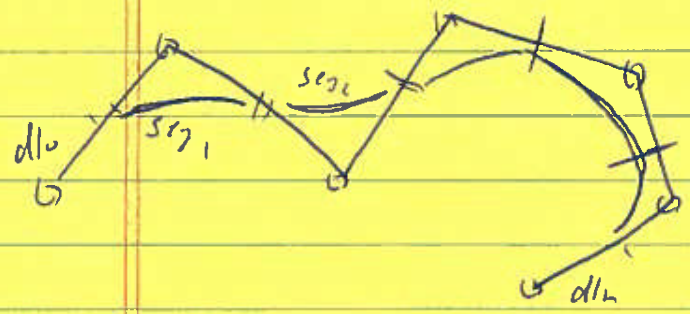
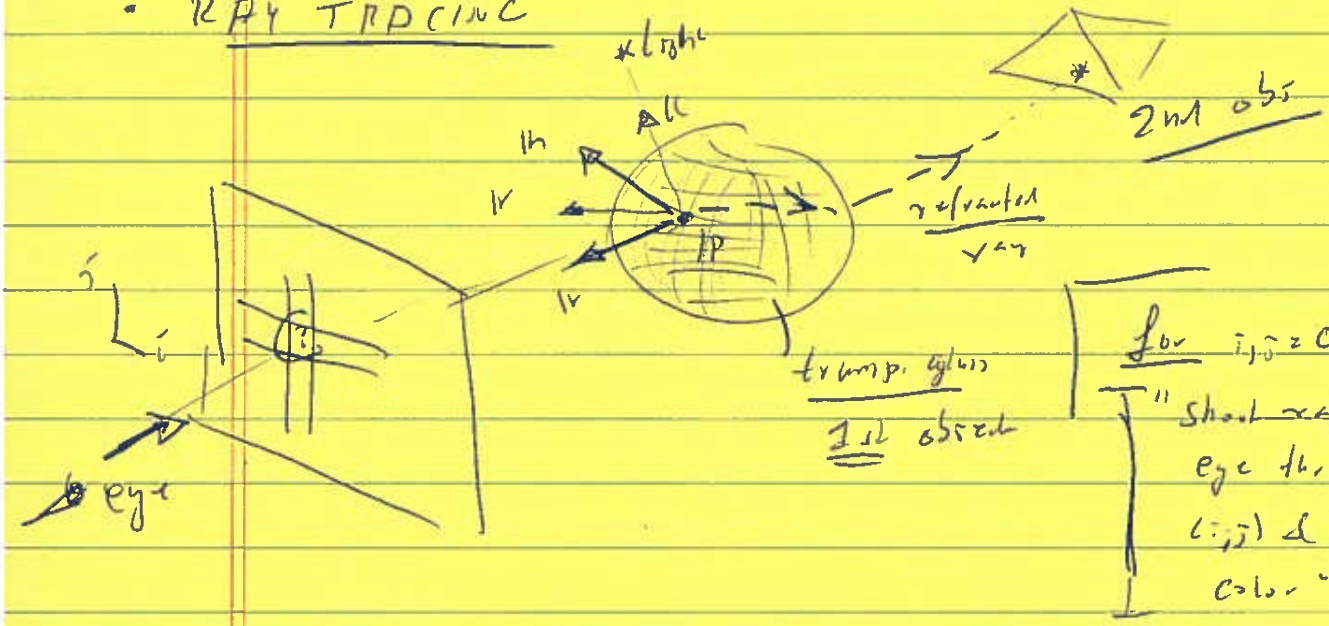
Lec 1

- **B-spline Curve**



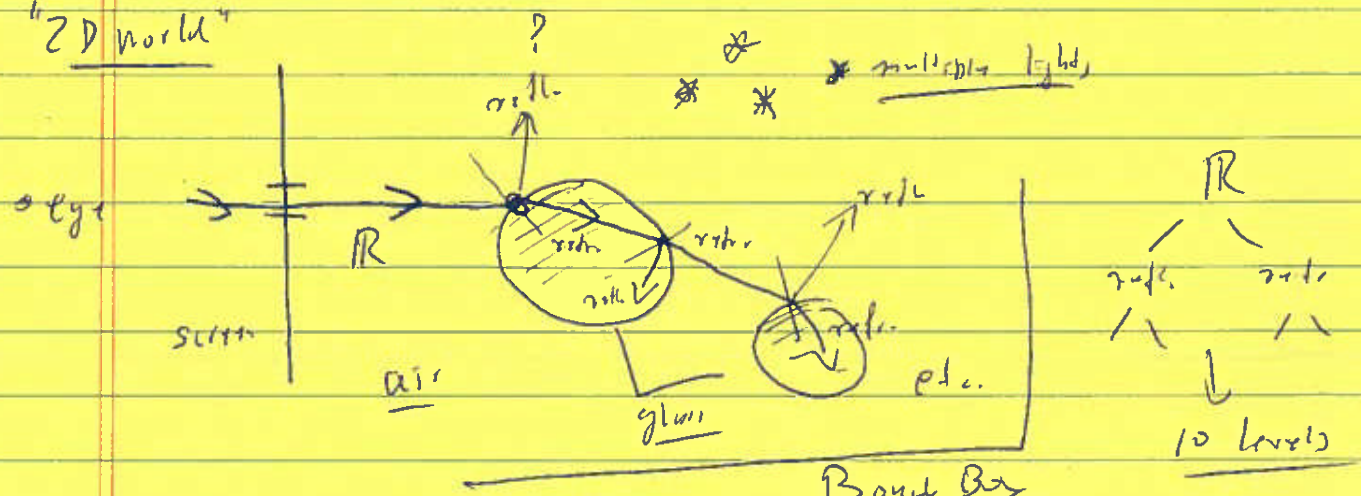
piecewise poly curve, consists of multiple segment

- **RAY TRACING**



for $i,j = 0,\dots,m(n)$ do
" shoot ray from eye through pix (i,j) & determine color "

⇒ Reflection, Refraction / Transparency etc.



WS 2015 — Lec 2

- NOTATION

$P = \begin{pmatrix} x \\ y \end{pmatrix} = (x, y)^T$ — transpose

$v = \vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = (v_x, v_y)^T$

$l = l_{\vec{v}} = \|v\| = \sqrt{v_x^2 + v_y^2}$

- LINES

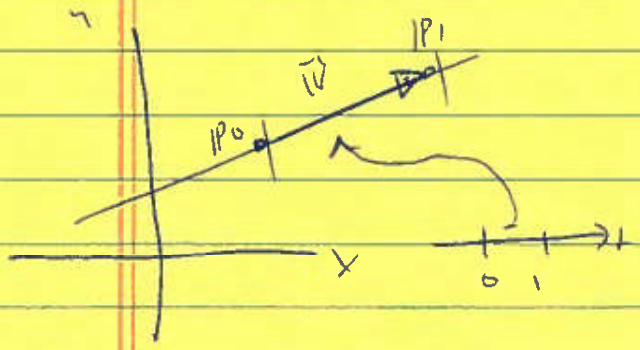
$y = mx + b = \frac{\Delta y}{\Delta x} x + b$ (slope), $\Delta x = 0$

EXPLICIT

IMPLICIT

$Ax + By + C = 0$

$y = 0x + 1$ · $x = 0$ ⇒ all directions can be represented



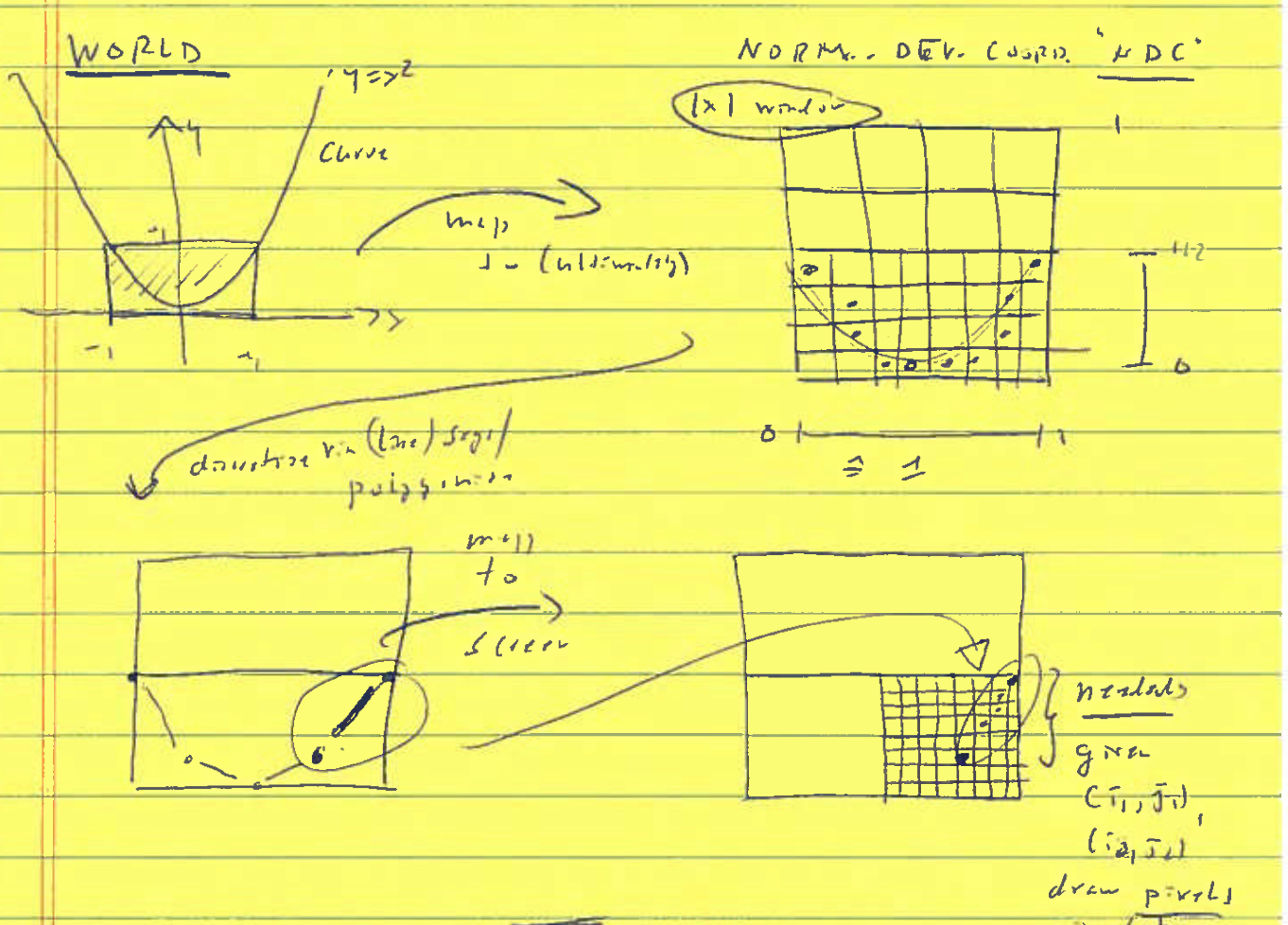
PARAMETRIC

$P = P(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = P_0 + t \cdot \vec{v}$

(in drawn line: $-\infty < t < \infty$)

Lec 2

- Curves → Curve drawing & line drawing

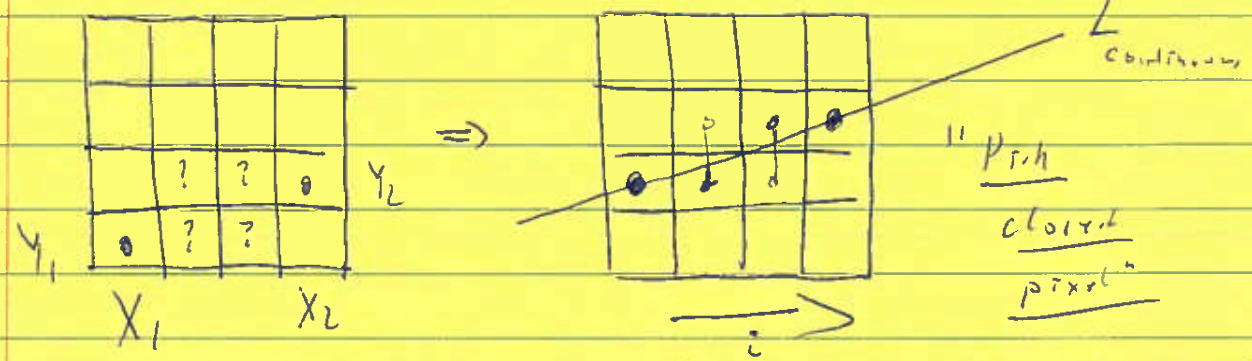


- LINE DRAWING ALG'I

i) DDA – dig. diff. analyzer Slow

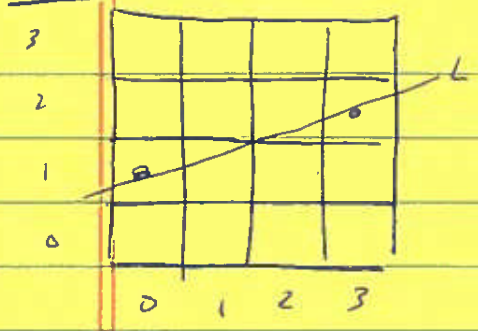
ii) "Bresenham's alg." FAST (INT's only)

PROBLEM: / WHAT'S DESIRED?



Lec 2

- Ex:



$(x_1, y_1) = (0, 1)$

$(x_2, y_2) = (3, 2)$

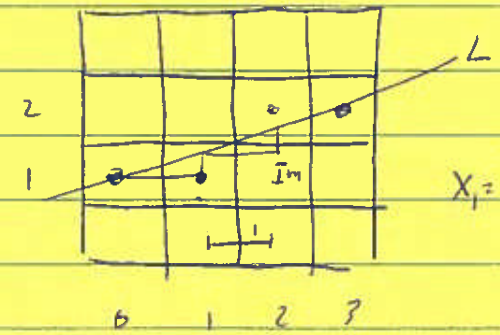
$\Rightarrow m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{3}$

ONLY COND. CASE FOR WHICH $0 < m < 1$

$\Rightarrow$ DDA: "move (x) 1 - by - 1 &
determine "closest" y-value

~~For~~ $(i = x_1)$ ~~to~~ $(i = x_2)$
- determine pixel closest to (y) for this i-value;
- Make Pix (i, y);

here: $x_1 = 0$, $x_2 = 3$, $m = 1/3$ (float!)

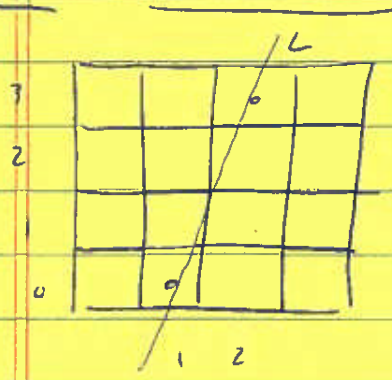


	i	y	Make Pix
$x_1 =$	0	$1 + 0m = 1.0$	(0, rd (1.0)) = (0, 1)
	1	$1 + 1m = 4/3$	(1, rd (4/3)) = (1, 1)
	2	$1 + 2m = 5/3$	(2, rd (5/3)) = (2, 2)
$x_2 =$	3	$1 + 3m = 2.0$	(3, rd (2.0)) = (3, 2)

Lec 2

- Other cases? → other slopes

Ex: $1 < m < \infty$ "steep lines"

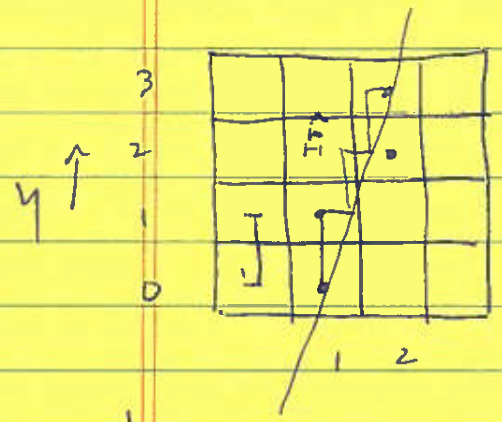


$(X_1, Y_1) = (1, 0)$, $(X_2, Y_2) = (2, 3)$

$\Rightarrow m = \frac{\Delta Y}{\Delta X} = 3$ "steep line"

⇒ INC Y 1-by-1 & determine prop. x-value

⇒ use $\hat{m} = 1/m = 1/3$



For $(\bar{j} = Y_1)$ to $(\bar{j} = Y_2)$ do
- determine pix. closest to $\bar{j}$;
- Make Pix ("corresp", $\bar{j}$);

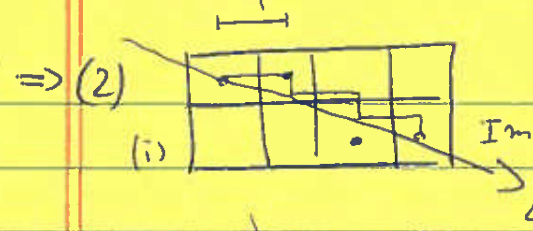
here:

	$\bar{j}$	x	Make Pix ( )
$Y_1 =$	0	$1 + 0\hat{m} = 1.0$	(rd(1.0), 0)
	1	$1 + 1\hat{m} = 4/3$	(rd(4/3), 1)
	2	$1 + 2\hat{m} = 5/3$	(rd(5/3), 2)
$Y_2 =$	3	$1 + 3\hat{m} = 2.0$	(rd(2.0), 3)

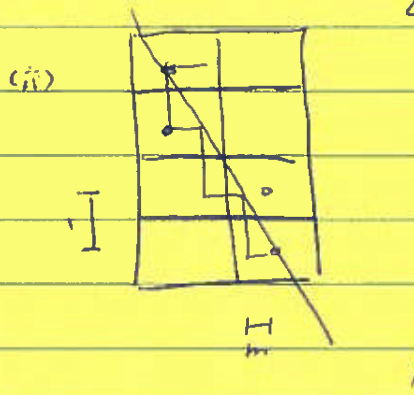
- (1) Always $X_1 \leq X_2$ (if NOT then SWAP end points)
- (2) Negative m: $-1 < m < 0$??? $-\infty < m < -1$?!! "-" think
- (3) SPECIAL: $m = 0$, $m = 1$, $m = -1$, $m = \infty$, $m = -\infty$?!?

Lec 2

$\Rightarrow$ (2)



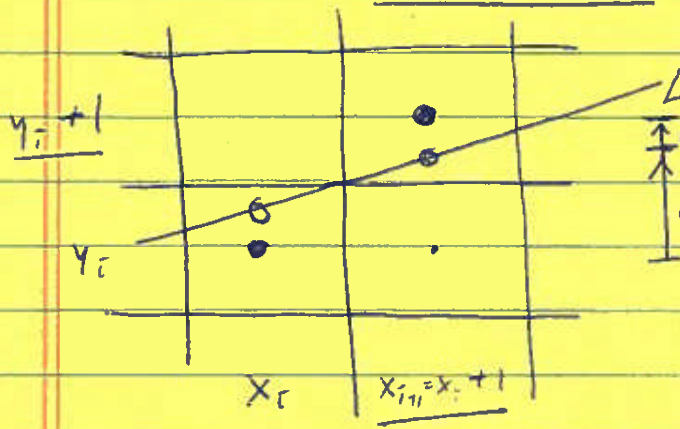
$\rightarrow$ INC i 1-by-1, and m (<0) to current y-value



$\rightarrow$ DEC j 1-by-1, and (positive) $\hat{m}$-value to current x-value

! • BRESEN HPM'S ALG $\rightarrow$ ONLY INT ARITHMETIC ! (Consider $0 < m < 1$ case only)

IDEA: "SIMPLY DETERMINE WHETHER CURR y-value TO BE INCREMENTED OR NOT ..."



"WHEN MOVING TO THE RIGHT, i.e., $x_{i+1} = x_i + 1$, WHAT y-value to use?"

Two cases only:

(i) $y_{i+1} = y_i$

or (ii) $y_{i+1} = y_i + 1$

'DECIDER': IF $(d_1 - d_2) < 0$ THEN $y_{i+1} = y_i$ ELSE $y_{i+1} = y_i + 1$ $\Rightarrow$ $d_1 - d_2 = ?$

$$(d_1 - d_2) = m(x_i+1) + b - y_i - \{y_i + 1 - (m(x_i+1) + b)\}$$

$$= 2mx_i - 2y_i + 2m + 2b - 1 \qquad 1 \cdot \Delta X$$

$$\Delta X(d_1 - d_2) = 2\Delta Y x_i - 2\Delta X y_i + \underbrace{2\Delta Y + 2\Delta X b - \Delta X}_{= \text{const.}} = P_i$$

"predicate P_i"

Lec 2

⇒ Predicate P_i determines whether to INC y-value by 1 or not!

⇓

IF $(P_i < 0)$ **THEN** $y_{i+1} = y_i$
ELSE $y_{i+1} = y_i + 1$

⇒ HOW TO COMPUTE $P_1, P_2, P_3, \ldots$?

→ Devise a recursive formula for P_i:

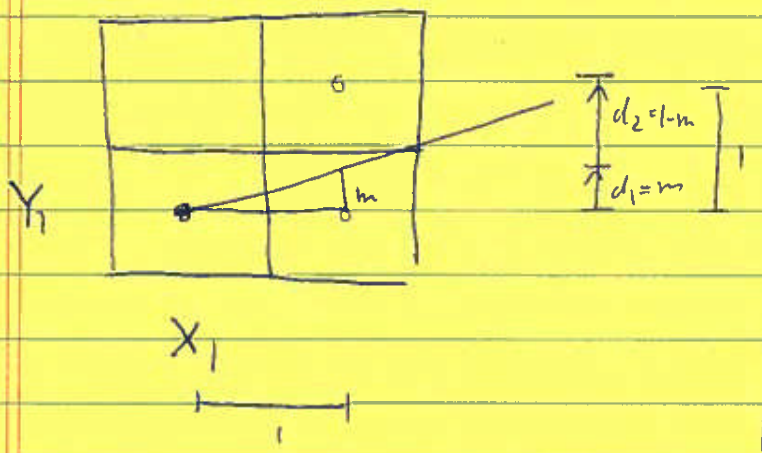
$$\begin{cases} P_i = 2\Delta Y x_i - 2\Delta X y_i + C \\ P_{i+1} = 2\Delta Y x_{i+1} - 2\Delta X y_{i+1} + C \end{cases}$$

$$P_{i+1} - P_i = 2\Delta Y(x_{i+1} - x_i) - 2\Delta X(y_{i+1} - y_i)$$

$$= 2\Delta Y - 2\Delta X(y_{i+1} - y_i)$$

$$\boxed{P_{i+1} = P_i + 2\Delta Y - 2\Delta X(y_{i+1} - y_i)} \quad (*)$$

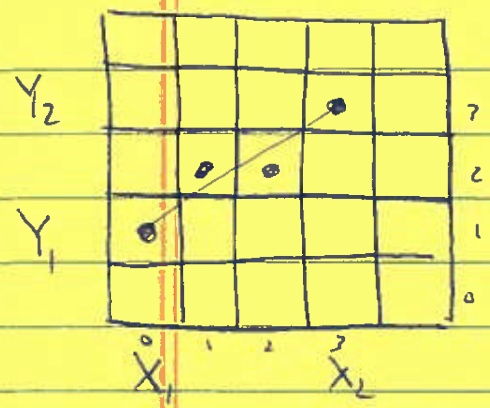
Next P_1:



$$P_1 = \Delta X(d_1 - d_2)$$
$$= \Delta X(m - (1-m))$$
$$= \Delta X(2m-1)$$
$$= 2\Delta Y - \Delta X$$

$$\boxed{P_1 = 2\Delta Y - \Delta X} \quad (**)$$

Ex.:



$(X_1, Y_1) = (0,1)$, $(X_2, Y_2) = (3,3)$

$\Rightarrow \Delta X = 3$

$\Delta Y = 2$

$$P_1 = 2\Delta Y - \Delta X$$

$$P_{i+1} = P_i + 2\Delta Y - 2\Delta X (y_{i+1} - y_i), \quad i = 1, \ldots$$

	i	PIX@	Predicate P_i	
START	1	(0, 1)	$P_1 = 4 - 3 = 1 \geq 0$	$\Rightarrow$ (+1)
	2	(1, 2)	$P_2 = 1 + 4 - 6(2-1) = -1 < 0$	$\Rightarrow$ (+0)
	3	(2, 2)	$P_3 = -1 + 4 - 6(2-2) = 3 \geq 0$	$\Rightarrow$ (+1)
END	4	(3, 3)	DONE	

$\{$ Slope $1 < m < \infty$ $\Rightarrow$ "INC y 1-by-1 and determine proper x-value"

Slope $(m) < 1$ $\Rightarrow$ Replace X by Y, Y by X, i by j:

$$P_1 = 2\Delta X - \Delta Y$$

$$P_{j+1} = P_j + 2\Delta X - 2\Delta Y (x_{j+1} - x_j), \quad j = 1, \ldots$$

All other cases: handled similarly

$\Rightarrow$ See book!