & -l \end{bmatrix}^{-1} \begin{bmatrix} V_{XR} \\ V_{yR} \\ \dot{\theta}_R \end{bmatrix}$$

where l = 1.

a) Using forward kinematics, build a MATLAB Simulink model of the robot.
b) Calculate the three wheels' velocities so that the robot is moving in the X_R direction with a velocity V_{XR} = 1m/s. Run the simulation to confirm your results.
c) Calculate the three wheels' velocities so that the robot is moving in the diagonal direction in XY plane with V_{XR} = 1m/s and V_{YR} = 1m/s. The robot should not change its heading angle. Run the simulation to confirm your results.

II. Practice

Task 1: Write a program to control the motor to spin one round. Comment on the result (for example, how do you do that? is there any error? Why?)

Task 2: Write a program that controls the robot to travel a distance of 100 cm. Comment on the result (for example, how do you do that? is there any error? Why?)

Task 3: Write a program to count the number of pulses generated in each second and calculate the speed of the motor (in revolutions per minute – RPM).

Task 4: Write a program to drive the robot along a circle having the diameter of 2 m.

Task 5: Write a program to drive the robot to travel half of a circle with the diameter of 2 m. Comment on the result (for example, how do you do that? is there any error? Why?)