

ADVANCED CONTROL SYSTEMS

Motion Control - Adaptive Control -

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Adaptive Control

PROJECT

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When our mathematical model is very close to the real manipulator dynamics, it is safe to resort to the inverse dynamics control scheme.

Kinds of uncertainties:

- ▶ dynamic parameters: masses, center of mass vectors, inertia matrices
- ▶ friction coefficients (*slowly time-varying*)
- ▶ payload at the end-effector in pick-and-place tasks (*poorly known and abruptly change*)

What we assume to know

- ▶ kinematic description, i.e. Denavit-Hartenberg parameters

The dynamic model can be written using the regressor matrix $Y(q, \dot{q}, \ddot{q})$

$$\begin{aligned} B(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) &= \tau \\ &= Y(q, \dot{q}, \ddot{q})\Theta. \end{aligned} \quad (1)$$

where Θ is a np vector of constant parameters (p parameters per link) and Y is an $(n \times np)$ matrix.

The regression matrix Y depends linearly on \ddot{q} , quadratically on \dot{q} (for the terms related to kinetic energy), and trigonometrically on q .

Remark. Knowing Θ is the same of knowing B, C, F, g .

Assumption. The desired trajectory $q_d(t)$ is known and twice-differentiable

Control goal. Design a control law τ such that

$$\begin{aligned}\tilde{q} &\rightarrow 0 \\ \dot{\tilde{q}} &\rightarrow 0\end{aligned}$$

despite uncertainty on the dynamic parameters (unknown, slowly time-varying as friction coefficients, abruptly time-varying as mass and inertia at the end-effector in pick-and-place tasks)

Requirement. We ask for *global stability*: despite how far are the initial estimates of the unknown/uncertain parameters from their true values and how large is the initial trajectory error

Assumption. The mathematical model is exactly equal to the robot dynamics equation.

Joint Space Inverse Dynamics Control I

$$\begin{aligned}\tau &= B(q)y + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) \\ &= B(q)(-K_P q - K_D \dot{q} + r) + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) \\ &= B(q)(-K_P q - K_D \dot{q} + \ddot{q}_d + K_D \dot{q}_d + K_P q_d) + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) \\ &= B(q)(\ddot{q}_d + K_P \tilde{q} + K_D \dot{\tilde{q}}) + C(q, \dot{q})\dot{q} + F\dot{q} + g(q)\end{aligned}$$

where $\tilde{q} = q_d - q$, $\dot{\tilde{q}} = \dot{q}_d - \dot{q}$.

Joint Space Inverse Dynamics Control II

$$\tau = B(q)\ddot{q}_d + C(q, \dot{q})\dot{q}_d + g(q) + F\dot{q}_d + K_D \dot{\tilde{q}} + K_P \tilde{q}$$

In case we have an uncertain knowledge of the robot dynamics, the control laws will look like

Joint Space Inverse Dynamics Control I

$$\tau = \hat{B}(q) \left(\ddot{q}_d + K_P \tilde{q} + K_D \dot{\tilde{q}} \right) + \hat{C}(q, \dot{q}) \dot{q} + \hat{F} \dot{q} + \hat{g}(q)$$

Joint Space Inverse Dynamics Control II

$$\tau = \hat{B}(q) \ddot{q}_d + \hat{C}(q, \dot{q}) \dot{q}_d + \hat{g}(q) + \hat{F} \dot{q}_d + K_D \dot{\tilde{q}} + K_P \tilde{q}$$

where $\hat{B}(q)$ approximates $B(q)$, $\hat{C}(q, \dot{q})$ approximates $C(q, \dot{q})$, ...

Such control laws are very difficult to be made *adaptive*, unless ...

We introduce the new signal \dot{q}_r (*reference velocity*)

$$\dot{q}_r = \dot{q}_d + \Lambda \tilde{q}$$

which sums the desired velocity \dot{q}_d with a error term related to the displacement between the desired position q_d and the actual position q .

The matrix Λ is a positive definite matrix.

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The matrix Λ is a positive definite matrix.

The time derivative of \dot{q}_r is

$$\ddot{q}_r = \ddot{q}_d + \Lambda \dot{\tilde{q}}$$

We define the new *control law* as

$$\tau = B(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma \quad (2)$$

where

$$\sigma = \dot{q}_r - \dot{q} = \dot{\tilde{q}} + \Lambda \tilde{q}$$

and K_D is a positive definite matrix.

Remarks.

- The term $K_D\sigma$ in

$$\tau = B(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma$$

is equivalent to a PD action on the tracking error

$$\begin{aligned} K_D\sigma &= K_D(\dot{q}_r - \dot{q}) \\ &= K_D(\dot{q}_d + \Lambda\tilde{q} - \dot{q}) \\ &= K_D\ddot{\tilde{q}} + K_D\Lambda\tilde{q} \\ &= \underbrace{K_D(\dot{q}_d - \dot{q})}_{D\text{-action}} + \underbrace{K_D\Lambda(q_d - q)}_{P\text{-action}} \end{aligned}$$

where $K_P := K_D\Lambda$ is the proportional gain.

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where $K_P := K_D\Lambda$ is the proportional gain.

- ▶ the control τ be re-written as

$$\begin{aligned} \tau &= B(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma \\ &= Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\Theta + K_D\sigma \end{aligned}$$

Substituting

$$\tau = B(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma$$

into

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = \tau$$

we end up with ($\sigma = \dot{q}_r - \dot{q}$)

$$B(q)\dot{\sigma} + C(q, \dot{q})\sigma + F\sigma + K_D\sigma = 0.$$

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$$B(q)\dot{\sigma} + C(q, \dot{q})\sigma + F\sigma + K_D\sigma = 0.$$

We consider the following Lyapunov function candidate as a function of σ, \tilde{q}

$$V(\sigma, \tilde{q}) = \frac{1}{2}\sigma^T B(q)\sigma + \frac{1}{2}\tilde{q}^T M\tilde{q}$$

with M a symmetric positive definite matrix.

$V(\sigma, \tilde{q})$ is a positive definite function (non negative and equal to zero only in $\sigma = 0$ and $\tilde{q} = 0$).

Since $\sigma = \dot{q}_r - \dot{q} = \ddot{\tilde{q}} + \Lambda \tilde{q}$, at the equilibrium point $\sigma = 0$ and $\tilde{q} = 0$ we also have

$$\ddot{\tilde{q}} = 0.$$

The time derivative of $V(\sigma, \tilde{q})$ along the trajectory of

$$B(q)\dot{\sigma} + C(q, \dot{q})\sigma + F\sigma + K_D\sigma = 0.$$

is

$$\begin{aligned}\dot{V}(\sigma, \tilde{q}) &= \sigma^T B(q)\dot{\sigma} + \frac{1}{2}\sigma^T \dot{B}(q)\sigma + \tilde{q}^T M\dot{\tilde{q}} \\ &= \sigma^T (-C(q, \dot{q})\sigma - F\sigma - K_D\sigma) + \frac{1}{2}\sigma^T \dot{B}(q)\sigma + \tilde{q}^T M\dot{\tilde{q}} \\ &= \sigma^T (-F\sigma - K_D\sigma) + \frac{1}{2}\sigma^T (\dot{B}(q) - 2C(q, \dot{q}))\sigma + \tilde{q}^T M\dot{\tilde{q}} \\ &= -\sigma^T (F + K_D)\sigma + \tilde{q}^T M\dot{\tilde{q}} \\ &= -\sigma^T F\sigma - \sigma^T K_D\sigma + \tilde{q}^T M\dot{\tilde{q}}\end{aligned}$$

Choosing $M = 2\Lambda K_D$

$$\dot{V}(\sigma, \tilde{q}) = -\sigma^T F \sigma - \sigma^T K_D \sigma + 2\tilde{q}^T \Lambda K_D \dot{\tilde{q}}$$

and by substituting

$$\begin{aligned}\sigma &= \dot{\tilde{q}} + \Lambda \tilde{q} \\ \dot{\tilde{q}} &= \sigma - \Lambda \tilde{q}\end{aligned}$$

we get

$$\dot{V}(\sigma, \tilde{q}) = -\sigma^T F \sigma - \dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q}, \quad (3)$$

Therefore, the time derivative is negative definite and vanishes only for $\tilde{q} = 0$, $\dot{\tilde{q}} = 0$.

Then the equilibrium point $[\tilde{q}^T \ \sigma^T]^T = 0$ is *globally asymptotically stable*.

We have just proved the following theorem

Theorem

Given the desired trajectory $q_d(t)$ for the robotic arm

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = \tau,$$

the control law

$$\left\{ \begin{array}{lcl} \tau & = & B(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma \\ \dot{q}_r & = & \dot{q}_d + \Lambda\tilde{q}, \quad \tilde{q} = q_d - q \\ \ddot{q}_r & = & \ddot{q}_d + \Lambda\dot{\tilde{q}}, \quad \dot{\tilde{q}} = \dot{q}_d - \dot{q} \\ \sigma & = & \dot{q}_r - \dot{q} \end{array} \right.$$

guarantees the global asymptotic stability of $(\tilde{q}(t), \dot{\tilde{q}}(t)) = (0, 0)$.

The *nominal* control law

$$\tau_n = B(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma$$

becomes

$$\tau = \hat{B}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{F}\dot{q}_r + \hat{g}(q) + K_D\sigma$$

when we use the estimated matrices \hat{B} , \hat{C} , \hat{F} , \hat{g} based on the available vector $\hat{\Theta}$ of Θ .

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Assumption: \hat{B} , \hat{C} , \hat{F} , and \hat{g} satisfy the same properties of B , C , F , and g .

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In particular

$$\tau_n = \underbrace{B(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + F\dot{q}_r + g(q)}_{Y(q, \dot{q}, \ddot{q}_r, \dot{q}_r)\Theta} + K_D\sigma$$

$$\tau = \underbrace{\hat{B}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{F}\dot{q}_r + \hat{g}(q)}_{Y(q, \dot{q}, \ddot{q}_r, \dot{q}_r)\hat{\Theta}} + K_D\sigma$$

$$\begin{aligned}\tau &= \hat{B}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{F}\dot{q}_r + \hat{g}(q) + K_D\sigma \\ &= Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\Theta} + K_D\sigma \\ &= Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\Theta + Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\Theta} + K_D\sigma\end{aligned}$$

where $\tilde{\Theta} = \hat{\Theta} - \Theta$ is the error parameter vector.

It is worth highlighting that *the matrix Y depends only on the joint desired accelerations and not on their actual values*, ($\dot{q}_r = \dot{q}_d + \Lambda\tilde{q}$, $\ddot{q}_r = \ddot{q}_d + \Lambda\dot{\tilde{q}}$).

Using the matrices in the dynamic model

$$\begin{aligned}\tau &= \hat{B}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{F}\dot{q}_r + \hat{g}(q) + K_D\sigma \\ &= \left(B(q) + \tilde{B}(q)\right)\ddot{q}_r + \left(C(q, \dot{q}) + \tilde{C}(q, \dot{q})\right)\dot{q}_r + \left(F + \tilde{F}\right)\dot{q}_r + (g(q) + \tilde{g}(q)) + K_D\sigma\end{aligned}$$

where

$$\tilde{B} = \hat{B} - B, \quad \tilde{C} = \hat{C} - C, \quad \tilde{F} = \hat{F} - F, \quad \tilde{g} = \hat{g} - g.$$

Substituting in the equations of motion

$$\begin{aligned} B(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) &= \dots \\ \dots &= \left(B(q) + \tilde{B}(q) \right) \ddot{q}_r + \left(C(q, \dot{q}) + \tilde{C}(q, \dot{q}) \right) \dot{q}_r + \left(F + \tilde{F} \right) \dot{q}_r + (g(q) + \tilde{g}(q)) + K_D \sigma \end{aligned}$$

and remembering that $\sigma = \dot{q}_r - \dot{q}$, $\dot{\sigma} = \ddot{q}_r - \ddot{q}$, we finally have

$$\begin{aligned} B(q)\dot{\sigma} + C(q, \dot{q})\sigma + F\sigma + K_D\sigma &= -\tilde{B}(q)\ddot{q}_r - \tilde{C}(q, \dot{q})\dot{q}_r - \tilde{F}\dot{q}_r - \tilde{g}(q) \\ &= -Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\Theta} \end{aligned}$$

Since we want to on-line estimate the value of $\hat{\Theta}$, we have to modify the Lyapunov function candidate

$$V(\sigma, \tilde{q}, \tilde{\Theta}) = \frac{1}{2}\sigma^T B(q)\sigma + \tilde{q}^T \Lambda K_D \tilde{q} + \frac{1}{2}\tilde{\Theta}^T K_{\Theta} \tilde{\Theta} \quad (4)$$

taking into account also the new component for the state $\tilde{\Theta}$, i.e. the *estimation error*.

If K_{Θ} is a symmetric positive definite matrix, the Lyapunov function candidate

$$V(\sigma, \tilde{q}, \tilde{\Theta}) = \frac{1}{2} \sigma^T B(q) \sigma + \tilde{q}^T \Lambda K_D \tilde{q} + \frac{1}{2} \tilde{\Theta}^T K_{\Theta} \tilde{\Theta} \quad (5)$$

is nonnegative and equal to zero only at $\sigma = 0$ (i.e. $\dot{q} = \dot{q}_d$), $\tilde{q} = 0$ (i.e. $q = q_d$), $\tilde{\Theta} = 0$ (i.e. $\hat{\Theta} = \Theta$); this implies that V is a positive definite function.

Computing the time derivative

$$\dot{V}(\cdot) = \sigma^T B(q) \dot{\sigma} + \frac{1}{2} \sigma^T \dot{B}(q) \sigma + \tilde{q}^T \Lambda K_D \dot{\tilde{q}} + \tilde{\Theta}^T K_{\Theta} \dot{\tilde{\Theta}}$$

along the trajectories of the system

$$B(q) \dot{\sigma} + C(q, \dot{q}) \sigma + F \sigma + K_D \sigma = -Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \tilde{\Theta}$$

we have

$$\begin{aligned} \dot{V}(\cdot) &= \sigma^T \left(-C(q, \dot{q}) \sigma - F \sigma - K_D \sigma - Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \tilde{\Theta} \right) + \frac{1}{2} \sigma^T \dot{B}(q) \sigma + \tilde{q}^T \Lambda K_D \dot{\tilde{q}} + \tilde{\Theta}^T K_{\Theta} \dot{\tilde{\Theta}} \\ &= -\sigma^T F \sigma - \dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q} - \sigma^T Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \tilde{\Theta} + \tilde{\Theta}^T K_{\Theta} \dot{\tilde{\Theta}} \\ &= -\sigma^T F \sigma - \dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q} - \tilde{\Theta}^T Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma + \tilde{\Theta}^T K_{\Theta} \dot{\tilde{\Theta}} \end{aligned}$$

$$\begin{aligned}\dot{V}(\cdot) &= -\sigma^T F \sigma - \dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q} - \tilde{\Theta}^T Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma + \tilde{\Theta}^T K_\Theta \dot{\tilde{\Theta}} \\ &= -\sigma^T F \sigma - \dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q} + \tilde{\Theta}^T \left(K_\Theta \dot{\tilde{\Theta}} - Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma \right),\end{aligned}$$

If we choose as dynamic equation for the estimation error

$$\dot{\tilde{\Theta}} = K_\Theta^{-1} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma,$$

then we also have

$$\dot{\tilde{\Theta}} = K_\Theta^{-1} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma,$$

since the nominal value Θ are assumed constant ($\dot{\tilde{\Theta}} = \frac{d}{dt} (\tilde{\Theta} - \Theta) = \dot{\tilde{\Theta}}$).

Finally

$$\dot{V}(\cdot) = -\sigma^T F \sigma - \dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q}$$

$$\dot{V}(\cdot) = -\sigma^T F \sigma - \dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q}$$

is a negative **semi**-definite function in σ , \tilde{q} , $\tilde{\Theta}$.

Since

$$\sigma = \dot{q}_r - \dot{q} = \dot{q}_d + \Lambda \tilde{q} - \dot{q} = \dot{\tilde{q}} + \Lambda \tilde{q} = (\dot{q}_d - \dot{q}) + \Lambda(q_d - q),$$

$$\dot{V}(\cdot) = -\sigma^T F \sigma - \dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q}$$

is a negative **semi**-definite function in σ , \tilde{q} , $\tilde{\Theta}$.

Since

$$\sigma = \dot{q}_r - \dot{q} = \dot{q}_d + \Lambda \tilde{q} - \dot{q} = \dot{\tilde{q}} + \Lambda \tilde{q} = (\dot{q}_d - \dot{q}) + \Lambda(q_d - q),$$

the control law and the adaptation law

$$\begin{cases} \tau &= \hat{B}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{F}\dot{q}_r + \hat{g}(q) + K_D\sigma \\ &= Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\Theta} + K_D\sigma \\ \dot{\hat{\Theta}} &= \Gamma Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\sigma, \quad (\Gamma := K_{\Theta}^{-1} \succ 0) \end{cases}$$

guarantee the global converge of the closed loop system toward $\sigma = 0$, $\tilde{q} = 0$, which implies **convergence to zero** of $\dot{\tilde{q}}(t)$ and $\tilde{q}(t)$, and **boundness** of $\hat{\Theta}(t)$.

We have just proved the following theorem

Theorem

Given the desired trajectory $q_d(t) \in \mathcal{C}^2$ for the robotic arm

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = \tau,$$

the control law

$$\begin{cases} \tau &= \hat{B}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{F}\dot{q}_r + \hat{g}(q) + K_D\sigma = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\Theta} + K_D\sigma \\ \dot{\hat{\Theta}} &= \Gamma Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\sigma, \quad (\Gamma := K_{\Theta}^{-1} \succ 0) \end{cases}$$

guarantees that the tracking error along the desired trajectory is globally asymptotically stable, i.e. $(\tilde{q}(t), \dot{\tilde{q}}(t)) \rightarrow (0, 0)$.

Remark.

Since

$$B(q)\dot{\sigma} + C(q, \dot{q})\sigma + F\sigma + K_D\sigma = -Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\Theta}$$

and $\sigma \rightarrow 0$, we have that

$$Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\Theta} \rightarrow 0$$

which does **not** mean that the estimation error $\tilde{\Theta}$ goes to zero, but only that $\hat{\Theta} - \Theta$ belong to the kernel of $Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$, i.e.

$$Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\Theta} \rightarrow Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\Theta$$

even though $\hat{\Theta} \nrightarrow \Theta$.

This adaptive control solves a so-called *direct adaptive control problem*: we find a control law that ensures limited tracking errors; we do not determine the actual parameters of the system (as in an *indirect adaptive control problem*).

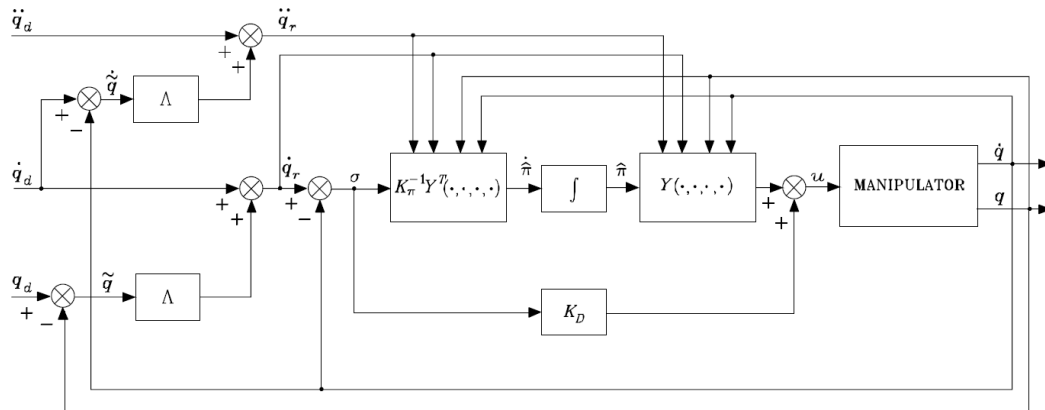


Figure: Joint Space Adaptive Control block scheme

Final observations:

- ▶ $Y\hat{\Theta}$ approximates the inverse dynamics control, i.e. it *approximates the compensation of nonlinear effects and joint decoupling*.
- ▶ $K_D\sigma$ stabilizes the tracking error via a *linear PD control action*.
- ▶ $\hat{\Theta}$ is updated via an *adaptive law of gradient type*, with a convergence rate depending on K_{Θ} .
- ▶ Choosing $\sigma = 0$ means adopting a pure inverse dynamics compensation.
- ▶ The proposed control scheme does not take into account neither external disturbances nor unmodelled dynamics. This implies that their effects on the output variables are attributed by the controller to parameter estimate mismatching.



To do

- Implement in Simulink the Adaptive Control law for the a 1-DoF link under gravity.
[choose the dynamic parameters and their initial estimates, and set the desired trajectory as (1) $q_d(t) = A \sin(\omega t)$, (2) $\ddot{q}_d(t)$ a periodic square wave $\pm A$]

Hint.

Plant

$$I\ddot{q} + F\dot{q} + \underbrace{mgd}_{G} \sin q = \tau$$

Linear parameterization

$$\tau = \underbrace{\begin{bmatrix} \ddot{q} & \dot{q} & \sin q \end{bmatrix}}_{Y(q, \dot{q}, \ddot{q})} \underbrace{\begin{bmatrix} I \\ F \\ G \end{bmatrix}}_{\Theta}$$

$$\dot{\hat{\Theta}} = \begin{bmatrix} \dot{\hat{I}} \\ \dot{\hat{F}} \\ \dot{\hat{G}} \end{bmatrix} = \begin{bmatrix} \gamma_1 & 0 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & 0 & \gamma_3 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \dot{q} \\ \sin q \end{bmatrix} (\dot{q}_r - \dot{q})$$

Reference velocity

$$\dot{q}_r = \dot{q}_d + \lambda \tilde{q} = \dot{q}_d + \frac{k_P}{k_D} (q_d - q)$$