Arpan Biswas, MDS202011, DMAT Quiz - 2

1) Suppose B is not the highest paid. Then we know that C is the highest paid. If C is the highest paid, then C cannot be the lowest paid. Then, we know that D is the highest paid. This is a contradiction since C and D cannot both be highest paid. Hence, B is highest paid.

Now, since B is highest paid, D cannot be the highest paid and therefore C is the lowest paid.

So, B is the highest paid and C is the lowest paid.

2) (a) If P is a knight, then he says the truth and he goes to school.

If P is a knave, then he is lying. So, the statement "If P is knight, P will go to school" is false. Hence, P is a knight and he doesn't go to shool. This is a contradiction. So, P cannot be a knave.

Hence, P goes to school.

(b) Suppose both P and Q are knights.
Then P is a knave which is a contradiction.
So both P and Q cannot be knights.

Suppose P is knave and Q is knight.
Then, what P says is false. This means that P is a knight and Q is a knight.
Again we get a contradiction.

Suppose P is a knave and Q is a knave.
~~Then~~ Then again it means that P is a knight and Q is a knight. This is a contradiction.

Finally, suppose P is a knight and Q is a knave.
Then "If Q is a knight then P is a knave" is true and hence there is no contradiction.
Therefore, P is a knight and Q is a knave.

(c) Suppose P is a knight. Then everyone is a knave including P. Thus we have a contradiction. So, P is a knave.

Suppose if Q is a knave. Then he is lying. So, there are more than one knights. But this is not possible since both P and Q are knaves and hence we have a contradiction. Thus, Q is a knight. Since Q is a knight, he is telling the truth and so R is a knave.

Thus, P is knave, Q is knight and R is knave.

3) (a)

1.	$\neg(\neg p \vee q)$	premise
2.	$\neg p$	assume
3.	$\neg p \vee q$	$\vee i_1$ 2
4.	$\perp$	$\perp i$ 1, 3
5.	$\neg\neg p$	$\neg i$ 2−4
6.	p	$\neg\neg e$ 5

(b) The sequent is not valid.

Suppose $p=1$, $q=1$, $s=0$ and $t=0$.

In this case, $p \to q$ and $s \to t$ holds but $p \vee s \to q \wedge t$ does not hold.

(c)

1.	$(p \land q) \to r$	premise
2.	$r \to s$	premise
3.	$q \land \neg s$	premise
4.	p	assume
5.	$q \lor \neg q$	LEM
6.	q	assume
7.	$p \land q$	$\land_i$ 4, 6
8.	r	$\to_e$ 7, 1
9.	s	$\to_e$ 8, 2
10.	$\neg s$	$\land_{e_2}$ 3
11.	$\perp$	$\perp_i$ 9, 10
12.	$\neg q$	assume
13.	q	$\land_{e_1}$ 3
14.	$\perp$	$\perp_i$ 12, 13
15.	$\perp$	$\lor_e$ 5, 6–11, 12–14
16	$\neg p$	$\neg_i$ 4–15

(d)

1.	$(p \vee r) \rightarrow (p \rightarrow q)$	premise
2.	p	premise
3.	$p \vee r$	$\vee i_1$ 2
4.	$p \rightarrow q$	$\rightarrow_e$ 3, 1
5.	q	$\rightarrow_e$ 2, 4

(e)

1.	$\phi_1 \rightarrow (\phi_2 \vee \phi_3)$	premise
2.	$\neg \phi_2$	premise
3.	$\neg \phi_3$	premise
4.	ϕ_1	assume
5.	$\phi_2 \vee \phi_3$	$\rightarrow_e$ 4, 1
6.	ϕ_2	assume
7.	$\perp$	$\perp_i$ 2, 6
8.	ϕ_3	assume
9.	$\perp$	$\perp_i$ 3, 8
10.	$\perp$	$\vee e$ 5, 6-7, 8-9
11.	$\neg \phi_1$	$\neg_i$ 4-10

4) (i) (a)

1.	$\neg p \to (q \wedge r)$	premise
2.	$p \vee \neg p$	LEM
3.	p	assume
4.	$p \vee (q \wedge r)$	$\vee i_1$ 3
5.	$\neg p$	assume
6.	$q \wedge r$	$\to_e$ 5, 1
7.	$p \vee (q \wedge r)$	$\vee i_2$ 6
8.	$p \vee (q \wedge r)$	$\vee_e$ 2, 3-4, 5-7

(i) (b)

1.	$p \to (q \to \neg p)$	premise
2.	$p \vee \neg p$	LEM
3.	p	assume
4.	$q \to \neg p$	$\to_e$ 3, 1
5.	$\neg p$	assume
6.	q	assume
7.	$\neg p$	copy 5
8.	$q \to \neg p$	$\to_i$ 6 - 7
9.	$q \to \neg p$	$\vee_e$ 2, 3-4, 5-8

(i)(c)

1.	$(\phi_1 \vee \phi_2) \wedge (\phi_1 \vee \phi_3)$	premise
2.	$\phi_1 \vee \phi_2$	$\wedge e_1$ 1
3.	$\phi_1 \vee \phi_3$	$\wedge e_2$ 1
4.	$\phi_1 \vee \neg \phi_1$	LEM
5.	ϕ_1	assume
6.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee i_1$ 5
7.	$\neg \phi_1$	assume
8.	ϕ_1	assume
9.	$\perp$	$\perp i$ 7, 8
10.	ϕ_2	$\perp e$ 9
11.	ϕ_2	assume
12.	ϕ_2	copy 11
13.	ϕ_2	$\vee e$ 2, 8-10, 11-12
14.	ϕ_3	assume
15.	ϕ_3	copy 14
16.	ϕ_3	

16.	ϕ_1	assume
17.	$\bot$	$\bot_i$ 7, 16
18.	ϕ_3	$\bot_e$ 17
19.	ϕ_3	$\vee_e$ 3, 16–18, 14–15
20	$\phi_2 \wedge \phi_3$	$\wedge_i$ 13, 19.
21.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee_{i_2}$ 20
22.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee_e$ 4, 5–6, 7–21

~~(ii) (a)~~

~~1. $\neg p \to (q \wedge r)$~~ ~~premise~~

(ii) (b)

1.	$p \to (q \to \neg p)$	premise
2.	q	assume
3.	p	assume
4.	$q \to \neg p$	$\to_e$ 3, 1
5.	$\neg p$	$\to_e$ 2, 4
6.	$\bot$	$\bot_i$ 3, 5
7.	$\neg p$	$\neg_i$ 3–6
8.	$q \to \neg p$	$\to_i$ 2 – 7

(ii) (a)

1.	$\neg p \rightarrow (q \wedge r)$	premise
2.	$\neg(p \vee (q \wedge r))$	assume
3.	p	assume
4.	$p \vee (q \wedge r)$	$\vee_{i_1}$ 3
5.	$\perp$	$\perp_i$ 2, 4
6.	$\neg p$	$\neg_i$ 3–5
7.	$q \wedge r$	assume
8.	$p \vee (q \wedge r)$	$\vee_{i_2}$ 7
9.	$\perp$	$\perp_i$ 2, 8
10.	$\neg(q \wedge r)$	$\neg_i$ 7–9
11.	$(q \wedge r)$	$\rightarrow_e$ 6, 1
12.	$\perp$	$\perp_i$ 10, 11
13.	$\neg\neg(p \vee (q \wedge r))$	$\neg_i$ 2–12
14.	$p \vee (q \wedge r)$	$\neg\neg_e$ 13

(ii)(c)

1.	$(\phi_1 \vee \phi_2) \wedge (\phi_1 \vee \phi_3)$	premise
2.	$\neg(\phi_1 \vee (\phi_2 \wedge \phi_3))$	assume
3.	ϕ_1	assume
4.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee i_1$ 3
5.	$\perp$	$\perp i$ 2, 4
6.	$\neg\phi_1$	$\neg i$ 3–5
7.	$\phi_1 \vee \phi_2$	$\wedge e_1$ 1
8.	ϕ_1	assume
9.	$\perp$	$\perp i$ 6, 8
10.	ϕ_2	$\perp e$ 9
11.	ϕ_2	assume
12	ϕ_2	$\vee e$ 7, 8–10, 11
13.	$\phi_1 \vee \phi_3$	$\wedge e_2$ 2
14.	ϕ_1	assume
15.	$\perp$	$\perp i$ 6, 14
16.	ϕ_3	$\perp e$ 15
17.	ϕ_3	assume

18. ϕ_3 $\vee_e$ 13, 14–16, 17

19. $\phi_2 \wedge \phi_3$ $\wedge_i$ 12, 18

20. $\phi_1 \vee (\phi_2 \wedge \phi_3)$ $\vee_{i_2}$ 19

21. $\bot$ $\bot_i$ 20, 2

22. $\neg\neg(\phi_1 \vee (\phi_2 \wedge \phi_3))$ $\neg_i$ 2–21

23. $\phi_1 \vee (\phi_2 \wedge \phi_3)$ $\neg\neg_e$ 22