Following the work of Scully and Zubairy, it is well known that given an operator

$$\mathscr{L} \equiv \varepsilon^{\dagger} \boldsymbol{\Lambda}$$

acting on a state vector such that

$$\mathscr{L} |\psi\rangle \equiv |\boldsymbol{\gamma}\rangle ,$$

$$\boldsymbol{\Lambda} |\varphi_{\boldsymbol{\xi}}\rangle \equiv \varepsilon_{\Lambda} \Lambda \left(\boldsymbol{\xi}\right) |\varphi_{\boldsymbol{\xi}}\rangle ,$$

with

$$\boldsymbol{\zeta}^{\dagger} \left(\boldsymbol{\xi}\right) \boldsymbol{\Lambda} \left(\boldsymbol{\xi}\right) \equiv \left[\alpha + \iota\theta \left(\boldsymbol{\xi}\right) \beta^{-\alpha}\right]^{\beta} ,$$

then by inspection,

$$\boxed{\rho \equiv \epsilon + |\varphi_{\boldsymbol{\kappa}}\rangle \langle\varphi_{\boldsymbol{\kappa}}| \zeta^{\dagger} \left(\boldsymbol{\kappa}\right) |\boldsymbol{\gamma}\rangle \langle\boldsymbol{\gamma}| \zeta \left(\boldsymbol{\xi}\right) |\varphi_{\boldsymbol{\xi}}\rangle \langle\varphi_{\boldsymbol{\xi}}| .}$$