FE312: Homework 6

1 Testing a Liquidity Factor Model

In their 2003 Journal of Political Economy paper, Lubos Pastor and Robert Stambaugh advocate the need for a liquidity factor. Further, they claim that their liquidity factor can explain part of the momentum. In this homework we will investigate their claims and in the process learn a bit more how we can implement the APT in practice.

Open the dataset HW6_Data.xlsx. It contains the following: The Pastor-Stambaugh liquidity factor (ps_level), innovations in the liquidity factor (ps_innov), excess returns on the market portfolios (MKTRF), the risk-free rate (RF) and returns on 10 Portfolios sorted on past 1 year returns (the 10 Momentum portfolios). Let us test a version of the APT that has two factors: the market portfolio and the Pastor-Stambaugh liquidity factor. We will investigate whether the momentum effect can be attributed to differences in exposure to the liquidity factor between winners and losers.

1. Plot the level of the Pastor-Stambaugh liquidity factor. High values of this factor indicate liquid months, low levels identify illiquid months. Identify the two months when the liquidity factor had its lowest value. What happened on those dates?

2. Pastor and Stambaugh have kindly provided us with an innovation series (ps_level). How do you think this series is constructed and how does it differ from the level liquidity factor (ps_level)?

3. Construct $\tilde{f}_{m,t} = MKTRF_t - E[MKTRF_t]$, i.e. innovations in returns to the market portfolio. Use the historical average return for the market to estimate $E[MKTRF_t]$

4. For each momentum portfolio ($R_1 \ldots R_{10}$) estimate the factor loadings on innovations to the market portfolio and the liquidity innovation series

$$R_{i,t} = a_i + b_{mkt,i}\tilde{f}_{m,t} + b_{liq,i}PS\_INNOV_t + \epsilon_{it}$$

From this regression, we will use the market loadings $b_{mkt,i}$ and the liquidity factor loadings $b_{liq,i}$. You should have 10 of each.

5. Is there any difference in liquidity factor loadings across the momentum portfolios? Interpret the estimated loadings on the liquidity factor for the 10 momentum portfolios. When liquidity is high, which portfolio goes up by more, winners or losers? Is the difference statistically significant? (HINT: for answering the latter question, create an eleventh portfolio, Winners - Losers and estimate its market and liquidity factor betas).

6. Estimate the historical average return of each of the 10 momentum portfolios,

$$\hat{\mu}_i = \frac{1}{T}\sum_{t=1}^{T} R_{i,t}.$$

7. Now, it's time to estimate the model: Regress the estimates of average returns on the 10 momentum portfolios on their loadings on the market and the liquidity factor:

$$\hat{\mu}_i = \lambda_0 + \lambda_{mkt} b_{mkt,i} + \lambda_{liq} b_{liq,i} + u_i$$

This is a regression with only 10 datapoints, one for each momentum portfolio. Report the estimates for the λ's, the associated t-statistics and the R^2 of this regression.

8. Interpret the estimated prices of risk λ_{mkt} and λ_{liq}. Does the sign make sense? Are they statistically different from zero? Also interpret the estimate of the risk-free rate, λ_0. Given that we are dealing with monthly data, does the magnitude make sense?

9. Based on your findings what do you conclude? Do you think that liquidity risk is priced, at least using the cross-section of momentum portfolios? Does liquidity provide an adequate explanation for momentum? Plot the historical average return of these portfolios $\hat{\mu}_i$ versus the model-predicted return $\lambda_0 + \lambda_{mkt} b_{mkt,i} + \lambda_{liq} b_{liq,i}$.

2 Factor Models

Suppose there are three stocks, A,B and C. Your analyst tells you that a 2 factor model with *uncorrelated factors* accurately describes returns. Factor 1 is industrial production and Factor 2 is energy prices. Note that f_1 and f_2 are factor surprises, that is $Ef_i = 0$ and the intercepts hence are the respective expected returns on the assets (e.g. $E(r_A) = 0.36$ is the expected return on stock A.

$$\begin{aligned} r_A &= 0.36 + 2f_1 + 4f_2 + \epsilon_A \\ r_B &= 0.225 + 3f_1 + 2f_2 + \epsilon_B \\ r_C &= 0.12 + 1f_1 + 1f_2 + \epsilon_C \end{aligned}$$

with variances given by $var(f_1) = 0.1$, $var(f_2) = 0.1$, $var(\epsilon_A) = 0.15$, $var(\epsilon_B) = 0.28$, $var(\epsilon_C) = 0.05$.

1) What is the covariance matrix between the three asset returns? [Hint: we are assuming that the factors are uncorrelated]

2) What is the risk-free rate implied by the absence of arbitrage?

3) What is the three R^2s that your analyst got when he regressed the returns of A, B and C respectively on the factor realizations?

4) Can you find two portfolios of A, B and C that have *only* f_1 or f_2 factor exposure respectively? What are their expected returns and standard deviations?

5) Let us now use the APT in asset allocation:

   (a) What is the optimal (=max SR) portfolio of A,B and C? Its Sharpe ratio? Its factor loadings on f1 and f2? [Hint: you'll have to calculate the asset correlations first and then input in Markowitz]

   (b) Suppose that your clients do not want any exposure to factor 2 risk. How does your optimal portfolio look now? [Hint: you'll have to introduce an additional constraint in Markowitz]

   (c) Now, revisit part (a), but now choose only between the two factor mimicking portfolios you constructed in part 4. Did you get a different answer?

6) Is it possible that the CAPM holds in this economy? What would the market capitalization of A, B and C have to be for both the APT and the CAPM to hold?