Finance 460 Equation Sheet

- Covariance Relations:

$$cov(a\tilde{x}, b\tilde{y}) = a \cdot b \cdot cov(\tilde{x}\tilde{y}) \qquad cov(\tilde{x}, \tilde{y} + \tilde{z}) = cov(\tilde{x}, \tilde{y}) + cov(\tilde{x}, \tilde{z}) \qquad var(a\tilde{x}) = a^2 var(\tilde{x})$$

- Correlation between $\tilde{x}$ and $\tilde{y}$

$$corr(\tilde{x}, \tilde{y}) = \frac{cov(\tilde{x}, \tilde{y})}{\sigma_x \, \sigma_y}$$

- Covariance of two securities when their residuals are uncorrelated:

$$cov(\tilde{r}_i, \tilde{r}_j) = \beta_i \beta_j \sigma_m^2 \quad \textbf{if } cov(\epsilon_i, \epsilon_j) = 0$$

- Statistical Functions:

$$var(\tilde{r}_A) = E[(\tilde{r}_A - \overline{r_A})^2] \qquad cov(\tilde{r}_A, \tilde{r}_B) = E[(\tilde{r}_A - \overline{r_A})(\tilde{r}_B - \overline{r_B})]$$

- Fraction of the your wealth you put in the risky assset 1, given a risk aversion coefficient of A:

$$w_1^* = \frac{E(\tilde{r}_1) - r_f}{A\sigma_1^2}$$

- The MVE portfolio weights when there are two risky assets A and B ($x_B = (1 - x_A)$):

$$x_A = \frac{E(\tilde{r}_A^e)\sigma_B^2 - E(\tilde{r}_B^e) cov(\tilde{r}_A^e, \tilde{r}_B^e)}{E(\tilde{r}_A^e)\sigma_B^2 + E(\tilde{r}_B^e)\sigma_A^2 - [E(\tilde{r}_A^e) + E(\tilde{r}_B^e)]\, cov(r_A^e, r_B^e)}$$

- The CAPM equation

$$E(\tilde{r}_i) = r_f + \beta_i \left[E(\tilde{r}_m) - r_f\right]$$

- The CAPM Beta:

$$\beta_i = \frac{cov(\tilde{r}_i, \tilde{r}_m)}{\sigma_m^2}$$

- The security market line

$$\tilde{r}_{i,t} - r_{f,t} = \alpha_i + \beta_i \cdot (\tilde{r}_{m,t} - r_{f,t}) + \tilde{\epsilon}_{i,t}$$

- Equation for the variance of portfolio a; and for the covariance of portfolios a and b:

$$var(\tilde{r}_a) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i^a w_j^a \sigma_{i,j} \qquad cov(\tilde{r}_a, \tilde{r}_b) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i^a w_j^b \sigma_{i,j}$$

- The covariance between two portfolios a and b that each load on two assets (1 and 2):

$$\begin{aligned} var(\tilde{r}_a) &= (w_1^a)^2 \cdot (\sigma_1)^2 + (w_2^a)^2 \cdot (\sigma_2)^2 + 2 \cdot w_1^a \cdot w_2^a \cdot cov(\tilde{r}_1, \tilde{r}_2) \\ cov(\tilde{r}_a, \tilde{r}_b) &= w_1^a \cdot w_1^b \cdot (\sigma_1)^2 + w_2^a \cdot w_2^b \cdot (\sigma_2)^2 + (w_1^a \cdot w_2^b + w_2^a \cdot w_1^b) \cdot cov(\tilde{r}_1, \tilde{r}_2) \end{aligned}$$

- Under a K-factor model for returns:

$$r_{i,t}^e = a_i + b_{i,1} f_{1,t} + ... + b_{i,K} f_{K,t} + u_{i,t},$$

  - The covariance between two assets i and j, assuming the two factors are uncorrelated:

$$cov(r_i^e, r_j^e) = b_{i,1}\, b_{j,1} var(f_1) + ... + b_{i,K}\, b_{j,K} var(f_2)$$

  - The variance of asset i can be decomposed

$$var(r_i^e) = b_{i,1}^2 var(f_1) + ... + b_{i,2}^2 var(f_2) + var(u)$$

  - The systematic variance equals

$$\sigma_{sys,i}^2 = b_{i,1}^2 var(f_1) + ... + b_{i,2}^2 var(f_2)$$

  - the R^2 equals $\sigma_{sys,i}^2 / var(r_i^e)$

- Under the APT with k-factors
  - Return generating process:

$$\tilde{r}_{i,t} = E(\tilde{r}_{i,t}) + b_{i,1} \tilde{f}_{1,t} + b_{i,2} \tilde{f}_{2,t} + \cdots + b_{i,k} \tilde{f}_{k,t} + \tilde{\epsilon}_{i,t}$$

  - The APT pricing equation:

$$E(r_i) = \lambda_0 + \lambda_1 \cdot b_{i,1} + \lambda_2 \cdot b_{i,2} + \cdots + \lambda_k \cdot b_{i,k}$$

- Managed Fund Performance Measures:
  1. The **Sharpe Measure** of Portfolio p:

$$S_p = \frac{r_p^e}{\sigma_p}$$

  2. The **Jensen Measure** is the α_p from the regression:

$$r_{p,t}^e = \alpha_p + \beta_p \cdot r_{m,t}^e + \epsilon_{p,t} \text{ (CAPM)}$$

$$r_{p,t}^e = \alpha_p + b_{1,p} \cdot r_{1,t}^e + b_{2,p} \cdot r_{2,t}^e + \cdots + b_{k,p} \cdot r_{k,t}^e + e_{p,t} \text{ (APT)}$$

     - Where $r_{k,t}^e$ is the excess return on the k'th factor mimicking portfolio.
  3. The **Appraisal Ratio** of portfolio p (CAPM and APT):

$$AR_p = \frac{\alpha_p}{\sigma(\epsilon_p)}$$

     - Sharpe Ratio of optimal portfolio C of *mkt* and p is: $SR_C = \sqrt{SR_m^2 + AR_p^2}$

Table 1: Legend

$\sigma_{i,j}$	covariance between i and j
$\sigma_{i,i} = \sigma_i^2$	variance of i
β_i	market beta of security i
r_f	return on riskless asset
r_i^e	excess return of security i
$\overline{r_i}$	expectation of r_i
w_i^a	weight place on security i in portfolio a