# FINC460/FE312 - Midterm Exam

NAME: ______________________________ SECTION: ______________

1. Please do not open this exam until directed to do so.
2. Please write your name and section number on the front of this exam, and on any examination books you use.
3. Please show all work required to obtain each answer. Answers without justification will receive no credit.
4. State clearly any assumptions you are making.
5. This is a closed book exam. No books or notes are permitted. Calculators are permitted. Laptops are permitted but you are only allowed to use Excel and a blank worksheet.
6. Brevity is strongly encouraged on all questions.
7. The exam is worth 115 points.
8. Relax, and good luck!

## Hints:

1. *Think through problems before you start working. Draw pictures.*
2. *If you get stuck on part of a problem, go on to the next part. You may need to use answers from earlier parts of the question to calculate answers to the later parts. If you weren't able to solve the earlier part, assume something.*
3. *Remember, setting up the problem correctly will get you most of the points.*

## Question 1 (40pts)

1. (8 points) Mean variance analysis suggests that, since investors like returns and dislike risk, more volatile assets (i.e. those with higher variance of returns) should earn higher expected returns.

2. (8 points) Consider a momentum trading strategy that buys stocks that outperformed in the last year and sells stocks that underperformed. Suppose that this strategy yields positive returns on average. This pattern violates the CAPM.

3. (8 points) If the CAPM holds, every security should receive a strictly non-negative weight in the mean-variance efficient portfolio.

4. (8 points) Since both expected returns and variances scale linearly with time, the investment horizon should not affect the choice between stocks and bonds.

5. (8 points) Since portfolio constraints reduce the maximum Sharpe ratio we can achieve, they are best avoided.

## Question 2 (75pts)

You have the following information

| Security | Expected Return | Beta | Standard Deviation | Market Capitalization |
|---|---|---|---|---|
| Risk-Free Asset | 3% | - | - | - |
| Market Portfolio | 6% | | 11.2% | 100b |
| Stock A | 7.8% | | 20% | |
| Stock B | 4.2% | | 10% | |
| Derivative C | 3% | | 5% | |

For all parts of this question, assume **(1) the CAPM properly prices all assets, (2) assets A, B and C are the only risky assets in the economy, and (3) assets A, B, and C are correlated only due to their (potential) common market exposure, that is the residuals $\varepsilon_i$ in a factor model regression are uncorrelated for $i \in \{A, B, C\}$.** You should assume that the risk-free rate is the same for borrowing or lending.

Answer the following set of questions. For some of the questions you may need the answer to previous parts to solve them. If you do not have it, assume something and move on.

1. (10 points) What are the market betas of assets A, B, and C so that their expected returns are consistent with equilibrium?

2. (10 points) What fraction of the total variance of stocks A, B and C is due to systematic – as opposed to idiosyncratic – risk?

3. (20 points) Which portfolio should you choose among these five assets?

   (a) (10 points) What is the highest Sharpe ratio you can achieve among **all** possible combinations?

   (b) (5 points) Assuming you have a risk aversion coefficient of 5, which combination of the five assets above should you hold? Specify the fraction of your wealth that you will put in each of the five assets.

   (c) (5 points) How would your answer change if your risk aversion coefficient was very large?

4. (20 points) Again, assume you have a risk aversion of 5. Now assume that you can only invest in a subset of the three risky assets (A, B or C) in combination with the risk-free asset.

   (a) (5 points) Suppose you can invest only in asset C and the risk free rate. How much would you invest in C? Why?

   (b) (5 points) Now, suppose that you could only invest in A and C. Focusing only on the risky part of your portfolio, what fraction would you allocate into assets A and C?

   (c) (5 points) Last, now suppose now that you could only invest in B and C. How does your answer change?

   (d) (5 points) Based on the above, can you think of **any** situation in which a mean-variance investor would like to invest a positive (or negative) amount on asset C? Why?

5. (15 points) What is the market capitalization of assets A, B and C?

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# Finance 460 Midterm Equation Sheet

- Covariance Relations:

$$cov(a\tilde{x}, b\tilde{y}) = a \cdot b \cdot cov(\tilde{x}\tilde{y}) \qquad cov(\tilde{x}, \tilde{y}+\tilde{z}) = cov(\tilde{x}, \tilde{y}) + cov(\tilde{x}, \tilde{z}) \qquad var(a\tilde{x}) = a^2 var(\tilde{x})$$

- Covariance of two securities when their residuals are uncorrelated:

$$cov(\tilde{r}_i, \tilde{r}_j) = \beta_i \beta_j \sigma_m^2 \quad \textbf{if } cov(\epsilon_i, \epsilon_j) = 0$$

- Statistical Functions:

$$var(\tilde{r}_A) = E[(\tilde{r}_A - \overline{r_A})^2] \qquad cov(\tilde{r}_A, \tilde{r}_B) = E[(\tilde{r}_A - \overline{r_A})(\tilde{r}_B - \overline{r_B})]$$

- Fraction of the your wealth you put in the risky assset 1, given a risk aversion coefficient of A:

$$w_1^* = \frac{E(\tilde{r}_1) - r_f}{A\sigma_1^2}$$

- The MVE portfolio weights when there are two risky assets $A$ and $B$ ($x_B = (1 - x_A)$):

$$x_A = \frac{E(\tilde{r}_A^e)\sigma_B^2 - E(\tilde{r}_B^e)cov(\tilde{r}_A^e, \tilde{r}_B^e)}{E(\tilde{r}_A^e)\sigma_B^2 + E(\tilde{r}_B^e)\sigma_A^2 - \left[E(\tilde{r}_A^e) + E(\tilde{r}_B^e)\right] cov(r_A^e, r_B^e)}$$

- The CAPM equation

$$E(\tilde{r}_i) = r_f + \beta_i \left[E(\tilde{r}_m) - r_f\right]$$

- The CAPM Beta:

$$\beta_i = \frac{cov(\tilde{r}_i, \tilde{r}_m)}{\sigma_m^2}$$

- The security market line

$$\tilde{r}_{i,t} - r_{f,t} = \alpha_i + \beta_i \cdot (\tilde{r}_{m,t} - r_{f,t}) + \tilde{\epsilon}_{i,t}$$

- The Systematic Variance of an Asset with a beta of $\beta_i$, assuming a single factor model:

$$\sigma_{sys,i}^2 = \beta_i^2 \cdot \sigma_m^2$$

- the $R^2$ is then $\frac{\sigma_{sys,i}^2}{\sigma_i^2}$

- Equation for *Merrill Lynch* adjusted $\beta$'s:

$$\beta_i^{Adj} = 1/3 + (2/3) \cdot \beta_i$$

- Equation for the variance of portfolio $a$; and for the covariance of portfolios $a$ and $b$:

$$var(\tilde{r}_a) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i^a w_j^a \sigma_{i,j} \qquad cov(\tilde{r}_a, \tilde{r}_b) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i^a w_j^b \sigma_{i,j}$$

  - For two assets(1 and 2):

$$var(\tilde{r}_a) = (w_1^a)^2 \cdot (\sigma_1)^2 + (w_2^a)^2 \cdot (\sigma_2)^2 + 2 \cdot w_1^a \cdot w_2^a \cdot cov(\tilde{r}_1, \tilde{r}_2)$$

$$cov(\tilde{r}_a, \tilde{r}_b) = w_1^a \cdot w_1^b \cdot (\sigma_1)^2 + w_2^a \cdot w_2^b \cdot (\sigma_2)^2 + (w_1^a \cdot w_2^b + w_2^a \cdot w_1^b) \cdot cov(\tilde{r}_1, \tilde{r}_2)$$

- Under a 1-factor model for returns

$$r_{i,t}^e = a_i + b_i f_{1,t} + u_{i,t}$$

The covariance between two assets under a 1-factor model

$$cov(r_i^e, r_j^e) = b_i b_j var(f_1)$$

Table 1: Legend

| | |
|---|---|
| $\sigma_{i,j}$ | covariance between i and j |
| $\sigma_{i,i} = \sigma_i^2$ | variance of i |
| $\beta_i$ | market beta of security i |
| $r_f$ | return on riskless asset |
| $r_i^e$ | excess return of security i |
| $E[r_i]$ | expectation of $r_i$ |
| $w_i^a$ | weight place on security i in portfolio a |