FINC460/FE312 Investments - Midterm Exam

NAME: ______________________________ SECTION: ______________

1. Please do not open this exam until directed to do so.
2. This exam is 1 1/2 hours long.
3. Please write your name and section number on the front of this exam, and on any examination books you use.
4. Please show all work required to obtain each answer. Answers without justification will receive no credit.
5. State clearly any assumptions you are making.
6. This is a closed book exam. No books or notes are permitted. Calculators are permitted. Laptops are permitted but you are only allowed to use Excel and a blank worksheet.
7. Brevity is strongly encouraged on all questions.
8. The exam is worth 115 points.
9. Relax, and good luck!

Hints:

1. *Think through problems before you start working. Draw pictures.*
2. *If you get stuck on part of a problem, go on to the next part. You may need to use answers from earlier parts of the question to calculate answers to the later parts. If you weren't able to solve the earlier part, assume something.*
3. *Remember, setting up the problem correctly will get you most of the points.*

Short questions (40pts)

Assess the validity of the following statements (True, False or Uncertain) and explain your answers. Each question is worth 8pts.

1. Since no investor would bear risk without reward, the CAPM implies that the expected return on any risky asset must be higher than the risk free return.

   False, an asset with negative beta will have an expected return lower than the risk free rate according to the CAPM.

2. The CAPM implies that all securities should have zero alpha with respect to the SP&500.

   False, the market is not the SP&500. It should also include private firms, real estate, human capital, etc.

3. Individual securities may lie on the mean-variance frontier as long as they covary negatively with other securities.

   False/Uncertain. In general (except for the case where there are only two risky assets) individual securities will not lie on the frontier sicne they have diversifiable risk.

4. Suppose that small stocks had higher average returns than large stocks. This pattern violates the CAPM.

   False, small stocks could just have higher betas.

5. The weight individual securities receive in the mean-variance efficient portfolio is directly proportional to their Sharpe Ratio.

   False, that would imply it is just about their mean and variance. How securities covary with other securities in the portfolio is the main determinant of their weight.

Question 2 (75pts)

For all of the following questions, assume that the CAPM holds, and that the expected return on the market is 9% per year and the annual volatility of the market is 0.108. A risk-free asset exists which is in zero net supply. Assume also that you know that two portfolios A and B on the minimum-variance frontier of risky-assets have annual expected returns and volatilities of

$$E(r_A) = 0.06 \qquad \sigma_A = 0.15$$
$$E(r_B) = 0.12 \qquad \sigma_B = 0.20$$

Recall that portfolios on the minimum-variance frontier are those consisting *only* of risky assets which, for a given level of expected return, achieve the lowest possible level of volatility.

1. (15 points) Assume that you want to purchase \$100 million worth of a mean-variance efficient portfolio of risky assets (that is, where none of the \$100 million is invested in the risk-free asset). Assuming you can only trade in the portfolios A and B, How much of A and how much of B should you buy (in dollars)?

   Since CAPM holds, we know market is MVE portfolio. We want to re-create the market using A and B (we can re-create the optimal portfolio using any two frontier returns). We know:
   $R_{mkt} = w_A R_A + (1 - w_A) R_B$
   for some weight w_A. We can solve for w_A by taking expectations. Then:
   $E[R_{mkt}] = w_A E[R_A] + (1 - w_A) E[R_B]$
   Plugging in and solving, $w_A = 1/2$. Thus we should put \$50 million in each asset.

2. (15 points) Find the covariance of the returns of the portfolios A and B, and the beta of asset A.

We know: $R_{mkt} = \frac{1}{2}R_A + \frac{1}{2}R_B$

Take variance of both sides: $var(R_{mkt}) = \frac{1}{4}\sigma_A^2 + \frac{1}{4}\sigma_B^2 + 2\frac{1}{2}\frac{1}{2}cov(A, B)$

$0.108^2 = \frac{1}{4}0.15^2 + \frac{1}{4}0.20^2 + \frac{1}{2}cov(A, B)$

cov(A,B)=-0.008

To find the beta, compute:

$\beta_A = cov(R_A, R_{mkt})/var(R_{mkt})$

$cov(R_A, R_{mkt}) = cov(R_A, \frac{1}{2}R_A + \frac{1}{2}R_B) = \frac{1}{2}\sigma_A^2 + \frac{1}{2}cov(A, B) = 0.00725$

Thus $\beta_A = 0.62$

3. (15 points) Calculate the annual return on the risk-free asset. (*Note:* you need the answer from 2 to solve this. If you do not have it, assume something. Also, there is a hard way, and an easy way to solve this problem. Think carefully before you proceed!)

Use the CAPM: $E[R_A] - r_f = \beta_A[E[R_{mkt}] - r_f]$
We know everything but r_f.
Rearranging,
$E[R_A] - \beta_A E[R_{mkt}] = r_f(1 - \beta_A)$
$r_f = \frac{E[R_A] - \beta_A E[R_{mkt}]}{(1-\beta_A)}$
r_f=1.11%

4. (15 points) Suppose that your optimal allocation is to put 70% of your wealth into the mean-variance efficient portfolio and 30% of your wealth into the risk-free asset. What is your coefficient of risk aversion?

Use $w = \frac{E[R_{mkt}] - r_f}{A\sigma^2_{mkt}}$

Or, $A = \frac{E[R_{mkt}] - r_f}{w\sigma^2_{mkt}}$

$A = \frac{0.09 - 0.0111}{0.7(0.108)^2}$

A=9.66

5. (15 points) Suppose that there are now a number of new hedge funds that advertise they are "market-neutral," meaning that they have a beta of zero. The hedge funds charge a fee equal to two percent of asset under management. If these hedge funds really are market-neutral, have no more information than the rest of the market (and the market is efficient), and have zero holdings of the risk-free asset, calculate their post-fee expected return and the lowest possible return variance they could have. Could they have a higher variance than this? (answer yes or no and explain).

Find a combo of R_A and R_B that has zero beta. $R_p = wR_A + (1-w)R_B$
So, $\beta_p = w\beta_A + (1-w)\beta_B$
We already have $\beta_A = 0.62$, and find β_B using the CAPM equation $\beta_B = \frac{E[R_B]-r_f}{E[R_{mkt}]-r_f} = 1.38$
Then choose w to set $\beta_p = 0$.
Solving, we get $w = \frac{\beta_B}{\beta_B - \beta_A} = 1.82$
Now we know $E[R_p] = r_f$ (it has zero beta), so after fees you get the risk free rate minus the fees: 1.11-2=-0.89%. The variance is: $var(R_p) = w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)cov(A,B) = 0.125$
If there are truly only 2 assets in the economy then no, they can't have a higher variance then this since there is only 1 unique combination of A and B that achieves this expected return. (In other words, we know the expected return must be the risk free because it's zero-beta. If I draw horizontal line from risk free it can only hit the minimum variance frontier formed by A and B in one place). If there are more risky assets (outside span of A and B) then yes we could have higher variance.