Problem Set 4
Discrete Mathematics 2020-21

Chennai Mathematical Institute

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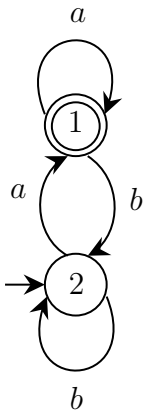
Don't panic.

Problem 1. Finite state automata can be used to model real life systems. Consider the robot Marvin, which is either depressed or happy. If you give him a problem to solve, he becomes happy no matter what his mood was earlier. It gets depressed if he stays idle for 5 minutes.

Assuming that he was depressed when he was manufactured, and happy when he died, Marvin's system can be modelled as follows:



State 1 : Happy
State 2 : Depressed

Action a : Give a problem to solve
Action b : Keep idle for 5 minutes

We all know that the Earth is (soon to be "was") a giant supercomputer designed by Deep Thought and built by Magratheans, to find the Ultimate Question of Life, the Universe and Everything. Model the Earth as a finite state automaton, with the following information:

Earth can either be computing or tired or dead. While computing, the Earth gets tired if you pollute it for 50 years, and dies if you pollute if for 100 years. When it is tired, you can make it start computing by planting trees, and it stays in the state as long as you plant the trees. Once it dies, there is no way to revive it. Assume that the Earth was green when manufactured and we want to keep it that way.

Problem 2. Robots manufactured by InfiniDim Enterprises are weak and can only stay in a finite number of states. Colin, the security robot, is one of them. Suppose he accepts a language L over some finite alphabet A. To break the security, one needs a robot which accepts

the reverse of the words that Colin can accept. Construct a robot to help you, i.e. produce a finite state automaton over the alphabet A, which accepts the language $rev(L)$, defined as,

$$rev(L) = \{u \in A^* | u = rev(v) \text{ for some word } v \in L\}$$

where, for a word $u = u_1u_2 \cdots u_n$, $rev(u) = u_nu_{n-1} \cdots u_2u_1$.

Problem 3. At the Restaurant at the End of the Universe, you find some blueprints (given below) to make some robots. Make the NFAs over the alphabet $\{0, 1, 2\}$ from those blueprints:

(i) $0 + 1$

(ii) $((0 + 1)^*000)^*$

(iii) $0^* + 1^*$

(iv) 00^*

(v) $((0^*1)^*2)^*0^*$

Now the world has gone to bed
Darkness won't engulf my head
I can see by infra-red
How I hate the night

Now I lay me down to sleep
Try to count electric sheep
Sweet dream wishes you can keep
How I hate the night

Problem 42. Zaphod Beeblebrox is really impressed by the Krikkit War Robots, but the natives of the planet Krikkit do not want to give him the robots. He has, however, managed to find the functionality of some of the robots. Make the automata (NFA) over the alphabet $\{0, 1\}$ for each of the following functionality:

(i) $\{ w \mid w$ contains 01 as a substring $\}$

(ii) $\{ w \mid w$ does not contain 01 as a substring $\}$

(iii) $\{ w \mid w$ has odd number of 0s $\}$

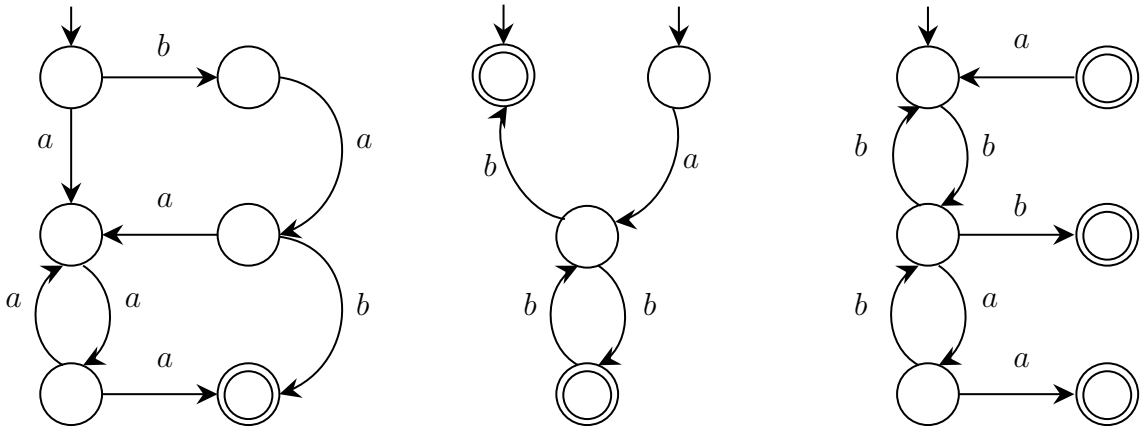
(iv) $\{ w \mid w$ has odd number of 0s *or* even number of 1s $\}$

(v) $\{ w \mid w$ has odd number of 0s *and* even number of 1s $\}$

(vi) $\{ w \mid$ every 0 in w is immediately followed by atleast one 1 $\}$

(vii) $\{ w \mid$ every 0 in w is eventually followed by atleast one 1 $\}$

Problem 5. Zaphod's engineers can not manufacture non-deterministic automata in real life. Which of the following finite state automata are deterministic? Build the deterministic automata for the non-deterministic ones. If possible, make the blueprints (regular expressions) for these as well, and keep them safe with you for the future generations.



So long, and thanks for all the fish!

Made with ♥ by your TAs.