

First Name: _____ **Last Name:** _____

Quiz 3

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- 5 minute individual quiz;
 - Answer the questions in the space provided. If you run out of space, continue onto the back of the page. Additional space is provided at the end;
 - **Show and explain all work;**
 - **Underline** the answer of each steps;
 - The use of books, personal notes, **calculator**, cellphone, laptop, and communication with others is forbidden;
 - By taking this quiz, you agree to follow the university's code of academic integrity.
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Some Laplace Transforms

f	$\mathcal{L}(f)$	f	$\mathcal{L}(f)$
1	$\frac{1}{s} \quad s > 0$	$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}} \quad s > 0$
$e^{-\alpha t}$	$\frac{1}{s+\alpha} \quad s > -\alpha$	$e^{-\alpha t} t^n$	$\frac{n!}{(s+\alpha)^{n+1}} \quad s > -\alpha$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2} \quad s > 0$	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2} \quad s > 0$
$e^{\alpha t} \sin(\omega t)$	$\frac{\omega}{(s-\alpha)^2+\omega^2} \quad s > \alpha$	$e^{\alpha t} \cos(\omega t)$	$\frac{s-\alpha}{(s-\alpha)^2+\omega^2} \quad s > \alpha$
$\sinh(\omega t)$	$\frac{\omega}{s^2-\omega^2} \quad s > \omega $	$\cosh(\omega t)$	$\frac{s}{s^2-\omega^2} \quad s > \omega $
$H_\alpha(t)$	$\frac{e^{-\alpha s}}{s} \quad s > 0$	$\delta_\alpha(t)$	$e^{-\alpha s} \quad s > -\infty$

Some Properties of the Laplace Transforms

Let $f, g : [0, +\infty) \rightarrow \mathbb{R}$ be piecewise continuous functions with piecewise continuous derivatives. Assume there exists $K \geq 0$ and $a_1, a_2 \in \mathbb{R}$ such that

$$|f(t)| \leq K e^{a_1 t}, \quad |g(t)| \leq M e^{a_2 t}, \quad \forall t \in [0, +\infty).$$

Then there holds

$$(i.) \quad \mathcal{L} \left(\frac{d^n}{dt^n} f(t) \right) (s) = s^n \mathcal{L}(f(t)) - s^{n-1} f(0) - \dots - s \frac{d^{n-2}}{dt^{n-2}} f(0) - \frac{d^{n-1}}{dt^{n-1}} f(0),$$

$$\forall s > a_1, (f \in C^{n-1}([0, \infty)), \frac{d^n}{dt^n} f \text{ piecewise continuous})$$

$$(ii.) \quad \mathcal{L} \left(\int_0^t f(\tau) d\tau \right) (s) = \frac{1}{s} \mathcal{L}(f(t)) (s), \quad \forall s > a_1,$$

$$(iii.) \quad \mathcal{L}((-1)^n t^n f(t)) (s) = \frac{d^n}{ds^n} \mathcal{L}(f(t)) (s), \quad \forall s > a_1,$$

$$(iv.) \quad \mathcal{L}(e^{-\alpha t} f(t)) (s) = \mathcal{L}(f(t)) (s + \alpha), \quad \forall s > a_1 + \alpha, \alpha \geq 0,$$

$$(v.) \quad \mathcal{L}(H_\alpha(t) f(t - \alpha)) (s) = e^{-\alpha s} \mathcal{L}(f(t)) (s), \quad \forall s > a_1, \alpha \geq 0,$$

$$(vi.) \quad \mathcal{L}((f * g)(t)) (s) = \mathcal{L}(f(t))(s) \cdot \mathcal{L}(g(t))(s), \quad \forall s > \max(a_1, a_2).$$

Exercise 1 100%

Find the solution of

$$y'' - 2y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 2$$

using the Laplace transform.

Quiz 3: solutions

Exercise 1 100%

Applying the Laplace tranform to both sides of the ODE one gets

$$s^2Y - 2sY + 2Y = s.$$

Hence,

$$Y(s) = \frac{s}{s^2 - 2s + 2}.$$

Since $s^2 - 2s + 2$ does not have a root in \mathbb{R} , $Y(s)$ is already a simple fraction and we have

$$Y(s) = \frac{s}{(s-1)^2 + 1} = \frac{s-1}{(s-1)^2 + 1} + \frac{1}{(s-1)^2 + 1}.$$

Using the provided table we directly get

$$y(t) = e^t \cos(t) + e^t \sin(t).$$