

# “Can Investment Shocks Explain the Cross-Section of Stock Returns?”: A Comment.

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## Abstract

A small but growing literature argues that heterogeneous exposure to capital embodied or investment-specific technology (IST) shocks earn a negative risk premium, and can account for differences in risk premia of firms with heterogeneous growth opportunities (Papanikolaou, 2011; Kogan and Papanikolaou, 2012a,b). Using the NIPA equipment price series to infer IST shocks, Garlappi and Song (2012) argue that these empirical findings are special to the post-1960 period. However, NIPA data does not adjust for quality pre-1960. Using an industry equilibrium model, we show theoretically that the quality-adjusted and non-adjusted equipment price series can be exposed to IST shocks with opposite signs. This lack of quality adjustment in pre-1960 period can reconcile the findings of Garlappi and Song (2012) with Papanikolaou (2011) and Kogan and Papanikolaou (2012a).

## 1 Introduction

Technological improvements embodied in new capital goods (IST shocks) are promising in understanding differences in risk premia among firms with different growth opportunities. In many cases, these technological improvements take the form of improvements in the quality of capital. For instance, a new laptop computer has substantially more processing power than a laptop computer purchased a decade ago, even though its dollar cost is not substantially different. Under certain conditions, the magnitude of these embodied shocks can be inferred from the growth rate of the price of new capital. For instance, the price per Gigahertz of computing power has

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declined dramatically over the last decade, signalling substantially improvements in computing technology. However, in this calculation quality adjustment is important. In particular, the nominal price of laptop computers has remained mostly flat in the last decade. Thus, if we were to infer improvements in computing technology using the non-adjusted price of equipment, we would conclude that in the last decade improvements in computing technology were minimal.

In this note, we clarify the relation between technology shocks embodied in new capital and the price of equipment. Using a standard industry equilibrium model, we show theoretically that a positive IST shock can lead to an *increase* in the non-adjusted price of equipment. Specifically, quality improvements in capital goods spur an increase in the demand for new capital, and therefore lead to an increase in its nominal price. However, the price of equipment adjusted for quality improvements is always *decreasing* in the IST shock.

Since the quality-adjusted and unadjusted series may load on IST shocks with opposite signs, inferences on the market price of IST shocks using the raw equipment price series can be misleading. For example, recent work by Garlappi and Song (2012) argues that the market price of risk of IST shocks is unstable over time, changing sign from positive over the 1930–1960 period to negative over the 1963–2010 period. Garlappi and Song conclude that investment-specific (IST) shocks are unlikely to be a priced risk factor, and that the findings of Kogan and Papanikolaou (2012a) are specific to the post-1963 period.

In their empirical analysis, Garlappi and Song use the changes in the NIPA relative price of equipment goods as a proxy for IST shocks.<sup>1</sup> Unfortunately, the NIPA equipment price series is not consistently adjusted for quality improvements in new capital goods. In particular, post 1960, NIPA measurements implement quality adjustment for certain types of equipment, such as computers. However, the pre-1960 series is not adjusted for quality changes.

To demonstrate how lack of quality adjustment can reconcile the findings of

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<sup>1</sup>Kogan and Papanikolaou use the quality-adjusted series of equipment prices of Gordon (1990), Cummins and Violante (2002) and Israelsen (2010), which implements a more systematic adjustment for equipment quality changes. Unfortunately, the quality-adjusted series of Gordon (1990), Cummins and Violante (2002) and Israelsen (2010) is not available prior to 1948, thus Garlappi and Song (2012) use the price of equipment series from NIPA. Specifically, their equation (1) defines the investment shock as

$$Ishock_t = \ln \left( \frac{P_I}{P_C} \right)_{t-1} - \ln \left( \frac{P_I}{P_C} \right)_t, \quad (1)$$

where  $P_I$  is the price deflator for equipment and software of gross private domestic investment (row 11 of NIPA table 1.1.9), and  $P_C$  is the price deflator for nondurable consumption goods (row 5 of NIPA table 1.1.9).

Garlappi and Song and Kogan and Papanikolaou, we use the general equilibrium model of Kogan, Papanikolaou, and Stoffman (2012). In addition to capital-embodied shocks, their model features a full cross-section of firms, which allows us to replicate the procedure of Garlappi and Song in simulated data. We construct measures of the IST shock in simulated data using both the adjusted and non-adjusted price of equipment. Using the benchmark calibration of Kogan et al. (2012), the model can replicate the main findings of Garlappi and Song in the 1930-1962 period. Specifically, value (high book-to-market) firms have higher loadings to the measure of the IST shock constructed using the non-adjusted price than growth (low book-to-market) firms. Constructing the IST shock using the quality-adjusted price leads to opposite results.

## 2 Quality adjustment and price of IST shocks

### 2.1 Data and measurement

Under certain restrictive conditions, the relative price of equipment can be used to infer the realizations of the IST shock: a favorable IST shocks corresponds to a decline in the *quality-adjusted* price of equipment (Greenwood, Hercowitz, and Krusell, 1997). In general, IST shocks and equipment price changes are imperfectly correlated (see e.g. Fisher, 2009), but, in the settings of Papanikolaou (2011) and Kogan et al. (2012) the correlation between IST shocks and the growth rate of the quality-adjusted price of equipment is still negative.

Quality adjustment, or lack thereof, can significantly affect the measurement of IST shocks in the data. NIPA attempts to adjust the price of equipment for quality improvements. However not all components of the price series are adjusted to the same degree (Moulton, 2001). Further, the degree of quality adjustment in the NIPA price series varies over time. Starting in 1985, the Bureau of Economic Analysis has introduced hedonic price indexes (in partnership with IBM) to control for quality changes in computers and peripherals (Justiniano, Primiceri, and Tambalotti, 2011). These adjustments have been extended back to 1959, and have been extended to several additional goods.

In summary, the NIPA price series is not consistently adjusted for quality. Further, the degree of quality adjustment differs pre- and post 1960. Consistent with this, Garlappi and Song (2012) report that the mean growth rate of the price of equipment is positive in the pre-1962 period (2.12%) and negative in the post-1962 period (-0.60%).

If we were to ignore the issue of quality adjustment, we would conclude that the pre-1960 period was a period of *negative* technical change on average.

Next, we illustrate that the relation between the raw price of equipment and IST shocks is ambiguous, and can change sign after quality adjustment.

## 2.2 An industry equilibrium model

Consider an industry equilibrium model with two sectors: a sector producing consumption goods and a sector producing capital. Financial markets are complete, and the riskless interest rate is constant at  $r$ . All the distributions below are under the risk-neutral pricing measure.

There is a single competitive representative firm in the consumption-good sector. The firm owns the entire capital stock of the industry. Capital stock is of heterogeneous quality, which is captured by  $\xi$ . The productivity of capital of vintage  $s$ ,  $\xi_s$ , follows a geometric Brownian motion

$$d\xi_t = \sigma_\xi \xi_t dZ_t. \quad (2)$$

Capital stock created at time  $s$  depreciates at rate  $\delta$ . Hence, the time- $t$  stock of the time- $s$  vintage is given by

$$K_{t,s} = e^{-\delta(t-s)} K_{s,s}. \quad (3)$$

The representative firm produces the consumption good according to a linear production function

$$Y_t = \int_{-\infty}^t \xi_s K_{t,s} ds = \int_{-\infty}^t \xi_s e^{-\delta(t-s)} K_s ds. \quad (4)$$

Thus, shocks to  $\xi_t$  are the only source of technological uncertainty. This formulation is a simplified version of the model in Kogan et al. (2012).

New capital is created by investment:

$$K_t = I_t. \quad (5)$$

New capital is produced in the investment sector using a fixed factor, land  $L$ .

$$I_t = L. \quad (6)$$

The vintage-specific shock  $\xi_s$  is embodied in new capital goods, and is isomorphic to the IST shock in Papanikolaou (2011) and Kogan et al. (2012). Specifically, define

the quality-adjusted stock of capital  $\hat{K}$  as

$$\hat{K}_t = \int_{-\infty}^t \xi_s K_{t,s} ds, \quad (7)$$

so the output of the consumption sector is

$$Y_t = \hat{K}_t. \quad (8)$$

The quality-adjusted capital stock  $\hat{K}$  grows through new investment and through technological advances captured by  $\xi$ , and depreciates at the same rate

$$d\hat{K}_t = (-\delta d\hat{K}_t + \xi_t I_t) dt. \quad (9)$$

Depending on the definition of investment, the model above is isomorphic to the formulations in Greenwood et al. (1997) and Papanikolaou (2011).<sup>2</sup> We thus refer to  $\xi$  as the IST shock.

Firms face a constant downward-sloping demand curve for the output of the industry. The spot price  $p_t^C$  of the output good is related to the quantity of output by the inverse demand curve

$$p_t^C = Y_t^{-\gamma}, \quad \text{where } \gamma \in (0, 1). \quad (10)$$

In equilibrium, consumption  $C_t$  equals output  $Y_t$ , and therefore

$$p_t^C = C_t^{-\gamma}. \quad (11)$$

Consumption firms are competitive, and there are no capital adjustment costs. As a result, the market price of new equipment  $p_t^I$  equals the marginal value of a unit of capital in the consumption sector; the marginal value of capital is equal to the present

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<sup>2</sup>If we define investment as  $I_t = K_t$ , we effectively put the investment shock in the capital accumulation equation as in Greenwood et al. (1997) since  $(-\delta d\hat{K}_t + \xi_t I_t) dt$ . Instead, if we define investment to be the change in the homogenous capital stock, net of depreciation,  $\hat{I}_t = \xi_t K_t$ , then the output of the investment sector is  $\hat{I}_t = \xi_t L$ , which is the formulation in the model of Papanikolaou (2011).

value of the cash flows produced by an additional unit of investment

$$\begin{aligned} p_t^I &= \mathbb{E}_t \left[ \int_t^\infty e^{-r(u-t)} p_u^C \xi_t du \right] = \xi_t \mathbb{E}_t \left[ \int_t^\infty e^{-r(u-t)} C_u^{-\gamma} du \right] \\ &= \xi_t \int_t^\infty e^{-r(u-t)} \mathbb{E}_t [C_u^{-\gamma}] du. \end{aligned} \quad (12)$$

Equation (12) refers to the raw price of new equipment, without any quality adjustment. The quality-adjusted price,  $\hat{p}_t^I$ , adjusts the price of equipment for improvements in quality because of changes in  $\xi_t$ :

$$\hat{p}_t^I \equiv \frac{p_t^I}{\xi_t} = \int_t^\infty e^{-r(u-t)} \mathbb{E}_t [C_u^{-\gamma}] du. \quad (13)$$

Next, we characterize the relation between the unadjusted and adjusted price of equipment (12) and (13) with the IST shock  $\xi$ .

**Proposition 1** *The unadjusted price of equipment  $p^I$  is positively related to the IST shock  $\xi$ . In contrast, the quality-adjusted price  $\hat{p}^I$  is negatively related to the IST shock  $\xi$ .*

$$\frac{\partial \ln p_t^I}{\partial \ln \xi_t} \in (0, 1), \quad \frac{\partial \ln \hat{p}_t^I}{\partial \ln \xi_t} \in (-1, 0). \quad (14)$$

**Proof:** See Appendix.

Proposition 1 states that a positive capital-embodied shock leads to an increase in the demand for capital, and therefore to an increase in the price of new equipment. However, once capital is converted into constant-quality units, its price falls. An increase in  $\xi$  implies that the economy can achieve the same level of output using a lower level of investment. Hence, quality improvements in new vintages of capital lead to a decline in the quality-adjusted price  $\hat{p}_t^I$ .

The model in this section illustrates that adjusting the price of new equipment for improvements in quality is crucial along several dimensions. First, the unadjusted price series may be positively rather than negatively correlated with IST shocks, as we demonstrate in the Proposition above. Second, imperfect adjustment for quality improvements can lead to misleading inferences about the volatility of the true underlying shocks. In particular, NIPA computes the price of equipment as a weighted average across different categories of investment goods. If not all of these categories are consistently adjusted for quality, then by taking a cross-sectional average, NIPA is effectively averaging over some variables that are increasing in  $\xi$  and other variables

that are decreasing in  $\xi$ . Hence, the resulting series could be substantially less volatile than the true fundamental disturbance  $\xi$ .

Constructing the IST shock while ignoring the issue of quality adjustment may account for some of the findings of Garlappi and Song (2012). In particular, their finding that the relation between the NIPA price of equipment and the stock portfolios sorted on the market-to-book ratios differs in sign between the pre- and post-1960 periods may be driven by the lack of quality adjustment in the pre-1960 sample. In addition, their finding that the mean growth rate of the unadjusted NIPA relative price series is positive in 1930-1960 is also consistent with a *positive* IST shock during that period.

## 2.3 Results from a general equilibrium model

In our industry equilibrium model above, the supply of investment goods is fixed, as it is determined by the supply of a fixed factor of production. More generally though, the supply of investment goods may respond positively to an increase in demand due to quality improvements. As a result, the increase in the non-adjusted price of equipment will be dampened. In this section, we use the general equilibrium model of Kogan et al. (2012), which in addition to capital-embodied shocks, features endogenous aggregate investment, as well as a full cross-section of firms. The latter component is important, as it allows us to study the differential exposure of firms with different book-to-market ratios to two different measures of IST shocks: i) the correct measure derived from the quality-adjusted price  $-\Delta \ln \hat{p}^I$ ; and ii) a measure that ignores improvements in quality  $-\Delta \ln p^I$ .

We use the benchmark calibration in Kogan and Papanikolaou (2012a) and the empirical procedure of Kogan and Papanikolaou (2012a) and Garlappi and Song (2012). Specifically, we simulate 3,000 firms and sort them into 10 portfolios at the end of every year based on their ratio of book value of capital to their market value. We report the stock return exposure of the decile portfolios with  $-\Delta \ln \hat{p}^I$  and  $-\Delta \ln p^I$ . We consider samples of 80 years, and repeat the procedure 1,000 times. We report the median exposure estimate across simulations, along with the 5% and 95% percentiles.

We show the results in Table 1. As we see in the first row, constructing the investment shock using the quality-adjusted price  $-\Delta \ln \hat{p}^I$  results in a decreasing pattern of exposures to the IST shock across B/M portfolios. Specifically, a positive IST shock – a decline in the quality-adjusted price  $\hat{p}^I$  – has a more negative impact on the stock returns of value firms (high B/M) than growth firms (low B/M). This pattern

of stock return exposures is consistent with the findings of Kogan and Papanikolaou (2012a), as well as the findings of Garlappi and Song (2012) in the post-1960 period.

In the second row of Table 1, we construct the investment shock ignoring quality adjustment,  $-\Delta \ln p^I$ . In this case, there seems to be an *increasing* pattern of exposures to the IST shock across B/M portfolios, resembling the findings of Garlappi and Song (2012) in the pre-1960 period, when the price of equipment is not adjusted for quality improvements. In the model, this increasing pattern is an artifact of imperfect quality adjustment: the exposure to the true capital embodied shock  $\xi$  is decreasing across book-to-market portfolios.



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# Appendix

## Proof of Proposition 1

First, we need the following mathematical fact.

**Fact 1** If

$$\frac{\partial \ln g(x, \alpha)}{\partial \ln x} \in (a, b), \quad (15)$$

then

$$\frac{\partial \ln \int_{\alpha'}^{\alpha''} g(x, \alpha) d\alpha}{\partial \ln x} \in (a, b). \quad (16)$$

**Proof:**

$$\begin{aligned} \frac{\partial \ln \int_{\alpha'}^{\alpha''} g(x, \alpha) d\alpha}{\partial \ln x} &= \frac{\int_{\alpha'}^{\alpha''} \frac{\partial g(x, \alpha)}{\partial \ln x} d\alpha}{\int_{\alpha'}^{\alpha''} g(x, \alpha) d\alpha} \\ &= \frac{\int_{\alpha'}^{\alpha''} \frac{\partial \ln g(x, \alpha)}{\partial \ln x} g(x, \alpha) d\alpha}{\int_{\alpha'}^{\alpha''} g(x, \alpha) d\alpha} \\ &= \int_{\alpha'}^{\alpha''} \frac{\partial \ln g(x, \alpha)}{\partial \ln x} \underbrace{\left( \frac{g(x, \alpha)}{\int_{\alpha'}^{\alpha''} g(x, \tilde{\alpha}) d\tilde{\alpha}} \right)}_{w(x, \alpha)} d\alpha \in (a, b), \end{aligned}$$

because  $\int_{\alpha'}^{\alpha''} w(x, \alpha) d\alpha = 1$ . ■

Next, consider the properties of

$$\mathbb{E}_t \left[ (C_u)^{-\gamma} \right] = \mathbb{E}_t \left[ \left( \hat{K}_t + \int_t^u \xi_s L ds \right)^{-\gamma} \right] = \mathbb{E}_t \left[ \left( \hat{K}_t + \xi_t \int_t^u L e^{-\frac{\sigma^2}{2}(s-t) + \sigma(Z_s - Z_t)} ds \right)^{-\gamma} \right]. \quad (17)$$

Denote the resulting expression by  $f(\hat{K}_t, \xi_t; u - t)$ . Note that

$$\frac{\partial \ln \left( \hat{K}_t + \xi_t \int_t^u L e^{-\frac{\sigma^2}{2}(s-t) + \sigma(Z_s - Z_t)} ds \right)^{-\gamma}}{\partial \ln \xi_t} = \frac{-\gamma \xi_t \int_t^u L e^{-\frac{\sigma^2}{2}(s-t) + \sigma(Z_s - Z_t)} ds}{\left( \hat{K}_t + \xi_t \int_t^u L e^{-\frac{\sigma^2}{2}(s-t) + \sigma(Z_s - Z_t)} ds \right)} \in (-\gamma, 0). \quad (18)$$

Then, using Fact 1 above,

$$\frac{\partial \ln f(\hat{K}_t, \xi_t; u - t)}{\partial \ln \xi_t} \in (-\gamma, 0), \quad (19)$$

and, using Fact 1 again,

$$\frac{\partial \ln \int_t^\infty \mathbb{E}_t \left[ e^{-r(u-t)} (C_u)^{-\gamma} \right] du}{\partial \ln \xi_t} \in (-\gamma, 0). \quad (20)$$

Since  $\gamma \in (0, 1)$ , we conclude that

$$\frac{\partial \ln p_t^I}{\partial \ln \xi_t} \in (0, 1), \quad \frac{\partial \ln \hat{p}_t^I}{\partial \ln \xi_t} \in (-1, 0). \quad (21)$$

■

Table 1: Univariate IST-shock exposures of the book-to-market decile portfolios: the importance of quality-adjustment

BM-sort	Lo	2	3	4	5	6	7	8	9	10	10ml
$-\Delta \ln \hat{p}_t^I$	-1.06 [-2.13, 0.46]	-1.31 [-2.32, 0.15]	-1.37 [-2.39, 0.07]	-1.39 [-2.40, -0.00]	-1.47 [-2.42, -0.12]	-1.61 [-2.55, -0.27]	-1.74 [-2.71, -0.40]	-1.86 [-2.86, -0.56]	-2.00 [-2.99, -0.69]	-2.18 [-3.17, -0.89]	-1.12 [-1.43, -0.92]
$-\Delta \ln p_t^I$	7.04 [5.33, 8.45]	7.05 [5.27, 8.65]	7.06 [5.35, 8.68]	7.06 [5.41, 8.63]	7.07 [5.39, 8.76]	7.05 [5.34, 8.78]	7.09 [5.26, 8.89]	7.11 [5.28, 9.03]	7.17 [5.37, 9.24]	7.24 [5.24, 9.48]	0.22 [-1.15, 1.73]

Output of the general equilibrium model in Kogan et al. (2012). The table shows the loadings of book-to-market decile portfolio returns on equipment price changes. The top row corresponds to quality-adjusted investment good prices, and the bottom row corresponds to the raw investment good prices. The point estimates are the median numbers across the 1,000 simulations, and the numbers in brackets are the estimated 90% confidence intervals, with the end points corresponding to the 5% and 95% percentiles across the simulated samples.