# FINC460 - Spring 2008 Final Exam

NAME: _____ SECTION: _____

1. Please do not open this exam until directed to do so.
2. This exam is 3 hours long.
3. Please write your name and section number on the front of this exam, and on any examination books you use.
4. Please show all work required to obtain each answer. Answers without justification will receive no credit.
5. State clearly any assumptions you are making.
6. This is a closed book exam. No books or notes are permitted. Calculators are permitted. Laptops are permitted but you are only allowed to use Excel and a blank worksheet.
7. Brevity is strongly encouraged on all questions.
8. The exam is worth 115 points.
9. Relax, and good luck!

## Hints:

1. *Think through problems before you start working. Draw pictures.*
2. *If you get stuck on part of a problem, go on to the next part. You may need to use answers from earlier parts of the question to calculate answers to the later parts. If you weren't able to solve the earlier part, assume something.*
3. *Remember, setting up the problem correctly will get you most of the points.*

## Short questions (25 points)

Assess the validity of the following statements (True, False or Uncertain) and explain your answers.

1. (5 points) Since the coupons need to be reinvested, coupon-paying bonds are always more sensitive to interest rates than a zero-coupon bond with the same maturity.

    - FALSE. All else equal, zero coupon bonds are more sensitive to changes in interest rates, since they have higher duration.

2. (5 points) Newly issued T-Bonds (on-the-run) are usually more expensive that previously issued T-Bonds with similar maturity (off-the-run). This is an example of an arbitrage opportunity.

    - TRUE/UNCERTAIN. The first part is correct. However, on the run bonds are also more liquid, so the difference in prices/returns might be due to liquidity.

3. (5 points) Even if the CAPM or APT hold, assets that are not well diversified may lie on the efficient frontier.

   - FALSE. The CAPM and APT both imply that idiosyncratic risk does not matter since investors can form well-diversified portfolios. No portfolio with idiosyncratic risk can lie on the frontier, since one can diversify away the idiosyncratic risk, and if these models are true there will be no loss in terms of expected return.

4. (5 points) Firms that issue new shares tend to have higher returns after their secondary offering than before. The fact that stock issuance predicts future returns is evidence against market efficiency.

   - TRUE/UNCERTAIN. If the secondary offering (SEO) is not accompanied by a change in risk (factor loadings or market beta) then this is true. However, if the SEO is used to finance new projects, which may alter the risk of the firm, in particular increase it, then it is possible that future returns are higher, even if markets are efficient. Also, the fact that SEO changes leverage cannot be an explanation, since in this case leverage decreases which should imply that equity is less risky and therefore *lower* expected returns for equity.

5. (5 points) If the APT holds, two stocks that have exactly the same loadings on all factors can have different levels of expected return if some of the factors earn zero risk premium.

   - FALSE. If the APT holds, two stocks with the same loadings will always have the same levels of expected return, no matter what the risk premia are.

## Question 1 (40 points)

You are a portfolio manager who intends to implement the APT. The first step is the selection of appropriate factors. You have some candidate factors in mind, and you have asked some analysts to replicate the analysis in Chen, Roll and Ross. The analyst has reported the variance of the factors, ($\sigma_i$), the factor risk premia ( $\lambda_i$) and the results of a t-test on the factor risk premia:

| Factor | $\sigma_i$ | $\lambda_i$ | t-stat |
|---|---|---|---|
| Industrial Production | 0.20 | 3.00% | 2.93 |
| Inflation | 0.23 | 1.00% | 1.01 |
| Oil Prices | 0.10 | -2.00% | -3.12 |
| Consumer Sentiment | 0.15 | 4.00% | 0.75 |

All the factors are uncorrelated, i.e. $cov(f_i, f_j) = 0$ for all $i \neq j$.

1. (5 points) *Briefly* discuss the results above. In particular

   (a) Which factors have a statistically significant risk premium?

   - The two factors for which the risk premium is statistically different from zero at the 5% level are Industrial Production and Oil Prices.

   (b) Does the sign of the factor risk premia make sense? Rationalize the sign of the risk premia in the cases where they are statistically different from zero.

   - Industrial Production earns a positive risk premium. This is intuitive, as states of the world when IP growth is high are states when the economy is in an expansion. If investors want to hedge out recessions they will require a positive premium to invest in an asset that does well in expansions. For oil prices, it makes

some economic sense that the premium is negative, if states when oil prices are high are 'bad' states that investors want to hedge. Thus they prefer buying assets that do well in those states, driving prices up and risk premia down.

2. (15 points) You decide to select as factors industrial production ($f_1$), and oil prices ($f_2$). In addition, you know that the risk-free asset has a return of 1%. Now suppose that all assets in the economy can either be classified as "growth" or "value" stocks. A value-weighted portfolio of all value stocks will have a return over the next year given by:

$$\tilde{r}_V = E(\tilde{r}_V) + 1.4\tilde{f}_1 - 1.2\tilde{f}_2$$

while the value-weighted portfolio of all growth stocks will have a return over the next year given by:

$$\tilde{r}_G = E(\tilde{r}_G) + 0.5\tilde{f}_1 - 0.4\tilde{f}_2$$

Finally, the sum of the market capitalizations of all of the value stocks is currently \$1.32 trilion, and the sum of the market cap of all growth stocks is currently \$3.68 trillion.

Based on the information above, and assuming that the two-factor APT that you have constructed holds,

(a) What is the expected excess return and standard deviation of the Value and Growth Portfolios?

- For risk premia, we use the APT formula, so

$$E(\tilde{r}_V) = \lambda_0 + 1.4\lambda_1 - 1.2\lambda_2 = 7.6\%$$

$$E(\tilde{r}_G) = \lambda_0 + 0.5\lambda_1 - 0.4\lambda_2 = 3.3\%$$

The standard deviations of these portfolios are

$$\sigma(\tilde{r}_V) = \sqrt{1.4^2\sigma_1^2 + 1.2^2\sigma_2^2} = 30.46\%$$

$$\sigma(\tilde{r}_G) = \sqrt{0.5^2\sigma_1^2 + 0.4^2\sigma_2^2} = 10.77\%$$

(b) What is the expected excess return and standard deviation of the market portfolio?

- The market portfolio has returns

$$\tilde{r}_M = \frac{1.32}{1.32+3.68}\tilde{r}_V + \frac{3.68}{1.32+3.68}\tilde{r}_G = E(\tilde{r}_M) + 0.7376\tilde{f}_1 - 0.6112\tilde{f}_2$$

with expected returns

$$\begin{aligned} E(\tilde{r}_M) &= \lambda_0 + 0.7376\lambda_1 - 0.6112\lambda_2 \\ &= 0.01 + 0.7376 \times 0.03 - 0.6112 \times (-0.02) = 4.4352\% \end{aligned}$$

and standard deviations of

$$\sigma(\tilde{r}_M) = \sqrt{0.7376^2\sigma_1^2 + 0.6112^2\sigma_2^2} = 0.160$$

3. (20 points) Find the combination of the Value and Growth portfolios with the maximum Sharpe Ratio. Does the CAPM hold in this economy?

- First, we form excess returns by subtracting the risk free rate, $E(\tilde{r}_V^e) = E(\tilde{r}_V) - \lambda_0$ and we need to compute the covariance between V and G:

$$cov(r_V^e, r_G^e) = 1.4 \times 0.5 \times \sigma_1^2 + (-1.2) \times (-0.4) \times \sigma_2 = 0.0328$$

then we use the formula

$$x_V = \frac{E(\tilde{r}_V^e)\sigma_G^2 - E(\tilde{r}_G^e)cov(\tilde{r}_V^e, \tilde{r}_G^e)}{E(\tilde{r}_V^e)\sigma_G^2 + E(\tilde{r}_G^e)\sigma_V^2 - [E(\tilde{r}_V^e) + E(\tilde{r}_G^e)]\, cov(r_V^e, r_G^e)}$$

$$x_V = \frac{0.066 \times 0.1077^2 - 0.023 \times 0.0328}{0.066 \times 0.1077^2 + 0.023 \times 0.3046^2 - [0.066 + 0.023] \times 0.0328} = -0.583$$

and $x_G = 1 - x_V = 1.583$, which are not equal to the market portfolio weights, 1.32/5 and 3.68/5. Therefore, the market portfolio *is not the tangency portfolio* and thus the CAPM does not hold.

## Question 2 (50 points)

You are managing the portfolio of a hedge fund specializing in fixed-income securities. Currently, you are considering investing in STRIPS issued by the US Government. You believe that over the next year, a two-factor APT holds for the fixed-income market:

$$\begin{aligned} r_t^2 &= 0.95\% + d_2\tilde{f}_{1,t} - 1.3\tilde{f}_{2,t} \\ r_t^5 &= 2.45\% + d_5\tilde{f}_{1,t} - 0.1\tilde{f}_{2,t} \\ r_t^{10} &= 4.80\% + d_{10}\tilde{f}_{1,t} + 1.6\tilde{f}_{2,t} \end{aligned}$$

The three assets are zero-coupon bonds with maturities of 2, 5 and 10 years. and $r_t^m$ denotes **returns** of the m-year STRIP.

The first factor is a level factor that measures **percentage** changes in the level of yields, i.e. $\frac{\Delta y}{1+y}$. The second factor is a slope factor that measures changes in the slope of the term structure. Assume that the economy is currently in a recession and the term structure is currently flat. The two factors are uncorrelated, $cov(f_1, f_2) = 0$ and the standard deviations of the factors are $\sigma_1 = 0.03$ and $\sigma_2 = 0.04$ respectively.

1. (5 points) Use the Duration approximation to compute the sensitivities of the three zero-coupon bonds on the first factor ($d_i$). For the remaining questions treat this approximation as the actual factor loadings.

   - Since these are zero-coupon bonds, their duration equals their maturity, therefore $d_2 = -2$, $d_5 = -5$ and $d_{10} = -10$.

2. (15 points) Compute the risk premia for the two term structure factors and the risk-free rate implied by the absence of arbitrage.

- We need to solve a system of 3 equations in 3 unknowns

$$\begin{aligned} 0.95\% &= \lambda_0 - 2\lambda_1 - 1.3\lambda_2 \\ 2.45\% &= \lambda_0 - 5\lambda_1 - 0.1\lambda_2 \\ 4.80\% &= \lambda_0 - 10\lambda_1 + 1.6\lambda_2 \end{aligned}$$

  The solution is $\lambda_0 = 1\%$, $\lambda_1 = -0.3\%$ and $\lambda_2 = 0.5\%$

3. (15 points) Now suppose that your analysts have concluded that the economy is more likely to come out of the recession than the market expects. As a result, you believe the term structure will steepen and that $E\tilde{f}_2 = -2\%$. On the other hand, you are unsure what will happen to the level of interest rates. Also, you believe that the market is correct in assessing that $cov(f_1, f_2) = 0$ and that the standard deviations of the factors are $\sigma_1 = 0.03$ and $\sigma_2 = 0.04$ respectively. Construct a bond portfolio that will have a 4% higher rate of return if you are correct and has no exposure to the level of interest rates.

   (a) What would the factor loadings of your portfolio be?

   - We want to hedge out our level exposure, so we will want $b_{p,1} = 0$. Also, we want a 4% return if we are correct and $f_2 = -2\%$, so we want $b_{p,2} = -2$.

   (b) What fraction of your portfolio should be invested in each of the three zero-coupon bonds?

   - Denote by $w_i$ the weight placed on bond with maturity i. Those

weights solve the equations:

$$\begin{aligned} w_2 + w_5 + w_{10} &= 1 \\ -2w_2 - 5w_5 - 10w_{10} &= 0 \\ -1.3w_2 - 0.1w_5 + 1.6w_{10} &= -2 \end{aligned}$$

The solution is $w_2 = 1.11$, $w_5 = 0.22$ and $w_{10} = -0.33$.

(c) What does *the market* think the Sharpe Ratio of your portfolio is?

- The market believes that your portfolio has expected return in excess of the riskless rate equal to $E(r_p^e) = -2\lambda_2 = -1\%$. The standard deviation of your portfolio equals $\sigma_p = \sqrt{2^2 \times \sigma_2^2} = 0.08$. Therefore your Sharpe Ratio is equal to

$$SR_p = \frac{-0.01}{0.08} = -0.125$$

(d) What do *you* think the Sharpe Ratio of your portfolio is?

- You agree on the market's assessment of risk so the denominator is the same. However, you think that your portfolio will have 4% higher returns, so the Sharpe Ratio, according to your information is

$$\hat{SR}_0 = \frac{-0.01 + 0.04}{0.08} = 0.375$$

4. (15 points) Suppose that another fixed income hedge fund has loadings on factors 1 and 2 of $b_1 = -4$ and $b_2 = 3$, respectively, and advertises that, because of its ability to time the bond market, it will have the "very-high" pre-expense expected return of 5% over the next year. However, you know that this hedge fund is not well-diversified (in the APT sense), and indeed has a residual standard deviation of 10%.

(a) What is the most in expenses that this fund's managers should be able to charge (assuming that investors are fully rational and understand the structure of the economy)? Explain.

- We could construct a portfolio with the same factor loadings that would yield a return of

$$\mu_{APT} = \lambda_0 - 4\lambda_1 + 3\lambda_2 = 3.7\%$$

He generates a return of 5%, so his $\alpha$ with respect to our model will be equal to $\alpha_{APT} = 5\% - \mu_{APT} = 1.3\%$. This is the *maximum* amount he can charge.

(b) What is the (APT based) pre-expense appraisal ratio of this fund?

- His Appraisal Ratio will be

$$AR = \frac{\alpha}{\sigma_\epsilon} = \frac{1.3\%}{10\%} = 0.13$$

(c) Assuming that this fund charges the maximum fee you calculated in (a), how much of this fund would rational investors add to their portfolio? Explain.

- If he charges the full amount, i.e. 1.3%, without considering his idiosyncratic risk, we would be indifferent between investing in his fund or not since his post expense $\alpha$ would be zero. Since he is adding no $\alpha$, just risk, we will invest nothing.