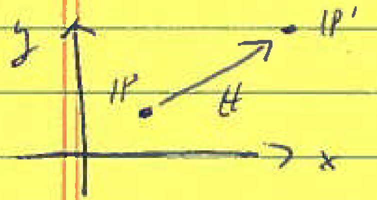
# LEC 4

## • HOMOGENEOUS COORDS.

**WHY?**

$$\mathbb{P} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \mathbb{P}_{hom} = \underline{\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}}$$



$$\mathbb{P}' = \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

$$= M x + t = \underline{\begin{pmatrix} x + t_x \\ y + t_y \end{pmatrix}}$$

$$\mathbb{P}'_{hom} = \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \left( \begin{array}{cc|c} 1 & 0 & t_x \\ 0 & 1 & t_y \\ \hline 0 & 0 & 1 \end{array} \right) \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = M_{hom} \mathbb{P}_{hom}$$

$$= \underline{\begin{pmatrix} x + t_x \\ y + t_y \\ 1 \end{pmatrix}}$$

$\Longrightarrow$ WRITE ALL <u>2D TRANSFORMS</u> USING <u>3×3 MATRIX</u>!

$$\boxed{\mathbb{P}' = \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \left( \begin{array}{cc|c} m_{11} & m_{12} & t_x \\ m_{21} & m_{22} & t_y \\ \hline 0 & 0 & 1 \end{array} \right) = M \mathbb{P}}$$

• **EX**: $Rot(\alpha, \mathbb{C})$: (centre $\mathbb{C}$)

$$\mathbb{P}' = Trans(t)\ Rot(\alpha)\ Trans(-t)\ \mathbb{P} \qquad // \ t = \mathbb{C}$$

$$= \begin{pmatrix} 1 & 0 & c_x \\ 0 & 1 & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C\alpha & -S\alpha & 0 \\ S\alpha & C\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -c_x \\ 0 & 1 & -c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & c_x \\ 0 & 1 & c_y \\ 0 & 0 & 1 \end{pmatrix} \left( \begin{array}{cc|c} C\alpha & -S\alpha & -c_x C\alpha + S\alpha c_y \\ S\alpha & C\alpha & -c_x S\alpha - c_y C\alpha \\ 0 & 0 & 1 \end{array} \right) \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \left( \begin{array}{cc|c} C\alpha & -S\alpha & c_x - c_x C\alpha + c_y S\alpha \\ S\alpha & C\alpha & c_y - c_x S\alpha - c_y C\alpha \\ 0 & 0 & 1 \end{array} \right) \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \underline{M \mathbb{P}'}$$

- General CONCATENATION

$$\mathbb{P}' = \begin{pmatrix} x' \\ y' \\ \vdots \end{pmatrix} = \underbrace{M_N \cdots M_1}_{} \mathbb{P} = M \mathbb{P}$$

ORDER MATTERS!
(Matrix multiplic. is [associative but] NOT commutative!) $\boxed{M_2 M_1 \neq M_1 M_2}$

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⟹ Compute M matrices for common, more complicated transformations, e.g.: scale/rot w.r.t. center $\mathbb{C}$.

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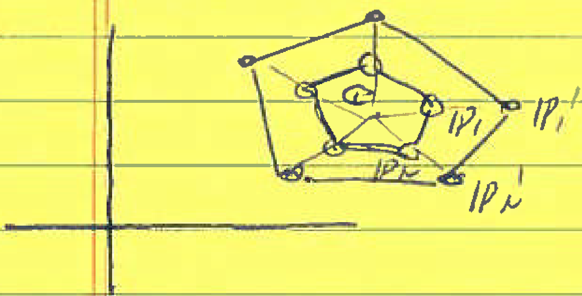
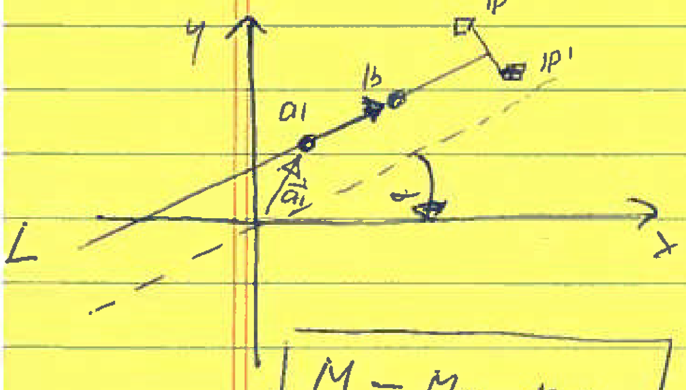
- Apply same transform. M to set of points:




$$\left( \left[ \mathbb{P}_1' \right] \ldots \left[ \mathbb{P}_N' \right] \right) = M \left( \left[ \mathbb{P}_1 \right] \ldots \left[ \mathbb{P}_N \right] \right)$$

$$\underline{\underline{P' = M P}}$$

- COMPLEX EX: Refl (L) ("refl. w.r.t. line L"):

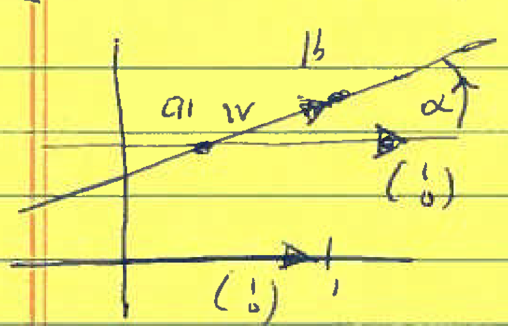



Line given by points $\mathbb{a}$, $\mathbb{b}$

$M_1 = \text{Trans}(-\mathbb{a})$
$M_2 = \text{Rot}(-\alpha)$
$M_3 = \text{Refl}\, x$
$M_4 = \text{Rot}(\alpha)$
$M_5 = \text{Trans}(\mathbb{a})$

$$\boxed{M_{\mathbb{P}} = M_5 \cdots M_1 \mathbb{P}}$$

{ Rem: What is rot. angle $\alpha$ ?




$$\mathbb{V} = \mathbb{b} - \mathbb{a} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\cos(\alpha) = \frac{\binom{1}{0} \cdot \mathbb{V}}{\left\|\binom{1}{0}\right\| \cdot \|\mathbb{V}\|} = \frac{v_x}{\sqrt{v_x^2 + v_y^2}}$$

$$\Longrightarrow \alpha = \ldots \qquad \}$$

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- "Row" vs. "Column" Notation (for positions)

1) OUR BOOK / CLASS:

$$\mathbb{P} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}, \quad \mathbb{P}' = \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & t_x \\ m_{21} & m_{22} & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$\Downarrow$ transpose both sides

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}^T = \left( \begin{pmatrix} m_{11} & m_{12} & t_x \\ m_{21} & m_{22} & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \right)^T$$

$$(x', y', 1) = \left( (x, y, 1) \begin{pmatrix} m_{11} & m_{21} & 0 \\ m_{12} & m_{22} & 0 \\ t_x & t_y & 1 \end{pmatrix} \right)$$

$\Downarrow$

2) OTHER BOOKS:

$$(x', y', 1) = (x, y, 1) \begin{pmatrix} m_{11} & m_{21} & 0 \\ m_{12} & m_{22} & 0 \\ t_x & t_y & 1 \end{pmatrix}$$

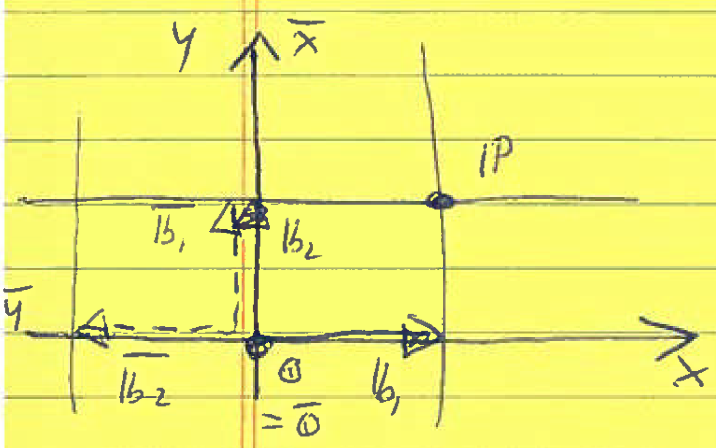
{ Rule

$$\mathbb{P}' = M\mathbb{P}$$

$$(\mathbb{P}')^T = (M\mathbb{P})^T = \mathbb{P}^T M^T \qquad \}$$

# LEC 4

## • CHANGE OF COORDINATE SYSTEM:




$$P_{sys1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$P_{sys2} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$Sys\,1 = \{ O, b_1, b_2 \}$$

$$Sys\,2 = \{ \bar{O} = O, \bar{b}_1, \bar{b}_2 \}$$

→ How to "transform Sys 1 to Sys 2"?

↳ $\boxed{\text{ROTATE SYS 1 BY } \alpha}$ $(= 90°)$

↳ THUS: USE ROT $(-\alpha)$

to get coords. of P w.r.t. Sys 2:

here: $\alpha = 90° = \pi/2$

$$\Rightarrow Rot(-\pi/2) = \left(\begin{array}{cc|c} 0 & 1 & 0 \\ -1 & 0 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$

$$\Rightarrow P_{sys2} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{= P_{sys1}}$$

$$= \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \checkmark$$