Numerical Methods
MATH 417 - 501
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Due January 24
Spring 2018

First Name: ____________________ **Last Name:** ____________________

Homework 1

Ex 1	Ex 2	Total

Exercise 1 **80%**

This homework involves developing simple matlab functions for the implementation of the bisection, the secant and the Newton's iterative algorithms.
To fix the idea, consider the problem of computing the golden number $\frac{1+\sqrt{5}}{2}$ which corresponds to the positive root (why?) of the polynomial

$$P(x) = x^2 - x - 1.$$

This polynomial can be implemented in *MATLAB* and passed to the iteration function as an inline (m-file function)

```
p = inline('-1+x.*(x-1)')
```

or anonymous function

```
p = @(x) -1+x.*(x-1)
```

The above form is preferred because requires less (costly) mulitplications. To have an idea of the considered function, use the following code to plot the function between a and b using n points

```
x = linspace(a,b,n); y=p(x); plot(x,y)
```

1. Write a matlab m-file which implements the bisection method for a zero of a given function f on an interval (a, b) up to a tolerance $\epsilon > 0$, i.e. until the iterate x_i satisfies

$$|f(x_i)| + \frac{|x_i - x_{i-1}|}{|x_i|} \leqslant \epsilon.$$

Also, it is always a could practice to limit the number of iterations by $Nmax$. The code should output (i) the approximate zero and (ii) the number of iterations performed and a warning if the maximum number of iterations was reached before reaching the desired tolerance. Its first line should read

```
function [r,N] = BISECTION(a,b,eps,Nmax,f)
```

2. Run your algorithm for $a = 0$, $b = 2$, $\epsilon = 10^{-r}$, $r = 2, 3, 4, 5$ and record the number of iterations.

3. Write a matlab m-file which implements the secant method for a zero of a given function f starting with the initial values x_0 and x_1 up to a tolerance $\epsilon > 0$ as above. If $Nmax$ denotes to maximum number of iterations, the code should output (i) the approximate zero and (ii) the number of iterations performed and a warning if the maximum number of iterations was reached before reaching the desired tolerance. Its first line should read

```
function [r,N] = SECANT(x0,x1,eps,Nmax,f)
```

4. Run your algorithm for $x_0 = 1$, $x_1 = 1.1$, $\epsilon = 10^{-r}$, $r = 2, 3, 4, 5$ and record the number of iterations.

5. Write a matlab m-file which implements the newton method for a zero of a given function f, its derivative g, starting with the initial values x_0 and up to a tolerance $\epsilon > 0$ as above. If $Nmax$ denotes to maximum number of iterations, the code should output (i) the approximate zero and (ii) the number of iterations performed and a warning if the maximum number of iterations was reached before reaching the desired tolerance. Its first line should read

```
function [r,N] = NEWTON(x0,eps,Nmax,f,g)
```

6. Run your algorithm for $x_0 = 1$, $\epsilon = 10^{-r}$, $r = 2, 3, 4, 5$ and record the number of iterations.

Exercise 2 **20%**

Assume that the locations of planets Mercury (M) and Earth (E) at day t are given (on the $xy - plane$) by

$$x_M(t) = -11.9084 + 57.9117\cos(2\pi t/87.97), \quad y_M(t) = 56.6741\sin(2\pi t/87.97)$$

and

$$x_E(t) = -2.4987 + 149.6041\cos(2\pi t/365.25), \quad y_E(t) = 149.5832\sin(2\pi t/365.25)$$

In both cases, they are elliptical orbits with the Sun at the focus of coordinates $(0, 0)$. Use the secant method (with relevant numerical parameters for you to chose) to compute the minimum Earth-Mercury separation over the next 1000 days, i.e. find

$$\min_{t\in[0,1000]} f(t),$$

where

$$f(t) := \sqrt{(x_E(t) - x_M(t))^2 + (y_E(t) - y_M(t))^2}.$$