First Name: ____________________ **Last Name:** ____________________

Homework 4

Exercise 1 30% (BY HAND)

Let $f(x) = x \ln |x|$ and $x_0 = 7.4$, $x_1 = 7.6$, $x_2 = 7.8$, $x_3 = 8.0$. Determine the most accurate three point formula approximation of $f'(7.8)$ and use the error bound formula to determine the error.

Exercise 2 30% (BY HAND)

Consider the second difference approximation

$$\frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

of $f''(x)$. Recall that there exists $\xi \in [x-h, x+h]$ such that

$$f''(x) - \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} = -f^{(4)}(\xi)\frac{h^2}{12}$$

- Assume that every evaluation $f(y)$ is perturbed by the roundoff error

  $$\bar{f}(y) = f(y) + e(y)$$

  where $|e(y)| \leqslant \epsilon$. Determine an error bound for

  $$\left| f''(x) - \frac{\bar{f}(x+h) - 2\bar{f}(x) + \bar{f}(x-h)}{h^2} \right|$$

  for any $x \in [a+h, b-h]$ provided $\max_{s\in[a,b]} |f^{(4)}(s)| \leqslant M$.
- Determine the optimal value of h (as a function of ϵ) which minimizes the error and deduce the smallest error achievable.

Exercise 3 40% (MATLAB)

(MATLAB)
Let $f(x) = e^x$ and consider the following two approximations of $f'(0)$:

- $f'(0) \approx r_1(h) = \frac{f(h)-f(0)}{h}$;
- $f'(0) \approx r_2(h) = \frac{1}{h}\left(-\frac{3}{2}f(0) + 2f(h) - \frac{1}{2}f2h)\right)$.

1. Compute the above two difference approximations and report the results and errors $e_i(h) = |r_i(h) - 1|$ for $i = 1, 2$ with $h = 0.1, 0.01, 0.001, ..., 10^{-16}$. Make sure that you print your results in scientific notation. You will have a table with 16 lines, each containing

   $$h, r_1(h), e_1(h), r_2(h), e_2(h).$$

   Also plot in a log-log scale $e_1(h)$ and $e_2(h)$ vs h. What do you deduce from this plot? The log-log plot can be easily obtained in matlab using the command

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loglog(H,E1,H,E2)
```

where H, $E1$ and $E2$ are the arrays (of dimension 16) containing the values of h, $e1$ and $e2$.