Investment-Specific Technological Change and Asset Prices

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April 13, 2008

Abstract

I show that investment-specific technological change is a source of systematic risk. Unlike neutral productivity shocks, the economy must invest resources to benefit from innovations in investment technology. A positive shock to investment technology is followed by a reallocation of resources from consumption to investment, leading to a negative price of risk. As a proxy for shock and a priced risk factor I use stocks that produce investment goods minus stocks that produce consumption goods. The value of assets in place minus growth opportunities falls after positive shocks to investment technology, which suggests an explanation for the value puzzle. I formalize these insights in a dynamic general equilibrium model with two sectors of production. The model's implications are supported by the data. The IMC portfolio earns a negative premium, predicts investment and consumption in a manner consistent with the theory, and helps price the value cross section.

1 Introduction

The second half of the twentieth century saw remarkable technological innovations, the majority of which took place in equipment and software. However, technological innovations

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affect output only to the extent that they are implemented through the formation of new capital stock. Since firms need to invest if they are to benefit from advances in technology, these innovations do not necessarily benefit all firms equally. I refer to these types of innovations as investment-specific technology shocks, thus differentiating it from the technological shock that affects all capital, as in the standard neoclassical model. In this paper, I argue that investment-specific technological change is a source of the systematic risk that is responsible for some of the cross-sectional variation in risk premia, both between different sectors in the economy and between value and growth firms.

I present a dynamic general equilibrium model that links investment-specific technological change to asset prices. In contrast to the standard one-sector model, shocks to investment technology (I-shocks) do not directly affect the production of the consumption good. Instead, they alter the real investment opportunity set in the economy by lowering the cost of new capital goods. Since the old capital stock is unaffected, the economy must invest to realize the benefits. As the economy trades off current against future consumption, there is a reallocation of resources from the production of consumption goods to investment goods. Because the economy is willing to give up consumption today, the marginal value of a dollar must be high in these states of the world. Therefore, stocks are more expensive if they pay off in states when real investment opportunities are good, or equivalently, when the investment-specific shock has a negative premium. The types of firms that are likely to do well in these states are firms that produce capital goods and firms with a lot of growth opportunities.

My model provides new empirical implications about the cross-section of stock returns. Since investment-specific technological change is not directly observable, direct empirical tests are difficult to implement. However, one of the advantages of my model is that by using the cross-section of stock prices, it provides restrictions that help identify investment-specific technological change. The model features two sectors of production, one that produces the consumption good and the other that produces the investment good. In this context, investment-specific technological change benefits firms that produce capital goods relative to firms that produce consumption goods. As a result, a portfolio of stocks producing investment goods minus stocks producing consumption goods (IMC) spans the investment shock. I construct an empirical equivalent of this portfolio and use it as a proxy for investment technology shocks.

The first implication of the model is that the investment-specific shock carries a negative premium. Therefore, that firms that are positively correlated with investment-specific shocks should have lower average returns. I show that sorting individual firms into portfolios on covariances with the IMC portfolio generates a spread in average returns that is not explained by either the CAPM or the Consumption CAPM (CCAPM). On the other hand, a two-factor model that includes the IMC portfolio successfully prices the spread. I repeat the procedure using the entire cross-section of stock returns, following Fama and French (1992). Overall, I find that the estimates of the risk premium are negative and similar, regardless of the test assets used.

The second implication is that the value of assets in place relative to the value of growth opportunities has a negative correlation with the I-shock. This negative correlation is important, because it offers a novel explanation for the value puzzle: a positive I-shock lowers the cost of new investment, which causes the value of future growth opportunities to increase and the value of assets in place to fall. As a result, growth stocks have lower expected returns because they do well in states in which real investment opportunities are good and the marginal value of wealth is high. I find that including the IMC portfolio in the (C)CAPM dramatically improves the ability of the model to price the cross-section of stocks sorted by book to market.

The model identifies the IMC portfolio as a proxy for investment-specific technological change. To examine this restriction more carefully, I first show that positive returns on the IMC portfolio are followed by an increase in the quantity of investment and a fall in the quality-adjusted relative price of new equipment. The fall in the price of new equipment indicates that the IMC portfolio at least partially reflects a shock to the supply of investment. Next, I consider the model's predictions about consumption and leisure. In the model, consumption falls in the short run but increases in the long run, and leisure temporarily falls. I find that both leisure and the discretionary component of consumption falls following positive returns on the IMC portfolio. The long-run response of consumption on IMC is positive, but not statistically significant.

As an additional robustness check, I use the observed investment-output ratio to further examine the implications of my model. Using the calibrated solution of my model, I invert the investment-output ratio to back out the normalized investment shock implied by the data. I find that the stochastic discount factor (SDF) implied by the model that uses the extracted shock is consistent with my earlier conclusions. Growth stocks and investment firms have higher correlation with the implied SDF than value stocks and consumption firms, implying lower returns.

The paper is organized as follows. In Section 2 I discuss related studies. In Section 3 I presents a general equilibrium model in which the capital stock in the investment sector is fixed, and in Section 4 I calibrate the model. In Section 5 I present the empirical results. Section 6 concludes. The Appendix contains all technical details.

2 Related studies

My work is motivated by two separate empirical observations, one in macroeconomics and one in finance. First, the macroeconomic literature documents a negative correlation between the price of new equipment and new equipment investment. Greenwood, Hercowitz, and Krussell (1997, 2000) interpret this finding as evidence for investment-specific technological change and show that it can explain both the long-run behavior of output and its short-run fluctuations. They calibrate a Real Business Cycle (RBC) model with investment-specific technological change, using the price of new equipment as a proxy for the realizations of the investment-specific shock. They show that this shock can account for a large fraction of both short- and long-run output variability, in magnitudes of 30% to 60%. Fisher (2006) treats the investment shock as unobservable and uses a similar model to derive long-run identifying restrictions on a Vector Auto Regression (VAR). In Fisher's model, the identified investment technology shock explains up to 62% of output fluctuations over the business cycle. Justiniano, Primiceri, and Tambalotti (2008) estimate a medium scale Dynamic Stochastic General Equilibrium (DSGE) model and find that the investment shock accounts for a large fraction of the business-cycle fluctuations in output and hours worked.

Second, in the finance literature, Makarov and Papanikolaou (2008) find that returns for those industries that produce final goods rather than investment goods have different statistical properties. To identify the latent factors that affect stock returns, they use an approach based on identification through heteroscedasticity. One of the factors they recover is highly correlated with the investment minus consumption portfolio and the Value minus Growth (HML) factor of Fama and French (1993). I provide a general equilibrium model that links these two facts by showing that the first implies the second, and that suggests an additional proxy for investment-specific shocks.

Jermann (1998) and Tallarini (2000) are earlier examples of work that explores the asset-pricing implications of general equilibrium models. These studies build on the neoclassical RBC model and focus on aggregate quantities and prices. In this environment these authors find that the equity premium and risk-free rate puzzles are exacerbated, because high risk aversion implies endogenously smooth consumption. They extend the production economy model to allow for cross-sectional heterogeneity in firms, with the explicit purpose of linking firm characteristics, such as book to market and size, to expected returns.

Zhang (2005) builds an industry equilibrium model and Gomes, Kogan, and Zhang (2003), Gourio (2005), and Gala (2006) build general equilibrium models in which investment frictions and idiosyncratic shocks result in ex-post heterogeneity in firms.

My work is most closely related to Gomes, Kogan, and Yogo (2006) who focus on ex-ante firm heterogeneity, i.e., heterogeneity that arises because of differences in the type of firm output rather than differences in productivity or accumulated capital. These authors build a general equilibrium model in which differences in the durability of a firm's output lead to differences in expected returns. They focus on differences between final goods producers; in contrast, I focus on differences between capital-good and final-good producers. My paper is also close to theirs in terms of the empirical methods used, since they also use the Input-Output tables from the Bureau of Economic Analysis to classify firms based on the type of output they produce. However, the general equilibrium models I mention above have a single aggregate shock and therefore imply a one-factor structure for the cross-section of stock returns. As a result, any difference in expected returns must be due to differences in market betas, since the conditional CAPM holds exactly.

Even though true betas are unobservable, Lewellen and Nagel (2004) argue that they could not covary enough with the market premium to justify the observed premia. In addition, models with one shock cannot generate the pattern documented by Fama and French (1993), in which value and growth stocks move together independent of the market portfolio. My model enriches the production technology of a standard general equilibrium asset pricing model by differentiating between types of technological shocks.

Although the production technology in my model is different from the models above, it has been used extensively in the macroeconomic literature. Investment-specific shocks were first considered by Solow (1960) in his growth model with vintage capital. Uzawa (1961) and Rebelo (1991) use a two-sector model to study endogenous growth. Boldrin, Christiano, and Fisher (2001) consider a similar model with two production sectors, habit preferences and investment-specific shocks. They calibrate their model to match the equity premium, but their main focus is on improving on the quantity dynamics. Jovanovic (2007) has no shocks to investment technology, but features "seeds" that can subsequently be converted into trees, and thus delivers similar implications about aggregate quantities. In contrast to these papers, my interest is on deriving implications about the cross-section of stock returns.

My paper is related to recent work that explores the effects of technological innovation on on asset prices. The closest example is Panageas and Yu (2006) who build a general equilibrium model with different vintages of capital, but they focus on the comovement between asset returns and consumption over the long-run. Most of the literature, for instance, Greenwood and Jovanocic (1999), Hobijn and Jovanovic (2001), DeMarzo, Kaniel and Kremer (2006), Pastor and Veronesi (2006), has not focused on the implications of technological innovation for the cross-section of asset prices.

My paper is also related to recent work that links the properties of firm cash flows to expected returns, for example Campbell and Vuolteenaho (2004), Lettau and Wachter (2006), Santos and Veronesi (2006b), and Lustig and Van Nieuwerburgh (2006). These models are based on the observation that growth firms' cash flows of have higher duration than do the cash flows produced by value firms, and thus may be more sensitive to changes in the financial investment opportunity set, in the spirit of Merton's ICAPM (1973). On the other hand, Bansal, Dittmar, and Lundblad (2006), Bansal, Dittmar, and Kiku (2007), Bansal, Kiku, and Yaron (2007) explore the fact that growth and value firms' cash flows have differential properties to explain the value premium. Because all these models are based on an exchange economy, changes in the financial investment opportunity set are either exogenously specified or arise through preferences. Finally, Gourio (2006) and Novy-Marx (2008) argue that the value firms face higher operating leverage and are thus riskier in bad times when the price of risk is high. My work complements the papers above by considering time-varying *real* investment opportunities in a model with production.

My paper is also related to the growing number of studies that explore the ability of the consumption-based model to explain the cross-section of expected returns. These studies focus on measurement issues (Ait-Sahalia, Parker, and Yogo, 2004; and Jagannathan and Wang, 2005); long horizons (Bansal, Dittmar, and Lundblatt, 2005; Parker and Julliard, 2005; and Malloy, Moskowitz, and Vissing-Jorgenson, 2006); conditional versions of the CCAPM (Lettau and Ludvigson, 2001; and Santos and Veronesi, 2006a); or multiple good economies (Lustig and Van Nieuwerburgh, 2005; Pakos, 2004; Piazzesi, Schneider, and Tuzel, 2006; and Yogo, 2006). My paper is also related to the research that explores the empirical implications of production-based models, such as Cochrane (1991, 1996); Li, Vassalou, and Xing (2006); Belo (2006) and Liu, Whited and Zhang (2007).

3 General equilibrium model

I build a general equilibrium model to formalize the idea that investment-specific shocks create a reallocation of resources between the consumption and the investment sector. The two-sector specification I consider is adapted from the model of Rebelo (1991) who studies endogenous growth. I first present a simplified version where the capital stock in the investment sector is fixed. In the Appendix, I solve the general model with two capital stocks and show that the main insights are robust.

3.1 Information

The information structure obeys standard technical assumptions. Specifically, there exists a complete $(\Omega, \mathcal{F}, \mathcal{P})$ probability space supporting the Brownian motion $Z_t = (Z_t^X, Z_t^Y)$. $\mathcal{P}$ is the corresponding Wiener measure, and $\mathcal{F}$ is a right-continuous increasing filtration generated by Z.

3.2 Firms and technology

Production in the economy takes place in two separate sectors, one producing the consumption good (numeraire) and one producing the investment good.

3.2.1 Consumption sector

The consumption goods sector (C-sector) produces the consumption good, C, with two factors of production, sector specific capital K_C and labor L_C

$$C_t \leq X_t K_{C,t}^{\beta_C} L_{C,t}^{1-\beta_C}, \tag{1}$$

where β_C is the elasticity of output with respect to capital. Output in the C-sector is subject to a disembodied productivity shock X that evolves according to

$$dX_t = \mu_X X_t dt + \sigma_X X_t dZ_t^X. \tag{2}$$

The X shock can be interpreted as a neutral productivity shock (N-shock) that increases the productivity of *all* capital in the consumption sector. This is the standard shock in existing one-sector general equilibrium models. The capital allocated to the C-sector depreciates at a rate δ, while the investment in consumption-specific capital is denoted by I_C. Thus, capital in the final goods sector evolves according to

$$dK_{C,t} = I_{C,t}\, dt - \delta\, K_{C,t}\, dt. \tag{3}$$

Investment in the C-sector is subject to adjustment costs. If the firm has capital K_C and wants to increase its capital by I_C, it consumes $c(\frac{I_C}{K_C})K_C$ total units of the investment good, where $c(\cdot)$ is an increasing and convex function. The value of a representative firm in the consumption sector equals

$$S_{C,t} = E_t \int_t^{\infty} \frac{\pi_s}{\pi_t} \left(X_s K_{C,s}^{\beta_C} L_{C,s}^{1-\beta_C} - w_s L_{C,s} - \lambda_s c\left(\frac{I_{C,s}}{K_{C,s}}\right) K_{C,s} \right) ds, \tag{4}$$

where w is the relative price of labor and λ is the relative price of the investment good, or equivalently the cost of new capital.

3.2.2 Investment sector

The investment goods sector (I-sector) produces the investment good using sector specific capital K_I and labor L_I. In the simplified model, the capital stock in the investment sector is fixed. One can therefore think of the investment sector as using land and labor to produce the investment good. The output of the I-sector can be used to increase the capital stock in the C-sector

$$c\left(\frac{I_{C,t}}{K_{C,t}}\right) K_{C,t} \leq Y_t K_I^{\beta_I} L_{I,t}^{1-\beta_I}. \tag{5}$$

The shock Y, which represents the investment shock, affects the productivity of the investment sector. A positive shock to Y increases the productivity of the investment sector, which implies that the economy can produce the same amount of new investment (I_C) using fewer resources (L_I). Therefore, a positive investment shock will imply a fall in the cost of producing new capital, whereas the old capital stock will be unaffected. The elasticity of output with respect to labor in the investment sector equals $1 - \beta_I$. The investment shock follows

$$dY_t = \mu_Y Y_t \, dt + \sigma_Y Y_t dZ_t^Y. \tag{6}$$

Firms in the investment sector represent claims on the land (K_I) used to produce investment goods. The value of a representative firm in the investment sector equals

$$S_{I,t} = E_t \int_t^{\infty} \frac{\pi_s}{\pi_t} \left(\xi_s Y_s K_I^{\beta_I} L_{I,s}^{1-\beta_I} - w_s L_{I,s} \right) ds, \tag{7}$$

where w is the wage and λ is the relative price of the investment good in terms of the consumption good.

Finally, if one defines the investment rate, $i_C \equiv \frac{I_C}{K_C}$, the adjustment cost function takes the form

$$c(i_C) = (c_1 + i_C)^\lambda - c_1^\lambda$$

and c_1 is chosen so that $c'(0) = 1$.

3.3 Households

There exists a continuum of identical households with recursive utility preferences. Households maximize a utility index J, that is defined recursively by;

$$J_0 = E_0 \int_0^\infty h(C_t, N_t, J_t) dt. \tag{8}$$

where C_t is consumption and N_t is leisure that the household enjoys in period t. Following Duffie and Epstein (1992), the aggregator is defined as:

$$h(C, N, J) = \frac{\rho}{1-\theta^{-1}} \left(\frac{(C N^\psi)^{1-\theta^{-1}}}{((1-\gamma)J)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - (1-\gamma)\, J \right)$$

Here ρ will play the role of the time-preference parameter, γ controls risk aversion, and θ the elasticity of intertemporal substitution. Utility is defined over the composite good $C\, N^\psi$, and ψ controls the relative shares of consumption and leisure. These preferences are exactly as in Angeletos and Panousi (2007), while in the time-additive case belong to the class studied by King, Plosser and Rebelo (1988).

Households supply $1 - N_t$ units of labor that can be freely allocated between the two sectors,

$$L_{C,t} + L_{I,t} = 1 - N_t. \tag{9}$$

Shifts in the allocation of labor between the two sectors allow the economy to intratemporally trade off consumption versus investment. [1]

Households trade a complete set of state contingent securities in the financial markets. Finally, the parameters in the model are assumed to satisfy

$$u \equiv \rho\theta\frac{1-\gamma}{\theta-1} - (1-\gamma)(\mu_X - \beta_C\delta) + \frac{1}{2}\sigma_X^2\, \gamma(1-\gamma) > 0 \tag{10}$$

[1] An alternative interpretation of L is as a perishable good which can be used as input in either of the two sectors or consumed directly, for example oil.

and

$$\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 > 0$$

The first restriction ensures that the value function for the social planner is bounded from below, whereas the second ensures that the state variables have a stationary distribution.

3.4 Discussion

3.4.1 Investment frictions

The model departs from the neoclassical growth model along two dimensions. The first is the introduction of convex adjustment costs. The second is that capital goods are produced in a different sector. Equation (5) implies that there is an upper bound on the investment rate in the consumption sector, because aggregate investment cannot exceed the output of the I-sector. This upper bound depends on labor allocated in the I-sector and the investment-specific shock Y.

My two-sector model can be reinterpreted as a one-sector model with stochastic adjustment costs. In a one-sector model without adjustment costs the relative price of the investment good is always one, because investment can be transformed into consumption in a linear fashion. The introduction of convex adjustment costs imposes some curvature on the investment-consumption possibility frontier, since the marginal cost of investment rises with the rate of investment. In contrast, my model features a stochastic investment-consumption possibility frontier. Specifically, allocating more labor to the investment sector increases investment at the cost of consumption, and the tradeoff depends on the state of technology in the investment sector, Y.

3.4.2 Labor-Leisure Tradeoff

The model departs from general equilibrium asset pricing models by incorporating a labor-leisure tradeoff. The model specifies a non-separable specification between leisure and consumption, which implies that marginal utility of consumption is increasing in labor supplied. When households work very hard, they really value that extra unit of consumption. If one chooses to interpret labor as an input good common to both sectors that is in fixed supply, the model also implies that households can derive utility from direct consumption of that good.

Although fairly common in the real business cycle literature, this specification has not been very common in asset pricing models. A possible reason for this is that in a one-sector model with only neutral shocks, this specification tends to attenuate the equity premium. Following a positive shock, investors consume more but they also work harder, which means that marginal utility falls by less than it otherwise would had labor supply been fixed.

In contrast, here, the non-separability between consumption and leisure will increase the volatility of marginal utility and thus enhance risk premia. Following a positive investment shock, households consume less and work harder, that is the two effects reinforce each other.

3.5 Competitive equilibrium

Definition 1. *A competitive equilibrium is defined as a collection of stochastic processes* C^*,N^*, K_C^*, L_I^*, L_C^*, I_C^*, π^*, λ^*, w^* *such that (i) households chose* C^* *to maximize (8) given* w^* *and* π^* *(ii) firms choose* I_C^*, L_I^* *and* L_C^*, *given* π^*, w^* *and* λ^*, *to maximize (4) and (7) (iii)* K_C^* *satisfies equation (3) given* I_C^* *(iv) all markets clear according to (1),(5) and (9)*

In this section I focus on the social planners problem, that is the problem of optimal allocation of labor. I demonstrate in the Appendix that, as in other models with dynamically complete financial markets and no externalities, a competitive equilibrium can be constructed from the solution to the social planner's problem.

Proposition 1. *The social planner's value function is*

$$J(X, Y, K_C) = \frac{(X\, K_C^{\beta_C})^{1-\gamma}}{1-\gamma} f(\omega),$$

where $\omega \equiv \ln\left(\frac{Y}{K_C}\right)$ *and* $f(\omega)$ *satisfies the ODE*

$$\begin{aligned} 0 \;=\; & \left\{ \rho \frac{1-\gamma}{1-\theta^{-1}} f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \left(L_C^{*\,1-\beta_C}(1-L_C^*-L_I^*)^{\psi}\right)^{1-\theta^{-1}} + c^{-1}\left(e^{\omega} K_I^{\beta_I} L_I^{*\,1-\beta_I}\right)\left(\beta_C(1-\gamma) f(\omega) - f'(\omega)\right) + \right. \\ & \left. -u\, f(\omega) + (\mu_Y + \delta\beta_C) f'(\omega) + \frac{1}{2}\sigma_Y^2 (f''(\omega) - f'(\omega)) \right\}. \end{aligned}$$

The allocation of labor between the two sectors is given by $L_C^* = \frac{(1-\beta_C)(1-l(\omega))}{1+\psi-\beta_C}$ *and* $L_{I,t}^* = l(\omega)$ *where*

$$\begin{aligned} l(\omega) \;=\; & \arg\min_{l} \left(\rho \frac{1-\gamma}{1-\theta^{-1}} f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \left(\left(\frac{(1-\beta_C)(1-l)}{1+\psi-\beta_C} \right)^{1-\beta_C} \left(1 - \frac{(1-\beta_C)(1-l)}{1+\psi-\beta_C} - l \right)^{\psi} \right)^{1-\theta^{-1}} + \right. \\ & \left. + c^{-1} \left(e^{\omega} K_I^{\beta_I} l^{1-\beta_I} \right) \left(\beta_C (1-\gamma) f(\omega) - f'(\omega) \right) \right) \end{aligned}$$

The state variable, ω, has dynamics

$$d\omega_t = \left(\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - i_C(\omega)\right) dt + \sigma_Y dZ_t^Y.$$

where

$$i_C(\omega) = c^{-1}\left(e^{\omega} K_I^{\beta_I} l(\omega)^{1-\beta_I}\right)$$

Proof *See Appendix.*

I solve for equilibrium policies numerically, and the details of the solution are shown in the Appendix.

In equilibrium, there is only one state variable that determines optimal policy, ω, and it can be interpreted as the ratio of "effective" capital stocks in the two sectors. Most importantly, innovations to ω come only from the I-shock (Y). Finally, as long as $\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 > 0$, the state variable has a unique stationary distribution. This guarantees that in equilibrium one sector does not dominate the economy.

The main mechanism that determines the price of risk for the I-shock is the allocation of labor between the sectors, $l(\omega)$. The behavior of $l(\omega)$ can be understood from the first order conditions of the planner's problem evaluated. To obtain some intuition, consider the case where investors have time-separable preferences ($\gamma = \theta^{-1}$), and labor supply is fixed ($\psi = 1$).

$$\pi_t = e^{-\rho t} U_C, \tag{11}$$

$$\xi_t = \frac{J_{K_C}}{U_C}\frac{1}{c'(i_{C,t}^*)} = X_t K_{C,t}^{\beta_C - 1}\frac{\beta_C(1-\gamma)f - f'}{(1-\gamma)(1-l(\omega))^{\gamma(\beta_C-1)}}\frac{1}{c'(ae^{\omega_t}l(\omega_t)^{1-\beta_I})}, \tag{12}$$

$$\lambda Y \frac{\alpha K_I^{\beta_I}}{X K_C^{\beta_C}} = \frac{l(\omega)^{1-\beta_I}}{(1-l(\omega))^{1-\beta_C}}\frac{1-\beta_C}{1-\beta_I}. \tag{13}$$

Here, π is the shadow cost of the resource constraint in the C-sector, (1), and $\lambda\pi$ is the shadow cost of the resource constraint in the I-sector, (5). This implies that π is the state price density and that λ is the relative price of the investment good in terms of the consumption good.

Equation (11) is standard and states that, in equilibrium, the marginal valuations in each state equals the shadow cost of the resource constraint in the C-sector, which is the state price density.

Equation (12) states that the relative price of output in the I-sector, ξ, equals the marginal value of capital in the C-sector divided by the marginal installation cost and by marginal utility. In the one sector model without adjustment costs, the relative price of the investment good is always one and marginal utility equals the marginal value of capital. In my model, ξ is a function of the I-shock, because a positive investment shock increases the supply of the investment good and therefore lowers its relative price. In addition, ξ depends on the N-shock (X), because a positive shock to productivity in the consumption sector increases the demand for the investment good and therefore its relative price. This is the reason why, in equilibrium, the N-shock affects both sectors symmetrically.

Equation (13) states that in equilibrium, the marginal product of labor in both sectors must be equal. This condition determines $l(\omega)$. The effect of the investment-specific shock on the allocation of labor depends on how Y affects $\xi(Y,\cdot)Y$. As long as $\xi(Y,\cdot)Y$ is increasing in Y, a positive shock to Y increases the profits of firms in the investment sector as well as the marginal product of labor in the I-sector. Therefore, the allocation of labor to the I-sector, $l(\omega)$, must temporarily increase, inducing a fall in consumption. In the future, the capital stock in the consumption sector increases, reversing the short-run fall in consumption. The end result is that consumption displays a U-shaped response to a positive investment shock. Consumption initially falls, as more resources are allocated to the I-sector in order to take advantage of the improvement in technology. Eventually, the new technology starts bearing fruit and the growth rate of consumption increases.

In general, investors will evaluate states based on three things: their consumption in that state (C_t), their leisure (N_t) and their continuation utility (J_t). The decision how much to work and how to allocate between the two sectors affects consumption and leisure contemporaneously and their continuation utility through investment. In this economy, the pricing kernel or stochastic discount factor will take the form:

$$\frac{d\pi_t}{\pi_t} = -r_{f,t}\,dt - b_Y(\omega_t)\,dZ_t^Y - \gamma\,dZ_t^X \tag{14}$$

where

$$b_Y(\omega_t) = -\left((\theta^{-1}(1-\beta_C) + \psi(\theta^{-1}-1))\frac{l'(\omega)}{1-l(\omega)} + \frac{f'(\omega)}{f(\omega)}\frac{\gamma-\theta^{-1}}{\gamma-1} \right)$$

The function $b_Y(\omega_t)$ reflects the price of risk for the investment-specific shock. When solving the model, I find that $l(\omega)$ is an increasing function of ω. This is important because it means that both consumption and leisure temporarily *fall* after a positive I-shock, which tends to increase investors' valuation of that state. However, the labor allocation decision also affects investor's continuation utility through investment, so continuation utility will be higher after a positive I-shock. Given that $f'(\omega)/f(\omega) < 0$, if investors have preference for later resolution of uncertainty , i.e. $(\gamma\theta < 1)$, this will increase their valuation of that state, whereas if they have preferences for early resolution of uncertainty $(\gamma\theta > 1)$, this will tend to lower state prices.

4 Computation and Calibration

In this section I present the numerical solution of the model. The solution details are delegated to the Appendix.

4.1 Quantities

In figures 5(a)-5(f) I plot certain key variables of the model as a function of the state variable ω, holding K_C and X fixed at 1. On the one hand, both consumption and leisure are declining functions of ω. States where the productivity of the investment sector is high relative to the capital stock (high ω states), are high marginal valuation states, as shown in figure 5(f). The price of new capital goods is a declining function of ω, which increases Tobin's Q.

In figure 6, I plot the dynamic responses to an investment-specific shock, with the time-period being 1 year. All quantities are computed relative to a steady state benchmark $d\omega = 0$ as responses to a one-standard deviation shock in ω.

Figure 6(a) shows that consumption falls in the short run, as more resources are allocated to the investment sectors, but increases permanently in the long run relative to the old steady-state. Output, as shown in figure 6(c) displays a similar response. Figures 6(b) and 6(d) show that labor supply and investment initially increase and then fall back to its initial level. The price of new capital goods, in figure 6(e), falls after the shock and then slowly increases but at a level lower than the old steady state, because the new steady state features a higher level of capital accumulation so in equilibrium the price of new capital is lower. Tobin's Q in figure 6(f) increases following the investment shock and then falls back to its equilibrium level. Finally, in figure 6(g) one can see that the investors' marginal valuation (state price density) increases following the shock and then falls to a level below the old steady state, as the economy features more capital relative to the old steady state and therefore more consumption.

4.2 Asset prices

4.2.1 Investment and consumption firms

In the model there are two representative firms, one producing the consumption good and one producing the investment good. The market value of each firm is

$$
\begin{aligned}
S_t^C &= E_t \int_t^\infty \frac{\pi_s}{\pi_t} \left(X_s K_{C,s}^{\beta_C} (L_{C,s}^*)^{1-\beta_C} - w_s L_{C,s}^* - \xi_s c \left(\frac{I_{C,s}^*}{K_{C,s}^*} \right) K_{C,s}^* \right) ds, \qquad (15) \\
S_t^I &= E_t \int_t^\infty \frac{\pi_s}{\pi_t} \left(\xi_s \alpha Y_s K_I^{\beta_I} (L_{I,s}^*)^{1-\beta_I} \right) ds. \qquad (16)
\end{aligned}
$$

The value of each sector includes all cashflows accruing to the owners of the capital stock.

Proposition 2. *The ratio of the value of the investment goods sector, S_t^I, over the consumption goods sector, S_t^C, equals*

$$
\frac{S_t^I}{S_t^C} = \frac{\beta_I f'(\omega)}{\beta_C (1-\gamma) f(\omega) - f'(\omega)},
$$

and is an increasing function of ω.
Proof *See Appendix.*

A positive I-shock increases the productivity of the investment sector and there increases its value relative to the consumption sector. In figure 5(h) I plot the relative value of the investment sector as a function of ω, and in figure 6(k) I plot the response of the relative valuation of the two sectors following a one standard shock to investment-technology. Consistent with the proposition above, a positive shock to investment technology increases the value of investment firms relative to consumption firms. This has two important implications.

The first is that the relative value of the two sectors is a state variable in the economy, since it is a monotone transformation of ω. The same is not true for the relative price of the investment good, ξ, which also depends on the N-shock, as shown in equation (12). Therefore, the cross-section of stock prices may contain additional information about real investment opportunities relative to the prices of investment goods or Tobin's Q. This information can be used to identify investment-specific technological change in the data.

The second implication is that a portfolio of investment minus consumption stocks (IMC) is positively correlated with the I-shock. Hence, I can use returns on the IMC portfolio to test the asset pricing implications of the model, namely that the pricing kernel loads on the I-shock with a negative premium. If the model is true, then the IMC portfolio should have negative expected returns after adjusting for market risk. Additionally, stocks that load positively on the IMC portfolio should have lower expected returns than stocks with low loadings on IMC. Furthermore, if the IMC portfolio spans a systematic source of risk, it should be able to explain the variation of realized returns.

4.2.2 Market portfolio

The sum of the market values of the two sectors equals the market portfolio or the value of the entire dividend stream [2]

$$\begin{aligned} S_t^M = S_t^C + S_t^I &= E_t \int_t^\infty \frac{\pi_s}{\pi_t}(C_s - w_s)\, ds \\ &= \frac{(X_t K_{C,t}^{\beta_C})^{1-\gamma}}{1-\gamma} \left(\beta_C\,(1-\gamma) f(\omega) + (\beta_I - 1) f'(\omega)\right). \end{aligned} \tag{17}$$

Returns on the market portfolio are driven by innovations in the neutral shock, dZ^X, and the investment shock, dZ^Y. When solving the model, I find that the value of the market portfolio is positively correlated with the neutral shock and negatively correlated with the investment shock. The latter can be seen in figures 5(g). When computing the response of the value of the market portfolio to the investment-specific shock, I find that it falls in the short run but it increases in the long run. A positive I-shock increases future dividends but increases discount rates. If agents have preferences such that $\theta < 1$, the discount rate effect dominates and leads to a fall in the price of the dividend stream. Given that the I-shock has a negative price of risk, this helps increase the equity premium. As shown in figure 6(j), in the long-run, the investment shock leads to higher levels of capital accumulation and therefore to a higher value of the market portfolio.

The dynamic effects of the investment shock to the value of the stock market can be alternatively interpreted as follows. Suppose that we interpret a positive investment shock as the availability of a new type of capital which has the same productivity as the old capital but is cheaper. This will lower the valuation of existing or old capital, as it becomes obsolete, and therefore lower the value of the stock market. In the long run however, the economy will accumulate more of the new type of capital, which is "better", so the valuation of the stock market will rise.

[2] one needs to differentiate between the value of the dividend and consumption streams. I refer to the former as the 'market' portfolio and the latter as the wealth portfolio.

4.2.3 Value versus growth

Because markets are complete, claims to cashflows can be decomposed in such a way as to create the analog of value and growth firms. The value of a firm can be separated into the value of assets in place and the value of future growth opportunities, as in Berk, Green and Naik (1999). When investment is frictionless, existing assets represent the entire value of the firm, since a perfectly elastic supply of capital means that all future projects have zero NPV. In contrast, frictions in investment prevent the supply of capital from being perfectly elastic, hence future growth opportunities have additional value.

I focus on the value of assets in place and growth opportunities in the C-sector, since in the baseline model the capital stock in the I-sector is fixed. The value of assets in place in the consumption sector equals the value of all dividends accruing from existing assets

$$S_t^V = \max_{\hat{L}_{C,s}} E_t \int_t^\infty \frac{\pi_s}{\pi_t} \left(X_s (K_{C_t} e^{-\delta(s-t)})^{\beta_C} \hat{L}_{C,s}^{1-\beta_C} - w_s \hat{L}_{C,s} \right) ds. \tag{18}$$

In the absence of arbitrage, the value of growth opportunities must equal the residual value

$$S_t^G = S_t^C - S_t^V. \tag{19}$$

This decomposition creates fictitious value and growth firms in the economy without the cost of modeling individual firms explicitly.

Lemma 1. *The relative value of assets in place over growth opportunities in the consumption sector equals*

$$\frac{S_t^V}{S_t^G} = \frac{g(\omega)}{\beta_C(1-\gamma)f(\omega) - f'(\omega) - g(\omega)},$$

where the function $g(\omega)$ satisfies the ODE in the Appendix.

Proof *See Appendix.*

Lemma 1 implies that innovations to S_t^V/S_t^G are independent of the neutral productivity shock, dZ^X, and are spanned by the investment-specific shock, dZ^Y.

I find that the relative value of assets in place minus the value of growth opportunities in the consumption sector is decreasing in ω, as shown in figures 5(i) and 6(l). This is important because it offers a novel explanation for the value effect. A positive I-shock lowers the cost of new capital in the C-sector, increasing the value of future growth opportunities relative to the value of installed assets. Therefore, since the I-shock carries a negative premium, this implies that growth stocks have lower average returns than value stocks.

In addition to explaining the value premium, my model can explain the findings of Fama and French (1993) that the HML portfolio can explain the cross-section of realized as well as expected stock returns, or that it represents a source of systematic risk not spanned by the market portfolio. Lemma 1 implies that a portfolio of value minus growth stocks also spans the investment-specific shock, creating comovement between value and growth firms. Conversely, models that explain the value premium with only one aggregate shock, cannot generate comovement in value and growth firms independent of the market portfolio. The presence of a second aggregate shock highlights one crucial difference between this paper and Gomes, Kogan and Zhang (2003) and Gala (2006) who argue that value firms are riskier than growth firms in bad times, and hence a conditional CAPM should price the value spread. In this paper, the value premium arises due to exposure to a second aggregate shock, the I-shock, and therefore the conditional CAPM does not hold.

The decomposition of aggregate value into value of assets in place and future growth opportunities though stylized, helps focus on the direct effects of investment shocks on aggregate dynamics, instead of on indirect effects through the aggregation heterogenous firms. A potentially interesting extension would explore firm-level investment decisions in more detail by incorporating firm level productivity shocks and possibly other frictions, as for example in Kogan and Papanikolaou (2008).

4.3 Parameters

I calibrate the model using the parameters in table 7. The value of θ falls within the confidence interval reported by Hall (1988), and falls between the values used in Campanale, Castro and Clementi (2007) and Bansal, Kiku and Yaron (2007). The value $\psi = 0.25$ is from Christiano and Eichenbaum (1992). I pick a conservative value for $\gamma = 3$, while the values for β_I and β_C imply that capital earns a share of 30% of output which is a fairly standard parametrization. The remaining parameters are picked to roughly match the first two moments of investment and consumption growth, the level of the risk-free rate, and the investment to output ratio.

4.4 Results

I compute unconditional moments as follows. First, I back out the conditional moments M_t from the solution of the model and its higher derivatives. Next, I compute the stationary distribution of ω. The invariant distribution, $p(\omega)$, is the solution to the Kolmogoroff Forward equation:

$$0 = -(\mu_Y - \frac{1}{2}\sigma_Y^2 + \delta - i_C(\omega))p'(\omega) + i'_C(\omega)p(\omega) + \frac{1}{2}\sigma_Y^2\, p''(\omega)$$

subject to

$$\int_{-\infty}^{\infty} p(\omega)d\omega = 1$$

Finally, I compute the unconditional moments $\overline{M} = E(M(\omega))$ as

$$\overline{M} = \int M(\omega)p(\omega)d\omega$$

In general, these are very different than the conditional moments evaluated at the mean state of nature, i.e. $M(E(\omega))$, which is what one obtains when log-linearizing the model around $E(\omega)$. In particular, the risk premia are often off by a factor of two and three. This point is also raised by Campanale, Castro and Clementi (2007).

The unconditional moments generated by the model are show in Table **??**. One can see that the model does a reasonable job capturing the targeted moments. In addition, the model's implied Tobin's Q is reasonably close to the data (1.37 vs 1.53), and so is the average investment rate, (0.25 vs 0.23). The model generates an equity premium of 4.1%, while matching the volatility of the stock market.

In addition, the model also implies that investment firms have returns that are on average 1.9% lower than consumption firms, which is close to the number in the data (1.6%). Moreover, the model predicts that a portfolio that is long the value of growth opportunities will underperform a portfolio that is long the value of assets in place by 4.4%. In the absence of within-sector firm heterogeneity, I refer to this as the fictitious value premium in the model. If one views growth firms as deriving a higher fraction of their value from growth opportunities than value firms, this number provides an upper bound on the actual value premium in the model.

Finally, the model underperforms along two dimensions. The first is that it delivers a highly volatile risk-free rate, with volatility roughly twice than what's in the data (6.7% vs 3.2%). The second is that the correlation between consumption and investment growth is too low (15% vs 54%). The low correlation stems from the high volatility of the investment shock (Y), that leads to a substitution between consumption and investment.

5 Empirical evidence

The model in the previous section has a number of empirical implications about the cross-section of stock returns. I provide evidence that these implications are supported by the data.

The first implication is that investment-specific technological change earns a negative risk premium. To see this, note that the pricing kernel implied by the model can be linearized as

$$\pi = a - b_Y dZ^Y - b_X dZ^X. \tag{20}$$

This is the empirical equivalent of equation (14) linearized around $E(\omega)$. The model implies that shocks to investment technology, dZ^Y, have a negative price of risk, or equivalently that $b_Y < 0$. A positive investment-specific shock lowers the cost of new capital and thus acts as a shock to the real investment opportunities in the economy, increasing the marginal value of wealth and therefore state prices. As a result, firms that covary positively with the investment-specific shock should have, ceteris paribus, lower average returns.

The second implication is that a portfolio that is long the value of assets in place and short the value of growth opportunities is negatively correlated with the I-shock, and should therefore earn a positive premium. To the extent that value firms have more assets in place and fewer growth opportunities than growth firms, sorting stocks on book to market should produce a positive spread in returns that is explained by their covariance with a proxy for the I-shock. Sorting stocks on book to market is well known to produce portfolios that are mispriced by the CAPM. This is the well documented value puzzle.

However, in order to test the empirical implications of the model, it is necessary to identify investment-specific technological change in the data. One of the advantages of the model is that it provides restrictions that identify investment-specific shocks. As shown in Propositions 1 and 2, random fluctuations in the ratio of the value of the investment goods sector, $S_{I,t}$, to that of the consumption goods sector, $S_{C,t}$, are driven only by the investment-technology shock, dZ_t^Y, and are unrelated to the neutral productivity shock, dZ_t^X. In other words, the investment shock is spanned by a zero-investment portfolio that is long the investment goods sector and short the consumption goods sector. Thus, the cross-sectional implications of risk exposure to the investment shock can be investigated by including returns to this portfolio in standard factor pricing models. This enables me to identify innovations in investment-specific shocks by the return on an investment minus consumption portfolio. Later, I will further investigate this model-implied restriction and also use the investment to GDP ratio as an alternative.

5.1 Investment minus consumption portfolio

Ideally, the distinction between firms producing investment and consumption goods would be clear and the new factor portfolio would be straightforward to obtain. However, many companies produce both types of goods. In order to overcome this difficulty, I classify industries as consumption or investment good producers using information from the US Department of Commerce's NIPA tables. [3] I classify industries as investment or consumption producers based on the sector they contribute the most value, with the full procedure described in the Appendix.

The composition details are displayed in table 1. The sector producing consumption goods is much larger than the sector producing investment goods, both in number of firms and in term of market capitalization. Further, the consumption and investment portfolio have fairly similar ratios of book to market equity. As a robustness check, I construct an investment minus consumption portfolio using the data provided by Gomes, Kogan and Yogo (2006). I create this portfolio, labeled IMC_{GKY}, by subtracting from the investment portfolio the non-durables portfolio. The correlation of the IMC portfolio with IMC_{GKY} and the Fama-French factors is displayed in table 2. The two proxies are highly correlated. The IMC portfolios have negative correlation with HML and small but positive correlation with the market. The negative correlation with HML is consistent with the model, since the value of assets in place minus growth opportunities is negatively correlated with the I-shock. Finally, the IMC portfolio has a positive correlation with the SMB factor, which stems from the fact that investment firms tend to have smaller market capitalization.

Table 3 shows average returns on the three IMC portfolios along with their CAPM alpha and their correlation with HML, broken down by decade. The IMC portfolios have negative average returns, and a negative CAPM alpha. This is consistent with the results of Gomes, Kogan and Yogo (2006) and Makarov and Papanikolaou (2006). More importantly, the IMC portfolios have higher correlation with HML in the later half of the sample, during which average returns and CAPM alphas on IMC and HML are higher in magnitude. This evidence is consistent with the dramatic surge in innovations in investment goods over the last few decades, such as personal computers and the internet. An increase in the volatility of the investment shock would translate into a higher premium for HML and IMC and would

[3] A similar procedure is followed by Chari, Kehoe, and McGrattan (1996), Castro, Clementi and MacDonald (2006) and Gomes, Kogan and Yogo (2006).

increase the correlation between the two.[4]

5.2 Cross-sectional tests

If I-shocks are spanned by the investment minus consumption portfolio and earn a negative premium , then sorting stocks on their covariances with IMC into portfolios should produce a negative spread in expected returns that is not explained by the market portfolio. Moreover, the IMC portfolio should be able to price this spread. Table 5 presents summary statistics on 10 portfolios of stocks constructed by sorting on covariances with IMC. This sort produces an almost monotone decline in average returns and a spread of roughly 2.0% annually. However, it is more informative to consider CAPM alphas because the portfolios have different risk profiles, as evidenced by the increasing pattern of both the standard deviation and the market beta of each portfolio. The pattern displayed by the pricing errors is more striking. The pricing errors of the portfolios decline almost monotonically from 2.6% to -2.4% annually. The pattern is dampened but not eliminated if one looks at alphas from the Fama-French three factor model. Finally, including the IMC portfolio in the CAPM significantly reduces the pricing errors. The GRS F-test rejects both the CAPM and the three factor model at the 1% level, whereas it fails to reject the model with the market and IMC portfolio at the 10% level.

The pricing kernel in equation (20) summarizes all the cross-sectional asset pricing implications of the model. The restrictions on the rate of return of all traded assets that is imposed by no arbitrage,

$$E[\pi R] = 1, \tag{21}$$

can be used to estimate (20) by the generalized method of moments. Accordingly, I estimate the model using two-stage GMM with the details described in the Appendix[5]. I report the mean absolute pricing error (MAPE), the sum of squared pricing errors (SSQE) and the J-test of the over-identifying restrictions of the model, namely that all the pricing errors are zero.[6] I use returns on the CRSP value-weighted portfolio and monthly non-durable consumption

[4] In the model, IMC and HML are perfectly correlated, because they are both spanned by the investment-specific shock. In the presence of additional aggregate shocks, this need no longer be the case. Nevertheless, an increase in the volatility of the investment shock would increase the correlation between IMC and HML, which is consistent with the evidence in the data.

[5] The estimated parameters from the first and second stage are qualitatively and quantitatively similar.

[6] I choose to report the sum of squared errors rather than the normalized sum of squared errors (R^2) because in the absence of a constant term it is not clear what the benchmark (i.e. the normalization term) is.

growth from NIPA as empirical proxies for the N-shock and focus on the period 1961-2005. I use the return on the IMC portfolio as proxy for the I-shock.[7] I compare the performance of the two models incorporating IMC with the CAPM, the CCAPM and the Fama-French three factor model. Finally, the model implies that HML derives its pricing ability through its exposure to the investment shock. I therefore include both HML and IMC in the same specification in order to see if each factor has additional pricing ability in the presence of the other. Because estimating (20) using the entire cross-section of stock returns can be problematic, the literature focuses on a particular subset of assets, which are portfolios of stocks sorted on economically meaningful characteristics.

This paper examines on whether investment-specific shocks are an important component of the pricing kernel. I therefore focus on the estimate of b_Y, rather than on the overall ability of the model to price each cross-section, which might depend on the particular choice of proxy for the neutral productivity shock. Most importantly, because (20) must hold for *all* traded assets in the economy, estimates of b_Y should be robust to using different test assets. To this end, I estimate (20) using a number of different cross-sections.

5.2.1 Risk-sorted portfolios

I consider as test assets portfolios of stocks sorted first by industry and then covariance with the IMC portfolio.[8] The construction of these portfolios is standard and is described in the Appendix. I estimate (20) using the above set of test assets.[9]

Table 6 reports estimates using 24 portfolios sorted first on industry and then on IMC covariance. The CAPM and (C)CAPM perform rather poorly in this cross-section, with risk prices on the market portfolio and consumption growth not statistically different from zero. Both models generate large absolute pricing errors, though the J-tests fail to reject each model due to the large standard error of the pricing errors. The Fama-French three factor model performs significantly better, with smaller absolute pricing errors and with half the variance of pricing errors. However, this improvement is mostly driven by the negative and

[7] Results using the IMC_{GKY} portfolios qualitatively and quantitatively very similar and are available upon request.

[8] Nagel, Lewellen and Shanken (2006) caution that if the cross-section of test assets has a strong factor structure, then some factors may appear to price the test assets arbitrarily well if they are correlated with the common factors. This is an argument against evaluating an asset pricing model only on a set of portfolios with a strong factor structure (i.e. the 25 BM/ME) or portfolios that are only sorted on covariances with the proposed factor.

[9] In an earlier version of the paper, I also reported results for portfolios sorted first on Size or Market Capitalization and then covariance with the IMC portfolio. The results are very similar to the cross-section of Industry-IMC covariance sorted portfolios, so to conserve space they are not reported.

statistically significant premium on SMB, whereas the premium on HML is positive but not statistically significant. The two models that include the IMC portfolio are substantially more successful than the (C)CAPM. The estimated risk prices on IMC are negative and statistically different from zero. The estimated risk price on the market is also positive and significant, but the same is not true for the price of consumption growth. Both models produce lower absolute pricing errors and smaller variation of pricing errors than the Fama-French three factor model, even though they have fewer number of factors. The last column includes both IMC and HML along with the market portfolio. The estimated premium on IMC is negative at -4.26 and -4.33 respectively and statistically significant from zero, whereas the premium on HML is negative and not statistically significant. [10]

The main results of this section can also be graphically displayed. Figures 4(b), 4(c) and 4(d) plot the CAPM pricing errors for the above sets of portfolios against their covariance with the IMC portfolio. One can see that all sorts produce a significant spread in pricing errors of around 6%-8% annually, and that there is a strong negative relationship between pricing errors and covariances with the IMC portfolio. This suggests that the CAPM omits a significant source of systematic risk, specifically risk associated with the IMC portfolio. Most importantly, the relationship is negative, which is consistent with a negative premium on I-shock.

5.2.2 Book to market sorted portfolios

Table 9 presents estimation results for the 25 Fama-French portfolios sorted on book to market and market capitalization. As is well known, the CAPM performs poorly in pricing this cross-section, generating large pricing errors, both in terms of absolute magnitude and variation across assets. Even though the estimated price of risk on the market portfolio is statistically significant, the mean absolute pricing error is 2.8%. The CCAPM is performing somewhat better with a mean absolute pricing error of 2.1%. The estimated price of risk on consumption growth is positive, statistically significant and equal to 68.7, with the magnitude consistent with the literature on the equity premium puzzle. The Fama-French three-factor model performs significantly better, with the sum of squared pricing errors equal to 0.65 vs 3.01 for the CAPM and 1.69 for the CCAPM.

Including the IMC portfolio in the (C)CAPM dramatically improves the performance of both models. More importantly, the estimated premium on the IMC portfolio is negative at

[10] In order to assess the magnitude of this number one should compare it with the in-sample mean-variance ratio of the IMC portfolio, which is around $-0.016/0.09^2 \approx -2$, or within one standard error of the estimate.

-5.60 and -9.32 respectively, and statistically different from zero. The overall performance of the model however depends on the choice of proxy for the N-shock. The model that includes the market portfolio does substantially worse, with the sum of squared errors equal to 2.22. This is partially due to the fact that IMC cannot explain the size premium. If SMB is added to the specification, then the model performs as well as the Fama-French three factor model with a sum of squared errors of 0.61, but with a substantially higher estimated premium on IMC at -16.8. The model with only consumption growth and IMC performs slightly better than the Fama-French model, generating a sum of squared errors of 0.62 with only two factors. Finally, including the IMC portfolio in the Fama-French model does not improve the performance of the model, with the estimated price of risk on IMC still negative but not statistically significant.

The results of this section suggest that the IMC portfolio embodies similar pricing information as HML, which is consistent with the model. As an additional test, Table 10 reports estimation results using 25 BM/IMC covariance sorted portfolios. The CAPM and CCAPM perform rather poorly, generating mean absolute pricing errors of 2.3%-2.4% and sum of squared errors of 2.22 and 1.87 respectively. The Fama-French model performs significantly better, generating a mean absolute pricing error of 1.2% and a sum of squared errors of 0.57. As before, including the IMC portfolio in the (C)CAPM dramatically improves the pricing performance of the model, reducing the variation in pricing errors by half to 1.07 and 0.72 respectively. Again, the estimated premium on IMC is negative at -3.13 and -5.11 and statistically significant. Finally, the HJ-test rejects all models except the one featuring consumption growth and the IMC portfolio.

The main results of this section can also be graphically displayed. Figures 2(e) and 2(f) plot the CAPM pricing errors for the above sets of portfolios against their covariance with the IMC portfolio. As before, there is a negative relationship between the pricing errors and covariances with IMC, suggesting that the CAPM omits a systematic source of risk that may be proxied by the IMC portfolio.

5.2.3 Individual stocks

In this section I repeat the cross-sectional tests using the entire cross-section of returns. The problem when using individual stocks is that covariances are measured with error, which biases the estimated risk premium towards zero. To alleviate measurement error I follow the procedure of Fama and French (1992) of first sorting stocks into portfolios and then assigning the portfolio covariance with IMC to the individual stocks. The full procedure is described in the Appendix.

The estimation results are shown on table 11. The first panel presents results for the entire sample. The estimated premium on IMC is -3.69 and statistically significant, even when the log book to market ratio and the log market capitalization is included in the specification. Most importantly, the estimate of b_Y is very close to the estimates obtained using different sets of test assets.

In the second and third panels of table 11, I present estimates using the first and second half of the sample. I find that the estimated premium on IMC is -5.54 in the second half of the sample and -1.91 in the first half, with t-statistics of -2.59 and -1.76 respectively. On the one hand, this is consistent with the negative premium on IMC being significantly stronger in the second half of the sample, as suggested by table 3, and with the fact that the rate of technological innovation in investment goods has been significantly higher in the last 25 years. On the other hand, care must be taken when interpreting the sub-sample results. The reason is that during the first sub-sample the number of stocks in each portfolio is much smaller, so each portfolio is less diversified. It is very likely that measurement error is higher during the first sub-sample, which would artificially make the premium smaller.

5.3 Does IMC proxy for investment-specific technological change?

Proposition 2 identifies the IMC portfolio as a proxy for the investment-specific shock. However, it is possible that this portfolio captures other sources of systematic risk or perhaps proxies for the systematic mispricing of some stocks. If the IMC portfolio is a valid proxy for investment-specific shocks, then it must satisfy the following.

First, IMC must be a factor that drives the time series of realized returns. If stocks A and B have high average returns because they are exposed to similar risk, then they must also move together. Makarov and Papanikolaou (2007) show that one of the factors driving the cross-section of industry portfolios is highly correlated(87%) with a portfolio of investment minus consumption goods industries. In their work the factors are identified through heteroscedasticity and not by imposing any economic structure.

Second, there must be a link between IMC and aggregate price and quantity of investment. Specifically, if this shock captures investment-specific productivity shocks, then it must predict an increase in investment and a fall in the quality-adjusted price of the investment good.

Third, if IMC is part of the pricing kernel, there must be some a link, at some horizon, between cashflows or returns on this portfolio and aggregate consumption or leisure. Even though in the model consumption can adjust instantaneously via the allocation of labor across sectors, this is a simplification. In reality, it might take several quarters for consumption to adjust. If the IMC portfolio captures an investment-specific shock, it should predict a short run fall and a long-run increase in the growth rate of consumption. In addition, it should predict a short-run increase in labor supply.

5.3.1 Response of investment

I explore how investment and the price of new investment goods respond to returns on the IMC portfolio. In particular, I look at investment in non-residential structures and equipment and software, and decompose the latter into investment in information equipment and investment in industrial equipment. I use the investment quantity indices from BEA. Regarding the price of new equipment, quality-adjustment is an issue. The reason is that if investment-specific shocks also represent an increase in the *quality* of the investment good, then its price might increase. This makes it difficult to disentangle quality-improving productivity shocks from demand-side shocks, because both could predict an increase in prices and quantities. Unfortunately, obtaining quality adjusted-prices of investment goods is somewhat problematic. According to Moulton (2001), NIPA currently incorporates hedonic methods to quality-adjust computers, semiconductors and digital telephone switching equipment, but other types of equipment deflators are not adjusted. Consequently, I use the price index for computers and peripheral equipment relative to the GDP deflator as a proxy for the quality-adjusted price for new equipment.

I estimate

$$x_{t+k} - x_t = \alpha_0 + \beta_k R_{IMC,t} + \gamma_{1,k} R_{MKT,t} + \gamma_{2,k} \Delta x_t + \epsilon_t, \tag{22}$$

where x_t denotes log investment or the log relative price deflator at time t, and $R_{IMC,t}$ and $R_{MKT,t}$ denote returns on IMC and the market portfolio respectively. The IMC portfolio is correlated with the market portfolio, which is known to predict investment and consumption. Therefore I control for returns to the market portfolio.

Figures 7(a)-7(c) plot the β_k coefficients along with 5% confidence intervals using HAC standard errors adjusted by Newey-West. I find that investment in equipment and non-residential structures sharply increases following positive returns on the IMC portfolio. The increase in investment following positive returns on IMC is statistically significant for up to 5-6 quarters forward. At the same time, the quality adjusted price of equipment falls following positive returns on IMC, but the response is statistically significant for only one quarter ahead. Investment in non-residential structures also displays a strong response, which is consistent with the findings of Gort, Greenwood and Rupert (1999) who document significant technological advances in structures.

5.3.2 Response of Consumption and Leisure

In the model, the cross-sectional heterogeneity in expected returns between investment and consumption firms stem from the differences in correlations with the stochastic discount factor, which partially depends on consumption and leisure. In this section, I look at the cumulative response of consumption and leisure on returns to the IMC portfolio, controlling for the market portfolio

$$x_{t+k} - x_t = \alpha_0 + \beta_k R_{IMC,t} + \gamma_{1,k} R_{MKT,t} + \gamma_{2,k} \Delta x_t + \epsilon_t, \tag{23}$$

where here x_t denotes log consumption or labor/leisure. I employ the quantity indices from BEA on consumption expenditures excluding food and energy. As measures of labor supply I use the Civilian Unemployment Index and total hours worked in the non-farm business sector.

Figure 7(d) plots the estimated β_k coefficients along with 5% confidence intervals using HAC errors adjusted by Newey-West. Consumption sharply falls following positive returns on the IMC portfolio. This fall is reversed after 10-14 quarters. Figures 7(e) and 7(f) plots the β_k coefficients for unemployment and labor supply. Consistent with the model, unemployment falls whereas hours worked increase following positive returns on the IMC portfolio.

The pattern is consistent with the spirit of the model. A positive shock to investment technology does not affect output immediately, but it leads to a gradual reallocation of resources from the sector producing consumption goods to the sector producing investment goods. The increase in investment will eventually be reflected in an increase in output and consumption growth.

5.4 The Investment to GDP ratio

A possible drawback of the cross-sectional asset pricing tests is that they relies too heavily on the model's implication that a portfolio of investment minus consumption producing firms is a good proxy for investment-specific shocks. One cannot ex-ante rule out an alternative where the value premium is driven by behavioral biases, and the same biases affect investment firms differently than consumption firms.

As an additional robustness check, I use the observed investment to GDP ratio along with aggregate consumption and compute the exact stochastic discount factor that is implied by the model, i.e. equation (14), rather than the linearized version in (20).

The model features two state variables, ω and $x = XK_C^{\beta_C}$. The level of consumption $C(\omega_t, x_t)$ and the investment-output ratio $IY(\omega_t, x_t)$ implied by the model can be inverted to obtain estimates $\hat{\omega}_t$ and $\hat{x}_t$ given the observed quantities, $\hat{C}$ and $\hat{IY}$.

$$\begin{aligned} IY(\omega_t, x_t) &= \hat{IY} \\ C(\omega_t, x_t) &= \hat{C} \end{aligned}$$

Given estimates $\hat{\omega}_t$ and $\hat{x}_t$, I compute the stochastic discount factor implied by the model and then repeat some of the cross-sectional tests.

First, I use the full solution of the model to construct estimates of $\hat{\pi} = \pi(\hat{\omega}_t, \hat{x}_t)$. Second, I compute the covariance of portfolio excess returns and the implied stochastic discount factor, $C_i \equiv \widehat{cov(\hat{\pi}_t, R^e_{i,t})}$. Third, I form estimates of the expected excess return on each portfolio, $\mu_i = ER^e_{i,t}$. If the stochastic discount factor prices the assets correctly, it must be that

$$ER^e_{i,t} = -cov(\hat{\pi}_t, R^e_{i,t})$$

Therefore, I estimate using GMM,

$$\mu_i = a + bC_i + u_i.$$

If the model is correct, then in the above regression $a = 0$, $b = -1$ and $u_i = 0 \forall i$.

Figure 7 presents the results for the cross-section of BM sorted portfolios.[11] The model is rejected, even though the R^2 in the above regression are fairly high for the book-to-market sorted portfolios (77-80%). The model is rejected because b is significantly different than -1, and in fact is between -20 and -30. The reason for this is that the estimated covariances of portfolio returns with the implied SDF, $\hat{\pi}$, display the right pattern (i.e. the are decreasing in the BM sort) but their magnitude is too small. However, given that $\hat{\pi}$ is constructed using aggregate data on investment, consumption and output, this is hardly surprising, as stock returns in general display low correlation with macroeconomic aggregates. Finally, note here that even though the 25 portfolios have essentially a two-factor structure, it is correlation

[11] Given that this cross-section has a high spread in average returns, it has been the main focus of the literature. To conserve space, I report results for the 10 BM and 25 ME/BM portfolios, while results using other cross-sections are available upon request.

with a *single* factor, i.e. the implied pricing kernel, that has the right pattern.

6 Conclusion

I extend the standard general equilibrium models used in finance to incorporate investment-specific technological change. Investment-specific shocks act as a shock to the real investment opportunities in the economy. In equilibrium, an increase in productivity of the capital goods sector is followed by a reallocation of resources from consumption to investment. This results in a negative price of risk for investment-specific shocks. In addition, the investment-specific shock affects the relative cost of new capital as well as the relative profitability of firms. In particular, a positive investment-specific shock increases the value of investment good producers and growth firms relative to consumption good firms and value firms. In contrast, a neutral productivity shock affects all firms symmetrically. The presence of two shocks creates heterogeneity in expected returns and also explains why HML is a factor in the time series of returns, or equivalently why value and growth stocks move together.

The implications of the model are largely supported by the data. Using the restrictions of the model, I employ a portfolio of investment minus consumption firms (IMC) to proxy for investment-specific technological change. The IMC portfolio predicts an increase in investment, a fall in the quality-adjusted price of new equipment and a short run fall in consumption. I find that, controlling for their exposure to the market portfolio, firms with high correlation with the IMC portfolio have lower average returns. Additionally, the IMC portfolio improves upon the ability of the (C)CAPM to price the cross-section of expected returns of portfolios of stocks sorted by book to market.

More than simply adding an additional source of systematic risk, a model with investment-specific technological change offers new insights about the relation of stock returns and macroeconomic sources of risk. Similar to Gomes, Kogan and Yogo (2006), my model recognizes an aspect of firm heterogeneity that is not directly related to accounting valuation ratios and implies that firms respond differently to some macroeconomic shocks. This illustrates how the cross-section of stock returns can help identify different macroeconomic shocks.

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7 Tables and figures

Table 1: *Parameters*

Parameter	γ	θ	ψ	ρ	β_C	β_I	λ	δ	μ_Y	σ_Y	μ_X	σ_X
Value	2	0.35	3	0.001	0.3	0.3	1.75	0.14	0.23	0.5	0	0.0275

Table 2: *Unconditional Moments: Consumption, Output, Investment and Hours*

	var	mean	standard deviation	autocorrelation	Correlation $\dot{c}$	$\dot{y}$	$\dot{i}$
Data	$\dot{c}$	0.030	0.035	0.341			
Median		0.027	0.039	0.794			
$[5\%, 95\%]$		[0.010 0.041]	[0.027 0.062]	[0.634 0.878]			
	$\dot{y}$	0.029	0.048	0.088	0.798		
		0.028	0.046	0.704	0.901		
		[0.009 0.042]	[0.033 0.064]	[0.464 0.832]	[0.662 0.976]		
	$\dot{i}$	0.052	0.166	0.133	0.581	0.802	
		0.031	0.096	0.205	0.417	0.700	
		[0.009 0.067]	[0.054 0.261]	[-0.102 0.477]	[0.028 0.731]	[0.542 0.821]	
	$\dot{l}$	0.000	0.025	0.158	0.482	0.781	0.712
		0.000	0.019	0.113	0.215	0.332	0.507
		[-0.001 0.001]	[0.011 0.032]	[-0.087 0.385]	[-0.181 0.387]	[0.148 0.649]	[-0.017 0.812]

Table 1 shows the parameters used in calibration. Table 2 shows the unconditional moments of consumption, output, hours and investment growth generated by the model versus their empirical counterparts. The moments implied moments are calculated by simulating the model for 50 years for 100,000 simulations. I report the median value and the 5% and 95% confidence intervals.

Table 3: *Unconditional Moments: Prices and Ratios*

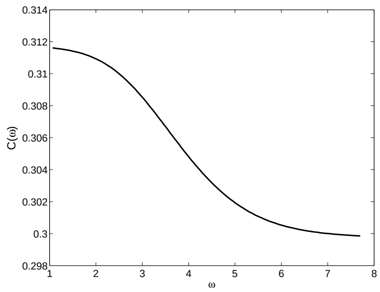
	Model	Model (sim)			Data	Data Source
	(inst)	median	5%	95%		
$ER_M - r_f$	0.041	0.048	[0.031	0.064]	0.066	CRSP
$\sigma(R_M)$	0.199	0.203	[0.192	0.213]	0.198	CRSP
ER_{IMC}	-0.019	-0.020	[-0.039	-0.015]	-0.016	CRSP, NIPA, author's calculations
$\sigma(R_{IMC})$	0.098	0.097	[0.089	0.109]	0.099	CRSP, NIPA, author's calculations
ER_{VMG}	0.044					
$\sigma(R_{VMG})$	0.225					
r_f	0.019	0.021	[0.001	0.053]	0.017	Kenneth French's website, St Louis Fed
$\sigma(r_f)$	0.067	0.048	[0.030	0.071]	0.029	Kenneth French's website, St Louis Fed
I/K	0.254	0.241	[0.121	0.345]	0.231	CAPEX(data128) over PPE(data8) in Compustat
Tobin's Q	1.376	1.369	[1.181	1.546]	1.531	MKCAP Equity + Book Debt over Book Assets
I/Y	0.191	0.201	[0.167	0.241]	0.173	Private Fixed Investment over GDP (excl. Gov.)

Table 4: *Variance Decomposition*

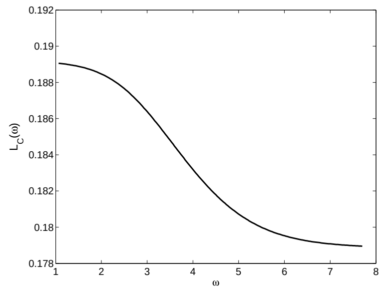
	X			Y		
	median	5%	95%	median	5%	95%
C	0.867	[0.795	0.925]	0.133	[0.075	0.205]
Y	0.242	[0.099	0.567]	0.758	[0.433	0.901]
L	0.000	[0.000	0.000]	1.000	[1.000	1.000]
I	0.051	[0.010	0.871]	0.949	[0.129	0.990]
Q	0.000	[0.000	0.000]	1.000	[1.000	1.000]
r_f	0.000	[0.000	0.000]	1.000	[1.000	1.000]
MKT	0.051	[0.021	0.102]	0.949	[0.898	0.989]
IMC	0.000	[0.000	0.000]	1.000	[1.000	1.000]
VMG	0.000	[0.000	0.000]	1.000	[1.000	1.000]

Table 3 shows the unconditional moments for asset prices and other variables generated by the model versus their empirical counterparts. The moments implied moments are calculated in two ways: The first computes instantaneous moments by directly computing the unconditional moments implied by the differential equations characterizing the solution and the invariant distribution of ω. The second, by simulating the model for 50 years for 100,000 simulations, where I report the median value and the 5% and 95% confidence intervals. Table 4 reports the variance decomposition, based on a VAR, for the key variables in the model.

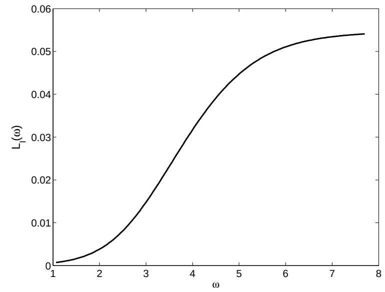
Figure 1: *Model Solution*



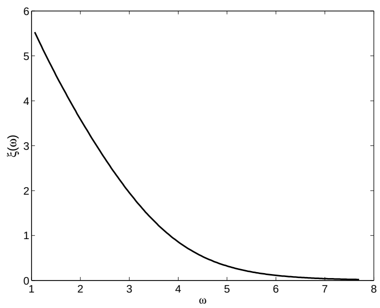
(a) Consumption



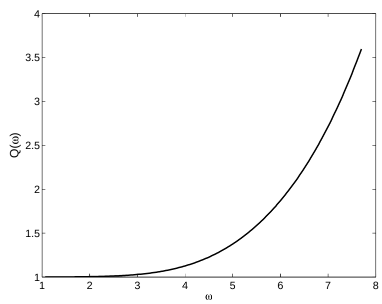
(b) Labor in C (L_C)



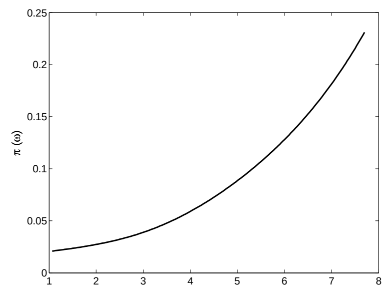
(c) Labor in I (L_I)



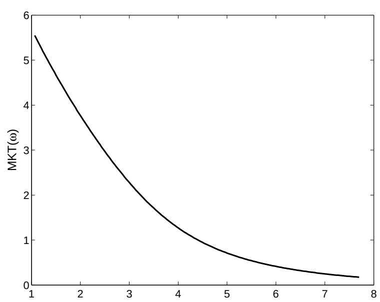
(d) Price of Capital Goods



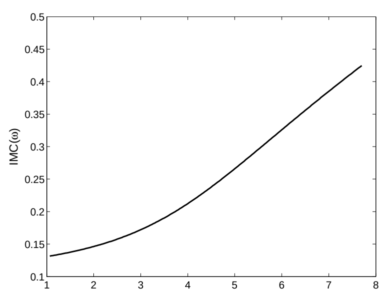
(e) Tobin's Q



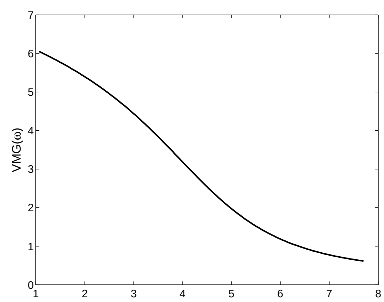
(f) State Price Density



(g) Market Portfolio: Price



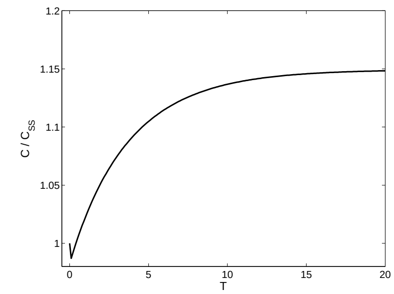
(h) Value of Investment firms relative to Consumption firms



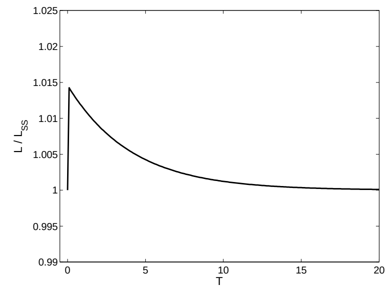
(i) Value of Assets in place relative to Growth Opportunities

The model is solved using a grid of 1200 points. Without loss of generality, I evaluate the above aggregate quantities and prices and $K_C = 1$ and $X = 1$.

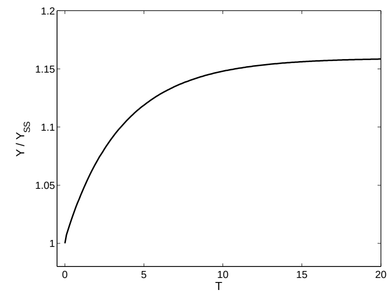
Figure 2: *Dynamic responses to the Investment-Specific shock*



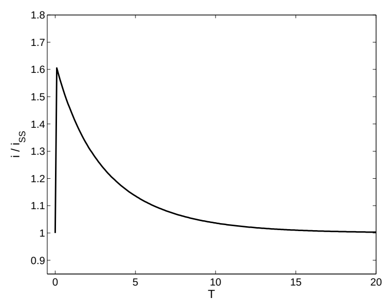
(a) Consumption



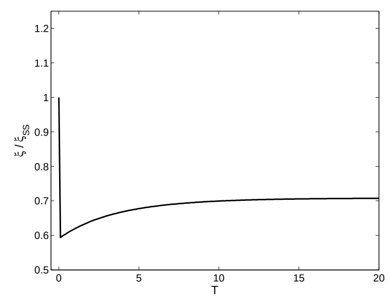
(b) Labor Supply



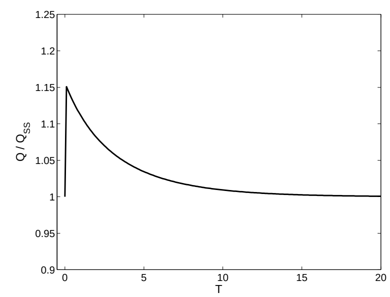
(c) Output



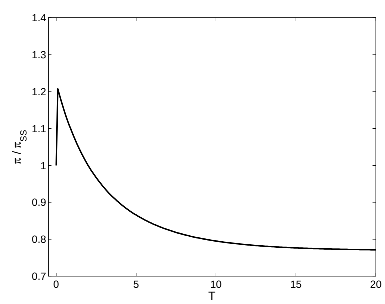
(d) Investment rate



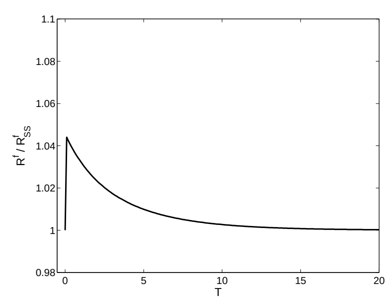
(e) Price of Capital Goods



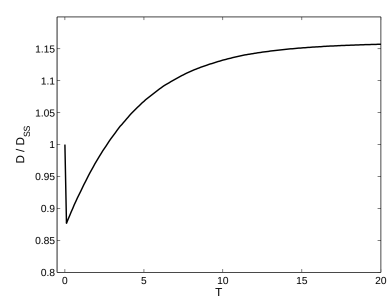
(f) Tobin's Q



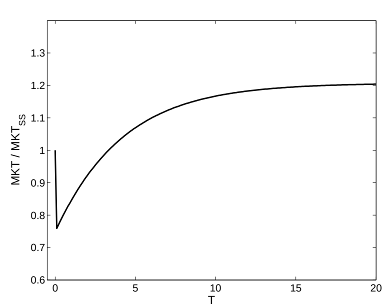
(g) State Price



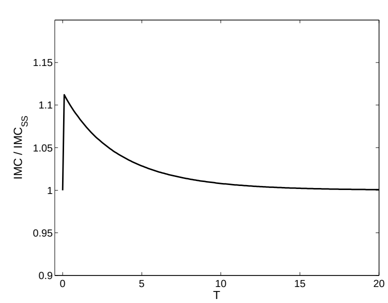
(h) Risk-free rate



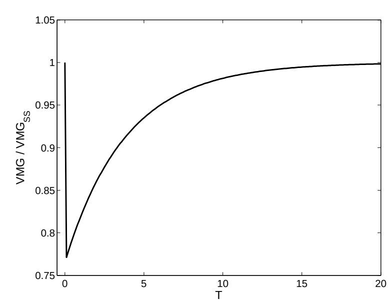
(i) Aggregate Dividend



(j) Market Portfolio: Price

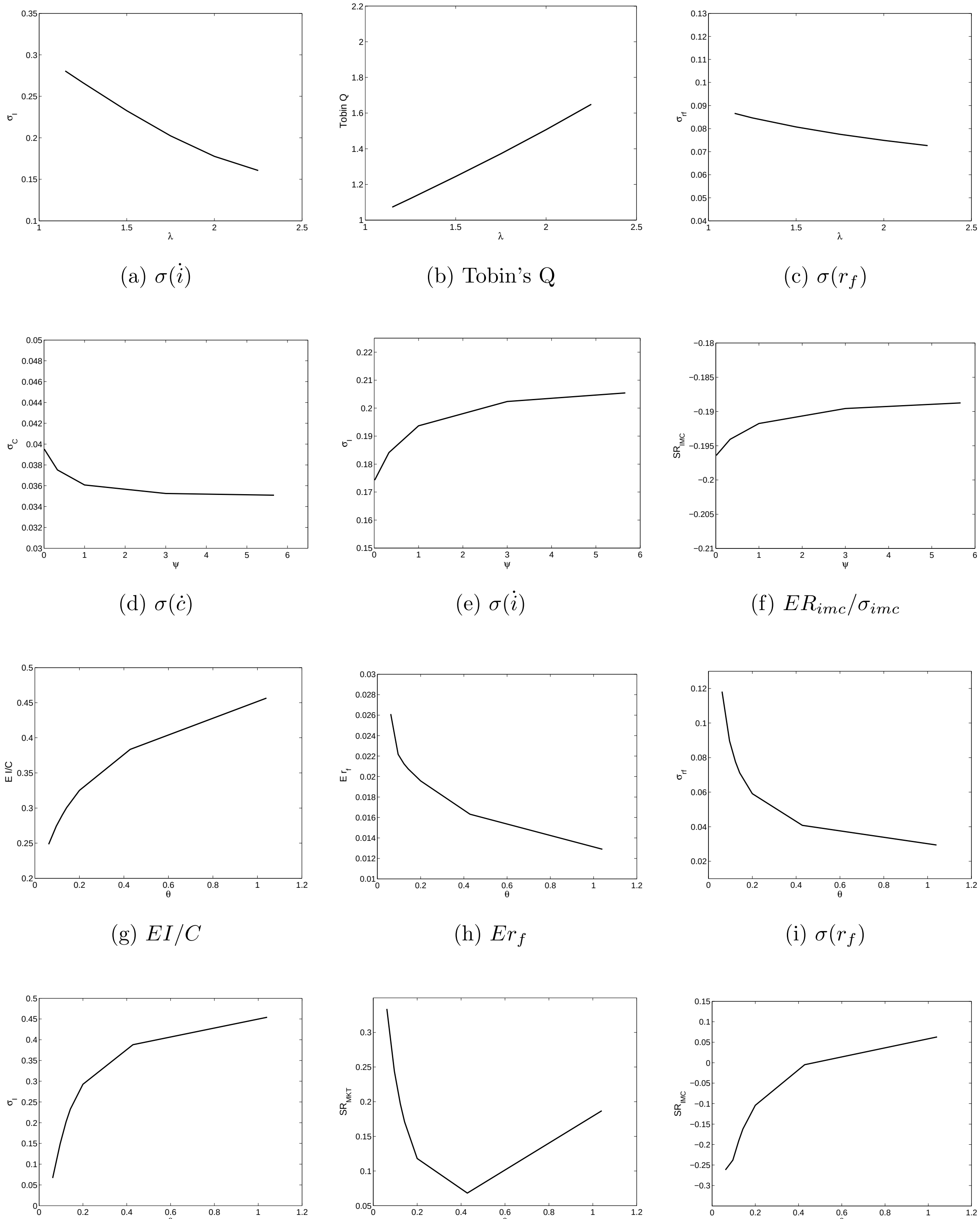


(k) Value of Investment firms relative to Consumption firms



(l) Value of Assets in place relative to Growth Opportunities

Figure 3: *Comparative Statics with respect to λ, ψ and θ*



(a) $\sigma(\dot{i})$ (b) Tobin's Q (c) $\sigma(r_f)$

(d) $\sigma(\dot{c})$ (e) $\sigma(\dot{i})$ (f) ER_{imc}/σ_{imc}

(g) EI/C (h) Er_f (i) $\sigma(r_f)$

(j) $\sigma(\dot{c})$ (k) ER_{mkt}/σ_{mkt} (l) ER_{imc}/σ_{imc}

Table 5: *IMC Portfolio: Composition*

IMC	Consumption Portfolio			Investment portfolio		
	mean	min	max	mean	min	max
# of firms	2467	153	5858	832	44	2055
ME (USb)	2140	33	10951	529	2	4903
Book-to-Market Equity	0.62	0.32	1.14	0.59	0.12	1.06
Debt-to-Asset	0.18	0.10	0.24	0.16	0.05	0.22

Table 6: *IMC Portfolio: Correlations*

1961:2005	IMC	IMC_{GKY}	MKT	SMB	HML	MOM
IMC						
IMC_{GKY}	77.5%					
MKT	22.9%	36.0%				
SMB	47.9%	49.9%	25.0%			
HML	-49.1%	-37.5%	-34.5%	-25.0%		
MOM	-1.8%	-5.6%	-5.7%	-1.4%	-13.0%	

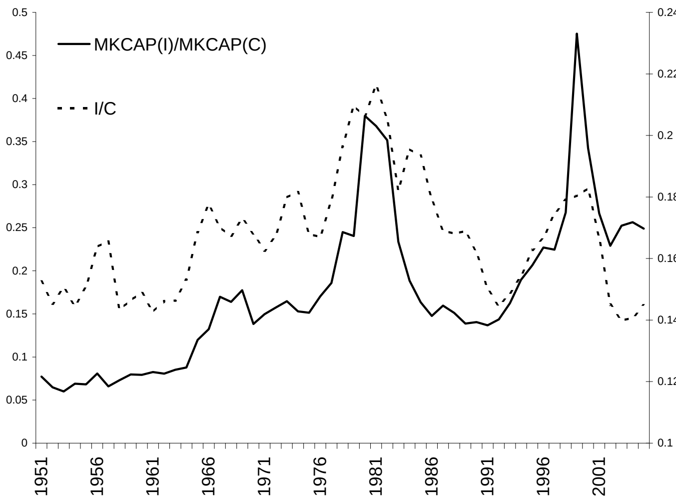
Table 5 reports composition details for the investment minus consumption portfolio (IMC) constructed using the NIPA tables. Data on book equity, long-term debt and total assets are from COMPUSTAT. Table 6 reports correlations with the above portfolios with the excess returns on the CRSP value-weighted index, the Fama-French (1993) factors, Carhart's momentum factor and IMC_{GKY}. IMC_{GKY} is the investment minus consumption constructed using the data provided by Gomes, Kogan and Yogo (2006). I form IMC_{GKY} by subtracting the non-durables portfolio from the investment portfolio. I report results for the whole sample (1961-2005).

Table 7: *Summary Statistics*

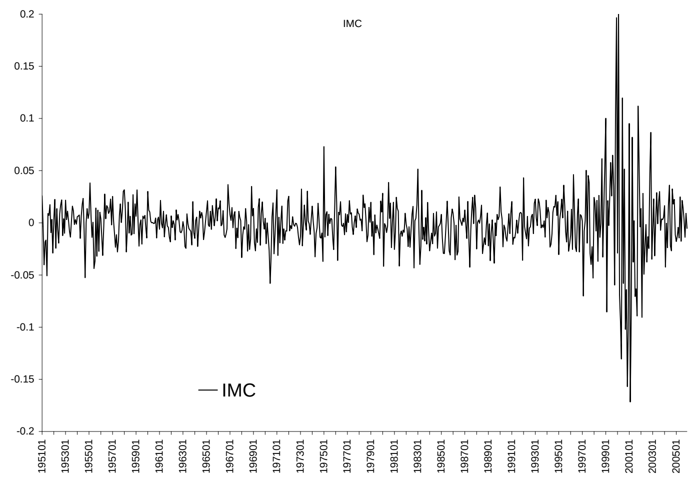
Period	IMC		IMC_{GKY}		HML	
	E(R)	α	E(R)	α	E(R)	α
1961 - 1970	-2.11	-2.22	-0.38	-1.30	6.37	6.83
	(1.65)	(1.68)	(2.43)	(2.32)	(2.24)	(2.24)
1971 - 1980	0.98	0.63	0.80	0.22	3.95	4.51
	(1.90)	(1.87)	(2.18)	(2.05)	(3.13)	(3.07)
1981 - 1990	-5.91	-5.93	-10.70	-11.28	6.83	8.30
	(1.93)	(1.96)	(2.80)	(2.78)	(2.85)	(2.40)
1991 - 2000	1.77	-3.46	-1.05	-6.17	3.46	9.42
	(5.28)	(5.15)	(4.91)	(4.75)	(4.04)	(3.56)
2001 - 2005	-4.10	-5.27	-0.51	-1.11	9.41	9.93
	(7.14)	(5.14)	(8.14)	(5.95)	(4.87)	(3.36)
1961 - 2005	-1.63	-2.74	-2.62	-4.12	5.63	7.15
	(1.58)	(1.53)	(1.64)	(1.56)	(1.49)	(1.37)

Table reports average annualized excess returns, CAPM alphas and correlations with HML for the following three portfolios. Standard errors are reported in parentheses. IMC is the investment minus consumption portfolio constructed using the NIPA tables, where industries are classified as investment or consumption based on which sector they contribute the most. Sample includes data from 1961 to 2005. IMC_{GKY} is the investment minus consumption constructed using the data of Gomes, Kogan and Yogo (2006). I form IMC_{GKY} by subtracting a weighted average of the services and the non-durables portfolio from the investment portfolio. I report results for the whole sample and for each decade.

Figure 4: *IMC*



(a) IMC Ratio



(b) IMC returns

The top figure shows the ratio of market values of investment over consumption industries (left axis) and the ratio of private fixed nonresidential investment over personal consumption expenditures from the NIPA tables (right axis). The bottom figure shows returns for the IMC portfolio, with the solid line classifying industries according to the sector they contribute most, and the dotted line drops common industries.

Table 8: *Summary Statistics: 10 portfolios sorted on IMC beta*

IMC Beta	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo	9 - 2
Excess Return (%)	6.50	7.82	7.12	6.78	6.95	5.81	6.08	5.92	4.82	5.02	-1.48	-3.00
	(2.15)	(2.55)	(2.46)	(2.67)	(2.84)	(2.83)	(3.19)	(3.42)	(3.86)	(4.42)	(3.90)	(2.61)
σ (%)	13.27	15.73	15.16	16.47	17.50	17.41	19.64	21.04	23.79	27.23	24.03	16.08
μ/σ (%)	14.14	14.36	13.57	11.89	11.47	9.64	8.94	8.12	5.85	5.32	-1.78	-5.39
β_{MKT}	0.63	0.91	0.87	0.96	1.02	1.00	1.14	1.22	1.35	1.48	0.85	0.44
	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)	(0.07)	(0.05)
α(%)	2.99	2.77	2.29	1.45	1.30	0.27	-0.25	-0.83	-2.66	-3.20	-6.19	-5.44
	(1.45)	(1.00)	(1.01)	(1.01)	(1.05)	(1.23)	(1.22)	(1.43)	(1.63)	(2.24)	(3.25)	(2.30)
R^2(%)	57.47	84.72	83.33	86.01	85.85	83.37	85.27	84.43	81.23	74.80	31.55	18.79
β_{MKT}	0.80	1.00	0.96	1.00	1.03	1.05	1.10	1.15	1.21	1.32	0.52	0.21
	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)	(0.07)	(0.05)
β_{SMB}	-0.34	-0.22	-0.19	-0.15	-0.06	0.08	0.07	0.21	0.26	0.49	0.83	0.48
	(0.03)	(0.03)	(0.03)	(0.03)	(0.06)	(0.04)	(0.05)	(0.05)	(0.06)	(0.07)	(0.09)	(0.08)
β_{HML}	0.33	0.15	0.16	0.02	-0.01	0.24	-0.09	-0.08	-0.29	-0.20	-0.53	-0.44
	(0.04)	(0.03)	(0.03)	(0.04)	(0.05)	(0.05)	(0.06)	(0.06)	(0.08)	(0.07)	(0.10)	(0.10)
α(%)	1.31	2.16	1.50	1.56	1.46	-1.52	0.26	-0.64	-1.12	-2.67	-3.98	-3.28
	(1.17)	(0.88)	(0.88)	(1.02)	(1.08)	(1.17)	(1.18)	(1.30)	(1.52)	(2.10)	(2.81)	(2.01)
R^2(%)	73.21	88.42	86.79	87.12	85.99	85.13	85.71	85.89	84.58	79.74	53.20	39.64
β_{MKT}	0.84	1.03	0.98	1.02	1.00	1.05	1.04	1.08	1.11	1.19	0.34	0.08
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)	(0.04)
β_{IMC}	-0.50	-0.28	-0.26	-0.15	0.05	-0.11	0.25	0.34	0.57	0.71	1.21	0.86
	(0.03)	(0.03)	(0.03)	(0.03)	(0.06)	(0.04)	(0.05)	(0.04)	(0.05)	(0.07)	(0.08)	(0.06)
α(%)	0.37	1.28	0.93	0.69	1.56	-0.33	1.06	0.94	0.34	0.51	0.14	-0.94
	(1.03)	(0.83)	(0.85)	(0.99)	(1.10)	(1.18)	(1.10)	(1.17)	(1.20)	(1.80)	(2.27)	(1.58)
R^2(%)	79.33	89.77	87.88	87.24	85.98	84.02	87.75	88.41	90.25	85.30	70.67	62.92

Table reports summary statistics of the simple monthly returns of 10 portfolios created by sorting stocks on their covariance with IMC. The pre-sorting covariances are estimated using weekly returns and 5 year windows. Portfolios are rebalanced every 5 years on June. Panel A reports mean excess returns over the 30-day T-bill rate, the standard deviation of returns and their beta and alpha with respect to the CAPM. Panel B reports beta and alpha for the Fama-French three factor model. Panel C reports beta and alpha for the model including the market portfolio and the IMC portfolio. Standard errors are shown in parenthesis.

Table 9: *Cross-Sectional Tests: 24 portfolios sorted on Industry and IMC covariance*

Factor Price						
Market	1.18		2.98	2.06		2.66
	(1.01)		(1.24)	(1.08)		(1.24)
SMB			-5.77			
			(2.34)			
HML			2.76			-1.37
			(2.73)			(3.68)
C_{ND}		16.40			28.25	
		(23.66)			(22.72)	
IMC				-4.26	-4.33	-4.49
				(1.66)	(1.75)	(1.99)
MAPE(%)	1.72	1.72	1.23	1.23	1.08	1.13
$SSQE$(%)	0.98	0.98	0.54	0.54	0.46	0.45
J-test	36.9	36.0	25.0	23.9	22.3	26.5
	(0.033)	(0.041)	(0.248)	(0.352)	(0.443)	(0.188)

Table reports the estimation results from cross-sectional tests. I estimate the parameters of the pricing kernel,

$$\pi = a - b\,F.$$

The set of test assets includes simple monthly returns of 24 portfolios created by sorting stocks first on 8 industries and then on their covariance with IMC. The pre-sorting covariances are estimated using weekly returns and 5 year windows. Portfolios are rebalanced every 5 years on June. The factors considered are the market portfolio, Fama and French's SMB and HML factors, the IMC portfolio, non-durable consumption growth (C_{ND}). The sample includes monthly data from 1961 to 2005. Estimation is by two-step generalized method of moments (GMM). HAC standard errors are computed by the Newey-West estimator with 3 lags of returns. I report second stage estimates of the factor premia (b), along with standard errors, annualized mean absolute pricing errors (MAPE) and the sum of squared pricing errors (SSQE) for the first stage. Furthermore, I report the J-test that all the pricing errors are zero along with it's p-value in parenthesis.

Table 10: *Cross-Sectional Tests: 25 portfolios sorted on ME and BM*

Factor Price							
Market	2.28		4.09	3.11	2.66		4.10
	(1.04)		(1.16)	(1.28)	(1.11)		(1.23)
SMB			3.14	8.16			4.43
			(1.46)	(2.19)			(2.21)
HML			9.09				8.43
			(1.76)				(2.46)
C_{ND}		68.28				98.87	
		(22.40)				(30.58)	
IMC				-16.44	-5.60	-9.32	-3.51
				(4.27)	(2.46)	(2.90)	(4.74)
MAPE(%)	2.80	2.08	1.22	1.21	2.55	1.33	1.15
$SSQE$(%)	3.01	1.69	0.65	0.61	2.22	0.62	0.58
J-test	107.4	92.8	75.6	79.6	102.7	50.6	73.1
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)

Table reports the estimation results from cross-sectional tests. I estimate the parameters of the pricing kernel,

$$\pi = a - b\,F.$$

The set of test assets includes simple monthly returns of the 25 portfolios created by sorting stocks on first market capitalization and then on book to market using NYSE quintiles. The data come from Kenneth French's website. The factors considered are the market portfolio, Fama and French's SMB and HML factors, the IMC portfolio, non-durable consumption growth (C_{ND}). The sample includes monthly data from 1961 to 2005. Estimation is by two-step generalized method of moments (GMM). HAC standard errors are computed by the Newey-West estimator with 3 lags of returns. I report second stage estimates of the factor premia (b), along with standard errors, annualized mean absolute pricing errors (MAPE) and the sum of squared pricing errors (SSQE) for the first stage. Furthermore, I report the J-test that all the pricing errors are zero along with it's p-value in parenthesis.

Table 11: *Cross-Sectional Tests: 25 portfolios sorted on BM and IMC beta*

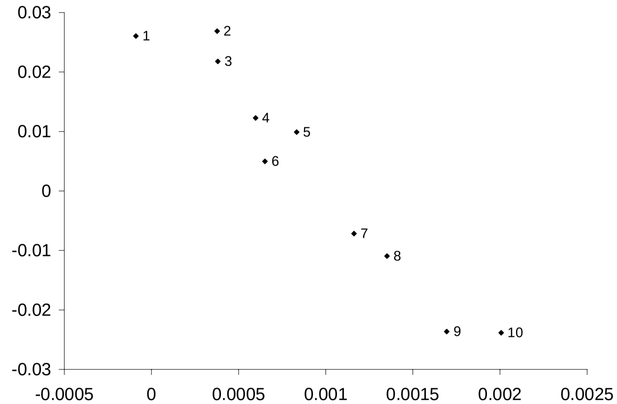
Factor Price						
Market	2.77		4.64	3.30		4.82
	(1.08)		(1.29)	(1.18)		(1.29)
SMB			-0.23			
			(2.27)			
HML			8.06			7.61
			(2.36)			(2.46)
C_{ND}		80.43			140.11	
		(29.45)			(52.67)	
IMC				-3.13	-5.11	-1.02
				(2.00)	(2.49)	(1.76)
MAPE(%)	2.42	2.34	1.20	1.72	1.54	1.21
$SSQE(\%)$	2.22	1.84	0.57	1.17	0.92	0.57
J-test	53.4	40.6	38.8	49.3	29.7	39.0
	(0.001)	(0.018)	(0.015)	(0.001)	(0.158)	(0.014)

Table reports the estimation results from cross-sectional tests. I estimate the parameters of the pricing kernel,

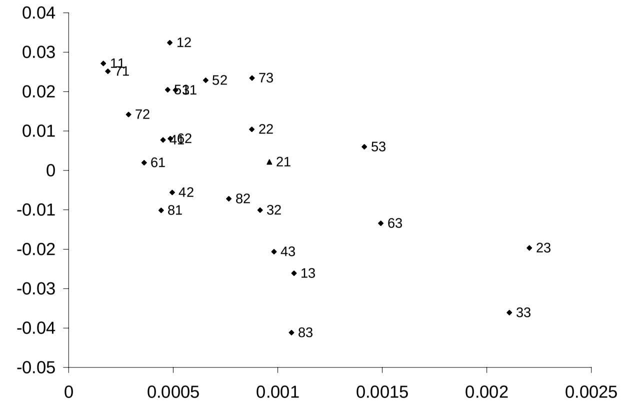
$$\pi = a - b\,F.$$

The set of test assets includes simple monthly returns of 25 portfolios created by sorting stocks first on their book to market ratio based on NYSE breakpoints and then on their covariance with IMC. The pre-sorting covariances are estimated using weekly returns and 5 year windows. Portfolios are rebalanced every 5 years on June. The factors considered are the market portfolio, Fama and French's SMB and HML factors, the IMC portfolio, non-durable consumption growth (C_{ND}). The sample includes monthly data from 1961 to 2005. Estimation is by two-step generalized method of moments (GMM). HAC standard errors are computed by the Newey-West estimator with 3 lags of returns. I report second stage estimates of the factor premia (b), along with t-statistics, annualized mean absolute pricing errors (MAPE) and the sum of squared pricing errors (SSQE) for the first stage. Furthermore, I report the J-test that all the pricing errors are zero along with it's p-value in parenthesis.

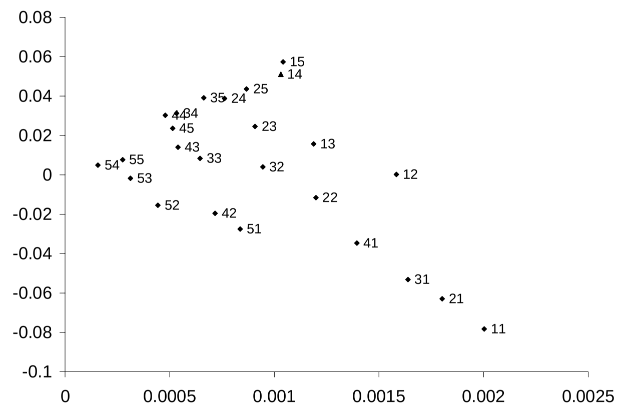
Figure 5: *CAPM pricing errors vs covariance with IMC*



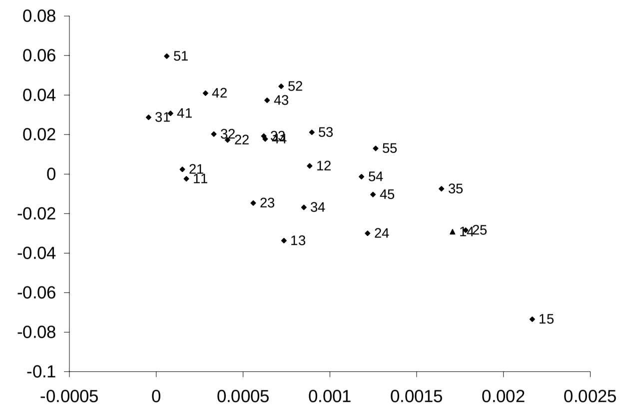
(a) 10 IMC



(b) 24 IND/IMC



(c) 25 BM/ME



(d) 25 BM/IMC

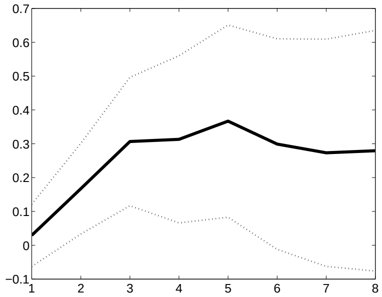
Figure plots the CAPM pricing errors of the test portfolios considered in tables 5 through 10 versus their covariances with the IMC portfolio. CAPM Pricing errors are the first stage pricing errors from the cross-sectional GMM tests. The constructions of these portfolios is standard and is described in the Appendix.

Table 12: *IMC and Expected Returns - Entire Cross-section of stocks*

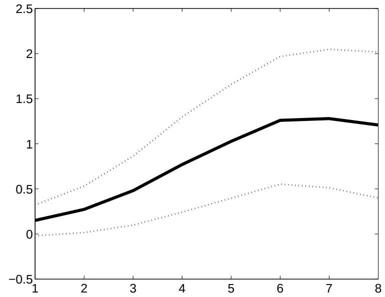
	$cov(R_i, R_{MKT})$	$cov(R_i, R_{IMC})$	$\ln ME$	$\ln BM$
Panel A: 1961 - 2005				
	0.626			
	(0.39)			
	1.086		-0.116	0.265
	(0.73)		(-3.28)	(4.49)
	3.023	-3.694	-0.117	0.260
	(1.94)	(-2.16)	(-3.37)	(4.45)
Panel B: 1961 -1981				
	1.149			
	(0.56)			
	0.122		-0.128	0.126
	(0.07)		(-2.75)	(1.46)
	1.047	-1.916	-0.129	0.120
	(0.53)	(-1.76)	(-2.80)	(1.39)
Panel C: 1982 - 2005				
	0.082			
	(0.03)			
	2.090		-0.103	0.410
	(0.89)		(-1.93)	(5.14)
	5.080	-5.546	-0.105	0.406
	(2.11)	(-2.59)	(-2.00)	(5.19)

Table reports results from Fama and MacBeth (1973) cross-sectional regressions of simple monthly returns of all NYSE, AMEX, and Nasdaq stocks on covariances and characteristics. Covariances are estimated using a procedure similar to Fama and French (1992). Specifically, for each individual stock I estimate the covariance of its returns with IMC using 5 years of weekly log excess returns. At the end of a five year period, stocks are then sorted into 100 pre-ranking covariance centiles. I then compute the equal-weighted monthly log excess returns on these 100 portfolios over the next 5 years. This procedure is repeated every 5 years, forming a time-series of returns on these 100 portfolios. I then reestimate covariances for the portfolios formed from the pre-ranking sorts using 5 years of monthly data to obtain post-ranking covariances. The post-ranking covariance estimate for a given portfolio is then assigned to each stock in the portfolio. Portfolio assignments are updated every 5 years. Every month the cross-section of stock returns in excess of risk free rate is then regressed on a constant (not reported), the covariance with the excess return on the CRSP value-weighted index, the covariance with the return on the IMC portfolio, the log of market capitalization (ME) and the log of Book to Market (BM). I report the time series average of the regression coefficients along with t-statistics using Fama-McBeth errors. Panel A reports results for the full sample, whereas Panels B and C report results for the first and second subperiod.

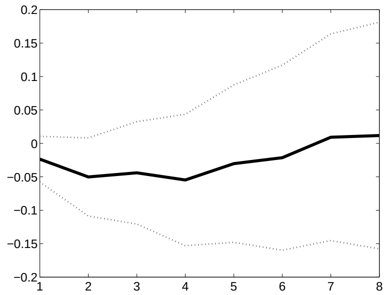
Figure 6: *Cumulative response on IMC: Investment and Consumption*



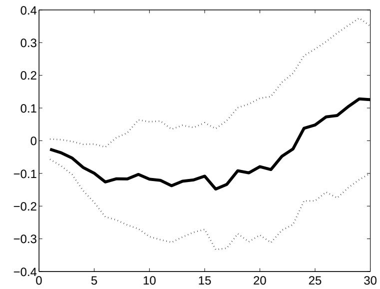
(a) Investment in equipment and software



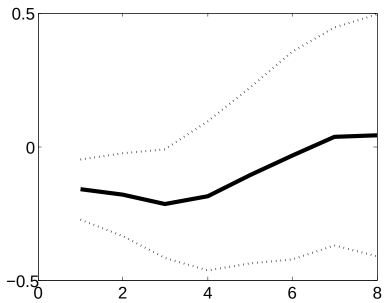
(b) Investment in Non-Residential structures



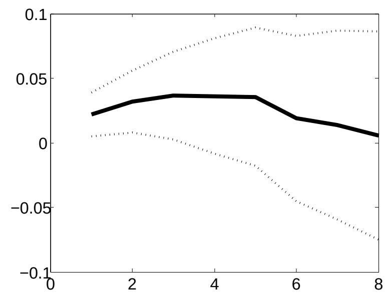
(c) Relative price: Information processing equipment and software



(d) Personal consumption expenditures excluding food and energy



(e) Civilian Unemployment



(f) Hours worked of All persons

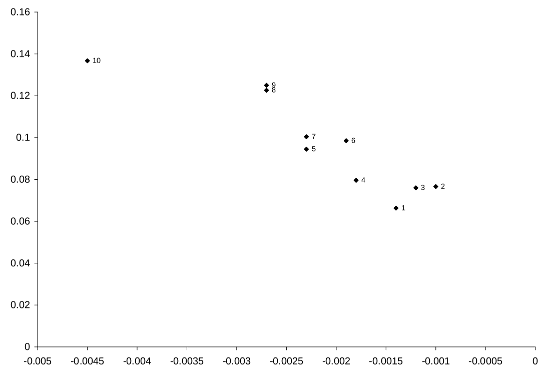
Figure plots the coefficients β_k in the regression of:

$$x_{t+k} - x_t = a + \beta_k IMC_t + \gamma_1 MKT_t + \gamma_2 \Delta x_t + e_t k$$

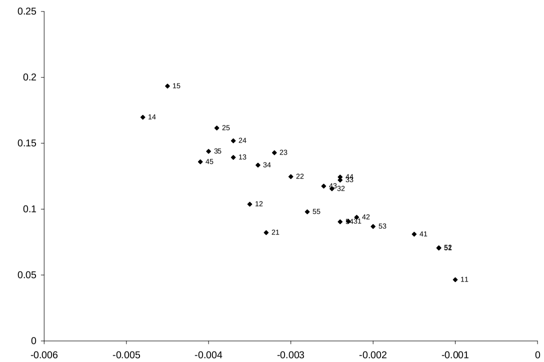
along with two HAC standard error bands. I compute HAC standard errors errors using Newey-West, with the truncation lag equal to the length of the overlap plus two quarters. The sample includes quarterly data in the 1951:2005 period. The quantity and price indices are from the NIPA tables in the BEA website. The price index for Information processing equipment and software is relative to the GDP deflator.

Figure 7: *The Investment to GDP ratio and the Cross-Section of Stock Returns*

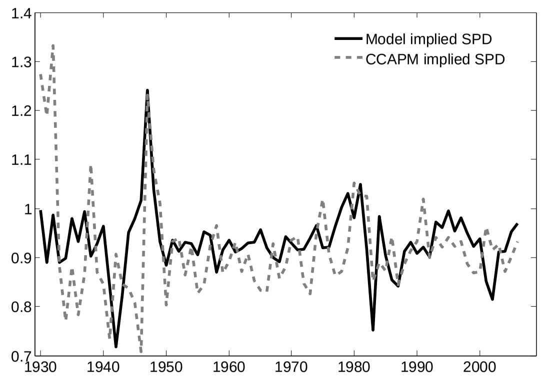
(a) 10 portfolios sorted on BM



(b) 25 portfolios sorted on ME/BM



(c) Time Series of Model-implied SDF



(d) GMM results

	BM10	BM25
a	0.05	0.03
	(0.02)	(0.02)
b	-21.29	-29.54
	(6.60)	(5.55)
R^2	0.81	0.77
J-test	16.17	53.12
p-value	0.04	0.00

Panels (a) and (b) plot average returns on the 10 BM and 25 ME/BM sorted portfolios versus their covariance with the SDF implied by the model computed in section (5.4) by inverting the observed investment to GDP ratio (Private Non-residential Fixed Investment over Gross Domestic Product). Panel (c) plots the model implied SDF versus the SDF implied by the CCAPM, and panel (d) presents estimation results of

$$E(R_i^e) = a + bC_i + u_i$$

where R_i^e denotes excess returns on the above portfolios over the risk-free rate and C_i denotes covariances with the model implied SDF.

8 Appendix

8.1 Proof of Proposition 1

The Hamilton-Jacobi-Bellman equation for the social planner's optimization problem is:

$$\begin{aligned} 0 &= \max_{L_I, L_C, i_C, N} \{h(C,N,J) + (i_C - \delta)J_{K_C}K_C + J_X X \mu_X + \\ &+ \frac{1}{2}J_{XX}X^2\sigma_X^2 + \mu_Y J_Y Y + \frac{1}{2}J_{YY}Y^2\sigma_Y^2\Big\} \end{aligned}$$

where

$$h(C,N,J) = \frac{\rho}{1-\theta^{-1}}\left(\frac{(CN^{\psi})^{1-\theta^{-1}}}{((1-\gamma)J)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - (1-\gamma)J\right)$$

subject to:

$$\begin{aligned} C &\leq XK_C^{\beta_C}L_C^{1-\beta_C} \\ c(i_C)K_C &\leq YK_I^{\beta_I}L_I^{1-\beta_I} \\ L_C + L_I &\leq 1 - N \end{aligned}$$

Denote the lagrange multipliers of the above constraints as $(\pi, \xi\pi, w\pi)$. Replacing the labor market clearing condition $N = 1 - L_C - L_I$ in the Bellman equation, one can see that the first order condition with respect to L_C does not depend on the value function directly:

$$\begin{aligned} L_C^* &= \arg\max_{L_C} CN^{\psi} \\ &= \arg\max_{L_C}(XK_C^{\beta_C}L_C^{1-\beta_C})(1-L_I-L_C)^{\psi} \\ &= \frac{(1-\beta_C)(1-L_I)}{1+\psi-\beta_C} \end{aligned}$$

Replacing the above in the HJB equation along with the constraint on investment:

$$\begin{aligned} 0 &= \max_{L_I}\left\{h\left(XK_C^{\beta_C}\left(\frac{(1-\beta_C)(1-L_I)}{1+\psi-\beta_C}\right)^{1-\beta_C}, 1-L_I-\frac{(1-\beta_C)(1-L_I)}{1+\psi-\beta_C}, J\right) - \delta J_{K_C}K_C + \right. \\ &+ \left. c^{-1}(YK_I^{\beta_I}L_I^{1-\beta_I})J_{K_C} + J_X X\mu_X + \frac{1}{2}J_{XX}X^2\sigma_X^2 + \mu_Y J_Y Y + \frac{1}{2}J_{YY}Y^2\sigma_Y^2\right\} \end{aligned}$$

We will look for a guess of the form:

$$J = \frac{(XK_C^{\beta_C})^{1-\gamma}}{1-\gamma}f(\omega) \qquad \omega \equiv \ln Y - \ln K_C$$

Using our guess, The HJB equation becomes

$$\begin{aligned} 0 &= \max_{L_I}\left\{\rho\frac{1-\gamma}{1-\theta^{-1}}f(\omega)^{\frac{\gamma-\theta^{-1}}{\gamma-1}}\left(\left(\frac{(1-\beta_C)(1-L_I)}{1+\psi-\beta_C}\right)^{1-\beta_C}\left(1-\frac{(1-\beta_C)(1-L_I)}{1+\psi-\beta_C}-L_I\right)^{\psi}\right)^{1-\theta^{-1}} + \right. \\ &\left. + c^{-1}\left(e^{\omega}K_I^{\beta_I}{L_I}^{1-\beta_I}\right)(\beta_C(1-\gamma)f(\omega) - f'(\omega)) - u\,f(\omega) + (\mu_Y + \delta\beta_C)f'(\omega) + \frac{1}{2}\sigma_Y^2(f''(\omega) - f'(\omega))\right\}. \end{aligned}$$

where

$$u \equiv \rho\theta\frac{1-\gamma}{\theta-1} - (1-\gamma)(\mu_X - \beta_C\delta) + \frac{1}{2}\sigma_X^2\,\gamma(1-\gamma) > 0 \tag{24}$$

I solve the optimization problem numerically, as described in Section B. It is straightforward to construct the competitive equilibrium from the solution to the planner's problem. Prices (π, ξ, w) can be recovered from the planner's first order conditions:

$$\begin{aligned}
\pi_t &= \exp\left(\int_0^t h_J(C_s, N_s, J_s)\,ds\right) h_C(C_t, N_t, J_t) \\
\xi_t &= \frac{J_K(X_t, Y_t, K_{C,t})}{\pi_t}\frac{1}{c'(i_{C,t})} \\
w_t &= (1-\beta_C)X_t K_{C,t}^{\beta_C} L_{C,t}^{-\beta_C}
\end{aligned}$$

8.2 Proof of Proposition 2

Consider the value of a firm in the C-Sector. The firm buys new capital and hires labor to maximize its value

$$\begin{aligned}
\pi_0 S_0^C &= E_0 \int_0^\infty \max_{L_{C,s}, i_{C,s}} \pi_s \left(X_s K_{C,s}^{\beta_C} L_{C,s}^{1-\beta_C} - w_s L_{C,s} - \xi_s c(i_C) K_C\right) \\
&= E_0 \int_0^\infty \pi_s \left(\beta_C X_s K_{C,s}^{\beta_C} (L_{C,s}^*)^{1-\beta_C} - \lambda_s c(i_{C,s}^*) K_{C,s}\right) ds
\end{aligned}$$

The planner's lagrangian evaluated at the optimum can be written as:

$$\mathcal{L}_0 = E_0 \int_0^\infty h(C_s^*, N_s^*, J_s^*) - \pi_s(C_s - X_s K_{C,s}^{\beta_C} {L_{C,s}^*}^{1-\beta_C}) - \lambda_s \pi_s \left(c(i_{C,s}^*) K_{C,s} - Y_s K_I^{\beta_I} {L_I^*}^{1-\beta_I}\right) ds$$

The envelope theorem implies that

$$\frac{\partial \mathcal{L}}{\partial K_C} = \frac{\partial J}{\partial K_C}$$

also note that

$$\frac{\partial K_{C,s}}{\partial K_{C,0}} K_{C,0} = K_{C,s}$$

Therefore

$$\frac{\partial J}{\partial K_{C,0}} K_{C,0} = E_0 \int_0^\infty \pi_s \left(\beta_C X_s K_{C,s}^{\beta_C} (L_{C,s}^*)^{1-\beta_C} - \xi_s c(i_{C,s}^*) K_{C,s}\right) ds$$

Similarly, the value of a firm in the investment sector is:

$$\begin{aligned}
\pi_0 S_0^I &= E_0 \int_0^\infty \pi_s \left(\xi_s Y_s K_I^{\beta_I} (L_{I,s})^{1-\beta_I} - w_s L_{I,s}\right) ds \\
\pi_0 S_0^I &= E_0 \int_0^\infty \pi_s \left(\xi_s \beta_I Y_s K_I^{\beta_I} (L_{I,s}^*)^{1-\beta_I}\right) ds
\end{aligned}$$

Moreover, because

$$\frac{\partial J_0}{\partial Y_0} Y_0 = E_0 \int_0^\infty \pi_s \left(\xi_s Y_s K_{I,s}^{\beta_I} (L_{I,s}^*)^{1-\beta_I}\right) ds$$

this implies

$$\pi_0 S_0^I = \beta_I J_Y Y_0$$

therefore

$$\frac{S_0^I}{S_0^C} = \frac{\beta_I f'(\omega)}{\beta_C(1-\gamma)f(\omega) - f'(\omega)}.$$

Also, $\frac{S_t^I}{S_t^C}$ is increasing in ω if

$$\frac{f''}{f'} > \frac{f'}{f}.$$

To see that this is the case, note that the above condition implies that $\frac{f'}{f}$ is strictly decreasing. In the regions where $f'' < 0$, the above inequality holds because the LHS is positive while the RHS is negative, so we only need to focus on the case where $f'' > 0$, where both sides are negative. Now let's consider the cases where:

1. case $\frac{f'''}{f''} > \frac{f''}{f'}$

   In this case $\frac{f''}{f'}$ is a decreasing function. Meanwhile, the slope of $\frac{f'}{f}$ depends on whether $\frac{f''}{f'} > \frac{f'}{f}$ or $\frac{f''}{f'} < \frac{f'}{f}$. We can exclude the case where

   $$\frac{f''}{f'} < \frac{f'}{f}$$

   since that would mean that $\frac{f'}{f}$, which asymptotes to 0 as $\omega \to -\infty$ is increasing with ω. Moreover, the two curves cannot cross, since if $\frac{f'}{f}$ is below $\frac{f''}{f'}$ that it is decreasing and if $\frac{f'}{f}$ is above then it is increasing. Thus the only possibility is $\frac{f''}{f'} > \frac{f'}{f}$.

2. case $\frac{f'''}{f''} < \frac{f''}{f'}$

   In this case $\frac{f''}{f'}$ is an increasing function. On the other hand, if $\frac{f''}{f'} > \frac{f'}{f}$ then $\frac{f'}{f}$ is decreasing. But we can rule out this case because $\lim_{\omega \to -\infty} \frac{f'}{f} = 0$ and the curves cannot intersect. We can also rule out the other case where $\frac{f''}{f'} > \frac{f'}{f}$, because this would imply that they are both increasing functions but by $\lim_{\omega \to -\infty} \frac{f'}{f} = 0$ and $\frac{f'}{f} \in (1-\gamma, 0)$ this is not possible.

From the above, the only possibility then is that $\frac{f''}{f'} > \frac{f'}{f}$.

8.3 Proof of Proposition 3

Consider the value of a firm in the C-sector that plans to not invest in the future

$$\pi_0 S_{C,0}^V = E_0 \int_0^\infty \pi_s \left(X_s (K_{C,0} e^{-\delta s})^{\beta_C} L_{C,s}^{1-\beta_C} - w_s L_{C,s} \right) ds$$

its labor decision yields

$$(1-\beta_C) X_s (K_{C,0} e^{-\delta s})^{\beta_C} L_{C,s}^{1-\beta_C} = w_s L_{C,s}$$

Let $\Lambda_{s,t} = \exp(\int_t^s i_{C,u}^* \, du)$. Now consider a firm who follows the optimal investment policy, its first order condition is

$$(1-\beta_C) X_s {K_{C,s}}^{\beta_C} {L_{C,s}^*}^{1-\beta_C} = w_s L_{C,s}^*$$

dividing through, this yields

$$L_{C,s} = L_{C,s}^* \Lambda_{s,0}^{-1}$$

which implies

$$
\begin{aligned}
\pi_0 S_0^V &= E_0 \int_0^\infty \pi_s X_s \, K_{C,0}^{\beta_C} e^{-\delta\beta_C \, s} \hat{L}_{C,0}^{1-\beta_C} \, ds \\
\pi_0 S_0^V &= E_0 \int_0^\infty \pi_s \beta_C X_s \, K_{C,s}^{\beta_C} L_{C,0}^{1-\beta_C} \Lambda_{s,0}^{-1} \, ds \\
\pi_0 S_0^V &= E_0 \int_0^\infty \exp\left(\int_0^s h_J(C,N,J) - i_{C,u} \, du \right) \beta_C h_C(C,N,J) C ds \\
&= \frac{X_0^{1-\gamma} K_0^{\beta_C(1-\gamma)}}{1-\gamma} \times \\
& \quad E_0 \int_0^\infty \exp\left(\int_0^s \hat{\rho}_u du \right) \rho\beta_C(1-\gamma) L_{C,s}^{(1-\beta_C)(1-\theta^{-1})} N_s^{\psi(1-\theta^{-1})} f(\omega_s)^{\frac{\gamma\theta-1}{\theta(\gamma-1)}} \, ds \\
&= \frac{X_0^{1-\gamma} K_{C,0}^{\beta_C(1-\gamma)}}{1-\gamma} g(\omega_0)
\end{aligned}
$$

The Feynman-Kac theorem implies that $g(\omega_t)$ can be computed as the solution to the ODE:

$$
0 = \rho\beta_C(1-\gamma) L_C(\omega)^{(1-\beta_C)(1-\theta^{-1})} N(\omega)^{\psi(1-\theta^{-1})} f(\omega)^{\frac{\gamma\theta-1}{\theta(\gamma-1)}} + \hat{\rho}(\omega) g(\omega) + \mathcal{D}_\omega g(\omega)
$$

where

$$
\hat{\rho}(\omega_u) = h_J(C_u, N_u, J_u) + (\beta_C(1-\gamma)-1) i_{C,u} - \delta\beta_C(1-\gamma) + (1-\gamma)\left(\mu_X - \frac{1}{2}\sigma_X^2\right) + \frac{1}{2}(1-\gamma)^2 \sigma_X^2.
$$

8.4 Numerical Solution

8.4.1 Markov-Chain Approximation

The solution method closely follows Kushner and Dupuis (1993). For exposition purposes consider the case where $\gamma = \theta^{-1}$ and $\psi = 0$. In this case the HJB equation becomes:

$$
0 = \min_l \left\{ (1-l)^{(1-\gamma)(1-\beta_C)} - (u + \beta_C(\gamma-1) c^{-1}(e^\omega l^{1-\beta_I})) f(\omega) + f'(\omega)(\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - c^{-1}(e^\omega l^{1-\beta_I})) + \frac{1}{2}\sigma_Y^2 f''(\omega) \right\}
$$

and ω follows

$$
d\omega = (\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - c^{-1}(e^\omega l^{*1-\beta_I})) \, dt + \sigma_Y dZ_t^Y
$$

I discretize the state space, creating a 1200 point grid for ω and f with $h = \Delta\omega$. Then the following approximations can be used

$$
f'(\omega_n) \approx \frac{f_{n+1} - f_{n-1}}{2h} \qquad \text{and} \qquad f''(\omega_n) \approx \frac{f_{n+1} + f_{n-1} - f_n}{h^2}.
$$

I then approximate the HJB equation as

$$
f_n = \min_l \left\{ e^{-\beta(\omega_n;l)\Delta t^h} \left[p_-(\omega_n;l) f_{n-1} + p_+(\omega_n;l) f_{n+1} \right] + (1-l)^{(1-\gamma)(1-\beta_C)} \Delta t^h \right\} \tag{25}
$$

where

$$\beta(\omega_n; l) = u + \beta_C(\gamma - 1)c^{-1})e^{\omega_n} l^{1-\beta_I}) \qquad p_-(\omega_n; l) = \tfrac{1}{2} + h\,\frac{\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - c^{-1}(e^{\omega_n} l^{1-\beta_I})}{2\sigma_Y^2}$$
$$p_+(\omega_n; l) = \tfrac{1}{2} - h\,\frac{\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - c^{-1}(e^{\omega_n} l^{1-\beta_I})}{2\sigma_Y^2} \qquad \Delta t^h = \frac{h^2}{\sigma_Y^2}$$

and I have used the approximation $\frac{1}{1+\beta(\omega_n;l)\Delta t^h} \approx e^{-\beta(\omega_n;l)\Delta t^h}$. This corresponds to an Markov Chain approximation to ω, where

$$p(\omega = \omega_n + h|\omega = \omega_n) = p_+(\omega_n; l) \qquad \text{and} \qquad p(\omega = \omega_n - h|\omega = \omega_n) = p_-(\omega_n; l)$$

are the transition probabilities and the time interval is Δt^h. The markov chain is locally consistent because

$$\begin{aligned} E(\Delta\omega_n|\omega_n) &= (\mu_Y + \delta - \frac{1}{2}\sigma_Y^2 - c^{-1}(e^{\omega_n} l^{1-\beta_I}))\Delta t^h \\ E((\Delta\omega_n - E\Delta\omega_n)^2|\omega_n) &= \sigma_Y^2\Delta t^h + o(\Delta t^2) \end{aligned}$$

Note that care must be taken when choosing h to ensure that the probabilities are non-negative for all admissible controls $l \in [0, 1]$ at all points in the grid. Alternative differencing schemes that produce positive probabilities can also be used.

Using an initial guess for f, say f^i, one can numerically compute the minimum in(25). Then, given $l_n^{*\,i}$, one can start from $n = 0$ and recursively compute the update on f using the Gauss-Seidel algorithm:

$$f_n^{i+1} = e^{-\beta(\omega_n; l_n^{*\,i})\Delta t^h}\left[p_-(\omega_n; l_n^{*\,i})f_{n-1}^{i+1} + p_+(\omega_n; l_n^{*\,i})f_{n+1}^{i}\right] + (1 - l_n^{*\,i})^{(1-\gamma)(1-\beta_C)}\Delta t^h \tag{26}$$

I impose a reflecting barrier on ω at the boundaries of the grid. This reduces to $f_0 = f_1$ and $f_N = f_{N-1}$, since there is no discounting at the boundary and

$$p(\omega = \omega_0 + h|\omega = \omega_0) = 1$$

$$p(\omega = \omega_N - h|\omega = \omega_N) = 1$$
$$\Delta t^h(\omega_N) = \Delta t^h(\omega_0) = 0$$

Finally, because the minimum in (25) is costly to compute, I iterate a couple of times on (26) before updating the policy function.

8.5 Data

8.5.1 IMC

I use the 1997 BEA Standard Make and Use Tables at the detailed level. I use the standard make (table 1) and use (table 2) tables. The uses tables enumerates the contribution of each IO commodity code to Personal Consumption Expenditures (IO code F01000) and Gross Private Fixed Investment (IO code F02000). I use the make tables along with the NAICS-IO map to construct a mapping between 6-digit NAICS Codes to IO commodity codes. I then use the uses table to create a map from IO codes to Investment or Consumption. Because some industries contribute to both PCE and GPFI, I follow two schemes to create a unique link. The first assigns industries to the sector they contribute the most value in terms of producer's prices excluding transportation costs. The second scheme classifies industries as consumption or investment if they contribute only in one sector.

I use COMPUSTAT to create a PERMNO-NAICS link and form value weighted portfolios using simple returns on all common stocks traded on NYSE, AMEX and Nasdaq. I construct two portfolios, using the first (IMC) and the second (IMCX) classification scheme. Examples of Investment industries are

IO Code	Description
213111	Drilling oil and gas wells
333111	Farm machinery and equipment manufacturing
333295	Semiconductor machinery manufacturing
334111	Electronic computer manufacturing
334220	Broadcast and wireless communications equipment
336120	Heavy duty truck manufacturing

Examples of Consumption industries are

IO Code	Description
1111B0	Grain farming
221100	Power generation and supply
311410	Frozen food manufacturing
312110	Soft drink and ice manufacturing
325611	Soap and other detergent manufacturing
334300	Audio and video equipment manufacturing

The full list of IO codes and their assignments into industries is available from the author's website.

8.5.2 10 IMC sorted portfolios

The portfolios are formed from ordinary common equity, traded in NYSE, AMEX, or Nasdaq, in the CRSP Monthly Stock Database. For every stock, I estimate pre-ranking covariances on IMC based on a 5-year window using weekly log excess returns. I sort stocks into IMC covariance deciles. I construct value-weighted portfolios using simple monthly returns, tracking the portfolios for 5 years until next re-balancing.

8.5.3 24 IND/IMC sorted portfolios

The portfolios are formed from ordinary common equity, traded in NYSE, AMEX, or Nasdaq, in the CRSP Monthly Stock Database. For every stock, I estimate pre-ranking covariances on IMC based on a 5-year window using weekly log excess returns. Stocks are sorted into eight industries based on their two-digit SIC codes: (1) nondurables manufacturing, (2) durables manufacturing, (3) other manufacturing, (4) nondurables retail, (5) durables retail, (6) services, (7) finance, and (8) natural resource. Within each industry, stocks are then sorted into three portfolios based on their IMC covariance using breakpoints of 30th and 70th percentiles. I construct value-weighted portfolios using simple monthly returns, tracking the portfolios for 5 years until next re-balancing.

8.5.4 25 BM/IMC sorted portfolios

The portfolios are formed from ordinary common equity, traded in NYSE, AMEX, or Nasdaq, in the CRSP Monthly Stock Database. For every stock, I estimate pre-ranking covariances on IMC based on a 5-year window using weekly log excess returns. Using book to market equity from COMPUSTAT and the pre-ranking covariances, including only stocks that have full observations, I first sort stocks into BM quintiles using NYSE breakpoints and then into IMC covariance quintiles. Portfolios are formed in June every 5 years. I construct value-weighted portfolios using simple monthly returns, tracking the portfolios for 5 years until next re-balancing.

8.5.5 25 ME/BM portfolios

The data come from Kenneth French's web page.

8.5.6 Macroeconomic Data

Monthly data come from the website of the St Louis Fed. Quarterly quantity data and price deflators come from the website of the Bureau of Economic Analysis, specifically from the NIPA tables 5.3.3, 5.3.4, 2.3.3, 1.1.3 and 1.1.4.

8.6 Empirical Methodology

8.6.1 GMM Cross-Sectional tests

I estimate the linear factor model using two-step GMM. As Cochrane (2001) illustrates, if one is interested in estimating a linear model for the SDF of the following form:

$$m = a - bF$$

One can test the moment restriction

$$E[mR^e] = 0. \tag{27}$$

Letting the vector of unknown parameters $\theta = [b, \mu_F]$ and the data $X_t = [R_t^e, F_t]$, where F is the factors and R^e are excess returns. If one is using excess returns, then the mean of the pricing kernel is unidentified. If the model is correct, then (27) must hold at the true parameter values:

$$E[g(X, \theta_0)] = 0$$

where

$$g(X, \theta) = \begin{bmatrix} R_t^e - R_t^e(F_t - \mu_F)'b \\ F_t - \mu_F \end{bmatrix}$$

I use the first stage weighting matrix

$$W = \begin{bmatrix} kI_N & 0 \\ 0 & \hat{\Sigma}_{ff}^{-1} \end{bmatrix}$$

Following Gomes, Kogan and Yogo (2006) I pick $k = \det(\hat{\Sigma}_R)^{-1/N}$. The first stage weighting matrix puts equal weight in each of the N asset pricing restrictions. I compute the spectral density matrix for the second stage using the Newey-West estimator with 3 lags of returns.

I use the covariance rather than the beta representation because I am interested in the marginal ability of IMC to price each cross-section. As Cochrane 2001 illustrates, one can test whether a factor is priced, given the other factors in the specification by $b \neq 0$.

8.6.2 Fama-Mcbeth Cross-Sectional tests

I run Fama and MacBeth (1973) cross-sectional regressions of simple monthly returns of all NYSE, AMEX, and Nasdaq stocks on covariances and characteristics. Covariances are estimated using a procedure similar to Fama and French (1992). Specifically, for each individual stock I estimate the covariance of its returns with IMC using 5 years of weekly log excess returns. At the end of a five year period, stocks are then sorted into 100 pre-ranking covariance centiles. I then compute the equal-weighted monthly log excess returns on these 100 portfolios over the next 5 years. This procedure is repeated every 5 years, forming a time-series of returns on these 100 portfolios. I then re-estimate covariances for the portfolios formed from the pre-ranking sorts using 5 years of monthly data to obtain post-ranking covariances. The post-ranking covariance estimate for a given portfolio is then assigned to each stock in the portfolio. Portfolio assignments are updated every 5 years. Every month the cross-section of stock returns in excess of risk free rate is then regressed on a constant, the covariance with the excess return on the CRSP value-weighted index, the covariance with the return on the IMC portfolio, the log of market capitalization (ME) on December of year t-1 and the log of Book to Market (BM) of year t-1.