Finance 460 Midterm Equation Sheet

- Covariance Relations:

$$cov(a\tilde{x}, b\tilde{y}) = a \cdot b \cdot cov(\tilde{x}\tilde{y}) \qquad cov(\tilde{x}, \tilde{y} + \tilde{z}) = cov(\tilde{x}, \tilde{y}) + cov(\tilde{x}, \tilde{z}) \qquad var(a\tilde{x}) = a^2 var(\tilde{x})$$

- Covariance of two securities when their residuals are uncorrelated:

$$cov(\tilde{r}_i, \tilde{r}_j) = \beta_i \beta_j \sigma_m^2 \quad \textbf{if } cov(\epsilon_i, \epsilon_j) = 0$$

- Statistical Functions:

$$var(\tilde{r}_A) = E[(\tilde{r}_A - \overline{r_A})^2] \qquad cov(\tilde{r}_A, \tilde{r}_B) = E[(\tilde{r}_A - \overline{r_A})(\tilde{r}_B - \overline{r_B})]$$

- Fraction of the your wealth you put in the risky assset 1, given a risk aversion coefficient of A:

$$w_1^* = \frac{E(\tilde{r}_1) - r_f}{A\sigma_1^2}$$

- The MVE portfolio weights when there are two risky assets A and B ($x_B = (1 - x_A)$):

$$x_A = \frac{E(\tilde{r}_A^e)\sigma_B^2 - E(\tilde{r}_B^e) cov(\tilde{r}_A^e, \tilde{r}_B^e)}{E(\tilde{r}_A^e)\sigma_B^2 + E(\tilde{r}_B^e)\sigma_A^2 - [E(\tilde{r}_A^e) + E(\tilde{r}_B^e)]\, cov(r_A^e, r_B^e)}$$

- The CAPM equation

$$E(\tilde{r}_i) = r_f + \beta_i\, [E(\tilde{r}_m) - r_f)]$$

- The CAPM Beta:

$$\beta_i = \frac{cov(\tilde{r}_i, \tilde{r}_m)}{\sigma_m^2}$$

- The security market line

$$\tilde{r}_{i,t} - r_{f,t} = \alpha_i + \beta_i \cdot (\tilde{r}_{m,t} - r_{f,t}) + \tilde{\epsilon}_{i,t}$$

- The Systematic Variance of an Asset with a beta of β_i, assuming a single factor model:

$$\sigma_{sys,i}^2 = \beta_i^2 \cdot \sigma_m^2$$

- the R^2 is then $\frac{\sigma_{sys,i}^2}{\sigma_i^2}$
- Equation for *Merrill Lynch* adjusted β's:

$$\beta_i^{Adj} = 1/3 + (2/3) \cdot \beta_i$$

- Equation for the variance of portfolio a; and for the covariance of portfolios a and b:

$$var(\tilde{r}_a) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i^a w_j^a \sigma_{i,j} \qquad\qquad cov(\tilde{r}_a, \tilde{r}_b) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i^a w_j^b \sigma_{i,j}$$

  - For two assets(1 and 2):

$$var(\tilde{r}_a) = (w_1^a)^2 \cdot (\sigma_1)^2 + (w_2^a)^2 \cdot (\sigma_2)^2 + 2 \cdot w_1^a \cdot w_2^a \cdot cov(\tilde{r}_1, \tilde{r}_2)$$

$$cov(\tilde{r}_a, \tilde{r}_b) = w_1^a \cdot w_1^b \cdot (\sigma_1)^2 + w_2^a \cdot w_2^b \cdot (\sigma_2)^2 + (w_1^a \cdot w_2^b + w_2^a \cdot w_1^b) \cdot cov(\tilde{r}_1, \tilde{r}_2)$$

- Under a 1-factor model for returns

$$r_{i,t}^e = a_i + b_i f_{1,t} + u_{i,t}$$

  The covariance between two assets under a 1-factor model

$$cov(r_i^e, r_j^e) = b_i b_j var(f_1)$$

Table 1: Legend

$\sigma_{i,j}$	covariance between i and j
$\sigma_{i,i} = \sigma_i^2$	variance of i
β_i	market beta of security i
r_f	return on riskless asset
r_i^e	excess return of security i
$\overline{r_i}$	expectation of r_i
w_i^a	weight place on security i in portfolio a