
Asset Allocation II

FE-312 Investments



NORTHWESTERN
UNIVERSITY

In this lecture, we will cover two important topics that arise when we try to apply asset allocation theory in practice.

1. How to choose the inputs (or avoid garbage in–garbage out)
2. How to choose portfolios for the long-run (or for more than one ‘period’)

- ▶ MV Optimization gives always the right answer, conditional on
 - ▶ the inputs
 - ▶ the assumptions
- ▶ Figuring out which inputs to use is going to be the hardest problem.
- ▶ Some of these inputs (e.g. average returns) are crucial, yet they are very difficult to estimate.
- ▶ Common practice is to use recent history to estimate these moments.
 - ▶ When deciding on how much data to use, we are trading off estimation error versus the likelihood that the past is not a good indicator of the future.

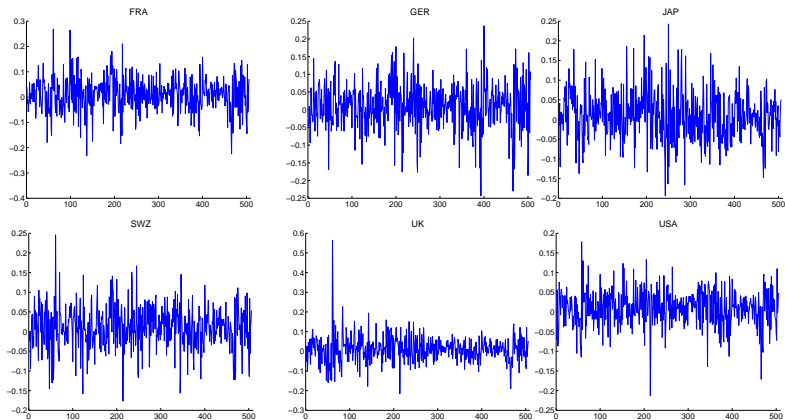
Example

- ▶ Consider the problem of investing in the following 6 international equity indices: **France, Germany, Japan, Switzerland, UK** and the **USA**.
- ▶ Download monthly historical index returns from MSCI.
- ▶ Compute historical mean and covariance matrix

$$\hat{\mu} = \begin{pmatrix} 1.02 \\ 1.00 \\ 0.94 \\ 1.06 \\ 1.03 \\ 0.88 \end{pmatrix} \quad \text{and} \quad \hat{\Sigma} = \begin{pmatrix} 44.0 & 30.9 & 17.9 & 23.8 & 26.5 & 17.0 \\ 30.9 & 41.8 & 16.3 & 24.6 & 22.1 & 16.0 \\ 17.9 & 16.3 & 39.0 & 15.2 & 16.4 & 10.0 \\ 23.8 & 24.6 & 15.2 & 28.8 & 21.3 & 13.7 \\ 26.5 & 22.1 & 16.4 & 21.3 & 41.8 & 17.0 \\ 17.0 & 16.0 & 10.0 & 13.7 & 17.0 & 20.5 \end{pmatrix}$$

- ▶ Feed the inputs into Markowitz, assume monthly risk-free rate of 5%/12

Return series



- We have ≈ 500 months of data, ranging from 1970 to 2012

Portfolio Optimization using Markowitz

Number of securities:

No	Name	Fraction	Expected Return	Standard Deviation
1	'FRA'	-2.03%	1.02%	6.6%
2	'GER'	-3.31%	1.00%	6.5%
3	'JAP'	16.64%	0.94%	6.2%
4	'SWZ'	49.78%	1.06%	5.4%
5	'UK'	7.16%	1.03%	6.5%
6	'USA'	31.76%	0.88%	4.5%

100.00%

Correlations	1	2
1	1.0	0.7
2	0.7	1.0
3	0.4	0.4
4	0.7	0.7
5	0.6	0.5
6	0.6	0.5

Corr OK? YES

Results:

Portfolio's Expected Return	0.0098
Portfolio's Standard Deviation	0.0434

Risk Free Rate

Risk Aversion Coefficient: A=

Slope of CAL

Weight on optimal risky portfolio: x^* =

- ▶ The MV-Optimizer returns the following set of weights

$$w^* = \begin{pmatrix} -0.020 \\ -0.033 \\ 0.166 \\ 0.498 \\ 0.071 \\ 0.318 \end{pmatrix}$$

- ▶ However, when we do MV-optimization, the computer thinks that the expected returns and covariance that we input is the truth (i.e. $\hat{\mu} = \mu$ and $\hat{\Sigma} = \Sigma$).
- ▶ Is there a way to quantify how far our estimated mean (or covariance) is from the truth?
- ▶ We can use statistics to build a confidence interval for the true mean return μ

Standard error of the mean

- ▶ A 95% confidence interval for the mean can be constructed as

$$\hat{\mu} \pm 1.96 \times s(\mu)$$

where $s(\mu) = \sigma/\sqrt{T}$, where T is the number of observations we used.

- ▶ In our case, the confidence interval is

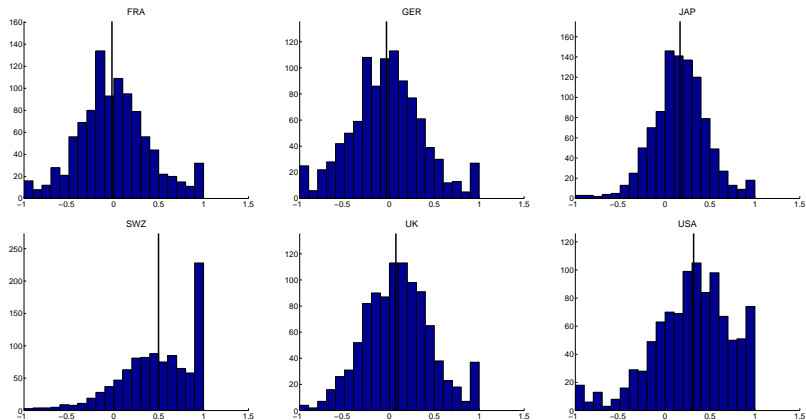
$$CI = \begin{pmatrix} 1.02 \\ 1.00 \\ 0.94 \\ 1.06 \\ 1.03 \\ 0.88 \end{pmatrix} \pm 1.96 \times \begin{pmatrix} 0.295 \\ 0.287 \\ 0.278 \\ 0.238 \\ 0.287 \\ 0.201 \end{pmatrix}$$

- ▶ Not very comforting. And we did not even take the estimation error in the covariance matrix into account!

Monte Carlo simulations help quantify the effect of measurement error on our decision problem

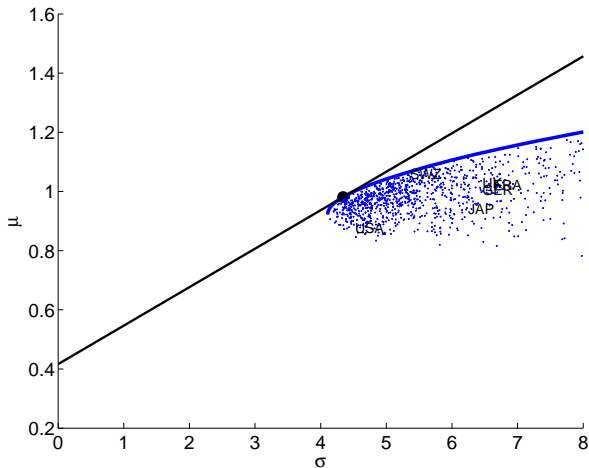
- ▶ Estimate the mean and covariance of asset returns using available data
- ▶ Compute the MV-efficient portfolio
- ▶ Suppose that this is now the truth. Let's get into the brain of an investor who imperfectly observes this 'truth' and makes investment decisions.
 1. Simulate one realization of asset returns using the historical mean and covariance matrix.
 2. Estimate the mean and covariance matrix for this simulation. The investor in this simulation believes these are the right inputs to Markowitz.
 3. Feed these inputs into Markowitz – estimate weights of the optimal portfolio from the perspective of this investor.
 4. Simulate the performance of this 'optimal' portfolio using the 'true' moments of returns.
- ▶ Repeat (1)-(4) a large number of times
- ▶ Plot distribution of optimal portfolio weights across simulations

Sensitivity Analysis



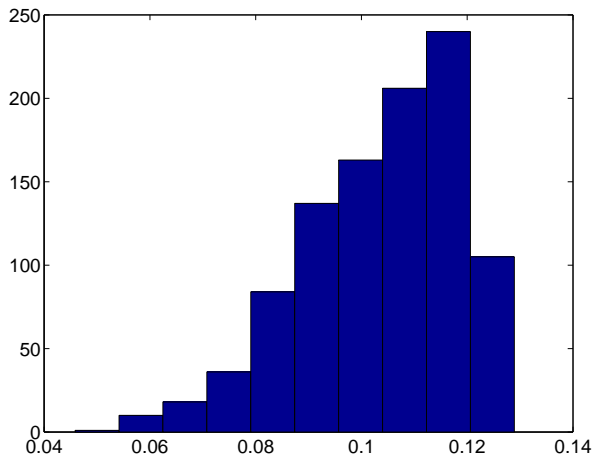
- ▶ Weights restricted to $[-1, 1]$. Black line represents optimal allocation using historical moments. Histogram displays variability of portfolio weights across simulations.

Sensitivity Analysis



- What we think is the MVE portfolio may be far from the truth.

Realized Sharpe Ratios across simulations



- Compare the realized Sharpe Ratio to the one implied by the MV optimizer (0.13)

There are (at least) two ways of getting around this problem:

1. Use more conservative estimates for expected returns. We could “shrink” the expected returns away from the historical return towards some prior belief about the mean.
2. Impose constraints on the portfolio weights to ensure that we do not take extreme positions.

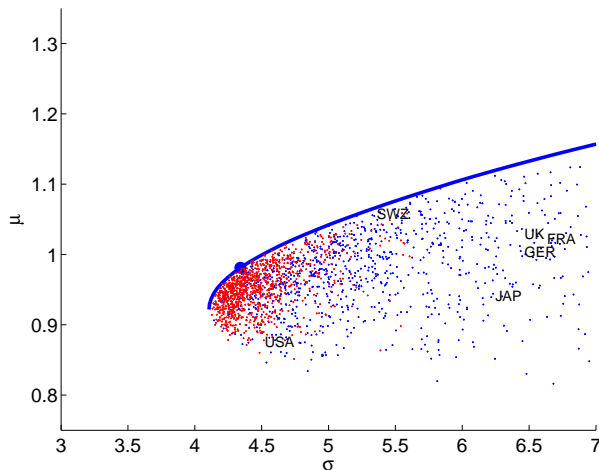
- ▶ The root cause of the problem is that expected returns are measured with considerable error. Since our estimate of the mean return will vary considerably across samples, so will the ‘optimal’ portfolio weights.
- ▶ One solution is to shrink our estimates towards a baseline. Suppose that we have this strong prior belief that:
 - ▶ The average return across all stock markets is the same
 - ▶ It is equal to 1% per year.
- ▶ In this case, we could shrink our estimates towards $\mu_0 = 1\%$ (our prior belief) by

$$\hat{\mu} = w \mu_0 + (1 - w) \tilde{\mu},$$

where $\tilde{\mu}$ would be the (random) estimate of mean returns in each simulation.

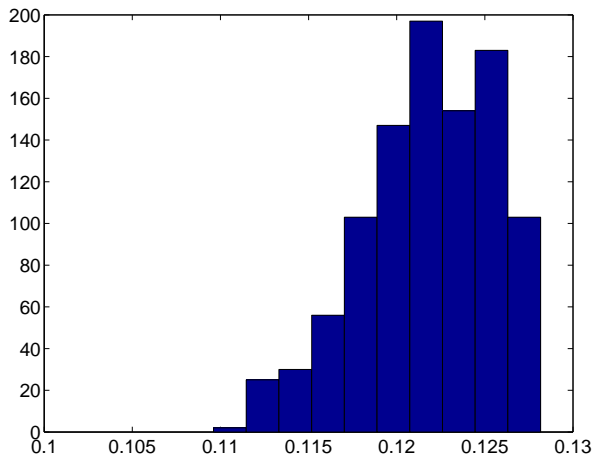
- ▶ Notice here that μ_0 need not correspond to the true mean return in each simulation. After all, we do not know what it is!
- ▶ We are free to pick w depending on how much faith in the data vs our prior belief we have. Let's start with $w = 1/2$.

Portfolio Optimization: Shrinkage Methods



- Even though we are using the ‘wrong’ expected return, now the performance of our optimizer is much better across simulations!

Realized Sharpe Ratios across simulations



- Compare the realized Sharpe Ratio to the one implied by the MV optimizer (0.13). Much better than before...

- ▶ In the previous example, we made two arbitrary choices: μ_0 and w . How should we choose these in practice?
- ▶ By definition, μ_0 contains information that is *in addition* to whatever historical data we used to estimate mean returns. This information can come from many sources, including,
 1. an economic (i.e. equilibrium) model, such as the CAPM
 2. our subjective beliefs (for instance, based on reading the news)
 3. ...
- ▶ The weight w that we attach to our prior belief μ_0 will depend on our confidence on these inputs, which in an ideal world is tied to a measure of statistical precision (e.g. the standard error of the mean estimate).

- Shrinkage methods can be viewed as a special case of Bayesian Analysis. Suppose you are trying to estimate the mean return, and you have two sources of information:
 1. Historical data: You estimate the historical mean to be $\tilde{\mu} = 1.2\%$ per month, with a standard error of $s = 0.2\%$.
That gives you a 90% confidence interval of $1.2\% \pm 1.65 \times 0.2\%$
 2. An analyst report that you downloaded from Morningstar. The analyst expects the index to rise by approximately $\hat{\mu}_0 = 2\%$ per month. To use this information however, you need an estimate of its precision (informativeness). This should be either on the analyst report, or based on your own beliefs. Suppose that you have a confidence interval, and that you back out a standard error of $v = 1\%$ — implying a range of estimates of $2\% \pm 1.65 \times 1\%$
- As a Bayesian, the ‘optimal’ way of combining this information is

$$\hat{\mu} = \mu_0 \frac{1/v^2}{1/v^2 + 1/s^2} + \tilde{\mu} \frac{1/s^2}{1/v^2 + 1/s^2} = 0.0123$$

- ▶ Consider the previous problem, but now introduce portfolio constraints.
 - ▶ no short-selling
 - ▶ no more than 30% allocation in one country
- ▶ We can introduce these constraints into Markowitz.
- ▶ Constraints clearly shrink the set of feasible portfolios.
 - ▶ Hopefully it buys us some robustness instead.

Portfolio Optimization with Constraints

Number of securities:

No	Name	Fraction	Expected Return	Standard Deviation
1	'FRA'	1.44%	1.02%	6.6%
2	'GER'	5.00%	1.00%	6.5%
3	'JAP'	20.63%	0.94%	6.2%
4	'SWZ'	30.00%	1.06%	5.4%
5	'UK'	12.93%	1.03%	6.5%
6	'USA'	30.00%	0.88%	4.5%

100.00%

Correlations	1	2
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Corr OK? YES

Results:

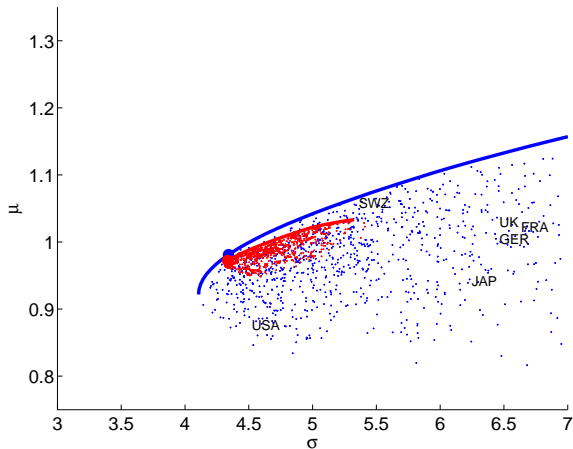
Portfolio's Expected Return	0.0097
Portfolio's Standard Deviation	0.0433

Risk Free Rate Risk Aversion Coefficient: A=

Slope of CAL Weight on optimal risky portfolio: x*=

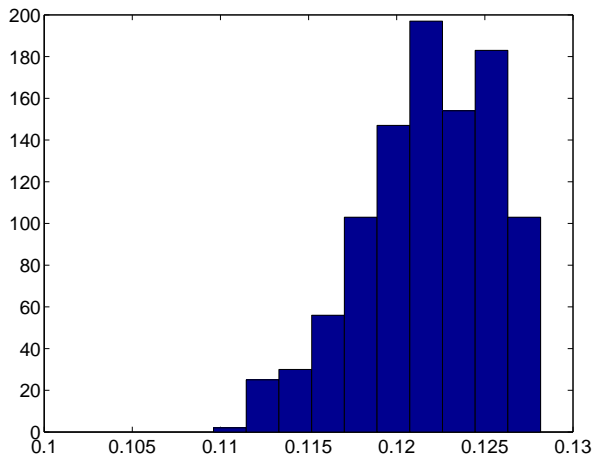
- ▶ Imposing short sales lowers our Sharpe ratio.
- ▶ But may lead to more robust estimates

Portfolio Optimization with Constraints



- Constrained frontier (red) lies inside the unconstrained frontier (blue)
- But the performance of the 'in-sample' MVE portfolio across simulations is closer to the frontier

Realized Sharpe Ratios across simulations



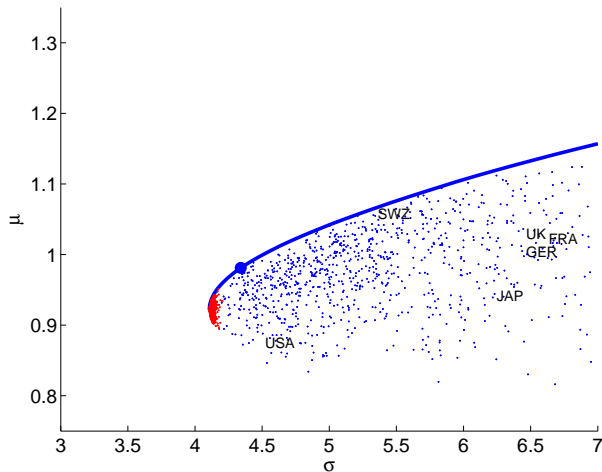
- Compare the realized Sharpe Ratio to the one implied by the MV optimizer (0.128)

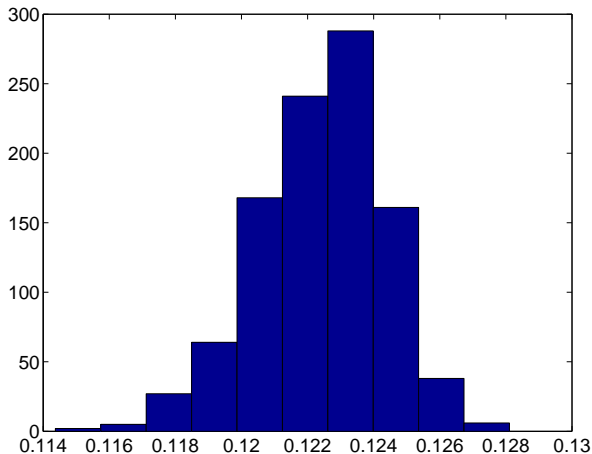
- ▶ Portfolio constraints are one way to prevent Markowitz from 'reading too much into historical data'.
- ▶ More generally, there may be other reasons why you would want to limit exposure to certain classes of assets.
 - i) Transaction costs.
 - ii) Liquidity.
 - iii) Non-tradeable income
- ▶ We should recognize that often some of the assumptions behind MV-Optimization are violated. Imposing portfolio constraints can limit potential problems.

The following alternatives do not involve estimating means, hence they can be more robust:

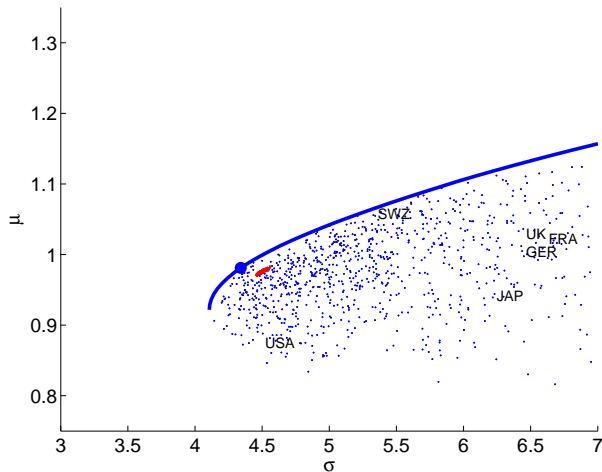
1. Minimum Variance Portfolio
2. Risk parity
3. Equal weights ($1/N$)

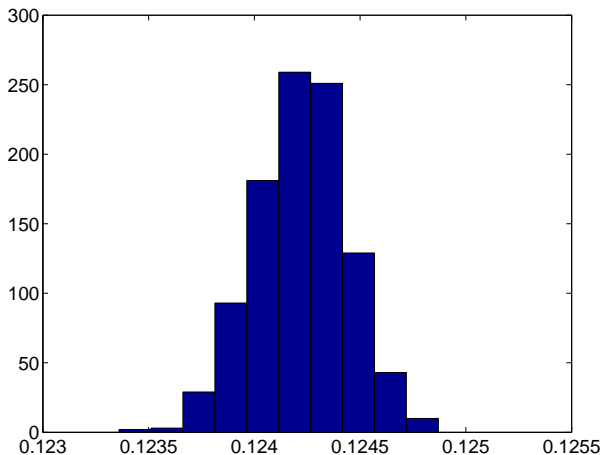
- ▶ Always take the farthest-left point on the M-SD frontier
- ▶ Optimal when all expected returns are the same
- ▶ That is, use it when you don't think you can forecast returns
- ▶ Still requires knowing all covariances, but they are easier to estimate than mean returns





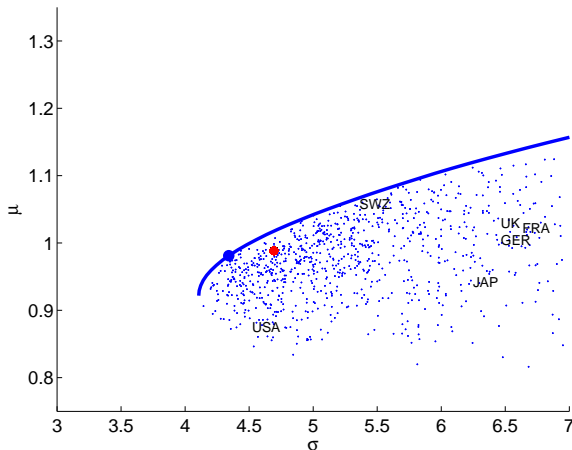
- ▶ Risk parity
 - ▶ Each asset gets weight proportional to $1/\sigma_i^2$
 - ▶ Contribution to risk from each asset is equalized
 - ▶ Would be the optimal only if covariances are zero and expected returns are all the same
 - ▶ Use it when you can't forecast returns or correlations
- ▶ Pros: avoids estimation of means
- ▶ Cons: ignores asset correlations!





- ▶ Another useful point of comparison is a ‘naive’ diversification strategy that assigns a weight of $1/N$ in each asset.
- ▶ This strategy is easy to implement
 - ▶ We do not need *any inputs* at all!
- ▶ How badly does it do across our simulations?

A 'naive' diversification strategy



- Since we fixed the weights, the performance of our strategy does not vary in our (long) sample
- The 'naive' diversification strategy has a Sharpe Ratio of 0.122

A 'naive' diversification strategy

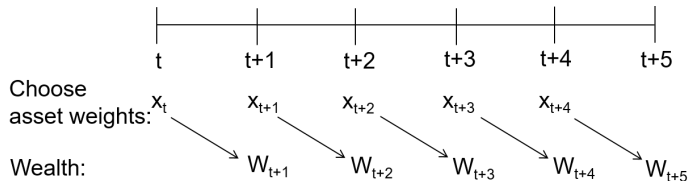
► Lessons:

- You do not need to fully optimize your weights to take advantage of diversification. Sometimes, a 'naive' strategy will do just fine.
- Will this always be the case? Not necessarily. Here, all the different indices we considered were relatively 'similar'. They had:
 - comparable risks (variances/covariances)
 - similar (i.e. statistically indistinguishable) average rates of return
- At the end of the day, we did not have a strong a priori view as to whether these assets were economically different. More importantly, the data did not make a compelling case that there were meaningful differences across these securities. Hence, in this case, the naive $1/N$ strategy did pretty ok.
- Sometimes however, the assets that you will be choosing over may be quite different: stocks vs bonds vs derivatives. In this case, a naive diversification strategy may end up doing rather poorly.

- ▶ The ideas behind mean-variance analysis are sound
 - ▶ Maximizing return and penalizing variance makes sense
 - ▶ Conditional on the inputs, we find the optimum
- ▶ You just need to be *very* careful about the inputs
- ▶ **When somebody shows you a ‘MV-optimal’ portfolio, you should have many questions to ask:**
 - ▶ Where did the inputs come from? How long was the sample?
 - ▶ How robust are the results to alternative assumptions?
 - ▶ Can we use a simple method that is easier to estimate?
- ▶ Same concerns about MV – past history doesn’t predict future – should also make you question risk parity, minimum-variance, etc.

- ▶ How does the dynamic context differ from the static problem? That is, does the long horizon lead to different portfolio weights than a short horizon?
- ▶ Notation:
 - ▶ x_t : portfolio weight in risky assets at time t (could be a vector)
 - ▶ $W_{t+1}(x_t)$ = wealth at time $t+1$, which is a function of the time- t portfolio weight. For example, in the case of one risky asset \tilde{r} and risk-free rate r^f , we have that

$$W_{t+1}(x_t) = W_t (x_t r_{t+1} + (1 - x_t) r^f)$$



- Investor maximizes the expected utility of end-of-period wealth W_T by choosing a dynamic series of portfolio weights. This is a dynamic trading strategy, which could change due to pre-determined variables (like changing investor constraints or liabilities) or to changing investment opportunity sets (like different returns in booms vs busts).

- ▶ Mathematically, the problem can be represented as

$$\max_{x_t} E[u(W_T)]$$

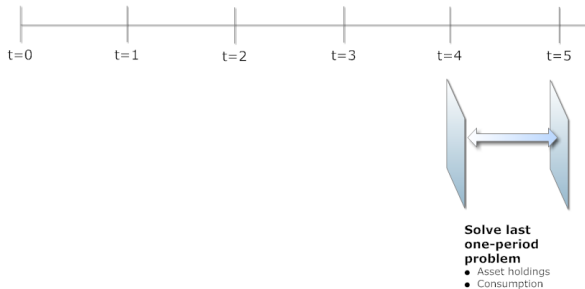
subject to

$$W_{t+1} = W_t(1 + r_{p,t+1}(x_t))$$

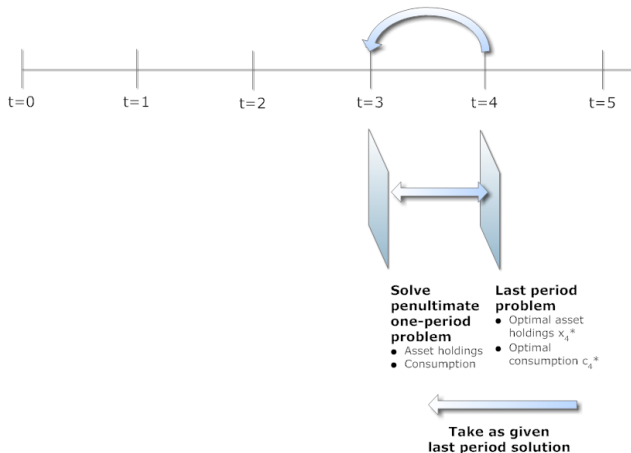
where the notation emphasizes that the return on the portfolio at the end of the period $t + 1$ is a function of the portfolio weight chosen at the beginning of the period t . We choose the set of portfolio weights x_t from t to $T - 1$. This is an optimal control problem. It is solved by *dynamic programming*.

- ▶ The optimal dynamic trading strategy is completely known at the start, even though it changes dynamically through time:
 - ▶ Utility could change [and the strategy optimally responds]
 - ▶ Returns could be predictable [and the strategy optimally responds]

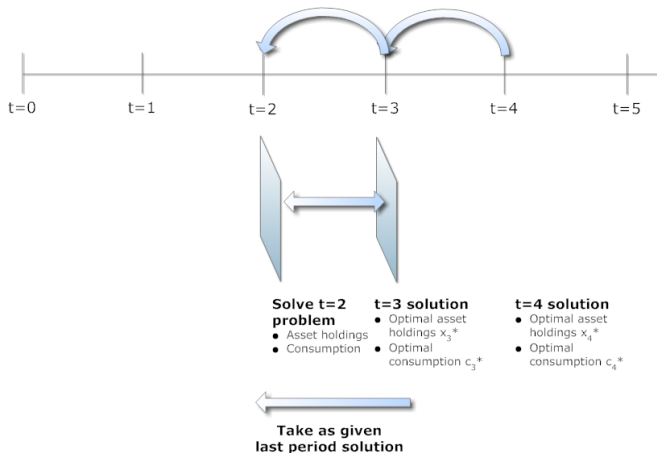
Investing for the Long-run



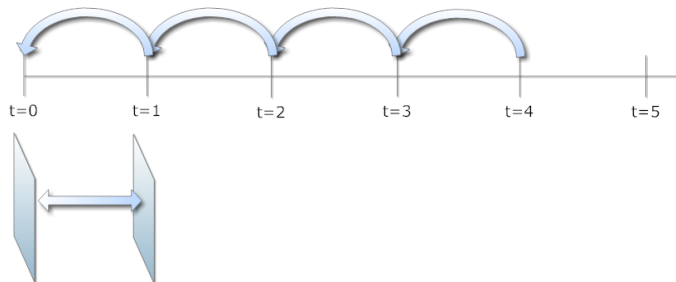
Investing for the Long-run



Investing for the Long-run



Investing for the Long-run



Solve $t=0$ problem

- Asset holdings
- Consumption

$t=1$ solution

- Optimal asset holdings x_1^*
- Optimal consumption c_1^*

$t=2$ solution

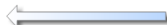
- Optimal asset holdings x_2^*
- Optimal consumption c_2^*

$t=3$ solution

- Optimal asset holdings x_3^*
- Optimal consumption c_3^*

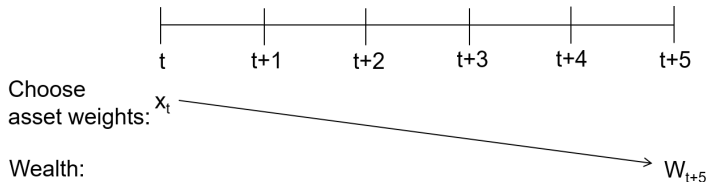
$t=4$ solution

- Optimal asset holdings x_4^*
- Optimal consumption c_4^*



Take as given
last period solution

- Contrast this to the problem of a buy-and-hold investor



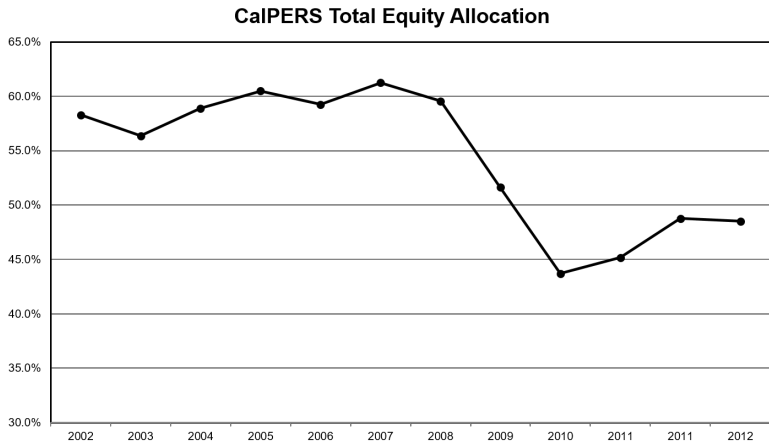
- In a one-period problem, the investor chooses portfolio weights only once at the beginning of the period, which results in end-of-period wealth. This is also called a buy-and-hold problem.
- Optimal dynamic strategies must always do at least as well as buy-and-hold portfolios. Why?

- ▶ Suppose that stock returns are not predictable and volatility is constant. Also, assume that the risk-free rate is also constant.
- ▶ In this case, it turns out that the dynamic portfolio choice problem becomes a series of identical *static* problems
- ▶ That is, the investor acts *myopically*. The future is always the same, so at each date she solves the same problem.
- ▶ Is this different from a buy-and-hold problem over the entire period?
- ▶ Absolutely! Although the dynamic problem is a series of one-period problems, it involves rebalancing back to the same portfolio weight.

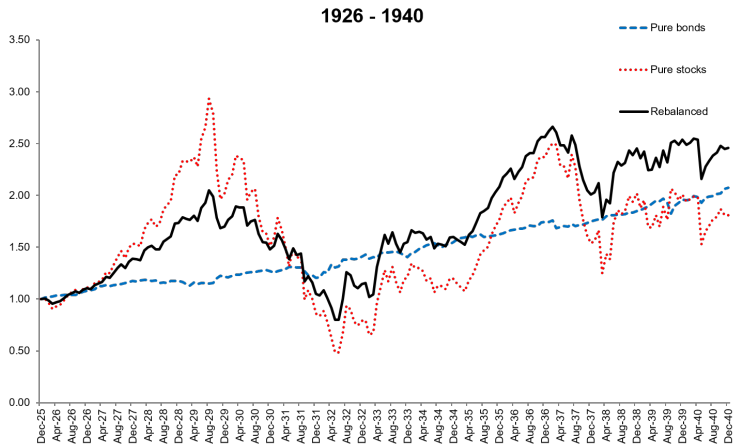
- ▶ Rebalancing is a counter-cyclical (relative to wealth) strategy
 - ▶ If equity has increased over the last period, the equity proportion is too high relative to optimal → sell equity
 - ▶ If equity has decreased over the last period, the equity proportion is too low relative to optimal → buy equity
 - ▶ This counter-cyclicality also has advantages if returns exhibit mean reversion, but mean reversion is not necessary to capture a rebalancing premium
- ▶ Buying low and selling high is how any investor, long or short horizon, makes money!
- ▶ Most investors are pro-cyclical, and chase returns. They invest precisely at the wrong time: when prices are high, investors are drawn by **past** high returns, but **future** expected returns are not necessarily high (in fact, they are typically lower).

- ▶ Long-term investing fallacies
 - ▶ A long-run investor never buys and holds
- ▶ Long-term investing is first and foremost about being a good short-term investor
- ▶ Additional benefits of rebalancing
 - ▶ Ride out short-term fluctuations in prices, and profit from periods of elevated risk aversions and mispricing
- ▶ Acquire distressed assets when investors with over-stretched risk capacity or constraints have to sell

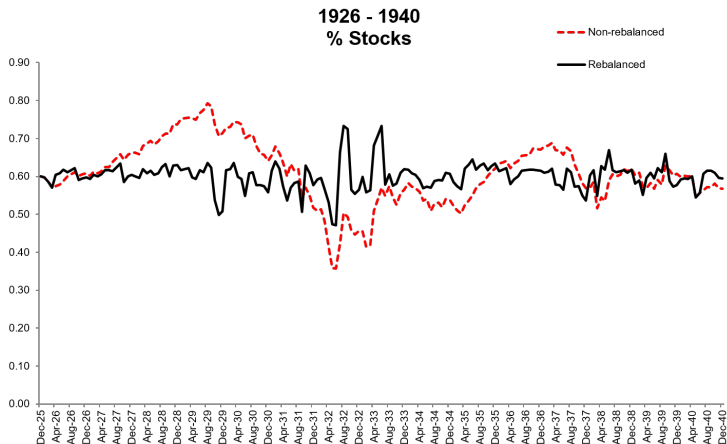
Investment Mistakes at CalPERS



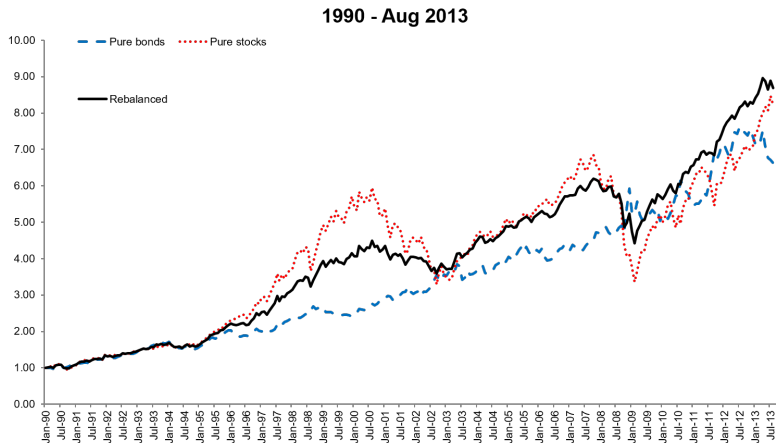
Rebalancing: Great Depression



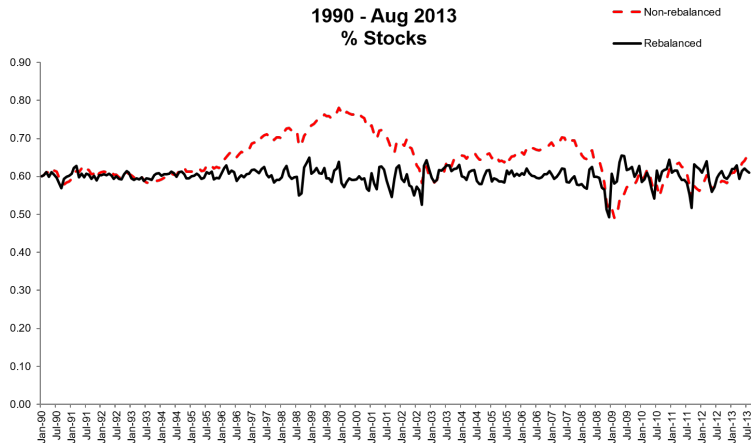
Rebalancing: Great Depression



Rebalancing: 1990–2014



Rebalancing: 1990–2014



- ▶ The previous discussion focused on the case where means and volatilities were constant—the world was the same every period
- ▶ If returns are predictable (say, high volatility this period is followed by high volatility next period), then the static portfolio weight changes every period.
- ▶ Is this a relevant case? Here is a simple test to see whether returns are predictable:
 - ▶ finance textbooks mention the square-root of time rule:
$$\sigma(\text{K-period return}) = \sqrt{K} \times \sigma(\text{1-period return})$$
 - ▶ How well does this hold in practice?

Horizon	$var(R_H)$	$\frac{var(R_H)}{var(R_1)}$
1-month	0.003	
1-year	0.040	13.54
2-year	0.086	28.82
3-year	0.117	39.34
4-year	0.147	49.17
5-year	0.165	55.47
10-year	0.240	80.63

- ▶ Table shows variance of H-period returns on market portfolio since 1927
- ▶ What is going on here? Does the finance textbook give the right advice?
 - ▶ Which implicit assumption typically made is wrong?
- ▶ In general
$$var(R_1 + R_2) = var(R_1) + var(R_2) + cov(R_1, R_2)$$
- ▶ Is the last term zero? Can we sign it?

- ▶ With a long horizon, the optimal portfolio strategy over risky assets can be written as

$$\text{Optimal Weight}(t) = \text{Myopic Weight}(t) + \text{Hedging Demand}(t)$$

which changes over time as the means and standard deviations of returns change. This was originally formulated by Samuelson and Merton (Nobel prizes in 1970 and 1997).

- ▶ Myopic weight = one-period solution
- ▶ Hedging Demand weight = portfolio through which a long-horizon investor takes advantage of predictable returns. It is labeled hedging demand because it “hedges” against changes in the investment opportunity set.
- ▶ For example, you may, all else equal, want to have more money when expected returns are high μ because the return to investment is higher. In general, solving this problem is quite complicated.

Investing over the Life Cycle: Common Advice

- ▶ The ubiquitous advice from financial professionals is to hold fewer stocks the older you are. A popular rule is the ‘100 minus your age’ rule to be held in equities.
- ▶ Example Vanguard Target Retirement Funds (eg 2050 Fund)
[“Life-cycle” funds]

	25 yrs before Retirement	1 year before Retirement	In Retirement
Stocks	90%	50%	30%
Bonds	10%	50%	65%
Cash	0%	0%	5%

Target Date Funds

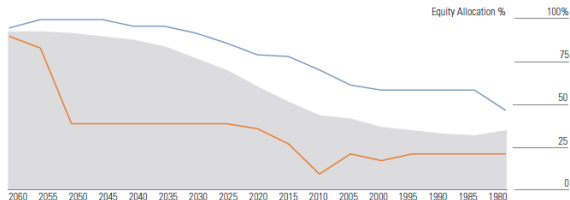
- ▶ Target date funds embody the idea of decreasing risky asset positions as you age. The deterministic mix as a function of years to maturity is called the glide path.
- ▶ Target date funds now constitute \$350 billion AUM

Exhibit 6

Industry Average
Target-Date Glide
Path

Data through 12/31/2012.
Source: Morningstar, Inc.

— 2012 Maximum
■ 2012 Average
— 2012 Minimum



- ▶ Labor is a non-traded asset and a form of wealth. Denote the present value of wages as H (standing for human capital). You are endowed with H . Financial wealth is W . Total wealth is $W + H$.
- ▶ If wages are risk free [like tenured Professors and Federal judges?], then having the wage is equivalent to holding risk-free assets. This counts in the total desired wealth allocation to risk-free assets. Consider mean-variance utility with a single risky asset and a risk-free asset. The optimal holding to the risky asset is:

$$w^* = \frac{1}{A} \frac{\mu - r_f}{\sigma^2} \left(\frac{W + H}{W} \right)$$

this formula adjusts for the fact that you should be optimizing over your *total* wealth, not just your financial wealth.

- ▶ An investor endowed with riskless human capital should tilt her portfolio toward stocks relative to an investor with only financial wealth

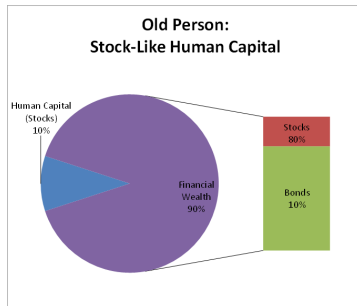
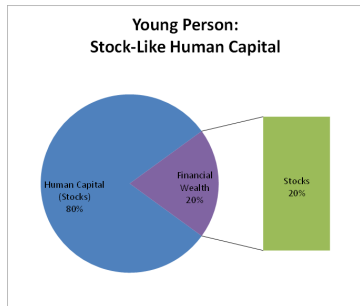
- ▶ Human capital is actually rather risky (unless you're a tenured prof or Federal judge)
- ▶ Palacios-Huerta (2003) finds white males with 1-5 years of experience and a college degree have annual human capital returns of 14.2%, with a standard deviation of human capital returns of 11.3%.
- ▶ Large disparity in human capital across professions. Christiansen, Joensen and Nielsen (2007) estimate human capital returns:

	Mean	Stddev
Basic School (9 years)	5%	17%
BA Humanities	-15%	28%
BA Social Sciences	19%	21%
MSc Economics	31%	15%
MSc Engineering	25%	17%
MA Law	25%	20%

- ▶ Suppose labor income is stochastic and correlated with equities (e.g. you work in asset management!). A positive correlation between labor income and equity returns reduces the optimal allocation to risky assets. Labor income then is equivalent to already holding some equities.
- ▶ Optimal asset allocation tilts the financial portfolio away from equities, and undoes the equity risk added by labor income
- ▶ Corollaries
 - ▶ Investors with income highly tied to the stock market should hold bonds (or other very low risk assets) in their financial portfolios
 - ▶ An investor should not hold a market weight of her employer's stock. In fact, she would want to underweight it compared to the market index weight (or even short it).
 - ▶ Your financial portfolio should tilt away from US stocks (if your income comes from a US source)

- ▶ Life-cycle models view asset allocation in the broader context of an allocation of total wealth (financial and human capital). When young, the value of human capital is high and then it decreases
- ▶ If labor income is like a bond (risk free) or uncorrelated with stocks, then investors will shift the risk composition (the total risk budget allowed by their risk aversion) toward bonds and away from stocks as they age. The financial planner may be correct!
- ▶ If labor income is risky — and positively correlated with equities—then human capital substitutes for equities and the investor compensates by increasing holdings of risk-free bonds. In this case, the investor does the opposite of what a financial planner recommends and shifts more financial wealth out of stocks into bonds as time passes.

Human capital and life-cycle investing



- ▶ All else equal, the life-cycle theory predicts
 - ▶ Bond-like labor income → Reduce equities as you age
 - ▶ Stock-like labor income → Increase equities as you age

What do People Actually Hold?

- ▶ A large fraction of households do not hold any equity (non-participation)
- ▶ Older working households hold higher fractions of equity than younger working households (household equity shares increase with age), and even at retirement the fraction of equity held is very high
- ▶ Thus, people seem to follow life-cycle model advice (if their labor income is bond-like)
- ▶ Rich households hold more equity than poor households

What do People Actually Hold?

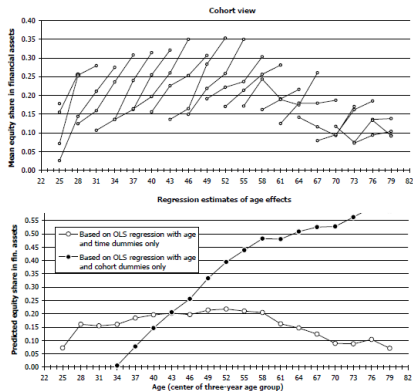
- ▶ Ameriks and Zeldes (2004) examine data from the Surveys of Consumer Finances and TIAA-CREF
- ▶ There is significant non-participation in equities. Only 50% of consumers own stocks, taking into account mutual funds and pensions.
Table 6:

Percent of U.S. Households Owning Stock, 1962-2001

Households owning stock through:	1962	1983	1989	1992	1995	1998	2001
(1) Direct ownership of publicly traded stock*	14.1%	19.1%	16.9%	17.0%	15.2%	19.2%	21.3%
(2) Direct + mutual funds	19.0	--	20.0	21.1	22.3	27.6	30.0
Upper bound, based on ownership of any taxable mutual funds	19.0	21.4	20.4	21.7	22.8	28.1	30.3
(3) Direct + mutual funds + trusts	19.1	--	21.0	22.1	23.2	27.9	30.4
Upper bound, based on ownership of any trust assets	19.9	22.7	22.2	23.8	24.8	28.6	30.9
(4) Direct + mutual funds + trusts + defined contribution (DC) pensions**	23.9*	--	30.2	31.8	37.7	44.8	48.3
Upper bound, based on ownership of any DC plan assets***	29.6	37.3	39.3	41.5	45.9	49.7	51.7
(5) Direct + mutual funds + trusts + DC pensions + IRAs (Comprehensive measure)	23.9*	--	33.4	37.7	41.3	49.4	52.2
Upper bound based on ownership of any IRA assets	29.6**	43.7	47.5	49.6	54.0	57.0	59.7

What do People Actually Hold?

Ameriks and Zeldes (2004) find no evidence that generally individuals tend to increase their equity portfolio shares as they age. There is some tendency for individuals to shift completely out of stocks when they annuitize or withdraw.



The figure shows equity shares in financial assets (Figure 7). The cohorts are 1989 1992 1995 1998. The regression estimates with cohort effects show a pronounced increase over time.

- ▶ Garbage in \rightarrow garbage out
- ▶ Long-term investors are not buy and hold
- ▶ Human capital is an important component of your wealth.