FINC460 - Fall 2008 Final Exam

NAME: ______________________________ SECTION: ______________

1. Please do not open this exam until directed to do so.
2. This exam is 3 hours long.
3. Please write your name and section number on the front of this exam, and on any examination books you use.
4. Please show all work required to obtain each answer. Answers without justification will receive no credit.
5. State clearly any assumptions you are making.
6. This is a closed book exam. No books or notes are permitted. Calculators are permitted. Laptops are permitted but you are only allowed to use Excel and only a blank worksheet. You are not allowed to use other spreadsheets with pre-entered formulas.
7. Brevity is strongly encouraged on all questions.
8. The exam is worth 115 points.
9. Relax, and good luck!

Hints:

1. *Think through problems before you start working. Draw pictures.*
2. *If you get stuck on part of a problem, go on to the next part. You may need to use answers from earlier parts of the question to calculate answers to the later parts. If you weren't able to solve the earlier part, assume something.*
3. *Remember, setting up the problem correctly will get you most of the points.*

Short questions (25 points)

Assess the validity of the following statements (True, False or Uncertain) and explain your answers.

1. (5 points) The mean-variance frontier created by N securities will always lie inside the mean-variance frontier created by N+1 securities.

   TRUE/UNCERTAIN - In general, the additional security will add a diversification effect and move the MVE frontier out. If not, the portfolios/assets that form the MVE under N assets are still possible, so the N+1 asset MVE must be (weakly) outside the N asset MVE frontier. If the additional asset is perfectly correlated with one of the N assets, then the MVE frontier of the N+1 assets would be the same as that for N assets.

2. (5 points) It is not possible for the APT and the CAPM to both hold at the same time.

   FALSE - Both can hold as long as the market portfolio is mean-variance efficient. There exists a unique combination of the APT factor mimicking portfolios that is MVE. If this portfolio coincides with the market, then both the APT and the CAPM will hold.

3. (5 points) Since larger firms have lower betas than small firms, the size anomaly is not an anomaly but is in fact consistent with the CAPM.

   FALSE/UNCERTAIN - To test this we could sort the large and small firms by betas and compare expected returns between large and small firms with the same betas. If these matched firms have different returns, then this phenomenon is an anomaly. Alternatively, check if the observed difference is due to the implied difference in returns due to differing betas. If $E[R_S - R_B] = (\beta_S - \beta_B)E[R_M - R_f]$ *holds, then the difference in returns is consistent with CAPM. In reality, the difference in returns has been greater than that implied by differences in betas.*

4. (5 points) Suppose that firms that increase their leverage, by issuing debt and buying back equity, have higher stock returns over the next 12 months. This is evidence of an inefficient market, since if increasing leverage is good news for the firm it should be immediately reflected in the stock price.

   FALSE - Levering up causes the firm's equity to be riskier, so the stocks will have higher betas and investors require a higher return for holding the security. This translates to a lower current stock price as cashflows haven't changed whereas the stock's beta has increased.

5. (5 points) An asset whose returns are uncertain can never have a negative risk premium, since investors will always prefer holding cash over it. *FALSE - In the CAPM, assets that correlate negatively with the market portfolio will have returns lower than the risk free rate, as they provide insurance. In general, assets that are positively correlated with factors that have a negative premium or assets that are negatively correlated with factors that carry a positive premium will have returns lower than the risk free rate.*

Question 1 (50 points)

You are a managing a fixed income portfolio for your clients. You believe that the following factor model holds for bond *returns*:

$$\begin{aligned} r_t^1 &= 1.1\% + 1\,\tilde{f}_{L,t} - 1\,\tilde{f}_{S,t} \\ r_t^5 &= 2.0\% + 5\,\tilde{f}_{L,t} \\ r_t^{10} &= 4.0\% + 10\,\tilde{f}_{L,t} + 10\,\tilde{f}_{S,t}, \end{aligned}$$

where r_t^n represents the return on a zero-coupon bond with maturity n, and $\tilde{f}_{i,t}$ represents factor i's surprise realizations. The two term structure factors, (f_L) and (f_S) are uncorrelated, i.e. $cov(f_L, f_S) = 0$, and $var(f_L) = 0.01$ and $var(f_S) = 0.005$.

1. (5 points) How would you characterize the two factors, (f_L) and (f_S), based on how bond returns load on these factors?

   f_L is a factor representing the level of interest rates where the loadings are the same as the duration of the bonds. f_S measures changes in the slope of the term structure.

2. (10 points) Construct two portfolios that exactly mimic the two term structure factors, f_L and f_S. What are the weights on these portfolios?

   In order to construct a portfolio that mimics the level factor, we need a portfolio that has loadings of 1 on f_L and 0 on f_S. This can be done by *solving the system of equations:*

$$w_1 + w_5 + w_{10} = 1$$
$$w_1 + 5w_5 + 10w_{10} = 1$$
$$-w_1 + 10w_{10} = 0$$

The weights on the three bonds are (w_1, w_5, w_{10})*=(1.143, -0.257, 0.114). Solving the parallel system for loadings of 0 on* f_L *and 1 on* f_S *yields portfolio weights of (1.571, -0.829, 0.257) for a portfolio that mimics the slope factor.*

3. (10 points) Find the risk free rate and the factor risk premia λ_S and λ_L implied by the absence of arbitrage opportunities.

   The APT pricing equation implies:

$$E[r^1] = \lambda_0 + \lambda_L - \lambda_S = .011$$
$$E[r^5] = \lambda_0 + 5\lambda_L = .02$$
$$E[r^{10}] = \lambda_0 + 10\lambda_L + 10\lambda_S = .04$$

   This implies factor prices are: $\lambda_0 = 0.01, \lambda_L = 0.002, \lambda_S = 0.001$

4. (15 points) Find the portfolio of the three zero coupon bonds that has the maximum Sharpe Ratio.

   To find the portfolio with the maximum Sharpe ratio, we can use the formula for MVE portfolio weights where the two risky assets are the *factors themselves.*

$$w_L = \frac{E(r_L)\sigma_S^2 - E(r_S)cov(r_L, r_S)}{E(r_L)\sigma_S^2 + E(r_S)\sigma_L^2 - [E(r_L) + E(r_S)]cov(r_L, r_S)}$$

$$w_L = \frac{(0.002)(0.005) - (0.001)(0))}{(0.002)(0.005) + (0.001)(0.01) - [(0.002) + (0.001)](0)} = 0.5$$

So $w_S = 1 - w_L = 0.5$. *From these weights we can back out weights on the three assets using the system of equations:*

$$\begin{aligned} w_1 + w_5 + w_{10} &= 1 \\ w_1 + 5w_5 + 10w_{10} &= 0.5 \\ -w_1 + 10w_{10} &= 0.5 \end{aligned}$$

The weights on the three bonds are (1.35714, -0.5429, 0.185714).

5. (10 points) Assume now that you believe that the Federal Reserve is likely to raise short term interest rates by 100bps but because this move signals a tougher inflation regime, long term rates will actually fall by 100 bps. Thus you believe that the yield curve is likely to flatten with the level being unaffected. That is, you believe that over the next year $\tilde{f}_L$, like everyone else, but that $\tilde{f}_S = 1\%$.

   - Construct a portfolio that takes advantage of this view but has no level factor risk.

     A portfolio that takes advantage of this view but has no level factor risk is a portfolio that exactly mimics the factor f_S *found above with weights (1.571, -0.829, 0.257).*

   - What is the Sharpe Ratio of this portfolio according to the market?

The market believes this portfolio has excess expected return equal to $E[r_p^e] = 1\lambda_S = 0.001$. *The standard deviation of the portfolio is* $\sigma_p = \sqrt{1^2 * \sigma_S^2} = 0.0707$. *The market thinks the Sharpe Ratio is:*

$$SR_p = \frac{0.001}{0.0707} = 0.01414$$

- What is the Sharpe Ratio on this portfolio according to you?

  You believe expected return on f_S *will have 1% higher returns, so the Sharpe Ratio according to your information is:*

$$SR_p = \frac{0.001 + 0.01}{0.0707} = 0.1556$$

Question 2 (40 points)

You are a partner in Roll and Rock Asset Management. Your firm uses the APT as a tool for managing money.

Your team of analysts has determined that there are three factors in the economy: industrial production (IP), inflation (IN), and consumer confidence (CF). Furthermore, your analysts have determined that, over the next year, the risk-permia and standard-deviations of the three factors will be as follows:

Factor	Risk Premium
Industrial Production (IP)	0.06
Inflation (IN)	-0.05
Consumer Confidence (CF)	0.00

In addition, they have determined that the loadings of the S&P 500 index on the three factors are as follows:

$$b_{IP} = 1.7 \qquad b_{IN} = 0.0 \qquad b_{CF} = 0.5$$

(Assume that the S&P 500 is a well-diversified portfolio.) They have also forecast the covariance matrix for the three factors for the coming year to be:

	f_{IP}	f_{IN}	f_{CF}
f_{IP}	0.01	0.00	0.00
f_{IN}	0.00	0.01	0.00
f_{CF}	0.00	0.00	0.04

Furthermore, assume that you can borrow or lend at the LIBOR rate of 5%/year.

1. (10 points) Comment on the sign of the risk premia for the three factors. Are they consistent with your intuition?

Industrial Production earns a positive risk premium. This makes sense because IP growth is high in 'good' states when the economy is expanding, so investors wanting to smooth consumption require a positive premium to invest in an asset that does well in expansions. An explanation for the negative risk premium on inflation is the fact that prices can rise due to scarcity in 'bad' states of the world, so assets paying off in these times need less risk premium. Finally, consumer confidence's zero risk premium reflects the intuitive idea that changes in investor sentiment contributes to volatility but does not affect returns.

2. (10 points) What do your analysts expect the return and the standard deviation of the S&P 500 to be over the next year?

   Using the APT pricing equation, we see:

$$\begin{aligned} E[r_{SP}] =& \lambda_0 + \lambda_{IP} b_{IP} + \lambda_{IN} b_{IN} + \lambda_{CF} b_{CF} \\ =& 0.05 + 0.06(1.7) + (-.05)(0) + 0(.5) = 15.2\% \end{aligned}$$

   The standard deviation is given by:

$$\sigma_{SP} = \sqrt{\sigma_{IP}^2 b_{IP}^2 + \sigma_{IN}^2 b_{IN}^2 + \sigma_{CF}^2 b_{CF}^2} = \sqrt{1.7^2 \sigma_{IP}^2 + .5^2 \sigma_{CF}^2} = 19.72\%$$

3. (10 points) Is the S&P 500 a mean-variance efficient portfolio?

   No, the S&P 500 is not mean-variance efficient. Recall that MVE returns have maximal return for a given level of variance. Clearly, this is not the case for the S&P 500 because it loads positively on Consumer Confidence which contributes positive risk, but no risk premium. Therefore, the S&P 500 contains unpriced risk and is not MVE.

4. (10 points) Assume you were to test the CAPM using the S&P 500 as

a proxy for the "market." Would you be likely to find that the CAPM held? Explain.

- If your answer to the above is no, please comment on whether, based on your results, the CAPM is truly incorrect.

  No. Since the S&P 500 is not mean-variance efficient, it is not a frontier return and consequently cannot be a pricing return in the CAPM framework. As such, we should not expect it to accurately price assets if we use it as a "proxy" for the market. This does not say that the CAPM is incorrect since we need a true market return (which is a MVE return) to correctly test the CAPM.

5. (BONUS 10 points) Assuming the CAPM holds, what are the loadings of the market portfolio on the three factors?

   If the CAPM holds, the market portfolio will be MVE and have the maximal Sharpe Ratio. Since Consumer Confidence only adds unpriced risk, we know the market portfolio will have a loading $b_{CF} = 0$. Now we can use the formula for a MVE portfolio between two risky assets to determine the loadings on Industrial Production and Inflation.

$$b_{IP} = \frac{E(r_{IP})\sigma_{IN}^2 - E(r_{IN})cov(r_{IP}, r_{IN})}{E(r_{IP})\sigma_{IN}^2 + E(r_{IN})\sigma_{IP}^2 - [E(r_{IP}) + E(r_{IN})]cov(r_{IP}, r_{IN})}$$

$$b_{IP} = \frac{(0.06)(0.01) - (-0.05)(0))}{(0.06)(0.01) + (-0.05)(0.01) - [(0.06) + (-0.05)](0)} = 6$$

   So we see the market portfolio has loadings $b_{IP} = 6$ and $b_{IN} = 1 - b_{IP} = -5$.

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