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## Lecture 6: Factor Investing and Performance Evaluation

FE-312 Investments



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- ▶ In the previous lecture, we saw several strategies that have historically beat the market.
- ▶ As we saw in the homework, the information in these portfolios can typically be summarized in a much smaller set of portfolios—the so called, factor portfolios.
- ▶ Example: consider the Size/Value/Momentum strategies we saw...

```
clear
load mebm; load FF; load MOM
ret=[mebm mkt];

N=size(ret,2);
ExpReturn = mean(ret)';
ExpCovariance=cov(ret);

p = Portfolio;
p = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setDefaultConstraints(p);
p.LowerBound=ones(N,1)*(-100); p.UpperBound=ones(N,1)*(100);

[pwgt,pbuy,psell] = estimateMaxSharpeRatio(p);
[prsk,pret] = estimatePortMoments(p,pwgt);

SR=(pret-mean(rf))/prsk
```

- ▶ The market portfolio has a SR of 0.115
- ▶ The portfolio that optimally combines the 25 ME/BM portfolios and the market has a SR of 0.45

```
clear
load mebm; load FF; load MOM
ret=[mebm mom mkt];

N=size(ret,2);
ExpReturn = mean(ret)';
ExpCovariance=cov(ret);

p = Portfolio;
p = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setDefaultConstraints(p);
p.LowerBound=ones(N,1)*(-100); p.UpperBound=ones(N,1)*(100);

[pwgt,pbuy,psell] = estimateMaxSharpeRatio(p);
[prsk,pret] = estimatePortMoments(p,pwgt);

SR=(pret-mean(rf))/prsk
```

- ▶ Adding the 10 momentum portfolios to the mix yields a SR of 0.52

```
clear
load mebm; load FF; load MOM
ret=[mebm(:,1) mebm(:,5) mebm(:,21) mebm(:,25) mom(:,1) mom(:,10)];

N=size(ret,2);
ExpReturn = mean(ret)';
ExpCovariance=cov(ret);

p = Portfolio;
p = setAssetMoments(p, ExpReturn, ExpCovariance);
p = setDefaultConstraints(p);
p.LowerBound=ones(N,1)*(-100); p.UpperBound=ones(N,1)*(100);

[pwgt,pbuy,psell] = estimateMaxSharpeRatio(p);
[prsk,pret] = estimatePortMoments(p,pwgt);

SR=(pret-mean(rf))/prsk
```

- ▶ But you can do *almost* as well by restricting attention to the corner portfolios
- ▶ Restricting the choice set to 6 (out of 36) portfolios still yields a SR of 0.41

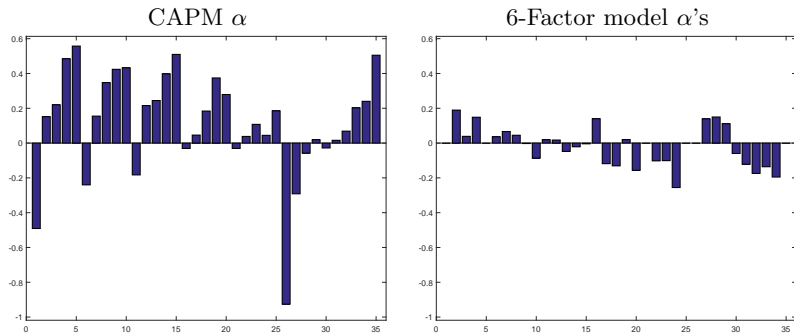
- ▶ An equivalent way of performing the previous exercise is to regress the excess returns of the 35 portfolios (25 ME/BM + 10 MOM) on *excess* returns of the small set of factors (the 6 corner portfolios) that we choose.
- ▶ That is, run 35 regressions,

$$R_{i,t} - r_f = \alpha_i + \beta_{i,1}R_{F1,t} + \cdots + \beta_{i,6}R_{F6,t} + \varepsilon_{i,t}$$

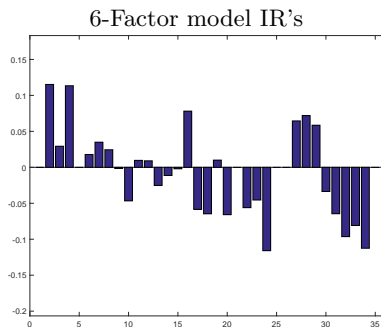
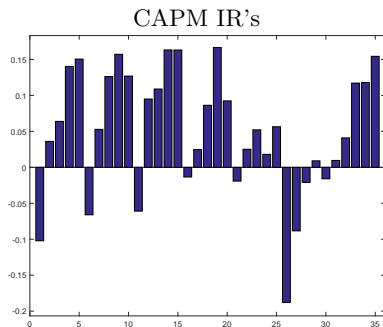
and compare the resulting alphas to what you would get from a CAPM-style regression

$$R_{i,t} - r_f = \alpha_i^{CAPM} + \beta_{i,M}(R_{M,t} - r_f) + \varepsilon_{i,t}$$

- ▶ Recall that  $\alpha_i^{CAPM}$  tells you something about how much you can increase your SR by combining this asset with the market. The alphas from the first regression have a similar interpretation: if they are large and non-zero, you can increase your SR over simply investing in your six factors.



- ▶ The graph on the left tells us that we can increase our SR substantially relative to just investing in the market
- ▶ The graph on the right tells us that most of these gains can be captured by investing in the 6 portfolios that we chose.



- To be more precise, the statistic that exactly summarizes how much you can increase the portfolio is the information ratio,

$$IR_i = \frac{\alpha_i}{\sigma(\varepsilon_i)}$$



- ▶ Factor models are useful for several reasons:
  1. Allow us to simplify the portfolio choice problem by choosing among a smaller set of assets that do ‘almost as well’ as the full menu
  2. Allow us to quickly evaluate whether we should be investing in a particular portfolio or asset class—given that we are already invested in the factors
  3. Allow us to compute a ‘fair’, or more appropriately, benchmark estimate for a securities expected return, given a set of ‘comparables’
- ▶ Importantly, factor models evaluate each strategy in terms of how it **covaries** with the factors in our model.
- ▶ Factor models can also be used in place of the CAPM (which is not doing that great in real data).

A factor model can be written as

$$r_{i,t} - r_{f,t} = \alpha_i + \sum_{j=1}^J \beta_{i,j} f_{j,t} + \varepsilon_{i,t}$$

- ▶ There are  $J$  factors. For now, think of each factor as a (zero-cost) portfolio.
- ▶ A useful factor model has a relatively smaller number of factors than number of assets ( $J \ll N$ )
  - ▶ Special case (Market model): only factor is  $f_1 = r_{m,t} - r_{f,t}$
- ▶ As we discussed previously, the  $\alpha_i$ 's in this regression—more precisely,  $\alpha_i / \sigma(\varepsilon_i)$  — tell you whether you should bother investing in asset  $i$ , given that you already hold ‘factors’  $i = 1 \dots J$  into your portfolio.
- ▶ Allows us to generalize the portfolio problem to allow many risks that investors may want to hedge (i.e they need not be simply mean-variance optimizers)

Assets are bundles of factor risk

- ▶ Corporate debt = real rate + inflation risk + term risk + credit risk
- ▶ Equity = real rate + inflation risk + term risk + credit risk + growth risk

*“Thus a long term corporate bond could actually be sold to three separate persons. One would supply the money for the bond; one would bear the interest rate risk; and one would bear the risk of default. The last two would not have to put up any capital for the bonds, although they might have to post some sort of collateral.”*

Fischer Black, 1970, quoted by Perry Mehrling

- ▶ Factor investing considers the factor premiums behind the assets; it looks through asset class ‘labels’ to the risk factors beneath
- ▶ The simple CAPM is a statement about a factor benchmark.
  - ▶ Consider a stock with a beta of 1.3

$$E[R_i] - r_f = \alpha + 1.3 (E[R_m] - r_f)$$
$$\underbrace{E[R_i]}_{\$1} = \alpha + \underbrace{\{-0.3r_f + 1.3E[R_m]\}}_{\$1}$$

- ▶ Factor benchmark is a short position of \$0.30 in cash (T-bills) and a leveraged position of \$1.30 in the market portfolio
- ▶ When you are buying a stock, you are buying its  $\alpha$  plus its factor exposure
- ▶ If the  $\alpha$  were zero, you are better off buying the ‘benchmark’

- ▶ If you are willing to make equilibrium arguments, you can derive restrictions on the  $\alpha$ 's
  1. Based on *mean-variance optimality*: **If the tangency portfolio is some combination of the factors  $f_1, f_2, \dots, f_j$ , then  $\alpha_i = 0$  for any asset**
  2. Based on *no arbitrage*: **If the  $\varepsilon$ 's are sufficiently uncorrelated,  $\alpha_i = 0$**
- ▶ This way, we can obtain a generalization of the CAPM: no longer assume market is the tangency portfolio but still use intuition from mean-variance optimality.
- ▶ I will use the term 'Arbitrage Pricing Theory' (APT) to refer to the *economic* restriction that the  $\alpha$ 's should be zero.

Just like the CAPM, we get a linear relation between expected (excess) returns and betas

- ▶ **The factor model pricing equation:**

$$E(r_i) - r_f = \sum_{j=1}^J \beta_{i,j} \lambda_j$$

$\lambda_j$  is the **risk premium** for exposure to factor  $j$  (more on this later)

- ▶ If the factors are portfolio returns,

$$E[f_j] = \lambda_j$$

- ▶ Implications:

- ▶ Any asset uncorrelated with the factors earns the risk-free rate.
- ▶ If not, then it is a ‘good’ deal, and we should either go long ( $\alpha > 0$ ) or short ( $\alpha < 0$ ).

- The beta of a portfolio is the portfolio of betas (recall previous lecture)

$$r_i = r_f + \sum_{j=1}^J \beta_{i,j} f_j + \varepsilon_i$$
$$\sum_i w_i r_i = r_f + \sum_{j=1}^J \left( \sum_i w_i \beta_{i,j} \right) f_j + \sum_i w_i \varepsilon_i$$

- So you can eliminate factor loadings in a portfolio by adding other portfolios

Suppose your current portfolio follows

$$r_a = \beta_{a,1} (r_{m,t} - r_{f,t}) + \beta_{a,2} f_{2,t} + \varepsilon_{a,t}$$

- You can eliminate exposure to the market factor by shorting the market. How much do you need to short?

$$w r_{a,t} + (1-w) r_{m,t} = \underbrace{(w \beta_{a,1} + 1 - w)}_{=0} (r_{m,t} - r_{f,t}) + w \beta_{a,2} f_{2,t} + w \varepsilon_{a,t}$$

- Solution:

$$w \beta_{a,1} + 1 - w = 0 \Rightarrow w = \frac{1}{1 - \beta_{a,1}}$$



- ▶ Another way you can use the factor model is as imposing additional constraints on portfolio optimization. Examples:
  - ▶ You are choosing a portfolio among size/value/momentum but you want to be market neutral (zero exposure to the market factor)
  - ▶ Same as above, but you want to track the market (i.e. you want a market  $\beta = 1$ )
  - ▶ You are managing the portfolio of an airline (oil) firm, and you want to hedge increases (decreases) in the 'oil' factor.

## What are factors?

Factors are **common** sources of risk that are in general **associated with risk premia** (a ‘useful’ factor has  $E[f_j] \neq 0$ )

- ▶ **Common:** affects many assets, not just one
  - ▶ Risk that can be diversified across many investments does not earn a premium (otherwise you could make money risklessly)
- ▶ **Associated with risk premia:** having a beta on the factor changes the expected return
  - ▶ Risk premium is the average return on the factor (if factor is a portfolio).
  - ▶ If the factor is not ‘investable’ then the risk premium would be the average return of a portfolio that is a ‘pure bet’ on that factor
    - ▶ Ex: a pure bet on inflation would go long TIPS and short T-Bonds.
  - ▶ Risk premium can be negative (if the factor does well in bad times – is insurance)
    - ▶ Ex: market volatility.

- ▶ Economic idea: assets that do poorly in bad times are risky, require a high return
  - ▶ “Bad times” is when the market falls (or consumption growth is low)
  - ▶ Corollary: you’ll accept a low or negative return for insurance
- ▶ Multifactor models: allow for more things to determine ‘bad’ times.
- ▶ To get some intuition about what determines factor risk premia, recall the basic equation

$$1 = E[mR]$$

- ▶ A multifactor model is simply a particular specification for  $m$

$$m = a - \sum_{j=1}^J \lambda_j f_j$$

Here, the risk premia on the factors are captured by  $\lambda_j$ . It is easy to show that if the factors are portfolios,  $E[f_j] \propto \lambda_j$ .

- Recall the pricing equation,

$$1 = E[mR]$$

- Since this also holds for the risk-free asset, we can also express it in terms for excess returns,

$$0 = E[mR^e].$$

- We can then define a factor model as a particular specification for  $m$ , say

$$m = 1 - \sum_{j=1}^J \gamma_j (F_j - E[F_j])$$

- In this case, we can show that

$$E[R_i^e] = \sum_{j=1}^J \beta_{i,j} \lambda_j$$

- Here, the risk premia on the factors are captured by  $\lambda_j$ . It is easy to show that  $\lambda_j \propto \gamma_j$

- To see this,

$$E[mR^e] = 0 \Rightarrow E[R^e] = -\text{cov}(m, R^e)$$

$$E[R^e] = -\sum_{j=1}^J \gamma_j \text{cov}(F_j, R^e)$$

$$E[R^e] = -\sum_{j=1}^J \gamma_j \text{var}(F_j) \underbrace{\frac{\text{cov}(F_j, R^e)}{\text{var}(F_j)}}_{\beta_j}$$

- Therefore

$$\lambda_j = \gamma_j \text{var}(F_j)$$

- A factor carries a high risk premium if
  - high  $\gamma_j$ , that is, high values of the factor indicate times are good (low  $m$ )
  - factor is very uncertain — that is,  $\text{var}(F_j)$  is very high
- Factors that signal bad times  $\gamma_j < 0$  will earn a negative risk premium

- ▶ Recall  $m$  represents a ‘re-weighting’ of objective probabilities that puts more weight in bad states (high  $m$ ) and less weight on good states (low  $m$ )
- ▶ States with high  $m$  means you’d like more money then (high marginal utility)
- ▶ Why? Simple answer could be that you are poor (low consumption growth so high marginal utility)
  - ▶ E.g. when your house burns down
- ▶ Factor models implicitly allow other variables to affect  $m$  (marginal utility).
- ▶ Example, you may view  $m$  as high when there are good investment opportunities (i.e. if you’re an investment fund (PE, HF, VC, etc.)
  - ▶ Example: during crashes there may be arbitrage opportunities; HFs want money in crashes

## The Worst ETFs of 2016

These funds lost a lot of money for shareholders this year.



Dan Caplinger (TMFGolagan)  
Dec 14, 2016 at 7:40AM

Topics Exchange-traded fund Market trend Investor

Millions of investors use exchange-traded funds to make money on profitable trends, but some ETFs performed disastrously in 2016. The funds listed below as the worst ETFs of 2016 were all among the bottom 25 in terms of year-to-date performance, according to Morningstar, and they represent three major adverse trends that hurt investors during the year. Let's go over what those trends were and how they hurt these poor-performing ETFs in 2016.

ETF	YTD Return
iPath S&P 500 VIX ST Futures ( <a href="#">NYSEMKT:VXX</a> )	(67%)
Direxion Daily Junior Gold Miners Bear 3x ( <a href="#">NYSEMKT:JDST</a> )	(98%)
iPath Global Carbon ETN ( <a href="#">NYSEMKT:GRN</a> )	(51%)

DATA SOURCE: MORNINGSTAR.

### Volatility plays perform horribly again

After several years with no major market downturns, many investors have looked to volatility ETFs as a way to bet on a future crash. Yet even though there were some periods of volatility in the stock market during 2016, the continuation of the bull market into an eighth year caused big losses for those who were counting on the long-awaited bear to come out of hibernation.

Some leveraged volatility plays lost 90% or more of their value during the year, and even the unleveraged iPath S&P 500 VIX ST Futures ETN fell by two-thirds in 2016. Among volatility plays, the inverse **VelocityShares Daily Inverse VIX ST ETN** (NYSEMKT: XIV) was a rare winner, climbing by 78% and taking advantage of the favorable conditions that caused most regular volatility ETF investors to suffer such massive losses. The year's events once again showed that for the most part, volatility ETFs are best suited for short-term calls, rather than buy-and-hold investments.

- Financial journalists are continuously surprised by the poor performance of ETFs that track the VIX.

## Factor risk premia: Example

iPath S&P 500 VIX ST Futures ETN (VXX) 24.07 -1.44 (-5.64%) As of 12:15 PM EST. NYSEArca Real Time Price. Market open.



- ▶ Year 2016 was not an exception. Performance since the ETF's inception in 2009 has not been so good.
- ▶ Is that surprising?



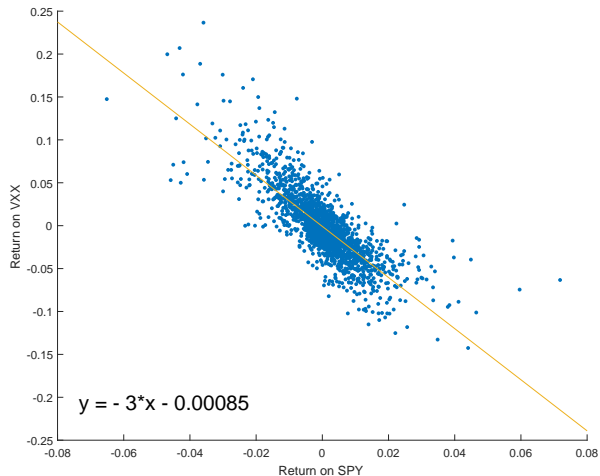
2009-2016		
	SPY	VXX
Mean	0.1568	-0.6802
(se) 0.0596	0.2201	
Standard Deviation	0.1683	0.6217

- ▶ If you had bought VXX in 2009, you would have lost 68% on average per year.
- ▶ Why would anyone buy this fund? Recall

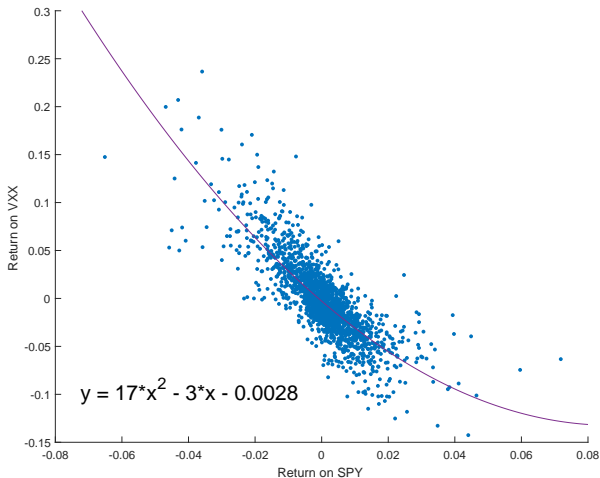
$$E[R^e] = -cov(m, R^e)$$

- ▶ What do you think the correlation between volatility and  $m$  is?
  - ▶ I.e., are increases in volatility good news or bad news?

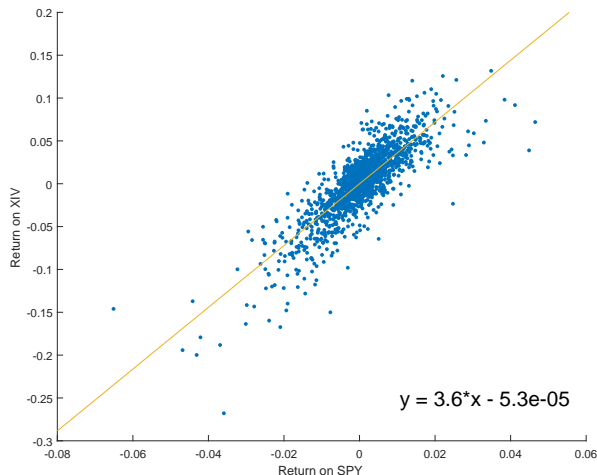
## VXX vs SPY



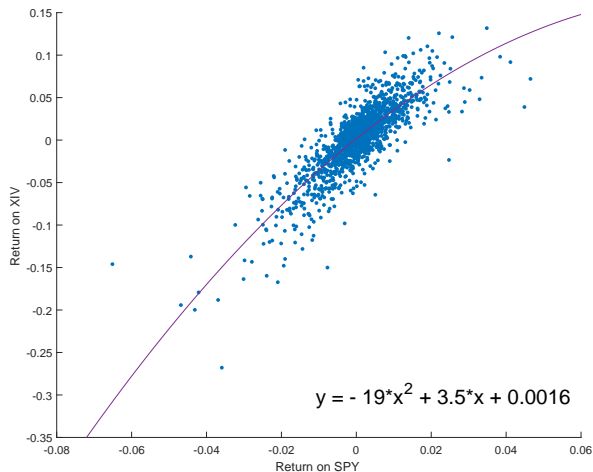
- VXX has a correlation with the market (SPY) of -80%



- More importantly, the relation is non-linear:
  - VXX pays off when the market crashes



- The inverse volatility fund (XIV) behaves exactly the opposite



- By buying XIV you are effectively selling crash insurance

- ▶ Economically motivated: Factors map into clearly identified risks.
  - ▶ **Pros:** Easy to understand; since based on economics less likely due to data mining (though not impossible)
  - ▶ **Cons:** Typically, they don't work so well ('real' economic variables have pretty low correlation with stocks, recall discussion on CCAPM)
  - ▶ Ideal to compute 'fair discount rates'
- ▶ Empirical factor models (i.e. adhoc)
  - ▶ **Pros:** They work well (at least in sample ...)
  - ▶ **Cons:** Typically weak economic foundation, hence no reason to think they are robust (i.e. not data mined). Not obvious why they should affect  $m$ .
  - ▶ Useful for performance evaluation

There are two ways you can use factor models

1. Ignore equilibrium restrictions ( $\alpha = 0$ ). Simply use the factor model to examine whether strategy  $i$  adds value (increases your SR) if you are already invested in portfolios (factors)  $j = 1 \dots J$ .
2. Use the factor model to derive a 'fair' compensation for risk (or appropriate discount rate)
  - ▶ If you do that, then you are really taking the equilibrium restrictions seriously
  - ▶ Easier to convince yourself if factors are economically motivated (and there is a good story why they affect  $m$ )

In practice, most people do (1). Even though the CAPM is not doing great, there is yet no broadly accepted factor model that can be used for (2). Economically motivated factor models are not always robust, while ad-hoc empirical models typically lack a good story about  $m$ .

## Example: The Fama-French 3 factor Model

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- ▶ Fama and French in their 1993 *Journal of Financial Economics* paper specified what has come to be called the *Fama-French 3 factor model*

$$R_i - r_f = \alpha_i + \beta_{mkt}(R_m - r_f) + \beta_{smb}R_{smb} + \beta_{hml}R_{hml} + \varepsilon_i$$

- ▶ Here,  $R_m$  is the market portfolio
- ▶ SMB and HML are the two Fama-French factors
  - ▶ SMB: small minus big
  - ▶ HML: high minus low book-to-market
- ▶ Despite claims of FF to the contrary, the FF3 model is (at least as proposed) an ad-hoc type of factor model.
  - ▶ It was motivated by the facts on size/value we saw last week



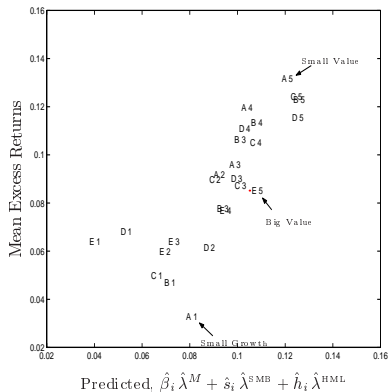
## The Fama-French 3 factor Model

- ▶ Fama and French split the universe of stocks into 6 portfolios, based on size (Market Capitalization) and value/growth (Book to Market):

Market Cap	Book-to-market		
	Low	Medium	High
Small	Portfolio 1: Small growth	Portfolio 2: Small core	Portfolio 3: Small value
Large	Portfolio 4: Large growth	Portfolio 5: Large core	Portfolio 6: Large value

- ▶ They then constructed 2 factors
  1. Small minus Big (SMB):  $(1/2SG + 1/2SV) - (1/2LG + 1/2LV)$
  2. High minus Low (HML):  $(1/2SV + 1/2LV) - (1/2SG + 1/2LG)$
- ▶ Their version of the APT includes as factors the market portfolio, SMB and HML.

# The Fama-French 3 factor Model



- ▶ The Fama-French model does a good job summarizing the value and size anomalies.
- ▶ This is not purely mechanical: it works because there is comovement among value and size stocks:
  - ▶ It works because 'value' firms have high exposures to the value factor

## Fama–French (1993) ‘explains’ many other ‘anomalies’

$$r_{i,t} - r_f = \alpha_i + \beta_{i,mkt}(r_{mkt} - r_f) + \beta_{i,HML}HML_t + \beta_{i,SMB}SMB_t + \varepsilon_{i,t}$$

- ▶ People had found many characteristics that predicted  $\alpha$  in the CAPM:  $\beta$ , size, leverage, earnings/price, book/market, dividend/price
- ▶ When you add  $HML$  and  $SMB$  to the model,  $\alpha$ 's shrink relative to the CAPM
- ▶ Interpretation: the value and size strategies subsume most of the other strategies above

# The Fama-French 3 factor Model

	Deciles									
	1	2	3	4	5	6	7	8	9	10
<b>BE/ME</b>	<b>Low</b>									<b>High</b>
Mean	0.42	0.50	0.53	0.58	0.65	0.72	0.81	0.84	1.03	1.22
Std. Dev.	5.81	5.56	5.57	5.52	5.23	5.03	4.96	5.06	5.52	6.82
<i>t</i> (Mean)	1.39	1.72	1.82	2.02	2.38	2.74	3.10	3.17	3.55	3.43
Ave. ME	2256	1390	1125	1037	1001	864	838	730	572	362
<b>E/P</b>	<b>Low</b>									<b>High</b>
Mean	0.55	0.45	0.54	0.63	0.67	0.77	0.82	0.90	0.99	1.03
Std. Dev.	6.09	5.62	5.51	5.35	5.14	5.18	4.94	4.88	5.05	5.87
<i>t</i> (Mean)	1.72	1.52	1.89	2.24	2.49	2.84	3.16	3.51	3.74	3.37
Ave. ME	1294	1367	1211	1209	1411	1029	1022	909	862	661
<b>C/P</b>	<b>Low</b>									<b>High</b>
Mean	0.43	0.45	0.60	0.67	0.70	0.76	0.77	0.86	0.97	1.16
Std. Dev.	5.80	5.67	5.57	5.39	5.39	5.19	5.00	4.88	4.96	6.36
<i>t</i> (Mean)	1.41	1.52	2.06	2.37	2.47	2.78	2.93	3.36	3.75	3.47
Ave. ME	1491	1266	1112	1198	990	994	974	951	990	652
<b>5-Yr SR</b>	<b>High</b>									<b>Low</b>
Mean	0.47	0.63	0.70	0.68	0.67	0.74	0.70	0.78	0.89	1.03
Std. Dev.	6.39	5.66	5.46	5.15	5.22	5.10	5.00	5.10	5.25	6.13
<i>t</i> (Mean)	1.42	2.14	2.45	2.52	2.46	2.78	2.68	2.91	3.23	3.21
Ave. ME	937	1233	1075	1182	1265	1186	1075	884	744	434

- The Fama-French model also does a good job explaining related patterns

# The Fama-French 3 factor Model

		Deciles									
		1	2	3	4	5	6	7	8	9	10
<b>BE/ME</b>	<b>Low</b>										<b>High</b>
$a$		0.08	-0.02	-0.09	-0.11	-0.08	-0.03	0.01	-0.04	0.03	-0.00
$t(a)$		1.19	-0.26	-1.25	-1.39	-1.16	-0.40	0.15	-0.61	0.43	-0.02
$R^2$		0.95	0.95	0.94	0.93	0.94	0.94	0.94	0.94	0.95	0.89
<b>E/P</b>	<b>Low</b>										<b>High</b>
$a$		-0.00	-0.07	-0.07	-0.04	-0.03	0.02	0.06	0.09	0.12	0.00
$t(a)$		-0.07	-1.07	-0.94	-0.52	-0.43	0.24	1.01	1.46	1.49	0.05
$R^2$		0.91	0.95	0.94	0.94	0.94	0.94	0.94	0.94	0.92	0.92
<b>C/P</b>	<b>Low</b>										<b>High</b>
$a$		0.02	-0.08	-0.07	-0.00	-0.04	0.00	0.00	0.05	0.06	0.01
$b$		1.04	1.06	1.08	1.06	1.05	1.04	0.99	1.00	0.98	1.14
$s$		0.45	0.50	0.54	0.51	0.55	0.50	0.53	0.48	0.57	0.92
$h$		-0.39	-0.18	0.07	0.11	0.23	0.31	0.36	0.50	0.67	0.79
$t(a)$		0.22	-1.14	-1.00	-0.04	-0.51	0.00	0.06	0.72	0.92	0.14
$t(b)$		51.45	61.16	62.49	64.15	59.04	61.28	60.02	63.36	58.92	46.49
$t(s)$		15.56	20.32	22.11	21.57	21.49	20.72	22.19	21.17	24.13	26.18
$t(h)$		-12.03	-6.52	2.56	4.28	7.85	11.40	13.52	19.46	24.88	19.74
$R^2$		0.93	0.95	0.95	0.95	0.94	0.94	0.94	0.94	0.94	0.92
<b>5-Yr SR</b>	<b>High</b>										<b>Low</b>
$a$		-0.21	-0.06	-0.03	-0.01	-0.04	-0.02	-0.04	0.00	0.04	0.07
$b$		1.16	1.10	1.09	1.03	1.03	1.03	1.00	0.99	0.99	1.02
$s$		0.72	0.56	0.52	0.49	0.52	0.51	0.50	0.57	0.67	0.95
$h$		-0.09	0.09	0.21	0.20	0.24	0.33	0.33	0.36	0.47	0.50
$t(a)$		-2.60	-0.97	-0.49	-0.20	-0.61	-0.25	-0.66	0.07	0.47	0.60
$t(b)$		59.01	70.59	67.65	65.34	56.68	68.89	62.49	54.12	50.08	34.54
$t(s)$		25.69	25.11	22.59	21.65	20.15	23.64	21.89	21.65	23.65	22.34
$t(h)$		-2.88	3.55	8.05	7.98	8.07	13.63	12.80	12.13	14.78	10.32
$R^2$		0.95	0.96	0.95	0.95	0.93	0.95	0.94	0.93	0.92	0.87

- ▶ The Fama-French model does not help explain the alphas in the momentum trading strategy
- ▶ Obviously no reason to stop at three factors...
- ▶ Carhart (1997) proposed as an extension the Carhart four-factor model, that includes a fourth factor, MOM, constructed as the 'winners' minus 'losers'

$$R_i - r_f = \alpha_i + \beta_{mkt}(R_m - r_f) + \beta_{smb}R_{smb} + \beta_{hml}R_{hml} + \beta_{umd}R_{umd} + \varepsilon_i$$

where  $UMD$  is the momentum factor, constructed in a similar way as HML and SMB

- ▶ But it never ends – go on Ken French's website and look
- ▶ Hou, Xue, and Zhang (2015) propose a factor model using the market plus sorts on size, investment/assets and return on equity (IBEX/CE) – four-factor model
- ▶ They argue this model explains a wide range of anomalies (see also a recent paper by Fama and French with similar ideas)
  - ▶ They explore trading strategies based on 80 different types of return-forecasting variables
  - ▶ They find that for many of these have  $\alpha = 0$ . What these means is that the proposed factors (simplified trading strategies) capture most of the profitability associated with them
  - ▶ That is, adding any of these additional strategies to your portfolio will not increase your SR
- ▶ Lessons:
  - ▶ All of these different trading strategies are not so different after all.

## Application: Do (active) Mutual Funds add value?

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- ▶ In the US, the size of financial assets under institutional management is over 75 trillion as of 2007.
- ▶ Substantial fraction is invested in active funds.
- ▶ Active funds have considerably higher fees than passive funds.
  - ▶ Vanguard S&P 500 Index fund has expenses of 0.20% per year.
  - ▶ Fidelity Magellan Fund has initial load of 3%, expenses of 0.95%.
- ▶ The average expense ratio of active funds is 130 basis points. (Carhart (1997))
- ▶ Is it worth it?



## How well do mutual fund managers perform?

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- ▶ Over the past 30 years,
  - ▶ the Wilshire 5000 has returned an average of 14.01% per year.
  - ▶ the average equity fund has returned 12.44% after fees.
- ▶ Operating expenses and fees amount to 1.7%.
- ▶ This seems to suggest that the average active manager adds very little/no value
- ▶ Fairer to compare actively managed funds with index funds, which have operating expenses of 0.3%. This reduces the under-performance margin to 1.27%.

## How well do mutual fund managers perform?

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- ▶ Yet there might be some managers who add value.
  - ▶ Can we identify them ex-ante?

## Does Warren Buffet have skill?

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- ▶ Berkshire Hathaway: large holding company, based around insurance
  - ▶ Insurance companies typically have large investment portfolios – have to hold assets in case of payouts
- ▶ Mean annual return since 1979: 17%. Standard deviation: 23%
- ▶ Sharpe ratio: 0.71 – 1.44 times higher than market over same period
- ▶ What has Warren Buffett's investment policy looked like? Could it have been replicated cheaply?
- ▶ Simple experiment: regress returns on Berkshire Hathaway on the market, *HML*, *SMB* and other factors

<i>Regression Statistics</i>				
Multiple R	0.46			
R Square	0.21			
Adjusted R Square	0.21			
Standard Error	6.09			
Observations	448			
ANOVA				
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>
Regression	1	4404.25	4404.25	118.76
Residual	446	16539.68	37.08	
Total	447	20943.93		
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Alpha	0.98	0.29	3.36	0.00
RmRf	0.70	0.06	10.90	0.00

Annual alpha: 11.71%

## Relative to the market, size and value

SUMMARY OUTPUT					
<i>Regression Statistics</i>					
Multiple R	0.52				
R Square	0.27				
Adjusted R Square	0.26				
Standard Error	5.88				
Observations	448				
ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	3	5605.80	1868.60	54.09	0.00
Residual	444	15338.13	34.55		
Total	447	20943.93			
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>
Alpha	0.81	0.28	2.85	0.00	0.25
RmRf	0.83	0.07	12.60	0.00	0.70
SMB	-0.28	0.10	-2.87	0.00	-0.48
HML	0.48	0.10	4.78	0.00	0.28

Annual alpha: 9.7%

## Relative to the market, size, value and profitability

SUMMARY OUTPUT					
<i>Regression Statistics</i>					
Multiple R	0.54				
R Square	0.29				
Adjusted R Square	0.28				
Standard Error	5.79				
Observations	448				
ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	4	6101.80	1525.45	45.53	0.00
Residual	443	14842.13	33.50		
Total	447	20943.93			
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>
Alpha	0.59	0.28	2.08	0.04	0.03
RmRf	0.89	0.07	13.32	0.00	0.76
SMB	-0.13	0.11	-1.26	0.21	-0.34
HML	0.42	0.10	4.27	0.00	0.23
RMW	0.50	0.13	3.85	0.00	0.24

RMW – Robust minus Weak profits (high minus low profitability)

Annual alpha: 7.1%

## Relative to the market and other strategies

SUMMARY OUTPUT					
<i>Regression Statistics</i>					
Multiple R	0.55				
R Square	0.31				
Adjusted R Square	0.30				
Standard Error	5.74				
Observations	448				
ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	6	6390.26	1065.04	32.27	0.00
Residual	441	14553.66	33.00		
Total	447	20943.93			
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>
Alpha	0.44	0.29	1.53	0.13	-0.13
RmRf	0.89	0.07	13.23	0.00	0.75
SMB	-0.19	0.11	-1.77	0.08	-0.40
HML	0.32	0.11	2.84	0.00	0.10
RMW	0.36	0.14	2.69	0.01	0.10
UMD	-0.02	0.07	-0.25	0.80	-0.15
BaB	0.27	0.09	2.89	0.00	0.08

BaB – betting against beta (long low-beta, short high-beta stocks)

Annual alpha: 5.3%

## Does Warren Buffet have skill? Yes, but...

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- ▶ BRK earns a CAPM alpha of nearly 12 percent per year
- ▶ However,  $\alpha$  falls by half once we control for simple investment strategies. So they're good, but not for 12% per year
- ▶  $R^2$  is only 0.31, though – 69% of variation in BRK returns is separate from these factors
  - ▶ Lots of idiosyncratic risk ( $\sigma_\varepsilon = 0.23 \times \sqrt{0.69} \approx 0.19$ )
  - ▶ Likely due to concentrated investments
  - ▶ Need to look at IR:  $\alpha/\sigma_\varepsilon \approx 0.06/0.19 = 0.21$



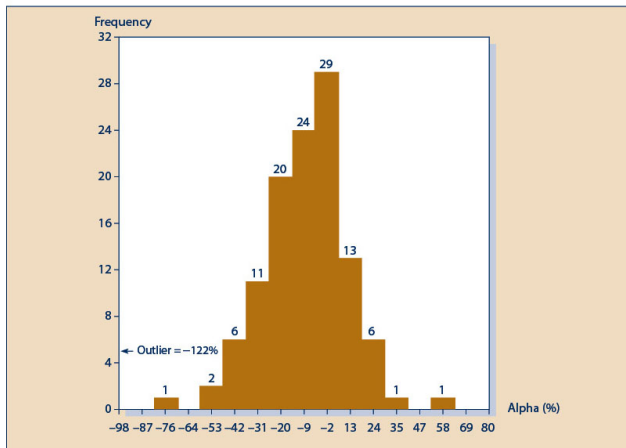
## Do (active) Mutual Funds add value?

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- ▶ Let's do this more broadly. What is the right factor model?
- ▶ The answer depends on what the alternatives are, i.e. what you can invest in yourself (without paying an active fund manager).
  - ▶ If the only thing you can do by yourself is to put your money into a checking account, then likely a good deal since you probably want *some* stock exposure.
  - ▶ If your alternative is to put money into a market index fund (or an ETF like SPY), then you want to compute the fund manager's  $\alpha$  (and IR) relative to a *factor* model with the market factor in it.
  - ▶ If you are able to trade some of these size/value/momentum strategies using ETFs (easier for size/value, harder for momentum), you want to use the Carhart 4F model.
- ▶ Also,  $\alpha$  is also the maximum you should be willing to compensate a portfolio manager. Otherwise, not worth it.

# How well do mutual fund managers perform?

**Figure 8.4**  
Frequency  
distribution of  
alphas.



Source: Michael C. Jensen, "The Performance of Mutual Funds in the Period 1945–1964," *Journal of Finance* 23 (May 1968). Reprinted by permission of the publisher, Blackwell Publishing Inc.

## Do (active) Mutual Funds add value?

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- ▶ Large dispersion in CAPM alphas
- ▶ Does this mean that fund managers add value?
- ▶ Not necessarily. Some might simply have gotten lucky!
  - ▶ Need to look at standard errors.
  - ▶ But 5% of all funds will still look like they significantly outperform even if all luck.
- ▶ Better test: Is there persistence in performance among managers?
- ▶ Sort mutual funds into portfolios based on their *past* performance.

# How well do mutual fund managers perform?

Figure: Table depicts performance over the subsequent 12 months relative to risk-free, relative to benchmark (CAPM) and relative to Carhart's Four-Factor model.

Portfolio	Monthly Excess Return	Std Dev	CAPM			4-Factor Model					
			Alpha	VWRF	Adj R-sq	Alpha	RMRF	SMB	HML	PR1YR	Adj R-Sq
1A	0.75%	5.45%	0.27% (2.06)	1.08 (35.94)	0.777	-0.11% (-1.11)	0.91 (37.67)	0.72 (19.95)	-0.07 (-1.65)	0.33 (11.53)	0.891
1B	0.67%	4.94%	0.22% (2.00)	1.00 (39.68)	0.809	-0.10% (-1.08)	0.86 (40.66)	0.59 (18.47)	-0.05 (-1.38)	0.27 (10.63)	0.898
1C	0.63%	4.95%	0.17% (1.70)	1.02 (44.65)	0.843	-0.15% (-1.92)	0.89 (49.76)	0.56 (20.86)	-0.05 (-1.61)	0.27 (12.69)	0.927
1 (high)	0.68%	5.04%	0.22% (2.10)	1.03 (43.11)	0.834	-0.12% (-1.60)	0.88 (50.54)	0.62 (23.67)	-0.05 (-1.86)	0.29 (13.88)	0.933
2	0.59%	4.72%	0.14% (1.75)	1.01 (57.00)	0.897	-0.10% (-1.78)	0.89 (66.47)	0.46 (22.95)	-0.05 (-2.25)	0.20 (12.43)	0.955
3	0.43%	4.56%	-0.01% (-0.08)	0.99 (70.96)	0.931	-0.18% (-3.65)	0.90 (76.80)	0.34 (18.99)	-0.07 (-3.69)	0.16 (11.52)	0.963
4	0.45%	4.41%	0.02% (0.33)	0.97 (85.70)	0.952	-0.12% (-2.81)	0.90 (90.03)	0.27 (18.18)	-0.05 (-3.12)	0.11 (9.40)	0.971
5	0.38%	4.35%	-0.05% (-1.10)	0.96 (93.93)	0.960	-0.14% (-3.31)	0.90 (89.65)	0.22 (14.42)	-0.05 (-3.27)	0.07 (6.18)	0.970
6	0.40%	4.36%	-0.02% (-0.46)	0.96 (91.94)	0.958	-0.12% (-2.82)	0.90 (86.16)	0.22 (14.02)	-0.04 (-2.37)	0.08 (6.01)	0.968
7	0.36%	4.30%	-0.06% (-1.39)	0.95 (92.90)	0.959	-0.14% (-3.09)	0.90 (85.73)	0.21 (13.17)	-0.03 (-1.62)	0.04 (2.89)	0.967
8	0.34%	4.48%	-0.10% (-1.86)	0.98 (85.14)	0.951	-0.13% (-2.52)	0.93 (75.44)	0.20 (10.74)	-0.06 (-3.16)	0.01 (0.84)	0.958
9	0.23%	4.60%	-0.21% (-3.24)	1.00 (67.91)	0.926	-0.20% (-3.11)	0.93 (60.44)	0.22 (9.69)	-0.10 (-3.80)	-0.02 (-1.17)	0.938
10 (low)	0.01%	4.90%	-0.45% (-4.58)	1.02 (46.09)	0.851	-0.40% (-4.33)	0.93 (42.23)	0.32 (9.69)	-0.08 (-2.23)	-0.09 (-3.50)	0.887
10A	0.25%	4.78%	-0.19% (-2.05)	1.00 (48.48)	0.864	-0.19% (-2.16)	0.91 (42.99)	0.33 (10.27)	-0.11 (-3.20)	-0.02 (-0.76)	0.891
10B	0.02%	4.92%	-0.42% (-3.84)	1.00 (40.67)	0.817	-0.37% (-3.45)	0.91 (35.52)	0.32 (8.24)	-0.09 (-2.16)	-0.09 (-2.99)	0.848
10C	-0.25%	5.44%	-0.74% (-5.06)	1.05 (32.16)	0.736	-0.64% (-4.49)	0.98 (28.82)	0.32 (6.29)	-0.04 (-0.73)	-0.17 (-4.09)	0.782

## Do (active) Mutual Funds add value? Unlikely.

---

- ▶ Some funds consistently beat the market.
- ▶ But not so once you control for size, value and momentum.
- ▶ In fact, momentum captures most of the performance of the winning funds.
- ▶ So, should you invest in mutual funds?
  - ▶ If your only alternative is buying the S&P 500, then the answer is probably yes.
  - ▶ If you can trade size, value and momentum yourself (using low-cost ETFs), then no.

- ▶ What if you can't short?
  - ▶ Then you need to run a restricted regression
  - ▶ Minimize sum of squared residuals forcing the coefficients to be positive (Excel won't do this automatically)
- ▶ Market timing?
  - ▶ Switch between treasury bills and the market portfolio depending on 'signals'
  - ▶ Merton (1981) shows that this is just like an option on the market index
  - ▶ Need to take into account non-linear strategies. Example

$$r_p - r_f = \alpha_p + \beta_p (r_m - r_f) + \gamma_p \times \max \{ r_m - r_f, 0 \} + \varepsilon_p$$

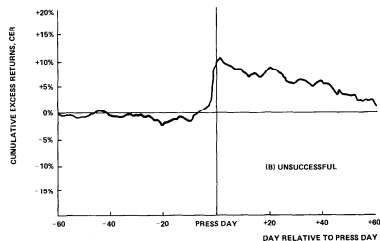
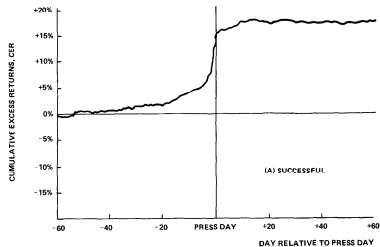
## Example: merger “arbitrage”

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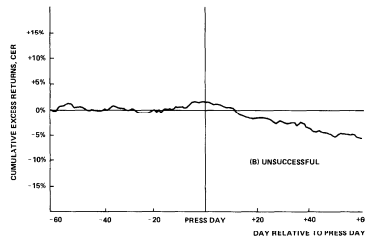
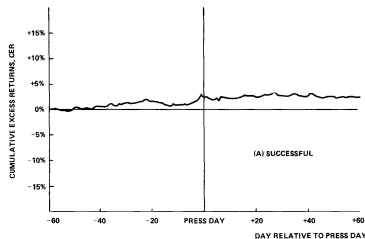
- ▶ In a merger, target is bought at a premium, say 20-30%.
- ▶ At announcement, the price of the target firm increases to a value close to the offer value.
- ▶ But, there remains a “deal spread”, typically around 3%
- ▶ Buy target, if stock deal hedge by shorting acquirer
- ▶ Not sure about the name: not a true arbitrage since it is risky!

# Market reaction at announcement day

## Target firms



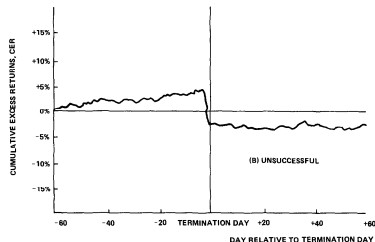
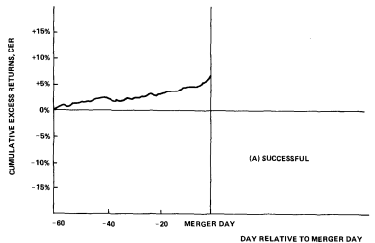
## Acquirer firms



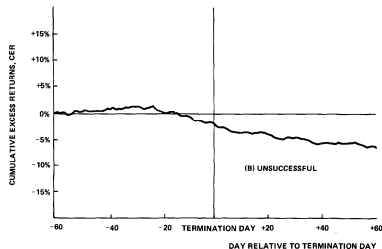
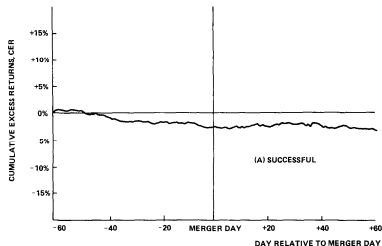


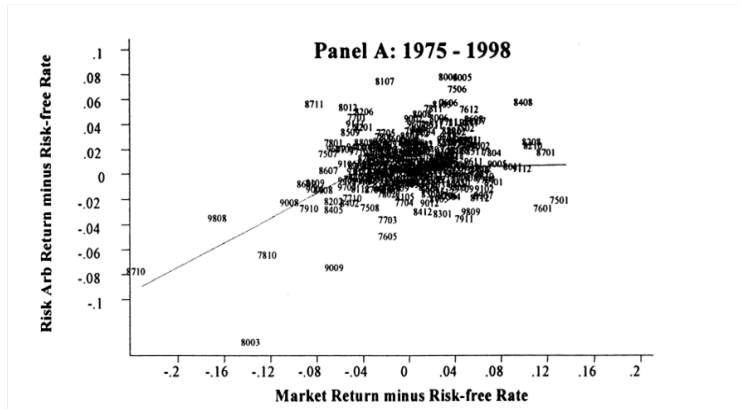
# Market reaction until merger day

## Target firms



## Acquirer firms





- ▶ Source: Mark Mitchell and Todd Pulvino, Journal of Finance
- ▶ Small consistent payoff; but a small probability of disaster. Just like the payoff of writing index puts!

- ▶ Even though it's an all-equity strategy (no option positions) dynamic trading gives an option-like payoff.
- ▶ Writing index puts earns a premium. It provides “disaster insurance” to the market. But no need to pay 2+20 to write index puts!
  - ▶ “Alpha”, “beta”, benchmark, performance evaluation should be relative to the strategy of writing index puts!
  - ▶ (Mitchell and Pulvino are now running a merger-arb hedge fund, so at least they think such alpha is there.)

- ▶ Factor models are a useful way of summarizing the data
- ▶ They help you answer the question: given that I am investing in these  $J$  strategies, should I add strategy  $J + 1$  to my portfolio?
- ▶ If they assume  $\alpha = 0$ , then factor models are also implicitly a statement about  $m$ . Typically that part is ignored.
- ▶ But perhaps you shouldn't. Another way to view this is as saying that the reason why a given strategy 'beats' the market is because it loses money when  $m$  is high (bad times).