Problem Set 3
Discrete Mathematics 2020-21

CHENNAI MATHEMATICAL INSTITUTE

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" I solemnly swear I am up to no good. ''

Your TAs (Also Fred, George and Harry)

Problem 1. Igor Karkaroff finds two trunks A and B in a cave. He knows that each of them either contains the golden snitch or a fatal trap.

Trunk A: "At least one of these two trunks contains the golden snitch."
Trunk B: "In A, there's a fatal trap."

Igor knows that either both the inscriptions are true, or they are both false. Can he choose a trunk being sure that he will find the golden snitch? If this is the case, which trunk should he open?

Problem 2. Albus Dumbledore has interviewed four wizards regarding the existance of a certain map. From the stories of the wizards, Dumbledore has concluded that if Moony is telling the truth, then so is Wormtail; Wormtail and Padfoot cannot both be telling the truth; Padfoot and Prongs are not both lying; and if Prongs is telling the truth then Wormtail is lying. For each of the four wizards, can Dumbledore determine whether that wizard is telling the truth or lying?

Problem 3. Make a truth table for each of these compound propositions. Note: $\oplus$ is the XOR operation that outputs true only when inputs differ (one is true, the other is false).

  a) $p \rightarrow \neg p$

  b) $p \leftrightarrow \neg p$

  c) $p \oplus (p \vee q)$

  d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

  e) $(p \oplus q) \rightarrow (p \oplus \neg q)$

Problem 4. Longbotton got himself stuck in the magical island of the elves and the goblins, where elves always speak the truth and goblins always lie.

a. Devise a question that allows him to determine whether a native is an elf.

b. What question should he ask A to determine whether A and B are the same type (i.e. both elves or both goblins)?

Problem 5. On the island of elves and goblins, Harry meets three creatures A, B and C from the island. Harry asks A, "Are you a liar?". A answers but his voice is drowned out by a clap of thunder. Harry asks B, "What did A say?". B answers, "A said he is a liar". C exclaims, "Don't believe what B said, he is lying". C then adds, "Also, A is a liar".

Which ones are liars?

Problem 6. Ron and Harry just become friends with Hermione, and they want to know when her birthday is. Hermione gives them a list of 10 possible dates:

May		15	16			19
June				17	18	
July	14		16			
August	14	15		17		

Hermione then tells Ron and Harry separately the month and the day of her birthday respectively.

Ron: I don't know when Hermione's birthday is, but I know that Harry doesn't know too.
Harry: At first, I don't know when her birthday is, but I know now.
Ron: Then I also know when her birthday is.

So when is Hermione's birthday?

Problem 7. Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 3$, then next problem set will be easy.

b) If $1 + 1 = 3$, then there will be no end semester exams for this course.

c) If $1 + 1 = 3$, then next statement is true.

d) If $1 + 1 = 2$, then this statement is true.

Albus Dumbledore, *Chief Warlock of the Wizengamot*

Problem 8. Suppose we know that:

- "if Parvati is thin, then Neville is not blonde or Padma is not tall"

- "if Padma is tall then Luna is lovely"

- "if Luna is lovely and Neville is blonde then Parvati is thin"

- "Neville is blonde"

Can we deduce that "Padma is not tall"?

Problem 9. Recent changes in the Hogwarts law say that one is eligible to be the Headmaster of Hogwarts only if they are at least 35 years old, are pure-blood, or at the time of their birth both of their parents were wizards, and they have taught at least 14 years in the school. Express your answer in terms of e: "One is eligible to be the Headmaster of Hogwarts," a: "One is at least 35 years old," b: "One is a pure-blood," p: "At the time of one's birth, both of their parents where wizards," and r: "One has taught at least 14 years in the school".

Problem 10. It is rumoured that there is gold buried on the island of elves (who always speak the truth) and goblins (who always lie). You ask one of the natives whether there is gold on the island. The native replies, "There is gold on this island is the same as I am an elf." The problem is

a. Can it be determined whether the native is an elf or a goblin?

b. Can it be determined whether there is gold on the island?

Problem 11. Voldemort, Dumbledore, and Grindelwald were powerful wizards, and hence perfect logicians. Each could instantly deduce all consequences of any set of premises. Also, each was aware that each of the others was a perfect logician. The three were shown seven stamps: two red ones, two yellow ones, and three green ones. They were then blindfolded, and a stamp was pasted on each of thier foreheads; the remaining four stamps were placed in a drawer. When the blindfolds were removed, Dumbledore was asked, "Do you know one colour the you definitely do not have?" Dumbledore replied, "No." Then Voldemort was asked the same question and replied, "No." Is it possible, from this information, to deduce the colour of Dumbledore's stamp, or of Voldemort's, or of Grindelwald's?

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Problem 12. Albus Dumbledore is planning a party, and he wants to know whom to invite. Among the people he would like to invite are three touchy friends. He knows that if Hermione attends, she will be unhappy if Ron is there, Ron will attend only if Harry will be there, and Harry will not attend unless Hermione also does. Which combinations of these three friends can he invite so as not to make someone unhappy?

Problem 13. To secretly open the chamber of secrets, Tom Riddle riddles Rubeus Hagrid by showing him a set of four cards placed on a table, each of which has a number on one side and a colored patch on the other side. The visible faces of the cards show 3, 8, red and brown. Riddle asks Hagrid which card(s) must Hagrid turn over in order to test the truth of the proposition that if a card shows an even number on one face, then its opposite face is red? Help Hagrid before Riddle runs aways.

Problem 14. Harry is presented three boxes. One contains Gillyweed, which he needs for the Triwizard tournament, the other two are empty. Each box has imprinted on it a clue as to its contents; the clues are:
Box 1 : "The Gillyweed is not here"
Box 2 : "The Gillyweed is not here"
Box 3 : "The Gillyweed is in Box 2"
Only one message is true; the other two are false. Which box has the Gillyweed?

Problem 15. Show the validity of the following sequents. Note: You might need to wait for next lecture to be able to solve some parts.

a) $(p \wedge q) \wedge r, s \wedge t \vdash q \wedge s$

b) $p \rightarrow (p \rightarrow q), p \vdash q$

c) $q \rightarrow (p \rightarrow r), \neg r, q \vdash \neg p$

d) $q \rightarrow r \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$

e) $p \vee q \vdash r \rightarrow (p \vee q) \wedge r$

f) $p \rightarrow q \vdash \neg q \rightarrow \neg p$

g) $p \rightarrow q, r \rightarrow \neg t, q \rightarrow r \vdash p \rightarrow \neg t$

h) $p \rightarrow q \vdash \neg p \vee q$

i) $\vdash \neg p \vee q \rightarrow (p \rightarrow q)$

j) $\vdash (p \rightarrow q) \vee (q \rightarrow r)$

Problem 16. **A**rthur Weasley, his wife Molly, their son Charles, their daughter Ginny, and Arthur's brother Bilius were involved in a murder. One of the five killed one of the other four. The following facts refer to the five people mentioned:

1. A man and a woman were together in a bar at the time of the murder.

2. The victim and the killer were together on a beach at the time of the murder.

3. One of Arthur's two children was alone at the time of the murder.

4. Arthur and his wife were not together at the time of the murder.

5. The victim's sibling was not the killer.

6. The killer was younger than the victim.

Which one of the five was the victim?

Problem 17. **N**o statement is true.