

**Instructions**

- The duration of the exam is 3 hours. You can refer to the prescribed textbook and the lecture notes.
  - To get full credit, you should justify your answers with valid arguments.
  - Questions 1 and 2 can be answered either via direct arguments or by using sequents + natural deduction rules.
  - No doubts/clarifications would be entertained during the examination. Make appropriate assumptions.
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1. A wants to determine the relative salaries of three coworkers using two facts. First, he knows that if B is not the highest paid of the three, then C is. Second, he knows that if C is not the lowest paid, then D is paid the most. Is it possible to determine the relative salaries of B, C and D from what A knows? If so, who is paid the most and who the least? Explain your reasoning. **(7 marks)**
2. There are three people P, Q, R in an island of knights and knaves. Knights always tell the truth and knaves always lie. Answer the following questions: **(10 marks)**
  - (a) P says "If I am knight, I'll go to school". Does P go to school?
  - (b) P says "If Q is a knight, then I am a knave". What are P and Q?
  - (c) P says: "We're all knaves". Q says: "No. Exactly one of us is a knight". What are P, Q and R?
3. Prove the validity of the following sequents using the natural deduction rules: **(15 marks)**
  - (a)  $\neg(\neg p \vee q) \vdash p$
  - (b)  $p \rightarrow q, s \rightarrow t \vdash p \vee s \rightarrow q \wedge t$
  - (c)  $(p \wedge q) \rightarrow r, r \rightarrow s, q \wedge \neg s \vdash \neg p$
  - (d)  $(p \vee r) \rightarrow (p \rightarrow q), p \vdash q$
  - (e)  $\phi_1 \rightarrow (\phi_2 \vee \phi_3), \neg\phi_2, \neg\phi_3 \vdash \neg\phi_1$
4. Consider the following sequents: **(18 marks)**
  - (a)  $\neg p \rightarrow (q \wedge r) \vdash p \vee (q \wedge r)$
  - (b)  $p \rightarrow (q \rightarrow \neg p) \vdash q \rightarrow \neg p$
  - (c)  $(\phi_1 \vee \phi_2) \wedge (\phi_1 \vee \phi_3) \vdash \phi_1 \vee (\phi_2 \wedge \phi_3)$
  - (i) Prove the validity of the above sequents using LEM.
  - (ii) Prove the validity of the above sequents with no application of the LEM rule.