Instructions

- *The duration of the exam is 4 hours. You can refer to the prescribed textbooks and the lecture notes.*
- *To get full credit, you should justify your answers with valid arguments.*
- *No doubts/clarifications would be entertained during the examination. Make appropriate assumptions.*

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1. Words are formed from the alphabet $\{a, b, c\}$. How many words of length 10 do not contain 7 consecutive cs? **(5 marks)**

2. (a) In how many ways can 50 *distinct* passes be distributed among 10 different representatives? There is no restriction on the number of passes that each representative receives. In other words, each representative can receive any number of passes between 0 to 10.

   (b) In how many ways can 50 *distinct* passes be distributed among 10 different representatives so that each of them receives exactly 5 passes?

   (c) In how many ways can 50 *identical* passes be distributed among 10 different representatives so that each representative receives exactly 5 passes? **(6 marks)**

3. Prove the validity of the following sequents using natural deduction rules (You cannot use De Morgan's laws directly in the proofs. If you plan to use De Morgan's laws, either you should derive them explicitly or cite the lecture number and slide number in which it was covered): **(15 marks)**

   (a) $\phi \vee \psi,\ \neg\phi \vee \psi \vdash \psi$

   (b) $\phi \vee \psi,\ \neg(\phi \wedge \psi) \vdash (\phi \wedge \neg\psi) \vee (\psi \wedge \neg\phi)$

   (c) $\neg(\phi \wedge \neg\psi), \neg(\psi \vee \neg\phi) \vdash \phi \wedge \psi$

4. Write regular expressions over the alphabet $\{a, b\}$ for the following languages over the alphabet $\{a, b\}$:

   (a) $\{w \in (a + b)^* \mid$ if w contains a then w contains b $\}$

   (b) $\{w \in (a + b)^* \mid w$ contains exactly one occurrence of the substring $ab\}$

   (c) $\{w \in (a + b)^* \mid w$ contains ab and ba as substrings $\}$

   (d) $\{w \in (a + b)^* \mid w$ does not contain $bb\}$

   (e) $\{w \in (a + b)^* \mid w$ contains an even number of as OR an odd number of bs$\}$ **(15 marks)**

5. Consider an alphabet $\Sigma = \{a, b, c\}$. Draw an NFA *with at most three states* (that is, number of states is ≤ 3) for the set of all words that contain at most two letters from Σ. For example, $aabb, ccc$ are words in this language. **(5 marks)**

6. Convert the following regular expressions over $\{0, 1\}$ to NFA: **(6 marks)**

   (a) $(0^*1^*)^* + 1$

   (b) $(1^* + 0)(00 + 1^*0)^*$

7. Convert the following NFA to an equivalent DFA: **(3 marks)**