SPATIAL CONVOLUTION AND CORRELATION

Filtering

- Filtering is a technique for modifying or enhancing an image.
  - Emphasize some features or remove other features
- Includes smoothing, sharpening, edge enhancement etc
- When filtering is used directly on the pixels it is called **spatial filtering**

Correlation and Convolution

- Correlation and Convolution are related concepts used in **linear spatial filtering**
  - When filtering is used directly on the pixels it is called **spatial filtering**
  - Term **linear** indicates that a pixel is replaced by the linear combinations of its neighbours

Correlation

- Correlation is a neighbourhood operation in which each output pixel is the weighted sum of neighbouring pixels
- The matrix of weights is called correlation kernel or filter or mask.

Correlation Example:

- Input Image

1	2	2	3	2
4	2	3	1	2
4	2	1	1	6
3	2	1	4	5
3	2	1	2	1

- Filter / mask:

4	5	1
5	1	2
3	1	3

Correlation Example: Apply correlation to pixel (1, 2) on the given below image using the given filter.

1	2	2	3	2
4	2	3	1	2
4	2	1	1	6
3	2	1	4	5
3	2	1	2	1

Input Image

4	5	1
5	1	2
3	1	3

Filter / mask

Output Pixel (1,2) = 2x4 + 2x5 + 3x1 + 2x5 + 3x1 + 1x2 + 2x3 + 1x1 + 1x3 = 46

Output Image

1	2	2	3	2
4	2	**46**	1	2
4	2	1	1	6
3	2	1	4	5
3	2	1	2	1

Correlation Example: On entire image

0	0	0	0	0	0	0
0	1	2	2	3	2	0
0	4	2	3	1	2	0
0	4	2	1	1	6	0
0	3	2	1	4	5	0
0	3	2	1	2	1	0
0	0	0	0	0	0	0

Input Image

4	5	1
5	1	2
3	1	3

Filter / mask

Output Image

15	34	30	33	22
25	61	46	67	38
39	67	56	59	42
38	60	46	42	66
24	42	32	38	52

Convolution

- Convolution is similar to correlation and so, each output pixel is the weighted sum of neighbouring pixels
- The difference is that, the matrix of weights (filter or mask) is rotated 180°

Convolution Example: On entire image

0	0	0	0	0	0	0
0	1	2	2	3	2	0
0	4	2	3	1	2	0
0	4	2	1	1	6	0
0	3	2	1	4	5	0
0	3	2	1	2	1	0
0	0	0	0	0	0	0

Input Image

3	1	3
2	1	5
1	5	4

4	5	1
5	1	2
3	1	3

Filter / mask

Output Image

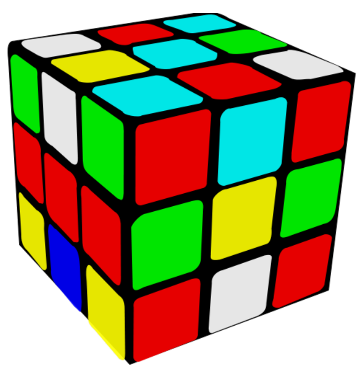
39	40	42	33	19
49	54	40	62	46
47	55	45	90	42
46	47	50	68	29
22	27	34	31	22

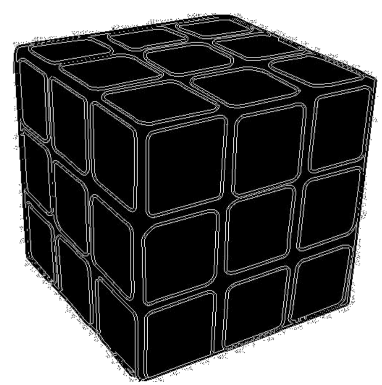
EDGE DETECTION

Edges

- Edges are those places in a image that corresponds to object boundaries
- Variation in intensity level between neighbourhood pixels constitute an edge

- Therefore the objective of edge detector is to measure intensity gradient





Edge Detection, Why?

- Image sharpening and enhancement
- Data extraction
- Object identification
  - Detect object shape, size etc
  - Finger print matching, medical diagnosis, number plate detection

Types of edges

- Generally edges are of three types:
  - Horizontal
  - Vertical
  - Diagonal edges

Edge Detection Steps

1. Smoothing : Noise reduction
2. Enhancement: Edge sharpening
3. Thresholding: Detecting which to discard and which to maintain
4. Localization: Determine the exact location of an edge

Edge detection methods

- First order derivative/Gradient Methods
  - Roberts Operator
  - Prewitt Operator
  - Sobel Operator
  - Robinson
  - Kirsch
- Second order derivative
  - Laplacian of Gaussian
  - Difference of Gaussian

Gradient Methods

- An image gradient is a directional change in the intensity or colour in an image
- It includes both magnitude and direction
  - Magnitude of the gradient is defined by

$$M = \sqrt{Gx^2 + Gy^2}$$

  - Direction is given by

$$\theta = tan^{-1}\left(\frac{Gx}{Gy}\right)$$

Roberts Edge Detector

- Roberts operator performs a simple two dimensional spatial gradient measurement on an image
- It uses a pair of 2x2 convolution mask

Gx =

1	0
0	-1

Gy =

0	1
-1	0

Roberts Edge Detector

- Algorithm:
  - Read the image
  - Convolve the image with Gx
  - Convolve the image with Gy
  - Compute the edge magnitude
  - Compare the edge magnitude with a threshold value
    - If edge magnitude is higher, assign it as a possible edge point

Roberts Edge Detector : Example

A =

3	6	1	5
3	5	7	0
4	3	4	3
2	5	7	5

Gx =

1	0
0	-1

Gy =

0	1
-1	0

Roberts Edge Detector : Example

A =

3	6	1	5
3	5	7	0
4	3	4	3
2	5	7	5

Gx =

1	0
0	-1

Gy =

0	1
-1	0

M = $\sqrt{Gx^2 + Gy^2}$

Gx = -2 Gy = 3

M = $\sqrt{(-2)^2 + 3^2}$ = 3.61 ≅ 4

4	4	2

Roberts Edge Detector : Example

A =

3	6	1	5
3	5	7	0
4	3	4	3
2	5	7	5

Gx =

1	0
0	-1

Gy =

0	1
-1	0

M =

4	4	2
1	4	6
1	4	4

Prewitt Edge Detector

- It uses 3x3 convolution mask

Gx =

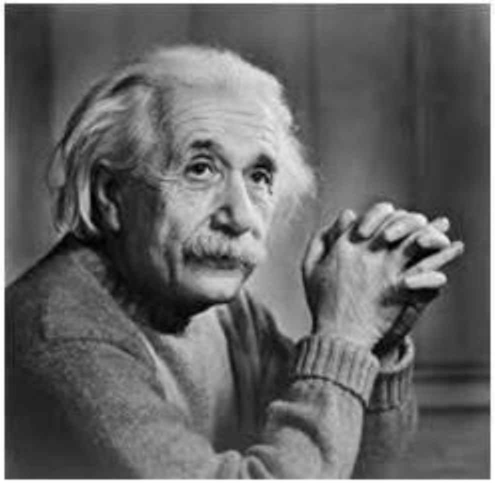
-1	0	1
-1	0	1
-1	0	1

Gy =

-1	-1	-1
0	0	0
1	1	1

- Prewitt method takes the central difference of the neighbouring pixels

Prewitt Edge Detector



After applying vertical mask

After applying horizontal mask

Sobel Edge Detector

- It uses 3x3 convolution mask

Gx =

-1	0	1
-2	0	2
-1	0	1

Gy =

-1	-2	-1
0	0	0
1	2	1

- Sobel method takes the central difference of the neighbouring pixels

Gradient based Edge Detection

Advantages

- Simple to implement
- Capable of detecting edges and their direction

Disadvantages

- Susceptible to noise
- Not very accurate

THANK YOU