1. - Plot the following points in the complex plane: i, $2-i$, $2+i$, -5.
   - Compute $|7-2i|$.
   - Compute the distance from $7-2i$ to 0 in the complex plane.
   - Compute $|2e^{6i}|$.
   - Compute $|(2e^{6i})(7-2i)|$.
2. (Multiplicative inverse) Let $z = a + ib$ be a nonzero complex number (so at least one of the real numbers a, b is nonzero). Then $1/z$ is the number such that $(z)(1/z) = 1$, and there is only one such number.
   - Show that
   $$1/z = \frac{a-ib}{a^2+b^2}$$
   [Hint: Just show that multiplying the right hand side by $a+ib$ produces the number 1, then the right hand side must be a correct formula for $1/z$.]
   - Compute real numbers a, b such that $1/(2+3i) = a + ib$.
   - Compute real numbers a, b such that $(1-2i)/(2+3i) = a + ib$.
   - If the polar form of z is $Re^{i\theta}$, then what is the polar form of $1/z$?
3. Let $z = 2 + i2$.
   - Find the polar form of z (i.e., find real numbers R and θ such that $R > 0$ and $2 + i2 = Re^{i\theta}$).
   - Find real numbers c and d such that $1/z = c + id$.
4. Now z is a complex number written in the form $z = e^{x+iy}$, where x and y are real.
   - Let $Re^{i\theta}$ be the polar form of z. Give formulas using x and y for R and $\tan(\theta)$. For which z are the formulas valid?
   - Compute the polar form of e^{2-3i}.
5.
6. Find a trig identity which for any real numbers θ expresses $cos(3\theta)$ in terms of $cos(\theta)$ and $sin(\theta)$. [Hint: $e^{i3\theta} = e^{i\theta}e^{i\theta}e^{i\theta}$.]
7. Complex conjugates) Let $z = a + ib$, then the complex conjugate z is defined to be $\overline{z} = a - ib$.
   - How are the locations of z and $\overline{z}$ in the complex plane related?
   - Check that $\overline{z}z = |z|^2 = a^2 + b^2$.
   - Show that if z is nonzero, then $1/z = (\overline{z})/(|z|^2)$. (Multiply z by this expression and check that the product is 1.)
   - Use the previous formula to find $1/z$ if $z = 2 + 3i$.
8. (Complex conjugation respects arithmetic)
   - Show that $(\overline{w})(\overline{z}) = \overline{wz}$. To do this, given real numbers a, b, c, d, simply compute to check that $(a-ib)(c-id) = \overline{(a+ib)(c+id)}$.
   - Similarly check that $\overline{w+z} = \overline{w} + \overline{z}$.
   - Use the first item to give an elementary proof that for any complex numbers w and z, we have $|wz| = |w||z|$. (HINT: $|wz|^2 = (wz)(\overline{wz})$, and $|w|^2|z|^2 = w\overline{w}z\overline{z}$.)
9. From the last problem, it follows that if $p(z)$ is a polynomial with real coefficients and w is a complex number such that $p(w) = 0$, then also $p(\overline{w}) = 0$, why? Check that any polynomial of the form $q(z) = (z-w)(z-\overline{w})$ is a polynomial with real coefficients.
10. (Roots of unity) Let n be a positive integer. The complex numbers $e^{2\pi i/n}$ has its nth power equal to 1. Likewise, if k is a nonnegative integer in the set $0, 1, ..., n-1$, then $e^{2\pi ki/n}$ also has its nth power equal to 1. Such a number is

called an nth root of unity. These numbers can be drawn on the unit circle in the complex plane.

- Draw all the fourth roots of unity on the unit circle.
- Draw (in another picture) all the eighth roots of unity.

11. Let $z = -1 + i$.
    - Write z in polar form.
    - Use the polar form to compute z^{16}.
    - If n is a positive integer and M is a positive real number, then the equation $z^n = M$ has exactly the following n solutions: $M^{1/n}e^{2\pi ki/n}$, $k = 0, 1, 2, ..., n-1$, why? Find all solutions of the equation $z^8 = 16$, and plot these solutions in the complex plane.
12. (Differentiation) All the usual formulas for differentiation work for polynomials, cosine, sine and the exponential function considered over complex numbers. For example, since $e^{iz} = cos(z) + isin(z)$, we can say the derivative of e^{iz} with respect to z is $ie^{iz} = -sin(z) + icos(z)$. (You can check this one by differentiating the power series.) These formulas in particular are true if we restrict inputs to real numbers (in this case one often writes t in place of z). Compute the second derivative of e^{iz} at $z = \pi/2$.
13. (DeMoivre) To understand why $e^{iz} = cos(z) + isin(z)$, compute by hand the first eight terms of these series, and compare.