Lecture 7: Multifactor Models – Examples

Investments

Factor Tilting

- **Factor Tilting** is forming a portfolio to take advantage of a forecast of a factor, $\tilde{f}$.
- We assume that in the return generating process equation for our portfolio:

$$\tilde{r}_{p,t} = E[\tilde{r}_{p,t}] + b_{p,1}\tilde{f}_{1,t} + \cdots + b_{p,n}\tilde{f}_{n,t} + \tilde{e}_{p,t}$$

  - $\hookrightarrow$ $E[\tilde{r}_{i,t}]$ represents the market's expectation
  - $\hookrightarrow$ $E[\tilde{f}_{1,t}] = 0$ *for the market*
  - $\hookrightarrow$ But, suppose we have superior information $E[\tilde{f}_{1,t}] \neq 0$, i.e. we can forecast f better than the market.

Factor Tilting

- The advantage of the multi-factor structure here is that we can express specific macro views, rather than expressing a view or forming a forecast about the whole market portfolio.
- Assuming we have this ability, we can earn superior profits by varying the factor betas/loadings of our portfolio.
  - $\hookrightarrow$ We want to increase the loading when we think that the factor is likely to have a positive realization
  - $\hookrightarrow$ We want to decrease the loading when we think that the factor is likely to be negative.

Factor Tilting - Example

Your analyst gives you the following information on three securities that are correctly priced according to a 2 factor APT model

$$\begin{aligned}\tilde{r}_A &= 0.12 + 1\cdot\tilde{f}_1 + 1\cdot\tilde{f}_2 + \tilde{e}_A\\ \tilde{r}_B &= 0.12 + 1\cdot\tilde{f}_1 + 2\cdot\tilde{f}_2 + \tilde{e}_B\\ \tilde{r}_C &= 0.12 + 3\cdot\tilde{f}_1 + 2\cdot\tilde{f}_2 + \tilde{e}_C\end{aligned}$$

- There are two common factors:
  - ↪ Factor 1 is a foreign income factor.
  - ↪ Factor 2 is a U.S. earnings price ratio factor.
- The way the model is constructed these factors are uncorrelated.
  - ↪ Here, the $E[r]$'s of 12% are what the market expects.
  - ↪ What are the factor risk premia (the λs)?

Factor Tilting - Example

- You believe very strongly that Japan will finally come out of its recession in the next few months and therefore exports of U.S. produced goods will rise more than the market expects. Moreover, you believe the earnings price ratio factor will not change at all in this time period, consistent with what analysts expect.

- Using the above three securities, construct ANY portfolio that takes advantage of all of these facts.
- What are

  i) the composition of the portfolio

  ii) the b's of the portfolio

  iii) the expected return on the portfolio.

Factor Tilting - Example

- We want to construct a portfolio with a lot of factor 1 exposure and no factor 2 exposure. Therefore, lets assume we want a loading of 10 on factor 1 and 0 on factor 2.
- Therefore, solve the three equations:

$$\begin{aligned} 1 \cdot w_A + 1 \cdot w_B + 3 \cdot w_C &= 10 \\ 1 \cdot w_A + 2 \cdot w_B + 2 \cdot w_C &= 0 \\ 1 \cdot w_A + 1 \cdot w_B + 1 \cdot w_C &= 1 \end{aligned}$$

- (The last equation is the usual restriction that the sum over the weights is one.) We can also write these equations as:

$$\begin{aligned} 1 \cdot w_A + 1 \cdot w_B + 3 \cdot (1 - w_A - w_B) &= 10 \\ 1 \cdot w_A + 2 \cdot w_B + 2 \cdot (1 - w_A - w_B) &= 0 \end{aligned}$$

Factor Tilting - Example

- The solution to this system of equations are the weights $w_A = 2$, $w_B = -5.5$, and $w_C = 4.5$. Then this portfolio will have a loading of 10 on factor 1 and a loading of 0 on factor 2.

  $\hookrightarrow$ We have constructed a *factor-mimicking portfolio*

- Assuming that you believe that the foreign income factor will rise by 2%, the expected return on this portfolio is:

$$2 \cdot 0.12 - 5.5 \cdot 0.12 + 4.5 \cdot 0.12 + 10 \cdot 0.02 = 0.32$$

- Is this the highest Sharpe-Ratio portfolio possible?
- In practice, we have more than three securities available to construct factor-mimicking portfolios. Is there an optimal way to construct factor-mimicking portfolios?

- Factor models are also a convenient way to manage portfolio risks.
- As a portfolio manager you can control specific risk exposures, customizing the portfolio to the needs of your clients.
- In order to hedge out your exposure to a factor:
  1. Estimate the loading of each asset with respect to the factor, $b_{i,k}$.
  2. Include an additional constraint: The factor loading of your portfolio should equal zero: $b_{p,k} = 0$.

- You believe the following four-factor model holds:

$$R_{it} = a_i + b_{M,i}\tilde{f}_{M,t} + b_{TS,i}\tilde{f}_{TS,t} + b_{CS,i}\tilde{f}_{CS,t} + b_{O,i}\tilde{f}_{O,t} + \varepsilon_{i,t}$$

where

- ↪ $\tilde{f}_{M,t}$ is the surprise return on the market portfolio.
- ↪ $\tilde{f}_{TS,t}$ is the surprise change in the slope of the yield curve.
- ↪ $\tilde{f}_{CS,t}$ is the surprise change in the credit spreads.
- ↪ $\tilde{f}_{O,t}$ is the surprise change in oil prices.

- The covariance matrix of the factors is

Covariance	MKT	TS	YS	OIL
MKT	2.100%			
TS		0.090%		
YS			0.030%	
OIL				0.760%

- Suppose you are choosing between 6 portfolios. You have estimated the following for each portfolio:

No	E (R)	$\sigma(R)$	Factor Loadings			
			market	**TS**	**YS**	**OIL**
SG	14.7%	0.306	1.439	5.487	0.279	0.043
SC	10.7%	0.208	0.872	3.419	-0.018	-0.019
SV	10.2%	0.210	0.786	3.925	1.250	-0.036
LG	11.2%	0.197	1.022	-0.602	2.617	-0.010
LC	9.7%	0.174	0.749	-0.221	-2.134	-0.062
LV	9.3%	0.169	0.672	-0.795	-1.202	-0.113

- What is the optimal portfolio that:
  - $\hookrightarrow$ Achieves an expected return of 12% and has zero exposure to Oil risk?
  - $\hookrightarrow$ Has the maximum Sharpe ratio ($r_f = 5\%$) and has loading of 1 with the market portfolio?

- First we calculate the correlation matrix implied by the 4-factor model:

	1	2	3	4	5	6
1	1	0.6821	0.6761	0.4662	0.4005	0.3132
2	0.6821	1	0.609	0.4107	0.3605	0.2804
3	0.6761	0.609	1	0.3804	0.2946	0.2203
4	0.4662	0.4107	0.3804	1	0.4221	0.4172
5	0.4005	0.3605	0.2946	0.4221	1	0.3937
6	0.3132	0.2804	0.2203	0.4172	0.3937	1

- Then we plug in the numbers into Markowitz, with some additional constraints on the loadings

Risk Management - Example

- Optimal Portfolio that achieves 12% return and has zero oil exposure.

Number of securities: 6

No	Name	Fraction	Expected Return	Standard Deviation
1	1 (SV)	0.29	14.7%	30.6%
2	2 (SC)	0.12	10.7%	20.8%
3	3 (SG)	-0.03	10.2%	21.0%
4	4 (LV)	0.47	11.2%	19.7%
5	5 (LC)	0.19	9.7%	17.4%
6	6 (LG)	-0.04	9.3%	16.9%
		1.00		

Correlations		2 2 (SC)	3 3 (SG)	4 4 (LV)	5 5 (LC)	6 6 (LG)
1	1 (SV)	0.68	0.68	0.47	0.40	0.31
2	2 (SC)	1.00	0.61	0.41	0.36	0.28
3	3 (SG)		1.00	0.38	0.29	0.22
4	4 (LV)			1.00	0.42	0.42
5	5 (LC)				1.00	0.39
		YES				

Portfolio's Expected Return	12.0%
Portfolio's Standard Deviation	18.4%

Cov(r_p,F_OI)/Var(F_OI) *(0.00000)*

FACTOR LOADINGS

	market	term_spread	yield spread	Oil
1	1.439	5.487	0.279	0.043
2	0.872	3.419	-0.018	-0.019
3	0.786	3.925	1.250	-0.036
4	1.022	-0.602	2.617	-0.010
5	0.749	-0.221	-2.134	-0.062
6	0.672	-0.795	-1.202	-0.113
PORTFOLIO	*1.097*	*1.614*	*0.922*	*0.000*

Portfolio Factor load =>

Covariance	market	term_spread	yield spread	Oil inflation
market	0.021			
term_spread		0.0009		
yield spread			0.0003	
Oil inflation				0.0076

Risk Management - Example

- Optimal Portfolio (i.e. max Sharpe Ratio) with market beta of 1.

Number of securities: 6

No	Name	Fraction	Expected Return	Standard Deviation
1	1 (SV)	0.24	14.7%	30.6%
2	2 (SC)	0.06	10.7%	20.8%
3	3 (SG)	-0.01	10.2%	21.0%
4	4 (LV)	0.33	11.2%	19.7%
5	5 (LC)	0.19	9.7%	17.4%
6	6 (LG)	0.19	9.3%	16.9%
		1.00		

Correlations		2 2 (SC)	3 3 (SG)	4 4 (LV)	5 5 (LC)	6 6 (LG)
1	1 (SV)	0.68	0.68	0.47	0.40	0.31
2	2 (SC)	1.00	0.61	0.41	0.36	0.28
3	3 (SG)		1.00	0.38	0.29	0.22
4	4 (LV)			1.00	0.42	0.42
5	5 (LC)				1.00	0.39
		YES				

Portfolio's Expected Return	11.4%
Portfolio's Standard Deviation	16.2%
Sharpe Ratio	0.393922705
risk free rate	5%

FACTOR LOADINGS

	market	*term_spread*	*yield spread*	*Oil*
1	1.439	5.487	0.279	0.043
2	0.872	3.419	-0.018	-0.019
3	0.786	3.925	1.250	-0.036
4	1.022	-0.602	2.617	-0.010
5	0.749	-0.221	-2.134	-0.062
6	0.672	-0.795	-1.202	-0.113
PORTFOLIO	***1.000***	***1.134***	***0.292***	***-0.026***

Portfolio Factor load =>

market loading ***1.00000***

Covariance	*market*	*term_spread*	*yield spread*	*Oil inflation*
market	0.021			
term_spread		0.0009		
yield spread			0.0003	
Oil inflation				0.0076

Risk Management using Factor-Mimicking Portfolios

- An alternative approach would have been to hedge the risk out by trading in factor-mimicking portfolios.
- The approach would be:
  1. Find the optimal portfolio, without imposing any constraints.
  2. Find the loading of your portfolio with respect to the factor you wish to hedge $(b_{p,f})$.
  3. Sell a fraction $b_{p,f}$ of the portfolio that mimics factor f. Since it is a zero-investment portfolio, no further action is necessary.
- The end result will be the same with either approach if the factor-mimicking portfolio is included in the set of assets you are optimizing over.