FINC460: Homework 8

1 Factor Models

Suppose there are three stocks, A,B and C. Your analyst tells you that a 2 factor model with *uncorrelated factors* accurately describes returns. Factor 1 is industrial production and Factor 2 is energy prices. Note that f_1 and f_2 are factor surprises, that is $Ef_i = 0$ and the intercepts hence are the respective expected returns on the assets (e.g. $E(r_A) = 0.36$ is the expected return on stock A.

$$\begin{aligned} r_A &= 0.36 + 2f_1 + 4f_2 + \epsilon_A \\ r_B &= 0.225 + 3f_1 + 2f_2 + \epsilon_B \\ r_C &= 0.12 + 1f_1 + 1f_2 + \epsilon_C \end{aligned}$$

with variances given by $var(f_1) = 0.1$, $var(f_2) = 0.1$, $var(\epsilon_A) = 0.15$, $var(\epsilon_B) = 0.28$, $var(\epsilon_C) = 0.05$.

1) What is the covariance matrix between the three asset returns? [Hint: we are assuming that the factors are uncorrelated]

2) What is the risk-free rate implied by the absence of arbitrage?

3) What is the three R^2s that your analyst got when he regressed the returns of A, B and C respectively on the factor realizations?

4) Can you find two portfolios of A, B and C that have *only* f_1 or f_2 factor exposure respectively? What are their expected returns and standard deviations?

5) Let us now use the APT in asset allocation:

(a) What is the optimal (=max SR) portfolio of A,B and C? Its Sharpe ratio? Its factor loadings on f1 and f2? [Hint: you'll have to calculate the asset correlations first and then input in Markowitz]

(b) Suppose that your clients do not want any exposure to factor 2 risk. How does your optimal portfolio look now? [Hint: you'll have to introduce an additional constraint in Markowitz]

(c) Now, revisit part (a), but now choose only between the two factor mimicking portfolios you constructed in part 4. Did you get a different answer?

6) Is it possible that the CAPM holds in this economy? What would the market capitalization of A, B and C have to be for both the APT and the CAPM to hold?