

Midterm Project

Thursday, November 4, 2021 8:09 AM

F: Whether to found company

M: market

$$P(M_0) = 0.5$$

$$P(M_1) = 0.3$$

$$P(M_2) = 0.2$$

V: profit

$$V(M, F)$$

	$M=M_0$	$M=M_1$	$M=M_2$
Found ($F=f_1$)	-7	5	20
Not found ($F=f_0$)	0	0	0

$C=1$ is cost of survey

S: outcome of survey

$$P(S|M)$$

	$S=S_0$	$S=S_1$	$S=S_2$
$M=M_0$	0.6	0.3	0.1
$M=M_1$	0.3	0.4	0.3
$M=M_2$	0.1	0.4	0.5

1. goal: calculate expected profit of company based on prior probability of market

$$E(V) = \sum_{i=0}^2 V(F=f_i, M=m_i) P(m_i)$$

$$= -7(0.5) + 5(0.3) + 20(0.2)$$

$$= -3.5 + 1.5 + 4$$

$$= 2$$

Yes, you should found the company if you think an expected profit of 2 is worth the efforts of founding a company. If you don't found it, you will have an expected profit of 0.

2. goal: calculate marginal probability of outcome of survey (S)

recall marginal pmf of S can be calculated by $P(S=s) = \sum_i P(s, m_i)$

$$\text{We're given } P(S|M) = \frac{P(S, M)}{P(M)} \rightarrow P(S, M) = P(M)P(S|M)$$

$$\text{Thus, we can rewrite } P(S=s) = \sum_i P(m_i) P(s|m_i)$$

$$P(S=S_0) = \sum_i P(m_i) P(S_0|m_i)$$

$$= 0.5(0.6) + 0.3(0.3) + 0.2(0.1)$$

$$= 0.3 + 0.09 + 0.02$$

$$= 0.41$$

$$P(S=S_1) = \sum_i P(m_i) P(S_1|m_i)$$

$$= 0.5(0.3) + 0.3(0.4) + 0.2(0.4)$$

$$= 0.15 + 0.12 + 0.08$$

$$= 0.35$$

$$= 0.15 + 0.12 + 0.08$$

$$= \boxed{0.35}$$

$$\begin{aligned} P(S=s_2) &= \sum_i P(m_i) P(s_2 | m_i) \\ &= 0.5(0.1) + 0.3(0.3) + 0.2(0.5) \\ &= 0.05 + 0.09 + 0.1 \\ &= \boxed{0.24} \end{aligned}$$

3. goal: find max amount entrepreneur should pay for survey (based on value of info of survey)

$$VPI_e(E_j) = (\sum_k P(E_j = e_{jk}|e) EU(\alpha_{ejk}|e, E_j = e_{jk})) - EU(\alpha|e)$$

We calculated $EU(\alpha|e)$ in #1 already. This is our expected utility if we don't survey.

So now let's focus on the first term in our above VPI equation.

Recall our oil drilling example in class.

For evidence, we used oil/no oil from the sample. Here, our evidence is S .

$$\begin{aligned} P(S=s_0) &= 0.41 \\ P(S=s_1) &= 0.35 \\ P(S=s_2) &= 0.24 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{evidence}$$

If $S=S_0$, expected gain is expected profit:

$$\begin{aligned} \text{Table 1} &\quad \downarrow \quad \text{Can obtain from Table 2} \\ &\quad \downarrow \quad \text{using } P(m|S) = \frac{P(m, S)}{P(S)} = \frac{P(s|m)P(m)}{P(S)} \\ \sum_i V(m_i, F) P(m_i | S_0) &= P(s|m) = \frac{P(s, m)}{P(m)} \xrightarrow{\text{see \#2}} P(s, m) = P(m)P(s|m) \\ &= V(m_0, F) P(m_0 | S_0) + V(m_1, F) P(m_1 | S_0) + V(m_2, F) P(m_2 | S_0) \\ &= V(m_0, F) \frac{P(s_0|m_0)P(m_0)}{P(s_0)} + V(m_1, F) \frac{P(s_0|m_1)P(m_1)}{P(s_0)} + V(m_2, F) \frac{P(s_0|m_2)P(m_2)}{P(s_0)} \\ &= -7 \frac{0.6(0.5)}{0.41} + 5 \frac{0.3(0.3)}{0.41} + 20 \frac{0.1(0.2)}{0.41} \\ &= -3.04878 \end{aligned}$$

Since this is less than 0, we just don't find the company, and our expected gain is 0.

Thus, if $S=S_0$, expected utility is 0.

If $S=s_1$, $\sum_i V(m_i, F) P(m_i | S_1)$

$$\begin{aligned} &= V(m_0, F) P(m_0 | S_1) + V(m_1, F) P(m_1 | S_1) + V(m_2, F) P(m_2 | S_1) \\ &= V(m_0, F) \frac{P(s_1|m_0)P(m_0)}{P(s_1)} + V(m_1, F) \frac{P(s_1|m_1)P(m_1)}{P(s_1)} + V(m_2, F) \frac{P(s_1|m_2)P(m_2)}{P(s_1)} \\ &= -7 \frac{0.3(0.5)}{0.35} + 5 \frac{0.4(0.3)}{0.35} + 20 \frac{0.4(0.2)}{0.35} \\ &= 3.2857 \end{aligned}$$

So if $S=s_1$, expected utility is 3.2857

If $S=s_2$, $\sum_i V(m_i, F) P(m_i | S_2)$

$$\begin{aligned} &= V(m_0, F) P(m_0 | S_2) + V(m_1, F) P(m_1 | S_2) + V(m_2, F) P(m_2 | S_2) \\ &= V(m_0, F) \frac{P(s_2|m_0)P(m_0)}{P(s_2)} + V(m_1, F) \frac{P(s_2|m_1)P(m_1)}{P(s_2)} + V(m_2, F) \frac{P(s_2|m_2)P(m_2)}{P(s_2)} \\ &= -7 \frac{0.1(0.5)}{0.24} + 5 \frac{0.3(0.3)}{0.24} + 20 \frac{0.5(0.2)}{0.24} \end{aligned}$$

$$\begin{aligned}
 &= V(m_0, F) \frac{P(S_0|m_0)P(m_0)}{P(S_0)} + V(m_1, F) \frac{P(S_1|m_1)P(m_1)}{P(S_1)} + V(m_2, F) \frac{P(S_2|m_2)P(m_2)}{P(S_2)} \\
 &= -7 \frac{0.1(0.5)}{0.24} + 5 \frac{0.3(0.3)}{0.24} + 20 \frac{0.5(0.2)}{0.24} \\
 &= 8.75
 \end{aligned}$$

So if $S = S_2$, expected utility is 8.75

$$\begin{aligned}
 \text{first term} &= \sum_s P(S=s) EU(S=s) \\
 &= 0.41(0) + 0.35(3.2857) + 0.24(8.75) \\
 &= 3.25
 \end{aligned}$$

The max amount an entrepreneur should pay for survey (based on value of info of survey) is $3.25 - 2 = 1.25$

4. goal: decide whether to found company based on conditioning on each of the 3 outcomes of survey

We calculated $E(V|S=S_0)$, $E(V|S=S_1)$, and $E(V|S=S_2)$ in #3 already.

$$E(V|S=S_0) = -3.04878 < 0 \rightarrow \text{do NOT found company}$$

$$E(V|S=S_1) = 3.2857 > 0 \rightarrow \text{found company}$$

$$E(V|S=S_2) = 8.75 > 0 \rightarrow \text{found company}$$

O is the expected gain from not founding company.