FINC460 - Midterm Exam

NAME: _____SECTION:_____

1. Please do not open this exam until directed to do so.

2. This exam is 1 1/2 hours long.

3. Please write your name and section number on the front of this exam, and on any examination books you use.

4. Please show all work required to obtain each answer. Answers without justification will receive no credit.

5. State clearly any assumptions you are making.

6. This is a closed book exam. No books or notes are permitted. Calculators are permitted. Laptops are permitted but you are only allowed to use Excel and a blank worksheet.

7. Brevity is strongly encouraged on all questions.

8. The exam is worth 130 points.

9. Relax, and good luck!

Hints:

1. *Think through problems before you start working. Draw pictures.*

2. *If you get stuck on part of a problem, go on to the next part. You may need to use answers from earlier parts of the question to calculate answers to the later parts. If you weren't able to solve the earlier part, assume something.*

3. *Remember, setting up the problem correctly will get you most of the points.*

Short questions (40pts)

Assess the validity of the following statements (True, False or Uncertain) and explain your answers. Each question is worth 8pts.

1. *If the CAPM holds, then the expected return on any risky asset must be higher than the risk free return.*

   FALSE. An asset with a negative market beta will have returns lower than the risk-free rate.

2. *According to the Black-Litterman model, in the absence of additional information, all investors should hold the market portfolio.*

   TRUE/UNCERTAIN. If the investors hold no views then they invest according to their priors, and the traditional BL model assumes that investors form priors according to the CAPM, in which case they should hold the market portfolio.

3. *According to the CAPM, holding everything else fixed, if the covariance of a security with the market portfolio doubles so will its beta and its risk premium.*

   TRUE. If the expected return and the variance of the market remains the same, if the covariance doubles so does the beta and the risk premium.

4. *The variance of a portfolio equals the average of the variances of the individual assets.*

   FALSE. The variance of a portfolio equals approximately 1/N times the *average variance* of the individual assets plus the *average* covariance of the individual assets with the portfolio. As N becomes large, only the covariance terms remain.

5. *XYZ is planning to pay a $ 10 dividend to shareholders. XYZ's stock price is expected to fall on the ex-dividend day by $10. This predictable drop in the stock price is a violation of efficient markets.*

   FALSE. The market expects XYZ's price to fall on the ex-dividend day by $10 *because* the market is efficient. If XYZ's price did not fall, there would be an arbitrage opportunity, since an investor could buy XYZ on the ex-dividend day, collect the dividend and then sell XYZ on the next day at the same price. If that were to happen, he would essentially get the value of the dividend for free.

Question 1 (40pts)

Assume that the CAPM holds. Now, consider a mutual fund AMQ, which has a correlation coefficient with the return on the market of zero. Further, the annual expected return on the market is 15% and the annualized standard deviation of the market is 30%. Also, the risk free rate is 5%/year. For simplicity, assume that AMQ charges no management fee, and has zero expenses.

1. (10pts) *Calculate the maximum Sharpe ratio you could earn with any portfolio.*

   Since the CAPM holds, the market is mean-variance efficient, and therefore has the highest possible Sharpe-ratio, which is equal to:

   $$\frac{E(r_m) - r_f}{\sigma_m} = \frac{0.1}{0.3} = 0.333$$

2. (10pts) *If your coefficient of risk aversion were 1, what portfolio should you hold? (assume you can trade in AMQ, the market portfolio, and the risk free asset)*

   You would choose to hold only the market and the risk-free asset (the market will contain some AQM, but you would not hold any AQM over and above what is in the market). Thus, your allocation choice is between a single risky asset (the market) and the risk-free asset, and we can use the second equation directly:

   $$w_1^* = \frac{E(\tilde{r}_1) - r_f}{A\sigma_1^2} = \frac{0.1}{1 \cdot 0.3^2} = 1.11 = 111\%$$

   Thus, you should borrow 11% of your weath at the risk-free rate (or, equivalently, invest -11% in the risk-free asset), and invest 111% of your

wealth in the market.

3. (5pts) *Find the expected return of AMQ.*

   Since the correlation of AQM with the market is zero, its market beta is also zero, and its expected return must be the risk-free rate of 5%.

4. (5pts) *Find the systematic standard deviation of AMQ.*

   Because AQM's market beta is zero, the systematic standard deviation is zero.

5. (10pts) *Assume that AMQ is on the minimum-variance frontier. Find the return variance of AMQ.*

   If AMQ is on the mean-variance frontier, then it will have no diversifiable risk. Given that AMQ has no systematic risk either, the return variance of AMQ will be zero.

Question 2 (50pts)

Assume that the table below illustrates your beliefs about the expected returns and betas of the market portfolio and two mutual funds A and B over the next year.

Asset	E(R)	β	σ
A	8%	0.5	15%
B	10%	1.5	35%
Market	9%	-	20%
Risk-free	2%	-	-

Additionally, you believe that returns of A, B and the market over the next year are represented by a single-factor model:

$$R_{i,t} - r_f = a_i + \beta_i(R_{m,t} - r_f) + \epsilon_{i,t}$$

where ϵ_A and ϵ_B are uncorrelated.

1. (5pts) *Find the R^2 of the above equation for both funds, A and B. What should the R^2 be according to the CAPM? Recall that the R^2 is defined as the "explained variance" (that is, the variance of $a_i + \beta_i(R_{m,t} - r_f)$), divided by the variance of the dependent variable $(R_{i,t} - r_f)$.*

   For A, the R^2 will be

   $$R_A^2 = \frac{0.5^2 \times 0.2^2}{0.15^2} = 44.4\%$$

   for B

   $$R_B^2 = \frac{1.5^2 \times 0.2^2}{0.35^2} = 73.5\%$$

   The CAPM says *nothing* about what the R^2 in this specification should be. Based on the above, fund A bears more idiosyncratic risk.

2. (5pts) *Calculate* $cov(A, B)$.

The covariance between the two funds will be:

$$cov(A, B) = 0.5 \times 1.5 \times 0.2^2 = 0.03$$

3. (10pts) *Plot the Security Market Line and find* α_A *and* α_B.

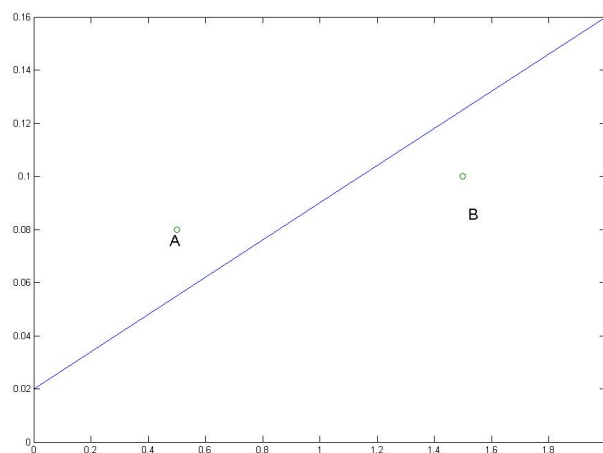Based on the CAPM, fund A should have an expected return equal to

$$E(R_A^{CAPM}) = 2\% + 0.5 \times (9\% - 2\%) = 5.5\%$$

while in fact it has $E(R) = 8\%$. Therefore $\alpha_A = 2.5\%$.
Similarly, fund B should have

$$E(R_B^{CAPM}) = 2\% + 1.5 \times (9\% - 2\%) = 12.5\%$$

while in fact it has $E(R) = 10\%$. Therefore $\alpha_B = -2.5\%$.

4. (10pts) *Is the market portfolio mean-variance efficient? Answer YES or NO and explain your answer.*

NO. The CAPM does not hold, since A and B do not lie on the SML. So the market portfolio is not mean-variance efficient and it does not have the highest Sharpe Ratio.

5. (15pts) *Now assume that your entire wealth is a portfolio of financial assets with a value of $1 million. Assume also that you can combine either A or B with the market portfolio, but you cannot hold both A and B. Also, you can freely short the market portfolio but not the two funds A or B, and you can borrow or lend at the risk-free rate of 2%/year.*

   Based on this, calculate:

   (a) *Your optimal portfolio of risky assets (i.e. the optimal allocation between the market and A or B).*

   (b) *The Sharpe-ratio of your portfolio.*

Since we cannot short fund B, it makes no sense to combine it with the market, since it has a negative alpha. Therefore we should be looking at fund A only. Fund A has excess return over the risk free rate of $E(R_A^e) = 8\% - 2\% = 6\%$, $var_A = 0.15^2 = 0.0225$ and covariance with the market of $\beta \times var(R_m) = 0.5 \times 0.2^2 = 0.02$. Therefore, the optimal allocation in A is given by the formula:

$$
\begin{aligned}
x_A &= \frac{E(\tilde{r}_A^e)\sigma_M^2 - E(\tilde{r}_M^e)cov(\tilde{r}_A^e, \tilde{r}_M^e)}{E(\tilde{r}_A^e)\sigma_M^2 + E(\tilde{r}_M^e)\sigma_A^2 - [E(\tilde{r}_A^e) + E(\tilde{r}_M^e)]\,cov(r_A^e, r_M^e)} \\
&= \frac{0.06 \times 0.2^2 - 0.07 \times 0.02}{0.06 \times 0.2^2 + 0.07 \times 0.15^2 - [0.06 + 0.07]\,0.02} = 0.72
\end{aligned}
$$

6. (5pts) *Based on your answer above, what is the maximum fee you would be willing to pay the manager of A or B?*

We should not pay the manager of A more than 2.5%, which is the excess return that he generates over the market.