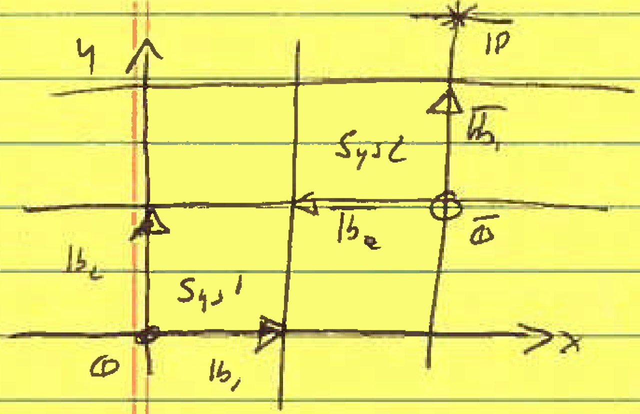
- COORD. SYS. TRANSFORM



$$IP_{sys\,1} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$IP_{sys\,2} = \begin{pmatrix} ? \\ ? \end{pmatrix}$$

Sys 1 ↦ Sys 2 :

① ROT SYS 1 by $\alpha = 90°$

② THEN translate sys by $t = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\Longrightarrow$ IP sys 2 = ??

- RULE : " Apply ① & ② in reverse order with "inverted" arguments :

$$IP_{sys\,2} = Rot(-\alpha) \cdot Transl(-t) \cdot IP_{sys\,1}$$

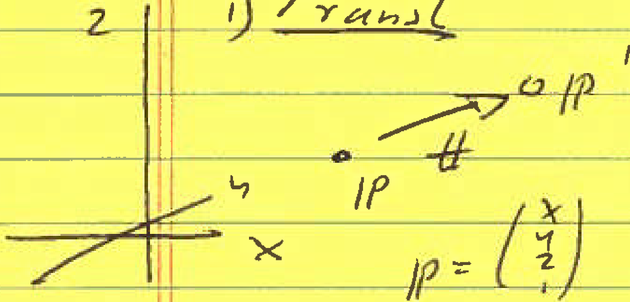
$$= \begin{pmatrix} c\text{-}\alpha & -s\text{-}\alpha & 0 \\ s\text{-}\alpha & c\text{-}\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \quad ✓$$

• 3D TRANSFORMS

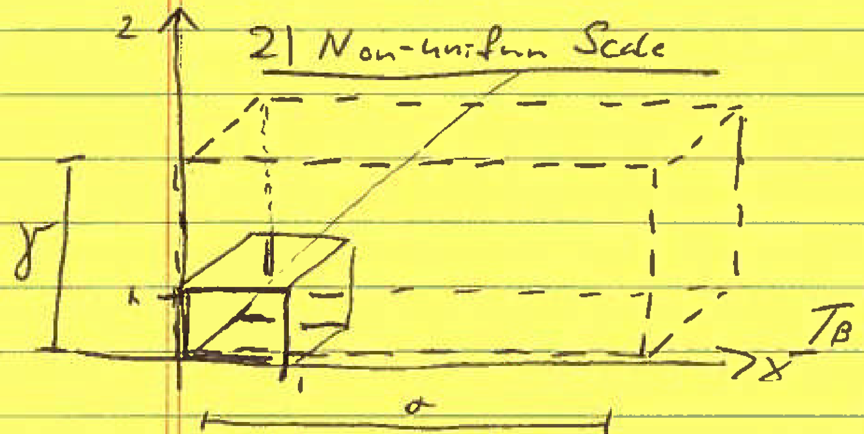
1) Transl



$$P = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\boxed{P' = \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & \cdot & \cdot & t_x \\ \cdot & 1 & \cdot & t_y \\ \cdot & \cdot & 1 & t_z \\ \cdot & \cdot & \cdot & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}}$$

2) Non-uniform Scale

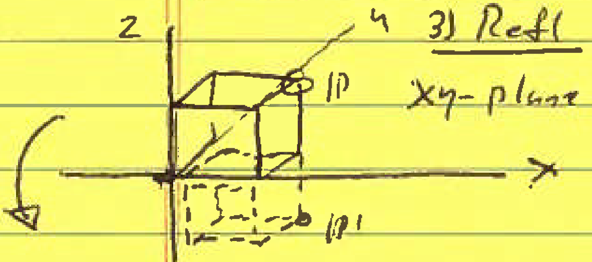


$$\boxed{\text{Scale}(\alpha, \beta, \gamma) = \left(\begin{array}{ccc|c} \alpha & \cdot & \cdot & \cdot \\ \cdot & \beta & \cdot & \cdot \\ \cdot & \cdot & \gamma & \cdot \\ \hline \cdot & \cdot & \cdot & 1 \end{array}\right)}$$

IMAGES OF BASIS VECTORS

3) Refl

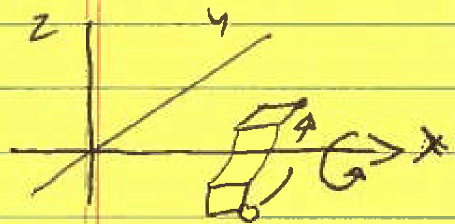
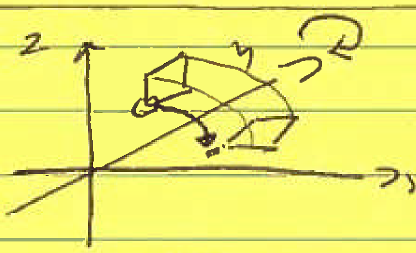
xy-plane



$$\boxed{\text{Refl } XY = \left(\begin{array}{ccc|c} 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & -1 & \cdot \\ \hline \cdot & \cdot & \cdot & 1 \end{array}\right)}$$

Refl XZ = ... , Refl YZ = ...

4) Rotation

i) Rot X	ii) Rot Y	iii) Rot Z
		

$$\text{Rot}X(\alpha) = \left(\begin{array}{ccc|c} 1 & \cdot & \cdot & 0 \\ \cdot & C\alpha & -S\alpha & 0 \\ \cdot & S\alpha & C\alpha & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$\text{Rot}Y(\beta) = \left(\begin{array}{ccc|c} C\alpha & \cdot & S\alpha & 0 \\ \cdot & 1 & \cdot & 0 \\ -S\alpha & \cdot & C\alpha & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \ !$$

$$\text{Rot}Z(\gamma) = \left(\begin{array}{ccc|c} C\alpha & -S\alpha & \cdot & 0 \\ S\alpha & C\alpha & \cdot & 0 \\ \cdot & \cdot & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

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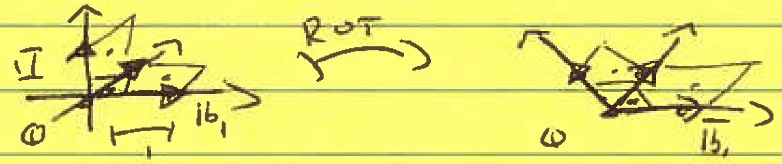
• **Remarks**: i) ROT.-MATRIX: $\boxed{\det(Rot) = 1}$

ii) $RotX(\alpha) \cdot RotY(\beta) \neq RotY(\beta) \cdot RotX(\alpha)$

iii) Rot. is ORTHONORMAL transformation:

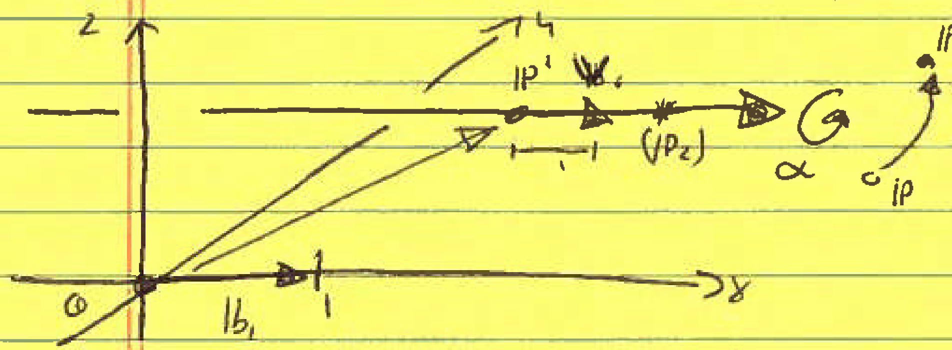


$$\boxed{Rot = \left(\begin{array}{ccc|c} \underbrace{[\bar{\mathbb{b}}_1] \; [\bar{\mathbb{b}}_2] \; [\bar{\mathbb{b}}_3]}_{\text{mutually orthogonal}} & & & \vdots \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}$$

$$\bar{\mathbb{b}}_i \cdot \bar{\mathbb{b}}_j = 0 \wedge \|\bar{\mathbb{b}}_i\| = 1$$

• COMPLICATED TRANSFORMS – CONCATENATION

Ex: "Rot about axis parallel to x-axis":



• given:

$\Downarrow$

$\mathbb{P}_1$, ~~…~~, α

$$\left.\begin{array}{lll} ① & Transl(-\mathbb{P}_1) & = M_1 \\ ② & RotX(\alpha) & = M_2 \\ ③ & Transl(\mathbb{P}_1) & = M_3 \end{array}\right\} M = M_3 M_2 M_1$$

$$\Rightarrow \mathbb{P}' = M \mathbb{P}$$

{ *Similar*: Rot about axis parallel to y- or z-axis }