MAT128a: Numerical Analysis, Fall 2015
Programming Project Four
Due: December 7, 2015

In this project, you will write three MATLAB functions which perform tasks involving Gaussian quadrature. There is a template file available on the website which defines each of these functions. Follow these templates exactly — we will grade the project by calling your functions and testing to see that they perform as expected.

Each of the functions you write should be placed in an ".m" file whose name is the same as the name of the function. You will submit your project by sending an email to the following address:

`mat128a_fall2015@math.ucdavis.edu`

Please send one email with three attachments, one for each of the ".m" files which comprise this project. You will get a reply with your score within two weeks of submission (probably much sooner). Please try to avoid making multiple submissions — wait to submit your project until you are completely satisfied with it. If you do submit more than once, only the last submission will be graded. Moreover, only submissions *received* before 11:59 PM on the due date will be considered.

Project description

1. Write a MATLAB function called "legeder" which takes as input a nonnegative integer n and a real number x in the interval $(-1, 1)$ and returns the value of the Legendre polynomial of degree n and its derivative at the point x. Repeatedly apply the recurrence relation

$$(k+1)P_{k+1}(x) = (2k+1)xP_k(x) - nP_{k-1}(x) \tag{1}$$

in order to evaluate P_n at x and use the formula

$$\frac{x^2-1}{n}P_n'(x) = xP_n(x) - P_{n-1}(x) \tag{2}$$

in order to evaluate its derivative.

2. Write a MATLAB function called "legeroot" which takes as input a real number z_0 and a nonnegative integer n and returns an approximation of the root of the Legendre polynomial P_n obtained via Newton's method. More specifically, you should compute the iterates $z_1, z_2, \ldots$ defined via the formula

$$z_{k+1} = z_k - \frac{P_n(z_k)}{P_n'(z_k)}. \tag{3}$$

The process should be terminated and the value of z_k accepted as the root when $|z_{k+1} - z_k| < 10^{-13}$.

3. Write a MATLAB function called "legequad" which takes as input a positive integer n and returns as output arrays containing the nodes and weights of the n-point Legendre quadrature rule on the interval $(-1, 1)$.