Ordinary Differential Equations
MATH 308 - 512
A. Bonito

March 28
Spring 2019

First Name: ____________________ **Last Name:** ____________________

Quiz 4

- 5 minute individual quiz;
- Answer the questions in the space provided. If you run out of space, continue onto the back of the page. Additional space is provided at the end;
- **Show and explain all work**;
- **Underline** the answer of each steps;
- The use of books, personal notes, **calculator**, cellphone, laptop, and communication with others is forbidden;
- By taking this quiz, you agree to follow the university's code of academic integrity.

Exercise 1 **40%**

Consider the following set of vectors

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

and determine whether they are linearly independent. If they are not, find a relation among them.

Exercise 2 **60%**

Find *one* eigenvalues and one associated eigenvector of the matrix

$$\begin{pmatrix} -3 & 3/4 \\ -5 & 1 \end{pmatrix}.$$

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Quiz 4: solutions

Exercise 1 **40%**

Let us try to find α_1, α_2, $\alpha_3 \in \mathbb{R}$ such that

$$\alpha_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

This is equivalent to

$$\begin{cases} \alpha_1 + \alpha_3 = 0 \\ \alpha_1 + \alpha_2 = 0 \\ \alpha_2 + \alpha_3 = 0 \end{cases}$$

which has the unique solution $\alpha_1 = \alpha_2 = \alpha_3 = 0$ so that the vectors are linearly independent.

Altenatively, we could have simply computed the determinant

$$\det \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \det \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} = \det \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix} = 2,$$

which is nonzero and therefore guarantee independence.

Exercise 2 **60%**

The eigenvalues are given by

$$\det(A - \lambda I) = 0.$$

This is

$$\det \begin{pmatrix} -3-\lambda & 3/4 \\ -5 & 1-\lambda \end{pmatrix} = 0$$

or computing the determinant

$$\lambda^2 + 2\lambda + 3/4 = 0$$

The solutions of the above equation are given by

$$\lambda_1 = -\frac{1}{2} \qquad \lambda_2 = -\frac{3}{2}.$$

The eigenvectors $\xi^1 = (\xi_1, \xi_2)^t$ associated with λ_1 satisfy

$$-5\xi_1 + \frac{3}{2}\xi_2 = 0$$

or

$$\xi^1 = \alpha \begin{pmatrix} 1 \\ -\frac{3}{10} \end{pmatrix}$$

for any constant α. For instance

$$\xi^1 = \begin{pmatrix} 10 \\ -3 \end{pmatrix}$$

The eigenvectors $\xi^2 = (\xi_1, \xi_2)^t$ associated with λ_2 satisfy

$$-3\xi_1 + \frac{3}{2}\xi_2 = 0$$

or

$$\xi^2 = \alpha \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$$

for any constant α. For instance,

$$\xi^2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$