Growth Opportunities, Investment-Specific Technology Shocks and the Cross-Section of Expected Returns*

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Abstract

The market value of a firm can be decomposed into two fundamental parts: the value of assets in place and the value of future growth opportunities. We propose a theoretically-motivated procedure for measuring heterogeneity in growth opportunities across firms. We identify firms with high growth opportunities based on the covariance of their stock returns with the investment-specific productivity shock. We find that, empirically, our procedure is able to identify economically significant and theoretically consistent differences in firms' investment behavior, as well as risk and risk premia in their stock returns. Our empirical findings are quantitatively consistent with a calibrated structural model of firms' growth.

1 Introduction

The market value of a firm can be decomposed into two fundamental parts: the value of assets in place and the value of future growth opportunities. If the systematic risk of growth opportunities differs from that of assets in place, heterogeneity in firms' growth option shares could help explain observed cross-sectional differences in stock returns. This basic observation underpins many of the theoretical models connecting firms' characteristics

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to the properties of their stock returns. Successful applications of this idea depend on the quality of empirical measures of growth opportunities. We propose a theoretically-motivated procedure for measuring growth option heterogeneity and document its empirical properties. We find that, empirically, our procedure is able to identify economically significant and theoretically consistent differences in firms' investment behavior, as well as risk and risk premia in their stock returns.

The literature on real determinants of economic growth has documented that a significant fraction of observed growth variability can be attributed to productivity shocks in the capital goods sector [Greenwood, Hercowitz and Krusell, 1997; Fisher, 2006]. Under certain assumptions, one can identify such shocks with the price of investment equipment. Greenwood et al. (1997) show that the historical series of investment-goods prices is negatively correlated with aggregate investment, both at business-cycle and lower frequencies. Our theoretical model predicts (see Proposition 2 below) that stock returns of firms for which growth options account for a relatively large fraction of their market value (high-growth firms) respond more to the investment-specific productivity shocks (I-shocks). Our empirical procedure is based on this intuition, relating unobservable asset composition (growth options relative to assets in place) to observed differences in stock price sensitivity to the I-shocks.

We sort firms on their stock return sensitivity to the I-shocks. Since the data on investment-goods prices is available only at the annual frequency, we replace the original I-shock series with returns on a factor-mimicking portfolio, constructed as a zero-investment portfolio long the stocks of investment-good producers and short the stocks of consumption-good producers (IMC). We identify a positive I-shock as a decline in prices of investment goods.

Since growth options are not directly observable, we use indirect metrics to assess the success of our procedure. In particular, the key metric is the response of firms' investment to the I-shock. Theoretically, firms with more growth options should invest relatively more in response to a positive I-shock. Moreover, in most standard models, high-growth firms tend to have higher Tobin's Q, higher average investment rates, and higher market betas. While none of the above relationships identify growth options perfectly, collectively, they provide a useful consistency check for any empirical measure of growth options.

We find that the mimicking portfolio betas (β_{IMC}) are able to identify heterogeneity in firms' investment and stock return sensitivity to the I-shocks. High-β_{IMC} firms invest more on average and their investment increases more in response to a unit decline in investment-goods prices. Economically, these effects are significant. The difference in investment-goods price sensitivity between the high-beta and the low-beta firms is three to five times larger than the sensitivity of an average firm. The average investment rate of low-beta firms is twenty percent less of that of the high-beta firms. Moreover, high-β_{IMC} firms tend to have higher Tobin's Q and higher market beta. These findings support our conjecture that β_{IMC} is a valid empirical proxy for the I-shock beta and that the latter captures cross-sectional differences in growth options across firms.

We show that our empirical findings are quantitatively consistent with a parsimonious structural model of investment. In our partial-equilibrium model, firms derive value from implementing positive-NPV projects, which arrive randomly. The price of capital goods varies stochastically, affecting firms' investment choices. Randomness in project arrival and expiration leads to cross-sectional heterogeneity in the firms' mix of growth opportunities and assets in place. Our model matches the key qualitative and quantitative features of the empirical data, including cross-sectional differences in firms' response to investment-specific shocks and their risk premia. In particular, we find that the beta of stock returns with respect to the investment-specific shock positively predicts firms' investment rates as well as the sensitivity of their investment to such shocks. These effects are consistent in magnitude with the corresponding empirical estimates. In our model, high-β_{IMC} stocks have relatively high market-to-book ratios (Tobin's Q), however, the investment-shock betas contain information about the firms' asset mix which is not reflected their Tobin's Q. We find that the cross-sectional differences in stock returns between portfolios sorted by investment-shock betas and market-to-book ratios are quantitatively similar to the data.

The rest of the paper is organized as follows. Section 2 relates our paper to existing work. In Section 3 we present the structural model of investment. Section 4 presents empirical results. In Section 5 we evaluate our model quantitatively using calibration.

2 Relation to the Literature

Our paper bridges and complements two distinct strands of the macroeconomic and finance literature. The first argues for the importance of investment-specific shocks for aggregate quantities and the second argues that differences in firm's mix between growth options and assets are important in understanding the cross-section of risk premia.

In macroeconomics, a number of studies have shown that investment-specific technological shocks can account for a large fraction of the variability output and employment, both in the long-run, as well as at business cycle frequencies [Greenwood et al., 1997; Greenwood, Hercowitz and Krusell, 2000; Boldrin, Christiano and Fisher, 2001; Fisher, 2006; Justiniano, Giorgio and Tambalotti, 2008]. Investment shocks can be modelled as either shocks to the marginal cost of capital as in Solow (1960) or as shocks to the productivity of a sector producing capital goods as in Rebelo (1991) or Boldrin et al. (2001). Given that investment shocks lead to an improvement in the real investment opportunity set in the economy, they are a natural place to start to understand the heterogeneity in the risk of growth options versus assets in place.

In financial economics, several studies have argued that decomposing value into assets in place versus growth opportunities may be useful in understanding the cross-section of risk premia [Berk, Green and Naik, 1999; Gomes, Kogan and Zhang, 2003; Carlson, Fisher and Giammarino, 2004; Zhang, 2005; Gala, 2006]. These studies argue that assets in place are riskier than growth options in bad times, which combined with a counter-cyclical price of risk may lead to value firms having higher returns on average than growth firms. Our work complements this literature by illustrating how a different mechanism can generate differences in risk premia between assets in place and growth options. Papanikolaou (2008) shows that in a two-sector general equilibrium model, investment shocks can generate a value premium. On the other hand, there is no other source of firm heterogeneity in his model, whereas we explicitly model firm heterogeneity in terms of the mix between growth options and assets in place.

Our work is also connected to the investment literature that links Tobin's Q, a measure of growth opportunities to firm investment. In order to generate a non-zero value for growth opportunities, some investment friction is often assumed such as convex or fixed adjustment costs, or investment irreversibility [Hayashi, 1982; Abel, 1985; Abel and Eberly, 1994; Abel and Eberly, 1996; Abel and Eberly, 1998; Eberly, Rebelo and Vincent, 2008]. In these models, marginal Q measures the valuation of an additional unit of capital invested in the firm, which in the finance literature is closely linked to the notion of growth options. Tobin's Q is often proxied by the market value of capital divided by it's historical cost. We contribute to this literature by introducing a new empirical measure of growth opportunities that relies on stock price changes rather than levels.

3 The Model

In this section we develop a structural model of investment. We show that the value of assets in place and the value of growth opportunities have different exposure to the investment-specific productivity shocks. Thus, the relative weight of growth opportunities in a firm's value can be identified by measuring the sensitivity of its stock returns to investment-specific shocks.

There are two sectors in our model, the consumption-good sector, and the investment-good sector. Investment-specific shocks enter the production function of the investment-good sector. We focus on heterogeneity in growth opportunities among consumption-good producers.

3.1 Consumption-Good Producers

There is a continuum of measure one of infinitely lived firms producing a homogeneous consumption good. Firms behave competitively and there is no explicit entry or exit in this sector.

Assets in Place

Each firm owns a finite number of individual projects. Firms create projects over time through investment, and projects expire randomly.[1] Let $\mathcal{F}$ denote the set of firms and $\mathcal{J}^{(f)}$ the set of projects owned by firm f.

Project j managed by firm f produces a flow of output equal to

$$y_{fjt} = \varepsilon_{ft} u_{jt} x_t K_j^{\alpha}, \tag{1}$$

where K_j is physical capital chosen irreversibly at the project j's inception date, u_{jt} is the project-specific component of productivity, ε_{ft} is the firm-specific component of productivity, such as managerial skill of the parent firm, and x_t is the economy-wide productivity shock affecting output of all existing projects. We assume decreasing returns to scale at the project level, $\alpha \in (0,1)$. Projects expire according to independent Poisson processes with the same arrival rate δ.

The three components of projects' productivity evolve according to

$$\begin{aligned}
d\varepsilon_{ft} &= -\theta_{\epsilon}(\varepsilon_{ft} - 1)\,dt + \sigma_e \sqrt{\varepsilon_{ft}}\,dB_{ft} \\
du_{jt} &= -\theta_u(u_{jt} - 1)\,dt + \sigma_u \sqrt{u_{jt}}\,dB_{jt} \\
dx_t &= \mu_x\, x_t\,dt + \sigma_x x_t\,dB_{xt},
\end{aligned}$$

[1] Firms with no current projects may be seen as firms that temporarily left the sector. Likewise, idle firms that begin operating a new project can be viewed as new entrants. Thus, our model implicitly captures entry and exit by firms.

where dB_{ft}, dB_{jt} and dB_{xt} are independent standard Brownian motions. All idiosyncratic shocks are independent of the aggregate shock, $dB_{ft} \cdot dB_{xt} = 0$ and $dB_{jt} \cdot dB_{xt} = 0$. The firm and project-specific components of productivity are stationary processes, while the process for aggregate productivity follows a Geometric Brownian motion, generating long-run growth.

Investment

Firms acquire new projects exogenously according to a Poisson process with a firm-specific arrival rate λ_{ft}. The firm-specific arrival rate of new projects is

$$\lambda_{ft} = \lambda_f \cdot \tilde{\lambda}_{f,t} \tag{2}$$

where $\tilde{\lambda}_{ft}$ follows a two-state, continuous time Markov process with transition probability matrix between time t and $t + dt$ given by

$$P = \begin{pmatrix} 1 - \mu_L\, dt & \mu_L\, dt \\ \mu_H\, dt & 1 - \mu_H\, dt \end{pmatrix}. \tag{3}$$

We label the two states as $[\lambda_H, \lambda_L]$, with $\lambda_H > \lambda_L$. Thus, at any point in time, a firm can be either in the high-growth $(\lambda_f \cdot \lambda_H)$ or in the low-growth state $(\lambda_f \cdot \lambda_L)$, and $\mu_H\, dt$ and $\mu_L\, dt$ denote the instantaneous probability of entering each state respectively. We impose that $E[\tilde{\lambda}_{f,t}] = 1$, which translates to the restriction

$$1 = \lambda_L + \frac{\mu_H}{\mu_H + \mu_L}(\lambda_H - \lambda_L) \tag{4}$$

When presented with a new project at time t, a firm must make a take-it-or-leave-it decision. If the firm decides to invest in a project, it chooses the associated amount of capital K_j and pays the investment cost $z_t x_t K_j$. The cost of capital relative to it's average productivity, z_t, is assumed to follow a Geometric Brownian motion

$$dz_t = \mu_z z_t\, dt + \sigma_z z_t\, dB_{zt}, \tag{5}$$

where $dB_{zt} \cdot dB_{xt} = 0$. The z shock represents the component of the price of capital that is unrelated to it's current level of average productivity, x, and is the investment-specific shock in our model. Finally, at the time of investment, the project-specific component of productivity is at its long-run average value, $u_{jt} = 1$.

Valuation

Let π_t denote the stochastic discount factor. The time-0 market value of a cash flow stream C_t is then given by $\mathrm{E}\left[\int_0^\infty (\pi_t/\pi_0) C_t \, dt\right]$. For simplicity, we assume that the aggregate productivity shocks x_t and z_t have constant prices of risk β_x, β_z, and the risk-free interest rate r is also constant. Then,

$$\frac{d\pi_t}{\pi_t} = -r \, dt - \beta_x \, d\, B_{x,t} - \beta_z \, d\, B_{z,t}. \tag{6}$$

This form of the stochastic discount factor is motivated by a general equilibrium model with with investment-specific technological shocks in Papanikolaou (2008). In Papanikolaou (2008), states with low cost of new capital are high marginal valuation states because of improved investment opportunities. This is analogous to a positive value of β_z. Our analysis below shows that empirical properties of stock returns imply a positive value of β_z. Finally, we choose a price of risk of the aggregate productivity shock x is positive, which is consistent with most equilibrium models and empirical evidence.

Firms' investment decisions are based on a tradeoff between the market value of a new project and the cost of physical capital. The time-t market value of an existing project j, $p(\varepsilon_{ft}, u_{jt}, x_t, K_j)$, is computed using the discounted cash flow formula:

$$p(\varepsilon_{ft}, u_{jt}, x_t, K_j) = \mathrm{E}_t \left[\int_t^\infty e^{-\delta(s-t)} \frac{\pi_s}{\pi_t} \varepsilon_{fs} u_{js} x_s K_j^\alpha \, ds \right] = A(\varepsilon_{ft}, u_{jt}) x_t K_j^\alpha, \tag{7}$$

where

$$A(\varepsilon, u) = \frac{1}{r+\delta-\mu_X} + \frac{1}{r+\delta-\mu_X+\theta_e}(\varepsilon-1) + \frac{1}{r+\delta-\mu_X+\theta_u}(u-1) + \frac{1}{r+\delta-\mu_X+\theta_e+\theta_u}(\varepsilon-1)(u-1)$$

Firms' investment decisions are straightforward because the arrival rate of new projects is exogenous and does not depend on their previous decisions. Thus, optimal investment decisions are based on the NPV rule. Firm f chooses the amount of capital K_j to invest in project j to maximize

$$p(\varepsilon_{ft}, u_{jt}, x_t, K_j) - z_t x_t K_j$$

Lemma 1 *The optimal investment K_j in project j, undertaken by firm f at time t is*

$$K^*(\varepsilon_{ft}, z_t) = \left(\frac{\alpha A(\varepsilon_{ft}, 1)}{z_t}\right)^{\frac{1}{1-\alpha}}.$$

The scale of firm's investment depends on firm-specific productivity, ε_{ft}, and the price of investment goods relative to average productivity, z_t. Because our economy features decreasing returns to scale at the project level, it is always optimal to invest a positive and finite amount.

The value of the firm can be computed as a sum of market values of its existing projects and the present value of its growth opportunities. The former equals the present value of cash flows generated by existing projects. The latter equals the expected discounted NPV of future investments. Following the standard convention, we call the first component of firm value *the value of assets in place*, VAP_{ft}, and the second component *the present value of growth opportunities*, $PVGO_{ft}$. The value of the firm then equals

$$V_{ft} = VAP_{ft} + PVGO_{ft}$$

The value of a firm's assets in place is simply the value of its existing projects:

$$VAP_{ft} = \sum_{j \in \mathcal{J}_f} p(e_{ft}, u_{jt}, x_t, K_j) = x_t \sum_{j \in \mathcal{J}_f} A(\varepsilon_{ft}, u_{j,t}) K_j^{\alpha}.$$

The present value of growth options is given by the following lemma.

Lemma 2 *The value of growth opportunities for firm i*

$$PVGO_{ft} = z_t^{\frac{\alpha}{\alpha-1}} \, x_t \, G(\varepsilon_{ft}, \lambda_{ft})$$

$$\begin{aligned} G(\varepsilon_{ft}, \lambda_{ft}) &= C \cdot E_t \left[\int_t^{\infty} e^{-\rho(s-t)} \, \lambda_{fs} \, A(\varepsilon_{fs})^{\frac{1}{1-\alpha}} \, ds \right] \\ &= \lambda_f \left(G_1(\varepsilon_{ft}) + \frac{\mu_L}{\mu_L + \mu_H} (\lambda_H - \lambda_L) \, G_2(\varepsilon_{ft}) \right), \qquad \tilde{\lambda}_{ft} = \lambda_H \\ & \lambda_f \left(G_1(\varepsilon_{ft}) - \frac{\mu_H}{\mu_L + \mu_H} (\lambda_H - \lambda_L) \, G_2(\varepsilon_{ft}) \right), \qquad \tilde{\lambda}_{ft} = \lambda_L, \end{aligned}$$

where

$$\rho = r + \frac{\alpha}{1-\alpha}(\mu_z - \sigma_z^2/2) - \mu_x - \frac{\alpha^2 \sigma_z^2}{2(1-\alpha)^2},$$

and

$$C = \alpha^{\frac{1}{1-\alpha}} \left(\alpha^{-1} - 1 \right).$$

The functions $G_1(\varepsilon)$ and $G_2(\varepsilon)$ solve

$$\begin{aligned} C \cdot A(\varepsilon, 1)^{\frac{1}{1-\alpha}} - \rho \, G_1(\varepsilon) - \theta_\epsilon(\varepsilon - 1) \frac{d}{d\varepsilon} G_1(\varepsilon) + \frac{1}{2} \sigma_e^2 \, \varepsilon \, \frac{d^2}{d\varepsilon^2} G_1(\varepsilon) &= 0 \\ C \cdot A(\varepsilon, 1)^{\frac{1}{1-\alpha}} - (\rho + \mu_H + \mu_L) \, G_2(\varepsilon) - \theta_\epsilon(\varepsilon - 1) \frac{d}{d\varepsilon} G_2(\varepsilon) + \frac{1}{2} \sigma_e^2 \, \varepsilon \, \frac{d^2}{d\varepsilon^2} G_2(\varepsilon) &= 0. \end{aligned}$$

In addition to the aggregate and firm-specific productivity, the present value of growth opportunities depends on the investment-specific shock, z, because the net present value of future projects depends on the cost of new investment. In summary, the firm value in our model is

$$V_{ft} = VAP_{ft} + PVGO_{ft} = x_t \sum_j A(\varepsilon_{ft}, u_{jt}) K_j^{\alpha} + z_t^{\frac{\alpha}{\alpha-1}} x_t G(\varepsilon_{ft}, \lambda_{ft}) \tag{8}$$

Risk and Expected Returns

Both assets in place and growth opportunities have constant exposure to systematic shocks dB_{xt} and dB_{zt}. However, their betas with respect to the productivity shocks are different. The value of assets in place is independent of the investment-specific shock and loads only on the aggregate productivity shock. The present value of growth option depends positively on aggregate productivity, and negatively on the unit cost of new capital. Thus, firm's betas with respect to the aggregate shocks are time-varying, and depend linearly on the fraction of firm value accounted for by growth opportunities. Since, by assumption, the price of risk of aggregate shocks is constant, expected excess return on a firm is an affine function of the weight of growth opportunities in firm value, as shown in the following proposition.

Proposition 1 *The expected excess return on firm f is*

$$ER_{ft} - r_f = \beta_x \sigma_x - \frac{\alpha}{1-\alpha} \beta_z \sigma_z \frac{PVGO_{ft}}{V_{ft}} \tag{9}$$

Many existing models of the cross-section of stock returns generate an affine relationship between expected stock return and firms' asset composition similar to (9). It is easy to see, in the context of our model, how the relationship (9) can give rise to a value premium. Assume that both prices of risk β_x and β_z are positive, which we justify in the following sections. Then growth firms, which derive a relatively large fraction of their value from growth opportunities, have relatively low expected excess returns because of their exposure to investment-specific shocks. To the extent that firms' book-to-market (B/M) ratios are partially driven by the value of firms' growth opportunities, firms with high B/M ratios tend to have higher average returns than firms with low B/M ratios.

3.2 Investment-Good Producers

There is a continuum of firms producing new capital goods. We assume that these firms produce the demanded quantity of capital goods at the current unit price z_t. We assume that profits of investment firms are a fraction ϕ of total sales of new capital goods.[2] Consequently, profits accrue to investment firms at a rate of $\Pi_t = \phi z_t \, x_t \, \overline{\lambda} \int_{\mathcal{F}} K_{ft} df$, where $\overline{\lambda} = \int_{\mathcal{F}} \lambda_{ft}$ is the average arrival rate of new projects among consumption-good producers. Even though λ_{ft} is stochastic, it has a stationary distribution, so $\overline{\lambda}$ is a constant.

Lemma 3 *The price of the investment firm satisfies*

$$V_{I,t} = \phi \Gamma \, x_t \, z_t^{\frac{\alpha}{\alpha-1}} \frac{1}{\rho_I} \tag{10}$$

where we assume

$$\rho_I \equiv r - \mu_X + \frac{\alpha}{1-\alpha} \mu_Z - \frac{1}{2} \frac{\alpha}{1-\alpha} \sigma_Z^2 - \frac{1}{2} \frac{\alpha^2 \, \sigma_Z^2}{(1-\alpha)^2} > 0$$

and

$$\Gamma \equiv \phi \, \overline{\lambda} \, \alpha^{\frac{1}{1-\alpha}} \left(\int A(e_f, 1)^{\frac{1}{1-\alpha}} df \right).$$

The value of the investment firms will equal the present value of their cashflows. If we assume that these firms incur proportional costs of producing their output, and given that the market price of risk is constant for the two shocks, their value will be proportional to cashflows or the aggregate investment expenditures in the economy. The stock returns of the investment firms will then load on the investment shock (z) as well as the common productivity shock (x).

We define an IMC portfolio in the model as a portfolio that is long the investment sector and short the consumption sector. The beta of firm f with respect to the IMC portfolio

[2] Alternatively, one can specify a production function of investment firms so that z_t is a market clearing price and their profit is a fraction ϕ of sales.

return is given by

$$\beta_{ft}^{imc} = \frac{cov_t(R_{ft}, R_t^I - R_t^C)}{var_t(R_t^I - R_t^C)}$$

where $R_t^I - R_t^C$ is the return on the IMC portfolio.

Proposition 2 *The beta of firm i with respect to the IMC portfolio return is given by*

$$\beta_{ft}^{IMC} = \beta_{0t} \left(\frac{PVGO_{ft}}{V_{ft}} \right) \tag{11}$$

where

$$\beta_{0t} = \frac{\overline{V}_t}{\overline{VAP}_t}$$

Proposition 2 is the basis of our empirical approach to measuring growth opportunities. The covariance of firm i's return with respect to the IMC portfolio return is proportional to the fraction of firm i's value represented by its growth opportunities. Firms that have few active projects but expect to create many projects in the future derive most of their value from their future growth opportunities. These firms are anticipated to increase their investment in the future, and their stock price reflects that. There is also an aggregate term in (11) that depends on the fraction of aggregate value that is due to growth opportunities, which affects the IMC portfolio's correlation with the z-shock.

4 Empirical Analysis

Our analysis in Section 3 suggests that the firm-specific measure of I-shock sensitivity could be used to measure growth opportunities as a fraction of firm value. Our theoretical model also predicts that returns on the IMC portfolio, which is long the stocks of investment-good producers and short the stocks of consumption-good producers, should be a valid proxy for the investment-specific shocks. In this section, we investigate empirically these two predictions of the model. We verify that IMC returns can be used to detect I-shock sensitivity at the firm level, and that cross-firm heterogeneity in I-shock sensitivity is correlated with heterogeneity in growth opportunities.

4.1 The Estimation Procedure

Investment-specific shocks

We use the quality-adjusted price of new equipment constructed by Cummins and Violante (2002) to measure investment-specific shocks. This series, and its predecessor constructed by Gordon (1990), have been used for this purpose in the real business cycle literature [Greenwood et al., 1997; Greenwood et al., 2000; Fisher, 2006]. To estimate innovations in this price series, we first remove the low frequency components via the HP-filter (Hodrick and Prescott (1997)), and then fit an AR(1) model to the resulting series using the entire sample. Innovation shocks are identified as the residuals from this AR(1) model. Thus, our measure of investment-specific technology shocks (Δz_t) is

$$\Delta z_t = p_t^{hp} - 0.741\, p_{t-1}^{hp}, \tag{12}$$

where p_t^{hp} is the HP-detrended logarithm of the quality-adjusted price of new equipment relative to the NIPA personal consumption deflator from Cummins and Violante (2002). Innovations in investment technology lead to a decline in the quality-adjusted price of new equipment, therefore we refer to a negative realization of Δz_t as a positive investment-specific shock.

Cummins and Violante (2002) extrapolate the quality adjustment of Gordon (1990) to construct a price series for the period 1943-2000. As a robustness check, we also estimate equation (12) using the raw relative price series (without HP-filtering) but including a time trend. The estimates of z_t obtained using these two alternative methods are more than 90% correlated and the results using either price series are quantitatively very similar. To save space, we only report the results based on the definition (12).

We observe I-shocks at annual frequency, and therefore cannot estimate the I-shock sensitivity of stock returns using the Δz_t series directly. We use instead the IMC portfolio introduced in Section 3, which serves as a mimicking portfolio for the investment-specific shocks. Here we follow Papanikolaou (2008), who also uses IMC returns as a factor-mimicking portfolio for investment-specific shocks. We classify firms as producing either investment or consumption goods according to the NIPA Tables, matching firms to industries according to NAICS codes. Gomes, Kogan and Yogo (2008) and Papanikolaou (2008) describe the details of this classification procedure.

The empirical relationship between the IMC portfolio returns and our measure of investment-specific shocks (Δz_t) is

$$-\Delta z_t = \underset{(1.85)}{0.0191} \quad R_t^{imc} + \underset{(2.41)}{0.0526} \quad R_{t-1}^{imc} + e_t \qquad R^2 = 11.4\%, \quad (13)$$

where we estimate Equation 13 in the 1960-2001 period and use Newey-West standard errors with 1 lag.

Estimation of β^{imc}

Following our model, we use the firm's beta with respect to the IMC portfolio returns as a measure of this firm's investment-specific shock sensitivity. For every firm in Compustat, we estimate a time-series of (β_{ft}^{imc}) from the following regression

$$r_{ftw} = \alpha_{ft} + \beta_{ft}^{imc} r_{tw}^{imc} + \varepsilon_{ftw}, \qquad w = 1 \dots 52. \quad (14)$$

Here r_{ftw} refers to the (log) return of firm f in week w of year t, and r_{ftw}^{imc} refers to the log return of the IMC portfolio in week w of year t. Thus, β_{ft}^{imc} is constructed using information only in year t.

We drop firms with fewer than 42 weekly stock-return observations per year, firms in the investment sector, financial firms (SIC codes 6000-6799), firms with missing values of CAPEX (Compustat item 128), PPE (Compustat item 8), Tobin's Q, CRSP market capitalization, firms whose investment rate exceeds 1 in absolute value, firms with Tobin's Q greater than 100, firms with negative book values and firms where the ratio of cashflows to capital exceeds 5 in absolute value. Our final sample contains 6,343 firms and 52,845 firm-year observations.

4.2 Empirical Findings

Summary statistics

Every year we split the universe of firms in the consumption-good sector into 10 portfolios based on their estimate of β^{imc}. Table 2 reports the summary statistics for firms in different β^{imc}-deciles. The patterns across the deciles are consistent with our interpretation of β^{imc} as measuring growth opportunities. High-β^{imc} firms tend to have higher investment rates (24.8% vs 19.4% for the low-β^{imc} firms), higher Tobin's Q (1.86 vs 1.48), higher R&D expenditures (6.3% vs 2.9%), and pay less in dividends (24% vs 15%), although the latter relationship is hump-shaped.

Response of firm-level investment to I-shocks

Since growth opportunities are not directly observable, it is difficult to assess the validity of an empirical proxy. However, if firms with high growth opportunities are indeed better positioned to take advantage of positive investment-specific shocks, they should increase investment more in response to a positive investment shock than firms with low growth opportunities.

We estimate sensitivity of firms' investment to the I-shocks using the following econometric specification:

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d \, D(\beta^{imc}_{f,t-1})_d + b_1 \, (-\Delta z_{t-1}) + \sum_{d=2}^{5} b_d \, D(\beta^{imc}_{f,t-1})_d \times (-\Delta z_{t-1}) + cX_{f,t-1} + \gamma_f + u_t, \tag{15}$$

where $i_t \equiv \frac{I_t}{K_{t-1}}$ is the firm's investment rate, defined as Capital Expenditures (Compustat item 128) over Property Plant and Equipment (Compustat item 8), Δz_t is our measure of the investment-specific shock, defined in Equation (12), and $D(x)_d$ is a β^{imc}-quintile dummy variable ($D(\beta^{imc}_{i,t-1})_n = 1$ if the firm's β^{imc} belongs to the quintile n in year $t-1$). X_{t-1} is a vector of controls, which includes Tobin's Q, lagged investment, leverage, cash flows and log of the firm's capital. Definitions of these variables are standard and are summarized in Table 1. We standardize all independent variables to zero mean and unit standard deviation using unconditional moments. Since the quality-adjusted price series ends in 2000, our sample covers the 1962-2000 period.

The coefficients $(a_1, \ldots, a_5)$ and $(b_1, \ldots, b_5)$ on the dummy variables measure differences in the level of investment and response of investment to an I-shock respectively. We estimate the investment response both with and without firm- and industry-level dummies, and both with and without controlling for commonly used predictors of firm-level investment. When computing standard errors we need to account for the fact that investment may contain an unobservable firm and time component. Following Petersen (2009), we cluster standard errors both by firm and time.[3]

We summarize the results in Table 3. The results show that for all specifications, firms with high β^{imc} invest more on average and their investment rate responds more to an investment-specific shock. A single-standard-deviation investment shock changes firm-level investment by 0.026 standard deviations on average. This number varies between -0.01 for the low-β^{imc} firms and 0.073 for the high-β^{imc} firms. When controlling for industry fixed effects and Tobin's Q, lagged investment rate, leverage, cash flows and log capital, the difference in coefficients on the I-shock between the extreme β^{imc} quintiles of firms diminishes somewhat to 0.051, and it drops to 0.03 once firm fixed effects are included in the specification.

The investment-specific shock captures a systematic component of investment across firms and therefore survives aggregation. To evaluate the economic significance of investment re-

[3]Cameron, Gelbach and Miller (2006) and Thomson (2006) propose estimating the variance-covariance matrix by $V = V_{firm} + V_{time} - V_{white}$, which combines the standard errors clustered by firm with the standard errors clustered by time.

sponse to the I-shock, we estimate equation (15) while aggregating firm-specific observations at the quintile level. Specifically, every year we form five portfolios of firms based on the lagged values of their β^{imc}, and construct portfolio-level investment rates and controls by averaging across firms within the portfolio. We present the results in Table 4. At the portfolio level, firm-specific idiosyncratic shocks to the investment rate are diversified away, and, on average, a single-standard-deviation I-shock increases investment rate by 0.173 standard deviations. This number ranges between -0.009 and 0.537 for the low- and high-β^{imc} quintile portfolios respectively. Thus, the investment rate of the portfolio of high-β^{imc} firms has three times the average sensitivity to the I-shocks.

Our empirical results show that a firm's β^{imc} predicts the response of its investment rate to investment-specific technology shocks extracted from the historical quality-adjusted equipment price index.

Robustness checks

As our first robustness check, we replace our measure of the I-shock, z_t by accumulated log returns on the factor-mimicking portfolio $\tilde{R}^{imc}_t = R^{imc}_t + R^{imc}_{t-1}$ and estimate

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d\, D(\beta^{imc}_{f,t-1})_d + b_1\, \tilde{R}_{imc,t} + \sum_{d=2}^{5} b_d\, D(\beta^{imc}_{f,t-1})_d \times \tilde{R}_{imc,t} + cX_{it-1} + \gamma_f + u_t. \quad (16)$$

We present the results in Table 5. A one-standard deviation shock to $\tilde{R}_{imc}$ increases firm-level investment on average by 0.093 standard deviations, but the response differs in the cross-section and ranges from 0.061 to 0.162 for the low- and high-β^{imc} quintiles respectively. These results provide further evidence that the IMC portfolio is a valid mimicking portfolio for the investment-specific technology shocks.

Next, we compare our measure of growth opportunities to two alternative measures. The first alternative is a commonly used empirical proxy for growth opportunities, Tobin's Q. Firms with abundant growth opportunities are firms for which the marginal value of a unit of capital inside the firm is greater than its replacement cost, or equivalently the marginal

Q is greater than one. A commonly used empirical proxy for marginal Q is the average Q, defined as the market value of the firm divided by the replacement cost of its capital, where the latter is measured as the book value of firm's assets.

The second alternative is the firm's market beta. We consider the market beta as a proxy for growth opportunities because of the intuition provided by the real options literature. Typically, the part of the firm's value that is due to growth opportunities behaves as a levered claim on assets in place, and therefore it has higher volatility and is more sensitive to aggregate shocks than assets in place. Thus, real options models predict that high-growth-opportunity firms have relatively high market betas.

We estimate equation (15) using either Tobin's Q or the market beta instead of β^{imc}. Since industries may differ in their cyclical behavior and thus their market risk, we sort firms on the ranking of their market beta within the industry, where industries are defined at the 2-digit SIC code level. Using Tobin's Q as an alternative measure of growth options leads to results that are qualitatively similar but weaker than those obtained with β^{imc}, as we show in Table 6. High- Tobin's Q firms increase their investment rate by 0.042 standard deviations in response to a single-standard-deviation I-shock compared to 0 for the low-Tobin's Q firms. Thus, the spread in the I-shock response across the Q-quintiles is half of that observed across the β^{imc}-quintiles.

Cross-sectional differences in market beta do not predict a statistically significant response of investment to I-shocks, as can be seen in Table 7. Thus, we conclude that the predictive ability of β^{imc} is not captured by the standard intuition of the real leverage argument.

The metric we use to evaluate the validity of our measure of growth opportunities is linked to how that measure is constructed, since we are using a measure of stock-price sensitivity to I-shocks to predict investment-rate sensitivity to the same shock. We therefore check whether β^{imc} also predicts cross-sectional heterogeneity in firms' investment response to other systematic shocks affecting the willingness of firms to investment. In particular, we consider unexpected changes in aggregate credit or liquidity conditions as a source of such shocks. Tightening credit conditions should have similar effect on investment as a negative investment-specific shock, leading to increased cost of investment. Thus, states with where credit is tight are effectively states with low real investment opportunities.

To this end, we consider the innovation in the spread between Baa and Treasury bonds as a measure of innovations in aggregate credit condition.[4] As before, we construct the innovations ($\Delta\xi$) in the HP-filtered credit spread

$$\Delta\xi_t = cr_t^{hp} - 0.32\, cr_{t-1}^{hp}, \tag{17}$$

where cr_t^{hp} is the HP-filtered spread between Baa and Treasury bonds. The first finding is that the correlation between z and ξ is positive, but not statistically significant (0.07 in the 1960-2000 sample). Such a low degree of correlation suggests that it is unlikely that both variables are driven by the same fundamental shock, for instance, it is not the case that investment good prices rise or fall depending on fluctuations in demand induced by changes in credit conditions.

We estimate cross-sectional differences in the firm-level investment response to changes in credit spreads across the β^{imc}-quintiles. Specifically, we estimate equation (15) with ξ_t replacing z_t. Table 8 reports the results. On average, firms increase investment when credit spreads fall, and the sensitivity of investment rate to credit shocks increases across the β_{imc}-quintiles. A single-standard-deviation positive credit shock increases the average firm-level investment rate by 0.08 standard deviations. The difference in investment rate responses between high- and low-β_{imc} quintiles of firms is statistically significant and equal to 0.081 standard deviations. With various additional controls, the latter estimate falls between 0.039 and 0.052.

We perform a number of additional robustness checks. First, it is possible that β^{imc}

[4]Philippon (2008) documents that a measure of Q constructed using bond market data outperforms measures of Q constructed using equity data.

captures firms' financial constraints, and not the differences in their real production opportunities. This possibility is consistent with our approach, since financially constrained firms, defined as firms with insufficient cash holdings and limited access to external funds, cannot take advantage of investment opportunities and as such have effectively low growth options. Thus, future growth opportunities depend both on the firm's financial constraints and its real investment opportunities. To sharpen the interpretation of our empirical results, we attempt to distinguish financial constraints from real frictions. We replicate our empirical analysis on a sample of firms relatively less likely to be constrained, namely, firms that have been assigned a credit rating by Standard and Poor's. This restricts our sample to $8,060$ firm-year observations. We find that our results hold in this sample, with the difference in the response of investment to the I-shock between the extreme β^{imc}-quintiles of 0.112. This estimate is in fact greater than the one obtained for the entire sample of firms, indicating that our findings are unlikely to be explained by financial constraints alone.

Second, we estimate β^{imc} using stock return return data, while the theory suggests using returns on the total firm value. Our findings could be explained by investment of highly levered firms being relatively sensitive to investment shocks. Additional empirical evidence suggests that this is not the case. First, as we show in Table 7, firms' market betas are much less informative about their investment rate response to I-shocks than their IMC betas. The argument based on leverage implies that both betas should have comparable predictive performance. Second, we approximate β^{imc} at the *asset* level (de-lever the equity-based estimates) under the assumption that firms' debt is risk-free. We re-estimate Equation 15 using de-levered β^{imc}. We find that the difference in investment responses between the high- and low-β^{imc} firms is statistically significant and equal to 0.07, regardless of whether we use book or market leverage.

Finally, we consider whether β^{imc} may be capturing inter-industry differences in technology, instead of capturing meaningful differences in growth opportunities.[5] We investigate

[5]This possibility is not addressed by the controls we use in estimation, since we do not allow the loadings

this possibility by defining β^{imc}-quintiles based on the firms' intra-industry β^{imc} ranking. We find that our results are mainly driven by intra- rather than inter-industry variation. The difference in investment responses between the firms in high- and low-β^{imc}-quintiles relative to their industry peers is statistically significant and lies between 0.062 and 0.081, depending on which industry classification we use.

To conserve space, we do not report the full details of these robustness checks and refer the reader to the web Appendix.

5 Calibration

We calibrate our model to approximately match moments of aggregate dividend growth and investment growth, accounting ratios, and asset returns. Thus, most of the parameters are chosen jointly based on the behavior of financial and real variables.

We pick $\alpha = 0.85$, the parameters governing the projects' cash flows ($\sigma_\varepsilon = 0.2, \theta_e = 0.35, \sigma_u = 1.5, \theta_u = 0.5$) and the parameters of the distribution of λ_f jointly, to match the average values and the cross-sectional distribution of the investment rate, the market-to-book ratio, and the return to capital (ROE).

We model the distribution of mean project arrival rates $\lambda_f = E[\lambda_{ft}]$ across firms as

$$\lambda_f = \mu_\lambda \delta - \sigma_\lambda \delta \log(X_f) \quad X_f \sim U[0,1], \tag{18}$$

We pick $\sigma_\lambda = \mu_\lambda = 2$. Regarding the dynamics of the stochastic component of the firm-specific arrival rate, $\tilde{\lambda}_{ft}$, we pick $\mu_H = 0.075$ and $\mu_L = 0.16$. We pick $\lambda_H = 2.35$, which according to (4) implies $\lambda_L = 0.35$. These parameter values ensure that the firm grows about twice than average in its high growth phase and about a third as fast in the low growth phase.

We set the project expiration rate δ to 10%, to be consistent with commonly used values for the depreciation rate. We set the interest rate r to 2.5%, which is close to the historical

on quintile dummies to interact with the industry fixed effects.

average risk-free rate (2.9%). We choose the parameters governing the dynamics of the shocks x_t and z_t to match the first two moments of the aggregate dividend growth and investment growth. We choose $\phi = 0.07$ to match the relative size of the consumption and investment sectors in the data.

Finally, the parameters of the pricing kernel, $\beta_x = 0.69$ and $\beta_z = 0.35$ are picked to match approximately the average excess returns on the market portfolio and the IMC portfolio. Given our calibration, the model produces a somewhat lower average return on the IMC portfolio -3.9% vs -2.1% in the $1962 - 2005$ sample. However, investment firms tend to be quite a bit smaller than consumption firms, so the size effect may the estimated return of the IMC portfolio upwards. In fact, when excluding the month of January, which is when the size effect is strongest, the average return on the IMC portfolio is -3.5%, whereas it's α with respect to the Small-minus-Big (SMB)) portfolio of Fama and French (1993) is -3.7%.

We simulate the model at a weekly frequency ($dt = 1/52$) and time-aggregate the data to form annual observations. Each simulation sample contains 2,500 firms for 100 years. We use the first half of each simulated sample for burn-in. We simulate 1,000 samples and report averages of parameter estimates and t-statistics across simulations.

5.1 Investment

We first evaluate how well our model accounts for the empirical properties of firms' investment. We estimate equation 15 using on the simulated data.

We define firm-level investment during year t as a sum of the investment expenses incurred throughout that year, i.e. $I_{ft} = \sum_{s \in t} x_s z_s K^*_{fs}$, where K^*_{fs} refers to the capital of project acquired by firm f at time s.

We define the book value of the firm as the replacement cost of its capital, $B_{ft} = z_t \, x_t \sum_{j \in \mathcal{J}_{ft}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{ft}$ denotes the set of projects owned by firm f at the end of year t.[6] In the simulated data, we compute

[6]As a robustness check, we also perform simulations with the book value of the firm defined as the

firm-level β^{imc} using the same methodology as in our empirical results, namely by estimating equation (14) using weekly data every year. We normalize all variables to zero mean and unit standard deviation and compute standard errors clustered by firm and time.

We report averages of the coefficient estimates and t-statistics across 1,000 simulations in Table 10. In simulated data, a single-standard-deviation investment shock leads to an increase in firm-level investment of 0.038 standard deviations. However, as in the actual data, the impact of investment shocks varies in the cross-section of firms from 0.019 to 0.083 between the low- and high-β^{imc} firms respectively. The difference in coefficients between the high- and low-β^{imc} firms drops to 0.026 when we include Tobin's Q and cash flows in the specification. Thus, the magnitude of investment response to I-shocks in the model is very similar to the empirical estimates in Table 3.

In the model, Tobin's Q, or the market-to-book ratio, also contains information about growth opportunities. To verify this, we estimate equation (15) in simulated data. We report simulation averages of coefficients and t-statistics in Table 11. In simulated data, the effect of a single-standard-deviation investment shock on firm investment varies from 0.09 for the top Q-quintile to 0.014 for the bottom quintile. The difference in coefficients between the high- and low-Q firms drops to 0.03 when Tobin's Q is included in the regression. From this, we conclude that in the model, Tobin's Q is a good proxy for growth opportunities. Of course, this is partly because it is measured with accurately in simulations, whereas in the data it might be contaminated by measurement error.

We conclude that our model is able to replicate the key empirical properties of firms' investment, both qualitatively and quantitatively. Next, we verify that the model also captures the properties asset returns reported in Section 4.

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cumulative historical investment cost of its current portfolio of projects. Our results are essentially the same under the two definitions.

5.2 Stock Returns

As we show in Proposition 1, cross-sectional differences in the relative value of growth opportunities of firms lead to cross-sectional differences in their risk premia. Furthermore, we show in Proposition 2 that unobservable growth opportunities can be measured empirically using the firms' betas with respect to the IMC portfolio returns. We now verify that our model implies empirically realistic behavior of stock returns in relation to the differences in growth opportunities (captured by β^{imc}) across firms.

We sort firms annually into 10 value-weighted portfolios based on the most recent value of their β^{imc}. Empirical estimates of β^{imc} use one-year moving-window regressions with weekly return. In simulations, we compute β^{imc} using equation (11). In each simulation, and for each portfolio p, we estimate average excess returns $E[R_{pt}] - r_f$, return standard deviations $\sigma(R_{pt})$, and regressions

$$R_{pt} - r_f = \alpha_p + \beta_{m,p}(R_{Mt} - r_f) + \epsilon_{pt} \tag{19}$$

and

$$R_{pt} - r_f = \alpha_p + \beta_{m,p}(R_{Mt} - r_f) + \beta_{imc,p}(R_{It} - R_{Ct}) + \epsilon_{p,t}, \tag{20}$$

where R_{It} and R_{Ct} denote returns on the portfolios of investment-good producers and consumption-good producers respectively.

Table 12 compares the properties of returns in historical and simulated data. The top panel replicates the findings of Papanikolaou (2008), who shows that sorting firms into portfolios based on β^{imc} results in i) a declining pattern in average returns; ii) an increasing pattern of return volatility and market betas; and iii) a declining patter of CAPM alphas. The difference in average returns and CAPM alphas between the high and low β^{imc} portfolios is -3.1% and -7.6% respectively. The high-β^{imc} portfolio has a standard deviation of 29% and a market beta of 1.6 versus 16% and 0.8 respectively for the low-β^{imc} portfolio. The bottom panel of Table 12 contains the corresponding simulation-based estimates. The difference in average returns and CAPM alphas between the high- and low-β^{imc} portfolios is -3.5% and -5.7% respectively. Moreover, the high-β^{imc} portfolio has both a higher standard deviation (20%)and market beta (1.2) than the low-β^{imc} portfolio (14% and 0.8 respectively). Furthermore, the estimates of α_p in (20) are close to zero for all portfolios, both in simulated and actual data. Thus, returns on the β^{imc}-sorted portfolios are well described by a two-factor pricing model that includes market returns and returns on the IMC portfolio.

As we discuss in the introduction, many papers use the decomposition of firm's value into asset in place and growth options in an attempt to explain the value premium puzzleIt is therefore useful to assess the ability of our model to replicate the empirical relationship between stock returns and the book-to-market ratio. As we show in Section 3, as long as book to market is a good proxy for $PVGO/V$, our model will exhibit a positive value premium. We now investigate the model's implications quantitatively.

The top panel in Table 13 replicates the empirical findings of Fama and French (1993). The difference in average returns and CAPM alphas between value firms and growth firms is 7.2% and 7.9% respectively. Moreover, with the exception of the extreme value portfolio, the CAPM beta tend to be negatively related to the book-to-market ratio. The bottom panel presents corresponding simulation results. The difference in average returns and CAPM alphas between the two extreme book-to-market portfolios is 4.3% and 6.3% respectively. Moreover, as in the data, the CAPM betas decline across the book-to-market deciles. Thus, our model replicates the failure of the CAPM to price the cross-section of book-to-market portfolios. We also report the estimates of equation (20), where we use both the market and the IMC portfolio as risk factors. The two-factor unconditional pricing model fails to price the cross-section of book-to-market portfolios empirically. The difference in estimated alphas between the extreme book-to-market deciles is 6.2%. The two-factor unconditional specification works rather well in simulated data, where the difference in alphas between the extreme book-to-market deciles is 0.8%.

6 Conclusion

In this paper we propose a novel measure of growth opportunities available to firms. Our measure relies on the idea that firms with abundant growth opportunities benefit more from investment-specific technological improvements than firms with few growth opportunities, and therefore, stock returns of high-growth firms have higher exposure to investment-specific technological shocks. Our empirical tests support this conjecture. Investment rates of high-growth firms, as identified by our measure, are relatively high on average and more sensitive to investment-specific shocks than investment rates of low-growth firms. Our measure of growth opportunities also captures cross-sectional differences in risk premia. Empirically, high-growth firms have lower average returns than low-growth firms. Such return differences are not explained by differences in market risk (CAPM), since, as discussed in Papanikolaou (2008), investment-specific shocks represent a distinct risk factor. The return premium on low-growth firms is distinct from the well-known value premium, and accounts for a fraction of the latter. We use calibration to show that the observed empirical patterns are quantitatively consistent with a stylized structural model of investment.

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Tables

Table 1: Data Definitions

Variable	Source
Investment (I)	Compustat item128
Capital (K)	Compustat item8
Book Assets (A)	Compustat item6
Book Debt (D)	Compustat item9
Book Preferred Equity (EP)	Compustat item56
Book Common Equity (EC)	Compustat item60
Operating Cashflows (CF)	Compustat item14+item18
Inventories (INV)	Compustat item76-78
Market Capitalization (MKCAP)	CRSP
R&D Expenditures (R&D)	Compustat item46
Cash Holdings (CASH)	Compustat item1
Dividends (DIV)	Compustat item19+item21
Share Repurchases (REP)	Compustat item115
Tobin's Q (Q)	(MKCAP + EP + D-INV)/(EC+EP + D)
Quality Adjusted Price of Investment Goods	Cummins and Violante (2002)
Consumption Deflator	NIPA

Table 2: Summary Statistics: β^{imc} sorted portfolios

β^{imc} sort	I/K	CASH/A	D/A	Tobin's Q	ROE	log(K)	R&D/A	DIV/CF
Low	19.4%	10.2%	18.5%	1.48	7.6%	3.78	2.9%	23.8%
2	19.2%	9.5%	18.6%	1.43	9.1%	4.44	2.5%	25.4%
3	19.3%	9.6%	18.5%	1.52	9.2%	4.38	2.7%	24.0%
4	19.4%	9.7%	18.2%	1.47	9.4%	4.44	2.9%	27.5%
5	20.1%	9.5%	18.4%	1.56	9.4%	4.36	2.9%	29.8%
6	20.3%	9.7%	18.5%	1.61	9.2%	4.31	3.0%	23.0%
7	21.0%	10.4%	18.4%	1.60	9.1%	4.16	3.5%	21.1%
8	21.3%	10.5%	19.0%	1.58	8.3%	4.02	3.6%	19.2%
9	23.0%	11.5%	19.4%	1.70	7.3%	3.71	4.3%	17.7%
High	24.8%	13.4%	19.1%	1.86	4.0%	3.22	6.3%	14.9%

Table 2 shows summary statistics for 10 portfolios of firms sorted by β^{imc}_{t-1}. β^{imc}_{t} refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. I/K is investment over Capital, CASH/A refers to Cash Holdings over Assets, D/A is Debt over Assets, Q refers to Tobin's Q, ROE refers to Cashflows over lagged Book Assets, log K is log Capital, R&D/A refers to Research and Development over Book Assets and DIV/CF refers to Dividends over Cashflows. When computing DIV/CF, we drop firms with negative cashflows. We report equal-weighted averages across portfolios.

Table 3: Response of I/K to I-Shock: Firms sorted by β^{imc}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.1490	-0.0906	-0.0847	-0.0602	0.0181
		(-6.04)	(-4.93)	(-3.61)	(-3.41)	(0.89)
$D(\beta_{imc})_2$		0.0537	0.0246	0.0265	0.0154	0.0078
		(3.62)	(2.26)	(2.23)	(1.60)	(0.66)
$D(\beta_{imc})_3$		0.1236	0.0628	0.0656	0.0414	0.0236
		(6.99)	(5.47)	(4.32)	(3.64)	(1.84)
$D(\beta_{imc})_4$		0.1887	0.1002	0.1064	0.0679	0.0286
		(8.14)	(6.38)	(5.77)	(4.71)	(1.84)
$D(\beta_{imc})_5$		0.3057	0.1514	0.1907	0.1065	0.0552
		(10.40)	(8.18)	(8.41)	(6.44)	(2.55)
$-\Delta z_{t-1}$	0.0256	-0.0099	-0.0179	-0.0088	-0.0155	0.0049
	(1.47)	(-0.48)	(-1.05)	(-0.57)	(-1.29)	(0.39)
$D(\beta_{imc})_2 \times (-\Delta z_{t-1})$		0.0188	0.0156	0.0101	0.0107	0.0001
		(2.53)	(3.08)	(1.41)	(1.94)	(0.01)
$D(\beta_{imc})_3 \times (-\Delta z_{t-1})$		0.0272	0.0207	0.0181	0.0158	0.0078
		(2.10)	(3.09)	(1.46)	(2.01)	(0.71)
$D(\beta_{imc})_4 \times (-\Delta z_{t-1})$		0.0517	0.0360	0.0343	0.0269	0.0253
		(2.88)	(3.34)	(2.60)	(2.84)	(2.71)
$D(\beta_{imc})_H \times (-\Delta z_{t-1})$		0.0836	0.0510	0.0503	0.0340	0.0302
		(3.57)	(4.12)	(2.41)	(2.40)	(1.86)
Observations	47996	47996	47996	47996	47996	47996
R^2	0.001	0.014	0.229	0.166	0.290	0.446
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 3 shows estimates of

$$i_{it} = a_1 + \sum_{d=2}^{5} a_d \, D(\beta^{imc}_{i,t-1})_d + b_1\,(-z_{t-1}) + \sum_{d=2}^{5} b_d \, D(\beta^{imc}_{i,t-1})_d \times (-z_{t-1}) + cX_{it-1} + \gamma_i + u_{it},$$

where $i_t \equiv I_t/K_{t-1}$ is firm Investment over the lagged Capital stock, on the innovation in the quality-adjusted price of new equipment z_t, and a vector of controls X_t which includes lagged values of log Tobin's Q, Cashflows over lagged Capital, log Book Equity over Book Assets, and log Capital. β^{imc}_t refers to the firm's beta with respect to the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. $D(\beta^{imc}_{i,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in terms of β^{imc}_{t-1}. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by firm and year. Sample period is 1963-2001 and excludes firms in the producing investment goods and financial firms (SIC6000-6799).

Table 4: **Data: Response of I/K to I-Shock: Portfolios sorted by β^{imc}**

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)
Constant		-0.5061	-0.3604	-0.4133	-0.2569
		(-4.49)	(-3.10)	(-4.20)	(-2.49)
$D(\beta_{imc})_2$		0.1355	0.1054	0.1120	0.0906
		(1.51)	(1.21)	(0.92)	(0.84)
$D(\beta_{imc})_3$		0.4637	0.3235	0.4043	0.2632
		(4.13)	(2.30)	(3.24)	(2.04)
$D(\beta_{imc})_4$		0.8425	0.5980	0.6915	0.4289
		(6.17)	(3.32)	(4.77)	(2.60)
$D(\beta_{imc})_5$		1.5767	1.1325	1.2972	0.7919
		(8.85)	(4.23)	(5.58)	(2.52)
$-\Delta z_{t-1}$	0.1728	-0.0089	-0.0582	-0.0240	-0.0732
	(1.98)	(-0.08)	(-0.53)	(-0.27)	(-0.84)
$D(\beta_{imc})_2 \times (-\Delta z_{t-1})$		0.0367	0.0309	0.0366	0.0281
		(0.86)	(0.59)	(0.69)	(0.51)
$D(\beta_{imc})_3 \times (-\Delta z_{t-1})$		0.0971	0.0857	0.0699	0.0562
		(1.39)	(1.03)	(1.04)	(0.74)
$D(\beta_{imc})_4 \times (-\Delta z_{t-1})$		0.2376	0.1868	0.1870	0.1288
		(2.87)	(1.84)	(2.22)	(1.31)
$D(\beta_{imc})_5 \times (-\Delta z_{t-1})$		0.5367	0.4642	0.4872	0.4046
		(3.58)	(3.66)	(3.13)	(3.20)
Observations	185	185	185	185	185
R^2	0.031	0.406	0.458	0.465	0.523
Controls (i_{t-1})	N	N	Y	N	Y
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y

Table 4 shows estimates of

$$i_{it} = a_1 + \sum_{d=2}^{5} a_d\, D(\beta^{imc}_{i,t-1})_d + b_1\, \tilde{R}_{imc,t-1} + \sum_{d=2}^{5} b_d\, D(\beta^{imc}_{i,t-1})_d \times \tilde{R}_{imc,t-1} + cX_{it-1} + \gamma_i + u_{it},$$

where $i_{it} \equiv I_{it}/K_{it-1}$ is Investment over the lagged Capital stock, on the innovation in the quality-adjusted relative price of new equipment z_t, and a vector of controls X_t which includes lagged values of log Tobin's Q, Cashflows over lagged Capital, log Book Equity over Book Assets, and log Capital. β^{imc}_t refers to the firm's beta with respect to the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. The innovation z_t is computed as the innovation of an AR(1) model on the HP-detrended quality-adjusted price of investment goods divided by the consumption deflator from Cummins and Violante (2002). Every year, we sort firms into 5 portfolios based on $\beta^{imc}_{i,t-1}$. Portfolio-level variables are constructed by averaging across firms within the portfolio. $D(\beta^{imc}_{i,t-1})_d$ is a portfolio indicator variable, denoting the portfolio containing firms in the d-th quintile of β^{imc}_{t-1}. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by year. Sample period is 1963-2001 and excludes firms producing investment goods and financial firms (SIC6000-6799).

Table 5: Response of I/K to R_{imc}: sorted by β^{imc}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.1588	-0.0953	-0.1016	-0.0708	-0.0103
		(-7.71)	(-5.88)	(-5.04)	(-4.55)	(-0.54)
$D(\beta_{imc})_2$		0.0502	0.0244	0.0275	0.0173	0.0098
		(3.54)	(2.37)	(2.42)	(1.86)	(0.90)
$D(\beta_{imc})_3$		0.1141	0.0587	0.0626	0.0403	0.0220
		(6.22)	(4.99)	(4.40)	(3.71)	(1.78)
$D(\beta_{imc})_4$		0.1813	0.0968	0.1031	0.0660	0.0272
		(8.00)	(6.53)	(5.99)	(4.93)	(1.84)
$D(\beta_{imc})_5$		0.2822	0.1356	0.1740	0.0935	0.0439
		(9.65)	(8.43)	(8.10)	(6.39)	(2.10)
$\tilde{R}^{imc}_{t-1}$	0.0925	0.0612	0.0388	0.0703	0.0491	0.0582
	(4.86)	(4.10)	(3.64)	(3.90)	(3.86)	(3.71)
$D(\beta_{imc})_2 \times \tilde{R}^{imc}_{t-1}$		-0.0002	-0.0045	0.0032	-0.0012	0.0040
		(-0.02)	(-0.63)	(0.34)	(-0.15)	(0.53)
$D(\beta_{imc})_3 \times \tilde{R}^{imc}_{t-1}$		0.0239	0.0163	0.0220	0.0164	0.0242
		(1.26)	(1.24)	(1.51)	(1.36)	(2.07)
$D(\beta_{imc})_4 \times \tilde{R}^{imc}_{t-1}$		0.0374	0.0300	0.0288	0.0258	0.0305
		(2.57)	(3.04)	(2.61)	(3.01)	(2.87)
$D(\beta_{imc})_5 \times \tilde{R}^{imc}_{t-1}$		0.1010	0.0818	0.0709	0.0653	0.0729
		(4.27)	(6.87)	(4.72)	(7.66)	(5.01)
Observations	52845	52845	52845	52845	52845	52845
R^2	0.009	0.022	0.243	0.176	0.304	0.453
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 5 shows estimates of

$$i_{it} = a_1 + \sum_{d=2}^{5} a_d\, D(\beta^{imc}_{i,t-1})_d + b_1\, \tilde{R}_{imc,t-1} + \sum_{d=2}^{5} b_d\, D(\beta^{imc}_{i,t-1})_d \times \tilde{R}_{imc,t-1} + cX_{it-1} + \gamma_i + u_{it},$$

where $i_t \equiv I_t/K_{t-1}$ is firm Investment over the lagged Capital stock, on cumulative log returns on the IMC portfolio, $\tilde{R}_{imc,t-1} \equiv \sum_{l=1}^{2} R_{imc,t-l}$, and a vector of controls X_t which includes lagged values of log Tobin's Q, Cashflows over lagged Capital, log Book Equity over Book Assets, and log Capital. $D(\beta^{imc}_{i,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in term of β^{imc}_{t-1}. β^{imc}_t refers to the firm's beta with respect to the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by firm and year. Sample period is 1963-2004 and excludes firms producing investment goods and financial firms (SIC6000-6799).

Table 6: Response of I/K to I-Shock: Firms sorted by Tobin's Q

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.2619	-0.1707	-0.2334	-0.1616	-0.2017
		(-11.03)	(-8.96)	(-9.08)	(-7.22)	(-9.05)
$D(Q)_2$		0.0398	0.0136	0.0669	0.0358	0.0903
		(2.58)	(1.21)	(2.79)	(1.75)	(4.45)
$D(Q)_3$		0.1347	0.0719	0.1725	0.1071	0.1872
		(6.96)	(5.46)	(5.11)	(3.73)	(7.27)
$D(Q)_4$		0.3673	0.2203	0.3288	0.2161	0.3432
		(14.48)	(11.93)	(8.15)	(6.42)	(11.33)
$D(Q)_5$		0.7113	0.4487	0.5605	0.3780	0.5790
		(23.72)	(20.26)	(9.19)	(7.25)	(13.01)
$-\Delta z_{t-1}$	0.0256	-0.0001	-0.0175	0.0009	-0.0135	0.0070
	(1.47)	(-0.01)	(-1.23)	(0.08)	(-1.19)	(0.52)
$D(Q)_2 \times (-\Delta z_{t-1})$		0.0289	0.0204	0.0196	0.0155	0.0132
		(2.13)	(1.87)	(1.86)	(1.61)	(1.10)
$D(Q)_3 \times (-\Delta z_{t-1})$		0.0295	0.0239	0.0246	0.0203	0.0197
		(1.78)	(1.67)	(1.45)	(1.36)	(1.16)
$D(Q)_4 \times (-\Delta z_{t-1})$		0.0228	0.0250	0.0124	0.0161	0.0066
		(0.82)	(1.22)	(0.55)	(0.90)	(0.27)
$D(Q)_5 \times (-\Delta z_{t-1})$		0.0415	0.0536	0.0407	0.0497	0.0207
		(2.27)	(3.40)	(2.74)	(3.66)	(1.48)
Observations	47996	47996	47996	47996	47996	47996
R^2	0.001	0.075	0.255	0.176	0.296	0.455
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 6 shows estimates of

$$i_{it} = a_1 + \sum_{d=2}^{5} a_d\, D(Q_{i,t-1})_d + b_1\,(-z_{t-1}) + \sum_{d=2}^{5} b_d\, D(Q_{i,t-1})_d \times (-z_{t-1}) + cX_{it-1} + \gamma_i + u_{it},$$

where $i_t \equiv I_t/K_{t-1}$ is firm Investment over lagged Capital, on the innovation in the quality-adjusted relative price of new equipment z_t, and a vector of controls X_t which includes lagged values of log Tobin's Q, Cashflows over lagged Capital, log Book Equity over Book Assets, and log Capital. $D(Q_{i,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in terms of Tobin's Q. The innovation z_t is computed as the innovation of an AR(1) model on the HP-detrended quality-adjusted price of investment goods divided by the consumption deflator from Cummins and Violante (2002). Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by firm and year. Sample period is 1963-2001 and excludes firms producing investment goods and financial firms (SIC6000-6799).

Table 7: Response of I/K to I-Shock: Firms sorted by β^{mkt}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.1591	-0.0961	-0.1040	-0.0777	-0.0075
		(-6.38)	(-4.94)	(-4.10)	(-3.91)	(-0.36)
$D(\beta_{mkt})_2$		0.0465	0.0217	0.0266	0.0202	0.0200
		(2.69)	(1.76)	(1.68)	(1.59)	(1.48)
$D(\beta_{mkt})_3$		0.1194	0.0595	0.0748	0.0532	0.0362
		(7.01)	(5.17)	(5.06)	(4.79)	(2.66)
$D(\beta_{mkt})_4$		0.2110	0.1082	0.1451	0.0969	0.0784
		(9.31)	(7.82)	(6.63)	(6.12)	(5.16)
$D(\beta_{mkt})_5$		0.3412	0.1752	0.2434	0.1517	0.1128
		(12.68)	(9.22)	(10.21)	(8.32)	(5.45)
$-\Delta z_{t-1}$	0.0256	0.0150	-0.0037	0.0047	-0.0064	0.0145
	(1.47)	(0.90)	(-0.25)	(0.33)	(-0.56)	(0.97)
$D(\beta_{mkt})_2 \times (-\Delta z_{t-1})$		0.0074	0.0128	0.0074	0.0121	-0.0006
		(0.50)	(0.93)	(0.43)	(0.76)	(-0.05)
$D(\beta_{mkt})_3 \times (-\Delta z_{t-1})$		0.0036	0.0119	0.0048	0.0102	-0.0001
		(0.25)	(0.91)	(0.34)	(0.77)	(-0.01)
$D(\beta_{mkt})_4 \times (-\Delta z_{t-1})$		0.0154	0.0125	0.0148	0.0116	0.0094
		(0.88)	(1.25)	(0.96)	(1.09)	(0.77)
$D(\beta_{mkt})_5 \times (-\Delta z_{t-1})$		0.0322	0.0156	0.0219	0.0101	0.0075
		(1.56)	(1.00)	(1.14)	(0.65)	(0.33)
Observations	47996	47996	47996	47996	47996	47996
R^2	0.001	0.017	0.230	0.168	0.292	0.447
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 7 shows estimates of

$$i_{it} = a_1 + \sum_{d=2}^{5} a_d\, D(\beta^{mkt}_{i,t-1})_d + b_1\,(-z_{t-1}) + \sum_{d=2}^{5} b_d\, D(\beta^{mkt}_{i,t-1})_d \times (-z_{t-1}) + cX_{it-1} + \gamma_i + u_{it},$$

where $i_t \equiv I_t/K_{t-1}$ is firm Investment over the lagged Capital stock, on cumulative log returns on the IMC portfolio, $\tilde{R}_{imc,t-1} \equiv \sum_{l=2}^{2} R_{imc,t-l}$, and a vector of controls X_t which includes lagged values of log Tobin's Q, Cashflows over lagged Capital, log Book Equity over Book Assets, and log Capital. $D(\beta^{mkt}_{i,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in term of β^{mkt}_{t-1}. β^{mkt}_t refers to the firm's beta with respect to the market portfolio in year t, estimated using non-overlapping weekly returns within year t. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by firm and year. Sample period is 1963-2004 and excludes firms producing investment goods and financial firms (SIC6000-6799).

Table 8: Response to innovations in Credit Spreads

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.1566	-0.0924	-0.1007	-0.0691	-0.1007
		(-7.29)	(-6.54)	(-4.60)	(-4.65)	(-4.60)
$D(\beta_{imc})_2$		0.0506	0.0245	0.0272	0.0171	0.0272
		(3.56)	(2.38)	(2.38)	(1.84)	(2.38)
$D(\beta_{imc})_3$		0.1152	0.0590	0.0630	0.0403	0.0630
		(6.49)	(5.09)	(4.54)	(3.79)	(4.54)
$D(\beta_{imc})_4$		0.1824	0.0969	0.1042	0.0665	0.1042
		(8.10)	(6.28)	(5.94)	(4.73)	(5.94)
$D(\beta_{imc})_H$		0.2872	0.1379	0.1811	0.0980	0.1811
		(9.84)	(7.22)	(8.23)	(5.77)	(8.23)
$-\Delta\xi_{t-1}$	0.0812	0.0515	0.0562	0.0414	0.0467	0.0414
	(5.29)	(3.38)	(4.44)	(2.39)	(3.80)	(2.39)
$D(\beta_{imc})_2 \times (-\Delta\xi_{t-1})$		0.0096	0.0064	0.0118	0.0099	0.0118
		(0.73)	(0.77)	(1.03)	(1.22)	(1.03)
$D(\beta_{imc})_3 \times (-\Delta\xi_{t-1})$		0.0299	0.0204	0.0237	0.0196	0.0237
		(1.66)	(1.78)	(1.59)	(1.71)	(1.59)
$D(\beta_{imc})_4 \times (-\Delta\xi_{t-1})$		0.0265	0.0175	0.0099	0.0098	0.0099
		(1.21)	(1.22)	(0.63)	(0.84)	(0.63)
$D(\beta_{imc})_H \times (-\Delta\xi_{t-1})$		0.0791	0.0519	0.0507	0.0389	0.0507
		(2.45)	(2.42)	(2.06)	(2.04)	(2.06)
Observations	52845	52845	52845	52845	52845	52845
R^2	0.007	0.018	0.244	0.170	0.303	0.170
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 8 shows estimates of the regression of the ratio of firm investment to its capital stock, $i_t \equiv I_t/K_{t-1}$, on the innovation in the spread between Baa and Aaa bonds, ξ_t, and a vector of controls X_{it} which includes lagged values of log Tobin's Q, Cashflows over lagged Capital, log Book Equity over Book Assets, and log Capital:

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d\, D(\beta^{imc}_{f,t-1})_d + b_1\,(-\xi_{t-1}) + \sum_{d=2}^{5} b_d\, D(\beta^{imc}_{f,t-1})_d \times (-\xi_{t-1}) + cX_{f,t-1} + \gamma_f + u_{ft},$$

The innovation z_t is computed as the innovation of an AR(1) model on the HP-detrended difference between Baa and Treasury bond yields. The data on bond yields are from the St. Louis Federal Reserve. β^{imc}_t refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. $D(\beta^{imc}_{f,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in terms of β^{imc}_{t-1}. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by firm and year. Sample period is 1963-2001 and excludes firms in the producing investment goods and financial firms (SIC6000-6799).

Table 9: Parameter values and Calibration

parameter	r	μ_x	σ_x	β_x	μ_z	σ_z	β_z	ϕ	α	δ
value	0.03	0.01	0.13	0.69	0.00	0.04	0.40	0.07	0.85	0.10
parameter	θ_ϵ	σ_e	θ_u	σ_u	μ_λ	σ_λ	μ_H	μ_L	λ_H	λ_L
value	0.35	0.20	0.50	1.50	2.00	2.00	0.07	0.16	2.35	0.35

Moment	Data	Model
$\mu(D_t)$	0.025 (D) 0.038 (P)	0.017
$\sigma(D_t)$	0.118 (D) 0.384 (P)	0.150
$\mu(I_t)$	0.043	0.035
$\sigma(I_t)$	0.163	0.243
$E(R_M) - r_f$	0.067	0.056
$\sigma(R_M - r_f)$	0.183	0.165
$E(R_{IMC})$	-0.021	-0.039
$\sigma(R_{IMC})$	0.099	0.115
$\rho(R_{IMC}, R_M - r_f)$	0.279	0.522
Market Cap of I rel to C	0.149	0.140
Investment over Capital (mean)	0.216	0.128
Investment over Capital (IQR)	0.391	0.168
Cashflows over Capital (mean)	0.317	0.248
Cashflows over Capital (IQR)	0.649	0.223
Market-to-Book (median)	1.569 (E) 1.317 (A)	1.988
Market-to-Book (IQR)	1.737 (E) 1.423 (A)	1.564
$\hat{\beta}^{imc}$ (median)	0.530	0.731
$\hat{\beta}^{imc}$ (IQR)	1.121	0.639

The top panel of Table 9 shows the parameters in our calibration. The bottom panel shows sample moments. We report mean and standard deviation of dividend growth [$\mu(DIV)$, $\sigma(DIV)$], mean and standard deviation of investment growth [$\mu(INV), \sigma(INV)$], mean and standard deviation of excess returns on the market portfolio [$E(R_M) - r_f$, $\sigma(R_M)$], mean and standard deviation of the investment minus consumption portfolio [$E(R_{imc})$, $\sigma(R_{imc})$], and the ratio of the market capitalization of the investment sector relative to the consumption sector. We report separate moments for dividends (D) and net payout (P). We report time series averages of the mean and inter-quintile range (IQR) of the investment rate and cashflows over capital, and the median and inter-quintile range of the market to book ratio. For market to book, we report moments separately for Equity (E) and Assets (A). Moments are estimated over the period 1927 to 2007 for the market portfolio and investment (Real, Nonresidential Investment in E&S). Moments of the investment rate, market to book and cashflows over capital are estimated using Compustat in the sample 1961-2006. Moments of IMC are estimated over the sample 1962-2004. Moments of market returns and dividend growth are from the long sample in Campbell and Cochrane (1999). Moments of net payout are from Larrain and Yogo (2008).

Table 10: Model: Response to I-Shock: sorted by β_{IMC}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)
Constant		-0.109	-0.106	-0.055	0.075
		(-18.76)	(-19.33)	(-4.81)	(4.42)
$D(\beta_{imc})_2$		0.024	0.023	0.013	-0.034
		(7.32)	(7.09)	(3.36)	(-4.65)
$D(\beta_{imc})_3$		0.053	0.050	0.016	-0.076
		(11.78)	(11.33)	(2.87)	(-5.60)
$D(\beta_{imc})_4$		0.102	0.098	0.029	-0.130
		(14.95)	(14.30)	(3.38)	(-5.90)
$D(\beta_{imc})_H$		0.351	0.341	0.200	-0.151
		(15.01)	(14.23)	(10.46)	(-4.24)
$-\Delta\, z_{t-1}$	0.038	0.019	0.019	0.013	-0.016
	(3.09)	(3.00)	(3.12)	(1.58)	(-2.41)
$D(\beta_{imc})_2 \times (-\Delta\, z_{t-1})$		0.005	0.005	0.004	-0.002
		(1.53)	(1.51)	(1.21)	(-0.66)
$D(\beta_{imc})_3 \times (-\Delta\, z_{t-1})$		0.010	0.010	0.009	-0.003
		(2.29)	(2.28)	(2.18)	(-0.67)
$D(\beta_{imc})_4 \times (-\Delta\, z_{t-1})$		0.019	0.019	0.018	-0.002
		(2.85)	(2.84)	(2.90)	(-0.32)
$D(\beta_{imc})_H \times (-\Delta\, z_{t-1})$		0.062	0.061	0.060	0.026
		(2.66)	(2.64)	(2.67)	(1.65)
R^2	0.002	0.021	0.023	0.033	0.067
Controls (i_{t-1})	N	N	Y	Y	Y
Controls (CF_{t-1}, K_{t-1})	N	N	N	Y	Y
Controls (Q_{t-1})	N	N	N	N	Y

Table 10 shows average coefficients and t-statistics across 1,000 simulations. We estimate a regression of the ratio of the firm investment to its book value, $i_t \equiv I_{ft}/B_{f,t-1}$, on the innovation in z_t and a vector of controls X_t, which includes lagged values of log Tobin's Q, cash flows over lagged capital, and log capital:

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d\, D(\beta^{imc}_{f,t-1})_d + b_1\,(-z_{t-1}) + \sum_{d=2}^{5} b_d\, D(\beta^{imc}_{f,t-1})_d \times (-z_{t-1}) + cX_{f,t-1} + u_{ft},$$

β^{imc}_t is the firm's beta with the investment minus consumption portfolio (IMC), estimated using equation 11. $D(\beta^{imc}_{f,t-1})_d$ is a dummy variable which takes the value of 1 if the firm f falls in the d-th quintile in terms of β^{imc}_{t-1}. Data are simulated at weekly frequency ($dt = 1/52$) and then aggregated to form annual values. Investment by firm is computed as the sum of the market value of new investment, i.e. $I_{ft} = \sum_{s\in t} x_s z_s K^*_{fs}$, where K^*_{fs} is the capital of project acquired by firm f at time s. Book Value is computed as the replacement cost of capital, $B_{ft} = z_t\, x_t \sum_{j\in\mathcal{J}_{ft}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{ft}$ denotes the set of projects owned by firm f at the end of year t. All variables have been standardized to zero mean and unit standard deviation. We report averages across simulations of coefficients and t statistics (in parenthesis). Standard errors are clustered by firm and time. Each simulation sample contains 2,500 firms for 50 years. We simulate 1,000 samples.

Table 11: Model: Response to I-Shock: sorted by Tobin's Q

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)
Constant		-0.147	-0.142	-0.083	0.202
		(-29.78)	(-31.28)	(-7.28)	(5.96)
$D(Q)_2$		0.048	0.045	0.022	-0.119
		(16.01)	(15.15)	(5.11)	(-7.15)
$D(Q)_3$		0.088	0.083	0.036	-0.203
		(20.17)	(18.99)	(4.97)	(-6.91)
$D(Q)_4$		0.142	0.135	0.057	-0.305
		(22.44)	(21.03)	(5.17)	(-6.68)
$D(Q)_H$		0.441	0.433	0.286	-0.398
		(16.75)	(16.00)	(12.50)	(-5.26)
$-\Delta \ln Z_{t-1}$	0.038	0.014	0.013	0.009	-0.031
	(3.09)	(2.51)	(2.67)	(1.33)	(-2.63)
$D(Q)_2 \times (-\Delta \ln Z_{t-1})$		0.008	0.008	0.007	0.000
		(2.87)	(2.91)	(2.38)	(-0.00)
$D(Q)_3 \times \Delta(-\Delta \ln Z_{t-1})$		0.015	0.015	0.013	-0.000
		(3.48)	(3.52)	(3.05)	(-0.02)
$D(Q)_4 \times (-\Delta \ln Z_{t-1})$		0.024	0.024	0.022	-0.000
		(3.78)	(3.79)	(3.50)	(0.06)
$D(Q)_H \times (-\Delta \ln Z_{t-1})$		0.076	0.076	0.075	0.033
		(2.93)	(2.90)	(2.88)	(1.73)
R^2	0.002	0.030	0.033	0.037	0.073
Controls (i_{t-1})	N	N	Y	Y	Y
Controls (CF_{t-1}, K_{t-1})	N	N	N	Y	Y
Controls (Q_{t-1})	N	N	N	N	Y

Table 11 shows average coefficients and t-statistics across 1,000 simulations. We estimate a regression of the ratio of the firm investment to its book value, $i_t \equiv I_{ft}/B_{f,t-1}$, on the innovation in z_t and a vector of controls X_t, which includes lagged values of log Tobin's Q, cash flows over lagged capital, and log capital:

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d \, D(Q_{f,t-1})_d + b_1 \, (-z_{t-1}) + \sum_{d=2}^{5} b_d \, D(Q_{f,t-1})_d \times (-z_{t-1}) + cX_{f,t-1} + u_{ft},$$

Tobin's Q is computed as the ratio of the market value of the firm divided by Book Value. $D(Q_{f,t-1})_d$ is a dummy variable which takes the value of 1 if the firm f falls in the d-th quintile in terms of Tobin's Q. Data are simulated at weekly frequency ($dt = 1/52$) and then aggregated to form annual values. Investment by firm is computed as the sum of the market value of new investment, i.e. $I_{ft} = \sum_{s \in t} x_s z_s K_{fs}$, where K_{fs} denotes the capital of project acquired by firm f at time s. Book value is computed as the replacement cost of capital, $B_{ft} = z_t \, x_t \, \sum_{j \in \mathcal{J}_{ft}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{ft}$ denotes the set of projects owned by firm f at the end of year t. All variables have been standardized to zero mean and unit standard deviation. We report averages across simulations of coefficients and t statistics (in parenthesis). Standard errors are robust to heteroscedasticity and clustered at the firm level. Each simulation sample contains 2,500 firms for 50 years.

Table 12: 10 portfolios sorted on IMC beta

	Data										
IMC Beta	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
$E(R) - r_f$ (%)	7.13	6.78	8.00	7.44	8.03	5.86	6.14	5.93	5.66	4.10	-3.03
	(2.83)	(2.96)	(3.56)	(3.17)	(3.34)	(2.24)	(2.25)	(1.88)	(1.49)	(0.89)	(-0.76)
σ (%)	15.92	14.47	14.22	14.86	15.21	16.53	17.27	19.96	23.99	29.25	25.09
β_{MKT}	0.81	0.79	0.80	0.87	0.90	1.01	1.05	1.17	1.39	1.60	0.79
	(19.19)	(25.18)	(31.44)	(33.75)	(33.30)	(53.43)	(59.87)	(44.38)	(33.19)	(26.29)	(8.56)
α(%)	2.39	2.17	3.30	2.37	2.80	-0.02	0.02	-0.92	-2.44	-5.22	-7.61
	(1.59)	(1.79)	(3.04)	(2.35)	(2.74)	(-0.02)	(0.02)	(-0.76)	(-1.45)	(-2.18)	(-2.26)
R^2(%)	63.29	72.54	78.03	83.24	84.76	90.50	89.81	84.10	81.52	72.63	23.86
β_{MKT}	1.00	0.97	0.97	0.99	1.01	1.07	1.03	1.04	1.13	1.17	0.17
	(23.54)	(40.14)	(58.99)	(42.25)	(54.10)	(56.66)	(46.05)	(42.81)	(36.84)	(31.59)	(2.81)
β_{IMC}	-0.43	-0.42	-0.38	-0.29	-0.27	-0.15	0.04	0.29	0.60	0.99	1.42
	(-10.79)	(-13.44)	(-16.37)	(-7.50)	(-8.45)	(-4.26)	(0.91)	(5.81)	(12.51)	(13.32)	(20.43)
α(%)	0.31	-0.03	1.34	0.89	1.37	-0.65	0.35	0.75	0.91	0.19	-0.12
	(0.22)	(-0.04)	(1.69)	(1.04)	(1.66)	(-0.74)	(0.38)	(0.57)	(0.74)	(0.12)	(-0.05)
R^2(%)	74.62	85.41	89.10	89.03	89.66	91.75	90.02	87.49	90.84	89.88	72.16
	Model										
IMC Beta	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
$E(R) - r_f$(%)	7.50	7.29	7.03	6.78	6.50	6.20	5.83	5.41	4.84	3.99	-3.51
	(3.72)	(3.50)	(3.30)	(3.10)	(2.88)	(2.67)	(2.42)	(2.15)	(1.81)	(1.34)	(-2.50)
σ(%)	14.35	14.80	15.16	15.54	15.98	16.47	17.04	17.75	18.71	20.38	10.51
β_{MKT}	0.82	0.87	0.89	0.92	0.95	0.98	1.02	1.06	1.11	1.19	0.36
	(22.83)	(29.91)	(37.78)	(48.46)	(66.69)	(91.27)	(102.76)	(75.40)	(48.45)	(30.98)	(5.02)
α(%)	2.70	2.24	1.81	1.38	0.92	0.44	-0.13	-0.79	-1.65	-2.98	-5.67
	(4.63)	(4.80)	(4.73)	(4.48)	(3.90)	(2.40)	(-0.82)	(-3.33)	(-4.39)	(-4.70)	(-4.85)
R^2(%)	91.28	94.71	96.57	97.81	98.70	99.21	99.30	98.92	97.64	94.51	34.79
β_{MKT}	0.96	0.98	0.99	0.99	1.00	1.01	1.01	1.02	1.02	1.03	0.06
	(52.55)	(68.83)	(80.04)	(86.59)	(93.31)	(92.06)	(89.09)	(87.59)	(84.15)	(83.47)	(2.43)
β_{IMC}	-0.33	-0.27	-0.22	-0.17	-0.11	-0.05	0.02	0.10	0.21	0.38	0.71
	(-11.57)	(-12.57)	(-12.16)	(-10.76)	(-8.04)	(-3.82)	(1.04)	(6.03)	(11.85)	(21.23)	(18.19)
α(%)	0.29	0.27	0.21	0.14	0.08	0.03	-0.04	-0.08	-0.11	-0.07	-0.36
	(0.92)	(1.07)	(0.92)	(0.64)	(0.40)	(0.10)	(-0.21)	(-0.36)	(-0.45)	(-0.23)	(-0.79)
R^2(%)	97.82	98.80	99.15	99.31	99.37	99.38	99.35	99.32	99.22	99.20	91.86

The top panel of Table 12 reports asset-pricing tests on 10 portfolios sorted on β^{imc}_{t-1}. β^{imc}_t refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. The construction of the IMC portfolio is detailed in Papanikolaou (2008). We use monthly data from January 1965 through December 2005. Standard errors are computed using NW with 1 lag, to adjust for autocorrelation in returns. t-statistics are computed in parenthesis. We report annualized estimates of mean returns and alphas. The bottom panel reports the corresponding estimates for simulated data. We report means across 1,000 simulations of coefficients and t-statistics. Each simulation sample contains 2,500 firms and has a length of 50 years. Returns of the market portfolio are computed as average return of the investment and consumption sectors, weighted by their market capitalization at time.

Table 13: 10 portfolios sorted on BE/ME

	Data										
BE/ME	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
$E(R) - r_f$ (%)	4.19	5.82	6.75	7.10	6.67	7.64	8.55	8.98	10.15	11.39	7.20
	(1.44)	(2.18)	(2.55)	(2.71)	(2.73)	(3.14)	(3.57)	(3.79)	(3.94)	(3.85)	(2.87)
σ (%)	18.44	16.90	16.75	16.56	15.48	15.38	15.13	15.00	16.30	18.70	15.89
β_{MKT}	1.09	1.04	1.02	0.98	0.90	0.90	0.84	0.83	0.89	0.97	-0.12
	(43.46)	(52.78)	(53.02)	(33.74)	(31.46)	(32.82)	(25.01)	(25.42)	(22.68)	(20.03)	(-1.88)
α(%)	-2.20	-0.22	0.79	1.38	1.44	2.38	3.64	4.11	4.93	5.73	7.93
	(-1.89)	(-0.27)	(0.95)	(1.29)	(1.29)	(2.35)	(2.96)	(3.36)	(3.59)	(3.05)	(2.94)
R^2(%)	85.75	91.45	90.52	85.36	81.68	83.56	75.37	75.21	73.21	65.40	1.50
β_{MKT}	1.02	1.05	1.08	1.07	1.00	0.99	0.96	0.96	1.00	1.03	0.01
	(36.12)	(53.48)	(54.19)	(40.51)	(37.25)	(40.08)	(31.29)	(33.81)	(27.66)	(20.39)	(0.09)
β_{IMC}	0.18	-0.04	-0.14	-0.22	-0.25	-0.21	-0.29	-0.29	-0.25	-0.14	-0.32
	(6.37)	(-1.43)	(-3.98)	(-5.73)	(-8.04)	(-7.54)	(-8.01)	(-7.69)	(-6.25)	(-2.53)	(-4.85)
α(%)	-1.08	-0.09	0.20	0.23	0.00	1.20	2.12	2.59	3.48	5.09	6.17
	(-0.93)	(-0.11)	(0.24)	(0.23)	(0.00)	(1.28)	(1.95)	(2.41)	(2.64)	(2.56)	(2.22)
R^2(%)	87.37	91.78	91.73	88.30	85.67	86.28	80.87	80.80	76.84	66.11	7.60
	Model										
BE/ME	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
$E(R) - r_f$(%)	3.62	4.65	5.26	5.72	6.12	6.46	6.78	7.06	7.40	7.90	4.28
	(1.21)	(1.76)	(2.12)	(2.40)	(2.66)	(2.89)	(3.11)	(3.31)	(3.53)	(3.83)	(2.98)
σ(%)	20.49	18.49	17.48	16.83	16.30	15.87	15.50	15.18	14.91	14.67	10.65
β_{MKT}	1.19	1.09	1.04	1.00	0.97	0.94	0.92	0.90	0.87	0.84	-0.34
	(29.75)	(48.67)	(75.39)	(98.94)	(87.67)	(64.14)	(48.75)	(38.70)	(31.12)	(24.01)	(-4.71)
α(%)	-3.35	-1.76	-0.85	-0.17	0.42	0.92	1.40	1.83	2.31	2.98	6.34
	(-5.16)	(-4.88)	(-3.70)	(-1.01)	(2.22)	(3.85)	(4.60)	(4.94)	(5.18)	(5.33)	(5.41)
R^2(%)	93.81	97.65	98.93	99.29	99.14	98.59	97.71	96.56	94.90	91.60	31.02
β_{MKT}	1.02	1.01	1.01	1.00	1.00	1.00	0.99	0.99	0.98	0.98	-0.04
	(78.45)	(76.73)	(81.33)	(88.22)	(92.69)	(93.95)	(90.01)	(81.18)	(69.01)	(54.13)	(-1.30)
β_{IMC}	0.41	0.20	0.09	0.00	-0.06	-0.12	-0.17	-0.22	-0.26	-0.33	-0.74
	(20.54)	(9.92)	(4.84)	(0.11)	(-4.58)	(-8.78)	(-11.19)	(-11.97)	(-12.06)	(-11.42)	(-17.24)
α(%)	-0.23	-0.30	-0.23	-0.17	-0.07	0.01	0.13	0.23	0.38	0.57	0.80
	(-0.92)	(-1.22)	(-1.09)	(-0.89)	(-0.43)	(0.01)	(0.63)	(1.06)	(1.47)	(1.83)	(1.69)
R^2(%)	99.22	99.13	99.27	99.34	99.39	99.37	99.27	99.12	98.77	97.88	91.17

The top panel of Table 13 reports asset-pricing tests on 10 portfolios sorted on Book to Market Equity. The data come from Kenneth French's website. The construction of the IMC portfolio is detailed in Papanikolaou (2008). We use monthly data from January 1965 through December 2005. Standard errors are computed using NW with 1 lag, to adjust for autocorrelation in returns. t-statistics are computed in parenthesis. We report annualized estimates of mean returns and alphas. The bottom panel reports the corresponding estimates for simulated data. Market Equity equals the value of the firm, V_{ft}, and book to market equals Book Value divided by Market Equity. Book Value is computed as the replacement cost of capital, $B_{ft} = z_t\, x_t \sum_{j \in \mathcal{J}_{ft}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{it}$ denotes the set of projects owned by firm f at the end of year t. We report means across 1,000 simulations of coefficients and t-statistics. Each simulation sample contains 2,500 firms and has a length of 50 years. Returns of the market portfolio are computed as average return of the investment and consumption sectors, weighted by their market capitalization at time.

7 Appendix

7.1 Proofs and Derivations

Proof of Lemma 1. That is, K_i is the solution to the problem:

$$\max_{K_i} A(\varepsilon_{ft}, 1) x_t K_i^{\alpha} - z_t\, x_t\, K_i.$$

The first order condition is

$$\alpha\, A(\varepsilon_{ft}, 1) K_i^{\alpha-1} = z_t.$$

■

Proof of Lemma 2. The value of growth options depends on the NPV of future projects. When a project is financed, the value added net of investment costs is

$$\left[\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right] z_t^{\frac{\alpha}{\alpha-1}} x_t\, A(\varepsilon_{ft}, 1)^{\frac{1}{1-\alpha}} = C z_t^{\frac{\alpha}{\alpha-1}} x_t\, A(\varepsilon_{ft}, 1)^{\frac{1}{1-\alpha}}.$$

The value of growth options for firm f equals the sum of the net present value of all future projects

$$\begin{aligned} PVGO_{it} &= E_t^{\mathcal{Q}}\left[\int_t^{\infty} e^{-r(s-t)} \lambda_{fs} C z_s^{\frac{\alpha}{\alpha-1}} x_s\, A(\varepsilon_{fs}, 1)^{\frac{1}{1-\alpha}}\, ds\right] \\ &= C z_t^{\frac{\alpha}{\alpha-1}} x_t\, E_t^{\mathcal{Q}}\left[\int_t^{\infty} e^{-\rho(s-t)} \lambda_{fs}\, A(\varepsilon_{fs}, 1)^{\frac{1}{1-\alpha}}\, ds\right] \\ &= C z_t^{\frac{\alpha}{\alpha-1}} x_t\, E_t\left[\int_t^{\infty} e^{-\rho(s-t)} \lambda_{fs}\, A(\varepsilon_{fs}, 1)^{\frac{1}{1-\alpha}}\, ds\right] \\ &= z_t^{\frac{\alpha}{\alpha-1}} x_t\, G(\varepsilon_{ft}, \lambda_{ft}), \end{aligned}$$

where $E_t^{\mathcal{Q}}$ denotes expectations under the risk-neutral measure $\mathcal{Q}$, where

$$\frac{d\mathcal{Q}}{d\mathcal{P}} = \exp\left(-\beta_x B_{x,t} - \beta_z B_{z,t} - \frac{1}{2}\beta_x^2\, t - \frac{1}{2}\beta_z^2\, t\right).$$

The second to last equality follows from the fact that λ_{ft} and ε_{ft} are idiosyncratic, and thus have the same dynamics under $\mathcal{P}$ and $\mathcal{Q}$.

Let $\mathbf{M}$ be the infinitesimal matrix associated with the transition density [Karlin and Taylor, 1975] of λ_{ft}:

$$\mathbf{M} = \begin{pmatrix} -\mu_L & \mu_L \\ \mu_H & -\mu_H \end{pmatrix}$$

The eigenvalues of $\mathbf{M}$ are 0 and $-(\mu_L + \mu_H)$. Let $\mathbf{U}$ be the matrix of the associated eigenvectors, and define

$$\Lambda(u) = \begin{pmatrix} 1 & 0 \\ 0 & e^{(-\mu_L+\mu_H)\,u} \end{pmatrix}$$

Then

$$E_t[\lambda_{fs}] = \mathbf{U}\,\Lambda(s-t)\,\mathbf{U}^{-1}\begin{bmatrix} \lambda_H \\ \lambda_f \cdot \lambda_L \end{bmatrix} = \begin{bmatrix} 1 + \frac{\mu_L}{\mu_L+\mu_H}(\lambda_H - \lambda_L)e^{-(\mu_L+\mu_H)(s-t)} \\ 1 - \frac{\mu_H}{\mu_L+\mu_H}(\lambda_H - \lambda_L)e^{-(\mu_L+\mu_H)(s-t)} \end{bmatrix}$$

and

$$\begin{aligned} G(\varepsilon_{ft}, \lambda_{ft}) &= C \cdot E_t\left[\int_t^\infty e^{-\rho(s-t)}\,\lambda_{fs}\,A(\varepsilon_{fs},1)^{\frac{1}{1-\alpha}}\,ds\right] = C \cdot E_t\left[\int_t^\infty e^{-\rho(s-t)}\,E_t[\lambda_{fs}]\,A(\varepsilon_{fs},1)^{\frac{1}{1-\alpha}}\,ds\right] \\ &= \begin{array}{ll} \lambda_f\left(G_1(\varepsilon_{ft}) + \frac{\mu_L}{\mu_L+\mu_H}(\lambda_H-\lambda_L)\,G_2(\varepsilon_{ft})\right), & \tilde{\lambda}_{ft} = \lambda_H \\ \lambda_f\left(G_1(\varepsilon_{ft}) - \frac{\mu_H}{\mu_L+\mu_H}(\lambda_H-\lambda_L)\,G_2(\varepsilon_{ft})\right), & \tilde{\lambda}_{ft} = \lambda_L \end{array} \end{aligned}$$

The second equality uses the fact the law of iterated expectations and the fact that $\lambda_{f,t}$ is independent across firms. The functions $G_1(\varepsilon)$ and $G_2(\varepsilon)$ are defined as

$$\begin{aligned} G_1(\varepsilon) &= C \cdot E_t \int_t^\infty e^{-\rho(s-t)}\,A(\varepsilon,1)^{\frac{1}{1-\alpha}}\,ds \\ G_2(\varepsilon) &= C \cdot E_t \int_t^\infty e^{-(\rho+\mu_L+\mu_H)(s-t)}\,A(\varepsilon,1)^{\frac{1}{1-\alpha}}\,ds. \end{aligned}$$

By the Feynman-Kac Theorem, $G_1(\varepsilon)$ and $G_1(\varepsilon)$ will satisfy the ODEs:

$$\begin{aligned} C \cdot A(\varepsilon,1)^{\frac{1}{1-\alpha}} - \rho G_1(\varepsilon) - \theta_\epsilon(\varepsilon-1)\frac{d}{d\,\varepsilon}\,G_1(\varepsilon) + \frac{1}{2}\sigma_e^2\,\varepsilon\,\frac{d^2}{d\,\varepsilon^2}\,G_1(\varepsilon) &= 0 \\ C \cdot A(\varepsilon,1)^{\frac{1}{1-\alpha}} - (\rho+\mu_H+\mu_L)G_2(\varepsilon) - \theta_\epsilon(\varepsilon-1)\frac{d}{d\,\varepsilon}\,G_2(\varepsilon) + \frac{1}{2}\sigma_e^2\,\varepsilon\,\frac{d^2}{d\,\varepsilon^2}\,G_2(\varepsilon) &= 0. \end{aligned}$$

■

Proof of Lemma 1. The risk premium on assets in place will be determined by the covariance with the pricing kernel:

$$E_t R_{ft}^V - r_f = -cov\left(\frac{dp_t}{p_t}, \frac{d\pi_t}{\pi_t}\right) = \beta_x \sigma_x$$

Similarly for growth options:

$$E_t R^G_{ft} - r_f = -cov\left(\frac{dPVGO_t}{PVGO_t}, \frac{d\pi_t}{\pi_t}\right) = \beta_x \sigma_x - \frac{\alpha}{1-\alpha}\beta_z \sigma_z$$

The risk premium on growth options will be lower than assets in place as long as $\beta_z > 0$. Consequently, expected returns excess returns of the firm will equal

$$E_t R_{ft} - r_f = \frac{VAP_{ft}}{V_{ft}}(ER^V_{ft} - r_f) + \frac{PVGO_{ft}}{V_{ft}}(ER^G_{ft} - r_f)$$

■

Proof of Lemma 3. Profits accruing to the I-sector can be written as

$$\begin{aligned}
\Pi_t &= \phi\, z_t\, x_t \int K_{it} di \\
&= \phi \left(\int A(e_i, 1)^{\frac{1}{1-\alpha}} di \right) \alpha^{\frac{1}{1-\alpha}}\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} \\
&= \Gamma \cdot x_t\, z_t^{\frac{\alpha}{\alpha-1}}
\end{aligned}$$

The price of the investment firm satisfies

$$\begin{aligned}
V_{I,t} &= E_t^Q \int_t^\infty \exp\{-r(s-t)\}\ \phi \Pi_s ds \\
&= \phi\Gamma\, E_t^Q \int_t^\infty \exp\{-r(s-t)\}\ x_s\, z_s^{\frac{\alpha}{\alpha-1}} ds \\
&= \phi\Gamma\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} E_t^Q \int_t^\infty \exp\Bigg\{ \left(-r + \mu_X - \frac{1}{2}\sigma_X^2 - \frac{\alpha\,\mu_Z}{1-\alpha} + \frac{1}{2}\frac{\alpha}{1-\alpha}\,\sigma_Z^2 \right)(s-t) + \\
&\quad + \sigma_X (B_s^x - B_t^x) + \frac{\alpha\,\sigma_Z}{\alpha-1}(B_s^z - B_t^z) \Bigg\} \\
&= \phi\Gamma\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} \int_t^\infty \exp\left\{ \left(-r + \mu_X - \frac{\alpha}{1-\alpha}\,\mu_Z + \frac{1}{2}\frac{\alpha}{1-\alpha}\,\sigma_Z^2 + \frac{1}{2}\frac{\alpha^2\,\sigma_Z^2}{(1-\alpha)^2} \right)(s-t) \right\} \\
V_{I,t} &= \phi\Gamma\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} \frac{1}{D_I}
\end{aligned}$$

■

Proof of Lemma 2. Returns on the IMC portfolio follow:

$$\begin{aligned} R_t^I - R_t^C &= (\cdot)\,dt + \sigma_X dB_t^X + \frac{\alpha}{\alpha-1}\sigma_Z dB_t^Z - \sigma_X dB_t^X - \frac{\overline{PVGO}_t}{\overline{V}_t}\frac{\alpha}{\alpha-1}\sigma_Z dB_t^Z \\ &= (\cdot)\,dt + \frac{\overline{VAP}_t}{\overline{V}_t}\frac{\alpha}{\alpha-1}\sigma_Z dB_t^Z \end{aligned}$$

whereas the return of firm i in the consumption sector is:

$$\begin{aligned} R_{ft} &= (\cdot)\,dt + \frac{VAP_{ft}}{V_{ft}}\sigma_X dB_t^x + \left(1 - \frac{VAP_{ft}}{V_{ft}}\right)\left(\sigma_X dB_t^x + \frac{\alpha}{\alpha-1}\sigma_Z dB_t^Z\right) + (\cdot)dZ_t^i + \sum_j (\cdot)dZ_t^j \\ &= (\cdot)\,dt + \sigma_X dB_t^x + \left(1 - \frac{VAP_{ft}}{V_{ft}}\right)\left(\frac{\alpha}{\alpha-1}\sigma_Z dB_t^Z\right) + (\cdot)dZ_t^i + \sum_j (\cdot)dZ_t^j \end{aligned}$$

so

$$cov_t(R_{ft}, R_t^I - R_t^C) = \left(\frac{PVGO_{ft}}{V_{ft}}\right)\left(\frac{\overline{VAP}_{ft}}{\overline{V}_{ft}}\right)\frac{\alpha^2}{(1-\alpha)^2}\sigma_Z^2$$

and

$$var_t(R_t^I - R_t^C) = \left(\frac{\overline{VAP}_{ft}}{\overline{V}_{ft}}\right)^2 \frac{\alpha^2}{(1-\alpha)^2}\sigma_Z^2$$

which implies that the beta of firm f with the IMC portfolio is increasing in firm f's growth options. ■