Lecture 6: CAPM and empirical evidence

Investments

Introduction

- In the last 20 years or so, a large body of empirical evidence has surfaced that challenges some of the basic assumptions of finance theory.
- Specifically, researchers have found evidence that challenges the capital asset pricing model.
- This evidence has been interpreted as saying that modern finance theory is “wrong”, markets are not efficient or investors are not rational.
- The goal of this lecture is to review some of this evidence and understand what it implies for modern portfolio theory.

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Who cares?

- The CAPM is the most often used model for figuring out the appropriate compensation for risk.
- It is widely used in
  a) Corporate Project Valuation
  b) Portfolio Management
  c) Evaluating Portfolio Managers
  d) Cost-of-capital determination
- Most of the evidence we will see challenges the CAPM.
  ↪ The failure of the CAPM implies that the market portfolio is not efficient.
  ↪ To understand what portfolio is efficient, you need to understand *how* it fails.
- We will revisit the question of the appropriate discount rate in a couple of lectures.

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Assumptions behind the CAPM

- The CAPM is an economic model that specifies what expected returns (and therefore prices) should be as a function of systematic risk.
- What is the argument that makes it hold? It is based on a proof by contradiction:
  1. Suppose that it didn't hold.
  2. Investors would get rewarded for bearing non-systematic risk.
  3. Non-systematic risk can, by definition, be diversified away.
  4. Investors would be getting something for nothing.
  5. Everyone should follow these strategies.
  6. Prices will eventually adjust and the anomalies disappear.
- Notice that there is an additional set of hidden assumptions. What are they?

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CAPM and the data

- If the CAPM is true, then *all* securities should lie in the SML.

$$E(\tilde{r}_i) = r_f + \beta_i \cdot [E(\tilde{r}_m) - r_f]$$

  - ↪ The relation of expected return and β_i is linear
  - ↪ *Only* β_i explains differences in returns among securities.
  - ↪ $E(R)$ of an asset with a $\beta = 0$ is r_f.
  - ↪ $E(R)$ of an asset with a $\beta = 1$ is the expected return on the market.

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CAPM and the data

- How can we test the CAPM? 2 Approaches:

1. Test $\alpha_i = 0$ in

$$R_{i,t} - r_f = \alpha_i + \beta_i (R_{m,t} - r_f) + \varepsilon_{i,t}$$

2. Given, $E(R_{i,t} - r_f)$ and β_i, test $\gamma_0 = 0$, $\gamma_1 > 0$ *and* $u_i = 0$ in

$$E(R_{i,t} - r_f) = \gamma_0 + \gamma_1 \beta_i + u_i$$

CAPM and the data: A first test

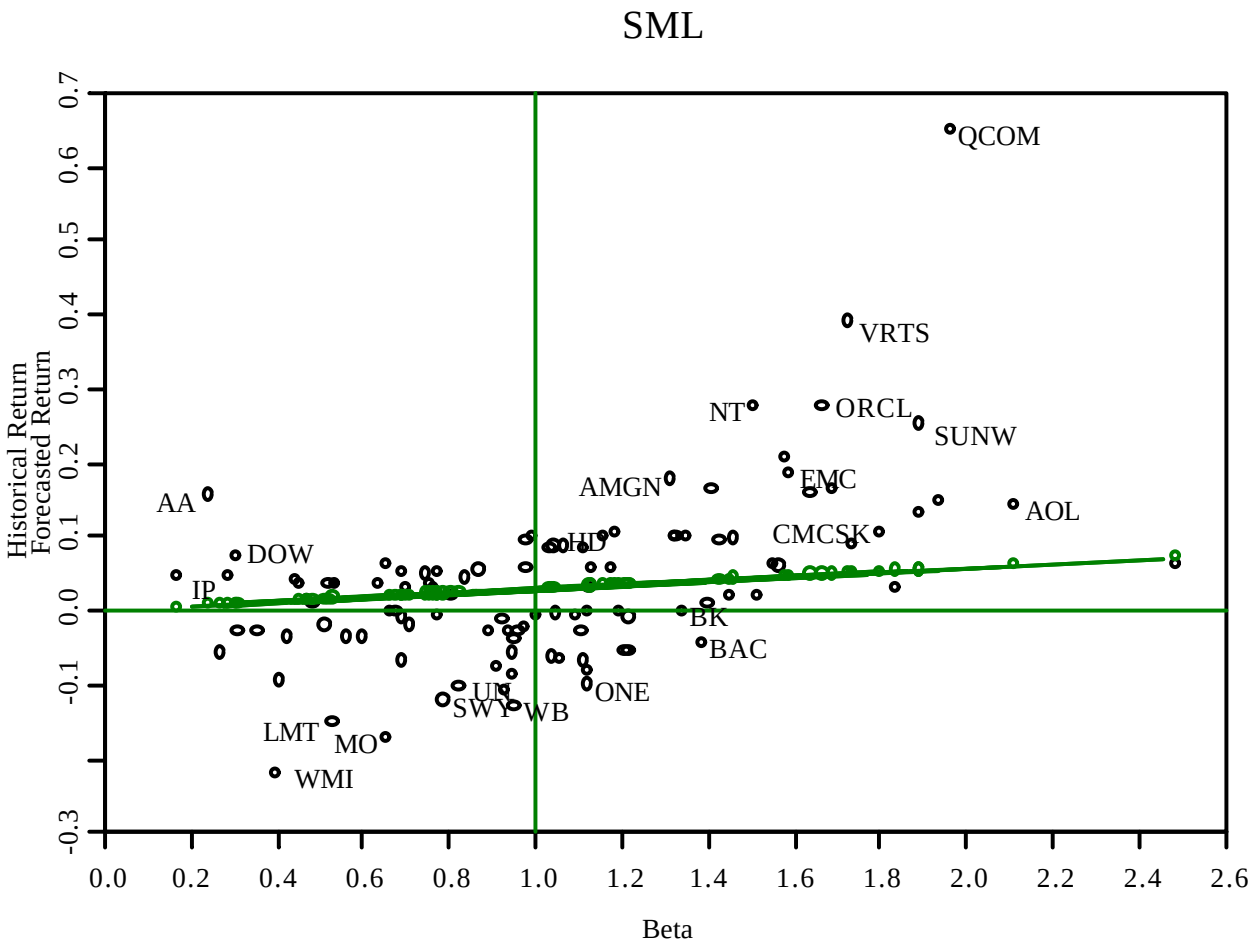
1. Collect the data.
   - ↪ We will use monthly data on 100 largest stocks
2. Estimate β_i and $E(R_{i,t} - r_f)$.
   - ↪ use a *first-pass* regression to estimate β_i
   - ↪ use historical average for $E(R_{i,t} - r_f)$
3. Set up a *second-pass* regression in Excel.
   - ↪ The dependent variable: $y_i = E(R_{i,t} - r_f)$
   - ↪ The independent variable: $x_i = \beta_i$
4. Results:

	Estimate	Standard Error	t-stat
γ_0	6.01%	1.8%	3.5
γ_1	0.17%	1.7%	0.1
R^2	2%		

5. What do these numbers mean?

CAPM and the data: A first test

- Average returns vs Betas for top 100 market-cap stocks.



CAPM and the data

- To test the CAPM one needs to specify what the market portfolio is.
  a) Only 1/3 non governmental tangible assets are owned by the corporate sector.
  b) Among the corporate assets, only 1/3 is financed by equity
  c) What about intangible assets, like human capital?
  d) International markets?
- Measurement error in β
  $\hookrightarrow$ Can be avoided by grouping stocks into portfolios.
- Measurement error in expected returns.

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CAPM and the data: A better test

- We can get around the measurement error problem by looking at diversified portfolios.
- We can sort firms into portfolios based on characteristics that we think should explain risk premia.
- Let's try this with market beta:
  1. For every year t, use past 5 years of data to estimate market beta.
  2. At the beginning of the year sort firms into 10 portfolios based on their estimated beta.
  3. Track the performance of these portfolios over the next year.
  4. At year $t+1$ repeat.
- This test was done by Black, Jensen and Scholes.

BJS Portfolio selection technique

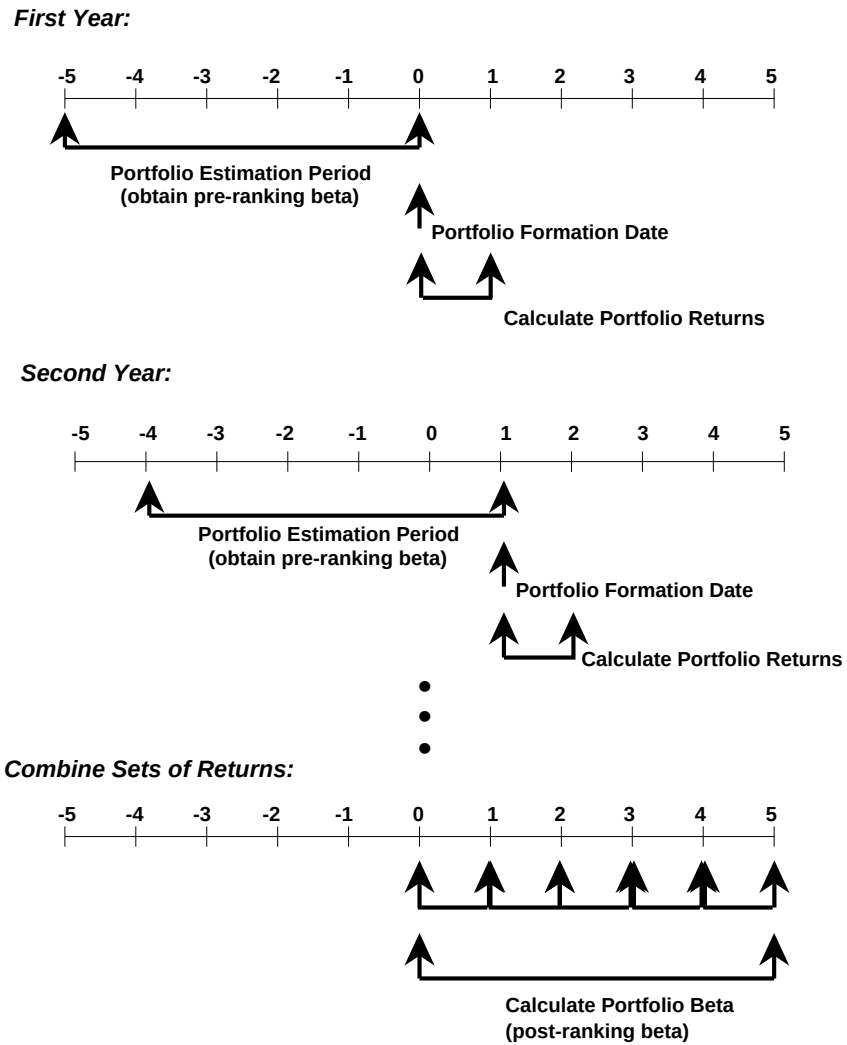
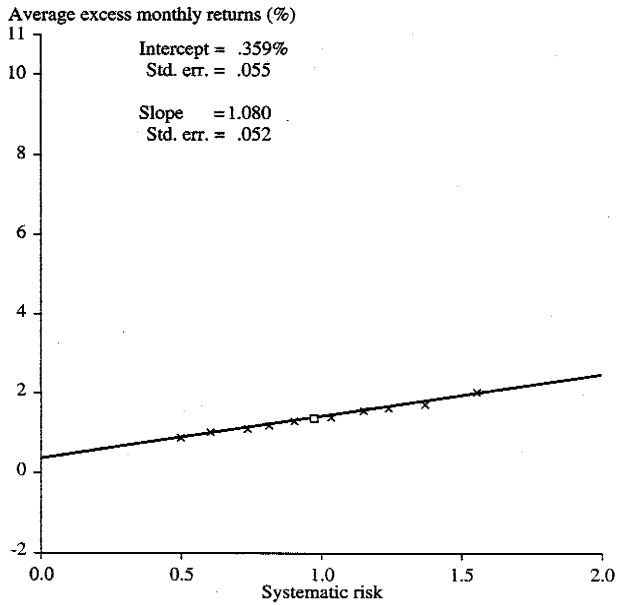


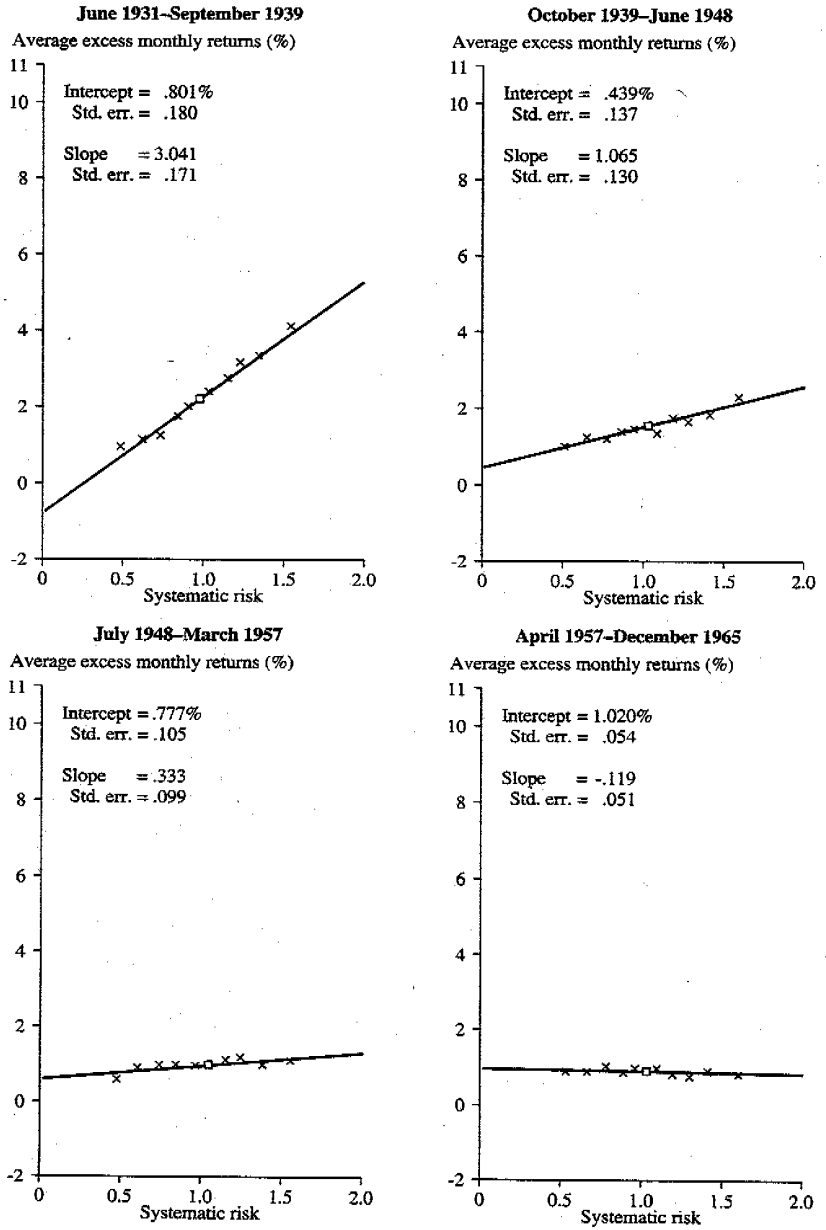
Table 2
Summary of Statistics for Time Series Tests, Entire Period (January, 1931-December, 1965)
(Sample Size for Each Regression =420)

Item*	*Portfolio Number* 1	2	3	4	5	6	7	8	9	10	$\overline{R}_M$
$\hat{\beta}$	1.5614	1.3838	1.2483	1.1625	1.0572	0.9229	0.8531	0.7534	0.6291	0.4992	1.0000
$\hat{\alpha} \bullet 10^2$	-0.0829	-0.1938	-0.0649	-0.0167	-0.0543	0.0593	0.0462	0.0812	0.1968	0.2012	
$t(\hat{\alpha})$	-0.4274	-1.9935	-0.7597	-0.2468	-0.8869	0.7878	0.7050	1.1837	2.3126	1.8684	
$r(\tilde{R}, \tilde{R}_M)$	0.9625	0.9875	0.9882	0.9914	0.9915	0.9833	0.9851	0.9793	0.9560	0.8981	
$r(\tilde{e}_t, \tilde{e}_{t-1})$	0.0549	-0.0638	0.0366	0.0073	-0.0708	-0.1248	0.1294	0.1041	0.0444	0.0992	
$\sigma(\tilde{e})$	0.0393	0.0197	0.0173	0.0137	0.0124	0.0152	0.0133	0.0139	0.0172	0.0218	
$\overline{R}$	0.0213	0.0177	0.0171	0.0163	0.0145	0.0137	0.0126	0.0115	0.0109	0.0091	0.0142
σ	0.1445	0.1248	0.1126	0.1045	0.0950	0.0836	0.0772	0.0685	0.0586	0.0495	0.0891

* $\overline{R}_M$ = average monthly excess returns, σ = standard deviation of the monthly excess returns, r = correlation coefficient.



BJS: Results


June 1931–September 1939
Average excess monthly returns (%)
Intercept = .801%
Std. err. = .180
Slope = 3.041
Std. err. = .171
Systematic risk
October 1939–June 1948
Average excess monthly returns (%)
Intercept = .439%
Std. err. = .137
Slope = 1.065
Std. err. = .130
Systematic risk
July 1948–March 1957
Average excess monthly returns (%)
Intercept = .777%
Std. err. = .105
Slope = .333
Std. err. = .099
Systematic risk
April 1957–December 1965
Average excess monthly returns (%)
Intercept = 1.020%
Std. err. = .054
Slope = -.119
Std. err. = .051
Systematic risk

CAPM and the data (1963-2008)

Market beta	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R)-r_f(\%)$	5.81	6.40	5.42	6.73	5.92	5.08	6.02	5.04	3.98	3.84	-1.97
	(2.51)	(3.04)	(2.72)	(3.77)	(3.13)	(2.65)	(2.43)	(1.95)	(1.19)	(0.98)	(-0.52)
$\sigma(\%)$	18.20	16.21	17.50	15.60	16.86	16.62	20.16	21.02	25.47	30.98	26.47
β^{mkt}	0.71	0.64	0.78	0.76	0.80	0.83	1.06	1.10	1.27	1.46	0.75
	(5.19)	(5.42)	(5.33)	(13.67)	(10.39)	(10.02)	(18.41)	(24.42)	(14.65)	(11.75)	(4.26)
$\alpha(\%)$	2.24	3.21	1.53	2.93	1.90	0.95	0.74	-0.44	-2.39	-3.47	-5.71
	(1.11)	(2.09)	(1.32)	(2.72)	(1.59)	(0.87)	(0.76)	(-0.43)	(-1.64)	(-1.48)	(-1.87)
$R^2(\%)$	49.70	50.28	64.13	76.94	73.46	80.12	88.95	88.09	81.13	72.10	25.93

- Sort firms into 10 portfolios based on their market beta β from

$$R^e_{it} = a_i + \beta_i R^e_{mt} + \varepsilon_{it}$$

- Use last year of weekly data to estimate β_i. Form portfolios at the end of December.

The CAPM and the data, bottom line

- The CAPM is not a very good model of risk and return, especially when we focus on the last 50 years:
  - ↪ High-beta firms have, if anything, lower average returns than low-beta firms.
- Why this is the case is a hotly contested topic.
- Next, we will see that the CAPM fails *in a particular way*
  - ↪ that is, there are particular *types* of firms that are mispriced
- Understanding *how* the CAPM fails will be the key to fixing it

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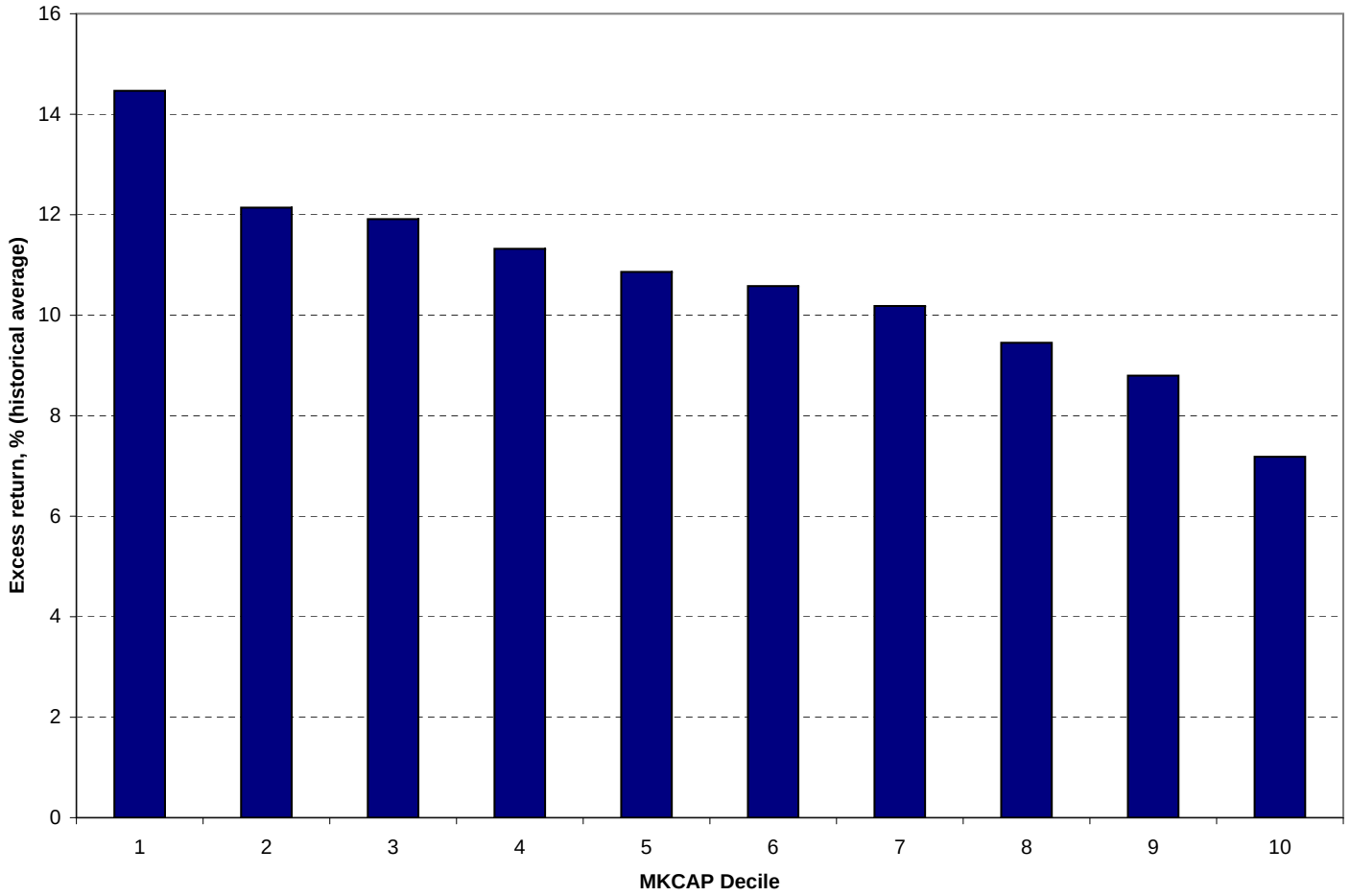
The Small Firm Effect

- A number of researchers, starting with Keim (1981) and Banz (1981) found that differences in *firm size* explained differences in expected returns.
  1. *firm size* was defined as total market capitalization.
  2. Small stocks (i.e. small cap stocks) outperformed large stocks (i.e. large cap stocks).
- In order to minimize measurement error, we will form portfolios of stocks based on their past market capitalizations

		# of Stocks			
Dec	MKCap(m$)	NYSE	AMEX	NASDAQ	Total
10	511,391	172	5	80	257
9	10,486	172	3	81	256
8	4,428	172	5	136	313
7	2,237	172	5	166	343
6	1,387	172	5	217	394
5	889	172	11	254	437
4	534	172	15	251	438
3	353	172	32	400	604
2	198	172	73	551	796
1	95	172	412	1,399	1,983

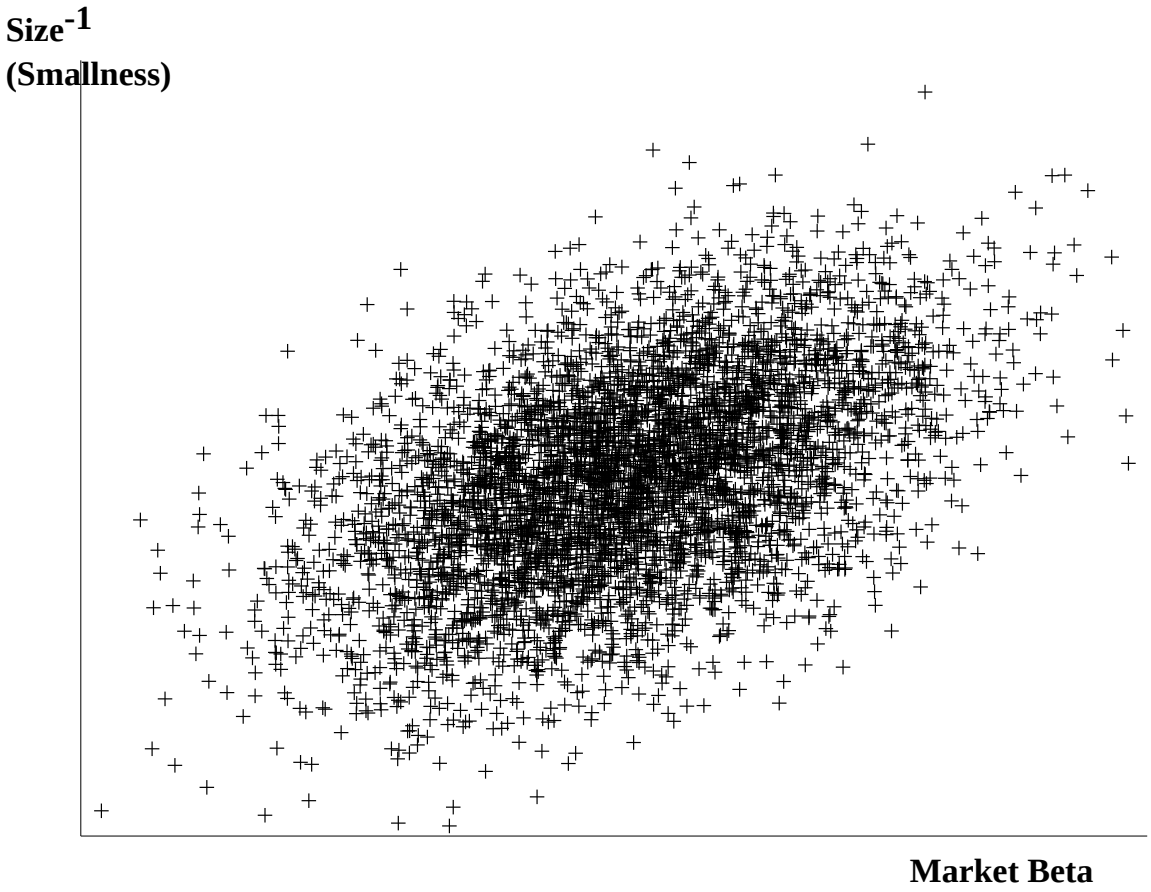
The Small Firm Effect

- Excess portfolio returns, historical average (1926-2007)



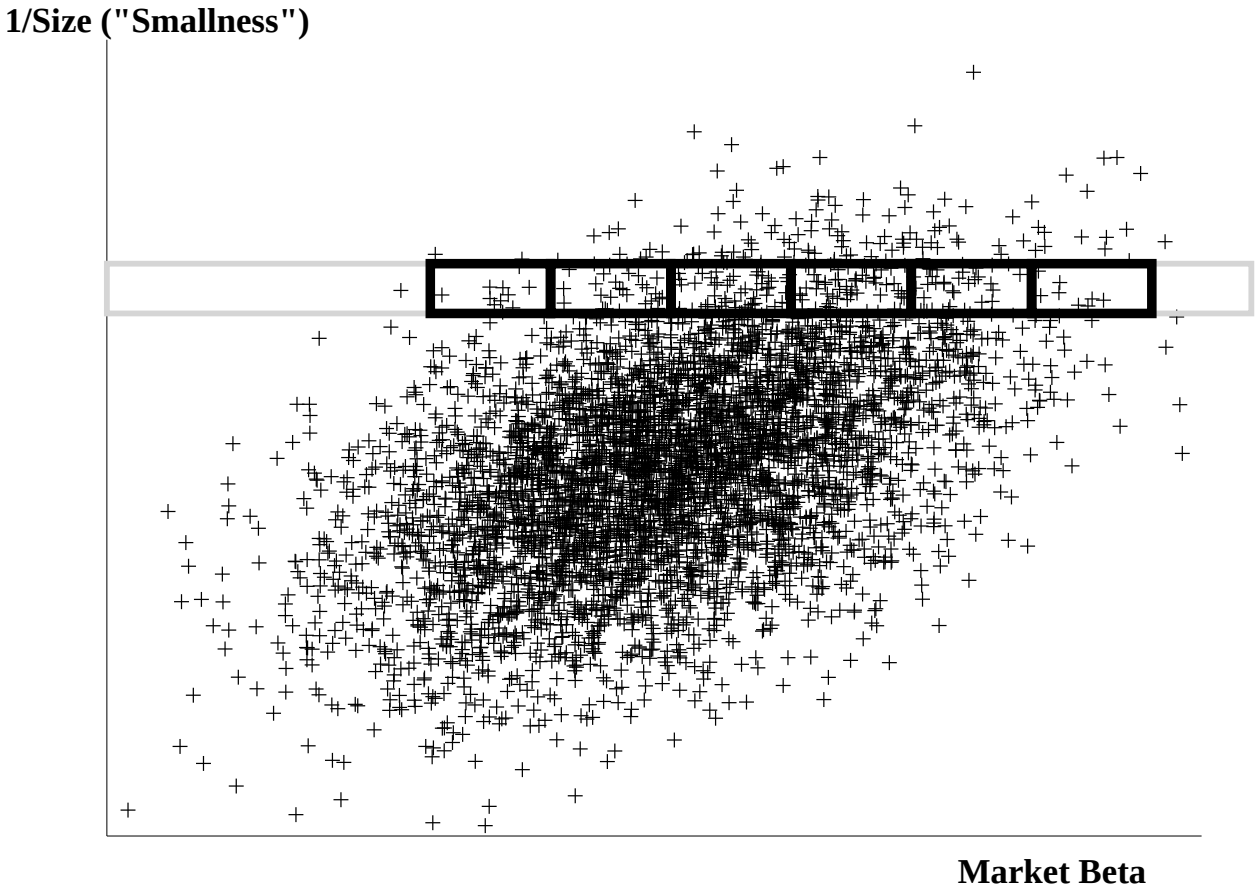
The Small Firm Effect

- Is this evidence inconsistent with the CAPM?
- Smaller firms tend to have higher β's



The Small Firm Effect

- To resolve this, Fama and French (1992) do a double sort, first on size, then on market β.



The Small Firm Effect

- They find that the relation between average returns and market β within size decile is generally negative.

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel A: Average Monthly Return (in Percent)											
All		1.22	1.30	1.32	1.35	1.36	1.34	1.29	1.34	1.14	1.10
Small-ME	1.78	1.74	1.76	2.08	1.91	1.92	1.72	1.77	1.91	1.56	1.46
ME-2	1.44	1.41	1.35	1.33	1.61	1.72	1.59	1.40	1.62	1.24	1.11
ME-3	1.36	1.21	1.40	1.22	1.47	1.34	1.51	1.33	1.57	1.33	1.21
ME-4	1.28	1.26	1.29	1.19	1.27	1.51	1.30	1.19	1.56	1.18	1.00
ME-5	1.24	1.22	1.30	1.28	1.33	1.21	1.37	1.41	1.31	0.92	1.06
ME-6	1.23	1.21	1.32	1.37	1.09	1.34	1.10	1.40	1.21	1.22	1.08
ME-7	1.17	1.08	1.23	1.37	1.27	1.19	1.34	1.10	1.11	0.87	1.17
ME-8	1.15	1.06	1.18	1.26	1.25	1.26	1.17	1.16	1.05	1.08	1.04
ME-9	1.13	0.99	1.13	1.00	1.24	1.28	1.31	1.15	1.11	1.09	1.05
Large-ME	0.96	0.99	1.01	1.12	1.01	0.89	0.95	0.95	1.00	0.90	0.68

The Small Firm Effect

- Other researchers have criticized these findings:
  1. Other measures of firm size, such as book value of assets, or number of employees have no similar predictive power.
  2. The size effect has more or less disappeared over the last 10-15 years.

- Because the effect is mostly there for small stocks, it could be due to a liquidity premium: Small stocks are less liquid (i.e. they have higher transaction costs) so investors may require higher rates of return in order to hold them.

The Value Effect

- Some researchers, starting with Graham and Dodd in the late 1930s, noticed that *value* stocks outperformed *growth* stocks.
  - ↪ **Definition:** *A value stock is a stock with a low market price relative to the book value of assets.*

    Some people believe these stocks are undervalued by the market and thus should present good investment opportunities.
  - ↪ **Definition:** *A growth stock is a stock with a high market price relative to the book value of assets.*

    Some people believe that these stocks are "glamor" stocks that are overvalued by the market, and as such the expected returns form holding them will be poor.

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The Value Effect

- Sort all stocks into 10 portfolios based on the *ratio of book value of equity to the market value of equity*.
- Re-balance portfolios every year.

	10 portfolios sorted on book-to-market equity, 1926-2007										
Sort	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R_i) - r_f$	6.77	8.01	7.85	7.98	8.93	9.37	9.66	11.41	12.12	13.17	6.40
	(2.22)	(2.13)	(2.07)	(2.34)	(2.18)	(2.39)	(2.59)	(2.70)	(2.95)	(3.62)	(2.57)
σ	19.98	19.16	18.59	21.06	19.60	21.46	23.29	24.22	26.48	32.49	23.10
α	-1.01	0.42	0.53	-0.23	1.38	1.13	0.92	2.46	2.43	1.97	2.98
(t)	(0.74)	(0.60)	(0.59)	(0.73)	(0.78)	(0.87)	(1.04)	(1.14)	(1.34)	(1.84)	(2.30)
β_{MKT}	1.01	0.98	0.95	1.06	0.98	1.06	1.13	1.16	1.25	1.45	0.44
(t)	(0.02)	(0.02)	(0.02)	(0.04)	(0.03)	(0.04)	(0.06)	(0.07)	(0.05)	(0.10)	(0.11)
$R^2(\%)$	89.80	92.74	91.86	89.94	88.02	87.23	83.44	80.80	79.38	70.35	12.89

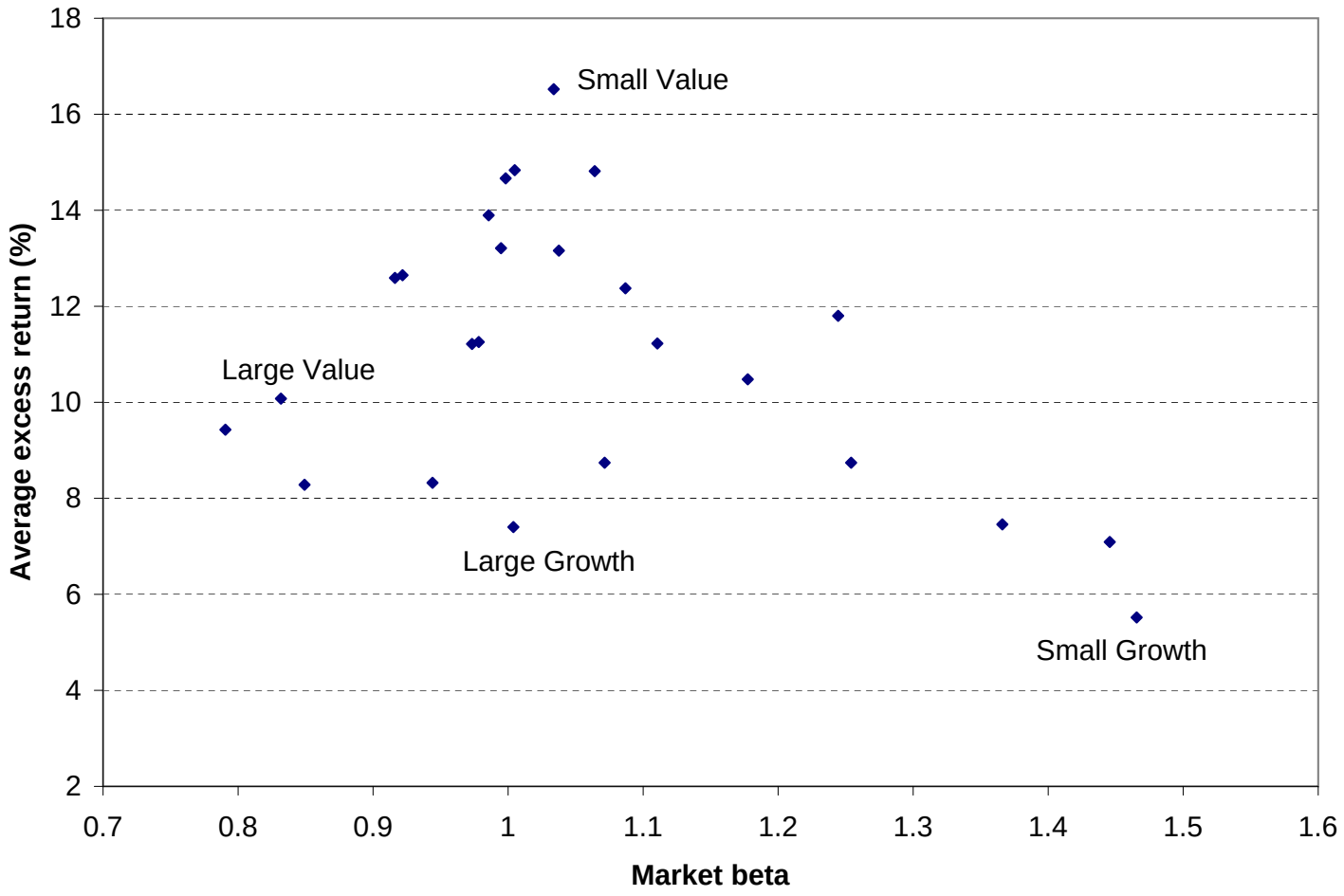
The Value Effect

- Puzzle is more pronounced in the post-war period

	10 portfolios sorted on book-to-market equity, 1962-2007										
Sort	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R_i) - r_f$	3.95	5.59	6.07	6.29	6.28	7.28	8.33	8.67	9.64	11.11	7.16
	(2.67)	(2.42)	(2.39)	(2.37)	(2.22)	(2.21)	(2.21)	(2.19)	(2.37)	(2.73)	(2.28)
σ	18.01	16.35	16.14	16.00	15.01	14.90	14.92	14.78	16.01	18.46	15.39
α	-2.07	-0.04	0.54	0.95	1.38	2.38	3.63	4.05	4.66	5.71	7.78
(t)	(1.05)	(0.73)	(0.76)	(0.95)	(1.01)	(0.92)	(1.10)	(1.12)	(1.23)	(1.71)	(2.44)
β_{MKT}	1.10	1.03	1.01	0.97	0.89	0.90	0.86	0.84	0.91	0.99	-0.11
(t)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)	(0.06)
$R^2(\%)$	86.02	91.34	90.27	85.63	81.92	83.31	76.58	75.20	74.27	65.82	1.07

Size and Value Effect together

- 25 portfolios sorted on Size and Book to Market
- Average excess portfolio returns vs market beta (1962-2007)



Explanations?

- Some potential explanations:

  distress risk Value stocks tend to be stocks that have underperformed in the past. A lot of them are in the verge of bankruptcy and may be particularly risky. Investors may require an additional risk premium in order to hold them.

  liquidity risk Small stocks are more illiquid and may thus command a higher premium.

- Are these plausible?

Related patterns

- Value and size effects are only the tip of the iceberg.
- The literature has documented a number of other patterns:
  - ↪ Firms with high investment rates under-perform firms with low investment rates
    Titman, Wei and Xie (2003)
  - ↪ Firms that issue new shares under-perform that do not.
    Loughran and Ritter (1995)
  - ↪ Firms that repurchase their shares over-perform that do not.
    Ikenberry, Lakonishok and Vermaelen (1995)
  - ↪ Firms with high idiosyncratic volatility under-perform firms with low idiosyncratic volatility
    Ang, Hodrick, Xing, and Zhang (2009)
- Are these separate phenomena?

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Idiosyncratic risk

Idiosyncratic risk	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R)-r_f(\%)$	5.02	5.96	6.40	7.28	5.84	5.60	3.59	3.42	1.01	-0.84	-5.86
	(2.45)	(2.60)	(3.23)	(2.89)	(1.99)	(1.72)	(0.97)	(0.87)	(0.20)	(-0.14)	(-0.95)
$\sigma(\%)$	15.35	17.38	17.82	20.69	25.59	26.55	30.27	32.75	39.33	42.44	37.05
β^{mkt}	0.77	0.92	0.93	1.07	1.29	1.26	1.45	1.53	1.67	1.76	0.98
	(12.71)	(44.95)	(18.91)	(19.60)	(15.81)	(10.53)	(10.81)	(9.59)	(9.23)	(8.20)	(3.97)
$\alpha(\%)$	1.14	1.34	1.73	1.92	-0.63	-0.68	-3.67	-4.24	-7.34	-9.64	-10.78
	(1.12)	(1.65)	(2.10)	(2.69)	(-0.42)	(-0.30)	(-1.41)	(-1.43)	(-1.71)	(-1.90)	(-1.83)
$R^2(\%)$	82.67	91.64	89.14	86.86	82.69	72.53	74.67	70.99	58.41	55.65	22.85

- Sort firms into 10 portfolios based on their idiosyncratic variance σ_ε^2 from

$$R_{it}^e = a_i + \beta_i R_{mt}^e + \varepsilon_{it}$$

- Use last year of weekly data to estimate σ_ε. Form portfolios at the end of December.

Short-Term Momentum

- In a 1993 *Journal of Finance* article, Jagadeesh and Titman show that firms with high (low) returns in the previous year tend to have higher (lower) returns in the following few months.

- The momentum effect seems short-lived in the data, lasting for only a few months.

Short-Term Momentum

- Form portfolios of stocks selected on their past return over the last 12 months.
- We will rebalance these portfolios every month.

	10 portfolios sorted on previous 12 month returns (1926-2007)										
Sort	Lo	2	3	4	5	6	7	8	9	Wi	Wi-Lo
$E(R_i) - r_f$	0.31	5.18	5.12	6.72	6.80	7.58	8.68	10.30	11.47	15.37	15.07
	(3.70)	(3.13)	(2.70)	(2.50)	(2.31)	(2.26)	(2.17)	(2.09)	(2.20)	(2.52)	(2.94)
σ	33.24	28.15	24.24	22.41	20.77	20.33	19.51	18.78	19.78	22.64	26.44
α	-11.52	-5.08	-3.91	-1.77	-1.18	-0.39	1.09	3.05	3.97	7.48	19.01
(t)	(1.65)	(1.31)	(1.12)	(0.94)	(0.79)	(0.66)	(0.70)	(0.70)	(0.81)	(1.33)	(2.44)
β_{MKT}	1.53	1.33	1.17	1.10	1.03	1.03	0.98	0.94	0.97	1.02	-0.51
(t)	(0.08)	(0.07)	(0.06)	(0.04)	(0.04)	(0.02)	(0.02)	(0.02)	(0.03)	(0.06)	(0.13)
R^2(%)	74.95	78.61	82.24	85.01	87.30	90.93	89.64	88.34	85.09	71.90	13.05

Short-Term Momentum

- At the moment, the momentum effect is one of the *most studied* anomalies in Finance.
- On the surface, momentum appears to challenge the efficient market hypothesis.
- This has led behavioral finance advocates to declare victory. They propose several behavioral explanations:
  1. under-reaction: bad news travels slowly.
  2. over-reaction: positive feedback.
  3. disposition effect: investors are reluctant to sell loser stocks.

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Short-Term Momentum

- Momentum also exists among different asset classes, not just individual stocks.
- It also exists among:
  - ↪ Commodities
  - ↪ Currencies
  - ↪ Sovereign bonds
  - ↪ Industry indices

Post-Earnings Announcement Drift

- Bernard and Thomas in the 1989 article in *Journal of Accounting Research* found evidence that stock prices were predictable based on *past* earnings announcements.
  1. They found that firms that had better than expected earnings had higher returns over the next few months.
  2. Firms that had worse than expected earnings in the past had lower returns going forward.
- They interpret this as evidence of market inefficiency due to investor *under-reaction* to earnings announcements.
- They formed portfolios of stocks based on past earnings surprises and tracked their performance.

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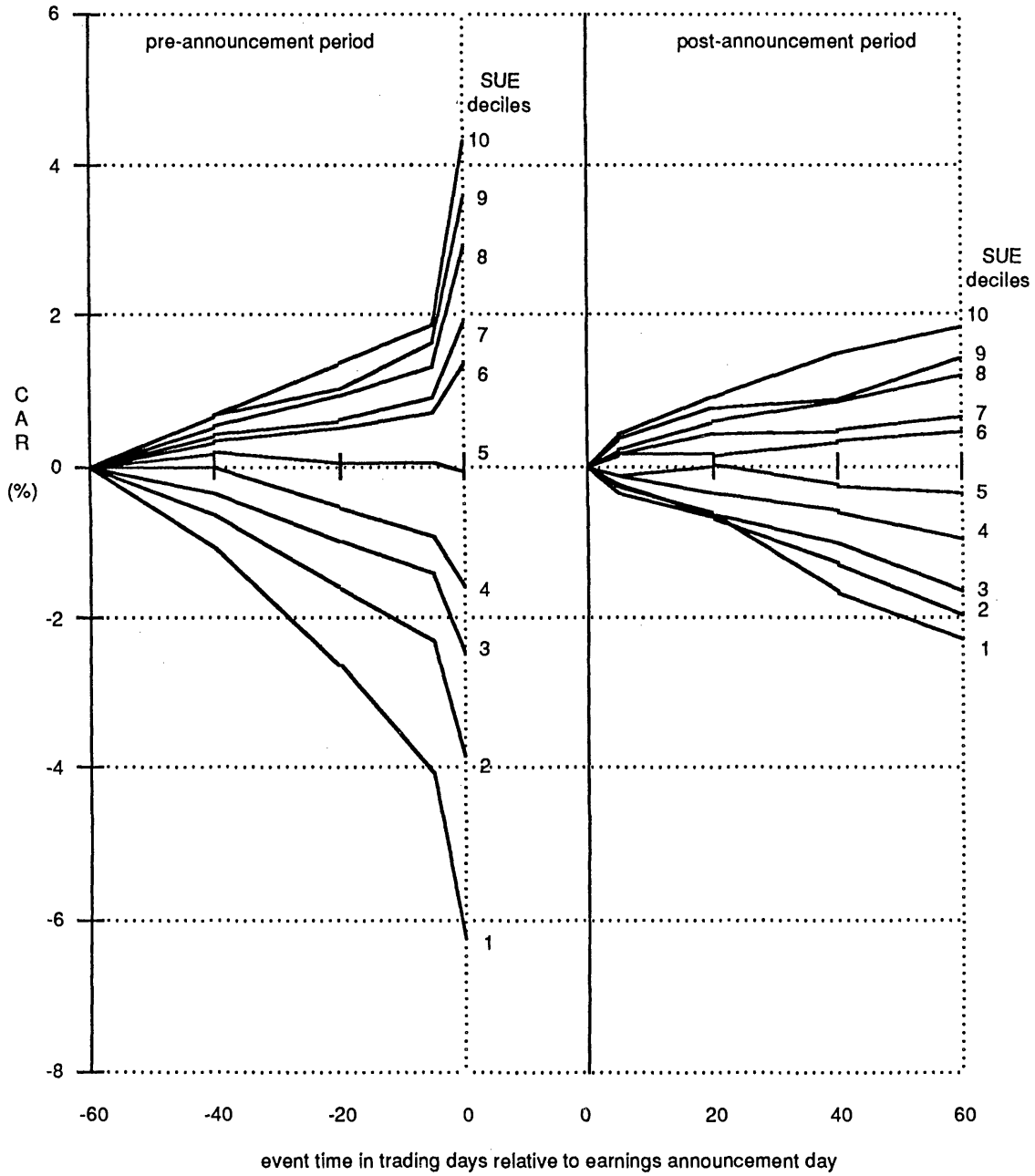
Post-Earnings Announcement Drift



The Profitability Effect

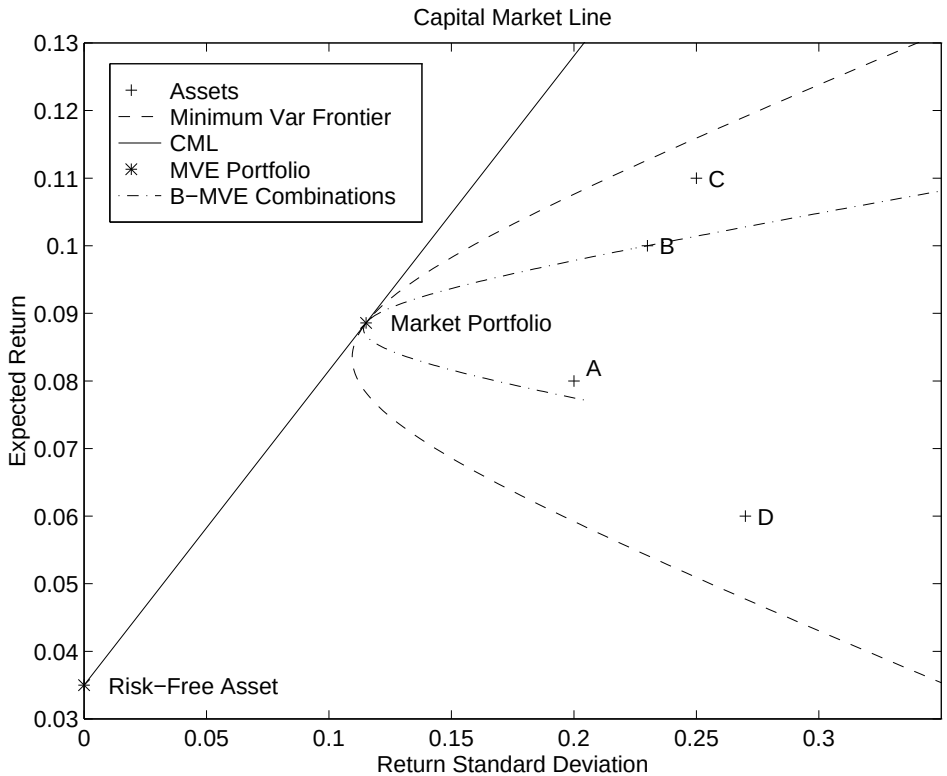
- A closely related pattern to the post-earnings announcement drift is the so-called profitability effect.
- Form portfolios of firms sorted on the past ratio of earnings before income and tax (EBIT) to book assets.
- Rebalance portfolios every year.

	10 portfolios sorted on return on assets, 1962-2007										
Sort	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	-0.19	2.52	5.69	5.17	6.73	6.96	4.42	6.63	4.43	6.26	6.46
	(-0.04)	(0.69)	(1.67)	(1.54)	(2.35)	(2.70)	(1.66)	(2.57)	(1.77)	(2.27)	(1.59)
$\sigma(\%)$	36.38	24.55	22.83	22.45	19.23	17.28	17.84	17.28	16.75	18.53	27.97
$\alpha(\%)$	-7.81	-3.46	0.18	-0.05	2.28	2.51	-0.14	2.11	0.01	1.69	9.50
(t)	(-2.17)	(-2.01)	(0.10)	(-0.02)	(1.23)	(2.54)	(-0.14)	(2.20)	(0.01)	(1.36)	(2.42)
β_{MKT}	1.52	1.20	1.10	1.04	0.89	0.89	0.91	0.90	0.88	0.91	-0.61
(t)	(7.31)	(9.51)	(8.91)	(7.79)	(8.58)	(13.34)	(13.69)	(14.89)	(22.69)	(12.63)	(-2.52)
$R^2(\%)$	56.79	76.89	75.47	70.07	69.54	85.77	84.62	88.72	90.14	79.02	15.32

The Roll Critique

The only test of the CAPM is whether the market portfolio is mean-variance efficient.

- CAPM will always hold if the market *proxy* that is used is MVE.
- If *proxy* is not MVE, relationship between $E(R)$ and β will not hold.



The Roll Critique

- Roll points out that, since the market portfolio is not identifiable, we cannot really test the CAPM.
  - ↪ The market proxies that we use do not include
    a) Real Estate
    b) Human Capital
- Roll concludes the CAPM is useless because it is not testable
  - ↪ Roll instead advocates the use of the APT, which we will see next.
- However, perhaps the usefulness of the Roll critique is in reminding us that if we find that the CAPM tests do not hold, then the so-called “market” (or really market proxy) is not MVE and, unless you have very special reasons for doing so, you should not hold the market proxy!

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Implications for portfolio choice

- Violation of the CAPM means that the market portfolio is not MVE.
  - ↪ If we believe these anomalies we should feed this information into the Markowitz model.
  - ↪ We can beat the market!
- Hedge funds have been doing exactly that for the last 15 years.

Beating the market

- Over the last 90 years, the market portfolio has generated an average excess return of 0.62% and a volatility of 5.46% per month
  - ↪ A Sharpe ratio of $0.62/5.46 = 0.1143$ in monthly terms.
- Can we do much better?
  - ↪ Let's put to work what we learned in this lecture
  - ↪ Consider six portfolios sorted on size and book-to-market.

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Beating the market

Portfolio	Mean	SE(mean)	Volatility	Correlations					
SG	1.00	0.25	7.81	1.00	0.95	0.90	0.85	0.82	0.81
S2	1.29	0.22	7.12	0.95	1.00	0.96	0.84	0.88	0.89
SV	1.50	0.26	8.36	0.90	0.96	1.00	0.79	0.88	0.92
LG	0.90	0.17	5.40	0.85	0.84	0.79	1.00	0.89	0.82
L2	0.96	0.18	5.82	0.82	0.88	0.88	0.89	1.00	0.94
LV	1.19	0.23	7.29	0.81	0.89	0.92	0.82	0.94	1.00

- Monthly data 1926-2010. Risk free rate has been 0.34% on average.
- How accurate are our estimates of historical returns?
- Assuming I believe this pattern is likely to hold in the future, what is the maximum Sharpe ratio I can achieve?

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Beating the market

Number of securities:	6

No	Name	Fraction	Expected Return	Standard Deviation
1	SG	-364%	1.00%	7.81%
2	S2	381%	1.29%	7.12%
3	SV	138%	1.50%	8.36%
4	LG	193%	0.90%	5.40%
5	L2	-144%	0.96%	5.82%
6	LV	-104%	1.19%	7.29%
		1.00		

Correlations		2	3
1		0.951	0.900
2			0.963
3			
4			
5			
		YES	

Portfolio's Expected Return	0.0248
Portfolio's Standard Deviation	0.0963

Risk Free Rate	0.0034	Risk Aversion Coefficient: A=	2.00
Slope of CAL	0.2218	Weight on optimal risky portfolio: x*=	115.09%

- I can achieve a Sharpe ratio that is twice the sharpe ratio of the market.
- I can do better by diversifying across strategies

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Beating the market

- If the CAPM does not hold, we can beat the market.
- Does this mean that this is free money?
- Only if we do not expose ourselves to additional systematic risks.
- Yet, it is possible that value and growth firms are differentially exposed to risks that are not fully captured by the market.
- To see this, notice that the 6 portfolios we saw earlier are not perfectly correlated, even though they are fully diversified.
- Also, their R^2s with the market are around 80%. What drives the other 20% of their variation?

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The Treynor-Black Model

- Suppose that you have N trading strategies whose excess returns follow

$$R_{it}^e = \alpha_i + \beta_i (R_{Mt} - r_f) + \varepsilon_{it}, \qquad \varepsilon_i \sim N(0, \sigma_i^2)$$

  where $cov(\varepsilon_i, \varepsilon_j) = 0$ (assume single factor model)

- The tangency (MVE) portfolio is

$$R_T = w_A R_A + (1 - w_A) R_M$$

- The active portfolio A has weights w_i in the individual assets:

$$w_i = \frac{\alpha_i / \sigma_i^2}{\sum_{i=1}^N \alpha_i / \sigma_i^2}$$

- What determines the weight w_A on the active portfolio?

The Treynor-Black Model

- The optimal weight on the active portfolio is

$$w_A = \frac{w_0}{1 + (1 - \beta_A) w_0}$$

where

$$w_0 = \frac{\alpha_A / \sigma_A^2}{E(R_M - r_F) / \sigma_M^2}$$

and

$$\alpha_A = \sum_{i=1}^{N} w_i \alpha_i, \quad \beta_A = \sum_{i=1}^{N} w_i \beta_i, \quad \sigma_A^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2$$

- The Sharpe ratio of the tangency portfolio is

$$S_T^2 = S_M^2 + \left(\frac{\alpha_A}{\sigma_A} \right)^2$$

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The state of affairs

- Is modern finance theory useless?
- Not really. Maybe just the CAPM...
  - ↪ ...or the version that we can test.
- We can use the same principles about the risk-return tradeoff to guide us into better models.
- These models will help us understand better these patterns in expected returns.
  1. If these are indeed anomalies, there is no reason why they should persist in the future.
  2. If there is a rational explanation, then these patterns may persist, but are not necessarily “free money”.

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Infinite monkey theorem

- **Infinite monkey theorem:** *Given enough time, a hypothetical chimpanzee typing at random would, as part of its output, almost surely produce one of Shakespeare's plays (or any other text).*