Some Laplace Transforms

f	$\mathcal{L}(f)$		f	$\mathcal{L}(f)$	
1	$\frac{1}{s}$	$s > 0$	$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$	$s > 0$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$s > -\alpha$	$e^{-\alpha t}\ t^n$	$\frac{n!}{(s+\alpha)^{n+1}}$	$s > -\alpha$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$s > 0$	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	$s > 0$
$e^{\alpha t}\sin(\omega t)$	$\frac{\omega}{(s-\alpha)^2+\omega^2}$	$s > \alpha$	$e^{\alpha t}\cos(\omega t)$	$\frac{s-\alpha}{(s-\alpha)^2+\omega^2}$	$s > \alpha$
$\sinh(\omega t)$	$\frac{\omega}{s^2-\omega^2}$	$s > \|\omega\|$	$\cosh(\omega t)$	$\frac{s}{s^2-\omega^2}$	$s > \|\omega\|$
$H_\alpha(t)$	$\frac{e^{-\alpha s}}{s}$	$s > 0$	$\delta_\alpha(t)$	$e^{-\alpha s}$	$s > -\infty$

Some Properties of the Laplace Transforms

Let $f, g : [0, +\infty) \to \mathbb{R}$ be piecewise continuous functions with piecewise continuous derivatives. Assume there exists $K \geq 0$ and $a_1,\ a_2 \in \mathbb{R}$ such that

$$|f(t)| \leq K e^{a_1 t}, \qquad |g(t)| \leq M e^{a_2 t}, \qquad \forall t \in [0, +\infty).$$

Then there holds

$(\imath.)$ $\mathcal{L}\left(\frac{d^n}{dt^n} f(t)\right)(s) = s^n \mathcal{L}\left(f(t)\right) - s^{n-1} f(0) - \ldots - s\frac{d^{n-2}}{dt^{n-2}} f(0) - \frac{d^{n-1}}{dt^{n-1}} f(0),$

$\forall s > a_1,\ (f \in C^{n-1}([0,\infty)),\ \frac{d^n}{dt^n} f$ piecewise continuous)

$(\imath\imath.)$ $\mathcal{L}\left(\int_0^t f(\tau)d\tau\right)(s) = \frac{1}{s}\mathcal{L}\left(f(t)\right)(s), \qquad \forall s > a_1,$

$(\imath\imath\imath.)$ $\mathcal{L}\left((-1)^n t^n f(t)\right)(s) = \frac{d^n}{ds^n}\mathcal{L}\left(f(t)\right)(s), \qquad \forall s > a_1,$

$(\imath v.)$ $\mathcal{L}\left(e^{-\alpha t} f(t)\right)(s) = \mathcal{L}\left(f(t)\right)(s+\alpha), \qquad \forall s > a_1 + \alpha,\ \alpha \geq 0,$

$(v.)$ $\mathcal{L}\left(H_\alpha(t) f(t-\alpha)\right)(s) = e^{-\alpha s}\mathcal{L}\left(f(t)\right)(s), \qquad \forall s > a_1,\ \alpha \geq 0,$

$(v\imath.)$ $\mathcal{L}\left((f * g)(t)\right)(s) = \mathcal{L}(f(t))(s) \cdot \mathcal{L}(g(t))(s), \qquad \forall s > \max(a_1, a_2).$