FINC460: Homework 2

1 Implementing Mean-Variance Analysis

1. Open the file HW2Data.xls. It contains the risk free rate and monthly returns of three stocks, IBM, GM and GE.

   a) Compute the average return for the 3 stocks.

   b) Compute the standard deviation of each stock.

   c) Compute the 95% confidence interval for the average return for each stock.

   d) Compute the correlation matrix

   e) Compute the Sharpe Ratio for each stock.

2. Now it is time to use some of these estimates to compute our optimal portfolio. Open the Markowitz spreadsheet and plug in the numbers that you found for the mean returns, standard deviation and correlations.

   a) What are the weights on the optimal portfolio of these stocks? (Hint: use solver)

   b) Draw the Capital Allocation Line.

   c) Assuming a risk aversion coefficient of 4, what is the optimal mix of risky vs riskless assets?

3. Now suppose that we go back in time to May 1st 1998. You are contemplating the same decision, but you obviously do not have all the data available.

   a) Using data available until May 1st 1998, repeat steps 1 and 2 above.

   b) Assuming you invested 100% in the optimal portfolio as of May 1st 1998 and you held on to this position until December 30 2006, compute the average realized return of this portfolio, its standard deviation and Sharpe ratio.

c) How well has your portfolio done? What happened?

4. Now, with the benefit of hindsight let's return to May 1st 1998. Repeat the steps above, but impose a constraint on your portfolio that no position can be greater than 50%, in absolute value, of your wealth. How did this 'constrained' portfolio do in the second half of the sample?

2 The tangible benefits of rebalancing

The point of this exercise is to quantify the benefits of rebalancing. Suppose that you are investing in two stock market indices, AAA and ZZZ. Both risky assets have returns that are normally distributed, with the same mean return (10%) and the same standard deviation (20%). Further, the correlation between the two risky returns is zero. Also, assume the risk-free rate is zero.

1. What is the optimal (i.e. mean-variance efficient) allocation among the two securities?

2. Now, we will examine the performance of two strategies over an investment horizon of 100 years. We will do so across many simulations (say 10,000 though you can choose any number you want). For each simulation,

   (a) Simulate a vector of returns for the two risky assets, AAA and ZZZ. Use your software's build in random number generator.

   For example, in Matlab, you can simulate a normal variable with mean μ and standard deviation σ via $x = \mu + \sigma \times$ randn, where randn simulates a standard $N(0, 1)$ variable.

   (b) Compute the returns to your initial wealth of the buy and hold strategy—that is, the strategy in which you allocated optimally at $t = 0$ and then did not rebalance.

   (c) Compute the returns to your initial wealth of the rebalanced strategy—that is, the strategy in which you rebalanced at the end of every year to the 'optimal' portfolio weights.

   (d) Compute the realized Sharpe ratio for each of the two strategies above. In addition, compute the average allocation to the first risky asset AAA in the buy and hold strategy (in the rebalancing strategy it should be constant by construction).

3. Plot a histogram of the realized Sharpe ratio across simulations of the two strategies. Also plot the mean portfolio weight to asset 1 of the buy and hold strategy. Discuss.

For the purposes of this exercise, you should use a matrix programming language like MATLAB or Octave. Alternatively, you can also use R. That said, it might be possible to do it in Excel, though it will be rather painful.

3 Human Capital

Let us revisit the portfolio problem in Lecture 1, but now let us take into account your human capital. As before, you can invest your wealth into either the riskless asset (1% per year) or in three stock market indices with the following characteristics:

Asset	$E(r)$	σ
US	8.0%	19%
Japan	3.7%	25%
Mexico	6.0%	15%

Correlations			
Assets	US	J	M
US	1.0	0.5	0.8
J	0.5	1.0	0.2
M	0.8	0.2	1.0

Now, let us add a fourth asset, your human capital. Assume that human capital accounts for 50% of your total wealth. This is an asset that you are endowed with, but that you cannot rebalance (you cannot issue shares on your human capital). Assume that the mean return to your human wealth is 19% per year, with a standard deviation of 21%. The correlation of your human capital returns with the return on US, Japan and Mexico stocks is 20%, 10% and 15%, respectively.

Solve the portfolio asset with human capital (HINT: add a fourth asset, with its weight fixed at 50%). Report the mean-variance efficient allocation in US, Japan and Mexico, both as a proportion of your total as well as your financial wealth. Compare the allocations with those in Lecture 1. Discuss.