Investments - Winter 2014 Final Exam

NAME: ______________________________ SECTION: ______________

1. Please do not open this exam until directed to do so.
2. This exam is 3 hours long.
3. Please write your name and section number on the front of this exam, and on any examination books you use.
4. Please show all work required to obtain each answer. Answers without justification will receive no credit.
5. State clearly any assumptions you are making.
6. This is a closed book exam. No books or notes are permitted. Calculators are permitted. Laptops are permitted but you are only allowed to use Excel and only a blank worksheet. You are not allowed to use other spreadsheets with pre-entered formulas.
7. Brevity is strongly encouraged on all questions.
8. The exam is worth 160 points.
9. Relax, and good luck!

Hints:

1. *Think through problems before you start working. Draw pictures.*
2. *If you get stuck on part of a problem, go on to the next part. You may need to use answers from earlier parts of the question to calculate answers to the later parts. If you weren't able to solve the earlier part, assume something and move on.*
3. *Remember, setting up the problem correctly will get you most of the points.*

Question 1 (50 points)

Assess the validity of the following statements (True, False or Uncertain) and explain your answers.

1. (10 points) Irrespective of whether the CAPM holds or not, all assets have zero alpha with respect to the mean variance efficient portfolio.

2. (10 points) If the CAPM holds, every security should receive a strictly non-negative weight in the mean-variance efficient portfolio.

3. (10 points) The momentum trading strategy represents an opportunity to make risk-less profits.

4. (10 points) In the APT, absence of arbitrage implies that all factors have a positive risk premium.

5. The weight individual securities receive in the mean-variance efficient portfolio is directly proportional to their Sharpe Ratio.

Question 2 (50 points)

Suppose that the Fama-French version of the APT is a good model of risk and return. The Fama-French model specifies that the expected return for each security i is given by

$$E[R_i]-r_f = \beta_{mkt} \underbrace{(E(R_m) - r_f)}_{\lambda_{mkt}} + \beta_{smb} \underbrace{(E(R_S) - E(R_L))}_{\lambda_{smb}} + \beta_{hml} \underbrace{(E(R_H) - E(R_L))}_{\lambda_{hml}}$$

Over the last 80 years, the SP&500 (R_M) has delivered an average excess return of 7.5% per year over the risk-free asset (r_f). The volatility of the stock market during this period was 19%. During the same period, small stocks (R_S) have outperformed large stocks (R_L) by 2%, and value stocks (R_V) have outperformed growth stocks (R_G) by 6%. The volatility of the small-minus-big strategy is 12% and the volatility of the value-minus-growth strategy is 15% per year. The risk-free rate is constant and equal to 2%.

These three trading strategies are all zero-investment strategies – that is, they are long-short positions – and they are constructed in such a way as to be uncorrelated with each other, that is $R_M - r_f$, $R_S - R_B$, and $R_H - R_G$ are uncorrelated with each other.

1. (10 points) Construct an estimate of the expected excess return on the three factors (λ_i), along with a 95% confidence interval. What additional assumptions do you need to make for your estimate to be valid?

2. (10 points) A private equity fund is considering its investment in a small startup. The startup is expected to generate 10 million of earnings in the first year, which are expected to grow at 5% per year indefinitely. The startup will pay all it's earnings as dividends. As a reminder, the Gordon-Growth formula is

$$P = \frac{D}{r - g},$$

where r is the discount rate and g is the dividend growth rate.

The fund manager wants to use the CAPM to obtain the appropriate discount rate to be used in valuing the startup. The fund has a market beta of 1, an SMB beta of 0.5 and a HML beta of -0.5. Compute the dollar difference in valuations between the CAPM and the APT.

3. (10 points) You are managing a successful mutual fund that caters to retail investors. Suppose that you think that the out-performance of small stocks relative to large stocks is a historical fluke that is unlikely to be repeated going forward. However, you think that your other estimates from part 1 are a valid estimate of the future expected returns and standard deviations. Compute the mean-variance efficient portfolio of all risky assets.

4. (10 points) You are considering offering your investors the portfolio you found in part 3. Suppose that their only other alternative is to hold a market index. They can combine these two portfolios with the risk free asset. What is the maximum fee you can charge, as a fraction of the money invested in your fund?

5. (10 points) Suppose your investors have access to your mutual fund and a risk-free asset. If they have a risk-aversion coefficient of 5, what fraction of their wealth will they allocate to your fund, assuming you charge them a zero fee? How much will they allocate if you charge them the maximum possible fee?

Question 3 (60 points)

You are a managing a fixed income portfolio for your clients. You believe that the following factor model holds for bond returns:

$$\begin{aligned} r_t^1 &= 1.1\% + 1\,\tilde{f}_{L,t} - 1\,\tilde{f}_{S,t} \\ r_t^5 &= 2.0\% + 5\,\tilde{f}_{L,t} \\ r_t^{10} &= 4.0\% + 10\,\tilde{f}_{L,t} + 10\,\tilde{f}_{S,t}, \end{aligned}$$

where r_t^n represents the return on a zero-coupon bond with maturity n, and $\tilde{f}_{i,t}$ represents factor i's surprise realizations. The two term structure factors, (f_L) and (f_S) are uncorrelated, i.e. $cov(f_L, f_S) = 0$, and $var(f_L) = 0.01$ and $var(f_S) = 0.005$.

Based on how bond returns load on these factors, we can characterize (f_L) and (f_S) as a 'level' and 'slope' factor respectively.

1. (10 points) Construct two portfolios that exactly mimic the two term structure factors, f_L and f_S. What are the weights of these portfolios on the three bonds?

2. (10 points) Construct a portfolio that has zero factor L and factor S risk.

3. (10 points) Find the risk free rate and the factor risk premia λ_S and λ_L implied by the absence of arbitrage opportunities.

4. (15 points) Find the portfolio that has the maximum Sharpe Ratio. What are the loadings of this portfolio to the two factors?

5. Assume now that you believe that the Federal Reserve is likely to raise short term interest rates by 100bps but because this move signals a tougher inflation regime, long term rates will actually fall by 100 bps. Thus you believe that the yield curve is likely to flatten with the level being unaffected. You believe, like everyone else, $E[\tilde{f}_L] = 0$ over the next year, but unlike the rest of the market, you think that $E[\tilde{f}_S] = 2\%$. Your share the same beliefs about variances as the rest of the market.

   (a) (5 points) Construct a portfolio that takes advantage of this view but has no L factor risk.

   (b) (5 points) What is the Sharpe Ratio of this portfolio according to the market?

   (c) (5 points) What is the Sharpe Ratio on this portfolio according to you?

Finance 460 Final Equation Sheet

- Covariance Relations:

$$cov(a\tilde{x}, b\tilde{y}) = a \cdot b \cdot cov(\tilde{x}\tilde{y}) \qquad cov(\tilde{x}, \tilde{y} + \tilde{z}) = cov(\tilde{x}, \tilde{y}) + cov(\tilde{x}, \tilde{z}) \qquad var(a\tilde{x}) = a^2 var(\tilde{x})$$

- Covariance of two securities when their residuals are uncorrelated:

$$cov(\tilde{r}_i, \tilde{r}_j) = \beta_i \beta_j \sigma_m^2 \quad \textbf{if } cov(\epsilon_i, \epsilon_j) = 0$$

- Statistical Functions:

$$var(\tilde{r}_A) = E[(\tilde{r}_A - \overline{r_A})^2] \qquad cov(\tilde{r}_A, \tilde{r}_B) = E[(\tilde{r}_A - \overline{r_A})(\tilde{r}_B - \overline{r_B})]$$

- Fraction of the your wealth you put in the risky asset 1, given a risk aversion coefficient of A:

$$w_1^* = \frac{E(\tilde{r}_1) - r_f}{A\sigma_1^2}$$

- The MVE portfolio weights when there are two risky assets A and B ($x_B = (1 - x_A)$):

$$x_A = \frac{E(\tilde{r}_A^e)\sigma_B^2 - E(\tilde{r}_B^e)cov(\tilde{r}_A^e, \tilde{r}_B^e)}{E(\tilde{r}_A^e)\sigma_B^2 + E(\tilde{r}_B^e)\sigma_A^2 - [E(\tilde{r}_A^e) + E(\tilde{r}_B^e)]\, cov(r_A^e, r_B^e)}$$

- The CAPM equation

$$E(\tilde{r}_i) = r_f + \beta_i\,[E(\tilde{r}_m) - r_f)]$$

- The CAPM Beta:

$$\beta_i = \frac{cov(\tilde{r}_i, \tilde{r}_m)}{\sigma_m^2}$$

- The security market line

$$\tilde{r}_{i,t} - r_{f,t} = \alpha_i + \beta_i \cdot (\tilde{r}_{m,t} - r_{f,t}) + \tilde{\epsilon}_{i,t}$$

- The Systematic Variance of an Asset with a beta of β_i, assuming a single factor model:

$$\sigma_{sys,i}^2 = \beta_i^2 \cdot \sigma_m^2$$

- the R^2 is then $\frac{\sigma_{sys,i}^2}{\sigma_i^2}$
- Equation for *Merrill Lynch* adjusted β's:

$$\beta_i^{Adj} = 1/3 + (2/3) \cdot \beta_i$$

- Equation for the variance of portfolio a; and for the covariance of portfolios a and b:

$$var(\tilde{r}_a) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i^a w_j^a \sigma_{i,j} \qquad cov(\tilde{r}_a, \tilde{r}_b) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i^a w_j^b \sigma_{i,j}$$

  - For two assets(1 and 2):

$$var(\tilde{r}_a) = (w_1^a)^2 \cdot (\sigma_1)^2 + (w_2^a)^2 \cdot (\sigma_2)^2 + 2 \cdot w_1^a \cdot w_2^a \cdot cov(\tilde{r}_1, \tilde{r}_2)$$

$$cov(\tilde{r}_a, \tilde{r}_b) = w_1^a \cdot w_1^b \cdot (\sigma_1)^2 + w_2^a \cdot w_2^b \cdot (\sigma_2)^2 + (w_1^a \cdot w_2^b + w_2^a \cdot w_1^b) \cdot cov(\tilde{r}_1, \tilde{r}_2)$$

- The APT return generating process:

$$\tilde{r}_{i,t} = E(\tilde{r}_{i,t}) + b_{i,1}\tilde{f}_{1,t} + b_{i,2}\tilde{f}_{2,t} + \cdots + b_{i,k}\tilde{f}_{k,t} + \tilde{\epsilon}_{i,t}$$

- The APT pricing equation:

$$E(r_i) = \lambda_0 + \lambda_1 \cdot b_{i,1} + \lambda_2 \cdot b_{i,2} + \cdots + \lambda_k \cdot b_{i,k}$$

- The covariance between two stocks, i and j under a 2-factor model

$$cov(R_i, R_j) = b_{i,1}\, b_{j,1} var(f_1) + b_{i,2}\, b_{j,2} var(f_2) + (b_{i,1}\, b_{j,2} + b_{j,1}\, b_{i,2})\, cov(f_1, f_2)$$

- The formula for the standard error of the mean is

$$\sigma_\mu = \frac{\sigma}{\sqrt{n}},$$

  where n is the number of observations and σ is the volatility of the underlying variable.