3.a. $\neg(\neg p \vee q) \vdash p$

1. $\neg(\neg p \vee q)$ premise
2. $\neg p$ assume
3. $\neg p \vee q$ $\vee i, 2$
4. $\perp$ $\perp i$ 1,3.
5. $\neg\neg p$ $\neg i$, 2-4
6. p $\neg\neg e$, 5.

$p \to q,\ s \to t \vdash p \vee s \to q \wedge t$

3.b. 1. $p \to q$ (premise)
2. $s \to t$ (premise)
3. $p \vee s$ assume
4. q. ~~$\to e$ 1,3~~
4. p assume

$\boxed{\text{Not valid}}$

c. $(p \wedge q) \to r,\ r \to s,\ q \wedge \neg s \vdash \neg p$

1. $(p \wedge q) \to r$. ~~assume~~ premise
2. $r \to s$ ~~assume~~ premise
3. $q \wedge \neg s$. ~~assume~~ Premise
4. p assume
5. q $\wedge e$, 3.
6. $p \wedge q$ $\wedge i$, 4,5
7. r. $\to e$ 1,6.
8. s $\to e$ 2,7
9. $\neg s$ ~~→~~ $\wedge e$, 3.
10. $\bot$ $\bot i$ 8,9.
11. $\neg p$. $\neg i$, 4-10

d. $(p \vee r) \to (p \to q),\ p \vdash q$.

1. $(p \vee r) \to (p \to q)$ premise
2. p premise

~~3. $p \vee r$ assume~~
~~4. $p \to q$. $\to e$, 1,3.~~
~~5. q. $\to e$ 4,2~~

3. $p \vee r$ $\vee i$, 2
4. $p \to q$ $\to e$ 1,3.
5. q. $\to e$, 4,1.

3 e. 1. $\phi_1 \to (\phi_2 \vee \phi_3)$ premise

2. $\neg \phi_2$ premise

3. $\neg \phi_3$. premise

4. ϕ_1 assume

5. $\phi_2 \vee \phi_3$ $\to e$ 1, 5

6. ϕ_2 assume

7. $\perp$ $\perp i$ 2, 6

8. ϕ_3 assume

9. $\perp$ $\perp i$ 3, 8

10. $\perp$

11. $\neg \phi_1$ $\neg i$ 4-10.

1. A wants to determine the relative salaries of three coworkers using 2 facts.

He knows that if B is not the highest paid of three, then C is.
if C is not the lowest paid., D is paid the most.

A knows. some premises.

let, B = B is the highest paid

C_1 = C is the highest paid C_2 = C is the lowest paid.

D = D is the highest paid.

1. $\neg B \to C_1$

2. $\neg C_2 \to D$.

3. ~~if~~ $\neg B$ assume.

1. First assume that B is not the highest paid.
then C is highest paid.
If C is not lowest paid, then D is highest paid and
A is highest paid.
But one can only be hight paid. So, contradiction.

So, B is highest paid.
Next assume that C is not lowest paid.
then D is highest paid
But again, it is a contradiction that B is highest paid.
Therefore C is lowest paid.
So, B is the highest paid. then D is paid.
So, Answer. B is the most then D then C is lowest paid.

2. There are three people P, Q, R in an island of knights and knaves. P = P is a knight

P says, "if I am a knight, I will go to school".

Let S = P goes to school.

1.	$P \to (P \to S) \land (P \to S) \to P$.	premise
2.	$P \to (P \to S)$	$\land e, 1$
3.	$(P \to S) \to P$	$\land e, 1$
4.	P	assume
5.	$P \to S$	$\to e\, 4, 2$.
6.	P	$\to e, 5, 3$.
7.	T	$\to T, 4, 6$.
8.	P	

9. $P \to S$ $\quad \to e, 8, 2$

10. S $\quad \to e\ 9, 8$

So, P will go to school.

Q: Q is a knight

b. P says "If Q is a knight then I am a knave".

1. $P \to (Q \to \neg P) \wedge (Q \to \neg P) \to P.$
2. $P \to (Q \to \neg P)$ $\quad \wedge e, 1$
3. $(Q \to \neg P) \to P$ $\quad \wedge e, 2$
4. $P \to (P \to \neg Q)$ $\quad$ M.T
5. $(P \to \neg Q) \to P$ $\quad$ M.T
6. P $\quad$ assume
7. $(P \to \neg Q)$ $\quad \to e\ 4, 6.$
8. P $\quad \to e, 7, 5$
9. T $\quad$ Ti 6,8
10. P $\quad \to e\ 10, 5$
11. $P \to \neg Q$ $\quad \to e\ 11, 10$
12. $\neg Q$ $\quad \wedge i\ 10, 12$
13. $P \wedge \neg Q.$

So, P is a knight and Q is a knave.

c. P says: "We're all knaves"
Q says: "No, exactly one of us is a knight"

$P \leftrightarrow \cancel{\neg p \wedge \neg q \wedge \neg r}$
$q \leftrightarrow (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \wedge (\neg p \wedge \neg q \wedge r)$

1. $P \rightarrow \neg p \wedge \neg q \wedge \neg r$
2. $q \rightarrow \cancel{(p \wedge \neg q \wedge \neg r) \wedge}$
$(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$

Let p is a knight,

3.	p	assume
4.	$\neg p$	$\wedge i$ 1
5.	$\perp$	$\perp i$ 3,4
6.	$\neg p$	

p is a knave.
so, his statement is False.
so, $\neg(\neg p \wedge \neg q \wedge \neg r)$ is true.
so, $\cancel{\neg(p \wedge}$ $\neg(\neg p \wedge \neg q \wedge \neg r)$
so, $p \vee q \vee r$. is true
some of us are knights.
q, says, exactly one of us is a knight.
let q is a knave,
so, $\neg((p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r))$:
$\neg(p \wedge \neg q \wedge \neg r) \wedge \neg(\neg p \wedge q \wedge \neg r) \wedge \neg(\neg p \wedge \neg q \wedge r)$
$\neg(p \wedge \neg q \wedge \neg r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee \neg r)$.
$(\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r)$

But this cannot be true ~~q~~

So, our assumption is wrong

so, ~~p~~ q is knight.

and as q's statement is true,

there is only one knight. so, r is also knave.

4.(a) $\neg p \rightarrow (q \wedge r) \vdash p \vee (q \wedge r)$ $\boxed{\text{LEM}}$

1. $\neg p \rightarrow (q \wedge r)$ premise
2. $\neg p$ assume
3. $q \wedge r$ $\rightarrow e$ 1,2
4. $p \vee (q \wedge r)$ $\vee i_2$ 3
5. p assume
6. $p \vee (q \wedge r)$ $\vee i_1$ 5
7. $p \vee \neg p$ LEM
8. $p \vee (q \wedge r)$ $\vee e$, 7, 2–4, 5–6

L.EM.

4.(b) $p \to (q \to \neg p) \vdash q \to \neg p$

1.	$p \to (q \to \neg p)$	premise
2.	$p \vee \neg p$	LEM
3.	p	assume
4.	$q \to \neg p$	$\to e$, 1,3.
5.	$\neg p$	assume
6.	q	assume
7.	$\neg p$	copy
8.	$q \to \neg p$	$\to i$, 6-8.
9.	$q \to \neg p$	$\vee e$ 2, 3-4, 5-8.

(NOT LEM)

4.(a) 1. $\neg p \to (q \wedge r) \vdash p \vee (q \wedge r)$

2.	$\neg(p \vee (q \wedge r))$	assume
3.	$\neg p \wedge \neg(q \wedge r)$	De-Morgan
4.	$\neg p$.	$\wedge e$ 3.
5.	$\neg(q \wedge r)$	$\wedge e$,3.
6.	$(q \wedge r)$	$\to e$ 1,4.
7.	$\bot$	$\bot i$ 5,6.
8.	$p \vee (q \wedge r)$	$\bot_e$ 2, 2-7

4.b. $p \to (q \to \neg p) \vdash q \to \neg p$.

1.	$p \to (q \to \neg p)$	~~Assume~~ premise
2.	q.	Assume
3.	p.	assume
4.	$q \to \neg p$	$\to e$ 1,3.
5.	$\neg p$	$\to e$ 4,2.
6.	$\perp$	$\perp i$ 3,5
7.	$\neg p$	$\neg i$ 3.
8.	$q \to \neg p$	$\to i$ 2-7

4.(c) $(\phi_1 \vee \phi_2) \wedge (\phi_1 \vee \phi_3) \vdash \phi_1 \vee (\phi_2 \wedge \phi_3)$.

1.	$(\phi_1 \vee \phi_2) \wedge (\phi_1 \vee \phi_3)$	premise.
2.	$(\phi_1 \vee \phi_2)$	$\wedge e$, 1
3.	$(\phi_1 \vee \phi_3)$	$\wedge e$, 1
4.	ϕ_1	assume
5.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee i$, 4
6.	ϕ_2	Assume
7.	ϕ_1	assume
8.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee i$, 7
9.	ϕ_3	Assume
10	$\phi_2 \wedge \phi_3$	$\wedge i$ 6,9
11.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee i$ 10
12.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee e$ 3, ~~6~~-8, 9-11
13.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee e$ 2, 4-5, 6-12