

Investment shocks, firm characteristics and the cross-section of expected returns*

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Abstract

Recent developments in real business cycle models emphasize the role of changes in the technology for producing and installing new capital goods as an important driver of fluctuations. Investment-specific technology (IST) shocks represent a systematic source of risk, and therefore firm heterogeneity in dimensions correlated with firms' exposure to IST shocks is likely to give rise to comovement in stock returns and heterogeneity in risk premia. In this paper we argue that firms differing in past investment rates or profitability are likely to have different exposure to IST shocks, and thus return patterns obtained by sorting firms on such characteristics can be interpreted in a framework emphasizing the impact of IST shocks on the cross-section of stock returns. We model firms as portfolios of assets in place and growth opportunities, with growth opportunities having endogenously higher exposure to IST shocks. We show that firms' past investment, as well as their profitability, are informative about the relative weight of growth opportunities in their value, and therefore these characteristics are correlated with firms' exposure to IST shocks. Our model reproduces quantitatively the key empirical relationships between firm investment, profitability, and expected stock returns.

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1 Introduction

Recent developments in real business cycle models emphasize the role of changes in the technology for producing and installing new capital goods as an important driver of fluctuations. Investment-specific technology (IST) shocks represent a systematic source of risk, and therefore firm heterogeneity in dimensions correlated with firms' exposure to IST shocks is likely to give rise to comovement in stock returns and heterogeneity in risk premia.

In this paper we study firm heterogeneity along two fundamental characteristics: firm investment rate and firm profitability. It has been documented in the literature that firms with high investment rates have lower subsequent returns relative to their peers, while firms with higher return on assets have higher subsequent returns (e.g., Titman, Wei, and Xie (2004), Chen, Novy-Marx, and Zhang (2010)). Moreover, firms with high or low values of each characteristic exhibit return comovement, which allows one to form return factors based on stock portfolios long stocks with the high value of each characteristic and short stock with the low value. Chen et al. (2010) show that firms' exposure to return factors constructed in this manner generates cross-sectional heterogeneity in risk premia, and, moreover, accounts for several of the previously documented return patterns obtained by sorting firms on other firm characteristics. In this paper we argue that firms with different past investment rates or profitability are likely to have heterogeneous exposure to IST shocks, and thus return patterns obtained by sorting firms on such characteristics can be interpreted in a framework emphasizing the impact of IST shocks on the cross-section of stock returns.

We start with an observation in Kogan and Papanikolaou (2010a,b) that investment-specific technology shocks have different impact on the two fundamental parts of firm value: the value of future growth opportunities (PVGO) rises relative to the value of assets in place (VAP) following a positive IST shock. Thus, firms that differ in their mix between PVGO and VAP also differ in their exposure to IST shocks. Intuitively, all else equal, firms with higher past investment rates are likely to be the firms with higher growth opportunities,

and thus higher exposure to IST shocks. Similarly, controlling for investment opportunities, firms with more profitable assets in place are likely to have higher VAP as a fraction of the total firm value, and thus lower exposure of firm value to IST shocks. We formalize this intuition in a structural model that builds on the framework of Kogan and Papanikolaou (2010b).

In Kogan and Papanikolaou (2010b), firms are exposed to an exogenous sequence of neutral and IST shocks, with IST shocks corresponding to changes in the price of new capital goods. In addition, each firm is endowed with a stochastic sequence of investment opportunities which it can implement by purchasing and installing new capital. The market value of a firm's growth opportunities, defined as the present value of future investment opportunities, increases following a reduction in the cost of physical capital. Consequently, firms with more growth opportunities have higher exposure to IST shocks than firms with fewer growth opportunities.

We extend the model in Kogan and Papanikolaou (2010b) in one critical dimension: we assume that firms' growth opportunities are not observable directly, and market participants must therefore learn about them by observing firms' investment decisions. As a result, firms' past investment rates are informative about their future investment opportunities and, consequently, their future expected stock returns. We quantify the potential magnitude of the theoretically predicted patterns by calibrating our model. We show that cross-sectional differences in firms' exposure to IST shocks in the model are consistent with the empirical patterns, confirming our theoretical argument above. Our calibrated model generates empirically plausible return spreads between firms with high and low investment rates and firms with high and low return on assets. Moreover, our model replicates the failure of the CAPM in pricing the cross-section of stock returns. CAPM fails in the model because differences in asset composition across firms are not fully captured by their market risk alone.

We provide further empirical support for our theory by estimating the stochastic discount factor implied by the model using two empirical measures of investment shocks: (1) returns

on a portfolio long the sector producing investment goods and short the sector producing consumption goods (IMC);¹ and (2) innovations in the price of new equipment. Using the test portfolios formed by sorting firms on their investment rates, return-on-assets and book-to-market ratios, we find that IST shocks carry a negative price of risk. A two-factor specification including one of the two measure of IST shocks, and either a market return factor or a neutral technological shock factor, improves upon traditional models such as the CAPM and CCAPM in pricing the cross-section of returns on the test portfolios.

2 Related Literature

The idea that growth opportunities may have different risk characteristics than assets in place, and thus that decomposing firm value into assets in place and growth opportunities may be useful for understanding the cross-section of risk premia, has been previously explored in the asset pricing literature (e.g., Berk, Green, and Naik (1999); Gomes, Kogan, and Zhang (2003); Carlson, Fisher, and Giammarino (2004); Zhang (2005)). In these models, the firms' mix of assets in place and growth opportunities leads to heterogeneous and time-varying risk exposures and thus a nontrivial cross-section of asset returns. Our work fits into the same general framework. With only a few exceptions, e.g., Berk et al. (1999), Santos and Veronesi (2009), most of the existing models of firm heterogeneity feature a single aggregate shock and thus imply that all return factors obtained by sorting firms on their characteristics, such as the book-to-market ratio, are conditionally perfectly correlated with the market portfolio. Berk et al. (1999) assume that firms' value is affected both by productivity and discount rate shocks. While they do not focus on return comovement as their primary object of interest, their model gives rise to comovement of returns on firms with similar characteristics. Our model shares some of the key conceptual elements with the framework of Berk et al. (1999),

¹Kogan and Papanikolaou (2010b) show that the portfolio long the sector producing investment goods and short the sector producing consumption goods (IMC portfolio) performs well as an empirical proxy for IST shocks. Firms' return exposures to this portfolio reflect cross-sectional differences in their investment dynamics and stock returns in a manner consistent with our theoretical framework.

but emphasizes a different source of aggregate risk. We focus on IST shocks as an intuitive and important source of systematic risk heterogeneity among assets in place and growth opportunities. We further relate cross-sectional differences in firms investment rates and profitability to differences in their value composition and thus show that nontrivial return comovement among firms with different investment rates and profitability can be explained by their heterogeneous exposure to IST shocks.

Our paper adds to the growing literature analyzing implications of investment-specific shocks for macroeconomics and capital markets (e.g., Greenwood, Hercowitz, and Krusell (1997, 2000); Christiano and Fisher (2003); Fisher (2006); Papanikolaou (2010); Justiniano, Primiceri, and Tambalotti (2010); Kogan and Papanikolaou (2010a,b)). In this paper we build on the modeling framework developed in Kogan and Papanikolaou (2010b). In addition, our results are closely related to Papanikolaou (2010), who argues that in a two-sector general equilibrium model investment shocks can generate a value premium as well as the value factor. Papanikolaou (2010) finds that, empirically, IST shocks have a negative price of risk, and shows how this result can be obtained in a general equilibrium production economy. This result supports our empirical findings in this paper, which we establish using a very different set of test assets. Finally, the model in Papanikolaou (2010) features no firm-level heterogeneity within the sectors, and therefore cannot be used to analyze the questions addressed here.

Our work is related to the asset pricing literature connecting Q-based theories of investment to stock return behavior (e.g., Cochrane (1991, 1996); Lyandres, Sun, and Zhang (2008); Liu, Whited, and Zhang (2009); Li, Livdan, and Zhang (2009); Chen et al. (2010); Li and Zhang (2010)). This literature focuses on the implications of firms' optimizing behavior for the relationship between their investment decisions and their expected returns. We offer a different perspective on such relationships. We study an explicit economic mechanism, heterogeneous exposure to IST shocks among growth opportunities and assets in place, through which risk premia and firm characteristics are jointly and endogenously determined. We

believe that our work complements existing studies and improves our understanding of the links between real investment and stock returns.

3 The Model

In this section we extend the structural model of investment in Kogan and Papanikolaou (2010b) by incorporating learning about firm-specific arrival rates of growth opportunities. To make the exposition largely self-contained, we describe all the elements of the model below, but we refer the readers to Kogan and Papanikolaou (2010b) for proofs of some of the technical results.

Our model has two aggregate shocks: a neutral productivity shock and an investment-specific shock. Assets in place and growth opportunities have different loadings on investment-specific shocks, which generates stock return comovement among firms with different characteristics. Given that investment shocks are priced, this heterogeneity in risk translates into cross-sectional differences in risk premia across firms based on the fraction of firm value due to growth opportunities.

There are two sectors in our model, the consumption-good sector, and the investment-good sector. We model investment shocks as shocks to the physical cost of new capital. We focus on heterogeneity in growth opportunities among consumption-good producers.

3.1 Consumption-Good Producers

There is a continuum of measure one of infinitely lived firms producing a homogeneous consumption good. Firms behave competitively and there is no explicit entry or exit in this sector. Firms are financed only by equity, hence the firm value is equal to the market value of its equity.

Assets in Place

Each firm owns a finite number of individual projects. Firms create projects over time through investment, and projects expire randomly.² Let \mathcal{F} denote the set of firms and \mathcal{J}_t the set of projects owned by firm f at time t .

Project j produces a flow of output equal to

$$y_{fjt} = u_{jt}x_tK_j^\alpha, \quad (1)$$

where K_j is physical capital chosen irreversibly at the project j 's inception date, u_{jt} is the project-specific component of productivity, and x_t is the economy-wide productivity shock affecting output of all existing projects. We assume decreasing returns to scale at the project level, $\alpha \in (0, 1)$.

A firm's projects expire independently at rate δ . At every instant, a measure δdt of projects expires, and thus δ captures the rate of depreciation of capital.

The two components of projects' productivity evolve according to

$$du_{jt} = -\theta_u(u_{jt} - 1)dt + \sigma_u\sqrt{u_{jt}}dB_{jt} \quad (2)$$

$$dx_t = \mu_x x_t dt + \sigma_x x_t dB_{xt}, \quad (3)$$

where dB_{jt} and dB_{xt} are independent standard Brownian motions. All idiosyncratic shocks are independent of the aggregate shock: $dB_{jt} \cdot dB_{xt} = 0$. The project-specific component of productivity follows a mean-reverting, stationary process, while the process for aggregate productivity follows a Geometric Brownian motion, generating long-run growth.

²Firms with no current projects may be seen as firms that temporarily left the sector. Likewise, idle firms that begin operating a new project can be viewed as new entrants. Thus, our model implicitly captures entry and exit by firms.

Investment

Firms acquire new projects exogenously according to a Poisson process with a firm-specific arrival rate λ_{ft} . The firm-specific arrival rate of new projects is

$$\lambda_{ft} = \lambda_f \cdot \tilde{\lambda}_{f,t} \quad (4)$$

where $\tilde{\lambda}_{ft}$ follows a two-state, continuous time Markov process with transition probability matrix between time t and $t + dt$ given by

$$P = \begin{pmatrix} 1 - \mu_L dt & \mu_L dt \\ \mu_H dt & 1 - \mu_H dt \end{pmatrix}. \quad (5)$$

We label the two states as $[\lambda_H, \lambda_L]$, with $\lambda_H > \lambda_L$. Thus, at any point in time, a firm can be either in the high-growth ($\lambda_f \cdot \lambda_H$) or in the low-growth state ($\lambda_f \cdot \lambda_L$), and μ_H and μ_L denote the transition rates between the two states. We impose that $E[\tilde{\lambda}_{f,t}] = 1$, which restricts the parameters to satisfy

$$1 = \lambda_L + \frac{\mu_H}{\mu_H + \mu_L}(\lambda_H - \lambda_L) \quad (6)$$

We assume that the current arrival rate of firm's projects is an unobservable, latent process. Firm's investment decisions are not contingent on $\tilde{\lambda}_{ft}$, while the market learns about $\tilde{\lambda}_{ft}$ by observing the arrival of new projects.

When presented with a new project at time t , a firm must make a take-it-or-leave-it decision. If the firm decides to invest in a project, it chooses the associated amount of capital K_j and pays the investment cost $z_t x_t K_j$. The cost of capital relative to its productivity, z_t , is assumed to follow a Geometric Brownian motion

$$dz_t = \mu_z z_t dt + \sigma_z z_t dB_{z,t}, \quad (7)$$

where $dB_{zt} \cdot dB_{xt} = 0$. The z shock represents the component of the price of capital that is unrelated to its current level of average productivity, x , and is the investment shock in our

model. Here, note that a positive investment shock is defined as a *drop* in z_t . Finally, at the time of investment, the project-specific component of productivity is at its long-run average value, $u_{jt} = 1$.

Valuation

As we discuss above, project arrival rate $\tilde{\lambda}_{ft}$ is a latent process, and the market learns about it by observing the arrival of new projects. Thus, the value of the firm depends on the probability of the firm being in the high growth state, $p_{ft} = \Pr_t(\tilde{\lambda}_{ft} = \lambda_H)$. Using the standard results on filtering for point processes (see Lipster and Shiryaev (2001)) we find that p_{ft} evolves according to

$$dp_{ft} = ((1 - p_{ft})\mu_H - p_{ft}\mu_L) dt + p_{ft} \left(\frac{\lambda_H}{p_{ft}\lambda_H + (1 - p_{ft})\lambda_L} - 1 \right) dM_t,$$

where M_t is a martingale defined by

$$dM_t = dN_t - (p_{ft}\lambda_H + (1 - p_{ft})\lambda_L) dt.$$

N_t denotes the cumulative number of projects undertaken by the firm: $dN_t = 1$ if the firm invests at time t and zero otherwise. Intuitively, dM_t is the investment innovation, which is informative about the firm's current growth opportunities.

We assume that financial markets are complete and the time-0 market value of a cash flow stream C_t is given by $E \left[\int_0^\infty (\pi_t/\pi_0) C_t dt \right]$, where π_t denotes the stochastic discount factor. For simplicity, we assume that the aggregate productivity shocks x_t and z_t have constant prices of risk, b_x and b_z respectively, and the risk-free interest rate r is also constant. Then,

$$\frac{d\pi_t}{\pi_t} = -r dt - b_x dB_{xt} - b_z dB_{zt}. \quad (8)$$

The above form of the stochastic discount factor is motivated by a general equilibrium model with IST shocks in Papanikolaou (2010). In Papanikolaou (2010), states with low

cost of new capital are high marginal valuation states because of improved investment opportunities. This is analogous to a positive value of b_z . Our analysis below shows that empirical properties of stock returns imply a positive value of b_z . Our analysis implies that the market price of the aggregate productivity shock x is positive, which is consistent with standard equilibrium models and empirical evidence.

Firms make investment decisions by trading off the market value of a new projects against the cost of required physical capital. The time- t market value of an existing project j , $p(u_{jt}, x_t, K_j)$, is computed using the discounted cash flow formula:

$$p(u_{jt}, x_t, K_j) = \mathbb{E}_t \left[\int_t^\infty e^{-\delta(s-t)} \frac{\pi_s}{\pi_t} (u_{js} x_s K_j^\alpha) ds \right] = A(u_{jt}) x_t K_j^\alpha, \quad (9)$$

where

$$A(u) = \frac{1}{r + \delta - \mu_X} + \frac{1}{r + \delta - \mu_X + \theta_u} (u - 1) \quad (10)$$

Firms' investment decisions are straightforward because the arrival rate of new projects is exogenous and does not depend on the firms' previous decisions. Thus, optimal investment decisions are based on the NPV rule: firm f chooses the amount of capital K_j to invest in project j to maximize

$$p(u_{jt}, x_t, K_j) - z_t x_t K_j.$$

Thus the optimal capital investment in the new project is given by

$$K^*(z_t) = \left(\frac{\alpha A(1)}{z_t} \right)^{\frac{1}{1-\alpha}}.$$

We compute the value of the firm as a sum of the market value of its assets in place, VAP_{ft} , and the present value of its growth opportunities, $PVGO_{ft}$. The value of the firm is equal to

$$V_{ft} = VAP_{ft} + PVGO_{ft}.$$

The value of assets in place is

$$VAP_{ft} = \sum_{j \in \mathcal{J}_f} p(e_{ft}, u_{jt}, x_t, K_j) = x_t \sum_{j \in \mathcal{J}_{ft}} A(u_{j,t}) K_j^\alpha.$$

The present value of growth opportunities is given by

$$\begin{aligned} PVGO_{ft} &= z_t^{\frac{\alpha}{\alpha-1}} x_t (p_{ft} G_H + (1 - p_{ft}) G_L), \\ G_H &= \lambda_f \left(G_1 + \frac{\mu_L}{\mu_L + \mu_H} (\lambda_H - \lambda_L) G_2 \right), \\ G_L &= \lambda_f \left(G_1 - \frac{\mu_H}{\mu_L + \mu_H} (\lambda_H - \lambda_L) G_2 \right), \end{aligned}$$

where

$$\rho = r + \frac{\alpha}{1 - \alpha} (\mu_z - \sigma_z^2/2) - \mu_x - \frac{\alpha^2 \sigma_z^2}{2(1 - \alpha)^2},$$

and

$$C = \alpha^{\frac{1}{1-\alpha}} (\alpha^{-1} - 1).$$

The constants G_1 and G_2 are given by

$$G_1 = C \cdot \rho^{-1} A(1)^{\frac{1}{1-\alpha}} \quad (11)$$

$$G_2 = C \cdot (\rho + \mu_H + \mu_L)^{-1} A(1)^{\frac{1}{1-\alpha}}. \quad (12)$$

Unlike the value of assets in place, the present value of growth opportunities depends on the IST shock, z , because the net present value of future projects depends on the cost of new investment.

In summary, the firm value in our model is

$$V_{ft} = VAP_{ft} + PVGO_{ft} = x_t \sum_{j \in \mathcal{J}_{ft}} A(u_{j,t}) K_j^\alpha + z_t^{\frac{\alpha}{\alpha-1}} x_t (p_{ft} G_H + (1 - p_{ft}) G_L) \quad (13)$$

Risk and Expected Returns

Both assets in place and growth opportunities have constant exposure to systematic shocks dB_{xt} and dB_{zt} . However, their betas with respect to the productivity shocks are different. The value of assets in place is independent of the IST shock and loads only on the aggregate productivity shock. The present value of growth opportunities depends positively on aggregate productivity, and negatively on the unit cost of new capital. Thus, firm's stock return betas with respect to the aggregate shocks are time-varying, and depend linearly on the fraction of firm value accounted for by growth opportunities. Since, by assumption, the price of risk of aggregate shocks is constant, expected excess return on a firm's stock is an affine function of the weight of growth opportunities in firm value,

$$ER_{ft} - r_f = b_x \sigma_x - \frac{\alpha}{1 - \alpha} b_z \sigma_z \frac{PVGO_{ft}}{V_{ft}} \quad (14)$$

Together, the expression for the risk premium in (14) and the firm value decomposition in (13) show why firm's lagged investment and its productivity contain information about its future excess returns. Everything else being equal, higher productivity raises the value of assets in place relative to that of growth opportunities, increasing the risk premium of firm's returns. Holding assets in place fixed, higher investment signals a higher value of growth opportunities, leading to higher p_{ft} and lower expected returns on the entire firm's value.

3.2 Investment-Good Producers

We now show how to construct a stock portfolio in the model mimicking IST shocks. For that we model firms producing investment goods. Our construction follows Kogan and Papanikolaou (2010b), and we reproduce the key results here for completeness.

There is a continuum of firms producing new capital goods. We assume that these firms produce the demanded quantity of capital goods at the current unit price z_t . We assume that profits of investment firms are a fraction ϕ of total sales of new capital goods. Consequently,

profits accrue to investment firms at a rate of $\Pi_t = \phi z_t x_t \bar{\lambda} \int_{\mathcal{F}} K_{ft} df$, where $\bar{\lambda} = \int_{\mathcal{F}} \lambda_{ft}$ is the average arrival rate of new projects among consumption-good producers. Even though λ_{ft} is stochastic, it has a stationary distribution, so $\bar{\lambda}$ is a constant.

The price of the investment firm is given by

$$V_{I,t} = \Gamma x_t z_t^{\frac{\alpha}{\alpha-1}} \frac{1}{\rho_I} \quad (15)$$

where

$$\rho_I \equiv r - \mu_X + \frac{\alpha}{1-\alpha} \mu_Z - \frac{1}{2} \frac{\alpha}{1-\alpha} \sigma_Z^2 - \frac{1}{2} \frac{\alpha^2 \sigma_Z^2}{(1-\alpha)^2} > 0$$

and

$$\Gamma \equiv \phi \bar{\lambda} \alpha^{\frac{1}{1-\alpha}} \left(\int A(1)^{\frac{1}{1-\alpha}} df \right).$$

Note that a positive IST shock, defined as a decline in z_t , benefits the investment-good producers. Investment firms in our model make no investment decisions, and therefore they derive their value entirely from assets in place. Were investment firms to acquire additional projects over time, their market value would include the present value of growth opportunities in addition to assets in place, analogously to the firms in the consumption-good sector. If so, the IST shock would affect the value of assets in place and growth opportunities differently, leading to cross-sectional heterogeneity in average returns *within* the investment sector.³ Given that the size of the investment sector is fairly small (it contributes 10-15 percent to the value of the market portfolio), our modeling choice provides tractability at the cost of leaving a minor fraction of firms out of our empirical tests.

We define the IMC portfolio in the model as a portfolio long the investment sector and short the consumption sector. As shown in Kogan and Papanikolaou (2010b), IMC portfolio returns are perfectly conditionally correlated with IST shocks in the model. This motivates our use of IMC returns as an empirical proxy for IST shocks in the estimation of the stochastic

³An illustration of this is in the Appendix of Papanikolaou (2010), who models capital accumulation in both the consumption and the investment sectors. Papanikolaou (2010) finds a spread in average returns between assets in place and growth opportunities within both sectors.

discount factor in Section 5.

4 Results

In this section we compare the output generated by the calibrated model to the data. We calibrate the model to match the means and standard deviations of aggregate quantities such as aggregate dividend growth and investment, as well as the mean and dispersion of firm-specific variables such as size, investment rate, Tobin's Q and return on assets. We then explore whether the model can replicate the observed empirical relationship between a firm's investment rate or return on assets and its expected return.

4.1 Calibration

In our calibration, we target moments of aggregate dividend and investment growth, firm-level characteristics, and asset returns.

We choose the parameters governing the dynamics of the shocks x_t and z_t to match the first two moments of the aggregate dividend and investment growth.

We pick the rate of decreasing returns at the project level ($\alpha = 0.85$), the parameters governing the projects' cash flows ($\sigma_u = 1.15, \theta_u = 0.1$) and the parameters of the distribution of λ_f jointly, to match the average values and the cross-sectional dispersion of firm size and the return to capital. We set the project expiration rate δ to 10%, to be consistent with commonly used values for the depreciation rate.

We model the distribution of mean project arrival rates $\lambda_f = E[\lambda_{ft}]$ across firms as

$$\lambda_f = \mu_\lambda \delta - \sigma_\lambda \delta \log(X_f) \quad X_f \sim U[0, 1], \quad (16)$$

We pick parameters characterizing the arrival rate of new projects to match the median and the cross-sectional dispersion of the investment rate and Tobin's Q in the cross-section. In particular, we pick $\sigma_\lambda = 1.5$ and $\mu_\lambda = 4$. Regarding the dynamics of the stochastic

component of the firm-specific arrival rate, $\tilde{\lambda}_{ft}$, we pick $\mu_H = 0.05$ and $\mu_L = 0.18$. We pick $\lambda_H = 4.20$, which according to (6) implies $\lambda_L = 0.13$. We set the interest rate r to 3.5%, which is close to the historical average risk-free rate (2.9%). We choose $\phi = 0.07$ to match the relative size of the consumption and investment sectors in the data.

Finally, parameters of the pricing kernel, $b_x = 0.65$ and $b_z = 0.375$ are picked to match approximately the average excess returns on the market portfolio and the value factor (HML). The positive sign of b_z implies that a positive IST shock (unexpected decline in z) is associated with an increase in the pricing kernel. Papanikolaou (2010) offers an example of a general equilibrium model that generates a positive value of b_z .⁴ He estimates b_z to fall in the range of 0.29-1.56, which contains our calibrated value.

We simulate the model at a weekly frequency ($dt = 1/52$) and time-aggregate the data to form annual observations. Each simulation sample contains 5,000 firms for 100 years. We drop the first half of each simulated sample to eliminate the dependence on initial values. We simulate 2000 samples and report medians of parameter estimates and t -statistics across simulations.

We define the market portfolio as the sum of market values of the consumption and the investment sector. We define the firm's investment rate as the sum of investment expenditures in a given year t , $INV_{f,t} = \sum_{\tau \in t} z_{\tau} x_{\tau} k^*(z_{\tau})$, divided by the lagged replacement cost of its capital stock, $K_{f,t-1} = z_{t-1} x_{t-1} \sum_{j \in \mathcal{J}_{f,t-1}} k_j$; a firm's return on assets equals cash flows accruing from existing projects in year t , $CF_{f,t} = \sum_{\tau \in t} \sum_{j \in \mathcal{J}_{f\tau}} u_{\tau} x_{\tau} k_j^{\alpha}$ divided by the lagged replacement cost of capital; Tobin's Q as the market value of the firm, $V_{f,t}$ divided by the replacement cost of capital, which equals in the model the reciprocal of the book-to-market ratio.

⁴In Papanikolaou (2010), a positive IST shock triggers a short-lived decline in consumption. In his model, if households have CRRA preferences or recursive Epstein-Zin with the product of the risk aversion and the EIS coefficients below one, the sign of b_z is positive. Papanikolaou (2010) finds that, empirically, consumption declines temporarily following a positive IST shock.

4.2 Data

Given the model and following Kogan and Papanikolaou (2010b), we use two observable proxies for investment shocks. The first is a portfolio of investment- minus consumption-producers (IMC). We first classify industries as producing either investment or consumption goods according to the NIPA Input-Output Tables. We then match firms to industries according to their NAICS codes. Gomes, Kogan, and Yogo (2009) and Papanikolaou (2010) describe the details of this classification procedure. Our second proxy uses the quality-adjusted series of new equipment prices constructed by Gordon (1990), Cummins and Violante (2002) and Israelsen (2010), normalized by the NIPA consumption deflator.⁵ We follow Kogan and Papanikolaou (2010b) and construct innovations to this time series by removing a time trend from the series of equipment prices and define investment-specific technological changes as changes in the detrended log relative price of new equipment goods. We allow the time trend to vary post 1982 given the evidence in Fisher (2006), who points out that the real equipment price experiences an abrupt increase in its average rate of decline in 1982. A potential explanation could be due to the effect of more accurate quality adjustment in more recent data (e.g., Moulton (2001)).

In our estimation of the stochastic discount factor we will also need proxies for the neutral productivity shock. We use two observable proxies for x . We construct our first proxy using changes in the total factor productivity of the consumption sector from Basu, Fernald, and Kimball (2006). We construct our second proxy by using returns to the market portfolio.⁶ This second proxy is a noisy measure of x , since it is also affected by the investment shock.

For every firm-year observation, we estimate IMC-beta using a univariate regression of firm returns on returns of the IMC portfolio at a weekly frequency, as in Kogan and

⁵Cummins and Violante (2002) extrapolate the quality adjustment of Gordon (1990) to construct a price series for the period 1943-2000. Israelsen (2010) extends the price series through 2008.

⁶Given that the market capitalization of the investment firms is small, about 14% on average, substituting the market portfolio with returns to the consumption sector leads to very similar results. We report results using the market portfolio mainly due to its popularity as a factor in CAPM-style regressions.

Papanikolaou (2010b).

Our sample covers the 1965-2008 period. We define the investment rate (IK) as capital expenditures (compustat capx) divided by lagged gross property, plant and equipment (compustat ppgt); return on assets (ROA) as operating income (compustat ib) divided by lagged book assets (compustat at); Tobin's Q as the sum of the market value of common equity (CRSP December market capitalization), the book value of debt (compustat dltd), the book value of preferred stock (compustat pstkrv), minus the book value of inventories (compustat invt) and deferred taxes (compustat txdb), divided by gross property, plant and equipment (compustat ppgt); book-to-market as the ratio of the market value of common equity divided by the book value of common equity (compustat ceq).

Given that in our model the capital-producing firms do not make any investment decisions and are modeled in reduced form, we omit them from the empirical analysis, along with financial firms and utilities.

4.3 Portfolios Sorted on Investment Rate

Our model reproduces the negative sign of the relationship between firms' past investment rates and their risk premia. In the model, cross-sectional differences in risk premia are determined by the relative weight of future growth opportunities in firm value, $PVGO/V$. Intuitively, investment by a firm leads the market to upgrade its estimate of the firm's current arrival rate of new projects. Thus, firms with higher investment rates tend to be richer in growth opportunities and therefore earn lower risk premia.

We form ten portfolios by sorting firms on their investment rate. We rebalance these portfolios in June of each year. The top panel of Table 3 summarizes the main empirical properties of the ten portfolios. Average excess returns are negatively related to past investment rates, with the highest-IK portfolio earning 4.5 percent less annually than the lowest-IK portfolio. These results are consistent with empirical findings in the literature. Furthermore, we find that the high-IK portfolios have much higher market betas than the

low-IK portfolios (1.56 for the highest IK decile vs 0.96 for the lowest), and the portfolio long the top IK decile and short the bottom decile has a CAPM alpha of -7.5 percent.

The high-IK decile portfolios have higher post-formation IMC beta than the low-IK portfolios, indicating their higher exposure to IST shocks. IMC betas of the extreme decile portfolios differ by 1.24. High-IK portfolios also have higher Tobin's Q, 3.3 for the highest-IK decile vs 0.8 for the lowest. These latter two facts confirm our intuition that firms' investment rates are informative about their growth opportunities.

We apply an identical estimation procedure to model-based simulated data. We show the results in the bottom panel of Table 3. In the model, sorting firms on IK leads to a declining pattern of average returns: the difference in terms of average returns between the high- and low-IK portfolio equals -2.3 percent in annual terms. In addition, high-IK firms also have higher market betas (1.07 vs 0.80). As a result, the CAPM fails to price the cross-section of IK portfolios, and the high- minus low-IK portfolio has a CAPM alpha equal to -3.9 percent.

The cross-sectional distribution of investment rates in the model is similar to that in the data. The spread in Tobin's Q across the IK decile portfolios is lower in the model than data. The model produces a strong increasing pattern of IMC betas across the IK portfolios. The corresponding empirical pattern is qualitatively similar and stronger in magnitude. The dispersion in the model is about half of what is observed in the data. Thus, in terms of firm characteristics the model gets halfway in generating the observed dispersion, implying that in the model, IK is a noisy signal of growth opportunities.

Finally, in the model, as in the data, sorting firms on their past investment rate produces a monotonically increasing pattern of loadings to the IMC portfolio, suggesting that the same forces could be generating comovement across both cross-sections.

4.4 Portfolios Sorted on Return to Assets

We explore whether the model can replicate the empirically documented relationship between a firm's current profitability, measured as the return on assets (ROA) and its subsequent

returns. Our intuition is that a firm’s current profitability is a noisy proxy for $1 - PVGO/V = VAP/V$, namely the fraction of firm value that is due to it’s existing assets. Highly profitable firms will, all else equal, derive a larger fraction of their value from existing projects. We thus expect that firms with high profitability will be firms that subsequently earn higher returns.

As before, we form ten portfolios of firms based on their return-to-assets (ROA). We rebalance portfolios in June every year. We show the results in the top panel of Table 4. In the data, the high-ROA outperforms the low-ROA portfolio by 6.4% a year. In addition, high-ROA firms have lower market betas than low-ROA firms (0.9 vs 1.5), and consequently the high- minus low-ROA portfolio has a CAPM alpha of 9.5%.

The empirical pattern of characteristics of the ROA-sorted portfolios is consistent with our intuition of ROA as a noisy proxy for VAP/V . The low-ROA portfolio has a higher exposure to the investment shock relative to the high-ROA portfolio (1.5 vs 0.3). Thus, a positive investment shock, measured as high returns on the IMC portfolio, is associated with higher current performance of low-ROA relative to high-ROA firms. In terms of Tobin’s Q, the ROA sort produces a U-shaped pattern. This is because Tobin’s Q is an imperfect measure of $PVGO/V$. There are two reasons why a firm may have high market values relative to it’s installed capital (K): if it has a high PVGO, or if it’s existing assets are highly profitable (VAP). These two opposite forces may result in a U-shaped pattern between ROA and Tobin’s Q.

Next, we investigate whether our model can replicate the above pattern by applying our empirical procedure to simulated data. We show the results in the bottom panel of Table 4. In simulated data, sorting firms on ROA leads to an increasing pattern of average returns: the high-ROA portfolio outperforms the low-ROA portfolio by 2.2% per year. As is the case in the actual data, the high-ROA portfolio has a lower market beta than the low-ROA portfolio (0.84 vs 1.28), resulting in a CAPM alpha of 4.3%.

Interestingly, the model reproduces the declining pattern of IMC-betas across the ROA

sort (1.60 vs 0.72), as well as the U-shaped relationship between ROA and Tobin's Q. The mechanism behind these patterns is that in the model dispersion in ROA results in dispersion in VAP/V , but not necessarily dispersion in marked values over capital. Another way to see this is that in the model, average and marginal Q differ. Average Q becomes a mixture of current profitability (VAP/K) and present value of future growth opportunities, ($PVGO/K$). Since high-ROA firms tend to have lower $PVGO/K$ but high VAP/K , this leads to a U-shaped pattern.

4.5 Portfolios Sorted on Book-to-Market

Given that the model is different than Kogan and Papanikolaou (2010b) and the fact that in the model average Q (the inverse of the book-to-market (BM) ratio) is a noisy measure of $PVGO/V$, we verify that our model, given the new parameterization, is also consistent with the empirical properties of high- and low-BM firms.

We show the results in Table 5. As in Kogan and Papanikolaou (2010b), the model matches the dispersion in average returns across BM-sorted portfolios, while producing somewhat higher dispersion in firm characteristics, namely Tobin's Q, investment rates and IMC-betas, than the data. Even though the dispersion is somewhat higher in the model, the qualitative patterns are similar: high book-to-market firms invest less on average, have lower IMC-betas and lower return on assets.

5 Estimating the Stochastic Discount Factor

Our asset pricing results are partially driven by our specific parameterization of the stochastic discount factor in equation (8). It is thus natural to explore whether there is empirical support for our parameter choice in the data. In particular, we are interested in whether the calibrated prices of risk in the model (β_x and β_z) are comparable to their empirical counterparts. Given observable proxies for the two shocks in our model (x and z) we could estimate equation (8) via the generalized method of moments (GMM).

We form the equivalent of equation (8), as

$$m = a - b_x \Delta x - b_z \Delta z. \quad (17)$$

In order to empirically estimate equation (17) we need a set of moment restrictions and observable proxies for the innovations in the x and the z shock.

Regarding the moment restrictions, the model implies that the SDF in equation (17) should price the cross-section of asset returns sorted by IK, ROA or BM. As an empirical proxy for the x shock, we use returns on the market portfolio, and changes in the (log) total factor productivity in the consumption sector, x^{ctfp} , from Basu et al. (2006). For the z shock, we use returns on the IMC portfolio and innovations in the relative price of new equipment, z^p from Israelsen (2010). We normalize our proxies for x and z to unit standard deviation.

We estimate equation (8) using the restrictions on the excess rate of return of any asset that is imposed by no arbitrage:

$$E[mR_i^e] = 0. \quad (18)$$

In our estimation, we use portfolio returns in excess of the risk free rate. As a result, the mean of the stochastic discount factor is not identified. Without loss of generality, we choose the normalization $E(m) = 1$.⁷ Using this normalization, the moment conditions (18) become

$$E[R_i^e] = -cov(m, R_i^e), \quad (19)$$

We estimate equation (17) using the no arbitrage restrictions for the set of 10 portfolios sorted on IK, ROA and BM separately, as well as the pooled cross-section of 30 portfolios.⁸ We report first-stage estimates using the identity matrix to weigh moment restrictions. We perform the same exercise in simulated data. We show the results in Table 6, where the left

⁷See Cochrane (2001), pages 256-258 for details.

⁸Papanikolaou (2010) estimates a similar specification for the SDF, using cross-sections of firms sorted on IMC-betas, BM, size/BM and industry portfolios

panel shows the results using actual data, whereas the right panel compares to estimated results using simulated data. As a measure of fit, we report sum of squared errors from the euler equations given by (19).

The top panel of Table 6 shows the estimation results from the pooled cross-section of assets. In the data, using either proxy for the investment shock z , the estimated prices of risk are negative and statistically significant, and equal to -0.85 and -0.64 using the innovations in the relative price of new equipment (z^p) and IMC portfolio respectively. One way to judge whether the calibrated prices of risk in the model (β_x and β_z) are consistent with the data is to perform the same exercise in the model. Doing so, produces a median estimate of -0.39 and -0.91 respectively, but a fairly wide range across simulations: the estimate of the price of risk using z^p lies outside the 95% confidence intervals across simulations, but not the estimate using IMC. This suggests that, if anything, the price of risk for the z shock is a bit lower in the model than the data.

Empirically, in terms of pricing errors, including proxies for z reduces the variance of pricing errors by a factor of two to five relative to using proxies for x alone. In the simulated data, the pricing errors of the single factor model (using either the tfp in the C-sector or returns to the market portfolio) are closed to the pricing errors in the data. However, since in the model 19 holds exactly up to estimation errors, the pricing errors of the two-factor model are close to zero.

The next three panels of Table 6 show the estimation results using only the cross-section of firms sorted by IK, ROA and BM respectively. The results above hold for each cross-section independently. The estimated price of risk of the z shock is negative and statistically significant and close in magnitude to the values implied by the model. Adding the proxies for the investment shock reduces the variance of pricing errors by a factor of two to six.

6 Tests of the Mechanism

Firms in our model differ in their exposure to systematic risk factors, and thus their risk premia, because of the differences in the relative weight of growth opportunities in firms' market value. We argue that firms' IK and ROA characteristics are correlated with their growth opportunities. Thus, IK and ROA characteristics are informative about average stock returns and stock return comovement because of their ability to approximate the weight of growth opportunities in firms' market value. In this section we directly test this mechanism.

In our model, all firms have identical exposures to neutral technological shocks but different exposures to IST shocks. In particular, firms with higher past investment and firms with lower past profitability have relatively high exposure to IST shocks. Thus, a portfolio long high-IK and short low-IK firms (or a portfolio long low-ROA and short high-ROA firms) forms a return factor highly correlated with IST shocks in our model. Returns on such portfolios predict firms' investment decisions, as IST shocks do. As we argue in Kogan and Papanikolaou (2010b), favorable IST shocks predict higher firm investment on average, with firms rich in growth opportunities exhibiting relatively high sensitivity.

Our first empirical test shows that IK and ROA return factors predict firms' investment in a manner consistent with our theory. We set up our test as a regression

$$i_{f,t} = a_1 + \sum_{d=2}^5 a_d D(\beta_{f,t-1}^{imc})_d + b_1 (\tilde{R}_{t-1}^z) + \sum_{d=2}^5 b_d D(\beta_{f,t-1}^{imc})_d \times (\tilde{R}_{t-1}^z) + cX_{f,t-1} + \gamma_f + u_{f,t}. \quad (20)$$

$\tilde{R}_{t-1}^z = \sum_{l=1}^2 R_{t-1}^z$ denotes the cumulative lagged return on the IK or the ROA factor ($R^z = [R^{cik}, R^{croa}]$). We construct the IK (ROA) factor as a zero-investment portfolio long firms in the highest IK quintile (lowest ROA quintile) and short firms in the lowest IK quintile (highest ROA quintile), excluding firms producing investment goods, utilities and financial firms. β_f^{imc} denotes firm f 's stock return beta with the investment minus consumption (IMC) portfolio constructed using weekly data (see Kogan and Papanikolaou (2010b) for details). The vector of controls X_t includes lagged values of cash flows over lagged capital, log book

equity over book assets, and log capital. We standardize all variables to zero mean and unit standard deviation and cluster errors by firm and year. Depending on the specification, we include industry (I) or firm (F) fixed effects.

We estimate equation (20) in actual and simulated data and summarize the results in Table 7.⁹ The left panel shows empirical results. Both the IK (top panel) and the ROA (bottom panel) factors predict heterogeneous response in investment rates among high- and low- growth firms, as identified by their IMC-betas. In historical data, a single standard deviation realization of the IK (ROA) factor is followed by a 0.08 (0.09) standard deviation difference in investment rate response between high- and low- growth firms. The right panel shows that model-based simulated data has very similar properties. A single standard deviation response to the IK (ROA) factor is followed by a 0.07 (0.07) standard deviation difference in investment rate response between high- and low- growth firms.

Our second empirical test investigates the relationship between IK and ROA, as firm characteristics, and firms' investment. In our model, both IK and ROA are correlated with the value of growth opportunities relative to total firm value. However, return on existing assets is not correlated with arrival of new projects, and therefore, in the cross-section, ROA is virtually unrelated to firms' investment responses to a z-shock. In contrast, firms' past investment rates are informative about their current project arrival rate, and therefore past IK predicts the response of firms' investment rates to a positive investment shock. We evaluate these properties of the model empirically by estimating

$$i_{f,t} = a_1 + \sum_{d=2}^5 a_d D(G_{f,t-1})_d + b_1 (\tilde{R}_{t-1}^{imc}) + \sum_{d=2}^5 b_d D(G_{f,t-1})_d \times (\tilde{R}_{t-1}^{imc}) + cX_{f,t-1} + \gamma_f + u_{f,t}, \quad (21)$$

where $\tilde{R}_{t-1}^{imc} = \sum_{l=1}^2 R_{t-1}^{imc}$ denotes cumulative past returns on the IMC portfolio (a proxy for the investment shock), and $G_f \in \{IK, ROA\}$.

Table 8 summarizes the estimation results for (21), both in historical and model-generated

⁹We omit the fixed effects in simulated data.

data. The top panel shows that heterogeneity in IK predicts differential response of firm investment to IST shocks, as measured by realized returns on the IMC portfolio. In the data (left panel), investment rate of high-*IK* firms relative to low-*IK* firms rises by 0.1 standard deviations in response to a single standard deviation positive IMC return. In the model (right panel), results are similar, with the corresponding number being 0.09. The bottom panel of the table shows that very different behavior is observed when sorting firms on their profitability. Both the empirical results and the corresponding model-based estimates are statistically indistinguishable from zero. Thus, we find empirical support for our model’s predictions for the relationship between *IK* or *ROA* on one hand and firms’ investment on the other hand.

7 Risk exposure versus characteristics

In this section we explore to what extent *IK* and *ROA* are correlated with expected returns because they capture firms’ exposure to the investment shock. First, we establish that there is a common source of comovement in portfolios of firms sorted on IMC-beta, *IK* and *ROA*. Second, we document that firm characteristics such as *IK*, *ROA* and Tobin’s *Q* contain information about *future* investment shock betas, in addition to past betas. Third, we establish that, once we control for these predicted investment shock betas, the power of *IK* and *ROA* to generate dispersion in average returns is greatly reduced.

7.1 Comovement

Recent work (Chen et al. (2010)) emphasizes the empirical importance of the factor structure of portfolios sorted on past investment rates and past profitability. In our model, aggregate dynamics is driven by two technological shocks (neutral and investment-specific) and firms with different past investment rates, or different past profitability, have heterogeneous exposure to these shocks. This result in a two-factor structure in returns on portfolios defined by sorting firms on the two characteristics above. Furthermore, our model suggests that by

directly sorting firms on their exposure to IST shocks, one should obtain a similar two-factor structure to the previous two sorts on firm characteristics.

To evaluate empirically our model’s implications for return comovement, we pool together ten portfolios sorted on past investment rates, ten portfolios sorted on past return on assets, and ten portfolios sorted on past IMC-beta, which is our empirical proxy for IST-shock exposure. We use the procedure due to Onatski (2009) in order to test for the number of factors in the data.¹⁰ The Onatski (2009) test fails to reject the hypothesis that the number of common factors in the data is two. Thus, we extract the first two principal components from the pooled cross-section of excess returns on these 30 portfolios (denoted by $f_1^{(30)}$ and $f_2^{(30)}$ respectively) and compare the resulting factors to those extracted independently from each of the three sets of ten portfolios.

Each of the factors constructed above represents a zero net investment portfolio. The Sharpe ratios of the two factors extracted from the pooled cross-section of 30 portfolios are 0.27 and -0.39 respectively. The second factor is positively correlated with the investment shock extracted using the price of new equipment (z^p) and the IMC portfolio returns, with correlations equal to 37% and 68% respectively (HAC t -statistics of 1.90 and 2.95).

Our results suggest that there is a common source of comovement across the set of portfolios sorted on IMC-beta, IK and ROA. To illustrate this, we extract the first two principal components from each of the three sets of ten portfolios, and regress them on the two factors extracted from the pooled cross-section of 30 portfolios. These regressions show high degree of commonality among the factor structures of the three sets of sorted portfolios. We summarize the results in Table 9. In each of the three sets of portfolio returns, the first principal component is almost perfectly spanned by $f_1^{(30)}$ and $f_2^{(30)}$. Moreover, in each cross-section the first principal component loads almost exclusively on $f_1^{(30)}$. The second principal

¹⁰Onatski (2009) provides the p-values of a test under the null hypothesis that the number of common factors k equals k_0 , under the alternative that $k_0 < k < k_1$. One can test for the number of factors by iteratively increasing k until the test fails to reject the null. In our empirical implementation, we specify the maximum number of factors (k_1) to equal 4, though varying k_1 from 4 to 8 leads to similar results.

component in each of the ten-portfolio cross-sections is spanned by $f_1^{(30)}$ and $f_2^{(30)}$ with an R^2 between 77% and 81%. Again, in each case, the second principal component loads almost exclusively on $f_2^{(30)}$, implying correlation with $f_2^{(30)}$ of approximately 90%.

The second principal component is highly correlated with the IMC portfolio, our model-implied measure of the investment shock. However, empirically, IMC may be a noisy measure of the investment shock, and thus may not fully capture the comovement across the 30 portfolios. To explore this possibility in more detail, we compare the residual comovement across the 30 portfolios once we remove their exposure to the market and IMC portfolio or the first two principal components. We plot the results in figure 1. The figure shows that, when removing the exposure to the market portfolio, there is substantial degree of comovement left, as measures by the steeply declining pattern of the first few eigenvalues. Removing the market and the IMC portfolio, reduces the degree of comovement left by more than half, but there is substantial comovement left. The Onatski (2009) test rejects the hypothesis that the number of common factors in the data is zero at the 5% level. Removing the first two principal components absorbs almost all the comovement across the 30 portfolios, as can be seen from the substantially flatter plot of the first 10 eigenvalues. Now, the Onatski (2009) test fails to reject the hypothesis that the number of common factors is zero at the 10% level.

The results above suggest that using the second principal component (PC2) of the pooled cross-section of returns may be closer empirically to the investment shock. When we repeat the same exercise as in Section 6 replacing returns on the IMC, IK and ROA portfolios with the second principal component, we find that the latter performs very similarly as regards to predicting dispersion in investment behavior. However, exposure to PC2 might more accurately measure exposure to the investment shock than beta-IMC and thus may have additional information regarding the cross-section of risk premia. We explore this possibility below in Section 7.2 below.

The result of this section suggest an intuitive explanation for return comovement among portfolios sorted on either past investment rates or past profitability. These char-

acteristics are related to firms' exposure to IST shocks, and this largely explains return comovement along both dimensions of firm heterogeneity.

7.2 Firm characteristics as proxies for risk

Here, we further explore to what extent firm characteristics such as Tobin's Q, investment rate (IK) or profitability (ROA) are associated with expected returns because they proxy a firm's exposure to the investment shock.

Firm betas are measured with error, while characteristics are not. Thus, it is possible that characteristics contain information about risk exposures incremental to estimated betas. One way to explore this possibility is to see whether firm characteristics predict future investment shock betas, controlling for lagged betas. Since we estimate betas using a non-overlapping window of one year, measurement error should be serially uncorrelated. If the true investment beta is persistent but uncorrelated with firm characteristics, then characteristics should have no ability to predict future betas.

To this end we estimate a panel regression of estimated betas with the IMC portfolio (and PC2) on firm characteristics, including year fixed effects:

$$\beta_{it}^i = \gamma_t + \gamma_0 \beta_{it-1}^i + \gamma_1 \log Q_{it-1} + \gamma_2 IK_{it-1} + \gamma_3 ROA_{it-1} + u_{it}, \quad (22)$$

where $i = \{imc, pc2\}$. We standardize all variables to unit mean and standard deviation.

We show the results in Table 10. Panel A shows the results for IMC-beta, panel B shows the results for PC2-beta. In both cases, all three characteristics predict future investment shock betas in a manner consistent with our theory. High investment rate and Tobin's Q firms have higher growth opportunities as a fraction of firm value, and thus have higher investment shock betas. In contrast, firms with high profitability are firms for which the value of existing assets contributes more to firm value, and thus have lower investment shock betas.

The question is then whether to what extent the cross-sectional dispersion in risk premia

that arises when we sort firms on IK and ROA is due to the predicted dispersion in investment risk exposures ($\hat{\beta}_{it}^i$). One way to answer this question is to sort firms on the fitted betas from equation 22 and then on the individual characteristics (IK, ROA). If the second sort produces substantial dispersion in average returns, this means that, controlling for subsequent risk exposures, characteristics have incremental role in predicting risk premia.

To this end, we first sort firms into deciles based on the predicted values of IMC-beta (PC2-beta) from the fourth column of table 10. Within each decile, we then sort firms into 10 portfolios based on IK or ROA. We then pool firms into portfolios based on the characteristics sort. This gives us 10 portfolios of firms sorted on IK or ROA relative to their peers with same values of $\hat{\beta}_{it}^i$. Table 11 shows moments for the portfolios sorted on IK and ROA, controlling for $\hat{\beta}_{it}^{imc}$. We can see that the dispersion in average returns and CAPM alphas is substantially smaller, once we control for the differential investment shock exposures of these portfolios. Sorting firms on IK (ROA) results in a difference of average return of -3.2% (3.0%) and CAPM alphas of -3.9% (5.3%), relative to an average return spread of -4.5% (6.5%) and CAPM alpha of -7.5% (9.5%) when we sort unconditionally on IK (ROA). Controlling for $\hat{\beta}_{it}^{pc2}$ further reduces the average return spread to -1.96% (2.2%) and CAPM alphas to -3.0% (3.3%) when sorting on IK (ROA). In all cases where we first control for the portfolio's investment shock exposure, none of the average returns or CAPM alphas are statistically different from zero at the 10% level.

The results of this section support the view that the reason why firm characteristics such as IK and ROA are informative about the cross-section of risk premia is because they are informative about the ratio of growth opportunities to firm value, which is endogenously related to exposures to the investment shock.

8 Conclusion

In this paper we propose an explanation of why firms differing in past investment rates or profitability have different expected returns, and why firms similar in these characteristics exhibit stock return comovement. Our analysis shows how investment-specific technology shocks can help generate these patterns in stock returns. We find empirical support for the qualitative predictions of our theory, and we develop a simple structural model of firm investment that captures the key empirical effects quantitatively. In summary, this paper adds to the growing literature analyzing the economic impact of investment-specific technology shocks by demonstrating the important effects of such shocks on the cross-section of stock returns and firm-level investment.

Tables

Table 1: Parameters

Parameter	Symbol	Value
<i>Technology</i>		
Growth rate of X-shock	μ_X	0.005
Volatility of X-shock	σ_X	0.140
Growth rate of IST shock	μ_Z	0.003
Volatility of IST shock	σ_Z	0.0373
Mean-reversion parameter of project-specific shock	θ_u	0.100
Volatility of project-specific shock	σ_u	1.150
<i>Production</i>		
Project DRS parameter	α	0.850
Share of profits captured by investment firms	ϕ	0.07
Depreciation rate of capital	δ	0.100
<i>Investment</i>		
Average project arrival rate	μ_λ	4.000
Dispersion project arrival rate	σ_λ	1.500
Project arrival rate in high-growth state	λ_H	4.200
Project arrival rate in low-growth state	λ_L	0.130
Transition probability into high-growth state	μ_H	0.050
Transition probability into low-growth state	μ_L	0.180
<i>Stochastic discount factor</i>		
Risk-free rate	r	0.03
Price of risk of X-shock	b_x	0.65
Price of risk of IST shock	b_z	0.375

Table 2: Calibration

Moment	Data	Model		
		Median	5%	95%
Mean of aggregate dividend growth	0.025	0.009	-0.072	0.148
Volatility of aggregate dividend growth	0.118	0.132	0.100	0.193
Mean of aggregate investment growth	0.047	0.023	-0.050	0.093
Volatility of aggregate investment growth	0.157	0.271	0.216	0.353
Correlation of dividend and investment growth	0.201	-0.089	-0.382	0.316
Mean excess return of market portfolio	0.059	0.057	0.050	0.133
Volatility of market portfolio return	0.161	0.184	0.141	0.247
Mean return of HML portfolio	0.061	0.042	0.024	0.054
Volatility of HML portfolio	0.141	0.0643	0.0332	0.100
Mean return of IMC portfolio	-0.019	-0.051	-0.117	-0.010
Volatility of IMC portfolio	0.112	0.123	0.086	0.182
Correlation of market and IMC portfolio	0.267	0.569	0.259	0.806
Relative market capitalization of I- and C-sector	0.149	0.169	0.100	0.220
Firm investment rate (median)	0.116	0.126	0.069	0.300
Firm investment rate (IQR)	0.157	0.147	0.066	0.241
Cashflows-to-Capital (median)	0.160	0.220	0.175	0.252
Cashflows-to-Capital (IQR)	0.234	0.336	0.304	0.373
Tobin Q (median)	1.410	2.440	1.524	3.918
Tobin s Q (IQR)	3.412	3.336	1.884	8.090
β^{imc} (median)	0.683	0.559	0.388	0.704
β^{imc} (IQR)	0.990	0.593	0.543	0.643
Firm size relative to average size (median)	0.201	0.630	0.513	0.667
Firm size relative to average size (IQR)	0.830	1.213	1.138	1.295

Table 2 compares sample moments to moments of simulated data. For simulated data, we report median values across simulations, along with the 5% and 95% percentiles. Moments of dividend growth are from the long sample in Campbell and Cochrane (1999). Aggregate investment is real private nonresidential investment in equipment and software, and the moments of investment growth are estimated over the sample 1927-2008. Stock return moments are estimated over the sample 1963-2008. Market portfolio includes the consumption and the investment sector, the IMC portfolio is constructed in Papanikolaou (2010). Moments of firm-specific variables are estimated using Compustat data. We report time series averages of the median and inter-quintile range (IQR) over the 1965-2008 period. The investment rate is capital expenditures (capx) over lagged capital (ppeg); cashflows-to-capital is operating income (ib) plus depreciation (dp) scaled by capital (ppeg); Tobin's Q, is the sum of market value of common equity from CRSP, book value of debt (dltt) and preferred stock (pstkrv) minus inventories (invt) and deferred taxes (taxdb) divided by capital (ppeg); β^{imc} is described in the text; firm size is CRSP December market capitalization.

Table 3: Portfolio Characteristics (10 IK portfolios)

	Data										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	7.33 (2.44)	6.99 (2.73)	7.35 (3.20)	7.23 (2.93)	6.85 (2.72)	5.38 (2.03)	6.27 (2.23)	3.86 (1.28)	3.05 (0.90)	2.82 (0.58)	-4.52 (-1.25)
$\sigma(\%)$	20.18	17.20	15.41	16.59	16.90	17.74	18.91	20.22	22.84	32.86	24.15
β_{MKT}	0.96 (10.10)	0.83 (10.76)	0.73 (10.24)	0.85 (13.50)	0.88 (14.31)	0.92 (18.16)	0.95 (16.04)	1.04 (15.94)	1.17 (18.42)	1.56 (10.59)	0.60 (3.10)
$\alpha(\%)$	2.53 (1.73)	2.84 (2.14)	3.71 (3.36)	2.99 (3.98)	2.44 (2.72)	0.78 (0.92)	1.52 (1.15)	-1.35 (-1.42)	-2.80 (-2.42)	-4.99 (-1.86)	-7.52 (-2.40)
$R^2(\%)$	73.39	75.43	72.34	84.91	88.57	86.82	81.94	85.85	85.16	73.15	20.05
β_{IMC}	0.09 (0.41)	0.15 (0.79)	0.01 (0.05)	0.15 (0.92)	0.35 (3.02)	0.33 (2.30)	0.26 (1.66)	0.35 (2.14)	0.71 (3.99)	1.33 (5.71)	1.24 (9.63)
IK (vw)	0.03	0.05	0.07	0.09	0.11	0.13	0.16	0.21	0.30	0.64	
Tobin's Q (vw)	0.83	0.70	0.85	1.04	1.20	1.50	1.83	1.89	2.62	3.26	
CF/K (vw)	0.07	0.08	0.10	0.12	0.14	0.17	0.19	0.21	0.24	0.30	
	Model										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	7.56 (3.43)	8.06 (3.60)	8.21 (3.66)	8.13 (3.59)	7.90 (3.44)	7.57 (3.22)	7.17 (2.96)	6.70 (2.67)	6.07 (2.29)	5.25 (1.83)	-2.31 (-2.08)
$\sigma(\%)$	15.82	16.07	16.13	16.29	16.53	16.91	17.42	18.07	19.06	20.53	8.15
β_{MKT}	0.80 (21.77)	0.81 (21.48)	0.81 (21.83)	0.83 (22.48)	0.84 (24.62)	0.87 (27.79)	0.90 (31.69)	0.94 (38.28)	0.99 (43.50)	1.07 (39.51)	0.27 (5.90)
$\alpha(\%)$	2.78 (4.28)	3.22 (4.79)	3.35 (5.02)	3.19 (4.97)	2.86 (4.74)	2.39 (4.28)	1.78 (3.57)	1.07 (2.39)	0.13 (0.27)	-1.13 (-2.14)	-3.91 (-4.57)
$R^2(\%)$	89.51	89.17	89.41	90.20	91.44	92.84	94.31	95.46	95.99	95.34	39.93
β_{IMC}	0.36 (2.44)	0.37 (2.40)	0.37 (2.47)	0.39 (2.60)	0.43 (2.81)	0.48 (3.06)	0.55 (3.49)	0.63 (3.93)	0.74 (4.57)	0.87 (5.45)	0.51 (16.02)
IK (vw)	0.05	0.09	0.11	0.12	0.14	0.15	0.18	0.22	0.28	0.41	
Tobin's Q (vw)	1.73	1.73	1.72	1.75	1.80	1.88	2.00	2.17	2.47	2.99	
ROA (vw)	0.29	0.28	0.27	0.27	0.26	0.26	0.25	0.24	0.23	0.19	

Table 3 shows characteristics for the 10 portfolios of firms sorted on investment rate. The top panel shows results from actual data, the bottom panel shows results from data simulated by the model. We report average returns in excess of the risk-free rate, as well CAPM alphas and univariate post-formation betas with respect to the market portfolio (β_t^{mkt}) and the investment minus consumption portfolio (β_t^{imc}). Estimation is done at annual frequencies in both the model and the data. We report the portfolio's investment rate, defined as the ratio of the sum of investment expenditures (capx) to the sum of book values of capital (ppeg); the portfolio's Tobin's Q, defined as the sum of market value of common equity from CRSP, book value of debt (dltt) and preferred stock (pstkrv) minus inventories (invt) and deferred taxes (taxdb) divided by the sum of book value of capital across firms; the portfolio's return on assets, defined as the sum of operating income (ib) divided by lagged book value of assets (at). The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949). Standard errors are computed using Newey-West with 1 lag to adjust for autocorrelation in returns. t -statistics are reported in parenthesis. Each simulation sample contains 5,000 firms and has a length of 50 years. We simulate 2,000 samples and report medians across simulations of coefficients and t statistics (in parenthesis). We simulate the model at a weekly frequency ($dt = 1/52$) and aggregate to form annual observations. When forming portfolios from simulated data, we only include firms with non-zero investment rates and non-zero values of capital.

Table 4: Portfolio Characteristics (10 ROA portfolios)

	Data										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	-0.19 (-0.04)	2.52 (0.69)	5.69 (1.67)	5.17 (1.54)	6.73 (2.35)	6.96 (2.70)	4.42 (1.66)	6.63 (2.57)	4.43 (1.77)	6.26 (2.27)	6.46 (1.59)
$\sigma(\%)$	36.38	24.55	22.83	22.45	19.23	17.28	17.84	17.28	16.75	18.53	27.97
β_{MKT}	1.52 (7.31)	1.20 (9.51)	1.10 (8.91)	1.04 (7.79)	0.89 (8.58)	0.89 (13.34)	0.91 (13.69)	0.90 (14.89)	0.88 (22.69)	0.91 (12.63)	-0.61 (-2.52)
$\alpha(\%)$	-7.81 (-2.17)	-3.46 (-2.01)	0.18 (0.10)	-0.05 (-0.02)	2.28 (1.23)	2.51 (2.54)	-0.14 (-0.14)	2.11 (2.20)	0.01 (0.01)	1.69 (1.36)	9.50 (2.42)
$R^2(\%)$	56.79	76.89	75.47	70.07	69.54	85.77	84.62	88.72	90.14	79.02	15.32
β_{IMC}	1.46 (6.36)	0.90 (4.77)	0.34 (1.68)	0.03 (0.15)	-0.03 (-0.26)	0.16 (0.99)	0.19 (0.99)	0.20 (1.47)	0.25 (1.85)	0.28 (1.81)	-1.19 (-5.11)
IK (vw)	0.14	0.11	0.10	0.09	0.10	0.10	0.10	0.12	0.14	0.17	
Tobin's Q (vw)	2.43	1.69	1.38	2.41	2.19	1.29	1.02	1.25	1.98	4.03	
CF/K (vw)	-0.40	0.04	0.08	0.18	0.19	0.14	0.14	0.17	0.23	0.34	
	Model										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	2.86 (0.76)	4.03 (1.16)	5.08 (1.63)	5.84 (2.06)	6.28 (2.40)	6.52 (2.59)	6.51 (2.67)	6.22 (2.60)	5.66 (2.33)	5.02 (2.02)	2.15 (1.26)
$\sigma(\%)$	27.64	25.34	22.79	20.73	19.27	18.41	17.90	17.52	17.72	18.00	12.48
β_{MKT}	1.28 (30.12)	1.19 (40.90)	1.07 (61.56)	0.98 (100.85)	0.90 (54.92)	0.86 (40.21)	0.83 (34.30)	0.81 (33.01)	0.82 (39.29)	0.84 (38.82)	-0.44 (-8.16)
$\alpha(\%)$	-3.56 (-4.02)	-1.93 (-3.12)	-0.32 (-0.99)	0.91 (3.18)	1.74 (4.40)	2.21 (4.72)	2.34 (4.52)	2.13 (4.17)	1.50 (3.16)	0.74 (1.54)	4.30 (3.52)
$R^2(\%)$	93.66	95.80	97.41	97.51	96.18	94.97	94.05	93.89	94.73	95.46	54.06
β_{IMC}	1.60 (11.24)	1.39 (9.60)	1.18 (8.19)	0.99 (7.01)	0.88 (6.44)	0.80 (5.97)	0.76 (5.68)	0.72 (5.62)	0.73 (5.58)	0.72 (5.87)	-0.87 (-28.35)
IK (vw)	0.12	0.09	0.09	0.09	0.10	0.10	0.10	0.11	0.11	0.14	
Tobin's Q (vw)	4.09	2.63	2.06	1.88	1.84	1.91	2.05	2.34	2.91	4.73	
ROA (vw)	0.02	0.06	0.12	0.17	0.21	0.26	0.30	0.36	0.47	0.82	

Table 4 shows characteristics for the 10 portfolios of firms sorted on investment rate. The top panel shows results from actual data, the bottom panel shows results from data simulated by the model. We report average returns in excess of the risk-free rate, as well CAPM alphas and univariate post-formation betas with respect to the market portfolio (β_t^{mkt}) and the investment minus consumption portfolio (β_t^{imc}). Estimation is done at annual frequencies in both the model and the data. We report the portfolio's investment rate, defined as the ratio of the sum of investment expenditures (capx) to the sum of book values of capital (ppeg); the portfolio's Tobin's Q, defined as the sum of market value of common equity from CRSP, book value of debt (dltt) and preferred stock (pstkrv) minus inventories (invt) and deferred taxes (taxdb) divided by the sum of book value of capital across firms; the portfolio's return on assets, defined as the sum of operating income (ib) divided by lagged book value of assets (at). The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949). Standard errors are computed using Newey-West with 1 lag to adjust for autocorrelation in returns. t -statistics are reported in parenthesis. Each simulation sample contains 5,000 firms and has a length of 50 years. We simulate 2,000 samples and report medians across simulations of coefficients and t statistics (in parenthesis). We simulate the model at a weekly frequency ($dt = 1/52$) and aggregate to form annual observations. When forming portfolios from simulated data, we only include firms with non-zero book values of capital.

Table 5: Portfolio Characteristics (10 BE/ME portfolios)

	Data										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	3.88 (1.29)	5.93 (2.25)	4.66 (1.73)	5.63 (2.26)	5.86 (2.22)	6.79 (2.45)	6.60 (2.28)	8.89 (3.04)	9.18 (2.77)	10.04 (2.98)	6.15 (2.16)
$\sigma(\%)$	20.16	17.68	18.09	16.73	17.72	18.57	19.45	19.60	22.20	22.62	19.12
β_{MKT}	1.01 (15.52)	0.91 (19.28)	0.94 (15.75)	0.85 (12.11)	0.88 (12.03)	0.89 (8.32)	0.91 (7.56)	0.92 (8.64)	1.00 (9.51)	1.03 (9.96)	0.02 (0.14)
$\alpha(\%)$	-1.16 (-1.01)	1.38 (1.41)	-0.06 (-0.06)	1.40 (1.36)	1.44 (1.21)	2.32 (1.75)	2.06 (1.18)	4.31 (2.66)	4.16 (2.11)	4.90 (2.29)	6.06 (2.02)
$R^2(\%)$	81.16	85.99	88.16	82.95	80.70	75.16	70.46	70.90	66.24	66.95	0.03
β_{IMC}	0.38 (2.54)	0.23 (1.67)	0.23 (1.37)	0.23 (1.47)	0.05 (0.33)	0.06 (0.30)	-0.00 (-0.02)	-0.05 (-0.31)	0.02 (0.11)	0.19 (1.08)	-0.19 (-1.41)
IK (vw)	0.16	0.16	0.13	0.11	0.10	0.10	0.10	0.10	0.10	0.10	
Tobin's Q (vw)	3.76	2.96	1.93	1.43	1.24	1.04	1.11	0.98	0.88	0.69	
CF/K (vw)	0.16	0.23	0.19	0.16	0.15	0.13	0.13	0.13	0.12	0.09	
	Model										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	3.56 (1.19)	4.42 (1.61)	5.13 (1.96)	5.75 (2.29)	6.37 (2.62)	6.93 (2.94)	7.47 (3.23)	7.96 (3.49)	8.44 (3.67)	9.17 (3.85)	5.61 (4.68)
$\sigma(\%)$	21.73	20.13	19.17	18.34	17.64	17.06	16.63	16.38	16.42	16.96	9.39
β_{MKT}	1.11 (29.08)	1.04 (35.98)	1.00 (42.37)	0.95 (45.34)	0.92 (42.72)	0.88 (36.22)	0.86 (29.99)	0.84 (26.37)	0.84 (24.80)	0.87 (25.75)	-0.24 (-3.87)
$\alpha(\%)$	-3.12 (-4.75)	-1.83 (-3.65)	-0.85 (-2.09)	0.03 (-0.10)	0.87 (2.09)	1.64 (3.64)	2.34 (4.65)	2.94 (5.26)	3.40 (5.79)	3.90 (6.92)	7.02 (6.69)
$R^2(\%)$	93.69	95.38	96.15	96.20	95.83	94.90	93.53	92.13	91.39	91.94	26.88
β_{IMC}	0.99 (5.50)	0.84 (4.73)	0.74 (4.30)	0.65 (3.89)	0.57 (3.51)	0.51 (3.19)	0.46 (2.98)	0.43 (2.83)	0.43 (2.84)	0.47 (3.09)	-0.55 (-6.59)
IK (vw)	0.25	0.15	0.12	0.11	0.10	0.10	0.09	0.08	0.08	0.06	
Tobin's Q (vw)	10.86	5.76	4.13	3.24	2.65	2.23	1.92	1.68	1.45	1.16	
ROA (vw)	0.73	0.52	0.44	0.39	0.35	0.32	0.28	0.25	0.21	0.15	

Table 5 shows characteristics for the 10 portfolios of firms sorted on investment rate. The top panel shows results from actual data, the bottom panel shows results from data simulated by the model. We report average returns in excess of the risk-free rate, as well CAPM alphas and univariate post-formation betas with respect to the market portfolio (β_t^{mkt}) and the investment minus consumption portfolio (β_t^{imc}). Estimation is done at annual frequencies in both the model and the data. We report the portfolio's investment rate, defined as the ratio of the sum of investment expenditures (capx) to the sum of book values of capital (ppeg); the portfolio's Tobin's Q, defined as the sum of market value of common equity from CRSP, book value of debt (dltt) and preferred stock (pstkrv) minus inventories (invt) and deferred taxes (taxdb) divided by the sum of book value of capital across firms; the portfolio's return on assets, defined as the sum of operating income (ib) divided by lagged book value of assets (at). The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949). Standard errors are computed using Newey-West with 1 lag to adjust for autocorrelation in returns. t -statistics are reported in parenthesis. Each simulation sample contains 5,000 firms and has a length of 50 years. We simulate 2,000 samples and report medians across simulations of coefficients and t statistics (in parenthesis). We simulate the model at a weekly frequency ($dt = 1/52$) and aggregate to form annual observations. When forming portfolios from simulated data, we only include firms with non-zero book values of capital.

Table 6: Asset pricing tests

		Data				Model									
		30 BM/INV/ROA portfolios													
b_x	Δx_t^{ctfp}	1.47 (2.78)		0.29 (0.89)		0.20 0.47 0.86 (2.80)				0.43 0.72 1.08 (5.13)					
	R_{MKT}		0.30 (2.54)		0.46 (3.32)		0.15 0.36 0.63 (2.69)					0.42 0.84 1.50 (4.97)			
b_z	$-\Delta z_t^p$			-0.85 (-2.57)						-0.75 -0.39 -0.06 (-4.73)					
	R_{IMC}				-0.64 (-3.28)							-1.60 -0.91 -0.61 (-5.06)			
SSQE (%)		1.93	2.43	0.99	0.52	0.09 0.61 1.16	0.68 1.40 2.45	0.04 0.10 0.32		0.06 0.16 0.48					
		10 IK portfolios													
b_x	Δx_t^{ctfp}	1.56 (2.66)		0.40 (0.99)		0.23 0.49 0.86 (3.11)				0.45 0.75 1.15 (5.23)					
	R_{MKT}		0.29 (2.53)		0.51 (3.71)		0.19 0.41 0.68 (3.06)					0.44 0.87 1.61 (5.14)			
b_z	$-\Delta z_t^p$			-0.71 (-2.65)						-0.93 -0.45 -0.07 (-2.95)					
	R_{IMC}				-0.70 (-3.15)							-1.60 -0.96 -0.63 (-5.17)			
SSQE (%)		0.70	0.80	0.48	0.14	0.01 0.11 0.33	0.11 0.27 0.53	0.00 0.01 0.03		0.00 0.01 0.04					
		10 ROA portfolios													
b_x	Δx_t^{ctfp}	1.28 (2.06)		0.07 (0.16)		0.18 0.45 0.86 (2.60)				0.39 0.66 0.98 (4.69)					
	R_{MKT}		0.24 (1.90)		0.41 (2.69)		0.13 0.33 0.58 (2.48)					0.38 0.76 1.37 (4.57)			
b_z	$-\Delta z_t^p$			-0.96 (-2.83)						-0.62 -0.29 0.04 (-2.00)					
	R_{IMC}				-0.57 (-2.45)							-1.36 -0.76 -0.47 (-6.50)			
SSQE		0.80	0.86	0.33	0.14	0.02 0.15 0.39	0.12 0.42 0.85	0.01 0.02 0.04		0.01 0.02 0.04					
		10 BM portfolios													
b_x	Δx_t^{ctfp}	1.54 (3.32)		0.27 (0.44)		0.19 0.46 0.85 (2.71)				0.51 0.84 1.32 (5.82)					
	R_{MKT}		0.40 (3.18)		0.48 (3.40)		0.14 0.35 0.62 (2.63)					0.50 1.02 2.00 (5.68)			
b_z	$-\Delta z_t^p$			-0.94 (-1.61)						-1.29 -0.55 -0.09 (-3.57)					
	R_{IMC}				-0.77 (-1.97)							-2.55 -1.30 -0.73 (-6.01)			
SSQE (%)		0.37	0.53	0.12	0.16	0.04 0.29 0.51	0.26 0.58 0.98	0.00 0.03 0.16		0.01 0.05 0.28					

Table 6 reports estimates of b_x and b_z from the model SDF: $m = a - b_x \Delta x - b_z \Delta z$. We use total factor productivity in the consumption sector (x^{ctfp}) and returns to the market portfolio (R_{mkt}) as proxies for the x shock, and innovations in the relative price of equipment (z_t^p) and returns to the investment minus consumption portfolio (R_{imc}) as proxies for the z shock. See main text for description of these variables. The left panel presents empirical estimates using annual data in the 1965-2008 period; we report first-stage estimates and HAC t-statistics in parenthesis. The right panel presents estimates from 2,000 simulations, each simulation has length 50 years; we report 5%, median (in bold) and 95% values across simulations of the point estimates along with HAC t-statistics in parenthesis.

Table 7: Response of firm investment to the IK and ROA factors

Dependent variable i_t	Data				Model			
	IK factor							
\tilde{R}_{t-1}^{cik}	0.031 (1.13)	0.002 (0.11)	0.002 (0.10)	-0.001 (-0.07)	0.050 (3.96)	0.030 (3.59)	0.025 (3.61)	0.024 (3.31)
$D(\beta_{imc})_2 \times \tilde{R}_{t-1}^{cik}$		-0.005 (-0.37)	-0.005 (-0.36)	0.009 (0.85)		0.009 (3.04)	0.007 (2.50)	0.008 (2.75)
$D(\beta_{imc})_3 \times \tilde{R}_{t-1}^{cik}$		0.017 (1.12)	0.017 (1.11)	0.016 (1.20)		0.015 (2.91)	0.010 (2.24)	0.013 (2.66)
$D(\beta_{imc})_4 \times \tilde{R}_{t-1}^{cik}$		0.035 (1.78)	0.035 (1.79)	0.022 (1.29)		0.018 (2.40)	0.016 (2.42)	0.018 (2.99)
$D(\beta_{imc})_H \times \tilde{R}_{t-1}^{cik}$		0.095 (4.28)	0.095 (4.30)	0.071 (3.91)		0.049 (2.79)	0.058 (3.23)	0.059 (3.46)
R^2	0.001	0.013	0.014	0.416	0.003	0.021	0.045	0.049
	ROA factor							
\tilde{R}_{t-1}^{croa}	0.057 (2.29)	0.023 (1.05)	0.023 (1.03)	0.024 (1.13)	0.046 (3.79)	0.027 (3.17)	0.023 (3.25)	0.022 (3.00)
$D(\beta_{imc})_2 \times \tilde{R}_{t-1}^{croa}$		0.001 (0.14)	0.002 (0.17)	0.001 (0.12)		0.010 (3.11)	0.007 (2.54)	0.008 (2.86)
$D(\beta_{imc})_3 \times \tilde{R}_{t-1}^{croa}$		0.016 (0.96)	0.016 (0.96)	0.005 (0.37)		0.016 (3.12)	0.011 (2.59)	0.014 (2.99)
$D(\beta_{imc})_4 \times \tilde{R}_{t-1}^{croa}$		0.052 (2.68)	0.053 (2.72)	0.025 (1.75)		0.019 (2.58)	0.017 (2.66)	0.019 (3.13)
$D(\beta_{imc})_H \times \tilde{R}_{t-1}^{croa}$		0.098 (3.22)	0.098 (3.25)	0.074 (3.26)		0.049 (2.94)	0.055 (3.10)	0.055 (3.30)
R^2	0.003	0.016	0.017	0.418	0.002	0.016	0.034	0.038
Industry/Firm FE	N	N	I	F	N	N	N	N
Controls (i_{t-1})	N	N	Y	N	N	N	Y	Y
Controls (CF, K)	N	N	N	Y	N	N	N	Y
Controls ($Q, E/A$)	N	N	N	Y	N	N	N	N

Table 7 shows estimates of

$$i_{f,t} = a_1 + \sum_{d=2}^5 a_d D(\beta_{f,t-1}^{imc})_d + b_1 (\tilde{R}_{t-1}^z) + \sum_{d=2}^5 b_d D(\beta_{f,t-1}^{imc})_d \times (\tilde{R}_{t-1}^z) + cX_{f,t-1} + \gamma_f + u_{f,t}$$

where $\tilde{R}^z = (\tilde{R}^{cik}, \tilde{R}^{croa})$ are empirical factors approximating IST shocks; $i_t \equiv I_t/K_{t-1}$ is firm investment over the lagged capital stock; $\tilde{R}_{t-1}^z \equiv \sum_{l=1}^2 R_{t-1}^z$; the vector of controls X_t includes lagged values of cashflows over lagged capital, log book equity over book assets, and log capital. $D(\beta_{i,t-1}^{imc})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in term of $\beta_{i,t-1}^{imc}$. β_t^{imc} refers to the firm's beta with respect to the investment minus consumption portfolio (IMC) in year t , estimated using non-overlapping weekly returns within year t . Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t -statistics in parenthesis using standard errors clustered by firm and year. The sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 8: Response of firm investment to I-shock: by IK or ROA

Dependent variable i_t	Data				Model			
	by IK quintiles							
\tilde{R}_{t-1}^{imc}	0.084 (4.17)	0.054 (3.33)	0.054 (3.33)	0.043 (2.47)	0.055 (4.56)	0.033 (3.99)	0.032 (3.96)	0.032 (3.87)
$D(IK)_2 \times \tilde{R}_{t-1}^{imc}$		0.009 (0.86)	0.009 (0.86)	0.010 (1.11)		0.022 (4.46)	0.021 (4.29)	0.021 (4.29)
$D(IK)_3 \times \tilde{R}_{t-1}^{imc}$		0.014 (1.25)	0.014 (1.25)	0.020 (1.90)		0.037 (5.28)	0.034 (5.04)	0.035 (5.04)
$D(IK)_4 \times \tilde{R}_{t-1}^{imc}$		0.032 (1.85)	0.032 (1.85)	0.029 (1.61)		0.055 (5.09)	0.048 (4.71)	0.050 (4.81)
$D(IK)_H \times \tilde{R}_{t-1}^{imc}$		0.095 (3.66)	0.094 (3.65)	0.089 (2.77)		0.077 (4.00)	0.052 (2.81)	0.056 (3.06)
R^2	0.007	0.268	0.268	0.473	0.004	0.112	0.123	0.123
	by ROA quintiles							
\tilde{R}_{t-1}^{imc}	0.084 (4.17)	0.081 (2.86)	0.080 (2.85)	0.068 (3.11)	0.055 (4.56)	0.063 (3.52)	0.062 (3.52)	0.063 (3.77)
$D(ROA)_2 \times \tilde{R}_{t-1}^{imc}$		-0.004 (-0.13)	-0.004 (-0.11)	0.005 (0.26)		-0.017 (-1.65)	-0.019 (-1.85)	-0.019 (-1.98)
$D(ROA)_3 \times \tilde{R}_{t-1}^{imc}$		0.000 (0.01)	0.001 (0.02)	0.012 (0.48)		-0.013 (-1.31)	-0.018 (-1.66)	-0.019 (-1.90)
$D(ROA)_4 \times \tilde{R}_{t-1}^{imc}$		-0.001 (-0.03)	-0.001 (-0.02)	0.007 (0.26)		-0.009 (-0.86)	-0.016 (-1.39)	-0.017 (-1.60)
$D(ROA)_H \times \tilde{R}_{t-1}^{imc}$		0.023 (0.67)	0.023 (0.69)	0.027 (0.90)		-0.001 (-0.12)	-0.013 (-1.25)	-0.012 (-1.23)
R^2	0.007	0.076	0.077	0.445	0.004	0.006	0.029	0.044
Industry/Firm FE	N	N	I	F	N	N	N	N
Controls (i_{t-1})	N	N	Y	N	N	N	Y	Y
Controls (CF, K)	N	N	N	Y	N	N	N	Y
Controls ($Q, E/A$)	N	N	N	Y	N	N	N	N

Table 8 shows estimates of

$$i_{f,t} = a_1 + \sum_{d=2}^5 a_d D(G_{f,t-1})_d + b_1 (\tilde{R}_{t-1}^{imc}) + \sum_{d=2}^5 b_d D(G_{f,t-1})_d \times (\tilde{R}_{t-1}^{imc}) + cX_{f,t-1} + \gamma_f + u_{f,t},$$

where $G_{f,t-1} \in \{i_{f,t-1}, ROA_{f,t-1}\}$; $i_t \equiv I_t/K_{t-1}$ is firm investment over the lagged capital stock; $\tilde{R}_{t-1}^{imc} \equiv \sum_{l=1}^2 R_{t-1}^{imc}$; the vector of controls X_t includes lagged values of cashflows over lagged capital, log book equity over book assets, and log capital. $D(G_{f,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile across $G_{f,t-1}$. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t -statistics in parenthesis using standard errors clustered by firm and year. The sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 9: Comovement

Cross-section	10 IK		10 ROA		10 IMC-beta	
	f_1	f_2	f_1	f_2	f_1	f_2
$\beta_{f_1^{(30)}}$	0.994 (61.71)	-0.003 (-0.04)	0.987 (37.15)	-0.028 (-0.51)	0.999 (60.21)	-0.086 (-1.47)
$\beta_{f_2^{(30)}}$	-0.034 (-2.28)	0.883 (14.22)	-0.024 (-1.12)	0.880 (16.30)	0.071 (4.94)	0.894 (17.18)
R^2	0.988	0.780	0.975	0.775	0.985	0.807

Table 9 shows the comovement of the first two principal components extracted from the cross-section of firms sorted on investment rate (IK), return on assets (ROA) or beta with respect to the IMC portfolio (IMC-beta) with the first two factors extracted from the pooled cross-section of assets, denoted by $f_1^{(30)}$ and $f_2^{(30)}$ respectively. We normalize all principal components to unit standard deviation. Each column reports the coefficients, t-statistics (in parentheses) and the R^2 of a linear regression of the first two principal components, f_1 and f_2 , extracted from the cross-section of excess returns on ten portfolios sorted on either IK, ROA, or IMC-betas, on $f_1^{(30)}$ and $f_2^{(30)}$. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 10: Firm characteristics and betas

β_t^{imc}	A: IMC-beta			
$\log Q_{t-1}$	0.072 (7.87)			0.061 (7.06)
IK_{t-1}		0.257 (7.36)		0.171 (5.78)
ROA_{t-1}			-0.217 (-3.60)	-0.163 (-2.84)
β_{t-1}^{imc}	0.216 (9.77)	0.223 (9.86)	0.227 (9.94)	0.212 (9.78)
R^2	0.235	0.230	0.228	0.237
β_t^{pc2}	B: PC2-beta			
$\log Q_{t-1}$	0.080 (4.87)			0.058 (3.78)
IK_{t-1}		0.314 (8.05)		0.218 (6.40)
ROA_{t-1}			-0.835 (-12.24)	-0.783 (-10.75)
β_{t-1}^{pc2}	0.186 (9.39)	0.190 (9.08)	0.177 (8.84)	0.167 (8.98)
R^2	0.215	0.213	0.221	0.226

Table 10 shows the results from a regression of β^{imc} and β^{pc2} on Tobin's Q, investment rate (IK) and return on assets (ROA). β^{imc} refers to the beta with the IMC portfolio and β^{pc2} refers to the betas with the second principal component extracted from the pooled cross-section of assets in section 7.1. We estimate betas using weekly returns. We include year fixed effects and cluster the errors by firm and year. We normalize all variables to unit standard deviation. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 11: Portfolios sorted on characteristics, controlling for predicted IMC-beta

	10 portfolios sorted on IK, controlling for predicted IMC-beta										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	6.27 (1.97)	6.56 (2.59)	6.84 (3.03)	6.37 (2.52)	5.58 (2.25)	6.18 (2.75)	5.91 (2.19)	4.51 (1.53)	3.68 (1.25)	3.08 (0.82)	-3.19 (-1.32)
$\sigma(\%)$	21.95	18.07	17.04	17.31	17.69	15.91	18.72	18.78	19.83	25.24	14.50
β_{MKT}	1.08 (14.52)	0.88 (12.30)	0.80 (10.68)	0.88 (20.57)	0.92 (13.40)	0.77 (10.00)	0.98 (18.63)	0.90 (10.05)	1.00 (19.79)	1.22 (12.43)	0.15 (1.30)
$\alpha(\%)$	1.07 (0.66)	2.32 (1.79)	2.96 (2.42)	2.10 (2.49)	1.16 (1.22)	2.44 (2.38)	1.19 (1.22)	0.15 (0.12)	-1.17 (-1.09)	-2.82 (-1.46)	-3.90 (-1.61)
$R^2(\%)$	79.56	78.31	73.67	86.25	88.58	78.28	90.14	76.60	84.87	77.68	3.38
	10 portfolios sorted on ROA, controlling for predicted IMC-beta										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	3.12 (0.67)	6.57 (1.89)	5.46 (1.89)	4.86 (1.68)	5.19 (2.04)	4.87 (1.81)	4.79 (1.83)	5.56 (2.29)	4.96 (1.94)	6.13 (2.12)	3.00 (0.78)
$\sigma(\%)$	32.81	25.36	20.57	19.88	17.38	19.19	18.20	15.93	17.50	19.10	23.67
β_{MKT}	1.40 (9.85)	1.13 (8.86)	0.97 (11.05)	0.96 (11.81)	0.84 (10.23)	0.97 (12.74)	0.94 (20.37)	0.80 (14.77)	0.90 (18.85)	0.92 (12.17)	-0.48 (-3.39)
$\alpha(\%)$	-3.66 (-1.16)	1.09 (0.52)	0.80 (0.51)	0.24 (0.18)	1.15 (0.99)	0.18 (0.16)	0.26 (0.29)	1.70 (1.50)	0.62 (0.69)	1.68 (1.22)	5.34 (1.48)
$R^2(\%)$	60.63	66.18	73.05	76.49	76.53	84.84	88.05	83.26	87.18	76.87	13.81

Table 11 shows characteristics for the 10 portfolios of firms sorted on investment rate (IK) and profitability (ROA), controlling for the predicted exposure to the investment shock (IMC-beta). We first sort firms into deciles based on the predicted values of IMC-beta from the fourth column of table 10, panel A. Within each decile, we then sort firms into 10 portfolios based on IK or ROA. We then pool portfolios across the predicted IMC-beta sort to form 10 portfolios sorted on IK or ROA. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949). Standard errors are computed using Newey-West with 1 lag to adjust for autocorrelation in returns. t -statistics are reported in parenthesis.

Table 12: Portfolios sorted on characteristics, controlling for predicted PC2-beta

	10 portfolios sorted on IK, controlling for predicted PC2-beta										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	4.99 (1.77)	7.00 (2.66)	5.90 (2.57)	6.99 (2.81)	6.39 (2.72)	5.86 (2.42)	5.52 (2.09)	4.10 (1.39)	4.49 (1.62)	3.03 (0.83)	-1.96 (-0.95)
$\sigma(\%)$	20.09	18.38	17.09	17.36	16.63	17.33	17.56	18.95	19.46	24.08	13.62
β_{MKT}	0.98 (12.75)	0.87 (10.58)	0.83 (12.68)	0.83 (10.06)	0.87 (17.26)	0.83 (8.28)	0.89 (23.83)	0.91 (11.27)	0.97 (13.62)	1.20 (15.21)	0.21 (2.28)
$\alpha(\%)$	0.24 (0.19)	2.81 (1.92)	1.89 (1.66)	2.97 (2.53)	2.21 (2.90)	1.84 (1.58)	1.22 (1.04)	-0.31 (-0.24)	-0.19 (-0.16)	-2.75 (-1.69)	-2.99 (-1.48)
$R^2(\%)$	79.45	73.49	78.26	75.76	89.96	76.47	85.36	76.59	82.28	81.71	8.03
	10 portfolios sorted on ROA, controlling for predicted PC2-beta										
	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	2.84 (0.75)	5.97 (1.77)	4.30 (1.63)	5.42 (2.04)	6.89 (2.19)	5.41 (2.31)	6.01 (2.20)	5.34 (2.23)	5.74 (2.13)	5.00 (1.71)	2.16 (0.83)
$\sigma(\%)$	26.73	23.23	18.49	18.01	21.35	17.26	18.27	16.40	18.40	19.52	17.92
β_{MKT}	1.17 (10.21)	1.06 (9.52)	0.83 (8.37)	0.85 (10.67)	1.03 (9.71)	0.89 (14.85)	0.93 (17.27)	0.85 (22.86)	0.90 (13.52)	0.94 (11.45)	-0.24 (-1.63)
$\alpha(\%)$	-2.82 (-1.37)	0.85 (0.44)	0.31 (0.18)	1.33 (0.99)	1.92 (1.50)	1.14 (1.13)	1.53 (1.50)	1.23 (1.44)	1.39 (1.23)	0.48 (0.34)	3.30 (1.30)
$R^2(\%)$	63.73	68.79	66.34	73.14	76.99	87.02	85.47	89.21	79.32	76.33	5.71

Table 12 shows characteristics for the 10 portfolios of firms sorted on investment rate (IK) and profitability (ROA), controlling for the predicted exposure to the second principal component from the pooled cross-section of assets (PC2-beta). We first sort firms into deciles based on the predicted values of PC2-beta from the fourth column of table 10, panel B. Within each decile, we then sort firms into 10 portfolios based on IK or ROA. We then pool portfolios across the predicted IMC-beta sort to form 10 portfolios sorted on IK or ROA. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949). Standard errors are computed using Newey-West with 1 lag to adjust for autocorrelation in returns. t -statistics are reported in parenthesis.

Figure 1: IMC and comovement of IMC-Beta/IK/ROA portfolios

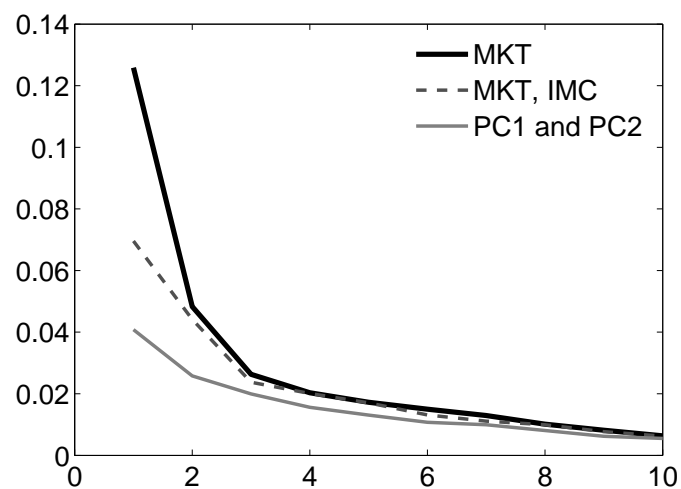


Figure 9 plots the first 10 eigenvalues of extracted from the pooled cross-section of 30 portfolios of firms sorted on investment rate (IK), return on assets (ROA) and beta with respect to the IMC portfolio (IMC-beta). We plot first 10 eigenvalues from the residuals of a regression of portfolio returns on i) the market portfolio (MKT); ii) the market and IMC portfolio (MKT+IMC); iii) the first two principal components extracted from the pooled cross-section. The sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

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