

Adverse selection, slow-moving capital, and misallocation[☆]William Fuchs^a, Brett Green^{a,*}, Dimitris Papanikolaou^b^a University of California, Berkeley, Haas School of Business, 545 Student Services #1900, Berkeley, CA 94720-1900, USA^b Northwestern University, Kellogg School of Management, 2100 Sheridan Road, Evanston, IL 60208, USA

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ABSTRACT

We embed adverse selection into a dynamic, general equilibrium model with heterogeneous capital and study its implications for aggregate dynamics. The friction leads to delays in firms' divestment decisions and thus slow recoveries from shocks, even when these shocks do not affect the economy's potential output. The impediments to reallocation increase with the dispersion in productivity and decrease with the interest rate, the frequency of sectoral shocks, and households' consumption smoothing motives. When households are risk averse, delaying reallocation serves as a hedge against future shocks, which can lead to persistent misallocation. Our model also provides a micro-foundation for convex adjustment costs and a link between the nature of these costs and the underlying economic environment.

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1. Introduction

To maximize output, resources need to be deployed efficiently. Changes in the economic environment, for instance, due to productivity shocks, often require the reallocation of resources across firms to maximize efficiency. Markets serve as the natural mechanism for reallocation. However, markets sometimes fail to function properly. For example, a firm may delay divestment of capital until it is able to recover its fair market value. In this paper, we propose a theory to explain slow movements in capital flows based on adverse selection. We then ask how firms' reallocation decisions depend on the economic environment and explore the implications for aggregate quantities.

Our economy features two sectors of production. Firms in both sectors use the same resource: capital. Each sector is subject to productivity shocks and, therefore, the relative productivity of these sectors changes over time, creating a reason for reallocating capital from the less

productive sector to the more productive one.¹ Capital reallocation takes place in a competitive market. Firms in the less productive sector sell their capital to firms in the more productive sector. Capital is heterogeneous in its quality (i.e., profitability), and firms privately observe the quality of the capital they own and operate, leading to an information asymmetry.

Following a productivity shock, if all capital were to trade immediately, the market price should reflect average quality. However, firms that own the most profitable capital units would refuse to trade at this price, causing the market to unravel as in *Akerlof (1970)*. In our dynamic economy, the equilibrium involves delays in capital reallocation. Following a productivity shock, firms in the less productive sector face a trade-off between selling their capital immediately or waiting to sell at a potentially higher price. Naturally, firms are more anxious to sell less profitable capital units. Firms in the more productive sector recognize this and offer lower prices initially. Firms with higher quality capital delay divesting longer to obtain a higher price. These delays in reallocation generate real economic costs, both at the firm level (lower profitability) and in the aggregate (lower output and total factor productivity (TFP)), due to misallocation of resources.

We demonstrate that delays in reallocation increase with the dispersion in capital quality and decrease with the level of interest rates. Increases in the dispersion of capital quality worsen the information asymmetry and, therefore, slow the equilibrium rate of reallocation. A decrease in interest rates lowers firms' cost of waiting for a higher price and thus also slows down reallocation. Thus, our model suggests a potential drawback of expansionary monetary policy. This last prediction is especially relevant in light of the 2007–08 financial crisis. Despite very low interest rates, many markets remained frozen well after the crisis ended.

Another implication of our model is that, when shocks are more persistent, firms reallocate capital more slowly. The intuition is that a firm looking to purchase capital today internalizes the inefficiency associated with selling capital in the future. As a result, they care not only about the quality of capital they buy, but also about its endogenous liquidity. This leads to an illiquidity discount in

capital prices, which in turn influences a firm's decision of when to sell its capital. In equilibrium, the illiquidity discount and the rate of reallocation are jointly determined. Higher quality capital takes longer to be reallocated and is therefore associated with a larger discount. As productivity shocks become more persistent, the discount falls, which increases the incentive for firms to wait for a higher price, thereby resulting in more delay in the reallocation process.

The baseline model features risk-neutral households. Thus, the interest rate is equal to the subjective discount rate. We introduce households with constant relative risk aversion (CRRA) utility to explore how our results extend to the case in which the stochastic discount factor varies endogenously over time. We obtain several new insights resulting from general equilibrium effects. First, households' desire to smooth consumption increases firms' cost of delay and translates into faster reallocation. Second, the model predicts that large downturns are followed by fast recoveries, whereas smaller downturns are followed by slower recoveries. Both of these predictions are in contrast to the predictions of models with convex adjustment costs. Third, when shocks are recurring, there is a motive for diversification. Interest rates adjust so that some firms choose to continue to operate capital in the inefficient sector indefinitely. As a result, the reallocation process stops even though some capital remains inefficiently allocated, leading to long-run persistence in misallocation.

We offer supporting evidence consistent with our theory. Because the model's predictions pertain to unobservable characteristics, implementing a direct test of the mechanism is inherently challenging.² To do so, we focus on the change in ownership from entrepreneurs to investors following a firm's initial public offering (IPO). Our model has two clear predictions. The first prediction is that owners of high-quality firms wait longer before selling. Thus, the length of time that has elapsed between a firm's incorporation and its IPO should be positively related to post-IPO measures of its profitability, after controlling for observable characteristics at the time of the IPO. The second prediction is that, because the equilibrium is fully separating, prices at the time of the IPO should incorporate all the information contained in the timing decision. Both predictions are supported by the data. We find that the age of the firm at the time of the IPO is strongly related to post-IPO measures of profitability. By contrast, we find no corresponding relation between firm age at IPO and subsequent changes in firm valuations, suggesting that these post-IPO increases in profitability are not news to investors.

The equilibrium dynamics of our model resemble those in models with convex adjustment costs. Depending on the degree of complementarity between capital quality and sectoral productivity, our model can generate aggregate

¹ The model is sufficiently flexible to admit multiple interpretations. Capital can represent physical capital, human capital (workers), or existing matches between physical and human capital, such as a division of a firm, whose productivity cannot be verified or contracted upon. Sectors in our model can be interpreted as industries, physical locations or firms. Productivity shocks can represent changes in the terms of trade, preferences, or technological progress. The exact mapping between the model and the real world depends on how the above terms are interpreted. For example, equipment used for construction during the real estate boom was put to use in the shale gas industry after 2008. As oil prices drop and real estate prices recover, machinery changes hands from oil prospectors back to real estate developers. Matches of workers and physical capital could also move together as firms or divisions are sold. As battery technology improves, both physical and human capital used by firms manufacturing gasoline-powered cars is reallocated to firms making electric vehicles. Similarly, there are many job-to-job transitions in the labor market. As social networking sites attract more users, programmers and entrepreneurs move from developing e-commerce websites to those focused on social networking.

² The reallocation decision of firms in our model operates based on unobservable characteristics. Absent this distinction, some of the model's predictions can appear to run counter to what intuition would suggest. For example, one might expect that, in contrast to our model, higher types should reallocate faster than lower types. However, this intuition refers to observable characteristics. If higher types can receive a higher price regardless of the timing of their reallocation decision, then they will naturally move more quickly.

dynamics that span a number of existing adjustment cost specifications used in macroeconomics; for instance, costs that penalize changes in either the level or the growth rate of reallocation. Our paper also relates to work that develops structural models of the firm.³ Many of these models also feature convex costs of adjusting durable factors of production: physical capital, labor, or intangible capital (see, for instance, Zhang, 2005; Eisfeldt and Papanikolaou, 2013; Belo, Lin, and Bazdresch, 2014). Our model can, therefore, provide a micro-foundation for these convex adjustment cost specifications. For instance, a common feature in many models is asymmetric adjustment costs, in which disinvestment is more costly than new investment (e.g., Zhang, 2005). Because divesting capital involves a sale of used assets, adverse selection is likely a major consideration. In addition, some models feature stochastic adjustment costs to either capital or labor (see, e.g., Eisfeldt and Rampini, 2006; Belo, Lin, and Bazdresch, 2014). Our work can help interpret the time variation in these costs as endogenous responses to the economic environment.

More broadly, our work is related to the voluminous literature studying the effect of information asymmetries on the market for capital. In financial economics, models with adverse selection are commonly used to study the sale of claims on firms' capital (see, for instance, Leland and Pyle, 1977; Myers and Majluf, 1984; Brennan and Kraus, 1987; Lucas and McDonald, 1990; Korajczyk, Lucas, and McDonald, 1991; Eisfeldt, 2004; Kurlat, 2013; Bigio, 2015; Li and Whited, 2015). Kurlat (2013) studies a setting in which entrepreneurs have private information about their projects. He shows that this is mathematically equivalent to a tax on capital, which leads to an amplification mechanism in response to aggregate shocks. Bigio (2015) incorporates a labor market in a related setting. Our work differs these studies by focusing on how the information friction alone can generate persistence in aggregate dynamics. Hence, in contrast to these two papers, the duration of the information asymmetry is endogenously determined in our model.

Li and Whited (2015) also explore capital reallocation in a setting where capital quality is persistent. In their model, the adverse selection problem is more severe than in our setting because firms looking to purchase used capital cannot observe from where it is being reallocated and hence there is no scope for signaling through delay. Therefore, in any equilibrium, the lowest quality capital is traded in every period. In contrast to our fully revealing equilibrium, they obtain multiple pooling equilibria. They then show how the endogenous wedge between new and used capital prices varies over the business cycle, which effectively leads to a countercyclical adjustment costs.

The fact that adverse selection can generate delays in trade between buyers and sellers is well understood within the dynamic adverse selection literature.⁴ Our baseline model builds on Fuchs and Skrzypacz (2013), who study the costs and benefits of temporarily closing the market.

Our contribution to this literature is to embed dynamic adverse selection into a production economy and study how general equilibrium forces affect the endogenous cost of reallocation and the resultant equilibrium dynamics. We show that the cost of reallocation rises as the interest rate drops, dispersion in quality increases, and productivity shocks become more permanent. Furthermore, stronger consumption smoothing motives speed up the rate of reallocation, but higher risk aversion can cause it to cease entirely. Our findings complement those of Guerrieri and Shimer (2014) and Chang (2014), who consider economies in which many markets exist simultaneously and firms with higher quality assets trade at higher prices but with lower probability. Our approach differs from theirs in that we analyze an economy with production and focus on aggregate dynamics in response to sectoral shocks.

Our work also connects to the literature studying cross-country differences in income levels. The current consensus is that these income differences are mostly accounted for by differences in TFP (Caselli, 2005). Further, these differences in TFP have been linked to differences in the degree of resource misallocation across countries. For instance, Hsieh and Klenow (2009) compare the dispersion in productivity between the US, India, and China and find substantially lower dispersion in the US. Using a fairly general model, they argue that if the dispersion in TFP in India and China were equal to US levels, TFP would be 30–60% higher. Part of this persistence in misallocation has been linked to the failure of markets (see, e.g., Banerjee and Duflo, 2005). In the absence of institutions that can serve to screen or verify the quality of resources being reallocated, the information asymmetry between buyers and sellers will increase. In our model, this manifests as a drop in average total factor productivity and the level of output.

Several papers provide empirical evidence showing a significant role for adverse selection in a variety of economic environments. Gibbons and Katz (1991) focus on the labor market, Lizzeri and Hendel (1999) study the market for durable goods, Finkelstein and Poterba (2004) examine the market for insurance, and Michaeli and Shaw (1994) focus on financial markets. Clearly, however, adverse selection is not the only mechanism that can generate disruptions in the efficient allocation of resources. The existing literature is rich with alternative theories; physical (convex and non-convex) costs, search, collateral constraints, learning, time-to-build, and other factors are likely to be important components in the allocation of new and existing capital. Indeed, one benefit of specifying an exogenous cost function is that it can embed multiple considerations. In contrast, by focusing on a particular friction, we are able to examine how these costs vary endogenously and interact with the economic environment.

The remainder of the paper is organized as follows. In Section 2, we illustrate how adverse selection generates slow movements in capital across sectors. In Section 3, we embed the mechanism into a stationary, general equilibrium model. Section 4 analyzes equilibrium of the model with risk-neutral households and studies the aggregate dynamics in response to transitory shocks as well as impulse responses to structural shifts. Section 5 extends our results to a setting with risk-averse households. Section 6

³ For summaries of recent work, see Strebulavev and Whited (2011); Kogan and Papanikolaou (2012).

⁴ See, for example, Janssen and Roy (2002), Hörner and Vieille (2009), Daley and Green (2012 forthcoming), and Fuchs, Öry, and Skrzypacz (2014).

demonstrates the connection of our theory to models with convex adjustment costs. Section 7 presents an empirical test and evidence that is consistent with our theory. Section 8 concludes. Proofs are in Appendix A.

2. Motivating example

To illustrate the main ideas in the paper, we start with an example. Consider an economy with two productive sectors, $i \in \{A, B\}$. Households are risk neutral and have an infinite horizon. The interest rate is fixed at r . There is a mass $M > 1$ firms in each sector. Firms cannot migrate across sectors. Firms maximize total discounted profits, which includes the purchase or sale of any capital.

There is a unit mass of capital. Capital is heterogeneous in its productivity, also referred to as *quality* or *type* and denoted by θ , which is distributed according to a uniform distribution with support $\Theta = [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}_{++}$. Output of the capital stock depends on sector productivity z_i and capital quality. Quality is observable only to the firm that owns and operates the capital. If the firm does not have any capital, it remains idle and produces zero output. For simplicity, we assume here that capital does not depreciate and there is no inflow of investment (the model in Section 3 incorporates such features).

A unit of capital of quality θ (θ -unit), operated by a firm in sector i , generates a flow of output per unit time equal to

$$\pi_i(\theta) = (\beta\theta^\alpha + (1 - \beta)z_i^\alpha)^{\frac{1}{\alpha}}, \quad (1)$$

where β captures the importance of capital quality in production and $(1 - \alpha)^{-1}$ represents the elasticity of substitution between capital quality and sector productivity.

We are interested in the process by which capital is reallocated from sector A to sector B. Therefore, assume that, at $t = 0$, all capital is allocated to firms in sector A and that sector productivity is higher in sector B, $z_B > z_A$, perhaps due to a demand shock or recent technological innovation in sector B. Prior to analyzing the role of adverse selection, we establish the frictionless benchmark. We then illustrate the key aspects of our mechanism and how adverse selection can endogenously generate reallocation costs. Finally, we compare our predictions to a model with exogenously specified costs to reallocating capital.

Remark 1. The model can alternatively be interpreted as being about the (re)allocation of human capital. Relabel “capital” as “workers,” “quality” as “ability,” and “prices” as “wages.” Sectors can be industries, locations, or firms. Workers are privately informed of their ability. Rather than being the firm’s decision of when to sell its capital, it becomes the worker’s decision of when to migrate to sector B. Firms in sector B do not observe workers ability but compete for workers from sector A through the timing and the wage they offer.

2.1. Benchmark: frictionless environment

In the absence of any reallocation costs, a social planner immediately reallocates all capital from sector A to sector B. In a decentralized economy, the same outcome obtains

without the information friction. To see this, suppose that θ is perfectly observable and, therefore, prices can be conditioned on capital quality θ . At any time, a sector B firm is willing to pay up to $\pi_B(\theta)/r$ to buy a θ -unit of capital. Because capital is scarce and sector B firms are identical and competitive, the price for a θ -unit gets bid up to exactly this amount. Each sector A firm sells at $t = 0$ at a price equal to the present value of the output the capital generates in sector B. Because no informational friction exists, all capital is immediately and efficiently reallocated.

2.2. Heterogeneous capital and adverse selection

Next, we introduce the informational friction: Capital is heterogeneous in its quality ($\underline{\theta} < \bar{\theta}$), which is privately observed by the firm that owns it. We study the competitive equilibrium of the decentralized economy in which reallocation decisions are made by firms. For a unit of capital to be reallocated, a transaction must take place. A firm in sector B must purchase the capital from a firm in sector A. This occurs in a dynamic marketplace. At every $t \geq 0$ a firm in sector A that wishes to sell its unit of capital can trade with firms in sector B that wish to purchase capital. There are no institutional frictions in the market (e.g., transactions costs or search). The only friction is an informational one. That is, buyers cannot observe the quality of capital in the market prior to purchasing it (or, alternatively, it is too costly to do so). Therefore, sector B firms face a potential adverse selection problem in the market for capital. We restrict attention to the case in which the adverse selection friction is binding, $\pi_A(\bar{\theta}) > \int \pi_B(\theta)dF(\theta)$; that is, the case in which a firm with the highest quality capital in sector A would prefer to retain its capital rather than trade at the average value to firms in sector B.

A competitive equilibrium of this environment can be characterized by a path of prices P_t and the time at which each unit of capital is reallocated, denoted by $\tau(\theta)$.⁵ We formalize our notion of equilibrium in Section 3 (see Definition 1). Roughly, it requires that, (i) given the path of prices, sector A firms with capital choose the optimal time to trade, (ii) firms in sector B make zero expected profits, and (iii) the market for capital clears.

Because quality is unobservable, prices cannot be conditioned on θ . Hence, instantaneous reallocation cannot be part of an equilibrium. To see why, suppose that all sector A firms sell their capital at $t = 0$. For sector B firms to break even requires that $P_0 = \frac{1}{r} \int \pi_B(\theta)dF(\theta)$. But given this price, a firm with capital of quality $\bar{\theta}$ in sector A would prefer to retain her capital. An alternative conjecture is that all firms with capital quality below some threshold trade at $t = 0$. In this case, the remaining capital in sector A is of discretely higher quality and the equilibrium price would jump upward. Clearly then, firms that sold capital at $t = 0$ did so suboptimally. Hence, we have ruled out the possibility that a mass of reallocation occurs at date zero.

Next, we construct an equilibrium in which the time at which capital is reallocated reveals its quality. Firms trade

⁵ We allow for the possibility that certain types of capital are never reallocated, in which case $\tau(\theta) = \infty$.

off the immediate gains from reallocation versus preserving the option to sell it in the future. Firms with lower-quality capital are effectively more anxious to sell, because their capital is less productive, and do so sooner than firms with higher-quality capital. Because higher-quality capital gets reallocated later, the market price of capital rises over time.

To construct this equilibrium, let χ_t denote the quality of capital that is reallocated at date t . For sector B firms to break even, it must be that

$$P_t = \frac{\pi_B(\chi_t)}{r}. \quad (2)$$

For this to be an optimal strategy, the firm that owns a χ_t -unit of capital must be locally indifferent between trading immediately or waiting an instant for a higher price:

$$\underbrace{rP_t - \pi_A(\chi_t)}_{\text{Cost of delay}} = \underbrace{\frac{d}{dt}P(t)}_{\text{Benefit}}. \quad (3)$$

The left-hand side of Eq. (3) corresponds to the cost that a firm with a χ_t -unit in sector A gives up by delaying trade. Using Eq. (2), the right-hand side can be rewritten as

$$\frac{d}{dt}P_t = \frac{\pi'_B(\chi_t)}{r} \dot{\chi}_t, \quad (4)$$

where $\dot{\chi}_t = \frac{d\chi_t}{dt}$ represents the rate of skimming.⁶ Because the distribution over types is uniform, $\dot{\chi}_t$ is proportional to the rate at which capital is reallocated to the more productive sector. Combining Eqs. (3) and (4) yields

$$\dot{\chi}_t = \frac{r(\pi_B(\chi_t) - \pi_A(\chi_t))}{\pi'_B(\chi_t)}. \quad (5)$$

This differential equation characterizes the equilibrium rate at which capital transitions to sector B . It is based on the first two equilibrium requirements: that sector A firms optimize their selling decisions and that sector B firms break even. One immediate observation from Eq. (5) is that the rate of reallocation is proportional to the gains from doing so (the productivity differential $\pi_B - \pi_A$), relative to a firms' benefit of delaying to get a higher price (the sensitivity of price to capital quality π'_B). Furthermore, the rate of reallocation is proportional to the interest rate. The larger is r , the more costly it is for firms to delay reallocation, and hence, the faster it occurs.

The boundary condition is pinned down by the market clearing condition, which requires the price at time zero to be at least $\pi_B(\underline{\theta})/r$. This implies that the lowest-quality capital trades immediately:

$$\chi_0 = \underline{\theta}. \quad (6)$$

For any set of production technologies $\{\pi_A, \pi_B\}$, Eqs. (5) and (6) pin down the equilibrium reallocation dynamics.

⁶ In a discrete-time analog of our setting where trade can occur at times $t = \{0, \Delta, 2\Delta, \dots\}$, an interval of types trade at each t . The equilibrium can be characterized by a sequence of increasing cutoff types $\{\chi_1, \chi_2, \dots\}$ such that all types $\theta \in (\chi_t, \chi_{t+1})$ trade at time t (where $\chi_0 = \underline{\theta}$). One can then interpret $\dot{\chi}_t$ as $\lim_{\Delta \rightarrow 0} \frac{\chi_{t+1} - \chi_t}{\Delta}$.

In sum, adverse selection inhibits the reallocation of capital, resulting in a slow transition of resources to the more productive sector. The equilibrium dynamics depend, in part, on the production technology and specifically, on the elasticity of substitution between capital quality and productivity. Using our constant elasticity of substitution (CES) formulation, we focus on three values for this elasticity, $\alpha \in \{0, 1, 2\}$.

First, as $\alpha \rightarrow 0$, the production technology tends to a Cobb–Douglas production function. In this case, the gains from reallocation are increasing with quality. Eq. (5) becomes

$$\dot{\chi}_t = \kappa \chi_t, \quad (7)$$

where $\kappa = (1 - (\frac{z_A}{z_B})^\beta)(1 - \beta)^{-1}r$. Combining with Eq. (6), the solution is given by

$$\chi_t = \underline{\theta} e^{\kappa t}, \quad (8)$$

where the above holds for $t \leq \tau(\bar{\theta})$, where $\tau \equiv \chi^{-1}$. Hence, the equilibrium reallocation rate is increasing over time until $\tau(\bar{\theta})$, at which point, all capital has been reallocated to sector B and the transition dynamics terminate.

Second, if $\alpha = 1$, the production technology is linear and hence, there are constant gains from reallocation. For $t < \tau(\bar{\theta})$, Eq. (5) becomes

$$\dot{\chi}_t = \left(\frac{1 - \beta}{\beta} \right) (z_B - z_A)r. \quad (9)$$

Because the right-hand side is a constant, the equilibrium reallocation rate is constant over time. Combining with Eq. (6), the solution is given by

$$\chi_t = \underline{\theta} + \left(\frac{1 - \beta}{\beta} \right) (z_B - z_A)rt. \quad (10)$$

For the case in which $\alpha = 2$, the differential equation does not admit an analytic solution. However, it is straightforward to compute it numerically. Moreover, it is easy to show that the equilibrium rate in this case decreases over time (see Proposition 1).

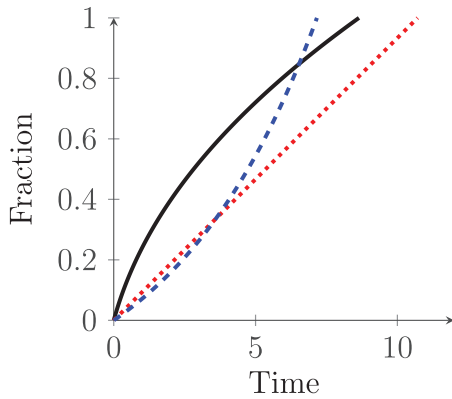
We plot the implied reallocation dynamics in Fig. 1. In all three cases, the quality and total fraction of capital that is reallocated increases over time. These properties are true regardless of the production technology. Panel B shows that the qualitative features of the equilibrium reallocation rate depend on the elasticity of substitution between factors. With constant gains from trade ($\alpha = 1$), the rate of reallocation is constant and the change in capital stock is linear over time. By contrast, the case of decreasing gains from trade ($\alpha = 2$) generates strictly concave dynamics for the capital stock, whereas in the case with increasing gains from trade ($\alpha = 0$) the model generates a convex path for the capital stock.

Proposition 1 formalizes the findings illustrated in Fig. 1.

Proposition 1. *Until all capital has been reallocated to the efficient sector,*

- If $\alpha < 1$, the equilibrium rate of reallocation is strictly increasing over time,
- If $\alpha = 1$, the equilibrium rate of reallocation is constant over time, and

Panel A: Fraction of capital reallocated



Panel B: Rate of capital reallocation

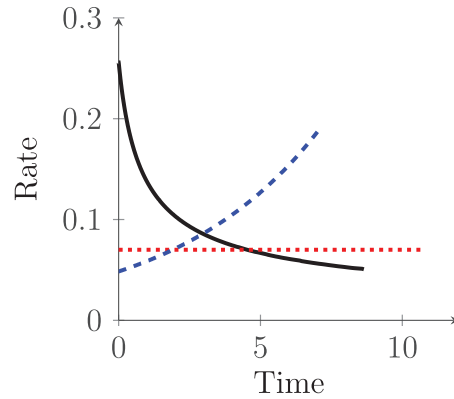


Fig. 1. Equilibrium reallocation with CES production technology for $\alpha = 0$ (dashed line), $\alpha = 1$ (solid line), and $\alpha = 2$ (dotted line). Panel A illustrates the capital quality that switches at time t , and Panel B illustrates the rate at which capital is reallocated.

- If $\alpha > 1$, the equilibrium rate of reallocation is strictly decreasing over time.

In what follows, we generalize the model outlined in Section 2.2 to allow for general production functions, transitory productivity shocks, and risk-averse households. In Section 6, we relate the dynamics in our model to those in models with convex adjustment costs.

3. Stationary model of capital reallocation

Our motivating example in Section 2 considers a single transitory period as reallocation occurs only once. Here, we allow sectoral productivity to vary stochastically over time. In this case, firms internalize the possibility of costly future reallocation in their decisions. Further, the frequency of these shocks affects the equilibrium prices of capital and, in turn, the reallocation dynamics.

3.1. Technology

Consumption goods are produced using capital. Capital can be located in one of two sectors (A and B). Capital is heterogeneous in its quality, where quality is indexed by θ . Quality is observable only to the owner of the capital unit. Capital quality is distributed according to $F(\theta)$, which is continuous with strictly positive density over the support $\Theta = [\underline{\theta}, \bar{\theta}]$. The flow output of a unit of capital depends on its quality θ , the sector in which it is currently allocated i , and the aggregate state x according to

$$y_t^i(\theta) = \pi_i(\theta, x)dt, \quad (11)$$

where π_i is strictly positive, increasing and twice differentiable in θ , with uniformly bounded first and second derivatives.⁷ We incorporate shocks to the model by allow-

ing the production technology to vary stochastically over time. Specifically, we introduce a Markov switching process $X(\omega) = \{X_t(\omega), 0 \leq t \leq \infty\}$ defined on the underlying probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $X_t(\omega) \in \{x_A, x_B\}$ represents the state of the economy at date t . Henceforth, we omit the argument ω and use a t subscript as a place holder for the argument (t, ω) . Unlike in Section 2, we let capital depreciate and allow new capital to flow into the most productive sector of the economy. Both occur at rate δ . The quality of capital inflows are also distributed according to F . The main purpose of capital inflows is to ensure that for any history of shocks, upon arrival of the next shock, the distribution of capital always has full support in the sector from which it is being reallocated.

3.2. Markets, information and prices

Reallocation of capital occurs in a competitive market. This market is open continuously at all $t \geq 0$. All firms observe the path of the exogenous state variable $X = \{X_s, 0 \leq s \leq \infty\}$. We let $\{\mathcal{F}_t\}_{t \geq 0}$ denote the filtration encoding the information observed by all firms prior to date t . In addition, a firm that currently owns a unit of capital privately observes its quality. The quality of each unit of capital is unobservable to all other firms. However, firms can observe to which sector the capital is currently allocated. For this reason, at each time t , there are two prices in the market—one for capital currently located in sector A , denoted by P_t^A , and one for capital currently located in sector B , denoted by P_t^B .

Financial markets are complete with respect to the underlying probability space. In equilibrium, a complete financial market can be implemented with a risk-free asset and a market index. The state-price density ξ (the price of Arrow-Debreu securities per unit of probability) evolves according to

$$\frac{d\xi_t}{\xi_t} = -r_t dt - \psi_t d\tilde{X}_t, \quad (12)$$

⁷ Formally, there exists m, M such that $0 < m < M < \infty$ and $\frac{\partial}{\partial q} \pi_i, \frac{\partial^2}{\partial q^2} \pi_i \in (m, M)$ for all (i, q, x) . Without imposing any structure on the distribution of capital quality, it is without loss to normalize $\pi_A(\theta, x_A) = \theta$. We have not done so here because at various points we put additional structure on the production technology.

where r_t is the risk-free rate of return and ψ_t is the price of risk associated with unexpected changes in the aggregate state ($d\tilde{X}_t \equiv dX_t - E_t[dX_t]$). We require that the state-price density satisfy the transversality condition that $\lim_{t \rightarrow \infty} \xi_t = 0$. Following convention, we refer to the growth in the state-price density as the stochastic discount factor (SDF).

3.3. Firms

A mass $M > 1$ of competitive firms is located in each sector. Firms maximize their market value by undertaking a capital allocation decision. Consider a sector i firm that purchases a unit of capital at date t . Upon doing so, the firm observes the capital quality, θ , and operates the capital until it is no longer optimal to do so. The decision facing the firm is when to reallocate (i.e., sell) its existing capital. Let $V_t^i(\theta)$ denote the firm's value for the unit of capital. Given an \mathcal{F}_t -adapt price process P_t^i , the firm's problem can be written as

$$V_t^i(\theta) = \sup_{\tau \geq t} E_t \left[\frac{1}{\xi_t} \int_t^\tau e^{-\delta(s-t)} \xi_s \pi_i(\theta, X_s) ds + e^{-\delta(\tau-t)} \xi_\tau P_\tau^i \right]. \quad (13)$$

3.4. Households

There exist a continuum of identical households, indexed by $h \in [0, 1]$. A household's problem is to choose a consumption process, $c^h = \{c_t^h : 0 \leq t \leq \infty\}$, that maximizes its lifetime utility,

$$\sup_c E_0 \left[\int_0^\infty e^{-\beta t} u(c_t) dt \right], \quad (14)$$

subject to the budget constraint,

$$w_0 \geq E_0 \left[\int_0^\infty \xi_t c_t dt \right]. \quad (15)$$

Here, $\beta > 0$ is the household's subjective discount rate and w_0 is the value of its initial endowment. We assume that u is a smooth, weakly concave function. We focus on the case of risk-neutral households in Section 4. In Section 5, we incorporate risk aversion.

3.5. Equilibrium concept

To define an equilibrium of the economy, we need the following notation and definitions. Aggregate consumption is denoted by $C_t = \int c_t^h dh$. We use $T_t^i(\theta)$ to denote the policy of a firm in sector i who acquires a unit of capital of quality θ at time t . The policy is admissible if it is both adapted to the filtration $\{\mathcal{F}_s\}_{s \geq 0}$ and weakly larger than t . Let $\Theta_t^i \equiv \{\theta : T_s^i(\theta) = t, s \leq t\}$ denote the set of capital qualities sold at date t from sector i . Finally, F_t^i denotes the distribution of capital quality, and $\theta_t^i \equiv \inf\{\theta : T_s^i(\theta) \geq t, s \leq t\}$ denotes the lowest quality of capital allocated to sector i at date t .

Definition 1. A competitive equilibrium consists of admissible policies, $T_t^i(\theta) : \Omega \rightarrow \mathbb{R}_+$ and \mathcal{F} -adapted consumption,

price and state density processes $c^h, P^i, \xi : [0, \infty] \times \Omega \rightarrow \mathbb{R}$ such that for each $i \in \{A, B\}$, $t \geq 0$, $\theta \in \Theta$, $j \neq i$, and $h \in [0, 1]$:

1. Firm's capital allocation decisions are optimal: $T_t^i(\theta)$ solves Eq. (13).
2. Household's consumption decisions are optimal: c^h solves Eq. (14) subject to Eq. (15).
3. The market for the consumption good clears: $C_t = Y_t \equiv \sum_i \int \pi_i(\theta, x) dF_t^i(\theta)$.
4. The market for capital clears: If $\Theta_t^i = \emptyset$, $P_t^i \geq \inf\{V_t^j(\theta) : \theta \geq \theta_t^j\}$, and
5. New firms make zero profit: If $\Theta_t^i \neq \emptyset$ then $P_t^i = \mathbb{E}[V_t^j(\theta) | \theta \in \Theta_t^i, \mathcal{F}_t]$.

Conditions 1–3 are straightforward. Condition 4 requires that the price for a unit of capital in sector i cannot be less than the lowest possible value for that unit of capital in sector j . If the price was strictly less, then all firms in sector j would demand capital at that price and demand would exceed supply. Besides having a natural economic interpretation, this condition rules out trivial candidate equilibria, such as one in which prices are always very low and trade never takes place. Condition 5 is motivated by free entry and says that the price of capital at time t must be equal to the expected value of the reallocated capital at time t , which implies that a firm purchasing a unit of capital cannot make positive (or negative) expected profits.

We first establish several standard, but useful, properties.

Lemma 1. In any competitive equilibrium, the state price density is proportional to the household's discounted marginal utility of consumption. We normalize the initial endowment so that $\xi_t = e^{-\beta t} u'(C_t)$.

In addition, the *skimming property* must hold. That is, lower-quality capital is reallocated faster than higher quality capital.

Lemma 2 (skimming). In any competitive equilibrium, the amount of time a firm in the inefficient sector delays reallocation following a productivity shift is weakly increasing in θ .

The intuition is the same as in the motivating example. For a given price offer, firms with lower-quality capital are more anxious to sell their capital, because their outside option to wait is less valuable due to lower output in the interim.

For both tractability and ease of exposition, we conduct our analysis within the class of *symmetric economies*. In a symmetric economy, the output of a firm depends only on the quality of its capital and whether that capital is allocated efficiently (i.e., to the more productive sector given the current state).

Definition 2 (symmetric economies). The economy is symmetric if there exists a pair of functions $\{\bar{\pi}, \underline{\pi}\}$ and scalar λ such that $\pi_i(\theta, x_i) = \bar{\pi}(\theta)$ for $i \in \{A, B\}$, $\pi_i(\theta, x_j) = \underline{\pi}(\theta)$, and $\lambda_{ij} = \lambda$ for $i \neq j$.

In Section 4, we restrict attention to symmetric economies. It is straightforward, though more notationally cumbersome, to extend results to a setting in which the economy is not symmetric. A symmetric economy is fully described by $\Gamma \equiv \{\underline{\pi}, \bar{\pi}, u, \beta, \delta, \lambda, F\}$. We refer to the production technology as a pair of functions $\{\underline{\pi}, \bar{\pi}\}$ each mapping Θ to \mathbb{R} . Unless otherwise stated, we assume there is no ambiguity in which sector is most efficient.

Assumption 1 (gains from trade). The production technology satisfies $\underline{\pi}(\theta) < \bar{\pi}(\theta)$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

This assumptions ensures that the market for capital does not completely break down (see Remark 2). We refer to the efficient sector at any given time t as the sector in which output is given by $\bar{\pi}$ at date t (i.e., i such that $X_t = x_i$).

4. Equilibrium with risk-neutral households

We begin by focusing on the setting with risk-neutral households, $u(c) = c$. In this case, $\xi_t = e^{-\beta t}$ (Lemma 1) and the short-term interest rate is simply equal to household's impatience, $r_t = \beta$. With the state-price density pinned down, the natural extension of the equilibrium from Section 2 can be characterized by two functions. The first is $\tau(\theta)$, which represents how long it takes a θ -unit of capital to be reallocated following a productivity shock (and provided that no other shocks arrive in the interim). The second is $\bar{V}(\theta)$, which is the (endogenous) value of an efficiently allocated unit of capital of quality θ . As in Section 2, we construct a *fully revealing equilibrium*, which requires that τ is strictly increasing in θ . Here again, it is sometimes easier to use the inverse of τ , which we denote by $\chi_t \equiv \tau^{-1}(t)$, which represents the quality of capital type that is reallocated a period of length t after the most recent shock.

To formalize the connection to the equilibrium objects in Definition 1, let $m_t \equiv t - \sup\{s \leq t : x_{s+} \neq x_{s-}\}$ denote the amount of time that has elapsed since the last shock arrived.

Definition 3. The firm strategies and capital prices that are consistent with (τ, \bar{V}) are given by

$$T_t^i(\theta) = \inf\{s \geq t : m_s = \tau(\theta), x_s \neq x_i\} \quad (16)$$

$$p_t^i = \begin{cases} \bar{V}(\chi(m_t)) & \text{if } x_t \neq x_i \text{ and } m_t < \tau(\bar{\theta}) \\ \bar{V}(\underline{\theta}) & \text{otherwise} \end{cases} \quad (17)$$

The main result of this section is stated in Theorem 1.

Theorem 1. In a symmetric economy with risk-neutral households and strict gains from trade, a unique (τ^*, V^*) exists such that the firm strategies and capital prices consistent with (τ^*, V^*) are part of a fully revealing competitive equilibrium.

To sketch the argument, we proceed with a heuristic construction of the equilibrium based on necessary conditions, which can be reduced to an initial value problem. This initial value problem has a unique solution, which proves that a unique candidate exists. We then verify that these necessary conditions are also sufficient.

According to the candidate equilibrium, the value a firm derives from capital depends only on its quality if it is efficiently allocated. If the unit of capital is inefficiently allocated, its derived value also depends the lowest quality of capital remaining in the inefficient sector (or equivalently, m_t). Let $\underline{V}(\theta, \chi)$ denote the value of an inefficiently allocated θ -unit when the lowest remaining quality of capital in the inefficient sector is $\chi \leq \theta$. According to (τ, \bar{V}) , the firm waits until $\chi = \theta$ to trade. Therefore, the evolution of \underline{V} for $\chi < \theta$ is given by

$$\begin{aligned} \beta \underline{V}(\theta, \chi) &= \underline{\pi}(\theta) - \delta \underline{V}(\theta, \chi) + \lambda (\bar{V}(\theta) - \underline{V}(\theta, \chi)) \\ &+ \frac{\partial}{\partial \chi} \underline{V}(\theta, \chi) \dot{\chi}_t. \end{aligned} \quad (18)$$

When $\theta = \chi$, a firm with a currently misallocated θ -unit of capital sells at a price equal to $\bar{V}(\chi)$. We abuse notation by letting $P(\theta)$ denote the price at which a firm in the inefficient sector sells a θ -unit to a firm in the efficient sector. Hence, a necessary boundary condition for \underline{V} is given by

$$\underline{V}(\theta, \theta) = P(\theta). \quad (19)$$

The (local) optimality condition, required to ensure that firm optimality holds, is that, when $\theta = \chi$, the firm with a θ -unit is just indifferent between selling immediately and waiting an instant. In other words, the firm's value function must smoothly paste to the path of prices

$$P'(\chi) = \frac{\partial}{\partial \chi} \underline{V}(\theta, \chi) \Big|_{\theta=\chi}. \quad (20)$$

For the zero profit condition to hold, the price at which capital transacts must be equal to its value in the efficient sector. This requires that

$$P(\theta) = \bar{V}(\theta). \quad (21)$$

Evaluating Eq. (18) at $\theta = \chi_t$ using Eqs. (19)–(21), we arrive at

$$\dot{\chi}_t = \frac{\rho \bar{V}(\chi_t) - \underline{\pi}(\chi_t)}{\bar{V}'(\chi_t)}, \quad (22)$$

where $\rho = \beta + \delta$, represents the firm's effective discount rate. Eq. (22) is analogous to Eq. (5), where π_B/r is replaced with \bar{V} . It is also worth noting that the rate at which productivity shocks arrive λ does not enter directly into Eq. (22). This is because the price the firm gets upon selling capital is equal to the value of that capital if another shock were to arrive (in which case the firm would retain possession). Nevertheless, λ does play an important role in determining the equilibrium capital values and prices.

4.1. Equilibrium value of capital

Consider an arbitrary (τ, V) , and note that the value of a unit of inefficiently allocated capital when $\chi = \underline{\theta}$ can be written as

$$\underline{V}(\theta, \underline{\theta}) = f(\tau(\theta)) \frac{\underline{\pi}(\theta)}{\rho} + (1 - f(\tau(\theta))) \bar{V}(\theta), \quad (23)$$

where

$$f(\tau) \equiv \int_0^\tau (1 - e^{-\rho t}) \lambda e^{-\lambda t} dt + e^{-\lambda \tau} (1 - e^{-\rho \tau}) \quad (24)$$

denotes the expected discount factor until either the state switches back or the capital gets reallocated to the other sector. Similarly, the value of an efficiently allocated θ -unit is given by

$$\bar{V}(\theta) = \frac{\rho}{\rho + \lambda} \frac{\bar{\pi}(\theta)}{\rho} + \frac{\lambda}{\rho + \lambda} \underline{V}(\theta, \underline{\theta}). \quad (25)$$

Solving Eqs. (23) and (25) jointly yields

$$\bar{V}(\theta) = g(\tau(\theta)) \frac{\bar{\pi}(\theta)}{\rho} + (1 - g(\tau(\theta))) \frac{\bar{\pi}(\theta)}{\rho}, \quad (26)$$

where $g(\tau) \equiv \frac{\lambda}{\rho + \lambda f(\tau)} f(\tau)$. The expression in Eq. (26) has an intuitive form. Capital spends some fraction of the time allocated efficiently and some fraction of the time misallocated. Therefore, its value is simply a weighted average of the value were it to be permanently efficiently allocated (i.e., $\frac{\bar{\pi}}{\rho}$) and permanently misallocated (i.e., $\frac{\underline{\pi}}{\rho}$). The amount of time it takes to get reallocated is determined by Eq. (22), which in turn depends on \bar{V} . This illuminates the nature of the fixed point. The solution turns out to be tractable. Substituting χ_t for θ into Eq. (26) and then substituting this expression for \bar{V} into Eq. (22) yields

$$\dot{\chi}_t = \frac{\rho(1 - g(t) + \frac{g'(t)}{\rho})(\bar{\pi}(\chi_t) - \underline{\pi}(\chi_t))}{g(t)\underline{\pi}'(\chi_t) + (1 - g(t))\bar{\pi}'(\chi_t)}. \quad (27)$$

As before, the boundary condition is pinned down by the fact that the lowest type must reallocate immediately after the productivity shock and, therefore,

$$\chi_0 = \underline{\theta}. \quad (28)$$

The regularity conditions imposed on $\bar{\pi}$ and $\underline{\pi}$ ensure a unique solution exists and that this solution is monotonically increasing (see Lemma A.1). The last step in the proof of Theorem 1 is to verify that the candidate satisfies the remaining equilibrium conditions. The zero profit condition follows from the fact that capital of quality θ trades at a price of $\bar{V}(\theta)$. Capital market clearing follows immediately from Eq. (17) and the fact that $\bar{V}(\theta)$ is equal to the value derived from a θ -unit. Finally, in Appendix A, we demonstrate that a firm that owns capital does not have a profitable deviation by showing that the Spence-Mirrlees condition holds for firms' objective function, which verifies firm optimality.

Remark 2 (complete market breakdown). Eq. (27) illustrates the importance of having strict gains from reallocating capital (Assumption 1) as it ensures that the numerator is strictly positive and thus χ_t , and hence τ , are strictly increasing. If $\underline{\pi}(\theta) \geq \bar{\pi}(\theta)$ over some interval of Θ , then, in equilibrium, the market for used capital would breakdown completely and the reallocation process would get “stuck”; capital with quality in and above the interval would never be reallocated. Whether the reallocation process from one sector to another is completed in finite time also depends on the gains from trade at the upper end of the distribution; if $\bar{\pi}(\bar{\theta}) > \underline{\pi}(\bar{\theta})$, then all capital gets reallocated in finite time; whereas if $\bar{\pi}(\bar{\theta}) = \underline{\pi}(\bar{\theta})$, then $\tau(\bar{\theta}) = \infty$.

Remark 3 (rate of reallocation). Throughout the paper, we refer to $\dot{\chi}_t$ as both the rate of skimming and the rate of reallocation. In general, the rate of reallocation also depends

on the distribution of capital quality. That is, given the equilibrium rate of skimming through types, χ_t , the rate of capital reallocation from sector i to sector j equals

$$\frac{dk^j(t)}{dt} = \dot{\chi}_t dF^i(\chi_t), \quad (29)$$

where F_t^i is the cumulative distribution of capital quality in sector i at time t . In all figures, F is uniformly distributed.

4.2. Reallocation following a permanent shock

A special case of the model is when the productivity shock is permanent. To study the transition dynamics for this case, let $\lambda = 0$, assume that all capital is originally allocated to sector A , and that the productivity shock occurs at $t = 0$ so that B is the more productive sector for all $t \geq 0$.

This situation is effectively the same as that in Section 2. Because sector B is more productive, capital transitions slowly from A to B due to adverse selection. Because there are no further technological shocks, firms in sector B retain the capital until it fully depreciates. Hence, firms have a value $\bar{\pi}(\theta)/\rho$ for a θ -unit of capital. Thus, in any fully revealing equilibrium, the rate at which a θ -unit of capital is reallocated does not impact the price at which it trades.

Proposition 2. Suppose that the productivity shock is permanent. Then, $g(t) = 0$ for all t and Eq. (27) reduces to

$$\dot{\chi}_t = \rho \frac{\bar{\pi}'(\chi_t) - \underline{\pi}'(\chi_t)}{\bar{\pi}'(\chi_t)}. \quad (30)$$

The expression for $\dot{\chi}$ in Eq. (30) is effectively the same as Eq. (5) in the example. Therefore, the equilibrium analyzed in the case of permanent productivity shocks is precisely the one characterized in Section 2. We revisit it here because it is useful for highlighting the economic environments under which various patterns in the rate of reallocation obtain. The numerator in Eq. (30) measures the magnitude of the productivity gains from reallocation as they depend on the quality of the capital. The larger the benefit of reallocation, the faster it takes place. The denominator measures the marginal productivity of capital quality in the efficient sector. It is perhaps surprising that higher marginal productivity of quality leads to slower reallocation. The intuition for this comes from the indifference condition of the cutoff type. The total change in prices with respect to time is given by

$$\frac{dP_t}{dt} = \frac{\bar{\pi}'(\chi_t)}{\rho} \dot{\chi}_t. \quad (31)$$

Fixing $\dot{\chi}_t$, increasing the marginal productivity of capital quality increases the rate at which prices increase over time. For the cutoff type to remain indifferent, the reallocation rate must decrease. Using Eq. (30) and noting that $\dot{\chi}_t$ is strictly positive results in Proposition 3.

Proposition 3. Suppose that the productivity shock is permanent. Then, the equilibrium rate of reallocation increases (decreases) over time until all capital has been reallocated if and only if $(\bar{\pi} - \underline{\pi})/\bar{\pi}'$ is increasing (decreasing) over $\theta \in \Theta$.

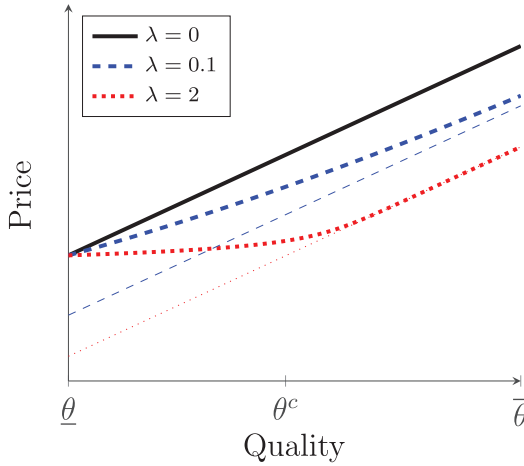


Fig. 2. The effect of transitory shocks on capital prices. The dashed line corresponds to $\lambda = 0.1$ and the dotted line corresponds $\lambda = 2$. The solid line represents the case when the shock is permanent ($\lambda = 0$), which also corresponds to the fully efficient value of capital. The fainter dashed (dotted) line represents the hypothetical value of a unit of capital if it is never reallocated for $\lambda = 0.1$ ($\lambda = 2$).

Increasing the sensitivity of output to quality increases the benefit of mimicking higher types. Therefore, owners of high-quality capital need to delay longer to distinguish from lower types.

4.3. Reallocation with transitory shocks

Here we examine the implications of transitory sectoral productivity shocks. In this case, firms investing in capital today become sellers of capital at some point in the future. Therefore, in considering their willingness to pay for a θ -unit of capital, firms must account for the potential costs associated with reallocation in the future.

We first explore the implications for the market price of capital. When the shock is permanent, the price at which a θ -unit trades is equal to $\bar{\pi}(\theta)/\rho$, which is the present value of the future output that it generates for a firm in the efficient sector. With transitory shocks, this is no longer the case. Instead, the market price of capital includes an endogenous illiquidity discount.

Proposition 4. *If the productivity shock is transitory ($\lambda > 0$), then the price at which a θ -unit of capital trades is strictly less than $\bar{\pi}(\theta)/\rho$ for all $\theta > \underline{\theta}$.*

Fig. 2 plots the equilibrium price at which capital trades as it depends on its quality and λ . As λ increases, the overall value and price of capital decreases. The discount can be measured by the difference between the full information price and the price at which capital sells when firms are privately informed, i.e., $\frac{\bar{\pi}(\theta)}{\rho} - \bar{V}(\theta)$. The size of the discount depends on capital quality. Because higher quality capital takes longer to be reallocated, it is associated with a larger discount.

Next, we turn to the implications for the equilibrium rate of reallocation. We ask how the price discount, driven by the transitory nature of shocks, impacts the reallocation

decision of firms. Because the presence of a discount reduces the gains from trade, intuition would suggest that the rate of reallocation should decrease with λ .⁸ Indeed, this force is at play. However, the intuition is incomplete because there is a second force: The nature of the illiquidity discount also affects sellers' incentives to delay trade. Because the discount is larger for higher types – since their capital is endogenously less liquid – firms with low-quality capital have less incentive to wait for a better price. Effectively, as λ increases, equilibrium prices become less sensitive to capital quality, which mitigates the severity of the adverse selection problem and tends to increase the rate of reallocation. **Proposition 5** formalizes this result.

Proposition 5. *Consider any two symmetric economies Γ_x and Γ_y , which are identical except that $\lambda_x < \lambda_y$. There exists a $\bar{t} > 0$ such that the rate of reallocation is strictly higher in Γ_y than in Γ_x prior to \bar{t} , i.e., $\chi'_y(t) > \chi'_x(t)$ for all $t \in [0, \bar{t}]$.*

The lowest-quality capital is always efficiently allocated, and therefore, $\rho \bar{V}(\underline{\theta}) = \bar{\pi}(\underline{\theta})$ in both Γ_x and Γ_y . Fixing the equilibrium strategies from Γ_x , consider the effect of an increase from λ_x to λ_y . Because the state is now switching more frequently and the rate of reallocation remains unchanged, firms with capital of quality $\theta > \underline{\theta}$ endure more misallocation, which gives them more incentive to imitate the lowest type. By construction, types arbitrarily close to $\underline{\theta}$ were indifferent in Γ_x between accepting $P(\underline{\theta})$ or waiting an instant. Hence, the increase in λ causes these types to strictly prefer to imitate $\underline{\theta}$. To restore the equilibrium in Γ_y , types near $\underline{\theta}$ must trade faster and the reallocation of capital increases, as in **Fig. 3**. For higher θ , the first effect (i.e., the reduction in the gains from reallocating) is larger and the rate of reallocation can decrease (as in Panel A) or increase (as in Panel B).

Fig. 3 also illustrates how the persistence of shocks affects the reallocation dynamics. In particular, more transient shocks tend to decrease the slope of $\dot{\chi}_t$, offsetting the effects of complementarity between quality and productivity α .

4.4. Response to a sectoral productivity shock

Next, we examine the response of aggregate quantities—output and productivity—to a sectoral productivity shock. The output of sector i at time t depends on the current distribution of project quality in that sector:

$$Y_t^i = \int y_t^i(\theta) dF_t^i(\theta), \quad (32)$$

where $y_t^i(\theta)$ denotes the output of a unit of capital of quality q in sector i at time t . Aggregate output is then equal to $Y_t = Y_t^A + Y_t^B$. We compute the average productivity of capital in each sector as

$$X_t^i = \frac{Y_t^i}{k_t^i}. \quad (33)$$

⁸ This is akin to what one could expect in a standard model with exogenous reallocation costs, in which increasing the volatility of sectoral shocks typically leads to more delay as the option value of waiting increases.

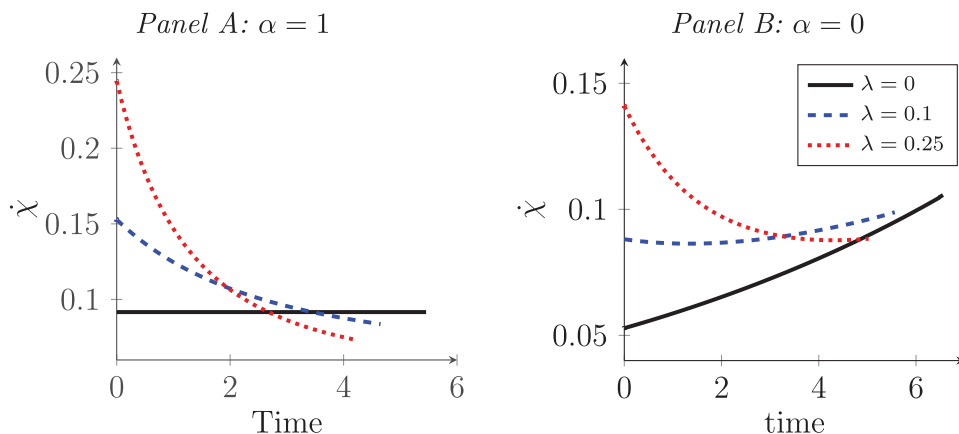


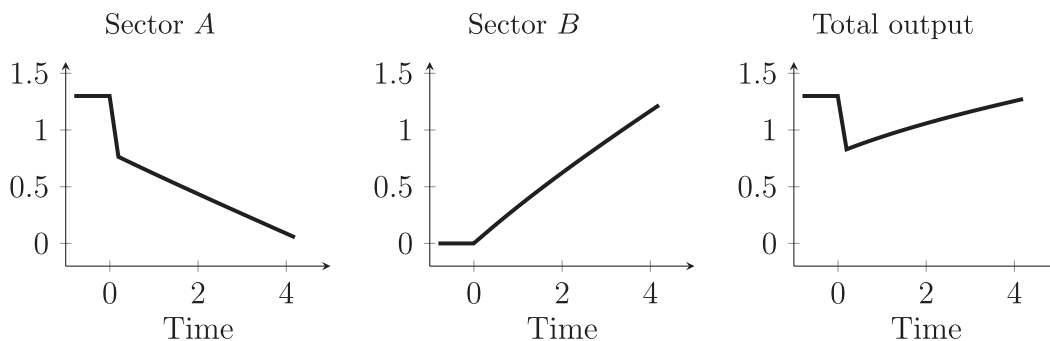
Fig. 3. Equilibrium reallocation with transitory shocks and CES production technology for $\alpha = 1$ (Panel A) and $\alpha = 0$ (Panel B). The other parameters used are $\beta = 0.45$, $r = 0.15$, $z_A = \frac{1}{2}$, $z_B = 1$, and $\Theta = [0.5, 1]$.

Because the quantity of aggregate capital is constant, aggregate productivity is equal to total output, $X_t = Y_t$. We focus on the case in which $\alpha = 1$, and the overall distribution of quality is distributed as a truncated normal on Θ . The results are illustrated in Fig. 4.

Because all capital is initially allocated in sector A, aggregate output falls on impact as shown in Panel A. As

the economy reallocates capital, output in sector A continues to fall while output in sector B rises. Once all capital is reallocated from sector A to sector B, total output is restored to the pre-shock level. The response of output to a sectoral shock is qualitatively similar to that of a model with adjustment costs. However, the behavior of total factor productivity exhibits dynamics that are markedly

Panel A: Output in response to sectoral productivity shock



Panel B: Productivity in response to sectoral productivity shock

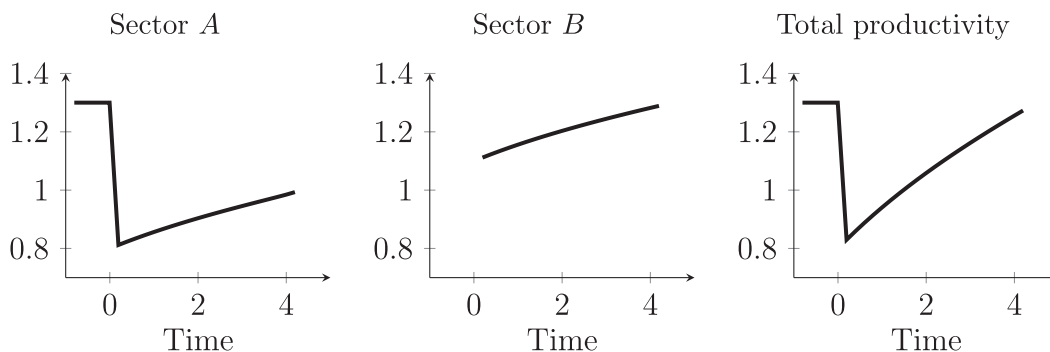
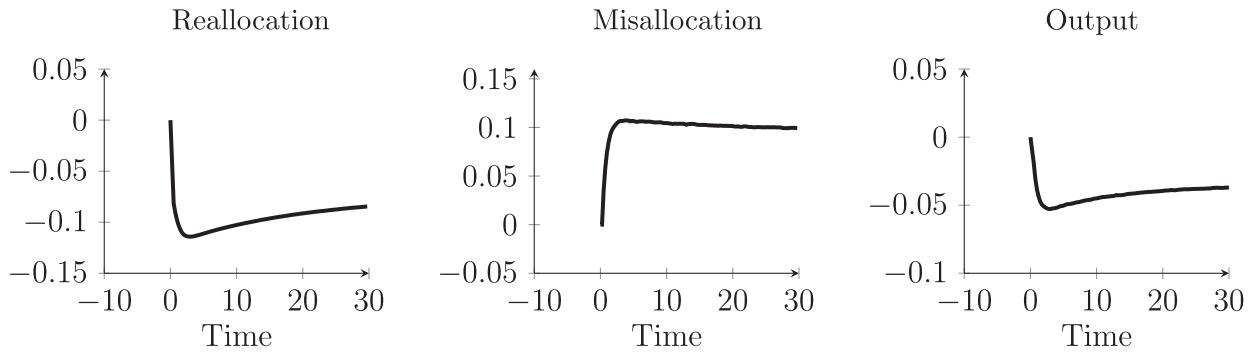


Fig. 4. Response to a sectoral productivity shock, where, at $t = 0$, sector B becomes the more productive sector. The figures uses constant gains from trade ($\alpha = 1$) and transitory shocks with $\lambda = 0.1$.

Panel A: Response to increase in quality dispersion ($\bar{\theta} - \underline{\theta}$)



Panel B: Response to reduction in discount rate (ρ)

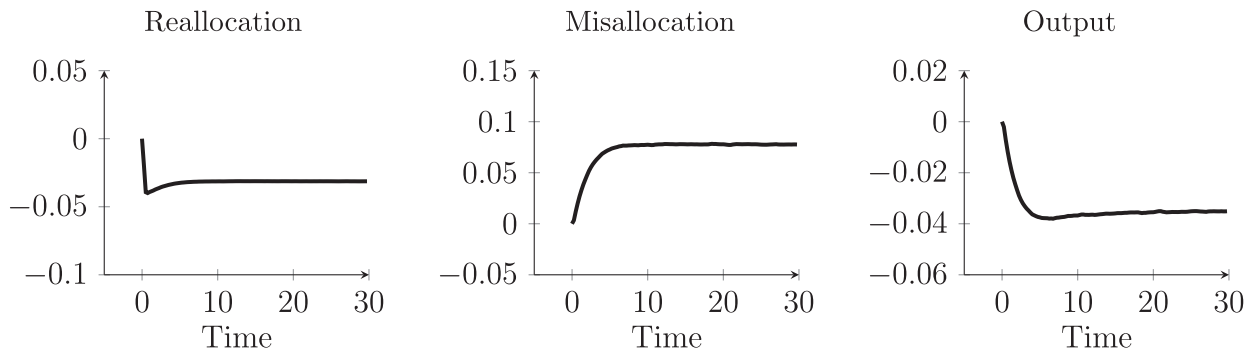


Fig. 5. Response to an increase in the dispersion of capital quality (Panel A) and a reduction in the discount rate (Panel B). Each graph plots the mean difference from the stochastic steady state across simulations. We construct impulse responses with respect to these structural changes as follows. We first simulate a sequence of sectoral productivity shocks assuming no structural shifts in parameters. Holding the sequence of sectoral productivity shocks fixed, we then permute the model by introducing an unanticipated parameter change at time 0 and compute the deviation across the two paths. We repeat this procedure one million times and report mean deviations over all simulations.

different from a model with adjustment costs. Panel B shows that productivity rises over time in both the sector from which capital exits (A) and the sector to which it is being reallocated (B). In contrast, in the standard adjustment cost models, productivity would either be flat or display opposite patterns in each sector.⁹

4.5. Response to changes in the economic environment

We examine output and productivity in response to unanticipated changes in two structural parameters of the model. First, we examine an increase in the dispersion of capital quality $\bar{\theta} - \underline{\theta}$. Second, we consider the effect of a change to firms' effective discount rate. To do so, we focus on the stationary model described in Section 4.3. We compute the level of misallocation at time t as the percentage of total potential output lost due to misallocation of capital, $1 - Y_t/\bar{Y}$, where $\bar{Y} = \int \bar{\pi}(\theta) dF(\theta)$ is the level of output in an economy without the adverse selection friction. We

compare the path of aggregate quantities as the economy transitions from the old to the new steady state.

Consider an unanticipated increase in the dispersion of capital quality, which derives from an expansion in the support of the quality distribution of new capital inflows, holding the mean quality constant. The quality of the existing capital stock is unaffected. We plot these impulse responses in Panel A of Fig. 5. The figure shows that an increase in the dispersion of the quality of new capital leads to lower rate of capital reallocation. Because capital is now more frequently misallocated, both aggregate output and productivity are lower. This effect operates at medium frequencies. Increasing the dispersion of quality for new capital does not have a discrete effect upon impact, because new capital flows in slowly and is initially efficiently allocated. However, upon the arrival of the next productivity shock, the distribution of quality in the divesting sector is now greater. This increase in the degree of adverse selection implies that the rate of reallocation is slower. As buyers become more uncertain about capital quality, sellers wait longer to sell to signal their type.

Next, we analyze the impact of a reduction in the firm's effective discount rate ρ . In our setting, a lower discount rate reduces the opportunity cost of delay for firms in the

⁹ With constant returns, average productivity of capital would be flat. With decreasing returns, productivity in sector A would increase, and productivity in sector B would decrease. Increasing returns to scale would generate the opposite pattern.

less productive sector [i.e., the left-hand side of Eq. (3)]. To distinguish themselves from firms with lower-quality capital, firms with higher quality capital must wait even longer. We plot these responses in Panel B of Fig. 5.

A fall in the discount rate leads to a slower rate of capital reallocation, more misallocation, and thus lower productivity. That the rate of misallocation increases and output decreases with a reduction in the interest rate lies in sharp contrast to the prediction of models with exogenously specified costs of reallocation. In those models, lowering the rate at which agents discount the future increases the present value of the benefits from reallocating capital, which leads to faster reallocation and an increase in efficiency. Our model, therefore, has different implications for how to use monetary policy to stimulate reallocation compared to models with exogenous reallocation costs.

The reduction in the discount rate leads to a lower flow of output. However, the effect on present discounted values, such as economic efficiency or welfare, are ambiguous. Inefficiency, defined as the fraction of the discounted output lost due to misallocation, can increase or decrease with ρ depending on parameter values and the fraction of misallocation capital. This result is in contrast to the implications of partial-equilibrium models with dynamic adverse selection (e.g., Janssen and Roy, 2002; Fuchs and Skrzypacz, 2013) in which the length of inefficient delay is inversely proportional to the discount rate and hence a change in the discount rate does not effect overall efficiency. The difference is due to general equilibrium effects and the transitory nature of shocks.

To summarize, it has been argued that shocks to reallocation costs, though difficult to interpret, can be useful for explaining features of the data (e.g., Eisfeldt and Rampini, 2006). Our results show that such shocks arise endogenously due to increased uncertainty about the quality of new investments rises or a reduction in the discount rate.

5. Risk-averse households

To this point, we have ignored general equilibrium effects on the interest rate by focusing on a setting with risk-neutral households. Let us now suppose that households exhibit CRRA utility: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. From Lemma 1, this implies that the state price density is given by

$$\xi_t = \exp(-\beta t) Y_t^{-\gamma}. \quad (34)$$

The crucial difference here is that the households' intertemporal rate of substitution now depends on total output and, therefore, the distribution of capital. As a result, the equilibrium rate of reallocation $\dot{\chi}_t$ depends on the distribution of capital in each sector and must be jointly determined with ξ_t .

We start by studying how the desire to smooth consumption over time affects reallocation dynamics in response to a permanent productivity shock. We illustrate how our results from Sections 2 and 4 can be extended and highlight two novel general equilibrium effects. First, the desire to smooth consumption increases the cost of delay and translates into faster reallocation. Second, the model predicts that large downturns are followed by fast

recoveries, whereas smaller negative shocks are followed by slower recoveries. Both of these predictions are in contrast to convex adjustment cost models in which the opposite prediction obtains.

We then reincorporate aggregate risk into the economy by allowing for multiple transitory shocks. We demonstrate that when households are sufficiently risk averse, some capital remains persistently misallocated. That is, the rate of reallocation reaches zero prior to all capital being reallocated. The intuition is that misallocated capital can serve as a hedge against a subsequent productivity shock. Thus, informational frictions not only generates delays in reallocation but also can freeze the reallocation process entirely.

5.1. Permanent shocks

Suppose that at $t < 0$ both sectors are equally productive. At $t = 0$, a productivity shock arrives that makes sector B relatively more productive ($\pi_B > \pi_A$). Capital then gradually flows from sector A to sector B . Our interest is in characterizing the equilibrium rate of reallocation and how it depends on γ as well as the initial distribution of capital across sectors. We allow capital to be arbitrarily distributed according to smooth, strictly positive density functions f_A, f_B over Θ . For simplicity, ignore both depreciation and new investment by setting $\delta = 0$. The primary additional consideration here is that ξ_t depends on output dynamics and, hence, the rate of reallocation, $\dot{\chi}_t$. Therefore, the equilibrium value of capital is determined endogenously. In this way, the effect of risk aversion is similar to the case with risk-neutral households and transitory shocks. However, here the mechanism works through the discount rate, whereas, with transitory shocks, the endogeneity worked through the cash flow channel.

To see this, recall that χ_t denotes the lowest-quality capital allocated to sector A at time t and therefore aggregate output and consumption can be written as

$$C_t = Y_t = \int_{\chi_t}^{\bar{\theta}} \pi_A(\theta) f^A(\theta) d\theta + \int_{\underline{\theta}}^{\chi_t} \pi_B(\theta) f^B(\theta) d\theta. \quad (35)$$

Hence, consumption grows according to

$$dC_t = (\pi_B(\chi_t) - \pi_A(\chi_t)) f^A(\chi_t) \dot{\chi}_t dt, \quad (36)$$

and thus $\dot{\chi}_t$ enters into the evolution of ξ_t ,

$$\frac{d\xi_t}{\xi_t} = -(\beta + \gamma C_t^{-1} (\pi_B(\chi_t) - \pi_A(\chi_t)) f^A(\chi_t) \dot{\chi}_t dt, \quad (37)$$

and leads to a short-term interest rate that is given by

$$r(\chi_t) = \beta + \gamma C_t^{-1} (\pi_B(\chi_t) - \pi_A(\chi_t)) f^A(\chi_t) \dot{\chi}_t. \quad (38)$$

The value of an efficiently allocated θ -unit of capital at time t (i.e., in state χ_t) can be written as

$$V(\theta, \chi_t) = v(\chi_t) \pi_B(\theta), \quad (39)$$

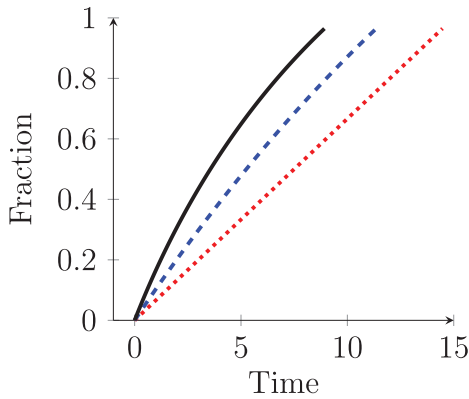
where $v(\chi_t)$ is simply the price of an perpetuity at time t . Using standard arguments, v satisfies

$$r(\chi_t) v(\chi_t) = 1 + \dot{\chi}_t v'(\chi_t), \quad v(\bar{\theta}) = \beta^{-1}, \quad (40)$$

and the zero-profit condition requires that

$$P_t = v(\chi_t) \pi_B(\chi_t). \quad (41)$$

Panel A: Fraction of capital reallocated



Panel B: Rate of reallocation

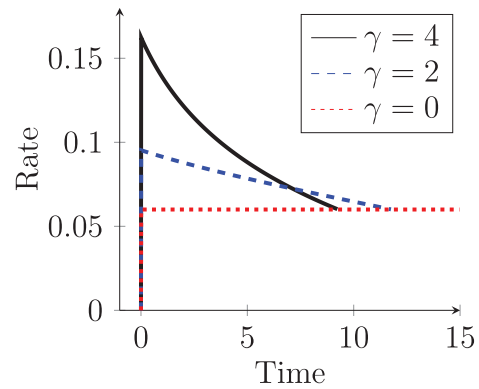


Fig. 6. This figure illustrates how the reallocation dynamics depend on γ for the case of a permanent productivity shock.

For a fixed $\dot{\chi}_t$, we have now fully characterized the equilibrium price. The next phase of the analysis follows closely that of Section 4. That is, we take the price as given and derive necessary conditions on $\dot{\chi}_t$. Analogous to Eq. (20), the optimality conditions for firms requires that their value function smoothly pastes to prices. Letting $\underline{V}(\theta, \chi_t)$ denote the value of an inefficiently allocated unit of capital, the (local) optimality condition requires that

$$\left. \frac{d}{dt} \underline{V}(\theta, \chi_t) \right|_{\theta=\chi_t} = \frac{d}{dt} P_t, \quad (42)$$

and value matching requires that

$$\underline{V}(\chi_t, \chi_t) = P_t. \quad (43)$$

Using the law of motion for \tilde{V} and \underline{V} along with Eqs. (41)–(43),

$$\dot{\chi}_t = \frac{1}{v(\chi_t)} \frac{\pi_B(\chi_t) - \pi_A(\chi_t)}{\pi'_B(\chi_t)}, \quad \chi_0 = \theta. \quad (44)$$

We are left with a pair of initial boundary problems [i.e., Eqs. (40) and (44)] to which the (unique) fixed point characterizes the equilibrium.

Theorem 2. *In any economy in which households have CRRA utility and the productivity shock is permanent, there exists a unique (τ^{**}, V^{**}) such that the firm strategies and capital prices consistent with (τ^{**}, V^{**}) are part of a fully revealing competitive equilibrium.*

The effect of consumption smoothing motives on equilibrium reallocation is illustrated in Fig. 6. Following a productivity shock, consumption drops and gradually increases over time. As a result, the interest rate jumps upward and then slowly decreases to its original level. The resulting increase in the interest rate makes it more costly for firms to delay reallocation. Hence, the rate of reallocation increases, especially just after the shock. The higher is γ , the stronger is the desire to smooth consumption and the stronger are these effects. Note that this prediction lies in contrast to models with exogenously specified adjustment costs in which an increase in the desire to smooth consumption leads to slower reallocation.

Another novel feature of the general equilibrium environment is that the rate at which the economy recovers from a productivity shock depends on the allocation of capital upon its arrival. To fix ideas, consider the case in which sector A experiences a negative shock to productivity at $t = 0$. If all capital is initially allocated in sector A when the shock arrives, then the economy suffers a severe drop in output but the rate of reallocation is high and the recovery process is relatively quick. On the other hand, if capital is more evenly split across the two sectors when the shock arrives, then the drop in output is smaller but the recovery process is slower. The intuition is that when there is more capital to reallocate, the growth rate of consumption is higher, which requires higher interest rates and lower v . This, in turn, raises the cost to firms in sector A from delaying the sale of their capital and increases $\dot{\chi}_t$.¹⁰ These dynamics are illustrated in Fig. 7.

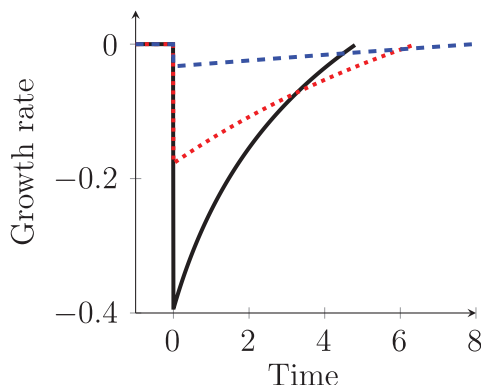
5.2. Transitory shocks and aggregate risk

In Section 5.1, the equilibrium dynamics are deterministic following the arrival of the productivity shock. The economic implications are driven by households' desire to smooth consumption over time. Here, we explore how aggregate risk and households' desire to smooth consumption across aggregate states affect reallocation dynamics. We do so by extending the analysis to a situation in which there are multiple transitory shocks. We focus attention on the reallocation dynamics from sector A to sector B when sector B is currently more productive ($\pi_B > \pi_A$), but sector A becomes more productive at some (random) point in the future.

In this case, aggregate risk plays an explicit role in the reallocation decision of firms. Using backward induction, we derive the system of differential equations characterizing the reallocation dynamics in Appendix B. We solve this system numerically by applying standard techniques. Fig. 8 illustrates an important finding from this exercise, namely,

¹⁰ A similar comparative static prediction obtains with respect to either the fraction that sectors A and B constitute of the larger (unmodeled) economy or the magnitude of the productivity shock.

Panel A: Growth rate of output



Panel B: Rate of reallocation

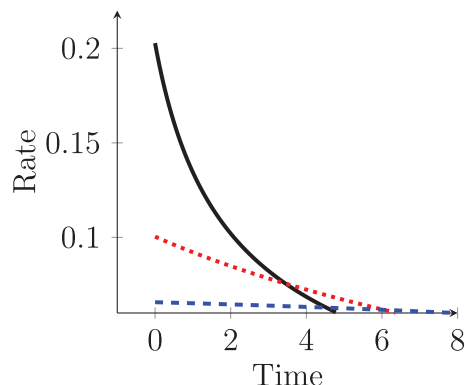
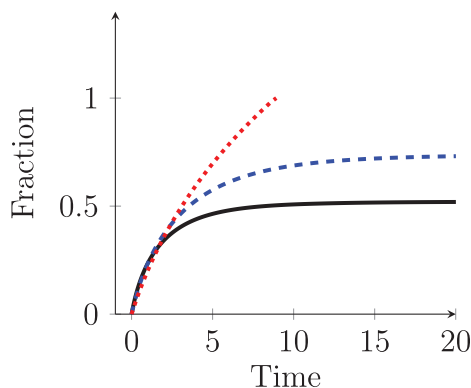


Fig. 7. Recovery from a negative productivity shock to sector A as it depends on the initial distribution of capital. The solid line corresponds to the case in which all of the capital is initially allocated in sector A. The dotted line (dashed line) correspond to the case in which 50% (10%) of the capital is initially allocated to sector A.

Panel A: Fraction of capital reallocated



Panel B: Rate of reallocation

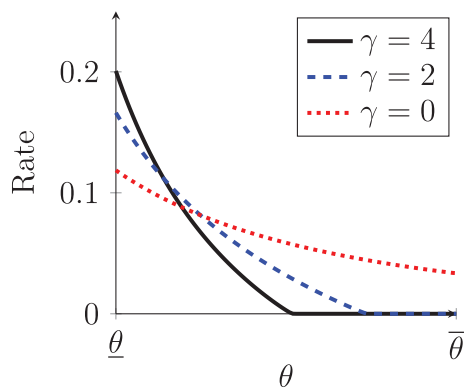


Fig. 8. Reallocation dynamics in the presence of aggregate risk. This figure plots the dynamics prior to the arrival of the last shock.

that the rate of reallocation reaches zero prior to all of the capital being reallocated to sector B. This implies that some capital remains *persistently misallocated*. The intuition is that misallocated capital can serve as a hedge against a subsequent productivity shock. Thus, informational frictions not only generates delays in reallocation but can also freeze the reallocation process entirely.

One could be tempted to consider a model in which shocks arrive indefinitely. We expect qualitatively similar results to obtain in such a model. However, analogous to macroeconomic models with heterogeneous households, solving for the equilibrium of such a model requires keeping track of the entire distribution of capital across sectors and thus an infinite dimensional state space. To solve such a model, one would need to develop an approximate solution method (e.g., Krusell and Smith, 1998). We leave this exercise for future work.

6. Comparison with exogenous adjustment cost specifications

Adjustment costs play a central role in both real business cycle models and structural models of the firm. Our

theory can motivate a variety of adjustment costs specifications, both at the macro and the micro level. For example, we can interpret A and B as referring to separate industries and capital in the model as referring to either physical capital or labor. Conversely, if we interpret A as an individual firm and B as a set of more productive firms, then our model is similar to one with convex adjustment costs at the firm level.¹¹

To explicitly illustrate the connection with adjustment cost models, we consider a version of the model in which capital is homogeneous (i.e., $\underline{\theta} = \bar{\theta}$), but there are exogenous costs to reallocating capital. We specify these costs as a function of the aggregate mass of capital being reallocated at a particular time and focus on the central planner's problem. We denote by k the capital stock in sector B.

We examine three formulations for these costs commonly used in the literature. The first formulation corresponds to the case in which adjustment costs are convex

¹¹ Under this interpretation, if capital refers to physical assets then it may be possible for firm A to alleviate the informational friction by committing to sell capital in bulk.

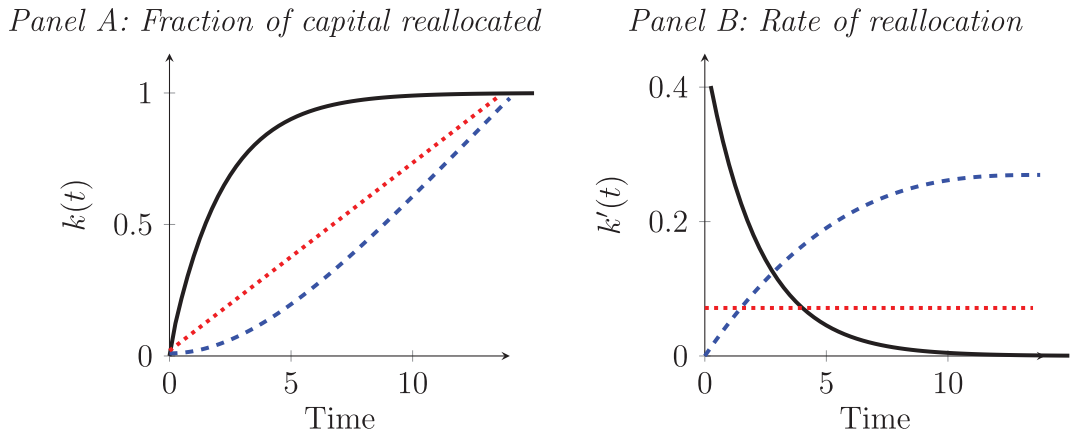


Fig. 9. Comparison across the 'kdot' (dotted), 'ik' (solid), and 'idot' (dashed) adjustment cost models.

in the rate of reallocation \dot{k} .

$$c(k) = \frac{1}{2} c(\dot{k})^2. \quad (45)$$

These costs are in line with the adjustment cost formulation in [Abel \(1983\)](#). We refer to this as the 'kdot' model. The second formulation is similar to [Eq. \(45\)](#), except that it specifies the adjustment cost in terms of the growth rate of capital being reallocated,

$$c(k, \dot{k}) = \frac{1}{2} c\left(\frac{\dot{k}}{1-k}\right)^2 (1-k). \quad (46)$$

This type of adjustment costs is commonly used in the literature studying investment and reallocation dynamics ([Abel and Eberly, 1994](#); [Eberly and Wang, 2009](#); [Eisfeldt and Rampini, 2006](#)). We refer to these costs as the 'ik' model. The third adjustment cost formulation penalizes changes in the flow rate of reallocation \dot{k} ,

$$c(\ddot{k}) = \frac{1}{2} c(\ddot{k})^2, \quad (47)$$

and is based on the adjustment costs proposed by [Christiano, Eichenbaum, and Evans \(2005\)](#), which penalize changes in investment. We refer to these costs as the 'idot' model. The reallocation dynamics for these three cases are plotted in [Fig. 9](#).¹²

We contrast our model's equilibrium dynamics to those implied by the models with different types of reallocation costs. As we compare [Fig. 9](#) with the dynamics implied by [Fig. 1](#), a striking similarity emerges. The 'idot' model of adjustment costs generates an increasing rate of reallocation in line with the case with increasing gains from trade ($\alpha = 0$), while the 'ik' model of adjustment costs generate a decreasing rate of reallocation in line with the case with decreasing gains from trade ($\alpha = 2$). When the gains from reallocation are constant ($\alpha = 1$), the dynamics match those of the 'kdot' model.

Relative to the first two formulations, the 'idot' model generates an S-shaped path for the capital stock and more

delayed responses of capital flow to a sectoral productivity shock. The rate of capital reallocation in the 'ik' model spikes on impact and decays smoothly over time. By contrast, in the 'idot' model, the rate of capital reallocation increases slowly over time. This slow increase occurs because the formulation in [Eq. \(47\)](#) severely penalizes large adjustments to the rate. [Christiano, Eichenbaum, and Evans \(2005\)](#) argue that this feature is crucial in explaining the response of aggregate investment to shocks.

In sum, the equilibrium dynamics implied by each of these exogenous adjustment cost models are similar to those predicted by our model. Our framework can thus be interpreted as providing a micro-foundation for a variety of adjustment cost specifications. An important caveat is that the theory we develop refers to frictions in the reallocation of existing resources, not how new resources are allocated. However, making new investments often requires raising capital through the sale of existing assets (or rights to cash flows generated by existing assets), in which case the same forces are likely to be at play. Extending our model to explore the impact of adverse selection on the dynamics of new investments is left for future work.

7. Empirical evidence

Here, we discuss how the model's predictions are related to the data. Developing direct tests of the mechanism is inherently challenging because the predictions pertain to unobservable characteristics. We start by presenting some new evidence that is consistent with our model, exploiting the fact that the unobservable quality of capital in our model is correlated with ex-post measures of profitability. We then relate the predictions of our model to stylized features of the data that have been documented in previous work.

7.1. An empirical test

We focus on the reallocation of a firm's cash flow rights from entrepreneurs to investors following a firm's initial public offerings. Risk aversion on the part of entrepreneurs can be viewed as a higher flow operating cost while the

¹² Detailed solutions available upon request.

firm is private and provides a motive for diversification and gains from trade with investors. Consumption smoothing motives and expansionary investment are other natural explanations.

IPOs offer an attractive setting to test our mechanism for two reasons. First, the amount of public information available about the firm is scarce prior to its IPO and, hence, an informational asymmetry between sellers (the entrepreneur) and buyers (investors) seems plausible. Second, IPOs are a setting in which ex-post measures of operating performance for the asset being traded are available. Thus, even though firm quality is unobservable, it should be correlated with post-IPO measures of the firm's profitability. We focus on testing the following two implications of our mechanism.

Prediction 1. Controlling for observable characteristics, entrepreneurs with more profitable firms wait longer to go public. Therefore, the length of time to IPO issuance should be positively correlated with post-IPO measures of profitability.

Prediction 2. In a fully revealing equilibrium, the market correctly interprets delay as a signal of profitability and prices adjust accordingly. Therefore, the length of time to IPO issuance should not be correlated with post-IPO stock returns.

We use the length of time elapsed between a firm's incorporation and its IPO as a proxy for how long entrepreneurs wait to go public and control for observable characteristics at the time of the IPO. These characteristics include firm size and profitability at the time of the IPO, along with IPO-year dummies, or IPO-year interacted with industry dummies. The details of our empirical specification are in [Appendix C](#).

We present our findings in [Table C.2](#). We find that the length of time from a firm's incorporation to its IPO is predictive of its future profitability (return on assets, or ROA) at horizons of up to five years. This predictive relation is robust to controlling for observable characteristics, including controls for firm size and its profitability at the time of the IPO, as well as IPO-year dummies, or IPO-year interacted with industry dummies. Columns (1)–(3) present results with different controls. The economic magnitudes are substantial. Focus on Column (3), which compares two firms that did an IPO at the same time, in the same industry, and have the same size and profitability at the year of the IPO. Our estimates imply that the firm that belongs in the 75th percentile in terms of the age at IPO experiences a 4.4–9.8% higher ROA than the firm at the 25th percentile over a one to five-year horizon. For comparison, the interquartile range in firm ROA ranges from 18.6–24% over a one to five-year horizon. This finding supports our model's prediction that entrepreneurs with higher-quality capital delay the sale of their capital for longer as a signal of quality to the market.

Importantly, even though a firm's age predicts future profitability, it does not predict the firm's stock returns following the IPO decision. This lack of return predictability, which is common across all specifications—see Columns (4) and (6)—implies that the higher ex-post profitability

associated with older firms does not represent news to the market. This finding suggests that, consistent with our model, the firm's price at the time of the IPO is fully revealing of its quality.

An alternative explanation for the first prediction could be due to heterogeneous entrepreneurial preferences. For instance, more risk-averse entrepreneurs should choose safer projects with lower profitability on average. Such entrepreneurs are likely to sell their firms faster than their less risk-averse counterparts. In this case, the timing of the IPO decision could be indicative of differences in the riskiness of these firms. This alternative theory would also imply that the timing should reveal differences in average ex-post returns to these firms. Because older firms are riskier, they should compensate investors with higher returns. This implication, which runs counter to Prediction 2, does not seem to be supported by the data.¹³

In sum, our empirical results are supportive of our mechanism. Naturally, these results are based on correlations and we do not establish causality. An exhaustive empirical analysis that establishes a causal link and allows us to distinguish between alternative theories is left for future work.

7.2. Existing supporting evidence

In addition to the empirical test conducted above, our model's predictions are consistent with a variety of indirect evidence in the existing literature.

In the context of reallocation of human capital, [Wagner and Zwick \(2012\)](#) exploit data from the German apprenticeship system to show the role of adverse selection. Consistent with our model, they find that workers who migrate to new firms quickly after completing their apprenticeship (i.e., early switchers) earn lower wages and are less productive than workers who stay with their existing firms. In the reallocation of physical capital, [Ramey and Shapiro \(2001\)](#) show the following stylized facts that are consistent with our model: (i) capital sells at a substantial discount relative to its replacement cost, (ii) this discount is smaller if capital sells to other aerospace firms, which presumably have better ability to evaluate its quality, and (iii) the process of selling used equipment is lengthy.

Our model implies that an increase in the degree of adverse selection due to, say, an increase in the dispersion in capital quality increases the cost of reallocation leading to lower output growth. Indeed, there is some evidence that, in general, financial crises are accompanied with an increase in misallocation of resources (see, for instance, [Oberfield, 2013; Ziebarth, 2013](#)). While the exact causes and consequences of financial crises are not yet fully understood, adverse selection appears to be an important component.

Our model also provides an economic explanation for why disinvestment should be more costly than investment,

¹³ Several other theories make predictions about the timing of the IPO decision (an incomplete list includes [Maksimovic and Pichler, 2001; Pastor and Veronesi, 2005; Pastor, Taylor, and Veronesi, 2009](#)). To the best of our knowledge, none of these theories makes explicit predictions about the timing of the decision and the firm's age.

a prominent feature in structural models of investment (see, e.g., [Abel and Eberly, 1994; 1996](#)). Disinvestment involves the sale of used capital, in which one naturally expects the adverse selection problem to be more severe, whereas investment often involves purchasing capital directly from its producers, in which the information friction is likely to be less severe (e.g., due to the reputational concerns of producers). Along these lines, [Cooper and Haltiwanger \(2006\)](#) estimate a structural model of convex and non-convex adjustment costs using plant-level data. Their estimates imply a substantial spread between the purchase and sale price of capital.

8. Conclusion

In this paper, we incorporate persistent adverse selection into a competitive decentralized economy to study the dynamics of capital allocation. The information friction leads to slow movements in capital reallocation, and it provides a micro-foundation for convex adjustment cost models. The model generates a rich set of dynamics for aggregate quantities. Importantly, our model illustrates how changes in the economic environment endogenously affect the costs of efficiently redeploying capital.

Clearly, adverse selection is not the only mechanism inhibiting the efficient allocation of capital. Nor is it the only way to rationalize these patterns. The existing literature is rich with explanations. Physical (convex) costs, search, financial frictions, learning, time-to-build, and other factors are likely to be important components in the allocation of new and existing capital. One key benefit of the adjustment cost approach is to absorb a variety of frictions into a single cost function. We abstract away from these considerations to highlight the key ideas of the paper. Incorporating these frictions into macroeconomic models that are suitable for calibration, and therefore providing a way to quantitatively assess the importance of each, is a promising area for future work. We view our work as an important step in this direction.

Appendix A. Proofs

Proof of Lemma 1. Follows immediately from the households' first order condition and the goods market clearing condition.

Proof of Lemma 2. Suppose the skimming property does not hold. Then, there exists (i, t) such that $t_1 \equiv T_t^i(\theta) > t_0 \equiv T_t^i(\theta')$ for some $\theta' > \theta$. Because θ prefers to wait at t_0 and θ' accepts, it must be that $V_{t_0}^i(\theta) \geq P_{t_0}^i = V_{t_0}^i(\theta')$. But since π_i is increasing, θ' could do strictly better by mimicking type θ , which violates [Eq. \(13\)](#).

The proof of [Theorem 1](#), relies on [Lemma A.1](#).

Lemma A.1. There exists a unique χ^* that satisfies [\(27\)](#) and [\(28\)](#). Furthermore, χ^* is strictly increasing.

Proof. [Eqs. \(27\)–\(28\)](#) define an initial value problem of the form

$$\chi'(t) = f(t, \chi(t)), \quad \chi(0) = \underline{q}. \quad (\text{A.1})$$

To verify existence and uniqueness of a solution, we apply the Picard-Lindelof Theorem (see [Zeidler, 1998](#), Theorem 3.A.) To do so, it is sufficient to verify several properties of f : (i) $f(t, x)$ is continuous on $[0, T] \times [\underline{q}, \bar{q}]$; (ii) f is bounded, and (iii) that $f(t, x)$ is Lipschitz. Property (i) is by inspection (since both g and π_i are continuously differentiable). Property (ii) follows immediately from the expression for g and the conditions placed on π_i . To demonstrate (iii), it suffices to show that $\frac{d}{dx}f(t, x)$ is bounded, which follows from the restriction that π_i have bounded first and second derivatives. \square

Proof of Theorem 1. From [Lemma A.1](#), there is a unique candidate (fully) revealing equilibrium. Thus, to prove the theorem, it suffices to check that the candidate satisfies the equilibrium conditions. The zero profit and capital market clearing conditions are satisfied by construction. To verify that firms optimize, note that no firm in the efficient sector strictly prefers to sell its capital because the price is $V(\underline{\theta})$, which is the least a firm can expect to earn by continuing to operate its capital. It remains to verify that there are no profitable deviations for firms in the inefficient sector. To see this, note that the seller's objective can be written as

$$u_\theta(t, P) = (1 - f(t)) \frac{\pi(\theta)}{\rho} + f(t)P \quad (\text{A.2})$$

and, therefore,

$$\frac{\partial}{\partial \theta} \left(\frac{\partial u_\theta / \partial P}{\partial u_\theta / \partial t} \right) = \frac{f'(\tau)}{f'(\tau)(P - \pi(\theta)/\rho)} > 0, \quad (\text{A.3})$$

which shows that the single-crossing condition is satisfied. In this case, a standard result ([Fudenberg and Tirole, 1991](#), chap. 7) is that the local incentive compatibility constraint and monotonicity of (τ, P) , which hold by construction, are sufficient to guarantee that no profitable global deviations exist.

Proof of Proposition 2. By inspection, $g(t) = 0$ for $\lambda = 0$. [Eq. \(27\)](#) then immediately reduces to [Eq. \(30\)](#).

Proof of Proposition 3. Taking the total derivative of the right-hand side of [Eq. \(30\)](#) with respect to time results in

$$\chi''(t) = \rho \cdot \frac{d}{d\chi} \left(\frac{\bar{\pi}(\chi) - \pi(\chi)}{\bar{\pi}'(\chi)} \right) \cdot \dot{\chi}_t.$$

Because $\dot{\chi}_t(t) > 0$ for all $t \in [0, \tau(\bar{\theta})]$, the derivative of $\dot{\chi}_t$ with respect to time has the same sign as the derivative of $\frac{\bar{\pi}(\chi) - \pi(\chi)}{\bar{\pi}'(\chi)}$ with respect to θ .

Proof of Proposition 1. Follows immediately from [Proposition 3](#) and the fact that, for the CES production technology, $\frac{d}{d\theta} \left(\frac{\bar{\pi} - \pi}{\bar{\pi}'} \right)$ is strictly positive for $\alpha < 1$, strictly negative for $\alpha > 1$, and equal to zero for $\alpha = 1$.

Proof of Proposition 4. This follows from [Eq. \(26\)](#) and the fact that (i) $\tau(\theta) > 0$ for all $\theta > \underline{\theta}$ and (ii) $g(t) > 0$ for all $t > 0$.

Proof of Proposition 5. Using a subscript to represent elements of the relevant economy, we have that

$$\begin{aligned} \dot{\chi}_2(0) - \dot{\chi}_1(0) &= (1 - g(0; \lambda_2) + g(0; \lambda_2)) \\ &\quad - (1 - g(0; \lambda_1) + g'(0; \lambda_1)) \end{aligned}$$

$$= g'(0; \lambda_2) - g'_1(0; \lambda_1) \\ > 0,$$

where the inequality follows from the fact that $\frac{d}{d\lambda} g'(0; \lambda) > 0$. Therefore, $\dot{\chi}_2(0) > \dot{\chi}_1(0)$. By the continuity and boundedness of $\dot{\chi}_1$ and $\dot{\chi}_2$, there must exist $\bar{t} > 0$ such that the inequality holds for $t \in [0, \bar{t}]$.

Proof of Theorem 2. The proof involves showing that there exists a unique candidate solution satisfying the joint system of differential equations and then verifying that the strategies and prices consistent with the candidate satisfy the equilibrium requirements.

Fix an economy, which can be represented by $\{f^A, f^B, \pi_A, \pi_B, \gamma, \beta\}$. Define

$$c(\chi) \equiv \int_{\chi}^{\bar{\theta}} \pi_A(\theta) f^A(\theta) d\theta + \int_{\underline{\theta}}^{\chi} \pi_B(\theta) f^B(\theta) d\theta.$$

Let (τ, V) denote an arbitrary candidate revealing equilibrium and note that the zero profit condition requires that $V(\theta, \chi) = \pi_B(\theta) v(\chi)$. Therefore, it is sufficient to characterize (τ, v) . Assuming τ is strictly increasing and therefore invertible, define $\chi_t \equiv \tau^{-1}$ and $\phi(\theta) = \frac{1}{\tau'(\theta)}$. From Eqs. (38), (40), and (44), any candidate revealing equilibrium must satisfy

$$\phi(\theta) = \frac{\pi_B(\theta) - \pi_A(\theta)}{\pi'_B(\theta) v(\theta)}, \quad \tau(\underline{\theta}) = 0 \quad (\text{A.4})$$

and

$$\left(\beta + \frac{\gamma}{c(\theta)} (\pi_B(\theta) - \pi_A(\theta)) f^A(\theta) \phi(\theta) \right) v(\theta) \\ = 1 + \phi(\theta) v'(\theta), \quad v(\bar{\theta}) = \beta^{-1}. \quad (\text{A.5})$$

Substituting the ordinary differential equation (ODE) from Eq. (A.4) into (A.5), and rearranging, we arrive at an initial value problem of the form

$$v'(\theta) = f(\theta, v(\theta)), \quad v(\bar{\theta}) = \beta^{-1}. \quad (\text{A.6})$$

The proof of existence and uniqueness of a solution to Eq. (A.6) follows closely the proof of Lemma 1 and is therefore omitted. Letting v^{**} denote this solution, substitute it into Eq. (A.4), and apply the same argument to get existence and uniqueness of τ^{**} . The next step is to show that τ^{**} is strictly increasing. From Eq. (A.4), it suffices to show that $v^{**} > 0$ for all θ . Suppose, to the contrary, that $v^{**}(\theta') < 0$ for some θ' . Because v^{**} is continuous and $v^{**}(\bar{\theta}) > 0$, there must exist a $\theta'' > \theta'$ such that $v^{**}(\theta'') = 0$, but this clearly violates Eq. (A.4). Therefore, τ^{**} is invertible. Let χ_t^{**} denote its inverse. Thus, we have shown there exists a unique candidate revealing equilibrium.

To verify the candidate is part of an equilibrium, specify that $\xi_t = \exp(-\beta t) c(\chi_t^{**})^\gamma$ and $c_t^h = c(\chi_t^{**})$. Household optimality and market clearing of the consumption good is immediate. That the capital market clears and new firms make zero profit in the candidate follows immediately from the fact that only χ_t^{**} trades at time t and the solution satisfies Eq. (41). Locally, firm optimality is by construction [i.e., Eq. (42)]. That the firm's strategy is optimal globally follows from the same arguments as used in the proof of Theorem 1.

Appendix B. Transitory shocks with risk-averse households

In this Appendix, we analyze the model with multiple transitory shocks and risk-averse households. Our analysis in Section 5.1 applies once the last shock arrives. Let us now consider the case in which there are two shocks. To fix ideas, suppose that at $t < 0$ all capital is allocated to sector A. At $t = 0$, a shock arrives that makes sector B more productive. But this shock is not permanent. At some random time $\tau > 0$, another shock arrives that makes sector A the more productive sector. As before, use $\bar{\pi}$ ($\underline{\pi}$) to denote the productivity of capital allocated efficiently (inefficiently).

One can think of the model as having two regimes. In the first regime ($t < \tau$), capital transitions from sector A to sector B. In the second regime ($t > \tau$), capital transitions back to sector A. We use subscripts to denote to which regime the object refers. For example, $T_1(\theta)$ denotes the time at which a sector A firm sells capital of quality θ to sector B in the first regime. Let θ_1 denote the lowest type remaining in sector A at the end of the first regime, i.e.,

$$\theta_1 \equiv \inf\{\theta : T_1(\theta) > \tau\}. \quad (\text{B.1})$$

B.1. Second regime

We proceed by backward induction. Note that all $\theta > \theta_1$ are efficiently allocated at the beginning of the second regime. Hence, for all $t \geq \tau$,

$$Y_t = \int_{\underline{\theta}}^{\bar{\theta}} \bar{\pi}(\theta) dF(\theta) - \int_{\chi_2(t)}^{\theta_1} (\bar{\pi}(\theta) - \underline{\pi}(\theta)) dF(\theta) \\ = c_2(\chi_2(t), \theta_1), \quad (\text{B.2})$$

where $\chi_2(t)$ denotes the lowest remaining type in the inefficient sector (sector B) during the second regime. Using the same argument as in Section 5.1, the solution consists of the rate at which types change

$$\phi_2(\chi, \theta_1) = \frac{\bar{\pi}(\chi) - \underline{\pi}(\chi)}{\bar{\pi}'(\chi)} \frac{1}{v_2(\chi; \theta_1)}, \quad (\text{B.3})$$

where $v_2(\chi; \theta_1)$ is the price of a perpetuity (that pays a constant flow of 1) in the current state and solves the ODE

$$(\rho + \gamma c_2(\chi, \theta_1)^{-1} (\bar{\pi}(\chi) - \underline{\pi}(\chi)) f(\chi) \phi_2(\chi, \theta_1)) v_2(\chi; \theta_1) \\ = 1 + \phi_2(\chi, \theta_1) v_2'(\chi; \theta_1). \quad (\text{B.4})$$

The boundary condition now becomes

$$v_2(\theta_1; \theta_1) = \rho^{-1}. \quad (\text{B.5})$$

The value of an efficiently allocated unit of capital of quality θ in the second regime is therefore equal to

$$\bar{V}_2(\theta, \chi_2, \theta_1) = v_2(\chi_2, \theta_1) \bar{\pi}(\theta). \quad (\text{B.6})$$

Next, we derive the value of an inefficiently allocated unit of capital during the second regime. It will suffice to compute this value evaluated at $\chi_2 = \underline{\theta}$ for all θ_1 , which is

given by

$$\begin{aligned} \xi_t \underline{V}_2(\theta, \underline{\theta}, \theta_1) &= \int_t^{T_2(\theta, \theta_1)} \xi_s \underline{\pi}(\theta) ds + \int_{T_2(\theta, \theta_1)}^\infty \xi_s \bar{\pi}(\theta) ds \\ &= \int_t^{T_2(\theta, \theta_1)} \xi_s (\underline{\pi}(\theta) - \bar{\pi}(\theta)) ds + \int_t^\infty \xi_s \bar{\pi}(\theta) ds \\ &= \xi_t \bar{V}_2(\theta, \underline{\theta}, \theta_1) - \int_t^{T_2(\theta, \theta_1)} \xi_s (\bar{\pi}(\theta) - \underline{\pi}(\theta)) ds \quad (\text{B.7}) \end{aligned}$$

where $T_2(\theta, \theta_1) = \int_\theta^\theta \frac{1}{\phi_2(y, \theta_1)} dy$ is the stopping rule used by a type θ seller in the second regime. Using a change of variables, Eq. (B.7) can be written as

$$\underline{V}_2(\theta, \underline{\theta}, \theta_1) = \bar{V}_2(\theta, \underline{\theta}, \theta_1) \quad (\text{B.8})$$

$$\begin{aligned} -(\bar{\pi}(\theta) - \underline{\pi}(\theta)) \int_\theta^\theta \exp(-\rho T_2(y, \theta_1)) \left(\frac{c_2(y, \theta_1)}{c_2(\theta, \theta_1)} \right)^{-\gamma} \\ \times \frac{1}{\phi_2(y, \theta_1)} dy, \quad (\text{B.9}) \end{aligned}$$

where, instead of integrating over time, we integrate over types that switch before type θ switches. Substituting in the expression for T_2 , one can calculate $\underline{V}_2(\theta, \underline{\theta}, \theta_1)$ in terms of \bar{V}_2 and ϕ_2 .

B.2. First regime

By the zero-profit condition, the price must equal the value of an efficiently allocated unit of capital in the first regime, denoted by \bar{V}_1 , which satisfies

$$\xi_t \bar{V}_1(\theta, \chi_1(t)) = E \left[\int_t^\tau \xi_s \bar{\pi}(\theta) ds + \xi_\tau \underline{V}_2(\theta, \underline{\theta}, \chi_1(\tau)) \right]. \quad (\text{B.10})$$

Here, $\chi_1(t)$ denotes the lowest quality of capital remaining in the inefficient sector during the first regime (sector A). Because $\chi_1(t)$ must be monotonic in a fully revealing equilibrium, we often omit t arguments and write functions in terms of the state variable χ_1 , using $\phi_1(\chi_1)$ to denote the rate of reallocation in the first regime. Aggregate consumption and output in the first regime is given by

$$c_1(\chi_1) \equiv \int_\theta^{\chi_1} \bar{\pi}(\theta) dF(\theta) + \int_{\chi_1}^{\bar{\theta}} \underline{\pi}(\theta) dF(\theta). \quad (\text{B.11})$$

From Lemma 1, the stochastic discount factor is $\xi_t = e^{-\rho t} c_1(\chi_1)^{-\gamma}$, which satisfies

$$\begin{aligned} \frac{d\xi_t}{\xi_t} &= -\rho dt - \gamma c_1(\chi_1)^{-1} \frac{\partial}{\partial \chi_1} c_1(\chi_1) \phi_1(\chi_1) dt \\ &\quad + \left(\left(\frac{c_2(\underline{\theta}, \chi_1)}{c_1(\chi_1)} \right)^{-\gamma} - 1 \right) dN_t, \quad (\text{B.12}) \end{aligned}$$

and

$$\begin{aligned} E_t \left[\frac{d\xi_t}{\xi_t} \right] &= -\rho dt - \gamma c_1(\chi_1)^{-1} \frac{\partial}{\partial \chi_1} c_1(\chi_1) \phi_1(\chi_1) dt + \lambda \left(\left(\frac{c_2(\underline{\theta}, \chi_1)}{c_1(\chi_1(t))} \right)^{-\gamma} - 1 \right) dt. \\ &\quad \underbrace{\hspace{10em}}_{-r_1(\chi_1) dt} \quad (\text{B.13}) \end{aligned}$$

Define the discounted price process \tilde{V} as

$$\begin{aligned} \tilde{V}(\theta, \chi_1) &\equiv c_1(\chi_1)^{-\gamma} \bar{V}_1(\theta, \chi_1) \\ &= E \left[\int_t^\tau e^{-\rho(s-t)} c_1(\chi_1(s))^{-\gamma} \bar{\pi}(\theta) ds \right. \\ &\quad \left. + e^{-\rho(\tau-t)} (c_2(\underline{\theta}, \chi_1(\tau)))^{-\gamma} \underline{V}_2(\theta, \underline{\theta}, \chi_1(\tau)) \right]. \quad (\text{B.14}) \end{aligned}$$

Note that

$$\tilde{V}_\chi \equiv \frac{d}{d\chi} (c_1(\chi)^{-\gamma} \bar{V}_1(\theta, \chi)) \quad (\text{B.15})$$

$$\begin{aligned} &= c_1(\chi)^{-\gamma} \bar{V}_\chi(\theta, \chi) - \gamma c_1(\chi)^{-(1+\gamma)} \\ &\quad \times (\bar{\pi}(\chi) - \underline{\pi}(\chi)) f(\chi) \bar{V}_1(\theta, \chi). \quad (\text{B.16}) \end{aligned}$$

By the martingale property, $\tilde{V}(\theta, \chi_1)$ satisfies the ODE

$$\begin{aligned} \rho \tilde{V} &= c_1(\chi_1)^{-\gamma} \bar{\pi}(\theta) + \tilde{V}_\chi \phi_1(\chi_1) \\ &\quad + \lambda (c_2(\underline{\theta}, \chi_1))^{-\gamma} \underline{V}_2(\theta, \underline{\theta}, \chi_1) - \tilde{V}. \quad (\text{B.17}) \end{aligned}$$

Or, after substituting for \tilde{V} ,

$$\begin{aligned} r_1(\chi) \bar{V}_1(\theta, \chi) &= \bar{\pi}(\theta) + \frac{\partial}{\partial \chi} \bar{V}_1(\theta, \chi) \phi(\chi) \\ &\quad + \lambda \left(\frac{c_2(\underline{\theta}, \chi)}{c_1(\chi)} \right)^{-\gamma} (\underline{V}_2(\theta, \underline{\theta}, \chi) - \bar{V}_1(\theta, \chi)). \quad (\text{B.18}) \end{aligned}$$

Since this is the value of an efficiently allocated unit of capital, the above equation holds only for $\theta \leq \chi$. For the boundary condition, consider what happens to an efficiently allocated unit of capital when all the capital has moved, but before the second shock hits, $\chi_1 = \bar{\theta}$ and $t < \tau$. The value of capital at the boundary must solve

$$\begin{aligned} \rho \bar{V}_1(\theta, \bar{\theta}) &= \bar{\pi}(\theta) + \lambda \left(\left(\frac{c_2(\underline{\theta}, \bar{\theta})}{c_1(\bar{\theta})} \right)^{-\gamma} \underline{V}_2(\theta, \underline{\theta}, \bar{\theta}) - \bar{V}_1(\theta, \bar{\theta}) \right) \quad (\text{B.19}) \end{aligned}$$

or, equivalently,

$$\bar{V}_1(\theta, \bar{\theta}) = \frac{1}{\rho + \lambda} \bar{\pi}(\theta) + \frac{\lambda}{\rho + \lambda} \left(\frac{c_2(\underline{\theta}, \bar{\theta})}{c_1(\bar{\theta})} \right)^{-\gamma} \underline{V}_2(\theta, \underline{\theta}, \bar{\theta}). \quad (\text{B.20})$$

Next, we solve for the value of an inefficient unit of capital. Following the same steps as before, the value of an inefficiently allocated unit \underline{V}_1 satisfies

$$\begin{aligned} r_1(\chi) \underline{V}_1(\theta, \chi) &= \underline{\pi}(\theta) + \phi_1(\chi) \frac{\partial}{\partial \chi} \underline{V}_1(\theta, \chi) \\ &\quad + \lambda \left(\frac{c_2(\underline{\theta}, \chi)}{c_1(\chi)} \right)^{-\gamma} (\bar{V}_2(\theta, \underline{\theta}, \chi) - \underline{V}_1(\theta, \chi)). \quad (\text{B.21}) \end{aligned}$$

Zero profit requires that

$$P_1(\chi) = \bar{V}_1(\chi, \chi). \quad (\text{B.22})$$

At the instant where a firm of θ trades, it has to be locally indifferent. So, at the boundary,

$$P_1(\chi) = \bar{V}_1(\chi, \chi) = \underline{V}_1(\chi, \chi) \quad (\text{B.23})$$

and

$$\begin{aligned} & \phi_1(\chi) \left. \frac{\partial}{\partial \chi} \underline{V}_1(\theta, \chi) \right|_{\theta=\chi} \\ &= \phi_1(\chi) \frac{d}{d\chi} P(\chi) \\ &= \left(\frac{\partial}{\partial \theta} \bar{V}_1(\theta, \chi)|_{\theta=\chi} + \frac{\partial}{\partial \chi} \bar{V}_1(\theta, \chi)|_{\theta=\chi} \right) \phi_1(\chi). \quad (\text{B.24}) \end{aligned}$$

Replacing the partial derivatives with respect to χ in the left-hand side and the right-hand side using the two ODEs for \underline{V}_1 and \bar{V}_1 [i.e., Eqs. (B.18) and (B.21)], we arrive at

$$\begin{aligned} & \bar{\pi}(\chi) - \underline{\pi}(\chi) - \lambda \left(\frac{c_2(\theta, \chi)}{c_1(\chi)} \right)^{-\gamma} \\ & \quad \times (\bar{V}_2(\theta, \underline{\theta}, \chi) - \underline{V}_2(\theta, \underline{\theta}, \chi)) \\ &= \left(\frac{\partial}{\partial \theta} \bar{V}_1(\theta, \chi)|_{\theta=\chi} \right) \phi_1(\chi). \quad (\text{B.25}) \end{aligned}$$

Therefore, in equilibrium, the rate of reallocation is given by

$$\begin{aligned} & \phi_1(\chi) \\ &= \max \left\{ \frac{\bar{\pi}(\chi) - \underline{\pi}(\chi) - \lambda \left(\frac{c_2(\theta, \chi)}{c_1(\chi)} \right)^{-\gamma} (\bar{V}_2(\theta, \underline{\theta}, \chi) - \underline{V}_2(\theta, \underline{\theta}, \chi))}{\frac{\partial}{\partial \theta} \bar{V}_1(\theta, \chi)|_{\theta=\chi}}, 0 \right\}. \quad (\text{B.26}) \end{aligned}$$

B.3. Numerical solution method

The numerical solution also works by backward induction. Starting in the second regime, we first solve for v_2 using Eqs. (B.4) and (B.5). Using Eq. (B.3), we can then find the rate of reallocation in the second regime. From this, we then solve for the equilibrium value functions in the second period (\bar{V}_2 and \underline{V}_2) using Eqs. (B.6) and (B.9). Substituting equation Eq. (B.3) into Eqs. (B.6) and (B.9), we obtain two non-linear ODEs in χ , with an initial condition at $\chi = \underline{\theta}$. We solve these ODEs numerically using an explicit Runge-Kutta (4, 5) formula, implemented in Matlab's ode45 solver. Moving back to the first regime, we solve for the two value functions in the first period (\bar{V}_1 and \underline{V}_1) using the same methodology, while taking the solutions (\bar{V}_2 and \underline{V}_2) as given.

Appendix C. Data and empirical methodology

Accounting data are from Compustat. Profitability (return to assets) is net income (Compustat: ni) divided by book assets (Compustat: at). Accounting variables in year t refer to variables corresponding to fiscal year ending in calendar year t . Industry is two-digit SIC code. Data on market capitalization and stock returns are from the Center for Research in Security Prices (CRSP). Market capitalization at year t is given by the absolute value of (CRSP: prc) times (CRSP: shrout) at the end of December of year t . Stock return for year t is the mean monthly return for calendar year t , annualized by multiplying it by 12. Data

Table C1

Descriptive statistics This table presents the descriptive statistics for our variables of interest.

Descriptive statistic	Mean	SD	p10	p25	p50	p75	p90
Firm age at IPO	13.51	18.19	2.00	3.00	7.00	15.00	32.00
Book assets, log	3.74	1.70	1.58	2.62	3.72	4.76	5.91
Profitability (ROA)	−0.06	0.31	−0.37	−0.10	0.03	0.09	0.15
Market capitalization, log	11.32	1.53	9.33	10.21	11.31	12.33	13.32
Returns	0.10	1.38	−1.34	−0.61	0.01	0.67	1.56

Table C2

Firm age at IPO versus ex-post profitability and stock returns.

Horizon (year after IPO)	Profitability (ROA)			Stock returns		
	(1)	(2)	(3)	(4)	(5)	(6)
One	0.102 [10.54]	0.030 [2.82]	0.032 [2.25]	0.040 [2.04]	0.018 [1.06]	0.021 [1.32]
Two	0.124 [9.56]	0.055 [4.68]	0.057 [4.58]	0.002 [0.14]	0.005 [0.34]	0.005 [0.40]
Three	0.127 [8.76]	0.054 [4.62]	0.051 [3.37]	0.005 [0.23]	0.011 [0.73]	0.005 [0.23]
Four	0.114 [8.18]	0.043 [3.44]	0.055 [3.22]	−0.013 [−0.99]	0.001 [0.07]	0.006 [0.38]
Five	0.148 [4.64]	0.035 [1.53]	0.071 [1.81]	0.002 [0.10]	0.016 [0.98]	0.008 [0.39]
Controls						
Book assets, log		y	y		y	y
Market capitalization, log		y	y		y	y
ROA		y	y		y	y
IPO-year	y	y		y	y	
INDxIPO-year		y	y		y	y

on IPOs and firm age are from Jay Ritter's website (<http://bear.warrington.ufl.edu/ritter/ipodata.htm>). We restrict the sample to those firms with non-missing observations on profitability, size, market capitalization, industry code, and book assets on the year of the IPO, leaving 6,004 firms (IPO events) covering the period 1975 to 2012. We winsorize all variables at the 0.5% and 99.5% percentiles using annual breakpoints.

C.1. Results

This table presents estimates of the coefficient b from the following empirical specifications

$$ROA_{ft+k} = b \log(1 + A_{ft}) + cZ_{ft} + u_{ft+k} \quad (C.1)$$

and

$$R_{ft+k} = b \log(1 + A_{ft}) + cZ_{ft} + u_{ft+k}. \quad (C.2)$$

Columns (1)–(3) present results with various controls for the specification in Eq. (C.1). Columns (4)–(6) present results with various controls for the specification in Eq. (C.2). A_{ft} is age of the firm at the time of the IPO, and ROA_{fs} and R_{fs} is profitability and stock returns, respectively, for firm f in year s . We examine horizons of up to five years following the IPO, $s = t \dots t + k$. The IPO-year corresponds to year t . We include a vector of controls that, depending on the specification, includes IPO-year fixed effects, firm profitability at year t , firm market capitalization at year t , book assets at year t , and industry-specific IPO-year dummies. We include t -statistics in brackets computed using standard errors clustered by IPO-year.

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