

## FE-312: Homework 3

1. You are given the following information: the variance of return on stock-1, stock-2, and the market portfolio are:  $\sigma_1^2 = 0.16$ ,  $\sigma_2^2 = 0.09$  and  $\sigma_M^2 = 0.04$ . The covariance between these assets are  $\sigma_{12}^2 = 0.02$ ,  $\sigma_{1M}^2 = 0.064$ , and  $\sigma_{2M}^2 = 0.032$ . Consider forming a portfolio 'p' that has 75% invested in stock-1 and 25% invested in stock-2.
  - (a) What is the variance of return for portfolio p?
  - (b) What are the betas of stock-1, stock-2, and p relative to the market (that is, what are  $\beta_{1M}$ ,  $\beta_{2M}$ , and  $\beta_{pM}$  respectively)?
  - (c) What are the  $R^2$  values for regressing returns of stock-1, stock-2, and p on the market portfolio?
2. Mr. Larson E. Rich has asked you for some financial advice. His retirement savings are currently invested as follows: \$20,000 in the riskless asset, \$40,000 in GM stock, and \$40,000 in Microsoft stock. He wants to know if this is a sensible portfolio. You decided to analyze it based on the CAPM. You want to find out if Mr. Rich's portfolio is on the Capital Market Line.

You look in a "Beta Book" and find that GM stock has a beta of 1.1 and its  $R^2$  of the regression to market is 0.40. Microsoft stock has a beta of 0.8 and its  $R^2$  of the regression to market is 0.30. Suppose further that the correlation between the return to GM stock and the return to Microsoft stock is 0.3.

- (a) If  $R_f$  is 4% and the expected excess return on the market ( $E[R_M] - R_f$ ) is 6%, what is the expected return on Mr. Richs portfolio?
- (b) If market return has a volatility of 20%, compute the volatility of Mr. Richs current portfolio.

**Hint:** You may use the information in the  $R^2$  values to calculate the standard deviation of each stocks return. Then use the information about correlations between the two stock returns to calculate the portfolio standard deviation.

- (c) Assuming that the CAPM is correct, find an efficient portfolio that has the same volatility as Mr. Richs current portfolio. What is the expected return on this portfolio? How does it compare to the expected return of his current portfolio? You may assume that that market return has a volatility of 20%,  $R_f$  is 4%, and  $(E[R_M] - R_f)$  is 6%.
3. There are three stocks in the economy ('A', 'B', and 'C') that are all uncorrelated with each other. The riskfree rate for borrowing and lending is 4% over the holding period. The table below summarizes the information for each stock regarding its expected return and variance:

Stock	Expected Return	Variance of Return
A	14%	0.004
B	12%	0.002
C	11%	0.002

- (a) Compute the tangency portfolio weights of these three stocks and the expected return and volatility of the tangency portfolio.
- (b) Suppose the CAPM holds true and these are the only three risky assets in the economy. If stock-A has a market capitalization of \$100 million, what are the market caps of stock-B and stock-C? [1-million = 1,000,000 =  $10^6$ ].
4. Load the file BetaSortedPortfolios.dta in Stata. We will use mean variance optimization to select among these 11 portfolios (10-beta sorted portfolios: dec1–dec10, plus the market: mkt). You can use your favorite portfolio optimizer for this exercise, either Excel or MATLAB.
- (a) Estimate the mean returns, standard deviations and correlations among these 11 portfolios. Plug the inputs into the mean variance optimizer. Report:
- The weights of the MVE portfolio among these 11 assets
  - The mean, standard deviation and Sharpe-Ratio of the MVE portfolio
- (b) Now, let's restrict ourselves to investing only among **two** risky portfolios: the market portfolio, and a portfolio that buys the lowest beta-decile stocks (dec1), shorts the stocks in the highest beta-decile (dec10) and uses the proceeds to buy the market.

Repeat the exercise above. Compare your Sharpe Ratio of this restricted strategy to the one obtained from investing in all 11 assets.

- (c) Now, using the portfolio you found above, run regressions of each of the 10 decile portfolios, and the market, on the *realized* returns on your portfolio. Make sure you run the regression in terms of *excess* returns over the risk-free rate.

Report the intercept and the slope, along with the standard errors. What does the pattern of the alphas (intercept) tell you?