Let

$$F(x)+C=\int f(x)dx \qquad f(x)+C=\int f'(x)dx \qquad g(x)+C=\int g'(x)dx$$

Here $F(x)+C$ is an antiderivative for $f(x)$, and $f(x)+C$ is an antiderivative for $f'(x)$.

Integration by substitution:

$$\int f(g(x))g'(x)dx=\int f(u)du=F(u)+C=F(g(x))+C$$

$u=g(x),\ du=g'(x)dx$

Integration by parts:

$$\int f(x)g'(x)dx=f(x)g(x)-\int g(x)f'(x)dx$$

This is often used when differentiating $f(x)$ makes it go away. Let $u=f(x)$ and $dv=g'(x)dx$ (u is something you can differentiate, dv is something you can integrate). Find $du=f'(x)dx$ and $v(x)=g(x)$.

$$\int f(x)g'(x)dx=\int udv=uv-\int vdu$$

Integration by partial fractions:

We'll look at this by example. Consider the integral

$$\int \frac{x-1}{x^2+x}dx$$

We write the integrand as the sum of two fractions that are *simpler*. The denominator is $x(x+1)$, and we attempt

$$\frac{x-1}{x^2+x}=\frac{A}{x}+\frac{B}{x+1}$$

I need to solve for A and B. Find a common denominator

$$\frac{x-1}{x^2+x}=\frac{A(x+1)}{x^2+x}+\frac{Bx}{x^2+x}$$

and set like terms in the numerator to be equal. The term with x is $x=Ax+Bx$ so $A+B=1$. The constant term is $-1=A$. The solution is $A=-1$ and $B=2$. Putting this back in the original integral:

$$\int \frac{x-1}{x^2+x}dx=\int\left(\frac{-1}{x}+\frac{2}{x+1}\right)dx=-\ln(|x|)+2\ln(|x+1|)+C=\ln\left(\frac{(x+1)^2}{|x|}\right)+C$$

We do not need the absolute values on $(x+1)^2$ because a squared number is never negative.

Now it's your turn. Some of these integrals are pretty easy and you should have them in memory. Some are harder and use the techniques we just went over.

1. $\int x^2 dx =$
   this is an indefinite integral, an antiderivative

2. $\int x^{-3} dx =$

3. $\int (1/x) dx =$

4. $\int_{-3}^{3} \sin(x) dx =$
   this is a definite, not an indefinite, integral

5. $\int_{0}^{\pi/2} \cos(2x) dx =$

6. $\int \tan(x) dx =$
   $\tan(x) = \sin(x)/\cos(x)$; substitution

7. $\int (x^2 + 1/x^2) dx =$

8. $\int_{-\infty}^{0} 6e^{2x} dx =$

9. $\int x \exp(x^2) dx =$

10. $\int (2x+3)\sin(x^2+3x) dx =$

11. $\int xe^x dx =$

12. $\int \ln(x) dx =$
by parts

13. $\int \frac{1}{(x^2-1)} dx =$

14. $\int (\ln(x)/x)\, dx =$

15. $\int \frac{x}{(x^2+3x+2)} dx =$

16. $\int \left[2x^3 + \frac{5}{x^4}\right] dx$