Lecture 3: Equilibrium

Investments

Introduction

- To operationalize Mean-Variance Analysis we need estimates of expected returns.
  - ↪ Last week, we saw that expected returns are very hard to estimate.
  - ↪ It will be useful to have a theory of what expected returns *should* be.
- The CAPM is an *equilibrium* model specifying a relation between expected rates of return and systematic risk (covariance) for all assets.
  - ↪ *Equilibrium* is an economic term that characterizes a situation where no investor wants to do anything differently.
  - ↪ Note that the Markowitz portfolio problem is relevant for each investor, regardless of whether the *equilibrium* argument, and the CAPM, is correct or not.

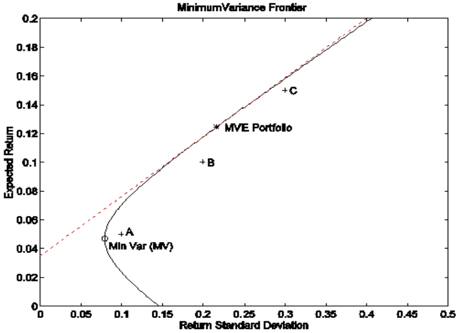
Equilibrium Pricing

- If everyone in the economy holds an efficient portfolio, then how should securities be priced so that they are actually bought 100% in equilibrium?
  - ↪ For example, if based on the prices/expected returns our model comes up with, we found that no maximizing investor would like to buy IBM, then something is wrong.
  - ↪ IBM would be priced too high (offer too low an expected rate of return).
  - ↪ The price of IBM would have to fall to the point where, in aggregate, investors want to hold exactly the number of IBM shares outstanding.
- So, what sort of prices (risk/return relationships) are feasible in equilibrium? The CAPM will give an answer.

The CAPM Assumptions

- A number of assumptions are necessary to formally derive the CAPM:
  1. No transaction costs or taxes.
  2. Assets are all tradable and are all infinitely divisible.
  3. No individual can effect security prices (perfect competition).
  4. Investors care only about expected returns and variances.
  5. Unlimited short sales and borrowing and lending.
  6. Homogeneous expectations.
- Assumptions 4 - 6 imply everyone solves the passive portfolio problem we just finished, and they all see the same efficient frontier!
- Some of these can be relaxed without too-much effect on the results.

MV Analysis and two fund seperation



- Markowitz: everyone holds a linear combination of two portfolios
  - ↪ the risk-free security
  - ↪ the tangency portfolio
- If everyone sees the same efficient frontier and CAL, then **everyone has the same tangency portfolio**

- What is the tangency portfolio?

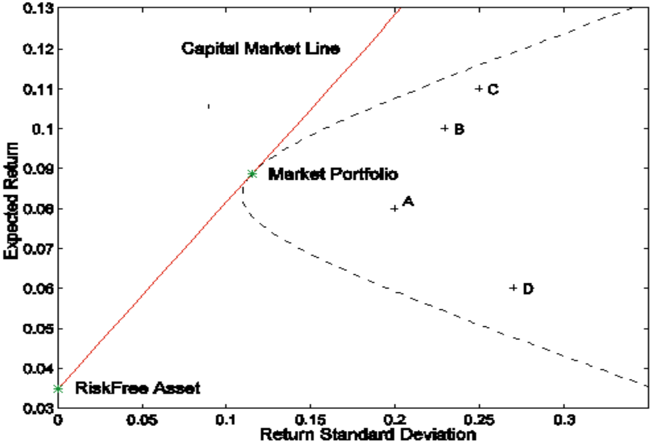
  1. Markowitz: Investors *should* hold the tangency portfolio.
  2. Equilibrium Theory (market clearing):
     - The risk-free asset is in zero supply:
       Borrowing and Lending must cancel out.
     - Average investor *must* want to hold the market portfolio.

- CAPM: the tangency portfolio **must be the market portfolio**.

$\hookrightarrow$ **Definition:** The "market" or total wealth portfolio is a portfolio of **all** risky securities held in proportion to their market value. This must be the sum over all securities, i.e. stocks, bonds, real-estate, human capital, etc.

$\hookrightarrow$ Here's where the assumption that all assets are tradeable comes in.

The Capital Market Line



- In equilibrium, every investor faces the same CAL.

What about Individual Assets?

- This CAL is called the *Capital Market Line* (CML). This line gives us the set of efficient or optimal risk-return combinations

$$E(\tilde{r}_e) = r_f + \left(\frac{E(\tilde{r}_m) - r_f}{\sigma_m} \right) \sigma_e$$

where $\tilde{r}_e$ is the return on any *efficient* portfolio (i.e. on the CML)

- Note that this says that all investors should only hold combinations of the market and the risk-free asset.

  $\hookrightarrow$ How does this relate to the increased popularity of index funds?

- However, the goal of the CAPM is to provide a theory for expected returns of *inefficient portfolios* (or individual assets) based on equilibrium arguments.

CAPM, a preview

- If investors want to hold the market portfolio, they should not profit by deviating
  - ↪ Investors only want to hold a security in their portfolio if it provides a reasonable amount of extra reward (expected return) in return for the risk (variance) it adds to the portfolio
  - ↪ Since no deviation is profitable, what each security adds to the risk of a portfolio must be exactly offset by what it adds in terms of expected return.
- Ratio of *marginal return* to *marginal variance* must be the same for all assets
  - ↪ What each asset adds in expected return is its *expected excess return*
  - ↪ What each adds in risk is proportional to its *covariance* with the portfolio we are holding (the market)
- This is the intuition for the standard form of the CAPM, which relates β (i.e. scaled covariance) to expected return.

- How does adding a small amount of security to the market portfolio affect its variance?

$$
\begin{aligned}
\sigma_m^2 &= \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j cov(\tilde{r}_i, \tilde{r}_j) \\
&= \sum_{i=1}^{N} w_i \left[\sum_{j=1}^{N} w_j cov(\tilde{r}_i, \tilde{r}_j)\right] \\
&= \sum_{i=1}^{N} w_i cov\left(\tilde{r}_i, \quad \underbrace{\left[\sum_{j=1}^{N} w_j \tilde{r}_j\right]}_{\text{return on the market}}\right)
\end{aligned}
$$

$\hookrightarrow$ The *marginal* increase in risk when you change the amount of a security in your portfolio is the **covariance with the portfolio return**.

A formal derivation of the CAPM

- Under our assumptions, all investors must hold the market portfolio
- Based on our notion of equilibrium, every investor must be content with their portfolio holdings; if this were not the case than the prices of the securities would have to change
  - $\hookrightarrow$ This is just a supply and demand argument; if some investors want to buy IBM, and no one wants to sell, prices will have to increase
  - $\hookrightarrow$ In equilibrium, everyone must be optimally invested
- No one can do anything to increase the Sharpe-ratio of their portfolio

A formal derivation of the CAPM

Suppose you currently hold the market portfolio, but decide to borrow a small additional fraction δ_{GM} of your wealth at the risk-free rate and invest it in GM

1. The return in your new portfolio

$$\tilde{r}_c = \tilde{r}_m - \delta_{GM} \cdot r_f + \delta_{GM} \cdot \tilde{r}_{GM}$$

2. So the expected return and variance will be:

$$\begin{aligned} E(\tilde{r}_c) &= E(\tilde{r}_m) + \delta_{GM} \cdot (E(\tilde{r}_{GM}) - r_f) \\ \sigma_c^2 &= \sigma_m^2 + \delta_{GM}^2 \cdot \sigma_{GM}^2 + 2 \cdot \delta_{GM} \cdot cov(\tilde{r}_{GM}, \tilde{r}_m) \end{aligned}$$

3. The changes in each of these are:

$$\begin{aligned} \Delta E(\tilde{r}_c) &= \delta_{GM} \cdot E(\tilde{r}_{GM} - r_f) \\ \Delta\sigma_c^2 &= 2 \cdot \delta_{GM} \cdot cov(\tilde{r}_{GM}, \tilde{r}_m) \end{aligned}$$

We ignore the δ_{GM}^2 term in the variance equation because, if δ is small (say 0.001), δ^2 must be so small that we can ignore it (0.000001).

A formal derivation of the CAPM

Now what if we invest δ more in GM, and invest just enough less in the IBM so that our portfolio variance stays the same.

1. The change in the variance is:

$$\Delta\sigma_c^2 = 2 \cdot (\delta_{GM} \cdot cov(\tilde{r}_{GM}, \tilde{r}_m) + \delta_{IBM} \cdot cov(\tilde{r}_{IBM}, \tilde{r}_m))$$

2. To make this zero, it must be the case that:

$$\delta_{IBM} = -\delta_{GM} \left(\frac{cov(\tilde{r}_{GM}, \tilde{r}_m)}{cov(\tilde{r}_{IBM}, \tilde{r}_m)} \right)$$

3. The change in the expected return of the portfolio will be:

$$\begin{aligned} \Delta E(\tilde{r}_c) &= \delta_{GM} \cdot E(\tilde{r}_{GM} - r_f) + \delta_{IBM} \cdot E(\tilde{r}_{IBM} - r_f) \\ &= \delta_{GM} \left[E(\tilde{r}_{GM} - r_f) - E(\tilde{r}_{IBM} - r_f) \left(\frac{cov(\tilde{r}_{GM}, \tilde{r}_m)}{cov(\tilde{r}_{IBM}, \tilde{r}_m)} \right) \right] \end{aligned}$$

A formal derivation of the CAPM

- However, we are holding the market portfolio, which is also the tangency portfolio. This portfolio has the highest Sharpe Ratio of *all* portfolios.

- Therefore, by definition, we **cannot** increase its expected return while keeping the variance constant.

  ↪ For this to be true it must be that:

$$\frac{E(\tilde{r}_{GM}) - r_f}{cov(\tilde{r}_{GM}, \tilde{r}_m)} = \frac{E(\tilde{r}_{IBM}) - r_f}{cov(\tilde{r}_{IBM}, \tilde{r}_m)} = \lambda$$

  ↪ λ is the ratio of the marginal benefit to the marginal cost.

A formal derivation of the CAPM

- Note that this also holds for portfolios of assets as well.
- We can use the market portfolio in place of IBM:

$$\frac{E(\tilde{r}_{GM}) - r_f}{cov(\tilde{r}_{GM}, \tilde{r}_m)} = \frac{E(\tilde{r}_m - r_f)}{cov(\tilde{r}_m, \tilde{r}_m)} = \frac{E(\tilde{r}_m - r_f)}{\sigma_m^2} = \lambda$$

which means that:

$$\begin{aligned} E(\tilde{r}_{GM}) - r_f &= \frac{E(\tilde{r}_m - r_f)}{\sigma_m^2} cov(\tilde{r}_{GM}, \tilde{r}_m) \\ &= E(\tilde{r}_m - r_f) \underbrace{\frac{cov(\tilde{r}_{GM}, \tilde{r}_m)}{\sigma_m^2}}_{\beta_{GM}} \end{aligned}$$

- This characterizes the **Security Market Line(SML)**.

The equity premium

- What determines the compensation for bearing market risk, $E(\tilde{r}_m) - r_f$

- If the CAPM is true, a mean-variance investor will allocate an amount
$$w_m^* = \frac{E(R_m) - r_f}{A\sigma_m^2}$$
to the market portfolio and the remainder to the risk-free rate

- Assume that all investors have the same risk aversion A. Since in equilibrium $w_m^* = 1$, it must be the case that
$$E(R_m) - r_f = A\sigma_m^2$$

- The compensation for market risk is increasing in risk aversion, and the amount of market risk. The same argument goes through if investors vary in risk aversion, replacing A above with an 'average' risk aversion coefficient for the economy

Summary

- By the definition of the tangent portfolio, investors should not be able to achieve a higher return/risk tradeoff (Sharpe Ratio) by combining the tangent portfolio with *any* other asset.

- This restriction implies a linear relationship between an asset's equilibrium return and its beta with the tangent portfolio:

$$E(\tilde{r}_i) - r_f = E(\tilde{r}_T - r_f) \times \beta_i$$

- The CAPM is the statement, that **in equilibrium**, the tangent portfolio is the market portfolio ($\tilde{r}_T = \tilde{r}_M$).

- One way to interpret this equation is as saying that the reward $(E(\tilde{r}_i) - r_f)$ must equal the amount of risk that is priced (β), times its price $(E\tilde{r}_M - r_f)$

SML - Example

Asset	$E(r)$	σ
A	8%	20%
B	10%	23%
C	11%	25%
D	6%	27%

Correlations				
Assets	A	B	C	D
A	1.0	0.0	0.3	-0.2
B	0.0	1.0	0.2	-0.2
C	0.3	0.2	1.0	-0.2
D	-0.2	-0.2	-0.2	1.0

- We also assume that $r_f = 3.5\%$.
- The resulting MVE portfolio has weights on the risky assets of:

$$w_{MVE} = \begin{bmatrix} 0.2515 \\ 0.3053 \\ 0.2270 \\ 0.2161 \end{bmatrix}$$

- CAPM: MVE portfolio is the market.

SML - Example

1. Calculate the βs for each of the four assets:

$$\beta_i = \frac{cov(r_i, r_m)}{\sigma_m^2}$$

   1.1 use the equation for the covariance of two portfolios

$$\begin{aligned} cov(r_A, r_m) &= cov(r_A, w_A r_A + w_B r_B + w_C r_C + w_D r_D) \\ &= w_A cov(r_A, r_A) + w_B cov(r_A, r_B) \\ &\quad + w_C cov(r_A, r_C) + w_D cov(r_A, r_D) \end{aligned}$$

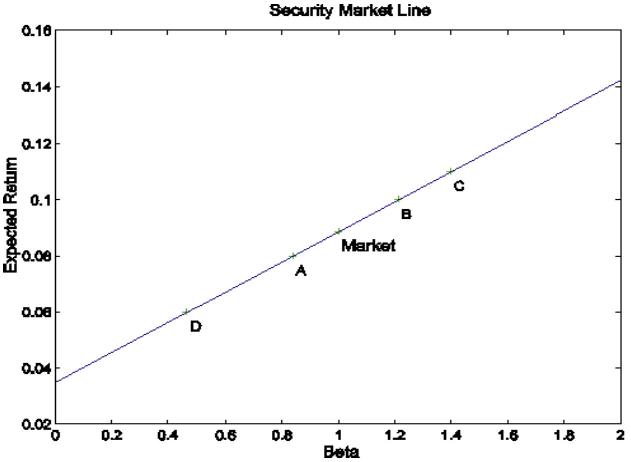
   1.2 use the equation for the variance of a portfolio:

$$\begin{aligned} \sigma_m^2 &= \sum_{i=A}^{D} \sum_{j=A}^{D} w_i w_j cov(r_i, r_j) \\ &= w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + w_C^2 \sigma_C^2 + w_D^2 \sigma_D^2 + \underbrace{2 w_A w_B cov(r_A, r_B) + \ldots}_{\text{6 covariance terms}} \end{aligned}$$

2. Plot expected excess returns vs βs (SML).

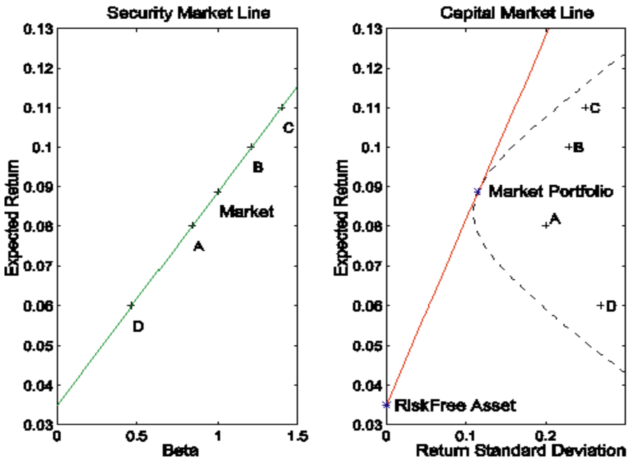
Kellogg School of Management

SML - Example



- Expected returns of all assets lie on the SML!
- What is the difference with the CAL?

SML vs CML



1. Every security lies on the SML
2. **Only** the market and the risk-free asset lie on the CML.
3. SML plots rewards vs systematic risk.
4. CML plots rewards vs total risk (systematic $+$ unsystematic).

The CAPM

- The CAPM is a theory making predictions about what the expected returns of *all* assets should be in equilibrium.
- It uses Mean-Variance Optimization as a pre-requisite.
- Its main prediction is that an asset's reward (expected return) should be proportional to the risk it adds to our portfolio (here, its market beta)
- The central insight that what should matter for asset returns is *covariance* rather than *variance* is of central importance to modern finance.
- This insight survives in all extensions of the CAPM

Beta is a Regression Coefficient

- Consider the regression equation:

$$\tilde{r}_i^e = \alpha_i + \beta_i \tilde{r}_m^e + \tilde{\varepsilon}_{i,t}$$

- The OLS regression coefficient for this regression is same as the definition of β_i we developed earlier

$$\beta_i = \frac{cov(\tilde{r}_i^e, \tilde{r}_m^e)}{\sigma_m^2},$$

  - $\hookrightarrow$ The part of $\tilde{r}_i^e$ that is "explained" by the market return is $\beta_i \tilde{r}_m^e$. This part is the *systematic* or *market* risk of the asset.
  - $\hookrightarrow$ The part of $\tilde{r}_i^e$ that is not correlated with the market return is $\tilde{\varepsilon}_{i,t}$. This part provides the *non-systematic* risk of the asset.

Systematic vs Idiosyncratic Risk

- We can decompose the total variance of asset i

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_\varepsilon^2$$

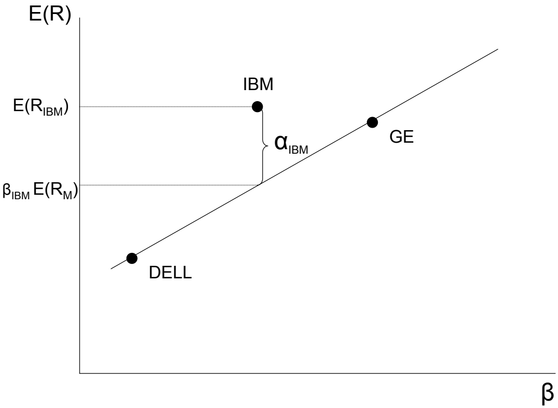
1. The systematic variance $\beta_i^2 \sigma_m^2$
2. The non-systematic variance σ_ε^2
   - Why is the correlation between the two parts equal to zero?
   - What is the R^2 of the regression?

$$R^2 = \frac{\beta_i^2 \sigma_m^2}{\sigma_i^2} = 1 - \frac{\sigma_\varepsilon^2}{\sigma_i^2}$$

- The CAPM implies that only the systematic component is *priced*.
- Why don't investors care about unsystematic risk?

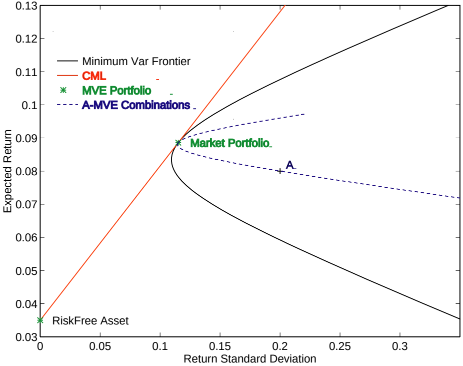
Deviations from the CAPM

- What must α_i be, according to the CAPM?
- α_i denotes the deviation of a security from the SML.



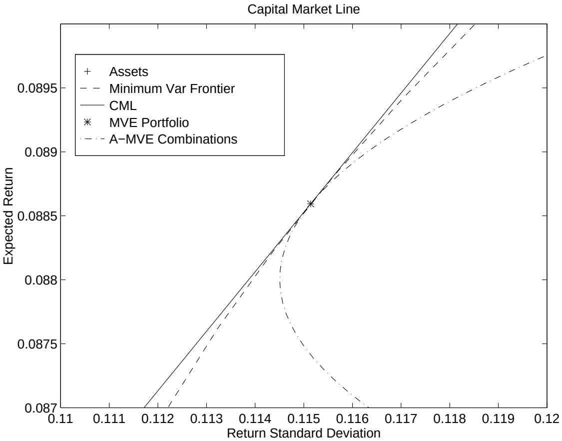
A graphical approach to understanding the CAPM

- Our derivation of the CAPM implies that a combination of any asset and the market must be efficient.
- We should be able to see this graphically. First, let's look at the possible combinations of the MVE portfolio (the market if the CAPM is true) and Asset A



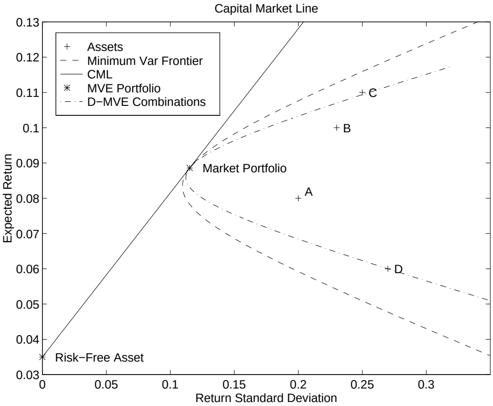
A graphical approach to understanding the CAPM

- It looks like all combinations fall inside the MVE frontier. The only way this can be the case is if the marginal return/variance is the same as for the market, that is if the combination line is tangent to the CML.



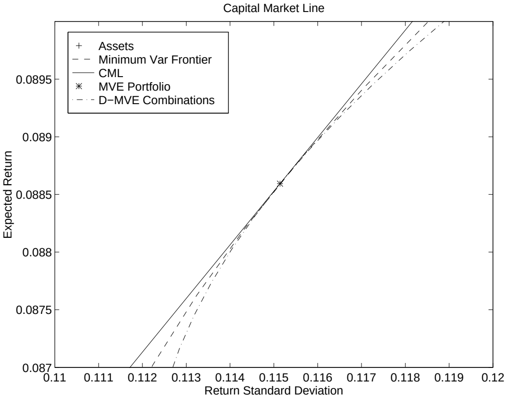
A graphical approach to understanding the CAPM

- These plots show the possible combinations of the MVE portfolio and asset D. We get the same tangency condition.
- The “combination” curve must be tangent to the CML for every asset.



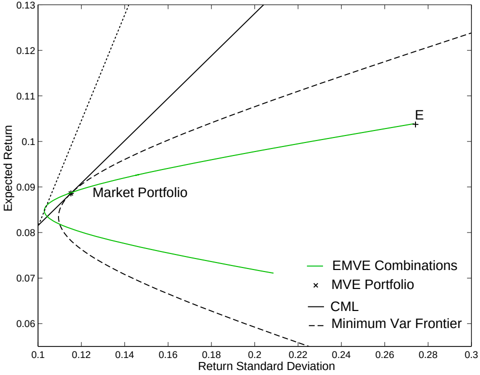
A graphical approach to understanding the CAPM

- Unless the ratio of marginal return to marginal variance is identical for all securities (and the market), investors will be unhappy holding the market portfolio, and prices will have to adjust to get to equilibrium.



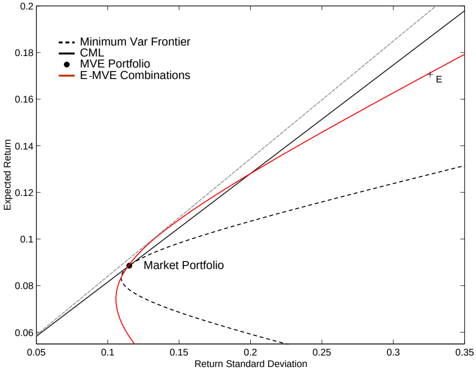
A graphical approach to understanding the CAPM

- What will these curves look like if the CAPM does not hold?
- If the expected return on new asset E is less than predicted by the CAPM, then the possible portfolios are:



A graphical approach to understanding the CAPM

- Alternatively, if the expected return on new asset E is greater than predicted by the CAPM,



Summary

- The CAPM is a theory that states what an asset's expected return should be in *equilibrium*.
- Whether the CAPM holds in the data is a hotly debated topic among academics.
- The CAPM need not be literally true to be useful.
  - $\hookrightarrow$ The CAPM can be used as a starting point for the expected returns in Markowitz
  - $\hookrightarrow$ If it is wrong, this means that we can "do better" than the market portfolio (assuming that expected return and standard deviation are what matter for us).

Using the CAPM in asset allocation

- We can use the *CAPM* to determine what the market believes expected returns *should be*
- Then we can combine our "views" with the CAPM-derived estimates to get portfolio weights.
  - ↪ The combination of these two approaches leads to the Black-Litterman model.
- To implement the CAPM, the key input we will need is the market β
- First, let's figure out how to get these...

1. Let $\tilde{r}_{i,t}$, $\tilde{r}_{m,t}$ and $r_{f,t}$ denote historical individual security, market and risk free asset returns (respectively) over some period $t = 1, 2, \ldots, T$.
2. The standard way to estimate a beta is to use a *characteristic line regression*:

$$\tilde{r}_{i,t} - r_{f,t} = \alpha_i + \beta_i(\tilde{r}_{m,t} - r_{f,t}) + \tilde{\varepsilon}_{i,t}$$

$\hookrightarrow$ The other form you will see this in is *the market model*:

$$\tilde{r}_{i,t} - E(\tilde{r}_{i,t}) = \alpha_i + \beta_i(\tilde{r}_{m,t} - E(\tilde{r}_{m,t})) + \tilde{\varepsilon}_{i,t}$$

Estimating β - Example

Here is an example of a (12 month) characteristic-line regression for GM Common stock (from *BKM*, Ch. 10):

Month	GM Return	Market Return	Monthly T-Bill Rate	Excess GM Return	Excess Market Return
January	6.06	7.89	0.65	5.41	7.24
February	−2.86	1.51	0.58	−3.44	0.93
March	−8.18	0.23	0.62	−8.79	−0.38
April	−7.36	−0.29	0.72	−8.08	−1.01
May	7.76	5.58	0.66	7.10	4.92
June	0.52	1.73	0.55	−0.03	1.18
July	−1.74	−0.21	0.62	−2.36	−0.83
August	−3.00	−0.36	0.55	−3.55	−0.91
September	−0.56	−3.58	0.60	−1.16	−4.18
October	−0.37	4.62	0.65	−1.02	3.97
November	6.93	6.85	0.61	6.32	6.25
December	3.08	4.55	0.65	2.43	3.90
Mean	0.02	2.38	0.62	−0.60	1.75
Std Dev	4.97	3.33	0.05	4.97	3.32

Regression Results	$r_{GM} - r_f = \alpha + \beta(r_M - r_f)$	
	α	β
Estimated coefficient	−2.590	1.1357
Standard error of estimate	(1.547)	(0.309)

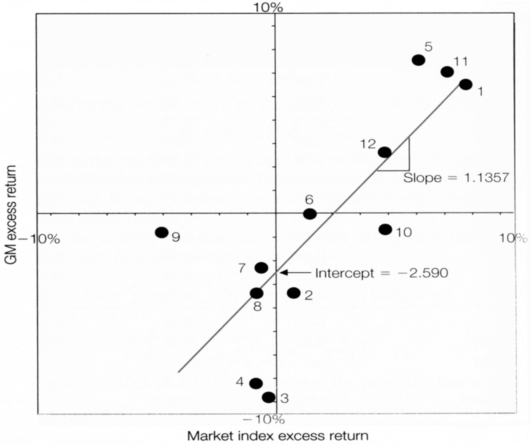
Variance of residuals = 12.601

Standard deviation of residuals = 3.550

R-SQR = 0.575

Estimating β - Example

This figure illustrates this still better:



- How do we read $\hat{\alpha}_i$, $\hat{\beta}_i$, and $\hat{\varepsilon}_{i,t}$ off of this *scatterplot*?

Estimating β

There are a number of Institutions that supply β's:

1. *Value-Line* uses the past five years (with weekly data) with the Value-Weighted NYSE as the market.
2. *Merrill Lynch* uses 5 years of monthly data with the S&P 500 as the market

Ticker Symbol	Security Name	June 1994 Close Price	Beta	Alpha	R-SQR	RESID STD DEV-N	Standard Error Beta	Standard Error Alpha	Adjusted Beta	Number of Observations
GBND	General Binding Corp	18.375	0.52	−0.06	0.02	10.52	0.37	1.38	0.68	60
GBDC	General Bldrs Corp	0.930	0.58	−1.03	0.00	17.38	0.62	2.28	0.72	60
GNCMA	General Communication Inc Class A	3.750	1.54	0.82	0.12	14.42	0.51	1.89	1.36	60
GCCC	General Computer Corp	8.375	0.93	1.67	0.06	12.43	0.44	1.63	0.95	60
GDC	General Datacomm Inds Inc	16.125	2.25	2.31	0.16	18.32	0.65	2.40	1.83	60
GD	General Dynamics Corp	40.875	0.54	0.63	0.03	9.02	0.32	1.18	0.69	60
GE	General Elec Co	46.625	1.21	0.39	0.61	3.53	0.13	0.46	1.14	60
JOB	General Employment Enterpris	4.063	0.91	1.20	0.01	20.50	0.73	2.69	0.94	60
GMCC	General Magnaplate Corp	4.500	0.97	0.00	0.04	14.18	0.50	1.86	0.98	60
GMW	General Microwave Corp	8.000	0.95	0.16	0.12	8.83	0.31	1.16	0.97	60
GIS	General MLS Inc	54.625	1.01	0.42	0.37	4.82	0.17	0.63	1.01	60
GM	General MTRS Corp	50.250	0.80	0.14	0.11	7.78	0.28	1.02	0.87	60 ◄
GPU	General Pub Utils Cp	26.250	0.52	0.20	0.20	3.69	0.13	0.48	0.68	60
GRN	General RE Corp	108.875	1.07	0.42	0.31	5.75	0.20	0.75	1.05	60
GSX	General SIGNAL Corp	33.000	0.86	−0.01	0.22	5.85	0.21	0.77	0.91	60

Estimating β

- Merrill uses *total* rather than *excess* returns:

$$\tilde{r}_{i,t} = a_i + b_i \tilde{r}_{m,t} + \tilde{e}_{i,t}$$

So, the a_i here is equal to:

$$a_i = \alpha_i + (1 - \beta_i) r_f$$

- ↪ where r_f is the average risk-free rate over the estimation period
- ↪ a_i is not equal to zero if the CAPM is true.
- ↪ However, b_i and β_i will be very close, as long as $cov(r_{f,t}, r_{m,t}) \approx 0$

Estimating β

- Note also that Merrill used adjusted β's, which are equal to:

$$\beta_i^{Adj} \approx 1/3 + (2/3) \cdot \hat{\beta}_i$$

- Why are they doing this?
  - $\hookrightarrow$ We may have a prior belief about what firm's betas should be.
  - $\hookrightarrow$ Alternatively, it could be that betas tend to mean-revert to one.
  - $\hookrightarrow$ Why $1/3$ and $2/3$? What should these numbers be based on?
  - $\hookrightarrow$ Other more advanced "shrinkage" techniques.

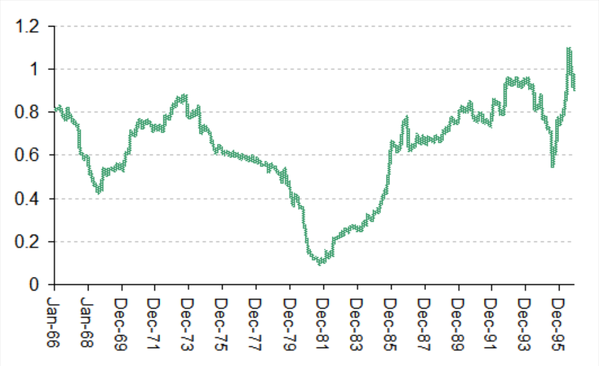
Estimating β

- Betas *may change over time*, so we use short windows of data (5-years)
- Possible reasons for time-varying betas
  1. changes in the firm's leverage
  2. changes in the type of a firm's operations
  3. the firm acquires targets in other industries
- We can use *rolling window* regressions to estimate a time series of βs:
  - ↪ at month t estimate β using months $t-60$ through $t-1$
  - ↪ at month $t+1$ estimate β using months $t-59$ through t.
- Alternatively, use more sophisticated statistical models that allow for time-variation in beta.

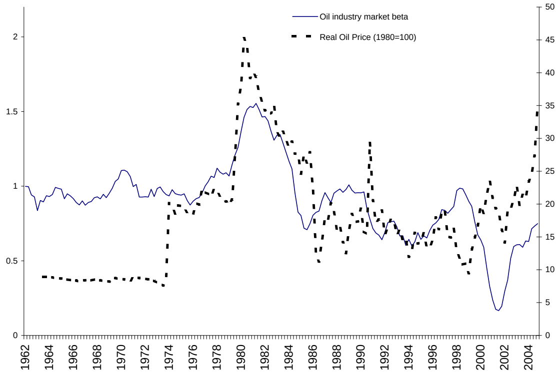
Estimating β

- To estimate market beta, typically we use monthly data
- Often, it is possible to get more efficient estimates by using higher frequencies: weekly, daily, or even intraday data.
- But! Higher frequencies brings a lot of other issues
  - ↪ Non-synchronous prices
  - ↪ Bid-ask bounce
- If some stocks are not frequently traded, they may not immediately respond to market movements.
- These stocks will falsely appear to be 'safe'

- βs can be quite volatile.
- Example: AT&T



- βs of industries also changes over time.
- Example: Oil Industry



Estimating β for new companies

- In the absence of historical data, how would we estimate β for new companies?
- Standard industry practice is to use “comparables”:
  - $\hookrightarrow$ Find a similar company, that is traded on an exchange and use the beta of that company.
- The following approach is in the same spirit but is more powerful.
  - $\hookrightarrow$ Often a comparable company cannot be found.
  - $\hookrightarrow$ Can yield a model-predicted beta from a sample of companies with similar characteristics.

Estimating β

- Which characteristics to use?
  - ↪ Industry
    - e.g. 1-digit SIC code
  - ↪ Firm Size
    - e.g. market capitalization
  - ↪ Financial Leverage
    - e.g. debt to assets
  - ↪ Operating Leverage
    - e.g. fixed costs (SGA?) to assets, ROA
  - ↪ Growth / Value
    - e.g. market to book, capital expenditures

Estimating β for new companies - SIC Industry codes

- How to code industries?
- Here, we used dummy variables at the 1-digit SIC code level

1-digit SIC	Description
0	Agriculture
1	Mining and construction
2	Manufacturing, non-durables
3	Manufacturing, durables
4	Transportation
5	Services, retail and wholesale trade
6	Services, finance
7	Services, business and entertainment
8	services, health-care

Estimating β for new companies - Methodology

1. Select a sample of companies to estimate the model.

2. Estimate $\beta_{i,t}$ for these companies using historical return data.

3. Regress estimated betas $\hat{\beta}_{i,t}$ on several characteristics that can drive betas. Example:

$$\hat{\beta}_{i,t} = a_0 + \sum_j \gamma_j INDUSTRY_{j,i} + a_1 FLEV_{i,t} + a_2 SIZE_{i,t} + a_3 OLEV_{i,t} + u_{i,t}$$

   ↪ $INDUSTRY_{j,i}$ is a dummy variable that take the value 1 if firm i belongs in industry j and 0 otherwise.

4. Suppose you are asked to find the beta for new company XYZ in the tech industry, given XYZ's characteristics. Your estimate will be

$$\hat{\beta}_{xyz,t} = a_0 + \gamma_{tech} + a_1 FLEV_{xyz,t} + a_2 SIZE_{xyz,t} + a_3 OLEV_{xyz,t}$$

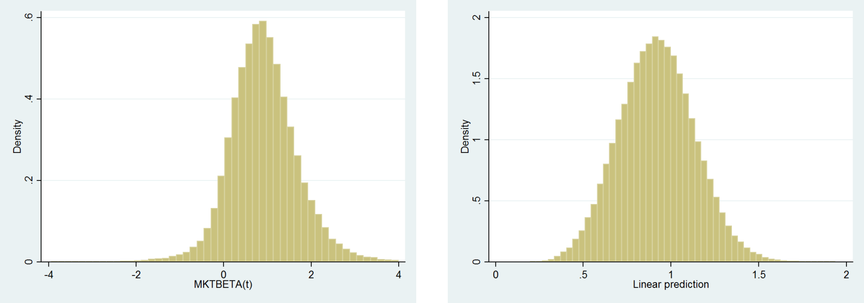
Estimating β for new companies - Example

```
. xi: areg mktbeta Llev Linv   LlogQ LlogV Lolev Lcf i.sic1,absorb(year) cluster(permno)
i.sic1            _Isic1_0-8          (naturally coded; _Isic1_0 omitted)

Linear regression, absorbing indicators              Number of obs =  103299
                                                     F( 14, 10965) =  287.36
                                                     Prob > F      =  0.0000
                                                     R-squared     =  0.1631
                                                     Adj R-squared =  0.1627
                                                     Root MSE      =  .72536

                              (Std. Err. adjusted for 10966 clusters in permno)
------------------------------------------------------------------------------
             |               Robust
     mktbeta |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        Llev |   .1814493   .0222703     8.15   0.000     .1377955    .2251032
        Linv |   .3263559   .0168192    19.40   0.000     .2933872    .3593246
       LlogQ |   .0728157   .0032753    22.23   0.000     .0663956    .0792358
       LlogV |   .0883986    .002254    39.22   0.000     .0839803    .0928169
       Lolev |   .0098227   .0161775     0.61   0.544    -.0218881    .0415335
         Lcf |  -.3496305   .0263879   -13.25   0.000    -.4013556   -.2979053
    _Isic1_1 |   .0928848   .0576064     1.61   0.107    -.0200341    .2058037
    _Isic1_2 |   .0821896   .0556701     1.48   0.140    -.0269339     .191313
    _Isic1_3 |   .2876742    .055479     5.19   0.000     .1789253    .3964231
    _Isic1_4 |   .1851213   .0575677     3.22   0.001     .0722783    .2979643
    _Isic1_5 |   .1617534   .0561379     2.88   0.004      .051713    .2717938
    _Isic1_6 |   .0596576   .0593802     1.00   0.315    -.0567383    .1760534
    _Isic1_7 |   .2545762   .0563962     4.51   0.000     .1440295    .3651228
    _Isic1_8 |   .1522056   .0582781     2.61   0.009       .03797    .2664412
       _cons |  -.4278493   .0616425    -6.94   0.000    -.5486797   -.3070189
-------------+----------------------------------------------------------------
        year |   absorbed                                      (45 categories)
```

Estimating β for new companies - Example



- Left panel: dispersion in estimated betas using data on stock return
- Right panel: dispersion in estimated betas using data on stock returns + firm characteristics
  (the predicted values from the regression of betas on firm characteristics)

- To create an optimal portfolio we need to estimate the minimum variance/efficient frontier, MVE portfolio, and the best capital allocation line (CAL).
- Where are the inputs coming from?
  1. One way to do this is to ignore market opinion, and to estimate the $E(r_i)$'s and $\sigma_{i,j}$'s for all assets.
     - For example, one could use past empirical data.
     - we have already seen that there are problems with this approach
  2. An alternative approach is to "accept" market opinion and simply hold the market portfolio.
     - This approach doesn't tell you what to do if you have information that you believe has not yet been incorporated into market prices.

The CAPM and Active Portfolio Management

- The approach we will take is to

  1. Calculate β's for the securities we plan to hold
  2. Using these β's, calculate $\sigma_{i,j}$'s and $E(r_i)$'s, assuming the CAPM holds exactly.
  3. Incorporate our information by "perturbing" the values away from the CAPM-calculated values.
  4. Using these estimates, determine our optimal portfolio weights using Markowitz.

- Note that if

  ↪ We start with all of the assets in market portfolio

  ↪ We use the unmodified $\sigma_{i,j}$'s and $E(r_i)$'s we get from step 2

  Then the weights we calculate in step 4 will be exactly those of the market portfolio. Why??

The CAPM and Active Portfolio Management

- Consider the characteristic line regression

$$\tilde{r}_{i,t} - r_{f,t} = \alpha_i + \beta_i(\tilde{r}_{m,t} - r_f) + \tilde{\varepsilon}_{i,t} \quad (1)$$

- To get the market's beliefs about expected returns, use the CAPM:

$$E(\tilde{r}_i) - r_f = \beta_i \cdot (E(\tilde{r}_m) - r_f) \quad (2)$$

  ↪ Given the estimates α_i , β_i and $E(\tilde{r}_{m,t} - r_f)$ from (1), if we calculate

$$\alpha_i + \beta_i E(\tilde{r}_{m,t} - r_f),$$

  is the same as estimating $E(\tilde{r}_{i,t} - r_{f,t})$ using historical averages. Why?

  ↪ The CAPM estimate of $E(\tilde{r}_{i,t} - r_{f,t})$ is obtained by imposing $\alpha_i = 0$.

- **NOTE** the important differences between (1) and (2):

  1. is about **realized return** – the everyday realization of uncertainty
  2. is about **average return**– the long-run expected gains from holding the asset

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Betas and the single-index model

- Since we estimated the betas anyway, we can also use the single-index model to simplify the estimation of the covariance matrix
- To construct the efficient frontier based on the Single Index Model (for 100 securities) we need estimates of the following:

r_f	1	1
$E(r_m)$	1	1
σ_m^2	1	1
α_i	N	100
β_i	N	100
$\sigma_{\varepsilon,i}^2$	N	100
Total	$3N+3$	303

$\hookrightarrow$ This is considerably smaller than the 5150 we had before.

$\hookrightarrow$ Where are the $E(r_i)$?

$\hookrightarrow$ Where are the $\sigma_{i,j} = Cov(\tilde{r}_i, \tilde{r}_j)$?

Example (cont)

- Monthly data for GE, IBM, Exxon (XON), and GM:

	Excess Returns					
	mean	std	alpha	beta	std_ε	R^2_{adj}
IBM	3.22%	8.44%	1.31%	1.14	7.13%	28.5%
XON	1.41%	4.03%	0.42%	0.59	3.28%	33.7%
GM	0.64%	7.34%	-1.06%	1.02	6.14%	30.0%
GE	2.26%	5.86%	0.53%	1.04	4.15%	49.9%
VW-Rf	1.67%	4.02%				
Rf	0.36%	0.05%				

↪ VW is the Value-Weighted index of all NYSE, AMEX, and NASDAQ common stocks.

↪ Rf is the (nominal) 1-month T-Bill yield, 4.394%/year (0.359%/month)

Example (cont)

- For expected returns, let's not impose the CAPM just yet.
- To get the correlation structure, we use:

$$\rho_{i,j} = \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j} \quad \forall i \neq j$$

	IBM	XON	GM	GE
IBM	1	0.32	0.30	0.39
XON	0.32	1	0.33	0.42
GM	0.30	0.33	1	0.40
GE	0.39	0.42	0.40	1

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Example (cont)

- Plugging the (1) “expected” returns, (2) return standard deviations, and (3) correlation matrix into the Excel spreadsheet we get the following weights for the tangency portfolio:

	Weight
IBM	29.6 %
XON	49.7 %
GM	-21.4 %
GE	42.4 %

- It seems unreasonable that we should hold such extreme portfolio positions.
  - $\hookrightarrow$ The equilibrium arguments we used in developing the CAPM suggest that the market knows something we don’t about future expected returns!

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Example (cont)

- Let's use the CAPM as a way of getting around this problem.
- This is equivalent to setting $\alpha_i = 0$ for all securities, or using the regression equation:

$$E(r_i) = r_f + \beta_i [E(r_m) - r_f]$$

and the past (average) return on the market to get "equilibrium" estimates of the expected returns.

Stock	CAPM $E(r_i^e)$
IBM	1.91 %
XON	0.99 %
GM	1.70 %
GE	1.73 %

Example (cont)

- With this "equilibrium" set of expected returns, we now get the portfolio weights:

	weights
IBM	13.6%
XON	33.3%
GM	16.4%
GE	36.6%

  - ↪ Why are the weights different?
  - ↪ Why are these not the market weights? When would these be the actual market weights?
  - ↪ Is this the portfolio you would want to hold, given that you were constrained to hold these four assets?

Example (cont)

- However, there may be times when we think that the market is a little wrong along one or more dimensions (a very dangerous assumption!)
- How can we combine our views with what the market expects?
  1. First, suppose that I think that the "market" has underestimated the earnings that IBM will announce in the next month, and that IBM's expected return is 2% higher than the market expects.
  2. Also, I have no information on the other three securities that would lead me to think that they are mispriced,
  3. and I believe that the past betas, and residual std dev's are good indicators of the relative future values.

Example (cont)

- In this case, we would use the same variance and covariance inputs, but would change the expected returns. The new portfolio weights would be:

Stock	$E(r_i^e)$	weights
IBM	3.91%	54.1%
XON	0.99%	17.7%
GM	1.70%	8.7%
GE	1.73%	19.5%

compared to the old allocation

	$E(r_i^e)$	weights
IBM	1.91%	13.6%
XON	0.99%	33.3%
GM	1.70%	16.4%
GE	1.73%	36.6%

Example (cont)

- Alternatively, suppose that I think the risk (β) of Exxon is increasing.

    1. I guess that Exxon's β will rise from 0.59 to 0.8.
    2. I also expect that Exxon's idiosyncratic risk σ_ε will not change.

- First, I should recalculate *almost* everything using the equations:

$$
\begin{aligned}
E(r_i) &= r_f + \beta_i[E(r_m) - r_f] \\
\sigma_i^2 &= \beta_i^2 \cdot \sigma_m^2 + \sigma_{\varepsilon,i}^2 \\
\rho_{i,j} &= \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j}
\end{aligned}
$$

Example (cont)

- The new correlations we come up with are:

	IBM	XON	GM	GE
IBM	1.00	0.44	0.30	0.39
XON	0.44	1.00	0.45	0.57
GM	0.30	0.45	1.00	0.40
GE	0.39	0.57	0.40	1.00

- as opposed to the old correlation matrix of:

	IBM	XON	GM	GE
IBM	1	0.32	0.30	0.39
XON	0.32	1	0.33	0.42
GM	0.30	0.33	1	0.40
GE	0.39	0.42	0.40	1

Example (cont)

- If, I believe that these are the new correlations, but that the market still believes that the past correlations represent the future (and will not discover this information over the next several months) then I would use the *old expected returns*, giving new portfolio weights of:

	old		new
	$E(r_i^e)$	weight	weight
IBM	1.91%	13.6%	17.0%
XON	0.99%	33.3%	16.7%
GM	1.70%	16.4%	20.5%
GE	1.73%	36.6%	45.7%

Example (cont)

- If, I believe that market knows that the β of Exxon is higher, and the expected return on Exxon is higher now to compensate for the increased risk:

	old		new	
	$E(r_i^e)$	weight	$E(r^e)$	weight
IBM	1.91%	13.6%	1.91%	9.2%
XON	0.99%	33.3%	1.34%	54.8%
GM	1.70%	16.4%	1.70%	11.1%
GE	1.73%	36.6%	1.73%	24.8%

- One other alternative is that I don't believe that the market yet knows that the risk of Exxon is higher, but will discover this in the next few months.
  1. What will happen as the market finds out?
  2. What should I do in this case?

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Forming 'views'

- Where is our information coming from?
  1. It could come from public sources of information that may or may not have been impounded in prices yet.
  2. It could be private, that is it could come from our own analysis of fundamentals.
- Nevertheless, we may not be 100% confident that our information is accurate.
- For instance, in the previous example, if I am only *somewhat* confident of my belief that the market will not adjust the price of Exxon properly, I might want to adjust the portfolio weights only part-way.

Forming 'views' - Example

Consider the Morningstar report on AIG:

Snapshot

American International Group AIG

Performance more

Growth of $10,000 09-05-03

Total Return%	**2000**	**2001**	**2002**	**YTD**
Stock	37.0	-19.3	-26.9	2.2
+/- Industry	10.5	-32.9	-25.3	-10.0
+/- S&P 500	46.1	-7.4	-4.9	-15.9

Key Stats

Last Close (09-05-03)	$58.95
Market Cap $Mil	153,793
Sales $Mil	73,501
Morningstar Style Box	Large Core
Industry	Insurance (Property)
Sector	Financial Services
Stock Type	Classic Growth

Morningstar Stock Grades

Growth	A
Profitability	B+
Financial Health	A-

Premium Features

Morningstar Rating
★★★

Business Risk
Avg

Fair Value Estimate
65.0

Economic Moat
Narrow

Analyst Report Summary
International growth and increased cross-selling should drive expansion for AIG. Read full analyst report

More Premium Features
- 1,000 Stock Analyst Reports
- Daily Analyst Notes

Forming 'views' - Example

1. Market price: $P_0 = 58.95$
2. Your view:
   2.1 $V_0 = (1+a)P_0 = 65.00 \rightarrow a = 10.26\%$
   2.2 $r_{CAPM} = [E(P_1) - V_0]/V_0$
3. You think the actual return will be:

$$\begin{aligned} r &= [E(P_1) - P_0]/P_0 \\ &= [E(P_1) - V_0 + V_0 - P_0]/P_0 \\ &= r_{CAPM}(V_0/P_0) + [V_0 - P_0]/P_0 \\ &= r_{CAPM} + a + r_{CAPM} \times a \\ &\approx r_{CAPM} + a \end{aligned}$$

Forming 'views' - Example

- If you were 100% confident in your view, then you would also think that AIG's expected return will be higher than the CAPM implied return by an amount $a = 10.26\%$.
- This is a very large number and will likely lead to extreme portfolio allocations.
- Suppose that you are only 10% confident in your view, then you might use $a = 0.1 \times 10.26\% = 1.26\%$.
- NEXT: The Black and Litterman model gives a systematic framework that enables you to incorporate your views with what the market's views in forming the optimal portfolio.

Black-Litterman model - Global Asset Allocation

- Focus on asset allocation between country indices.
- Use an international version of the CAPM.
- Calculate the "equilibrium" expected returns based on estimated variances and covariances. This is appropriate, since covariances can be estimated accurately, especially using daily data, for a small number of assets.
- The portfolio manager can specify any number of market "views" in the form of expected returns, and a variance (measure of uncertainty) for each of the views.
  - ↪ If the manager holds no views, she will hold the market portfolio.
  - ↪ If her views are high variance (low certainty), she will hold close to the equilibrium portfolio.
  - ↪ When her views are low variance, she will move considerably away from the market portfolio.

First, let's look at the sort of portfolio allocation we get if we use historical returns and volatilities as inputs:

Exhibit 1

Historical Excess Returns

(January 1975 through August 1991)

Total Historical Excess Returns

	Germany	France	Japan	U.K.	U.S.	Canada	Australia
Currencies	-20.8	3.2	23.3	13.4		12.6	3.0
Bonds CH	15.3	-2.3	42.3	21.4	-4.9	-22.8	-13.1
Equities CH	112.9	117.0	223.0	291.3	130.1	16.7	107.8

Annualized Historical Excess Returns

	Germany	France	Japan	U.K.	U.S.	Canada	Australia
Currencies	-1.4	0.2	1.3	0.8		0.7	0.2
Bonds CH	0.9	-0.1	2.1	1.2	-0.3	-1.5	-0.8
Equities CH	4.7	4.8	7.3	8.6	5.2	0.9	4.5

Annualized Volatility of Historical Excess Returns

	Germany	France	Japan	U.K.	U.S.	Canada	Australia
Currencies	12.1	11.7	12.3	11.9		4.7	10.3
Bonds CH	4.5	4.5	6.5	9.9	6.8	7.8	5.5
Equities CH	18.8	22.2	17.8	24.7	16.1	18.3	21.9

Note: Bond and equity excess returns are in U.S. dollars currency hedged (CH). Excess returns on bonds and equities are in excess of the London interbank offered rate (LIBOR), and those on currencies are in excess of the one-month forward rates. Volatilities are expressed as annualized standard deviations.

Black-Litterman model - Global Asset Allocation

also, we use historical correlations:

Exhibit 2

Historical Correlations of Excess Returns

(January 1975 through August 1991)

	Germany Equities CH	Germany Bonds CH	Germany Currency	France Equities CH	France Bonds CH	France Currency	Japan Equities CH	Japan Bonds CH	Japan Currency
Germany									
Equities CH	1.00								
Bonds CH	0.28	1.00							
Currency	0.02	0.36	1.00						
France									
Equities CH	0.52	0.17	0.03	1.00					
Bonds CH	0.23	0.46	0.15	0.36	1.00				
Currency	0.03	0.33	0.92	0.08	0.15	1.00			
Japan									
Equities CH	0.37	0.15	0.05	0.42	0.23	0.04	1.00		
Bonds CH	0.10	0.48	0.27	0.11	0.31	0.21	0.35	1.00	
Currency	0.01	0.21	0.62	0.10	0.19	0.62	0.18	0.45	1.00
U.K.									
Equities CH	0.42	0.20	−0.01	0.50	0.21	0.04	0.37	0.09	0.04
Bonds CH	0.14	0.36	0.09	0.20	0.31	0.09	0.20	0.33	0.19
Currency	0.02	0.22	0.66	0.05	0.05	0.66	0.06	0.24	0.54
U.S.									
Equities CH	0.43	0.23	0.03	0.52	0.21	0.06	0.41	0.12	−0.02
Bonds CH	0.17	0.50	0.26	0.10	0.33	0.22	0.11	0.28	0.18
Canada									
Equities CH	0.33	0.16	0.05	0.48	0.04	0.09	0.33	0.02	0.04
Bonds CH	0.13	0.49	0.24	0.10	0.35	0.21	0.14	0.33	0.22
Currency	0.05	0.14	0.11	0.10	0.04	0.10	0.12	0.05	0.06
Australia									
Equities CH	0.34	0.07	−0.00	0.39	0.07	0.05	0.25	−0.02	0.12
Bonds CH	0.24	0.19	0.09	0.04	0.16	0.08	0.12	0.16	0.09
Currency	−0.01	0.05	0.25	0.07	−0.03	0.29	0.05	0.10	0.27

Black-Litterman model - Global Asset Allocation

If you use our procedures from Lecture 2 and calculate and optimal portfolio, with $\sigma_p = 10.7\%$, you will get portfolio weights of:

Exhibit 3

Optimal Portfolios Based on Historical Average Approach

(percent of portfolio value)

Unconstrained

	Germany	France	Japan	U.K.	U.S.	Canada	Australia
Currency exposure	−78.7	46.5	15.5	28.6		65.0	−5.2
Bonds	30.4	−40.7	40.4	−1.4	54.5	−95.7	−52.5
Equities	4.4	−4.4	15.5	13.3	44.0	−44.2	9.0

With constraints against shorting assets

	Germany	France	Japan	U.K.	U.S.	Canada	Australia
Currency exposure	−160.0	115.2	18.0	23.7		77.8	−13.8
Bonds	7.6	0.0	88.8	0.0	0.0	0.0	0.0
Equities	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Black-Litterman model - Global Asset Allocation

- CAPM Based Estimates
- assuming market risk premium=7.15%

Country	Equity Index Volatility (%)	Equilibrium Portfolio Weight (%)	Equilibrium Expected Returns (%)
Australia	16.0	1.6	3.9
Canada	20.3	2.2	6.9
France	24.8	5.2	8.4
Germany	27.1	5.5	9.0
Japan	21.0	11.6	4.3
UK	20.0	12.4	6.8
USA	18.7	61.5	7.6

	Australia	Canada	France	Germany	Japan	UK
Canada	0.488					
France	0.478	0.664				
Germany	0.515	0.655	0.861			
Japan	0.439	0.310	0.355	0.354		
UK	0.512	0.608	0.783	0.777	0.405	
USA	0.491	0.779	0.668	0.653	0.306	0.652

Black-Litterman model - Global Asset Allocation

- There are N assets in the market and returns are normally distributed:

$$R \sim N(\mu, \Sigma)$$

- Here, μ is an unknown quantity, i.e. the investor is uncertain about what the expected returns are.
- She will form her “best guess” or in Bayesian terminology her *posterior* beliefs, by combining information from two sources:

  1. The *prior* Π belief, which in the Black-Litterman model is based on the CAPM:

  $$\mu \sim N(\Pi, \tau\Sigma)$$

     The variance of the prior, $\tau\Sigma$, denotes how much confidence the investor has on the CAPM. High values of τ means that he attaches less weight to the CAPM.

  2. The second will be her own views, as we will see next.

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Black-Litterman model - Global Asset Allocation

- Suppose that we think that Germany will have an expected return of 4%
- Black-Litterman allows you to express this view as

$$\mu_{ger} = q + \varepsilon, \qquad \varepsilon \sim N(0, \omega)$$

$\hookrightarrow$ Here, $q = 4\%$ is your belief about Germany's expected return.

$\hookrightarrow$ The variable ε allows for a margin of error in your view.

$\hookrightarrow$ Your confidence in your view is represented by ω, i.e. the variance in your margin of error. The higher ω is, the less confident we are.

Black-Litterman model - Global Asset Allocation

- The Black-Litterman model also allows you to form relative views.
- Suppose that we believe that Germany will outperform France by 5%
- Black-Litterman allows you to express this view as

$$P\mu = Q + \varepsilon, \qquad \varepsilon \sim N(0, \Omega)$$

↪ Here, P is a matrix capturing relative views:

  - For instance, if there are only two countries and you only form a relative view

$$P = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$$

↪ As before, Ω is the variance of your views.

Black-Litterman model - Global Asset Allocation

- Given our views and the CAPM prior, our “best guess” about what expected returns are is $\bar{\mu}$ is going to be a weighted average of the CAPM expected returns, Π and our views, Q.

$$\bar{\mu} = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q]$$

- In the case of one asset with prior π and one view, q, it simplifies to

$$\bar{\mu} = \pi \frac{1/(\sigma^2\tau)}{1/\omega + 1/(\sigma^2\tau)} + q \frac{1/\omega}{1/\omega + 1/(\sigma^2\tau)}$$

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Forming posterior beliefs - Example

- Suppose that you would like to form your best guess about GE's expected excess return going forward.
  - ↪ GE has a beta of 1.2 and a standard deviation of 0.33. Assuming the equity premium is 6%, this gives you (in excess of the risk-free rate)

$$\mu_{GE}^{capm} = 1.2 \times 6\% = 7.2\% \qquad \text{and} \quad var(\mu_{GE}^{capm}) = \tau \times 0.33^2 = 0.1 \times \tau$$

  - ↪ You estimated the historical average return of GE to be 8.3% over the last 10 years. This gives you another estimate or "view":

$$\mu_{GE}^{hist} = 8.3\% \qquad \text{and} \quad var(\mu_{GE}^{hist}) = \left(\frac{0.33}{\sqrt{10}}\right)^2 \approx 0.01$$

Forming posterior beliefs - Example

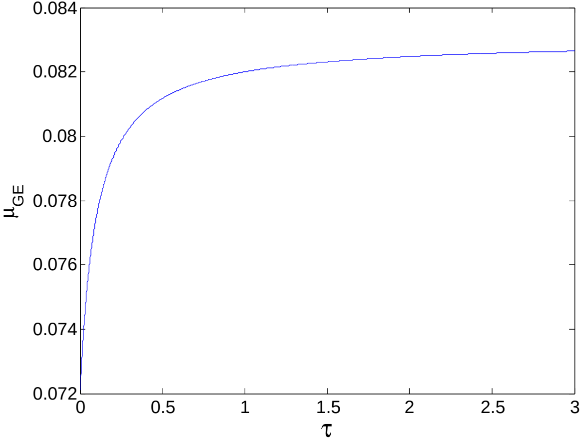
- Your best guess will be:

$$
\begin{aligned}
\bar{\mu}_{GE} &= \frac{\frac{1}{var(\mu_{GE}^{capm})} \times \mu_{GE}^{capm} + \frac{1}{var(\mu_{GE}^{hist})} \times \mu_{GE}^{hist}}{\frac{1}{var(\mu_{GE}^{capm})} + \frac{1}{var(\mu_{GE}^{hist})}} \\
&= \frac{\frac{1}{0.1\times\tau} \times 7.2\% + \frac{1}{0.01} \times 8.3\%}{\frac{1}{0.1\times\tau} + \frac{1}{0.01}}
\end{aligned}
$$

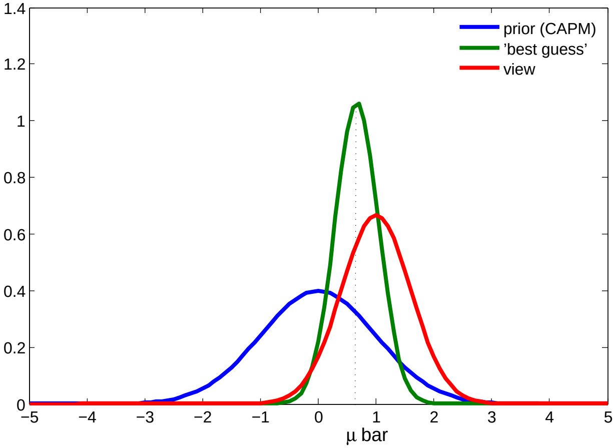
- It is a weighted average of your prior (the CAPM) and your view (the historical average return).
- The relative weights depend on the precision (the inverse of the variance) of the signals.
- Your confidence in the CAPM determines τ.

Forming posterior beliefs - Example

- Your posterior belief as a function of τ



Black-Litterman model - Global Asset Allocation



Black-Litterman model - Global Asset Allocation

- The optimal portfolio weights will be:

$$w^* = w_{MKT} + P'\Lambda$$

  where P are the investor's "view" portfolios, and Λ is a complicated set of weights.

- These weights have the following properties:

  1. The higher the expected return on a view, q, the higher the weight attached to that view, λ.
  2. The higher the variance of a view, ω, the lower the absolute value of the weight attached to the view.

- If you hold no view on an asset, then your optimal allocation will be the market weight.

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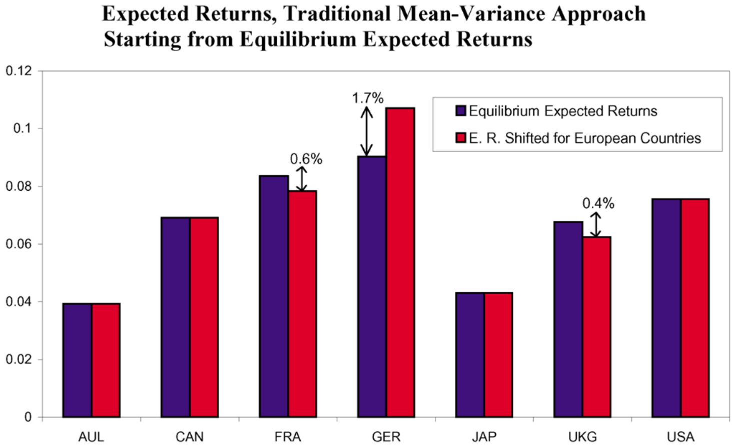
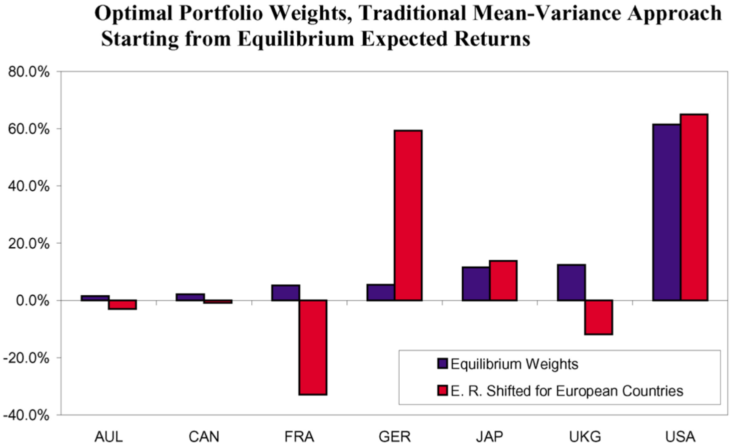
Black-Litterman model - Global Asset Allocation

- Suppose that we believe that Germany will outperform France and UK.



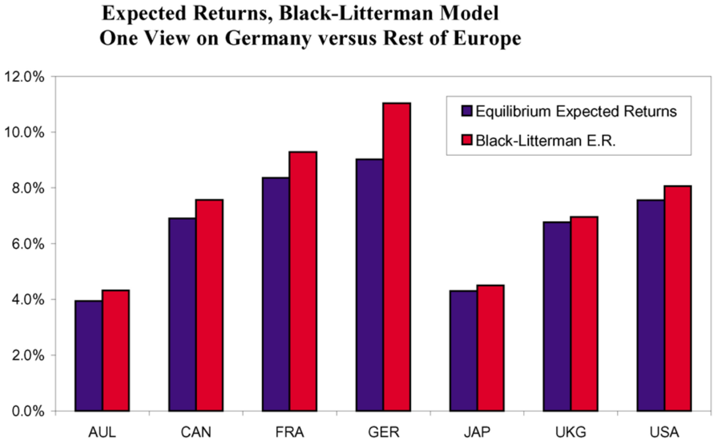
Black-Litterman model - Global Asset Allocation

- The optimal allocations according to *Markowitz* is:

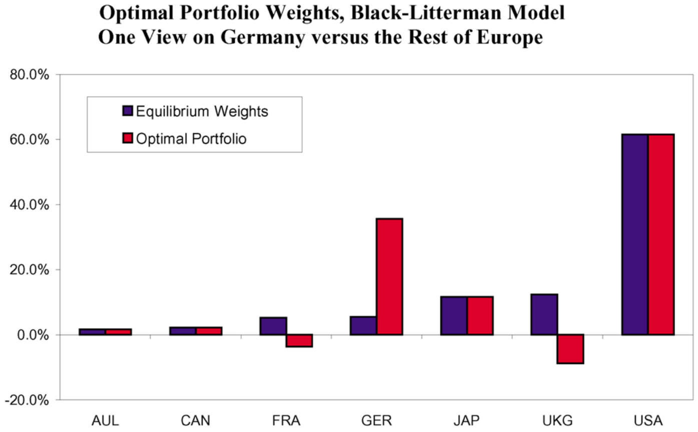


Black-Litterman model - Global Asset Allocation

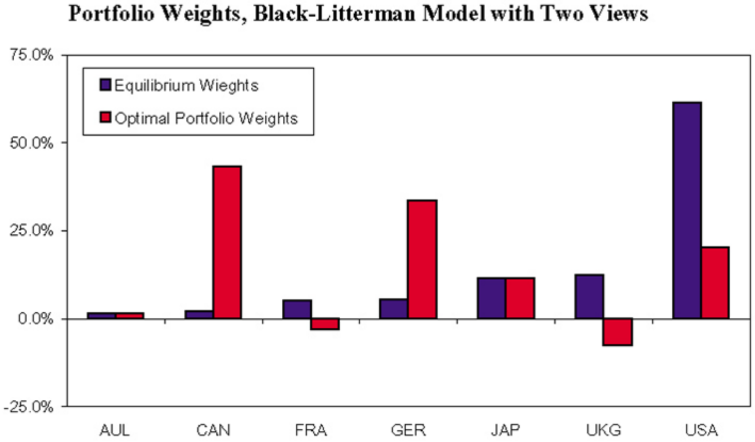
- Our view is that Germany will outperform France and UK.
- However, our best guess, $\bar{\mu}$, will be shifted for *all* countries.



- The Black-Litterman model internalizes the fact that assets are correlated. Your views about Germany vs France and UK translate into *implicit* views about other countries.



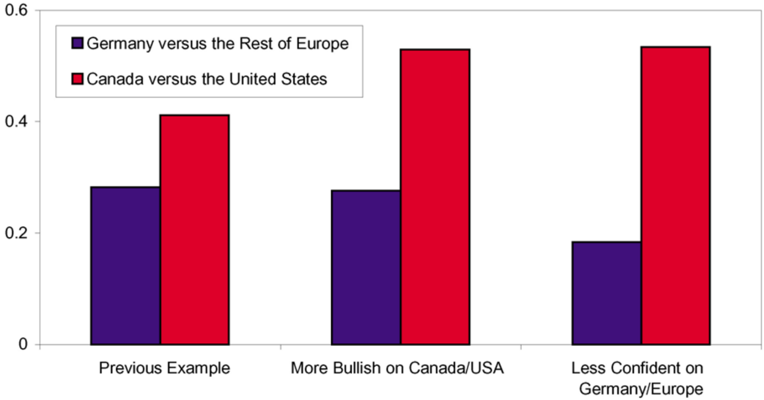
- Let’s add a view: Canada will outperform US by 3%:



weights depend

- How bullish is the view, i.e. (Q)
- How confident is the investor in the view (Ω).

Change in Optimal Weights as a Function of Confidence/Bullishness



Black-Litterman model vs Markowitz

- Black-Litterman
  1. The optimal portfolio equals the (CAPM) market portfolio plus a weighted sum of the portfolios about which the investor has views
  2. An unconstrained investor will invest first in the market portfolio, then in the portfolios about which views are expressed
  3. The investor will never deviate from the market weights on assets about which no views are held
  4. This means that an investor does not need to hold views about each and every asset!

- Markowitz
  1. Need to estimate vector of expected returns for all assets
  2. Estimation error often leads to unrealistic portfolio positions
  3. If we change the expected return for one asset, this will change the weights on *all* assets

Summary

- In order to use the CAPM in our mean-variance analysis, we need estimates of β. Regression analysis yields that
- The CAPM assumes that all investors hold the same views so all of them hold the market-portfolio
- The Black-Litterman model is one way to bring the information from the CAPM in our asset allocation decision