

Problem Set 4

Problem 1:

a)

Node A:(1 number) $P(A = T)$

Node B:(1 number) $P(B = T)$

Node C:(4 numbers) $P(C = T | A = T, B = T)$

$P(C = T | A = T, B = F)$

$P(C = T | A = F, B = T)$

$P(C = T | A = F, B = F)$

Node D:(2 numbers) $P(D = T | C = T)$

$P(D = T | C = F)$

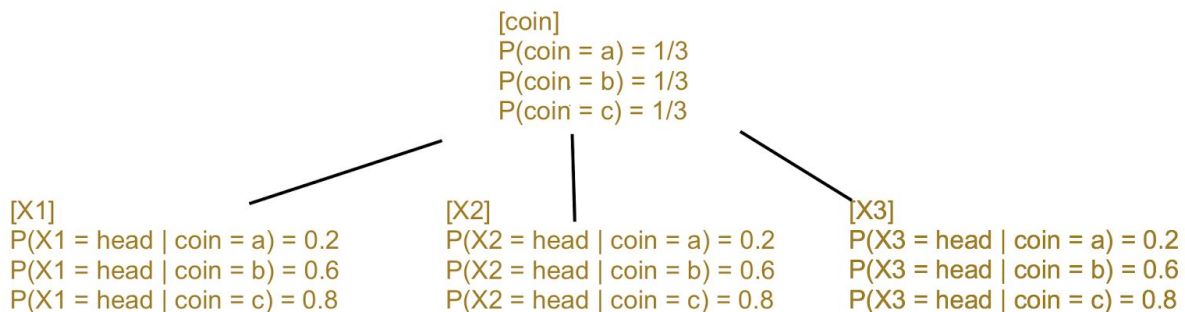
b)

Conditional independencies: A and D, B and D

Marginal independencies: A and B (C and D not given)

Problem 2:

a)



b)

$P(\text{coin} = a | X1=H, X2=H, X3=T)$

$= P(X1=H, X2=H, X3=T | \text{coin} = a) * P(\text{coin} = a) / P(X1=H, X2=H, X3=T)$

Note: treat $P(X1=H, X2=H, X3=T)$ as "const"; $X1, X2, X3$ are independent,

$= P(X1=H | \text{coin} = a) * P(X2=H | \text{coin} = a) * P(X3=T | \text{coin} = a) * P(\text{coin} = a) / \text{const}$

$= 0.2 * 0.2 * 0.8 * (1/3) / \text{const}$

$= 0.011 * \text{const}$

Similarly,

$P(\text{coin} = b | X1 = H, X2 = H, X3 = T) = 0.6 * 0.6 * 0.4 / (1/3) / \text{const}$

$= 0.048 * \text{const}$

$P(\text{coin} = c | X1 = H, X2 = H, X3 = T) = 0.8 * 0.8 * 0.4 / (1/3) / \text{const}$

$= 0.085 * \text{const}$

Therefore, coin "c" was most likely to have been drawn.

Problem 3:

a) 2nd network best describes the problem.

b) $P(M1|N) = P(M1, N)/P(N)$

= [sum $P(M1, N, F)$ over F] / $P(N)$ marginal

= $(P(M1|N,F)*P(N, F) + P(M1|N,\text{not } F)*P(N, \text{not } F)) / P(N)$

= $(P(M1|N,F)*P(N)*P(F) + P(M1|N,\text{not } F)*P(N)*P(\text{not } F)) / P(N)$ N and F independent

= $(P(M1|N,F)*P(F) + P(M1|N,\text{not } F)*P(\text{not } F))$

	$N = 1$	$N = 2$	$N = 3$
$M1 = 0$	$f+e(1-f)$	f	f
$M1 = 1$	$(1-e)(1-f)$	$e(1-f)$	0
$M1 = 2$	$e(1-f)$	$(1-e)(1-f)$	$e(1-f)$
$M1 = 3$	0	$e(1-f)$	$(1-e)(1-f)$
$M1 = 4$	0	0	$e(1-f)$

c)

		Possible numbers of star
$F1$	$F2$	5 or greater
$F1$	not $F2$	3 or 4
not $F1$	$F2$	impossible b/c $M1$ requires that N in $[0, 2]$, making $M2 = 0$
not $F1$	not $F2$	2

Problem 4:

Sample 1:

$P(A = \text{true}) = 0.2$, true b/c $0.1 < 0.2$

$P(B = \text{true} | A = \text{true}) = 0.1$, false b/c $0.2 > 0.1$

Prior sampling returns [true, false]

Sample 2:

$P(A = \text{true}) = 0.2$, false b/c $0.3 > 0.2$

$P(B = \text{true} | A = \text{false}) = 0.3$, false b/c $0.4 > 0.3$

Prior sampling returns [false, false]

Problem 5:

a)

Cloudy	Sprinkler	Rain	Wet Grass	Weight
T (0.3)	F (0.7)	T (0.4)	T	0.9
F(0.9).	T(0.2)	T(0.1)	T	0.99
F(0.9).	T(0.3)	F(0.8)	T	0.90
T(0.2).	F(0.5)	T(0.2)	T	0.90
T(0.3).	T(0.1)	F(0.9)	T	0.90

b)

$$P(\text{Rain} = T \mid \text{Wet Grass} = T) = (0.90 + 0.99 + 0.90) / (0.99 + 0.90 + 0.90 + 0.90) = 0.6078$$