11/02/2021

Quiz - 2 Solutions

1. The question gives us two conditions:

   (1) If B is not the highest, then C is.

   (2) If C is not the lowest, then D is highest.

Suppose B is the highest paid.

- From (2), this implies that D is the second and C is the third

Suppose B is not the highest paid

- From (1) this implies C is the highest
  - then, from (2) we get D is the highest

  This contradicts C getting paid the highest.

There fore the only possibility is B > D > C .

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2. a) If P is knight, his statement is true. P will go to school.

If P is knave, his statement is false. The negation of the statement is:

P is knight and P does not go to school.

This is a contradiction to P being knave.

Hence the only possibility is that P is knight. Hence P will go to school.

b) Suppose P is knight:

- This implies Q is knave.

Suppose P is knave. then his statement is false. The negation of the statement:

Q is knight and P is knight.

This is a contradiction to P being knave. Hence P is knight, Q is knave

c) Suppose P is knight. This is a contradiction to his own statement. Hence P is knave.

Since P is knave, the statement "We're all knaves" is false. Hence at least one of them is knight.

Since Q says No to P's statement, Q is telling the truth. Hence Q is knight.

Therefore Q's statement is true and exactly one of them is knight. This implies that R is knave.

Finally: P is knave, Q is knight and R is knave.

3.

a)

1.	$\neg(\neg p \vee q)$	premise
2.	$\neg p$	assumption
3.	$\neg p \vee q$	$\vee_i$ - 2
4.	$\perp$	$\perp_i$, 1, 3
5.	$\neg\neg p$	$\neg_i$ 2-4
6.	p	$\neg\neg_e$

b) $p \rightarrow q,\ s \rightarrow t \vdash p \vee s \rightarrow q \wedge t$

This is not a valid sequent.

$p = 1,\ s = 0,\ q = 1,\ t = 0$ makes the premises true, but the conclusion false

c)

1.	$(p \wedge q) \rightarrow r$	premise
2.	$r \rightarrow s$	premise
3.	$q \wedge \neg s$	premise
4	p	assumption
5.	q	$\wedge e_1$, 3
6.	$p \wedge q$	$\wedge i$ - 4,5
7.	r	$\rightarrow e$, 1, 6
8.	s	$\rightarrow e$, 2, 7
9.	$\neg s$	$\wedge e_2$, 3
10.	$\perp$	$\perp i$, 8, 9
11.	$\neg p$	$\neg i$, 4- 10

d)

1.	$(p \vee r) \rightarrow (p \rightarrow q)$	premise
2.	p	premise
3.	$p \vee r$	$\vee i$, 2
4.	$p \rightarrow q$	$\rightarrow e$, 1, 3
5.	q	$\rightarrow e$, 2, 4.

(e)

1.	$\phi_1 \rightarrow (\phi_2 \vee \phi_3)$	premise
2.	$\neg\phi_2$	premise
3.	$\neg\phi_3$	premise
4.	ϕ_1	assumption
5.	$\phi_2 \vee \phi_3$	$\rightarrow e$, 1, 4
6.	ϕ_2	assumption
7.	$\perp$	$\perp i$, 2, 6
8.	ϕ_3	assumption
9.	$\perp$	$\perp i$, 3, 8
10.	$\perp$	$\vee e$, 5, 6-7, 8-9
11.	$\neg\phi_1$	$\neg i$, 4-10

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4. (i) Using LEM.

a)

1.	$\neg p \rightarrow (q \wedge r)$	premise
2.	$p \vee \neg p$	LEM
3.	p	assumption
4.	$p \vee (q \wedge r)$	$\vee i$, 3
5.	$\neg p$	assumption
6.	$q \wedge r$	$\rightarrow e$, 1, 5
7.	$p \vee (q \wedge r)$	$\vee i$ - 6
8.	$p \vee (q \wedge r)$	$\vee e$, 2, 3-4, 5-7.

b)

1.	$p \to (q \to \neg p)$	premise
2.	$p \vee \neg p$	LEM
3.	p	assumption
4.	$(q \to \neg p)$	$\to e$, 1, 3
5.	$\neg p$	assumption
6.	q	assumption
7.	$\neg p$	copy 5
8.	$q \to \neg p$	$\to i$, 6, 7
9.	$q \to \neg p$	$\vee e$, 2, 3-4, 5-8.

c)

1.	$(\phi_1 \vee \phi_2) \wedge (\phi_1 \vee \phi_3)$	premise
2.	$\phi_1 \vee \neg\phi_1$	LEM
3.	ϕ_1	assumption
4.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee i$, 3
5.	$\neg\phi_1$	assumption
6.	$\phi_1 \vee \phi_2$	$\wedge e$, 1
7.	ϕ_1	assumption
8.	$\perp$	$\perp i$, 5, 7
9.	ϕ_2	$\neg e$, 8
10.	ϕ_2	assumption
11.	ϕ_2	copy 10
12.	ϕ_2	$\vee e$, 6, 7-9, 10-11
13.	$\phi_1 \vee \phi_3$	$\wedge e$, 2
14.	ϕ_1	assumption
15.	$\perp$	$\perp i$, 5, 14
16.	ϕ_3	$\perp e$, 15
17.	ϕ_3	assumption
18.	ϕ_3	copy 17
19.	ϕ_3	$\vee e$, 13, 14-16, 17-18
20.	$\phi_2 \wedge \phi_3$	$\wedge i$, 12, 13
21.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee i$, 20
22.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee e$, 2, 3-4, 5-21

ii) Without using LEM.

a)

1.	$\neg p \to (q \wedge r)$	premise
2.	$\neg(p \vee (q \wedge r))$	assumption
3.	p	assumption
4.	$p \vee (q \wedge r)$	$\vee i$ 3
5.	$\perp$	$\perp i$, 2, 4
6.	$\neg p$	$\neg i$, 3-5
7.	$q \wedge r$	$\to i$, 1, 6
8.	$p \vee (q \wedge r)$	$\vee i$, 7
9.	$\perp$	$\perp i$, 2, 8
10.	$\neg\neg(p \vee (q \wedge r))$	$\neg i$, 2-9
11.	$p \vee (q \wedge r)$	$\neg\neg e$, 10

b)

1.	$p \to (q \to \neg p)$	premise
2.	$\neg (q \to \neg p)$	assumption
3.	p	assumption
4.	$q \to \neg p$	$\to e$, 1, 3
5.	$\bot$	$\bot i$, 2, 4
6.	$\neg p$	$\neg i$ 3-5
7.	q	assumption
8.	$\neg p$	copy 6
9.	$q \to \neg p$	$\to i$, 7, 8
10.	$\bot$	$\bot i$, 2, 9
11.	$\neg\neg(q \to \neg p)$	$\neg i$, 2-10
12.	$q \to \neg p$	$\neg\neg e$ 11

c)

1.	$(\phi_1 \vee \phi_2) \wedge (\phi_1 \vee \phi_3)$	premise
2.	$\neg(\phi_1 \vee (\phi_2 \wedge \phi_3))$	assumption
3.	ϕ_1	assumption
4.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee i$, 3
5.	$\bot$	$\bot i$, 2, 4
6.	$\neg\phi_1$	$\neg i$ 3-5
7.	$\phi_1 \vee \phi_2$	$\wedge e$ 1
8.	ϕ_1	assumption
9	$\bot$	$\bot i$, 8, 6
10.	ϕ_2	$\bot e$, 9
11.	ϕ_2	assumption
12.	ϕ_2	copy 11
13.	ϕ_2	$\vee e$, 7, 8-10, 11-12
14.	$\phi_1 \vee \phi_3$	$\wedge e$, 1
15.	ϕ_1	assumption
16.	$\bot$	$\bot i$, 6, 15
17.	ϕ_3	assumption
18.	ϕ_3	copy
19.	ϕ_3	$\vee e$, 14, 15-16, 17-18
20.	$\phi_2 \wedge \phi_3$	$\wedge i$ 13, 19
21.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\vee i$, 20
22.	$\bot$	$\bot i$, 2, 21
23.	$\neg\neg(\phi_1 \vee (\phi_2 \wedge \phi_3))$	$\neg i$, 2-22
24.	$\phi_1 \vee (\phi_2 \wedge \phi_3)$	$\neg\neg e$