Some Laplace Transforms

$f(t)$	c_0	$F(s)$	$f(t)$	c_0	$F(s)$
1	0	$\frac{1}{s}$	$\frac{t^n}{n!}$	0	$\frac{1}{s^{n+1}}$
$e^{-\alpha t}$	$-\alpha$	$\frac{1}{s+\alpha}$	$e^{-\alpha t}t^n$	$-\alpha$	$\frac{n!}{(s+\alpha)^{n+1}}$
$\sin(\omega t)$	0	$\frac{\omega}{s^2+\omega^2}$	$\cos(\omega t)$	0	$\frac{s}{s^2+\omega^2}$
$e^{\alpha t}\sin(\omega t)$	α	$\frac{\omega}{(s-\alpha)^2+\omega^2}$	$e^{\alpha t}\cos(\omega t)$	α	$\frac{s-\alpha}{(s-\alpha)^2+\omega^2}$
$\sinh(\omega t)$	$\lvert\omega\rvert$	$\frac{\omega}{s^2-\omega^2}$	$\cosh(\omega t)$	$\lvert\omega\rvert$	$\frac{s}{s^2-\omega^2}$
$H_\alpha(t)$	0	$\frac{e^{-\alpha s}}{s}$	$\delta_\alpha(t)$	$-\infty$	$e^{-\alpha s}$

Some Properties of the Laplace Transforms

Let $f, g : [0, +\infty) \to \mathbb{R}$ be piecewise continuous functions with piecewise continuous derivatives. Assume there exists $M \geq 0$ and $c_1,\ c_2 \in \mathbb{R}$ such that

$$|f(t)| \leq Me^{c_1 t}, \qquad |g(t)| \leq Me^{c_2 t}, \qquad \forall t \in [0, +\infty).$$

Then there holds

$$(\imath.)\ \mathcal{L}\left(\frac{d^n}{dt^n}f(t)\right)(s) = s^n\mathcal{L}\left(f(t)\right) - s^{n-1}f(0) - \ldots - s\frac{d^{n-2}}{dt^{n-2}}f(0) - \frac{d^{n-1}}{dt^{n-1}}f(0),$$

$$\forall s > c_1,\ (f \in C^{n-1}([0,\infty)),\ \frac{d^n}{dt^n}f \text{ piecewise continuous})$$

$$(\imath\imath.)\ \mathcal{L}\left(\int_0^t f(\tau)d\tau\right)(s) = \frac{1}{s}\mathcal{L}\left(f(t)\right)(s), \qquad \forall s > c_1,$$

$$(\imath\imath\imath.)\ \mathcal{L}\left((-1)^n t^n f(t)\right)(s) = \frac{d^n}{ds^n}\mathcal{L}\left(f(t)\right)(s), \qquad \forall s > c_1,$$

$$(\imath v.)\ \mathcal{L}\left(e^{-\alpha t}f(t)\right)(s) = \mathcal{L}\left(f(t)\right)(s+\alpha), \qquad \forall s > c_1 + \alpha,\ \alpha \geq 0,$$

$$(v.)\ \mathcal{L}\left(H_\alpha(t)f(t-\alpha)\right)(s) = e^{-\alpha s}\mathcal{L}\left(f(t)\right)(s), \qquad \forall s > c_1,\ \alpha \geq 0,$$

$$(v\imath.)\ \mathcal{L}\left((f * g)(t)\right)(s) = \mathcal{L}(f(t))(s) \cdot \mathcal{L}(g(t))(s), \qquad \forall s > \max(c_1, c_2).$$