Quiz-1

1. The refrigerator contains at least 5 drinks of each type. There are 8 different types of drinks.

$$x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8 = 5$$

So, there are 5 balls and 7 bars. 0|0|0|0|0| | |

So, permuting them $^{12}C_5$ ways. $\frac{12!}{7!5!}$

2. $(0,0,0,0)$ to $(5,3,2,6)$ xyzw plane.

there are $\frac{16!}{5!3!2!6!}$ ways possible to move in positive, x, positive y, positive z or positive w direction.

total number of steps are ~~sta~~ taken = 16.

and 5 → x direction steps

3 → y direction steps

2 → z direction steps

6 → w direction steps.

3. x^4y^5 in $(2x-y)^9 + (x+4y)^9 = z$

coeff of x^4y^5 in $(2x-y)^9 = \binom{9}{4}(2x)^4(-y)^5$

$$= -\binom{9}{4}2^4$$

term x^4y^5 in $(x+4y)^9 = \binom{9}{4}(1)^4(4y)^5$.

coefficient of x^4y^5 in expression $= -\binom{9}{4}2^4 + \binom{9}{4}4^5$

$= \binom{9}{4}\left(4^5 - 2^4\right)$

4. Ten identical tokens have to be distributed among 4 persons such that each person gets at least one token.

let 4 persons get x_1, x_2, x_3 and x_4 tokens respectively

so, $x_1 + x_2 + x_3 + x_4 = 10$ and $x_1 \geq 1$, $x_1 - 1 \geq 0$

$x_2 \geq 1$, $x_2 - 1 \geq 0$

$x_3 \geq 1$, $x_3 - 1 \geq 0$

$x_4 \geq 1$, $x_4 - 1 \geq 0$

$X_1 + 1 + X_2 + 1 + X_3 + 1 + X_4 + 1 = 10$

$X_1 + X_2 + X_3 + X_4 = 6.$

let $x_1 - 1 = X_1$

$x_2 - 1 = X_2$

$x_3 - 1 = X_3$

$x_4 - 1 = X_4$.

there are 6 balls and 3 bars

o o | o o | o | o

so, number of ways $= \dfrac{9!}{6!\,3!}$

$= {}^9C_3$

5. (a) 15 distinguishable objects are distributed into 5 distinguishable box.

there is no restriction on the number of objects in a box.

~~so, $x_1 + x_2 + x_3 + x_4 + x_5 = 15 \rightarrow$ ~~ex~~ lets consider 15 identical objects.~~

~~so, no of permutation possible $\binom{19}{4}$~~

~~~~if the~~ But the objects are distinguishable, they can be permuted in 15! ways~~

~~so, total no. of possibilities. $15!\binom{19}{4}$~~

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15.

so, each of them are distinguishable and they have 5 choices.

So, possible number of permutation $= 5^{15}$.
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6.(b) the boxes have 1, 2, 3, 4, 5 objects.

so, now it can be permuted in ~~$\binom{15}{1} + \binom{14}{2}2! + \binom{12}{3}3!$~~.

$$\binom{15}{1} * \binom{14}{2} \times \binom{12}{3} \times \binom{9}{4} \times \binom{5}{5}$$

Because we can choose 1 object in box 1 in $\binom{15}{1}$ ways,
2 objects from the rest of the objects in $\binom{14}{2}$ ways,
3 objects from the rest 12 objects in $\binom{12}{3}$ ways,
4 objects from the rest 9 objects in $\binom{9}{4}$ ways
and then 5 objects will be left.
Now applying the product rule we will get the answer.

$$\binom{15}{1} \times \binom{14}{2} \times \binom{12}{3} \times \binom{9}{4} \times \binom{5}{5}$$

~~7.~~ (a)

7.(a) $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$

We are choosing ~~r objects~~ a team of r members from n players. and then choosing k captains out of it.

L.H.S

So, from n players, a team of r members can be chosen in $\binom{n}{r}$ ways. Then we can choose k captains from r players in $\binom{r}{k}$ ways.

R.H.S.

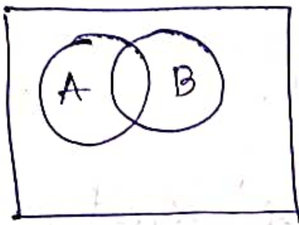
In R.H.S we are first choosing k captains from n players in $\binom{n}{k}$ ways, and then choosing the rest $(r-k)$ players from the rest of the players in $\binom{n-k}{r-k}$ ways.

So, LH.S = RH.S

8. In an exam containing 2 problems, 70% of them solved the first problem 80% solved the 2nd problem.

So, let A = solved 1st problem = 70%

B = solved 2nd problem = 80%



So, ~~there~~ total student who are solving the problems is 100%.

there might be some percentage of people, who will solve the both problems.

So, $A + B = 150\%$

~~the hole is total number of students who solved problems 100%~~

~~and the regions are pros A is solved by 150%~~

~~either first problem and/or 2nd problem is solved by the number of people.~~

~~as 150% is greater than 100% by 50%,~~

~~Who holes are the number of problems~~

There are two holes and 3 regions.

3 regions are $(A \cap B^c)$, $(B \cap A^c)$, $A \cap B$ ← people both solved problem 1 & 2.

people only solved problems 1

people only solved problem 2.

and 2 holes are A and B.

According to the PHP

So, there might be $A \cap B$ either in A or in B or in both which will be ~~eig~~ either in A and B.

$A + B = 150\%$

So, $A \cup B = 100\%$

So, $A \cap B = 50\%$. at least 50% will solve both 1st and 2nd

9. $D_n = n! - \binom{n}{1} D_{n-1} - \binom{n}{2} D_{n-2} \cdots - \binom{n}{n-1} D_1 - 1$

$n!$ is the total number of arrangements of n distinct objects.

D_n = Where n objects are diarranged.

let, 1 object is in right place. that object can be chosen in $\binom{n}{1}$ ways and $(n-1)$ objects are ~~diarran~~ deranged. that in D_{n-1} ways.

likely. we can choose 2 objects in right place and then Deranging $(n-2)$ objects. in $\binom{n}{2} D_{n-2}$ ways.

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all these are to be subtracted from $n!$, to get Derangement from n objects.

and at last we are considering ~~n~~ $(n-1)$ objects are placed in places and one object is deranged, in $\binom{n}{n-1} D_1$ ways.

And last we are considering all the objects will be placed in its own place. that can be done in 1 way.

Subtracting all these from $n!$ will provide derangement for n objects.

hence D_n is obtained.

10. There are 200 pigeons placed in 101 pigeon holes. Such that each pigeonhole contains at least one pigeon.

Let k_j be the number of pigeons till jth hole. Then ~~ff~~. $k_1, k_2, \ldots, k_{101}$ is an increasing sequence. with $1 \leq k_j \leq 200$.

then $k_1+100, k_2+100, \ldots, k_{100}+100$ is also an increasing sequence of distinct +ve integers.

with $101 \leq k_j+100 \leq 300$.

The, 202 pigeons. $k_1, k_2, \ldots, k_{101}, k_1+100, k_2+100 + \cdots k_{101}+100$

are all less than or equal to 300. Hence by Generalised PHP, ~~atleast~~ two of these integers are equal.

Because all of them are distinct.

there must be i, j with $k_i = k_j+100$. This means ~~exactly 100~~ there is some subset of ~~100~~ exactly 100 pigeons.

6. (a) 10, positive integers are there.
such sequence is,

5, 4, 3, 2, 1, 10, 9, 8, 7, 6.

(b) Any sequence of distinct +ve integers with length atleast $(r-1)(s-1)+1$ contains a monotonically decreasing subsequence of length s.

here $r=4$ and $s=6$.

so, $(r-1)(s-1)+1 = 3\times 5+1 = 16$