

First Name: _____ **Last Name:** _____

Quiz 2

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- 5 minute individual quiz;
 - Answer the questions in the space provided. If you run out of space, continue onto the back of the page. Additional space is provided at the end;
 - **Show and explain all work;**
 - **Underline** the answer of each steps;
 - The use of books, personal notes, **calculator**, cellphone, laptop, and communication with others is forbidden;
 - By taking this quiz, you agree to follow the university's code of academic integrity.
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Exercise 1 100%

Compute the Wronskian of $f(x) = e^x \sin(x)$ and $g(x) = e^x \cos(x)$. Deduce whether f and g are linearly independent (make sure to justify your answer).

Quiz 2: solutions

Exercise 1 100%

We compute

$$W(f, g)(x) = f(x)g'(x) - f'(x)g(x)$$

(fine if you use $f'(x)g(x) - f(x)g'(x)$ for the particular function given. Notice that

$$f(x) = e^x \sin(x) \quad \rightarrow \quad f'(x) = e^x \sin(x) + e^x \cos(x)$$

and

$$g(x) = e^x \cos(x) \quad \rightarrow \quad g'(x) = e^x \cos(x) - e^x \sin(x).$$

Therefore

$$W(f, g)(x) = e^{2x}(\sin(x) \cos(x) - \sin^2(x)) - e^{2x}(\sin(x) \cos(x) + \cos^2(x)) = -e^{2x}(\sin^2(x) + \cos^2(x))$$

and

$$W(f, g)(x) = -e^{2x}.$$

Because $W(f, g)(x) < 0$, f and g are linearly independent.