Ordinary Differential Equations
MATH 308H - 201
A. Bonito

January 24
Spring 2019

First Name: ____________________ **Last Name:** ____________________

Quiz 1

- 5 minute individual quiz;
- Answer the questions in the space provided. If you run out of space, continue onto the back of the page. Additional space is provided at the end;
- **Show and explain all work**;
- **Underline** the answer of each steps;
- The use of books, personal notes, **calculator**, cellphone, laptop, and communication with others is forbidden;
- By taking this quiz, you agree to follow the university's code of academic integrity.

Exercise 1 **20%**

For each differential equation, provide its order and say whether it is linear or not:

1.
$$\frac{dy}{dx}(x) = \frac{y(x)(2-3x)}{x(1-3y(x))}$$

2.
$$3\frac{d^2y}{dx^2}(x) + 4\frac{dy}{dx}(x) + 9y(x) = 2\cos(3y(x))$$

Exercise 2 **80%**

The quantity of a radiactive material $Q(t)$ disintegrates according to the following ODE

$$\frac{d}{dt}Q(t) = -rQ(t),$$

where $r > 0$.

- Find an exptession of $Q(t)$ at any time t.
- Find the time required for $Q(t)$ to decay to $1/p$ for some $p > 1$ its original amount.

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Quiz 1: solutions

Exercise 1 **20%**

1. First order, nonlinear.
2. Second order, nonlinear.

Exercise 2 **80%**

- The ODE is a first order linear ODE but is also separable. We can use either techniques seen in class. We go for the separable point of view and divide by $Q(t) > 0$:

$$\frac{1}{Q}Q' = -r.$$

Taking the anti-derivate and using the substitution rule yield

$$\int_{Q=Q(t)} \frac{1}{Q} dq = -rt + C$$

for any constant C. This implies

$$\ln |Q(t)| = -rt + C$$

or

$$|Q(t)| = Ce^{-rt}$$

for any positive constant. Because $Q(t)$ does not change sign (continuous) and we are interested in positive solutions, we deduce that

$$\boxed{Q(t) = Ce^{-rt}}$$

for any positive constant C, which is the desired expression.

- Notice that if Q_0 denotes the original amount, we are looking for t^* such that

$$Q_0 e^{-rt_*} = \frac{1}{p} Q_0$$

or

$$\boxed{t_* = \ln p/r}$$