Ordinary Differential Equations
MATH 308 - 512
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March 7
Spring 2019

First Name: ____________________ **Last Name:** ____________________

Quiz 3

- 5 minute individual quiz;
- Answer the questions in the space provided. If you run out of space, continue onto the back of the page. Additional space is provided at the end;
- **Show and explain all work**;
- **Underline** the answer of each steps;
- The use of books, personal notes, **calculator**, cellphone, laptop, and communication with others is forbidden;
- By taking this quiz, you agree to follow the university's code of academic integrity.

Some Laplace Transforms

f	$\mathcal{L}(f)$		f	$\mathcal{L}(f)$	
1	$\frac{1}{s}$	$s > 0$	$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$	$s > 0$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$s > -\alpha$	$e^{-\alpha t}\ t^n$	$\frac{n!}{(s+\alpha)^{n+1}}$	$s > -\alpha$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$s > 0$	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	$s > 0$
$e^{\alpha t}\sin(\omega t)$	$\frac{\omega}{(s-\alpha)^2+\omega^2}$	$s > \alpha$	$e^{\alpha t}\cos(\omega t)$	$\frac{s-\alpha}{(s-\alpha)^2+\omega^2}$	$s > \alpha$
$\sinh(\omega t)$	$\frac{\omega}{s^2-\omega^2}$	$s > \|\omega\|$	$\cosh(\omega t)$	$\frac{s}{s^2-\omega^2}$	$s > \|\omega\|$
$H_\alpha(t)$	$\frac{e^{-\alpha s}}{s}$	$s > 0$	$\delta_\alpha(t)$	$e^{-\alpha s}$	$s > -\infty$

Some Properties of the Laplace Transforms

Let $f, g : [0, +\infty) \to \mathbb{R}$ be piecewise continuous functions with piecewise continuous derivatives. Assume there exists $K \geqslant 0$ and $a_1,\ a_2 \in \mathbb{R}$ such that

$$|f(t)| \leqslant K e^{a_1 t}, \qquad |g(t)| \leqslant M e^{a_2 t}, \qquad \forall t \in [0, +\infty).$$

Then there holds

$$(\imath.)\ \mathcal{L}\left(\frac{d^n}{dt^n} f(t)\right)(s) = s^n \mathcal{L}\left(f(t)\right) - s^{n-1} f(0) - \dots - s\frac{d^{n-2}}{dt^{n-2}} f(0) - \frac{d^{n-1}}{dt^{n-1}} f(0),$$
$$\forall s > a_1,\ (f \in C^{n-1}([0,\infty)),\ \frac{d^n}{dt^n} f \text{ piecewise continuous})$$

$$(\imath\imath.)\ \mathcal{L}\left(\int_0^t f(\tau) d\tau\right)(s) = \frac{1}{s}\mathcal{L}\left(f(t)\right)(s), \qquad \forall s > a_1,$$

$$(\imath\imath\imath.)\ \mathcal{L}\left((-1)^n t^n f(t)\right)(s) = \frac{d^n}{ds^n}\mathcal{L}\left(f(t)\right)(s), \qquad \forall s > a_1,$$

$$(\imath v.)\ \mathcal{L}\left(e^{-\alpha t} f(t)\right)(s) = \mathcal{L}\left(f(t)\right)(s+\alpha), \qquad \forall s > a_1 + \alpha,\ \alpha \geqslant 0,$$

$$(v.)\ \mathcal{L}\left(H_\alpha(t) f(t-\alpha)\right)(s) = e^{-\alpha s}\mathcal{L}\left(f(t)\right)(s), \qquad \forall s > a_1,\ \alpha \geqslant 0,$$

$$(v\imath.)\ \mathcal{L}\left((f * g)(t)\right)(s) = \mathcal{L}(f(t))(s) \cdot \mathcal{L}(g(t))(s), \qquad \forall s > \max(a_1, a_2).$$

Exercise 1 **100%**

Find the solution of

$$y'' - 2y' + 2y = 0, \qquad y(0) = 0, \qquad y'(0) = 1$$

using the Laplace transform.

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Quiz 3: solutions

Exercise 1 **100%**

Applying the Laplace tranform to both sides of the ODE one gets

$$s^2Y - 1 - 2sY + 2Y = 0.$$

Hence,

$$Y(s) = \frac{1}{s^2 - 2s + 2}.$$

Since $s^2 - 2s + 2$ does not have a root in $\mathbb{R}$, $Y(s)$ is already a simple fraction and we have

$$Y(s) = \frac{1}{(s-1)^2 + 1}.$$

Using the provided table we directly get

$$y(t) = e^t \sin(t).$$