# Lecture 7: Multifactor Models – Examples

Investments



## **Factor Tilting**

- Factor Tilting is forming a portfolio to take advantage of a forecast of a factor,  $\tilde{f}$ .
- We assume that in the return generating process equation for our portfolio:

$$\tilde{r}_{p,t} = E[\tilde{r}_{p,t}] + b_{p,1}\tilde{f}_{1,t} + \dots + b_{p,n}\tilde{f}_{n,t} + \tilde{e}_{p,t}$$

- $\hookrightarrow E[ ilde{r}_{i,t}]$  represents the market's expectation
- $\hookrightarrow E[\tilde{f}_{1,t}] = 0$  for the market
- $\hookrightarrow$  But, suppose we have superior information  $E[\tilde{f}_{1,t}] \neq 0$ , i.e. we can forecast f better than the market.



## **Factor Tilting**

- The advantage of the multi-factor structure here is that we can express specific macro views, rather than expressing a view or forming a forecast about the whole market portfolio.
- Assuming we have this ability, we can earn superior profits by varying the factor betas/loadings of our portfolio.
  - $\hookrightarrow$  We want to increase the loading when we think that the factor is likely to have a positive realization
  - $\hookrightarrow$  We want to decrease the loading when we think that the factor is likely to be negative.



Your analyst gives you the following information on three securities that are correctly priced according to a 2 factor APT model

$$\tilde{r}_{A} = 0.12 + 1 \cdot \tilde{f}_{1} + 1 \cdot \tilde{f}_{2} + \tilde{e}_{A}$$

$$\tilde{r}_{B} = 0.12 + 1 \cdot \tilde{f}_{1} + 2 \cdot \tilde{f}_{2} + \tilde{e}_{B}$$

$$\tilde{r}_{C} = 0.12 + 3 \cdot \tilde{f}_{1} + 2 \cdot \tilde{f}_{2} + \tilde{e}_{C}$$

- There are two common factors:
  - $\hookrightarrow$  Factor 1 is a foreign income factor.
  - → Factor 2 is a U.S. earnings price ratio factor.
- The way the model is constructed these factors are uncorrelated.
  - $\hookrightarrow$  Here, the E[r]'s of 12% are what the market expects.
  - $\hookrightarrow$  What are the factor risk premia (the  $\lambda$ s)?



- You believe very strongly that Japan will finally come out of its recession in the next few months and therefore exports of U.S. produced goods will rise more than the market expects. Moreover, you believe the earnings price ratio factor will not change at all in this time period, consistent with what analysts expect.
- Using the above three securities, construct ANY portfolio that takes advantage of all of these facts.
- What are
  - the composition of the portfolio
  - ii) the b's of the portfolio
  - iii) the expected return on the portfolio.



- We want to construct a portfolio with a lot of factor 1 exposure and no factor 2 exposure. Therefore, lets assume we want a loading of 10 on factor 1 and 0 on factor 2.
- Therefore, solve the three equations:

$$1 \cdot w_A + 1 \cdot w_B + 3 \cdot w_C = 10$$
  
$$1 \cdot w_A + 2 \cdot w_B + 2 \cdot w_C = 0$$
  
$$1 \cdot w_A + 1 \cdot w_B + 1 \cdot w_C = 1$$

(The last equation is the usual restricution that the sum over the weights is one.) We can also write these equations as:

$$1 \cdot w_A + 1 \cdot w_B + 3 \cdot (1 - w_A - w_B) = 10$$
  
$$1 \cdot w_A + 2 \cdot w_B + 2 \cdot (1 - w_A - w_B) = 0$$



- The solution to this system of equations are the weights  $w_A = 2$ ,  $w_B = -5.5$ , and  $w_C = 4.5$ . Then this portfolio will have a loading of 10 on factor 1 and a loading of 0 on factor 2.
  - → We have constructed a factor-mimicking portfolio
- Assuming that you believe that the foreign income factor will rise by 2%, the expected return on this portfolio is:

$$2 \cdot 0.12 - 5.5 \cdot 0.12 + 4.5 \cdot 0.12 + 10 \cdot 0.02 = 0.32$$

- Is this the highest Sharpe-Ratio portfolio possible?
- In practice, we have more than three securities available to construct factor-mimicking portfolios. Is there an optimal way to construct factor-mimicking portfolios?



## Risk Management

- Factor models are also a convenient way to manage portfolio risks.
- As a portfolio manager you can control specific risk exposures, customizing the portfolio to the needs of your clients.
- In order to hedge out your exposure to a factor:
  - 1. Estimate the loading of each asset with respect to the factor,  $b_{i,k}$ .
  - 2. Include an additional constraint: The factor loading of your portfolio should equal zero:  $b_{p,k} = 0$ .



You believe the following four-factor model holds:

$$R_{it} = a_i + b_{M,i} \tilde{f}_{M,t} + b_{TS,i} \tilde{f}_{TS,t} + b_{CS,i} \tilde{f}_{CS,t} + b_{O,i} \tilde{f}_{O,t} + \varepsilon_{i,t}$$

#### where

- $\hookrightarrow \tilde{f}_{M,t}$  is the surprise return on the market portfolio.
- $\hookrightarrow \tilde{f}_{TS,t}$  is the surprise change in the slope of the yield curve.
- $\hookrightarrow \tilde{f}_{CS,t}$  is the surprise change in the credit spreads.
- $\hookrightarrow \tilde{f}_{O,t}$  is the surprise change in oil prices.
- The covariance matrix of the factors is

Covariance	MKT	TS	YS	OIL
MKT	2.100%			
TS		0.090%		
YS			0.030%	
OIL				0.760%



Suppose you are choosing between 6 portfolios. You have estimated the following for each portfolio:

No	E (R)	$\sigma(R)$	Factor Loadings			
			market	TS	YS	OIL
SG	14.7%	0.306	1.439	5.487	0.279	0.043
SC	10.7%	0.208	0.872	3.419	-0.018	-0.019
SV	10.2%	0.210	0.786	3.925	1.250	-0.036
LG	11.2%	0.197	1.022	-0.602	2.617	-0.010
LC	9.7%	0.174	0.749	-0.221	-2.134	-0.062
LV	9.3%	0.169	0.672	-0.795	-1.202	-0.113

- What is the optimal portfolio that:
  - → Achieves an expected return of 12% and has zero exposure to Oil risk?
  - $\hookrightarrow$  Has the maximum Sharpe ratio ( $r_f=5\%$ ) and has loading of 1 with the market portfolio?



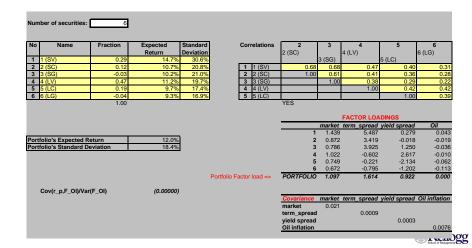
First we calculate the correlation matrix implied by the 4-factor model:

	1	2	3	4	5	6
1	1	0.6821	0.6761	0.4662	0.4005	0.3132
2	0.6821	1	0.609	0.4107	0.3605	0.2804
3	0.6761	0.609	1	0.3804	0.2946	0.2203
4	0.4662	0.4107	0.3804	1	0.4221	0.4172
5	0.4005	0.3605	0.2946	0.4221	1	0.3937
6	0.3132	0.2804	0.2203	0.4172	0.3937	1

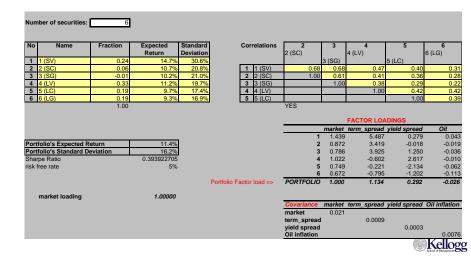
■ Then we plug in the numbers into Markowitz, with some additional constraints on the loadings



Optimal Portfolio that achieves 12% return and has zero oil exposure.



Optimal Portfolio (i.e. max Sharpe Ratio) with market beta of 1.



## Risk Management using Factor-Mimicking Portfolios

- An alternative approach would have been to hedge the risk out by trading in factor-mimicking portfolios.
- The approach would be:
  - 1. Find the optimal portfolio, without imposing any constraints.
  - 2. Find the loading of your portfolio with respect to the factor you wish to hedge  $(b_{p,f})$ .
  - 3. Sell a fraction  $b_{p,f}$  of the portfolio that mimics factor f. Since it is a zero-investment portfolio, no further action is necessary.
- The end result will be the same with either approach if the factor-mimicking portfolio is included in the set of assets you are optimizing over.

