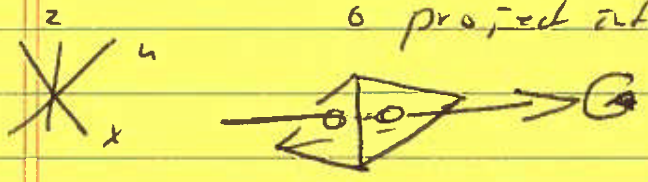
LEC 7

- PROJECTIONS ...

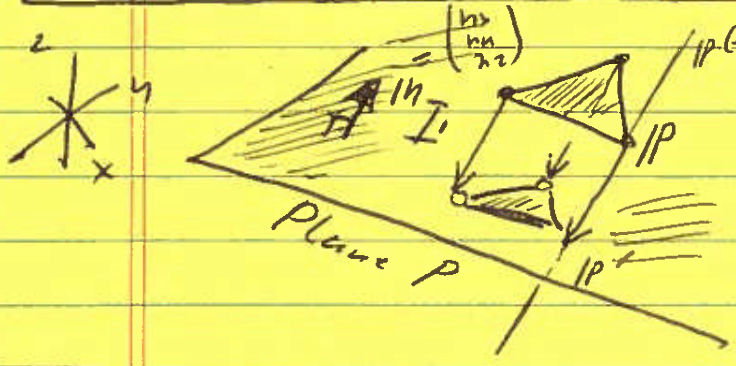
⇒ Axonometric project into xy- / xz- / yz- plane!

⇒ "Animation": ∘ Continually rotate an object by 1°, 2°, 3°, ..., 360°, ...
∘ around arbitrary axis in 3D space, &
6 project into coord. sys. planes



⇒ "Movie"

- ORTHO PROJ. ONTO ARBITRARY PLANE IN 3D SPACE:



① P: $Ax + By + Cz + D = 0$

② $P(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$

$= \vec{P} + t \cdot \vec{n}$

$= \begin{pmatrix} x \\ y \\ z \end{pmatrix} + t \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$

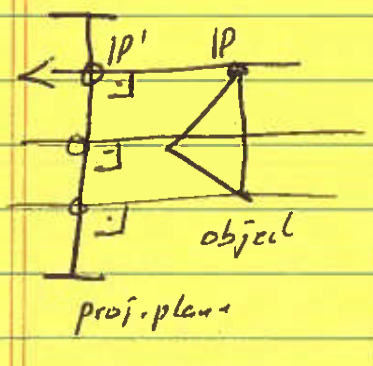
② into ①: $A \cdot (x + t n_x) + \dots + C \cdot (z + t n_z) = -D$

$\Rightarrow t = \dots \Rightarrow P(t) =$ ✓

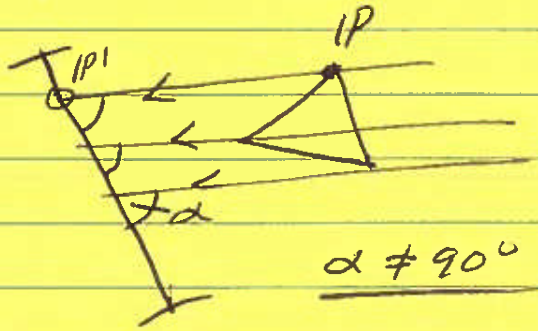
OR: Reduce to axonometric proj into coord. sys. plane

- OBLIQUE PROJS

ortho proj:



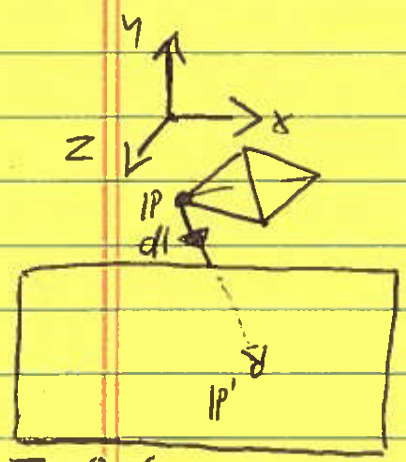
oblique:



$\alpha \neq 90°$

LEC 7

• OBLIQUE PROJ IN MATRIX FORM



Ex: oblique proj. into $z=0$ plane

given: $dl = \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}$, $\underline{dz \neq 0}$

$\Rightarrow$ Ray / line equation:

$$IP' = \begin{pmatrix} x' \\ y' \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + t \cdot \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}$$

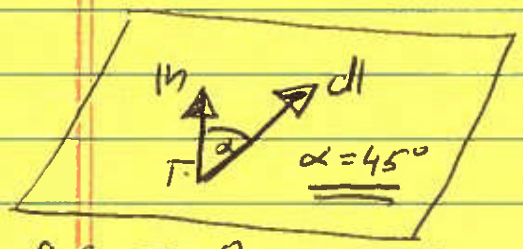
$$0 = z + t\,dz \Rightarrow \boxed{t = -z/dz}$$

$$\Rightarrow \left(\begin{array}{c} \begin{pmatrix} x' \\ y' \\ 0 \\ 1 \end{pmatrix} \end{array} = \left(\begin{array}{ccc|c} 1 & 0 & -dx/dz & 0 \\ 0 & 1 & -dy/dz & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}\right)$$

• SPECIAL / IMPORTANT ONES (OBLIQUE):

(a) CAVALIER: angle between ln and $dl = 45°$

(infinite possibilities)



e.g., $z=0 \Rightarrow ln = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

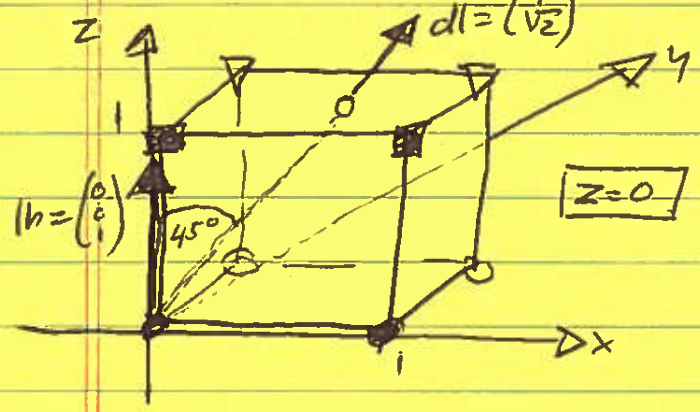
Ex: $z=0 \Rightarrow ln = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$,

$dl = \begin{pmatrix} 1 \\ 1 \\ \sqrt{2} \end{pmatrix}$

{check: $ln \cdot dl / 2 = \sqrt{2}/2 = \cos(\alpha) \Rightarrow \alpha = 45°$}

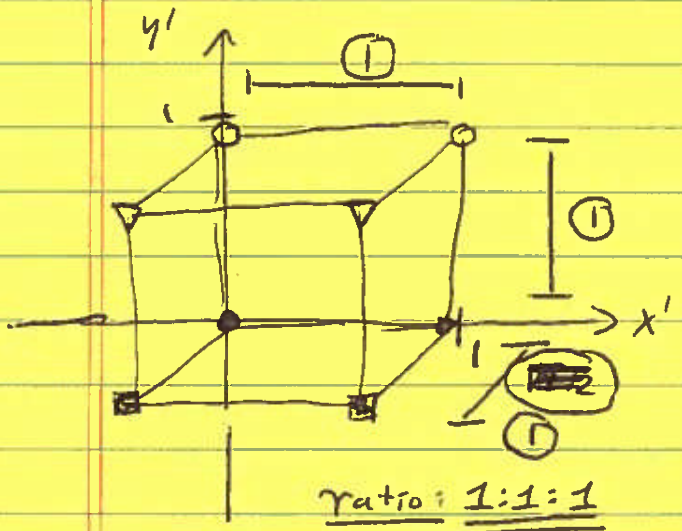
$$\Rightarrow CAV = \left(\begin{array}{ccc|c} 1 & 0 & -\sqrt{2}/2 & 0 \\ 0 & 1 & -\sqrt{2}/2 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

• **Ex:** Project UNIT CUBE using CAVALIER:



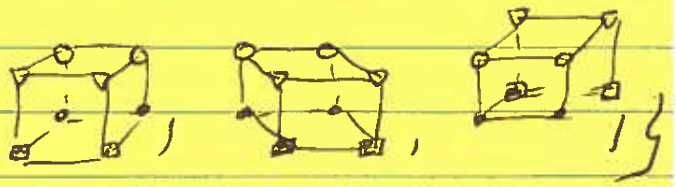
$$\begin{pmatrix} x' \\ y' \\ z'=0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -dx/dz \\ 0 & 1 & -dy/dz \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$dx = dy = dz = \sqrt{2}/2$



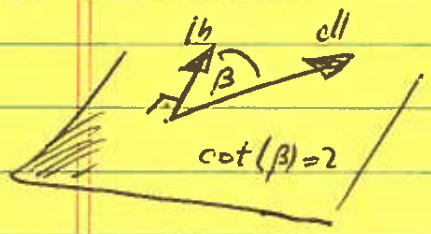
x	y	z	x'	y'
0	0	0	0	0
1	0	0	1	0
0	1	0	0	1
1	1	0	1	1
0	0	1	$-\sqrt{2}/2$	$-\sqrt{2}/2$
1	0	1	$1-\sqrt{2}/2$	$-\sqrt{2}/2$
0	1	1	$-\sqrt{2}/2$	$1-\sqrt{2}/2$
1	1	1	$1-\sqrt{2}/2$	$1-\sqrt{2}/2$

{ • ANIMATION: Rotate dl around z-axis:



}

(b) CABINET



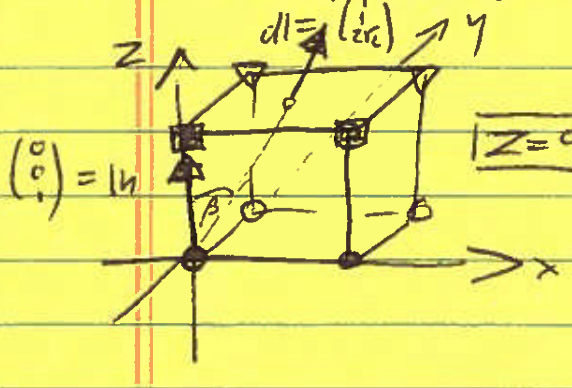
⇒ infinite possibilities for plane & direction vector dl

⇒ Ex $lh = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $dl = \begin{pmatrix} 1 \\ 2\sqrt{2} \end{pmatrix}$

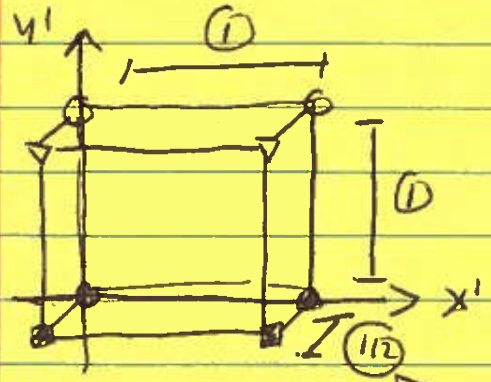
{ ⇒ ... $\cot(\beta) = 2$ }

$$\Rightarrow CAB = \left(\begin{array}{ccc|c} 1 & 0 & -\sqrt{2}/4 & 0 \\ 0 & 1 & -\sqrt{2}/4 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

Ex: CAB. proj. of UNIT CUBE:



$$\begin{pmatrix} x' \\ y' \\ z'=0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\sqrt{2}/4 \\ 0 & 1 & -\sqrt{2}/4 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



ratio: 1:1:1/2

x	y	z	x'	y'
0	0	0	0	0
1	0	0	1	0
0	1	0	0	1
1	1	0	1	1
0	0	1	$-\sqrt{2}/4$	$-\sqrt{2}/4$
1	0	1	$1-\sqrt{2}/4$	$-\sqrt{2}/4$
0	1	1	$-\sqrt{2}/4$	$1-\sqrt{2}/4$
1	1	1	$1-\sqrt{2}/4$	$1-\sqrt{2}/4$

consequence of $\cot(\beta) = 2$

⇒ Animation: rotate dl around z-axis

⇒ General: General oblique projection:
- any plane normal lh,
- any direction vector dl (not lying in the plane of projection.)