Ordinary Differential Equations | March 28
MATH 308H - 201 | Spring 2019
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First Name: ____________________ **Last Name:** ____________________

Quiz 4

- 5 minute individual quiz;
- Answer the questions in the space provided. If you run out of space, continue onto the back of the page. Additional space is provided at the end;
- **Show and explain all work**;
- **Underline** the answer of each steps;
- The use of books, personal notes, **calculator**, cellphone, laptop, and communication with others is forbidden;
- By taking this quiz, you agree to follow the university's code of academic integrity.

Exercise 1 **40%**

Consider the following set of vectors

$$\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$$

and determine whether they are linearly independent.

Exercise 2 **60%**

Find *one* eigenvalues and one associated eigenvector of the matrix

$$\begin{pmatrix} -3 & 3/4 \\ -5 & 1 \end{pmatrix}.$$

Quiz 4: solutions

Exercise 1 **40%**

To check independence, we compute

$$\det\begin{pmatrix}2 & 1 & -1\\ 1 & 1 & 2\\ 0 & 0 & 0\end{pmatrix} = \begin{pmatrix}2 & 1 & -1\\ 0 & \frac{1}{2} & \frac{5}{2}\\ 0 & 0 & 0\end{pmatrix} = 0.$$

We could also have used the row formula

$$\det(A) = \sum_{i=1}^{3}(-1)^{i+3}a_{3,i}\det(A(3,i)) = 0$$

because $a_{3,i} = 0$, $i = 1, 2, 3$.
Notice that we can therefore find nonzero $\alpha_i = 1, 2, 3$ such that

$$\alpha_1\begin{pmatrix}2\\1\\0\end{pmatrix} + \alpha_2\begin{pmatrix}0\\1\\0\end{pmatrix} + \alpha_3\begin{pmatrix}-1\\2\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}.$$

Indeed, this is equivalent to

$$\begin{cases} 2\alpha_1 - \alpha_3 = 0\\ \alpha_1 + \alpha_2 + 2\alpha_3 = 0\\ 0 = 0\end{cases}.$$

For instance $\alpha_1 = 1$, $\alpha_2 = -5$, $\alpha_3 = 2$ is a non trivial solution. The vectors are linarly dependent and

$$\begin{pmatrix}2\\1\\0\end{pmatrix} = 5\begin{pmatrix}0\\1\\0\end{pmatrix} - 2\begin{pmatrix}-1\\2\\0\end{pmatrix}.$$

Exercise 2 **60%**

The eigenvalues are given by

$$\det(A - \lambda I) = 0.$$

This is

$$\det\begin{pmatrix}-3-\lambda & 3/4\\ -5 & 1-\lambda\end{pmatrix} = 0$$

or computing the determinant

$$\lambda^2 + 2\lambda + 3/4 = 0$$

The solutions of the above equation are given by

$$\lambda_1 = -\frac{1}{2} \qquad \lambda_2 = -\frac{3}{2}.$$

The eigenvectors $\xi^1 = (\xi_1, \xi_2)^t$ associated with λ_1 satisfy

$$-5\xi_1 + \frac{3}{2}\xi_2 = 0$$

or

$$\xi^1 = \alpha \begin{pmatrix} 1 \\ -\frac{3}{10} \end{pmatrix}$$

for any constant α. For instance

$$\xi^1 = \begin{pmatrix} 10 \\ -3 \end{pmatrix}$$

The eigenvectors $\xi^2 = (\xi_1, \xi_2)^t$ associated with λ_2 satisfy

$$-3\xi_1 + \frac{3}{2}\xi_2 = 0$$

or

$$\xi^2 = \alpha \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$$

for any constant α. For instance,

$$\xi^2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$