

## ECS165 HW2

1a)

 $A^+ = \{A\}$  $B^+ = \{B, C\}$  $C^+ = \{C\}$  $D^+ = \{D\}$  $E^+ = \{E\}$  $AB^+ = \{A, B, C, D, E\} \leftarrow \text{key}$  $AC^+ = \{A, C, D, E\}$  $AD^+ = \{A, D\}$  $AE^+ = \{A, D, E\}$  $BC^+ = \{B, C\}$  $BD^+ = \{B, D\}$  $BE^+ = \{A, B, C, D, E\} \leftarrow \text{key}$  $CD^+ = \{C, D\}$  $CE^+ = \{C, E\}$  $DE^+ = \{D, E\}$  $ABC^+ = ABD^+ = ABE^+ = \{A, B, C, D, E\} \leftarrow \text{superkey}$  $ACD^+ = \{A, C, D, E\}$  $ACE^+ = \{A, C, D, E\}$  $ADE^+ = \{A, D, E\}$  $BCD^+ = \{B, C, D\}$  $BCE^+ = BDE^+ = \{A, B, C, D, E\} \leftarrow \text{superkey}$  $CDE^+ = \{C, D, E\}$  $ABCD^+ = \{A, B, C, D, E\} \leftarrow \text{superkey}$  $ABCE^+ = \{A, B, C, D, E\} \leftarrow \text{superkey}$  $ACDE^+ = \{A, C, D, E\}$  $BCDE^+ = \{A, B, C, D, E\} \leftarrow \text{superkey}$  $ABCDE^+ = \{A, B, C, D, E\} \leftarrow \text{superkey}$ Keys of R are AB, BE

1b)

check FD's (closure without each left hand side attributes, checking if follow)

$AC^+ = \{A, C\}$       so  $AC \rightarrow E$  doesn't follow the rest

$AE^+ = \{A, E\}$       so  $AE \rightarrow D$  doesn't follow the rest

$B^+ = \{B\}$               so  $B \rightarrow C$  doesn't follow the rest

$BE^+ = \{B, E, C\}$     so  $BE \rightarrow A$  doesn't follow the rest

check attributes:

$C^+ = \{C\}$               C in AC

$A^+ = \{A\}$               A in AC

$E^+ = \{E\}$               E in AE

$A^+ = \{A\}$               A in AE

$E^+ = \{E\}$               E in BE

$B^+ = \{B, C\}$           B in BE

Therefore, the original set of FD's:

$AC \rightarrow E$

$AE \rightarrow D$

$B \rightarrow C$

$BE \rightarrow A$

is a minimal basis for the FD's.

1c)

BCNF violations follow from the FD's:

AC->E

AE->D

B->C

AC->D

BD->C

ACD->E

ACE->D

Their left-hand-side attributes are not superkeys. (keys: AB, BE)

1d)

Choose AC to be X

Then  $X^+ = AC^+ = \{A, C, D, E\}$

R1(A, C, D, E)

{

~~AC → D~~ (deleted b/c it follows)

AC → E

AE → D <- violation

~~ACD → E~~ (deleted b/c it follows)

~~ACE → D~~ (deleted b/c it follows)

}

key:

$AC^+ = \{A, C, D, E\}$

$AE^+ = \{A, D, E\}$

R1's key: AC

R2(A, B, C)

{

B → C <- violation

}

key:

$AB^+ = \{A, B, C\}$

R2's key: AB

BCNF violation in R1, further decompose R1:

Choose AE to be X,

Then  $X^+ = AE^+ = \{A, D, E\}$

R3(A, D, E)

{

AE → D

}

key:

$AE^+ = \{A, D, E\}$

R3's key: AE

R4(A, C, E)

{

AC → E

}

key:

$AC^+ = \{A, C, E\}$

R4's key: AC

BCNF violation in R2, further decompose R2:

Choose B to be X,

Then  $X^+ = B^+ = \{B, C\}$

R5(B, C)

{

B → C

}

key:

$B^+ = \{B, C\}$

R5's key: B

R6(A, B)

{

}

key:

$AB^+ = \{A, B\}$

R5's key: AB

Therefore the BCNF decomposition:

R3(A, D, E)

R4(A, C, E)

R5(B, C)

R6(A, B)

with FD's:

AE → D

AC → E

B → C

1e)

3NF violations that follow from FD's:

AE->D

B->C

AC->D

BD->C

ACE->D

Left hand sides are not superkeys and right hand sides are not primes.

1f)

Find minimal basis:

from part b):

$AC \rightarrow E$

$AE \rightarrow D$

$B \rightarrow C$

$BE \rightarrow A$

Create relations with schemas:

$R1(A,C,E)$        $AC \rightarrow E$       key: AC

$R2(A,D,E)$        $AE \rightarrow D$       key: AE

$R3(B,C)$        $B \rightarrow C$       key: B

$R4(A,B,E)$       <- ABE is a superkey       $BE \rightarrow A$       key: BE

So 3NF decomposition is:

$R1(A,C,E)$

$R2(A,D,E)$

$R3(B,C)$

$R4(A,B,E)$

with FDs:

$AC \rightarrow E$

$AE \rightarrow D$

$B \rightarrow C$

$BE \rightarrow A$

1g)

List of all MVD's that follow FD's:

B->->C <-violation

AB->->C

AB->->D

AB->->E

AC->->D <-violation

AC->->E <-violation

AE->->D <-violation

BD->->C <-violation

BE->->A

BE->->C

BE->->D

ABC->->D

ABC->->E

ABD->->C

ABD->->E

ACD->->E <-violation

ACE->->D <-violation

BCE->->A

BDE->->A

BDE->->C

B->->ADE <-violation

AB->->DE

AB->->CE

AB->->CD

AC->->BE <-violation

AC->->BD <-violation

AE->->BC <-violation

BD->->AE <-violation

BE->->CD

BE->->AD

BE->->AC

ACD->->B <-violation

ACE->->B <-violation

BD->->C

ACD->->E

ACE->->D

B->->ADE

AC->->BE

AC->->BD

AE->->BC

BD->->AE

ACD->->B

ACE->->B

Therefore, 4NF violations:

B->->C

AC->->D

AC->->E

AE->->D

1h)

Choose  $AC \rightarrow E$  to be the 4NF violation,

$R1(A,C,E)$

{

$AC \rightarrow E$

$AC \twoheadrightarrow E$

}

key:  $AC$

$R2(A,B,C,D)$

{

~~$ABC \rightarrow D$~~  (deleted for minimal basis)

~~$ABD \rightarrow C$~~  (deleted for minimal basis)

~~$AB \rightarrow D$~~  (deleted for minimal basis)

$AB \rightarrow C$

$AC \rightarrow D$

$BD \rightarrow C$

$B \twoheadrightarrow C$  <- violation

$AB \twoheadrightarrow C$

$AB \twoheadrightarrow D$

$AC \twoheadrightarrow D$  <- violation

$BD \twoheadrightarrow C$  <- violation

}

key:  $AB$

key:  $AB$

Therefore, 4NF decomposition:

$R1(A,C,E)$

$R3(B,C)$

$R4(A,B,D)$

FDs and MVDs:

$AC \rightarrow E$

$B \rightarrow C$

$AB \rightarrow D$

$AC \twoheadrightarrow E$

$B \twoheadrightarrow C$

$AB \twoheadrightarrow D$

4NF violation in  $R2$ , further decomposition of  $R2$ :

Choose  $B \rightarrow C$

$R3(B,C)$

{

$B \rightarrow C$

$B \twoheadrightarrow C$

}

key:  $B$

$R4(A,B,D)$

{

$AB \rightarrow D$

$AB \twoheadrightarrow D$

}



1i)

$S(A,B,C) = \pi R(A,B,C,D,E)$

$B \rightarrow C$

~~$AB \rightarrow C$~~  (follow from the rest)

$B \twoheadrightarrow C$

~~$AB \twoheadrightarrow C$~~  (trivial)

$B \twoheadrightarrow A$

2a)

Use chase test to prove:  $CH \rightarrow R$  holds in Courses.

C	T	H	R	S	G
c	t1	h	r1	s1	g1
c	t2	h	r2	s2	g2

apply  $C \rightarrow T$

C	T	H	R	S	G
c	t1	h	r1	s1	g1
c	t1	h	r2	s2	g2

apply  $HT \rightarrow R$

C	T	H	R	S	G
c	t1	h	r1	s1	g1
c	t1	h	r1	s2	g2

The R column agrees.

Therefore,  $CH \rightarrow R$  holds.

2b)

Use chase test to prove:  $\text{CHR} \rightarrow \text{G}$  holds in Courses.

C	T	H	R	S	G
c	t1	h	r	s1	g1
c	t2	h	r	s2	g2

apply  $\text{C} \rightarrow \text{T}$

C	T	H	R	S	G
c	t1	h	r	s1	g1
c	t1	h	r	s2	g2

The only FD and MVD associated with G are:

$\text{CS} \rightarrow \text{G}$

$\text{CS} \twoheadrightarrow \text{G}$

Since elements in columns C+S do not agree, no further changes for column G.

Column G will never agree.

Therefore,  $\text{CHR} \rightarrow \text{G}$  does not hold in Courses.

2c)

$C^+ = \{C, T\}$	$CTR^+ = \{C, T, R\}$	Key for Courses: <u>HS</u>
$T^+ = \{T\}$	$CTS^+ = \{C, T, S, G\}$	
$H^+ = \{H\}$	$CTG^+ = \{C, T, G\}$	$R1(C, T, H)$
$R^+ = \{R\}$	$CHR^+ = \{C, T, H, R\}$	{
$S^+ = \{S\}$	$CHS^+ = \{C, T, H, R, S, G\}$ <-	$C \rightarrow T$
$G^+ = \{G\}$	superkey	$TH \rightarrow D$
$CT^+ = \{C, T\}$	$CHG^+ = \{C, T, H, R, G\}$	}
$CH^+ = \{C, T, H, R\}$	$CRS^+ = \{C, T, R, S, G\}$	key: <u>TH</u>
$CR^+ = \{C, T, R\}$	$CRG^+ = \{C, T, R, G\}$	
$CS^+ = \{C, S, G\}$	$CSG^+ = \{C, T, S, G\}$	$R2(C, H, R, G)$
$CG^+ = \{C, G\}$	$THR^+ = \{T, H, R\}$	{
$TH^+ = \{C, T, H, R\}$	$THS^+ = \{T, H, R, S\}$	$CH \rightarrow R$
$TR^+ = \{T, R\}$	$THG^+ = \{T, H, R, G\}$	$CHG \rightarrow R$ (deleted,
$TS^+ = \{T, S\}$	$TRS^+ = \{T, R, S\}$	follow others)
$TG^+ = \{T, G\}$	$TRG^+ = \{T, R, G\}$	$HRG \rightarrow C$
$HR^+ = \{C, H, R\}$	$TSG^+ = \{T, S, G\}$	}
$HS^+ = \{C, T, H, R, S, G\}$	$HRS^+ = \{C, T, H, R, S, G\}$	key: <u>CHG, HRG</u>
<-key	<-superkey	
$HG^+ = \{H, G\}$	$HRG^+ = \{C, T, H, R, G\}$	$R3(C, H, S)$
$RS^+ = \{R, S\}$	$HSG^+ = \{H, S, G\}$	{
$RG^+ = \{R, G\}$	$RSG^+ = \{R, S, G\}$	$HS \rightarrow C$
$SG^+ = \{S, G\}$	.....	}
$CTH^+ = \{C, T, H, R\}$		key: <u>HS</u>

So relations and FDs to run chase test for lossless join:

$R1(C, T, H)$   $R2(C, H, R, G)$   $R3(C, H, S)$   
 $C \rightarrow T$ ,  $TH \rightarrow C$ ,  $CH \rightarrow R$ ,  $HRG \rightarrow C$ ,  $HS \rightarrow C$

C	T	H	R	S	G
c	t	h	r1	s1	g1
c	t2	h	r	s2	g
c	t3	h	r3	s	g3

Since attribute G is not on the right hand side of any FDs, it is not possible to have G column marked with all lowercase letters.

Therefore,  $R1, R2, R3$  decomposition doesn't have a lossless join.

2d)

Find minimal basis:

FD's (check if any follows from the rest):

$C^+ = \{C\}$

$HR^+ = \{H, R\}$

$HT^+ = \{H, T\}$

$HS^+ = \{H, S\}$

$CS^+ = \{C, T, S\}$

attributes (check FD with any attribute removed from lhs still follows the rest)

$H^+ = \{H\}$   $R^+ = \{R\}$  in HR

$H^+ = \{H\}$   $T^+ = \{T\}$  in HT

$H^+ = \{H\}$   $S^+ = \{S\}$  in HS

$C^+ = \{C, T\}$   $S^+ = \{S\}$  in CS

Therefore, the original set of FDs is a minimal basis:

$C \rightarrow T$

$HR \rightarrow C$

$HT \rightarrow R$

$HS \rightarrow R$  <- schema with superkey

$CS \rightarrow G$

Create relations with schemas:

$R_1(C, T)$   $C \rightarrow T$  key: C

$R_2(C, H, R)$   $CH \rightarrow R, HR \rightarrow C$  key: CH, HR

$R_3(T, H, R)$   $TH \rightarrow R$  key: TH

$R_4(H, R, S)$   $HS \rightarrow R$  key: HS

$R_5(C, S, G)$   $CS \rightarrow G$  key: CS

Therefore, 3NF decomposition:

$R_1(C, T)$

$R_2(C, H, R)$

$R_3(T, H, R)$

$R_4(H, R, S)$

$R_5(C, S, G)$

with FDs:

$C \rightarrow T$

$CH \rightarrow R$

$HR \rightarrow C$

$TH \rightarrow R$

$HS \rightarrow R$

$CS \rightarrow G$

2e)

Find relations that are not in BCNF:

R1(C,T)     C->T     key: C

R2(C,H,R)     CH->R, HR->C     key: CH, HR

R3(T,H,R)     TH->R     key: TH

R4(H,R,S)     HS->R     key: HS

R5(C,S,G)     CS->G     key: CS

Because among all the FDs, the left hand sides are all superkeys,  
all the relations are in BCNF

2f)

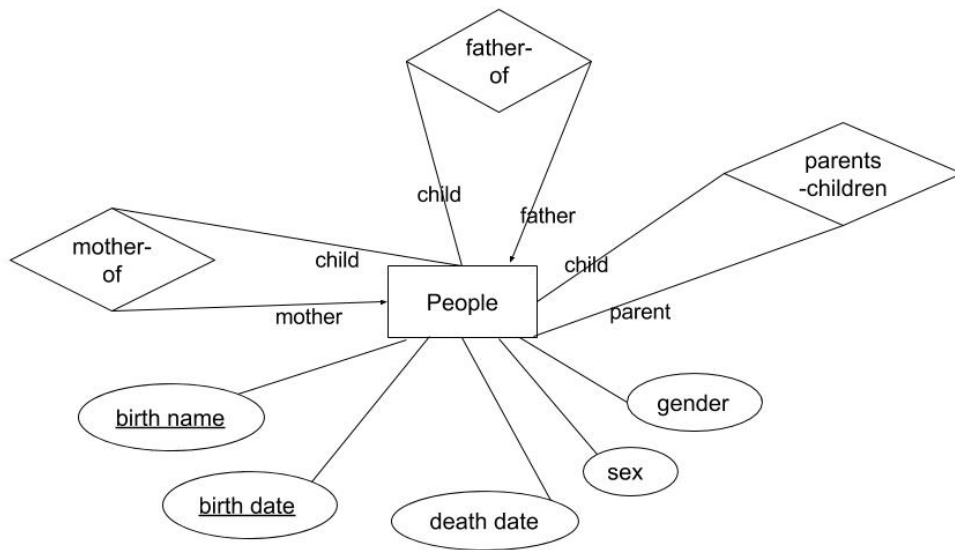
Find relations that are not in 4NF:

R1(C,T)	$C \twoheadrightarrow T$	trivial
R2(C,H,R)	$CH \twoheadrightarrow R, HR \twoheadrightarrow C$	trivial
R3(T,H,R)	$TH \twoheadrightarrow R$	trivial
R4(H,R,S)	$HS \twoheadrightarrow R$	trivial
R5(C,S,G)	$CS \twoheadrightarrow G$	trivial

Because all the MVDs follow from the FDs are trivial MVDs, there are no non-trivial MVDs. Hence, there are not violations.

All relations are in 4NF.

3)



Note:

A person only has one mother and one father

A right-end arrow on the line "mother" from relation "mother-of" to entity "People";

a lower-end arrow on the line "father" from relation "father-of" to entity "People".

Birth name + birth date is the key.