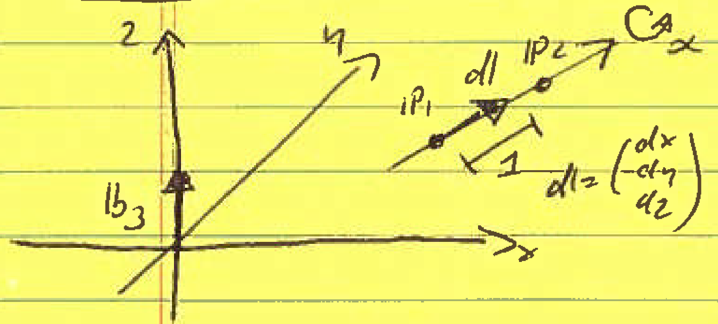
LEC 6

• ROT ABOUT GENERAL AXIS



given: $\mathbb{P}_1, \mathbb{P}_2, \alpha$

$$\Rightarrow dl = \frac{\mathbb{P}_2 - \mathbb{P}_1}{\|\mathbb{P}_2 - \mathbb{P}_1\|}$$

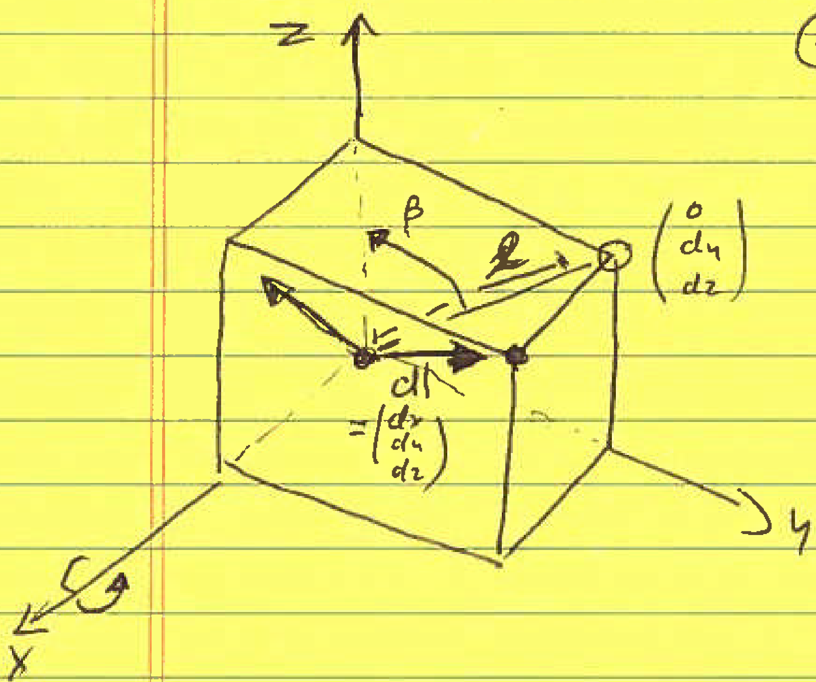
$$= \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{pmatrix} \Big/ \sqrt{(x_2-x_1)^2 + \dots + (z_2-z_1)^2}$$

$\Rightarrow$ "Reduce to Rot about z-axis"

i) $M_1 = \text{Transl}(-\mathbb{P}_1)$
ii) $M_2 =$ "Rot d into z axis"
iii) $M_3 = \text{Rot}Z(\alpha)$
iv) $M_4 =$ inverse of M_2
v) $M_5 = \text{Transl}(\mathbb{P}_1)$

$$\underline{M} = M_5 M_4 M_3 M_2 M_1$$

$\rightarrow$ ii) IS COMPLICATED ...



ⓐ Rotate dl about x-axis to move it into xz-plane:

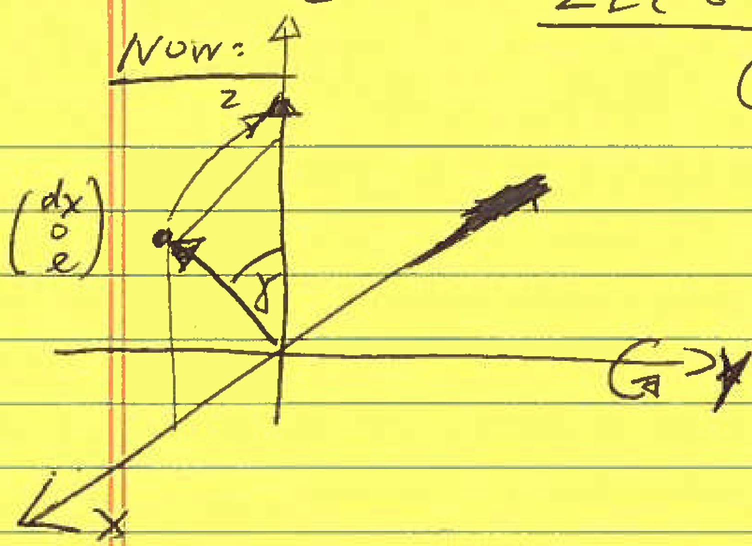
$$c\beta = \frac{dz}{\ell} = \frac{dz}{\sqrt{dy^2 + dz^2}}$$

$$s\beta = \frac{dy}{\ell} = \frac{dy}{\sqrt{dy^2 + dz^2}}$$

$$\Rightarrow M_{21} = \text{Rot}X(\beta)$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & dz/\ell & -dy/\ell & 0 \\ 0 & dy/\ell & dz/\ell & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

Now:



(b) "Rotate 'intermediate' vector into z-axis"

$c\gamma = l \Rightarrow \underline{c{-\gamma} = l}$

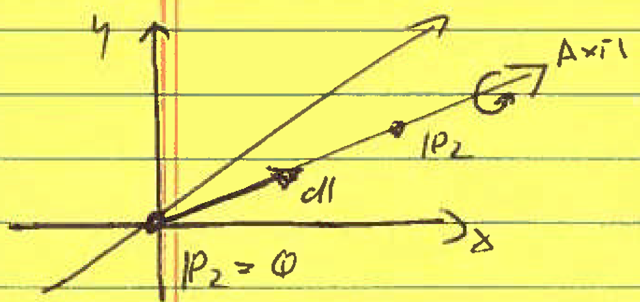
$s\gamma = dx \Rightarrow \underline{s{-\gamma} = -dx}$

$\Rightarrow M_{22} = RotY(-\gamma)$

$$= \left(\begin{array}{ccc|c} l & 0 & -dx & 0 \\ 0 & 1 & 0 & 0 \\ dx & 0 & l & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$\Rightarrow \boxed{M_2 = M_{22} \cdot M_{21}}$$

Ex:



given $P_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $P_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, α

$$\Rightarrow dl = \begin{pmatrix} \sqrt{3}/3 \\ \sqrt{3}/3 \\ \sqrt{3}/3 \end{pmatrix}$$

$\alpha = 45°$

(a) M_{21}: $l = \sqrt{d_y^2 + d_z^2} = \sqrt{6}/3$

$c\beta = \dfrac{\sqrt{3}/3}{\sqrt{6}/3} = \sqrt{2}/2 = d_z/l$

$s\beta = d_y/l = \ldots = \sqrt{2}/2$

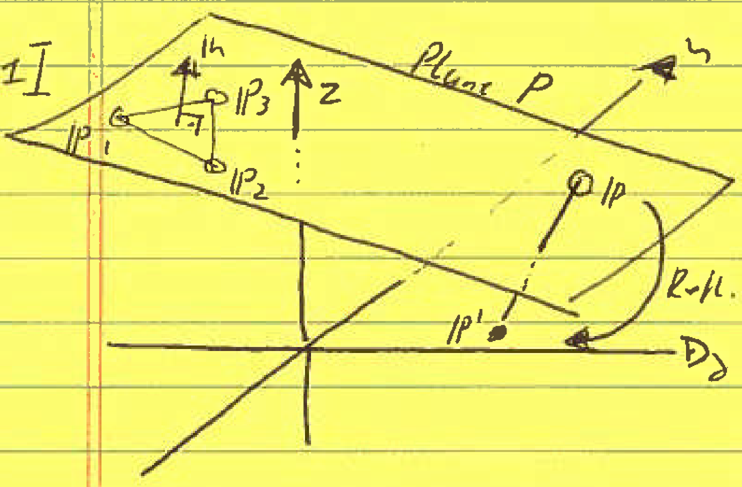
(b) M_{22}: $c(-\gamma) = l = \sqrt{6}/3$

$s(-\gamma) = -dx = -\sqrt{3}/3$

LEC 6

- REFL. W.R.T. GENERAL PLANE

1]



given: P_1, P_2, P_3 (points in plane P)

$\Rightarrow$ normal n =

$$n = \frac{(P_2 - P_1) \times (P_3 - P_1)}{\| (\ ) \times (\ ) \|} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$$

$\Rightarrow$ Reduce to ReflXY (= refl. w.r.t. xy-plane):

i) M_1 = Transl$(-P_1)$

ii) M_2 = "Rotate_n_into_zaxis"

iii) M_3 = ReflXY

iv) M_4 = "inverse of M_2"

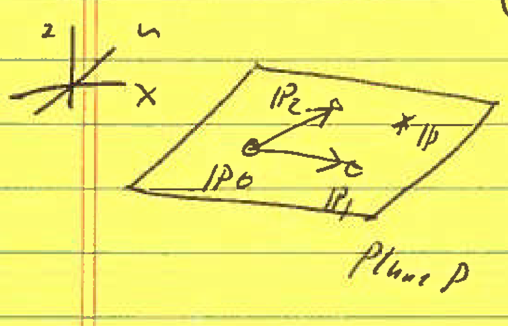
v) M_5 = Transl(P_1)

$M = M_5 M_4 M_3 M_2 M_1$

$$\boxed{M = M_5 M_4 M_3 M_2 M_1}$$

ii) $\longrightarrow$ SEE PREV. EX. (ROT. GEN. AXIS)

{Rem. "PLANE"



(a) IMPLICIT $\boxed{Ax + By + Cz + d = 0}$

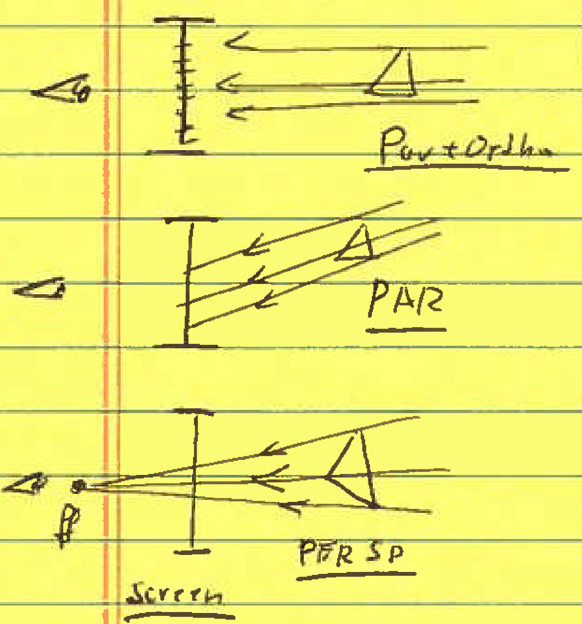
e.g., $x = 0$, $y = 0$, $z = 0$

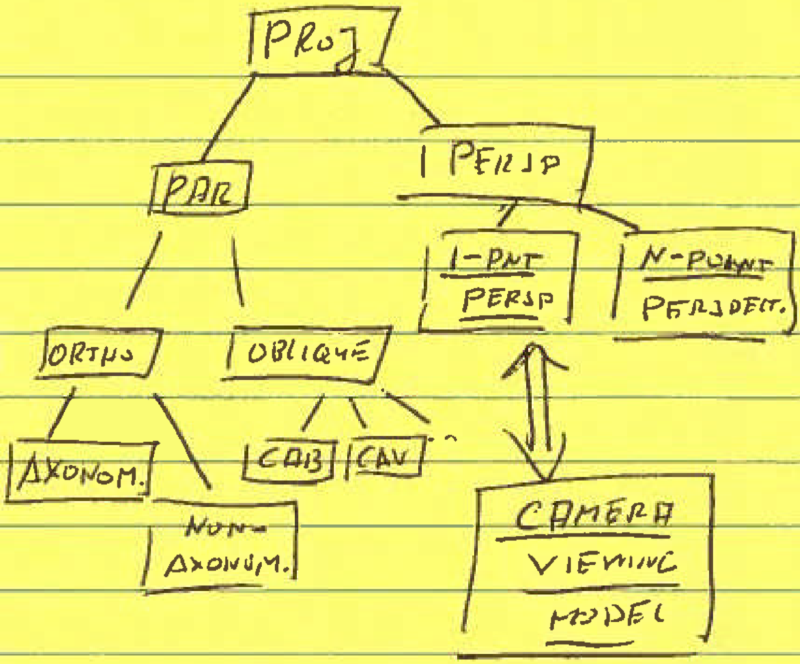
(b) EXPLICIT $\boxed{z = f(x,y) = mx + hy + b}$

(c) PARAMETRIC:

$$\boxed{P = P_0 + u \cdot (P_1 - P_0) + v \cdot (P_2 - P_0)}$$

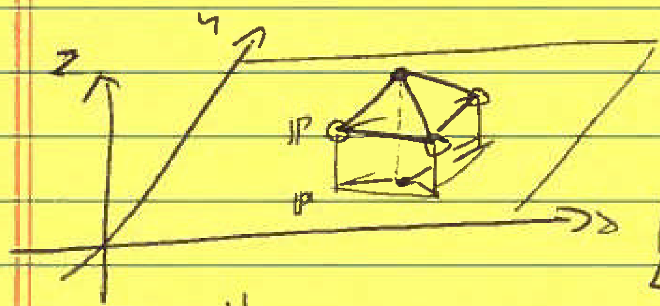
- **PROJECTIONS**





- SIMPLEST: AXONOMETRIC – PROJ. ONTO COORD. SYS. PLANES

EX. Proj XY = proj. onto xy-plane

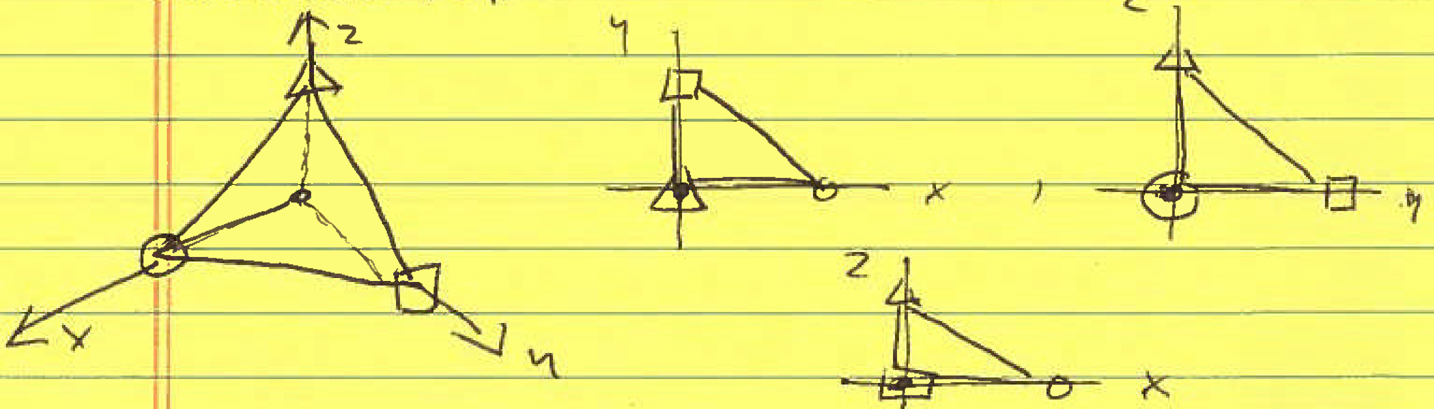


$\mathbb{p} = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}, \quad \mathbb{p}' = \begin{pmatrix} x \\ y \\ 0 \\ 1 \end{pmatrix}$

$$\boxed{\text{Proj}\,XY = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}$$

- OFTEN DONE: 3 Proj's onto coord. sys. planes

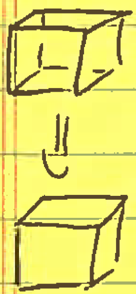
$$\Rightarrow \text{Proj}\,XZ = \left(\begin{array}{ccc|c} 1 & & & \\ & 0 & & \\ & & 1 & \\ \hline & & & 1 \end{array}\right), \quad \text{Proj}\,YZ = \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

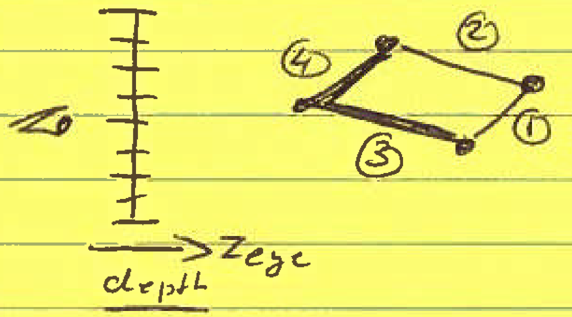


LEC 6

- HIDDEN LINE / SURFACE ALGORITHMS

Ex: "PAINTER'S ALGO" ("Paint from back to front")



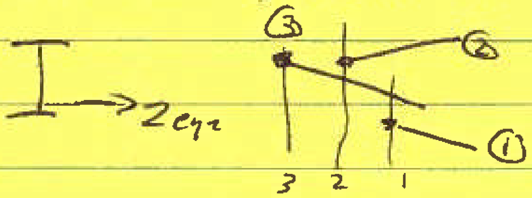


1) SORT BACK TO FRONT: 1, 2, 3, 4

2) RENDER IN THIS ORDER

- Here: SORT EDGES (POLYS) based on z_{eye}^{min}-values;

Ex:



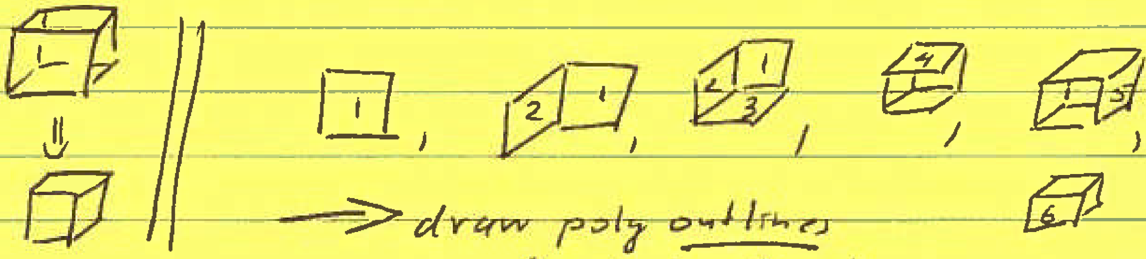
→ FOR: SCENES OF NON-INTERSECT. PLANAR POLYGONS!

ALGO (WIRE FRAME): 1) SORT POLYS W.R.T. DEPTH (= dist. from screen)

2) RENDER SORTED SET OF POLYS.:

i) ERASE EVERYTHING IN REGION OF NEW POLY;

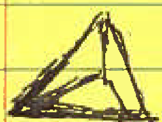
ii) DRAW EDGES OF NEW POLY;



→ draw poly outlines back to front

→ ONLY FOR ORTHO PROJECTIONS!

{Rem: VARYING EDGE THICKNESS BASED ON DEPTH:



→ Add'l. Points for project 2 }