Final Exam

4. (a) alphabet $= \{a, b\}$.

$\{w \in (a+b)^* \mid w$ has length atmost 3$\}$

$= \epsilon + (a+b) + (a+b)(a+b) + (a+b)(a+b)(a+b)$

here ϵ is to be accepted along with all possible combinations of $\{a,b\}$ taking length 1, 2 and 3. Thereby follows the solution.

4.(b). $\{w \in (a+b)^* \mid w$ doesnot contain aba substring$\}$

$b^* (a^* bb\, b^*) a^* b^*$

If there is an a, ba cannot come after that. so, either a will continue or it will be abb and then continuing the string. Thereby follows the answer.

4.(c) $\{ w \in (a+b)^* \mid$ If w starts with a ends with b$\}$

$= \epsilon + b(a+b)^* + a(a+b)^* b.$

ϵ to be accepted

Strings starts with b.

the string starts with a and ends with b.

4.(d). $\{w \in (a+b)^* \mid$ If w contains aa then w contains bb$\}$.

$= (a+b)^* aa (a+b)^* bb (a+b)^* + (a+b)^* bb (a+b)^* aa (a+b)^*$

$+ \cancel{b^*(a+b^*)}) (a+\epsilon)(ba+b)^*$

aa comes before bb

No aa.

bb comes before aa

$= (a+b)^* aa (a+b)^* bb (a+b)^* + (a+b)^* bb (a+b)^* aa (a+b)^*$

$+ (a+\epsilon)(ba+b)^*$

(Final Ans=)

4.(e). For even numbers of a's and even number's bs.

we have,

$$((ab+ba)(aa+bb)^*(ab+ba))^*$$

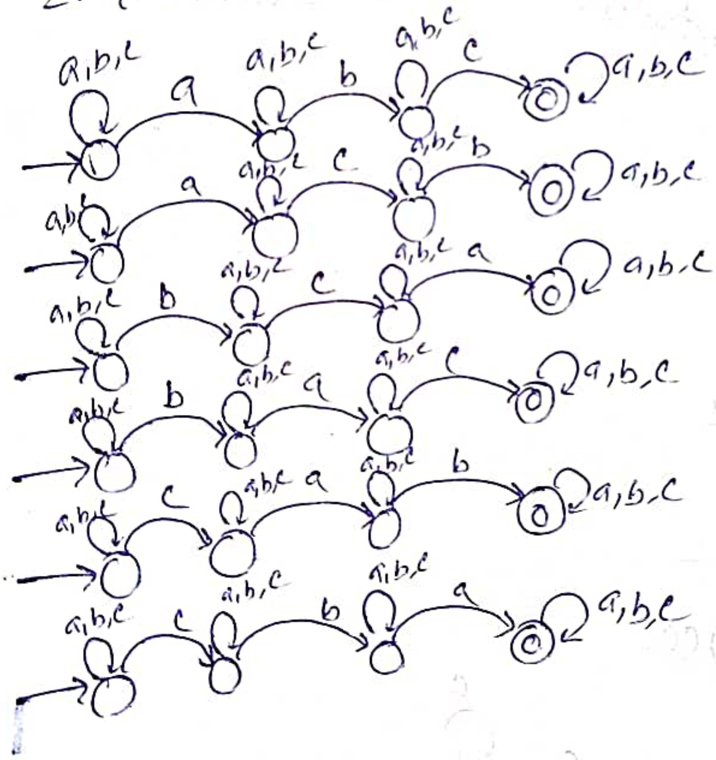
then for odd number of b's and even number of a's.

we need to add one extra b. in the above expression.
and aba which is one corner case.

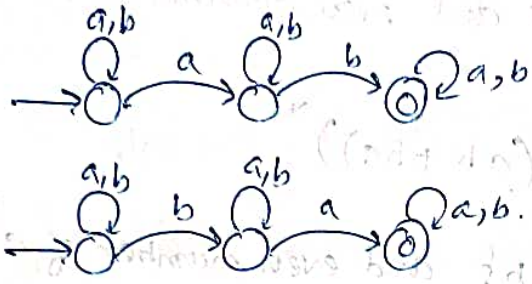
$$((ab+ba)(aa+bb)^*(ab+ba))^* \, b \, ((ab+ba)(aa+bb)^*(ab+ba))^* + aba$$

(Ans:-)

5.(a) $\Sigma = \{a,b,c\}$. $\{w \in \Sigma^* \mid w$ contains all the letters of $\Sigma \}$

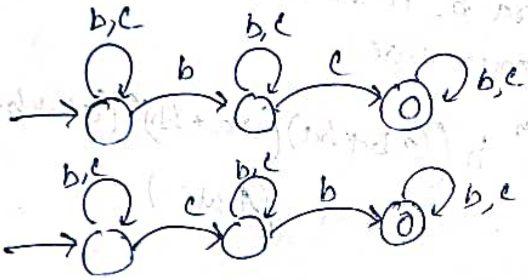


(b)



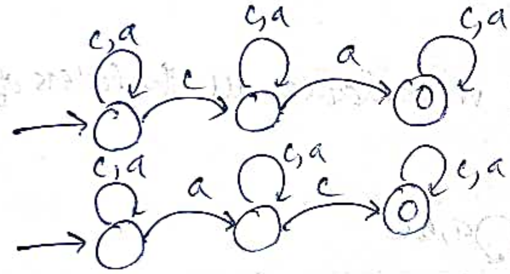
ab – strings

2 cases



bc – string

2 cases



ca – string

2 cases.

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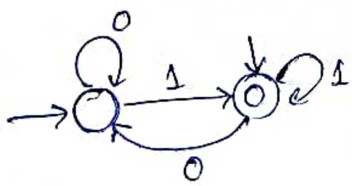
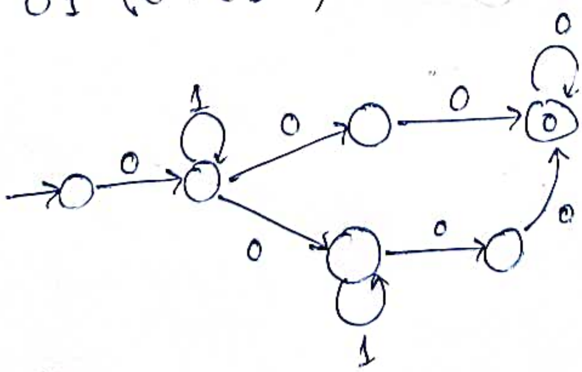
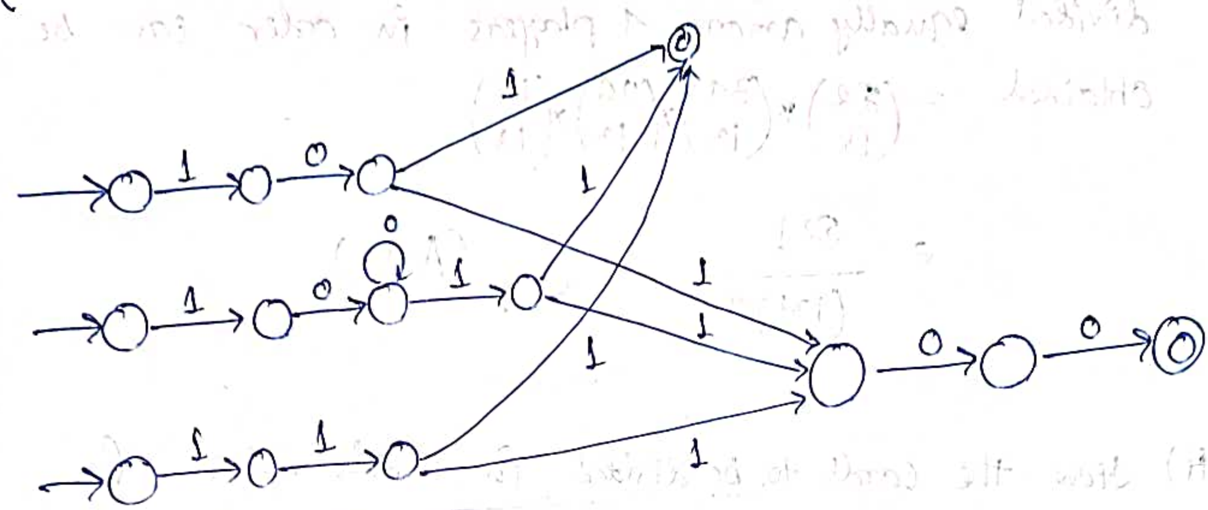
6. (a) $(0^* 1)^*$



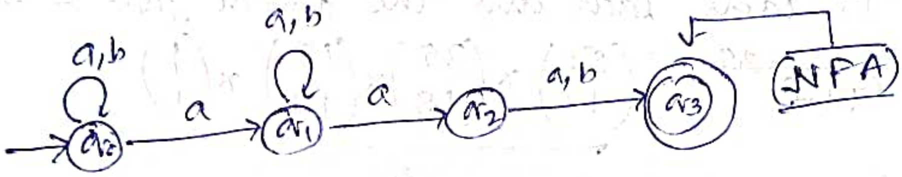
6.(b) $0 1^* (0 + 0 1^* 0) 0 0^*$

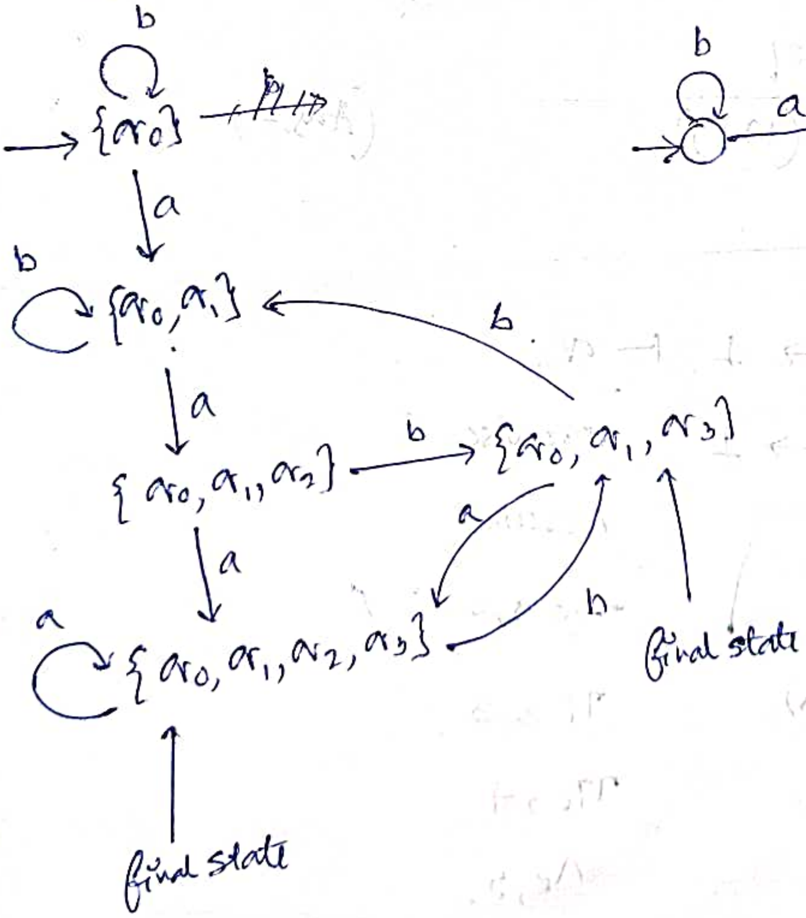


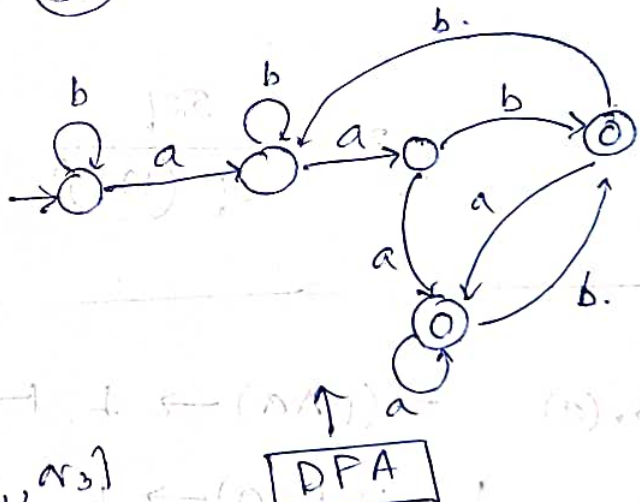
6.(c) $(10+10^*1+11)(1+100)$



7.







1. (i) The number of ways in which 52 cards can be divided equally among 4 players in order can be obtained $= \binom{52}{13} \times \binom{39}{13} \times \binom{26}{13} \times \binom{13}{13}$

$$= \frac{52!}{(13!)^4} \quad \text{(Ans:-)}$$

(ii) Now the cards to be divided in 4 sets, three of them having 17 cards each and the fourth just 1 card?

in $= \cancel{{}^{52}C_{17} \times {}^{35}} \quad \dfrac{\binom{52}{17} \times \binom{35}{18} \times \binom{18}{17} \times \binom{1}{1}}{3!}$

$$= \frac{52!}{3!\,(17!)^3} \quad \text{(Ans:-)}$$

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3. (a) $\neg(p \wedge q) \to \bot \vdash q.$

1.	$\neg(p \wedge q) \to \bot$	premise.
2.	$\boxed{\neg(p \wedge q)}$	Assume
3.	$\boxed{\bot}$	$\to e$ 1,2
4.	$\neg\neg(p \wedge q)$	$\neg i$ 2-3.
5.	$p \wedge q$	$\neg\neg e$, 4
6.	q	$\wedge e$, 5.

3.(b) $\neg(p \to q) \vdash p \wedge \neg q.$

1.	$\neg(p \to q)$	premise
2.	$q \vee \neg q$	LEM
3.	q	Assume
4.	p	Assume
5.	q	copy 3
6.	$p \to q$	$\to$i 4,5
7.	$\bot$	$\bot$i 1,6
8.	$p \wedge \neg q.$	$\bot$e, 7
9.	$\neg q.$	Assume
10.	$\neg p$	Assume
11.	$\neg p \vee q$	$\vee i_1$ 10
12.	p	Assume
13	$\neg p$	copy 10
14	$\bot$	$\bot$i 12,13
15.	q	$\bot$e 14.
16.	q	assume
17	$q.$	copy 16.
18.	q	$\vee$e 11, 13-15, 16-17
19.	$p \to q.$	$\to$i 12-18
20.	$\bot$	$\bot$i 19, 1
21.	$\neg\neg p$	$\neg$i 10-20
22.	p	$\neg\neg$e 21
23.	$p \wedge \neg q$	$\wedge$i 9,22
24.	$p \wedge \neg q$	$\vee$e 2, 3-8, 9-23.

3.(c) $(p \to \neg\phi_2) \to \phi_1,\ (q \to \neg\phi_1) \to \phi_2 \vdash \phi_1 \vee \phi_2$

1.	$(p \to \neg\phi_2) \to \phi_1$	premise
2.	$(q \to \neg\phi_1) \to \phi_2$	premise
3.	$\phi_1 \vee \neg\phi_1$	LEM
4.	ϕ_1	Assume
5.	$\phi_1 \vee \phi_2$	$\vee i$, 4
6.	$\neg\phi_1$	Assume
7.	$\neg\phi_1$	copy
8.	$p \to \neg\phi_2 \to \phi_1$	copy 1
9.	$\neg\phi_1$	copy 6
10.	$\neg(p \to \neg\phi_2)$	MT 8.9
11.	$p \wedge \neg\neg\phi_2$	Results of 3.6 from slide 10
12.	$\neg\neg\phi_2$	$\vee e_2$ 11
13.	ϕ_2	$\neg\neg e$ 12
14.	$\neg\phi_1 \to \phi_2$	$\to i$ 7-13.
15.	ϕ_2	$\to e$ 14,6.
16.	$\phi_1 \vee \phi_2$	$\vee i_2$, 15
17.	$\phi_1 \vee \phi_2$	$\vee e$ 3, 4-5, 6-16

2. ~~x digits cosen~~

There are x digits chosen from the set of even digits $\{0,2,4,6,8\}$ and y digits from the set $\{1,3,5,7,9\}$ such that $y \leq x$. Numbers are formed from these $x+y$ digits so that each digit is used exactly once.

So, No two odd digits are adjacent.
If we arrange x even digits, we can put y odd digits in $x+1$ places.

y odd digits can be arranged in $y!$ and x even ditits can be arranged in $x!$ ways.

So, placing y odd digits in $x+1$ places we can choose them in $\binom{x+1}{y}$ ways.

and all the $(x+y)$ digits can be arranged in,

$$\binom{x+1}{y} x!\, y! \text{ ways.}$$

now x can take values from 1 to 5.
and as $y \leq x$ y can take values from 0 to 5
hence taking summation over all the cases.

$$\sum_{x=1}^{5} \sum_{\substack{y=0 \\ x \geq y}}^{5} \binom{x+1}{y} x!\, y!. \qquad \text{(Ans:-)}$$