Instructions

- *The duration of the exam is 3.5 hours. You can refer to the prescribed textbook and the lecture notes.*
- *To get full credit, you should justify your answers with valid arguments.*
- *No doubts/clarifications would be entertained during the examination. Make appropriate assumptions.*

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1. (i) In how many ways can a pack of 52 cards be divided equally amongst four players in order.

   (ii) In how many ways can you divide these cards in four sets, three of them having 17 cards each and a the fourth just 1 card? **(5 marks)**

2. There are x digits chosen from the set of even digits $\{0, 2, 4, 6, 8\}$ and y digits from the set of odd digits $\{1, 3, 5, 7, 9\}$ such that $y \leq x$. Numbers are formed from these $x + y$ digits so that each digit is used exactly once. How many numbers can be formed such that no two odd digits are adjacent? **(5 marks)**

3. Prove the validity of the following sequents using natural deduction rules (You cannot use De Morgan's laws directly in the proofs. If you plan to use De Morgan's laws, either you should derive them explicitly or cite the lecture number and slide number in which it was covered): **(10 marks)**

   (a) $\neg(p \wedge q) \to \bot \vdash q$

   (b) $\neg(p \to q) \vdash p \wedge \neg q$

   (c) $(p \to \neg\phi_2) \to \phi_1,\ (q \to \neg\phi_1) \to \phi_2 \vdash \phi_1 \vee \phi_2$

4. Write regular expressions for the following languages over the alphabet $\{a, b\}$: **(15 marks)**

   (a) $\{w \in (a+b)^* \mid$ w has length at most $3\}$

   (b) $\{w \in (a+b)^* \mid w$ does not contain aba as a substring$\}$

   (c) $\{w \in (a+b)^* \mid$ if w starts with a, then w ends with $b\}$

   (d) $\{w \in (a+b)^* \mid$ if w contains aa then w contains $bb\}$

   (e) $\{w \in (a+b)^* \mid$ w contains an even number of as and an odd number of bs$\}$

5. Consider an alphabet $\Sigma = \{a, b, c\}$. Notice that we have a third letter c in the alphabet. Draw a DFA or an NFA for the following languages over this alphabet: **(8 marks)**

   (a) $\{w \in \Sigma^* \mid w$ contains all the letters of $\Sigma\}$

   (b) $\{w \in \Sigma^* \mid w$ contains exactly two letters from $\Sigma\}$

6. Convert the following regular expressions over $\{0, 1\}$ to equivalent NFA: **(9 marks)**

   (a) $(0^*1)^*$

   (b) $01^*(0 + 01^*0)00^*$

   (c) $(10 + 10^*1 + 11)(1 + 100)$

7. Determinize the following NFA: **(3 marks)**