FINC-460 MIDTERM EXAM

1. Question 1

(1) Not true - As idiosyncratic (i.e. diversifiable) variance should not earn any risk premium. Only higher exposure to systemic variance shoud earm higher expected returns.

(2) Not necessarily - it can be that the stocks that outperformed are higher beta stocks and stocks which underperformed were lower beta stocks (which would be consistent with CAPM).

(3) All assets are in positive net supply and if CAPM holds the market portfoilio that holds the assets in market weight proportions is efficient. So CAPM implies that all assets will have a positive weight in the mean varianc eefficient frontier.

(4) When stock returns are i.i.d., the optimal portfolio is independent of horizon. When stock returns are predictable, however the optimal portfolio allocation does depend on investment horizon. In this case, long horizon investors face less risk and hence invest more in stocks.

(5) Portfolio constarints serve a different purpose. Mean varaince optimized portfoilios are highly senstive to the expected return and covariance inputs. And often these inputs can have estimation errors. Constraints will help alleviate some of the problems with errors.

2. Question 2

(1) By CAPM, for any asset i, we have

$$\begin{aligned}
\beta_i &= \frac{E[R_i - r_f]}{E[R_M - r_f]} \\
\beta_A &= \frac{7.8 - 3}{6 - 3} \\
&= 1.6 \\
\beta_B &= \frac{4.2 - 3}{6 - 3} \\
&= 0.4 \\
\beta_C &= \frac{3 - 3}{6 - 3} \\
&= 0
\end{aligned}$$

(2)

$$\sigma_A^2 = \beta_A^2\sigma_M^2 + \sigma_{\epsilon,A}^2$$
$$0.2^2 = 1.6^2 * 0.1^2 + \sigma_{\epsilon,A}^2$$
$$\sigma_{\epsilon,A}^2 = 0.0144$$

For A, systemic variance is $\beta_A^2\sigma_M^2 = 0.0256$ and total variance is $\sigma_A^2 = 0.04$. So percentage of variance due to systematic shocks is given by $0.0256/0.04 = 64\%$. The remaining 36% is idiosyncratic.

For B, systemic variance is $\beta_B^2\sigma_M^2 = 0.0016$ and total variance is $\sigma_B^2 = 0.01$. So percentage of variance due to systematic shocks is given by $0.0016/0.01 = 16\%$. The remaining 84% is idiosyncratic.

For C, systemic variance is 0 as $beta_C = 0$ and total variance is $\sigma_C^2 = 0.0025$. So the total variance is idiosyncratic.

(3) a) Since CAPM holds the market portfolio is efficient and as such has the highest Sharpe ratio. So any portfolio of the risk free asset and the market will have the highest Sharpe ratio.

b) Risk aversion coefficient, $A = 5$. So the amount invested in the market will be

$$w = \frac{E[R - r_f]}{A\sigma^2} = \frac{6\% - 3\%}{5 * 0.1^2} = 0.6$$

The rest (0.4) will be invested in the risk free asset.

c) If the risk aversion coefficient was very large, the investor would still invest in the same securities (market, risk free) , just the proportion of assets invested in the market will reduce.

(4) a) We would iunvest only in the risk free - notice asset C has the same expected return as the risk free rate but has non-zero variance. So we would be exposed to idiosyncratic risk without any risk premuim.

b) The amount invested in asset C would depend on the covariance between asset A and C. Since A,C are uncorrelated we would invest only in asset A and not in asset C. Suppose we are invested 100% of our risky portfolio in asset A. Now suppose we replace w wt in the portfolio with asset C. We are replacing a positive sharpe ratio asset with a zero sharpe ratio asset that does not add any diversification benefit - this will be suboptimal. Similarly we would not want to borrow using C (short) either as we can borrow without risk in the risk free asset.

c) The same argument holds for com bination of B and C.

d) If C is correlated with A or B then it may become optimal for go long (if negatively correlated) or short (if positively correlated) in asset C.

(5) From the previous question we know that C will not be part of an efficient portfolio. So asset C must be in zero net supply and the market will be composed of assets A,B. Let w_A be the wt of asset A in the market portfolio. Then $1 - w_A$ will be the weight of asset B in the portfolio. They must satisfy the following equations

$$w_A\beta_A + (1 - w_A)\beta_B = 1$$
$$w_A\mathbb{E}[R_a] + (1 - w_A)\mathbb{E}[R_B] = \mathbb{E}[R_M]$$

Notice that these two are essentially the same. In addition the following can also be used

$$w_A^2\sigma_A^2 + (1 - w)^2\sigma_B^2 = \sigma_M^2$$

Last, we also know that

$$w_A + w_B = 1$$

Using any two of the above three equations we get

$$w_A = w_B = 0.5$$