Numerical Methods for PDEs
MATH 610 - 600
A. Bonito

Due April 23
Spring 2018

Programming Assigment 3

Exercise 1 **100%**

Euler method for finite element approximations for parabolic problems

Consider the following parabolic initial boundary value problem: find $u(x,y,t)$ such that

$$\begin{aligned} \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} + qu &= f(x,y,t) && \text{in} \quad \Omega, \quad t > 0, \\ \frac{\partial u}{\partial \nu} &= g && \text{on} \quad \Gamma, \quad t > 0, \\ u(x,y,0) &= u_0(x,y) && \text{in} \quad \Omega, \end{aligned}$$

where Γ is the boundary of Ω, and ν is the outer unit normal vector to Γ.

Solve the given below problems by approximating the corresponding boundary value problem using linear triangular finite elements on a partition of the domain generated by TRIANGLE and **explicit and implicit** Euler methods in time with uniform time step k. Consider meshes with $|\tau| \leqslant 1/m^2$, $m = 10, 20, 40$, where $|\tau|$ is the maximal area of the triangular elements. Also, in time t use meshes with approximately 20, 40, and 80 mesh points for implicit Euler. For the explicit Euler use time-steps that give stability and also large enough time-step for which the scheme is not stable.
Submit a report with the information regarding the given below problems.

Specifications

Use **double precision** arithmetics. You should use your program from the previous programming assignment.

Computational examples

Problem 1. Take $\Omega = (0,1) \times (0,1)$, $0 < t < 3$, $q = 5$, $g = 0$ and $f(x,y,t)$ is such that the exact solution is

$$u(x,y,t) = te^{-t} \cos(3\pi x) \cos(\pi y)$$

and $u_0(x,y) = 0$. Present in a table the L^2- and the H^1-norms of the error $u(x,y,t_n) - u_h^n(x,y)$ for $t = 1$ and $t = 3$.

Problem 2. Solve the problem with $q = 1$, $f(x) = 1$, Ω is a polygon with vertices $(0,0)$, $(0.5,0)$, $(1,1)$, $(0,2)$, and $u_0(x,y,t) = 0$. Plot the solution for $t = 1$ and $t = 3$.

Problem 3. The domain $\Omega = (0,1)^2 \setminus \bar{\Omega}_1$, where $\Omega_1 = \{|x - 0.5| < 0.25, \quad \{|y - 0.5| < 0.25\}$. Take $q = 1$, $g = 1$, $f(x,y) = xy$, and $u_0(x,y,t) = 0$. Plot the solution for $t = 1$ and $t = 3$.