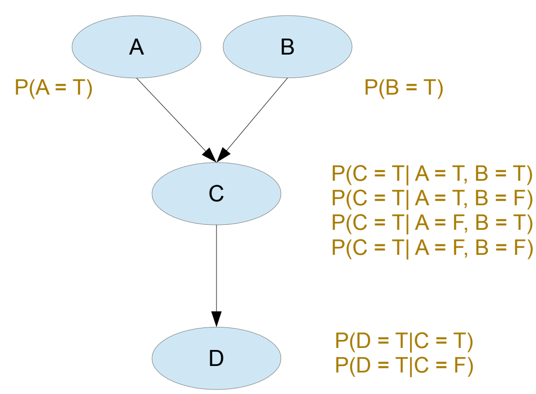
ECS 170: Problem Set 4

February 8, 2017

Your answers should be succinct.

1. Consider the Bayesian network in Figure 1.



conditionally independent: A and D, B and D
if neither C nor D are given, A and B are marginally independent

Figure 1: Network for problem 1.

   (a) How many numbers need to be stored in the probability tables to represent this model? Show your work by listing how many numbers are required for each node in the graph. 1 1 4 2

   (b) List all the independencies and conditional independencies asserted by the network structure.

2. (R&N 14.1) We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 20%, 60%, and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X_1, X_2, and X_3.

[coin]
P(coin = a) = 1/3
P(coin = b) = 1/3
P(coin = c) = 1/3

[X1]
P(X1 = head | coin = a) = 0.2
P(X1 = head | coin = b) = 0.6
P(X1 = head | coin = c) = 0.8

[X2]
P(X2 = head | coin = a) = 0.2
P(X2 = head | coin = b) = 0.6
P(X2 = head | coin = c) = 0.8

[X3]
P(X3 = head | coin = a) = 0.2
P(X3 = head | coin = b) = 0.6
P(X3 = head | coin = c) = 0.8

P(coin = a | X1 = head, X2 = head, X3 = tail)
= P(X1,X2,~X3 | coin = a)*P(coin = a)/P(X1,X2,~X3)
Note: treat P(X1,X2,~X3) as const, and because X1,X2,X3 are independent,
= P(X1 | coin = a)*P(X2 | coin = a)*P(~X3 | coin = a) * P(coin = a) / const
= 0.2*0.2*0.8*(1/3)/const
= 0.011*const
P(coin = b | X1 = head, X2 = head, X3 = tail) = 0.6*0.6*0.4/(1/3)/const
= 0.048*const
P(coin = c | X1 = head, X2 = head, X3 = tail) = 0.8*0.8*0.4/(1/3)/const
= 0.085*const

Therefore, coin c was most likely to have been drawn

(a) Draw the Bayesian network corresponding to this setup and define the necessary conditional probability tables.

(b) Calculate which coin was most likely to have been drawn from the bag if the observed flips come out heads twice and tails once.

3. (R&N 14.12) Two astronomers in different parts of the world make measurements M_1 and M_2 of the number of stars N in some small region of the sky, using their telescopes. Normally, there is a small possibility e of error by up to one star in each direction. Each telescope can also (with a much smaller probability f) be badly out of focus (events F_1 and F_2), in which case the scientist will undercount by three or more stars (or if N is less than 3, fail to detect any stars at all). Consider the three networks shown in Figure 2.

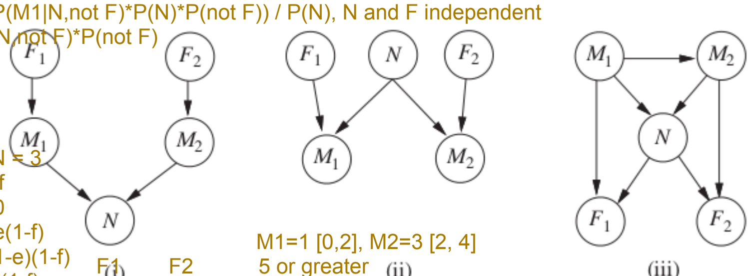
(a) Which network best captures the problem description? 2

(b) Assuming $N \in \{1, 2, 3\}$ and $M_1 \in \{0, 1, 2, 3, 4\}$, write out the conditional distribution for $P(M_1|N)$ in terms of e and f.

(c) Suppose $M_1 = 1$ and $M_2 = 3$. What are the possible numbers of stars if you assume no prior constraint on the values of N?

P(M1|N) = P(M1, N)/P(N) = sum P(M1, N, F) over F / P(N) , marginal
= (P(M1|N,F)*P(N, F) + P(M1|N,not F)*P(N, not F)) / P(N)
= (P(M1|N,F)*P(N)*P(F) + P(M1|N,not F)*P(N)*P(not F)) / P(N), N and F independent
= (P(M1|N,F)*P(F) + P(M1|N,not F)*P(not F)

	N = 1	N = 2	N = 3
M1 = 0	f+e(1-f)	f	f
M1 = 1	(1-e)(1-f)	e(1-f)	0
M1 = 2	e(1-f)	(1-e)(1-f)	e(1-f)
M1 = 3	0	e(1-f)	(1-e)(1-f)
M1 = 4	0	0	e(1-f)



M1=1 [0,2], M2=3 [2, 4]

F1	F2	5 or greater
F1	not F2.	3 or 4
not F1	F2	impossible b/c M1 requires that N in [0, 2],then M2 = 0
not F1	not F2	2

Figure 2: Three possible networks for the telescope problem.

4. Using prior sampling, compute 2 samples for $P(A, B)$. For your computation use the numbers $0.1, 0.2, 0.3, 0.4$ as your "random" numbers. For this and following problems, if the random number is less than or equal to the value in the probability table, then let the sampled value of the variable be true. Use the following probability tables:

- $P(A = True) = 0.2$
- $P(B = True|A = True) = 0.1$
- $P(B = True|A = False) = 0.3$

P(A = true) = 0.2, true b/c 0.1<0.2
P(B = true | A = true) = 0.1,false b/c 0.2 > 0.1
Prior sampling returns [true, false]

P(A = true) = 0.2, false b/c 0.3 > 0.2
P(B = true | A = false) = 0.3, false b/c 0.4 > 0.3
Prior sampling returns [false, false]

5. For this problem, use the network in Figure 3 and consider the query $P(\text{Rain} = True|\text{Wet Grass} = True)$.

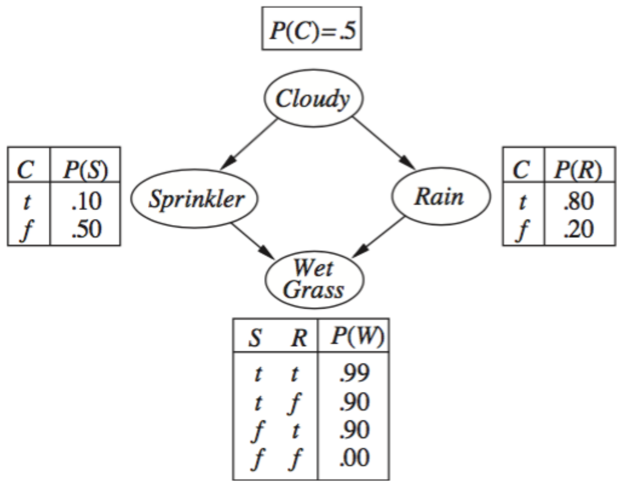


Figure 3: Bayesian network for problem 5.

(a) Generate 5 samples from the joint distribution using likelihood weighting. Fill in the table with your work (first one done for you). The numbers in the brackets are the corresponding random numbers drawn; you can choose to leave them out if you want. Assume the following random number stream: 0.3, 0.7, 0.4, 0.9, 0.2, 0.1, 0.9, 0.3, 0.8, 0.2, 0.5, 0.2, 0.3, 0.1, 0.9, 0.3, 0.2, 0.6, 0.3, 0.2, 0.4, 0.6, 0.8, 0.2, 0.5, 0.2, 0.6

Cloudy	Sprinkler	Rain	Wet Grass	Weight
T (0.3)	F (0.7)	T (0.4)	T	0.9
F(0.9).	T(0.2)	T(0.1)	T	0.99
F(0.9).	T(0.3)	F(0.8)	T	0.90
T(0.2).	F(0.5)	T(0.2)	T	0.90
T(0.3).	T(0.1)	F(0.9)	T	0.90

(b) According to the samples from part (a), what is the estimate for $P(\text{Rain} = True|\text{Wet Grass} = True)$?

P(Rain = T | Wet Grass = T) = (0.90+0.99+0.90)/(0.99+0.90+0.90+0.90) = 0.6078