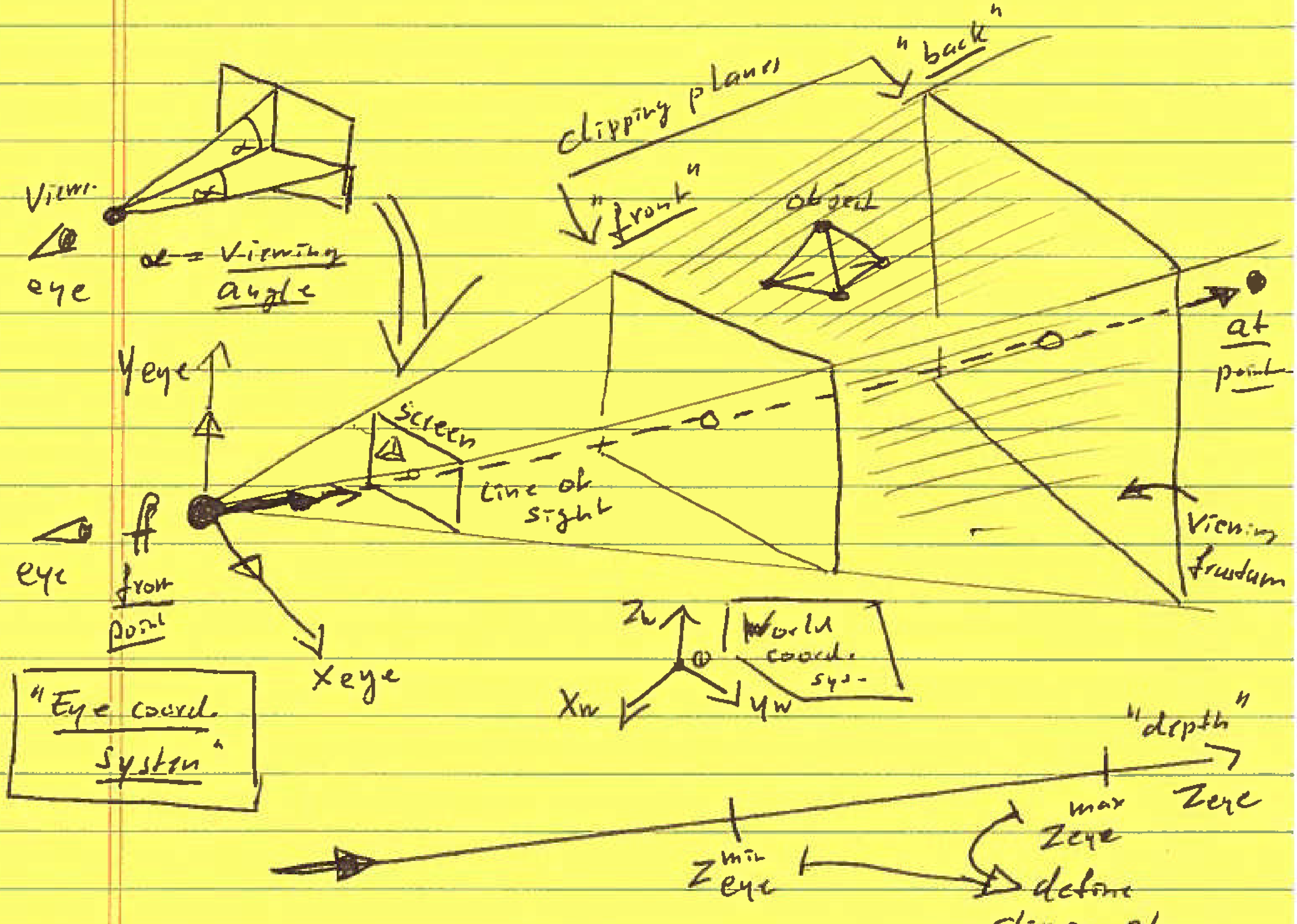
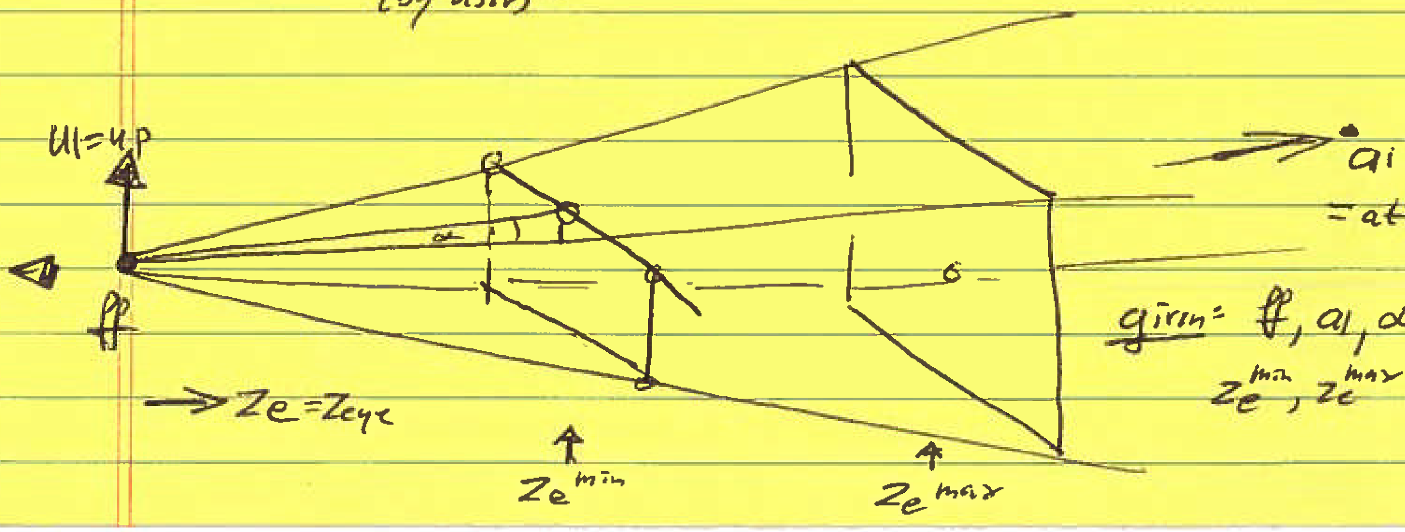
• CAMERA VIEWING MODEL

(See Nielson & Foley paper)

⇒ **THE** perspective projection used in COMP. GRAPHICS!



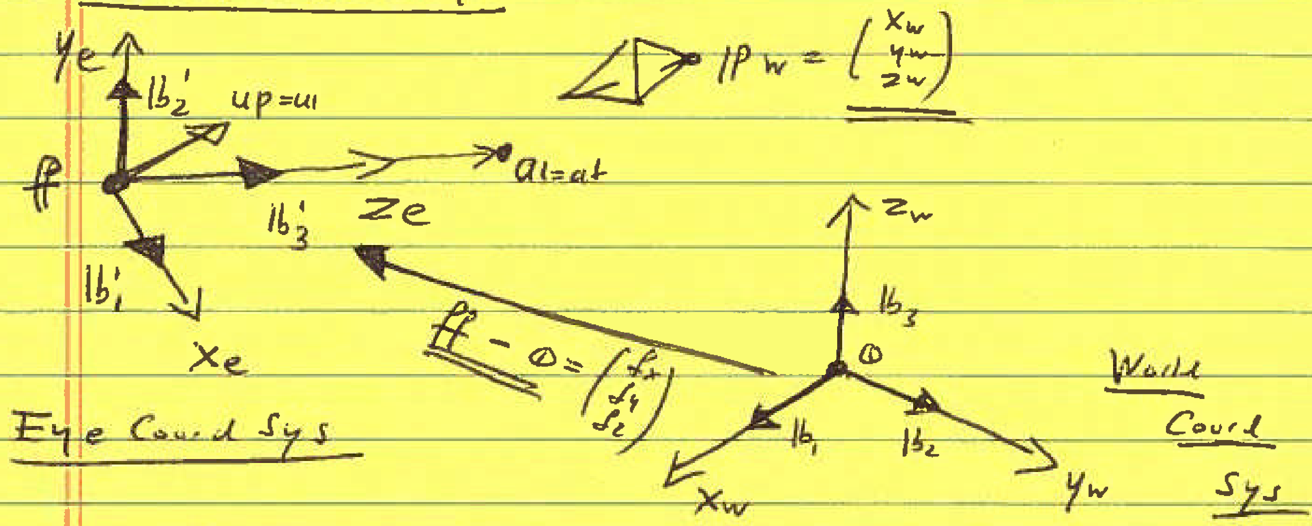
• SPECIFICATION (by user):



given: ff, ai, α, Z_e^{min}, Z_e^{max}

• Transformation(s) Defining the Camera Viewing Model

(I) From World To Eye



$$\boxed{\mathbb{b}_3' = \frac{\mathbb{a} - \mathbb{f}}{\|\mathbb{a} - \mathbb{f}\|}, \quad \mathbb{b}_1' = \mathbb{b}_3' \times \mathbb{u} / \|\mathbb{b}_3' \times \mathbb{u}\|, \quad \mathbb{b}_2' = \mathbb{b}_1' \times \mathbb{b}_3'}$$

= 3 normalized, mutually orthogonal basis vectors defining the LEFT-handed EYE coord. sys. ∎

⇒ Sys. transformations:

(i) Change of basis vectors / orientation:

$$M_1 = \left(\begin{array}{ccc|c} [\mathbb{b}_1'] & [\mathbb{b}_2'] & [\mathbb{b}_3'] & \begin{matrix}0\\0\\0\end{matrix} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

(ii) Change of origin / translation:

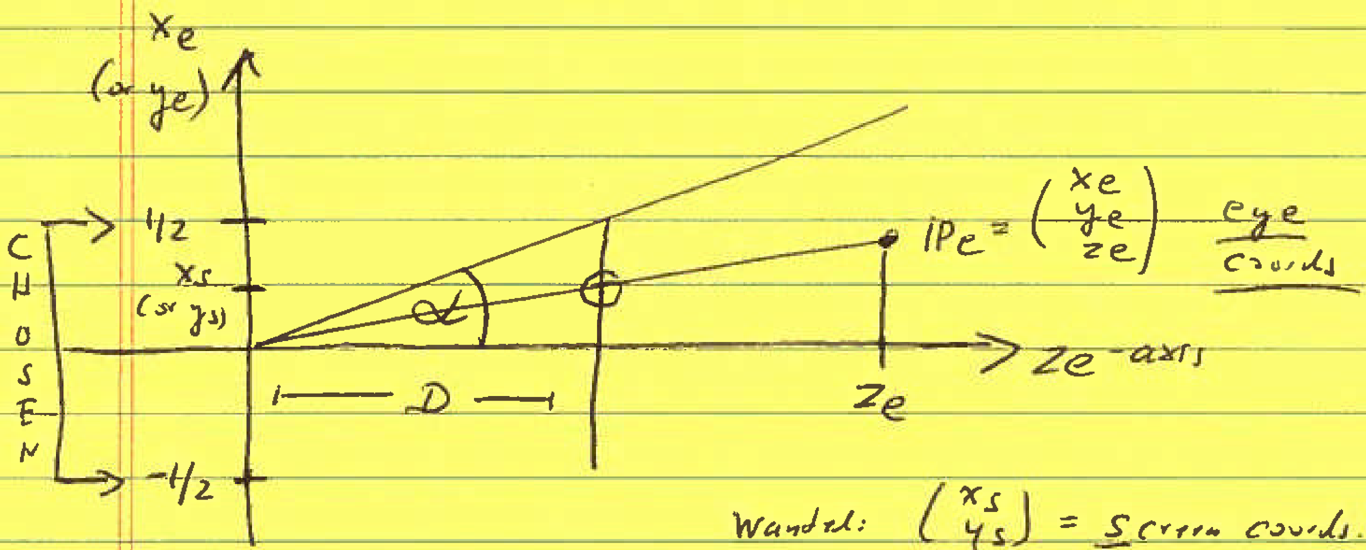
$$M_2 = \left(\begin{array}{ccc|c} 1 & 0 & 0 & f_x \\ 0 & 1 & 0 & f_y \\ 0 & 0 & 1 & f_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

⇒ APPLY INVERSE TRANSFORMS IN REVERSE ORDER TO OBTAIN COORDS OF $\mathbb{P}$ RELATIVE TO EYE:

$$\boxed{\mathbb{P}_e = M_1^{-1} M_2^{-1} \mathbb{P}_w} =$$

$$= \left(\begin{array}{c|c} [\ \mathbb{b}_1'\ ] & 0 \\ [\ \mathbb{b}_2'\ ] & 0 \\ [\ \mathbb{b}_3'\ ] & 0 \\ \hline 0\ 0\ 0 & 1 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 0 & 0 & -f_x \\ 0 & 1 & 0 & -f_y \\ 0 & 0 & 1 & -f_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \cdot \mathbb{P}_w$$

(II) From_Eye_To_NDC /* Mapping to Normalized Device Coordinates */

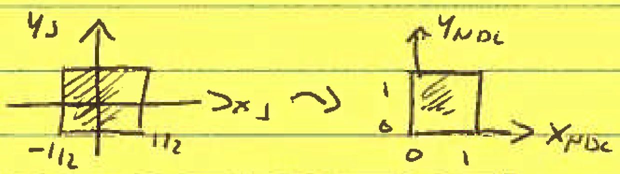


Wanted: $\begin{pmatrix} x_s \\ y_s \end{pmatrix}$ = Screen coords.

$$\Rightarrow \tan\alpha = \frac{1}{2D} \quad \Rightarrow D = \frac{1}{2\tan\alpha}$$

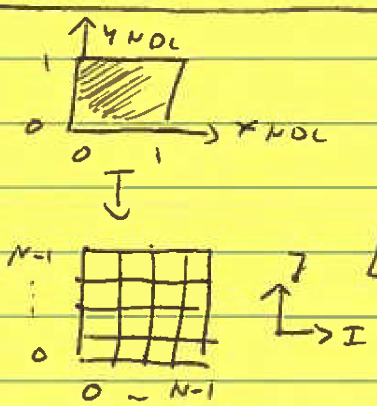
$$\Rightarrow \frac{x_s}{D} = \frac{x_e}{z_e} \quad \Rightarrow \boxed{\begin{aligned} x_s &= \frac{1}{2\tan\alpha}\,\frac{x_e}{z_e} \\ y_s &= \frac{1}{2\tan\alpha}\,\frac{y_e}{z_e} \end{aligned}}$$

Next: map to unit square:



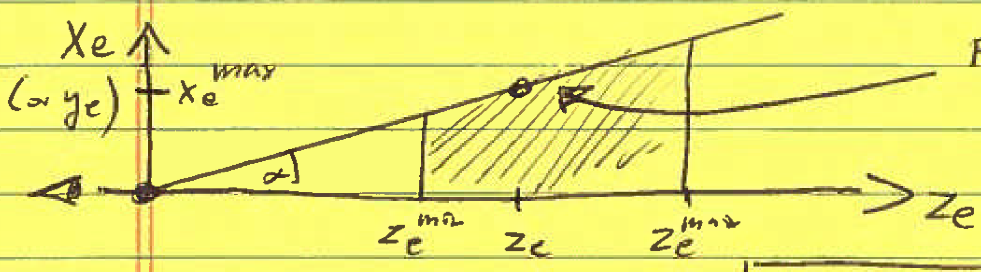
$$\boxed{\begin{pmatrix} x_{NDC} \\ y_{NDC} \end{pmatrix} = \begin{pmatrix} 1/2 + x_s \\ 1/2 + y_s \end{pmatrix}}$$

(III) From_NDC_To_Pixels /* map to N-by-N pixel region */



$$\boxed{\begin{pmatrix} I \\ J \end{pmatrix} = \begin{pmatrix} \text{round}\left((N-1)\cdot x_{NDC}\right) \\ \text{round}\left((N-1)\cdot y_{NDC}\right) \end{pmatrix}}$$

(IV) CLIPPING – AGAINST VIEWING FRUSTUM



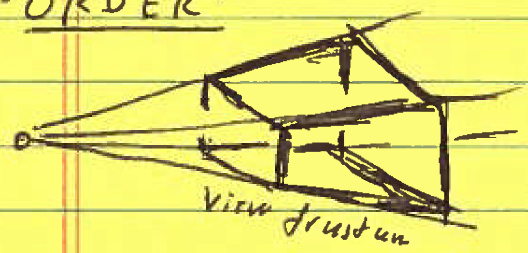
Point on boundary:

$$\boxed{x_e^{max} = z_e \tan(\alpha)}$$

CLIPPING CONDITIONS:

$$\boxed{\begin{array}{c} 0 < z_e^{min} \le z_e \le z_e^{max} \\ -z_e \tan\alpha \le x_e \le z_e \tan\alpha \\ -z_e \tan\alpha \le y_e \le z_e \tan\alpha \end{array}}$$

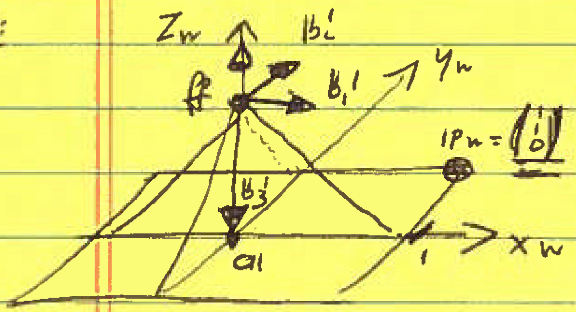
{ · ORDER:



1.) From – World – To – Eye /* All points */
2.) CLIP objects against view frustum
  ⇒ Keep only objects INSIDE
3.) – To – NDC
4.) – To – Pixels }

[See Lecture Notes from WQ 2009, p. 40: "Magic – M" transformation!]

Ex:



given: $\underline{\underline{f}} = \begin{pmatrix}0\\0\\1\end{pmatrix}$, $\underline{\underline{a}} = \begin{pmatrix}0\\0\\0\end{pmatrix}$, $\underline{\underline{u}} = \begin{pmatrix}0\\1\\0\end{pmatrix}$, $\underline{\underline{\alpha = 45°}}$

$$\Rightarrow \underline{\underline{b_3'}} = \begin{pmatrix}0\\0\\-1\end{pmatrix}, \quad \underline{\underline{b_1'}} = \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \underline{\underline{b_2'}} = \begin{pmatrix}0\\1\\0\end{pmatrix}$$

/* for EYE coords sys */

(I) World – To – Eye:

$$\underline{\underline{P_e}} = \left(\begin{array}{ccc|c} [1 & 0 & 0] & 0 \\ [0 & 1 & 0] & 0 \\ [0 & 0 & -1] & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot P_w = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix}1\\1\\0\\1\end{pmatrix} = \underline{\underline{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}}} \checkmark$$

(II) – To – NDC:

$$\begin{pmatrix} x_{NDC} \\ y_{NDC} \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} x_e/z_e \\ y_e/z_e \end{pmatrix} = \underline{\underline{\begin{pmatrix}1\\1\end{pmatrix}}} = \underline{\underline{P_{NDC}}} \checkmark$$

(III) – To – Pixels:

$$\begin{pmatrix} I \\ J \end{pmatrix} = \underline{\underline{\begin{pmatrix} N-1 \\ N-1 \end{pmatrix}}} = \underline{\underline{P_{Pix}}} \checkmark$$

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