

FINC312: Homework 2

Solution

1 Implementing Mean-Variance Analysis

1. You first need to choose an appropriate risk-free rate. I decided to use the most recent value of the risk free rate. You could have used the historical average, or better yet, the yield on a zero coupon bond that has the same maturity as your investment horizon. The expected return, standard deviations and sharpe ratios are in the table below:

	GE	GM	IBM
E(R)	1.34%	0.72%	1.41%
σ	6.26%	9.51%	8.87%
Sharpe Ratio	0.15	0.03	0.11

Remember, these are monthly numbers. If you want to get the annualized numbers, multiply expected returns by 12 and standard deviations by $\sqrt{12}$.

Assuming that the realized returns follow normal distributions, the 95% confidence interval for the average return is

$$(E(R) - 1.96 \times \text{stderr}, \quad E(R) + 1.96 \times \text{stderr})$$

You can calculate the standard errors as

$$\text{stderr} = \frac{\sigma}{\sqrt{\text{number of observations}}}$$

The correlation matrix is:

	GE	GM	IBM
GE	1.00	0.30	0.32
GM	0.30	1.00	0.24
IBM	0.32	0.24	1.00

2. Once you plug in this numbers into Markowitz, you should get something like this:

Number of securities:

No	Name	Fraction	Expected Return	Standard Deviation
1	GE	78%	0.01343	6%
2	GM	-10%	0.00720	10%
3	IBM	32%	0.01408	9%
		100.00%		

Correlations		1	2	3
1		1.0	0.3	0.3
2		0.3	1.0	0.2
3		0.4	0.2	1.0

Corr OK? YES

Results:

	0.0142
Portfolio's Standard Deviation	0.0612

Risk Free Rate Risk Aversion Coefficient: A=

Slope of CAL Weight on optimal risky portfolio: x*=

The CAL is the line that connects (0.4%,0) and (1.42%,6.12 %) .

3. Now let's go back to May 1st 1998.

a) Your estimates would look like this:

	GE	GM	IBM
E(R)	2.05%	1.09%	1.47%
σ	5.63%	7.67%	8.23%
SR	0.29	0.09	0.13

The correlation among securities also changed:

	GE	GM	IBM
GE	1.00	0.36	0.25
GM	0.36	1.00	0.30
IBM	0.25	0.30	1.00

Your optimal portfolio, using all data available as of May 1st 1998, would be this:

Number of securities:

No	Name	Fraction	Expected Return	Standard Deviation
1	GE	94%	0.02053	6%
2	GM	-9%	0.01093	8%
3	IBM	15%	0.01468	8%
		100.00%		

Correlations		1	2	3
1		1.0	0.4	0.2
2		0.3	1.0	0.3
3		0.4	0.2	1.0

Corr OK? YES

Results:

Portfolio's Expected Return	0.0205
Portfolio's Standard Deviation	0.0548

Risk Free Rate Risk Aversion Coefficient: A=

Slope of CAL Weight on optimal risky portfolio: x*=

- b) Your portfolio had an average realized return of 0.792% and a standard deviation of 6.813%. Using the risk free rate on Dec 29th 2006, you get the Sharpe Ratio of 0.0575.
- c) Hmm...we haven't done so well. GE performed very well until May 1st 1998, so our optimal portfolio puts a very high weight on GE. This makes our portfolio poorly diversified. Although all three securities did worse in the later years than in the earlier years, GE performed the worst among them.

4. The constrained portfolio would be this:

Number of securities:

No	Name	Fraction	Expected Return	Standard Deviation
1	GE	50%	0.02053	6%
2	GM	17%	0.01093	8%
3	IBM	33%	0.01468	8%
		100.00%		

Correlations		1	2	3
1		1.0	0.4	0.2
2		0.3	1.0	0.3
3		0.4	0.2	1.0

Corr OK? YES

Results:

Portfolio's Expected Return	0.0170
Portfolio's Standard Deviation	0.0506

Risk Free Rate Risk Aversion Coefficient: A=

Slope of CAL Weight on optimal risky portfolio: x*=

The constrained portfolio had an average realized return of 0.842% and a standard

deviation of 6.215%. Using the risk free rate on Dec 29th 2006, you get the Sharpe Ratio of 0.0666. The constrained portfolio did better than the unconstrained portfolio.