CSCI 6968 Weekly Participation 2

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1 Question

In poisson regression model we have $y \mid \mathbf{x} \sim \text{Poisson}(\exp(\theta^T \mathbf{x}))$.
So,

$$P_\theta(y_i|x_i) = \frac{\exp(-\exp(\theta^T x_i))\ [\exp(\theta^T x_i)]^{y_i}}{y_i!} \tag{1}$$

Now, by MLE finding the $\hat{\theta}$,

$$\begin{aligned}
\hat{\theta} &= \arg\max_\theta \ \left\{\prod_{i=1}^{n} P_\theta(y_i|x_i)\right\}^{\frac{1}{n}} \\
&= \arg\max_\theta \ \left\{\prod_{i=1}^{n} \frac{\exp(-\exp(\theta^T x_i))\ [\exp(\theta^T x_i)]^{y_i}}{y_i!}\right\}^{\frac{1}{n}} \\
&= \arg\max_\theta \ \frac{1}{n}\sum_{i=1}^{n}(\ln(\exp(-\exp(\theta^T x_i))) + y_i \ln(\exp(\theta^T x_i)) - \ln(y_i!)) \qquad (2) \\
&= \arg\max_\theta \ \frac{1}{n}\sum_{i=1}^{n} -\exp(\theta^T x_i) + \frac{1}{n}\sum_{i=1}^{n} y_i(\theta^T x_i) - \frac{1}{n}\sum_{i=1}^{n}\ln(y_i!) \\
&= \arg\max_\theta \ \frac{1}{n}\sum_{i=1}^{n} -\exp(\theta^T x_i) + \frac{1}{n}\sum_{i=1}^{n} y_i(\theta^T x_i)
\end{aligned}$$