FINC460/312 - Spring 2011 Final Exam

NAME: ______________________________ SECTION: ______________

1. Please do not open this exam until directed to do so.
2. This exam is 3 hours long.
3. Please write your name and section number on the front of this exam, and on any examination books you use.
4. Please show all work required to obtain each answer. Answers without justification will receive no credit.
5. State clearly any assumptions you are making.
6. This is a closed book exam. No books or notes are permitted. Calculators are permitted. Laptops are permitted but you are only allowed to use Excel and only a blank worksheet. You are not allowed to use other spreadsheets with pre-entered formulas.
7. Brevity is strongly encouraged on all questions.
8. The exam is worth 225 points.
9. Relax, and good luck!

Hints:

1. *Think through problems before you start working. Draw pictures.*
2. *If you get stuck on part of a problem, go on to the next part. You may need to use answers from earlier parts of the question to calculate answers to the later parts. If you weren't able to solve the earlier part, assume something.*
3. *Remember, setting up the problem correctly will get you most of the points.*

Short questions (50 points)

Assess the validity of the following statements (True, False or Uncertain) and explain your answers.

1. The CAPM implies that investors require a higher return to hold highly volatile securities.

   False. It is a *co-variance* (or beta) that matters for a stock, not the variance. A stock could have high variance but zero beta.

2. The mean-variance frontier created by N securities will either overlap or lie inside the mean-variance frontier created by N+1 securities.

   True. Adding assets can only expand the mean-variance frontier.

3. The data suggest that investors should only hold a diversified portfolio that is long value and short growth firms.

   False. Even though the portfolio long value and short growth has a high Sharpe ratio, we can do better by combining it with other assets (e.g. the market).

4. In the APT, absence of arbitrage implies that all factors have the same risk premium.

   False. APT says that factor prices are the same across different securities/markets. If the same risk did trade at a different price that would be an arbitrage opportunity. In contrast, each factor can have a different price of risk (λ) than other factors.

5. If market-makers were perfectly competitive, bid-ask spreads for all securities would be zero.

   False. Bid ask spreads should still exist, for two reasons. First, market makers need to protect themselves against better informed traders (adverse selection). Second, market makers may need to bear idiosyncratic risk (inventory risk). The bid-ask spread compensates market makers for adverse selection and inventory risk even if market makers were competitive.

Question 1 (50 points)

For this question, assume that the CAPM properly prices all assets. The risk-free rate is 3%. You know the expected returns and standard deviations of two portfolios: the market portfolio (M) and a portfolio of automobile firms (A):

Security	Expected Return	Standard Deviation
Risk-Free Asset	3%	-
Market Portfolio (M)	9%	15%
Automobile (A)	9%	20%

- (10 points) Draw the minimum-variance frontier of risky assets, along with the Capital Allocation Line and the location of the two portfolios, A and M.

  CAPM holds so market is the tangency portfolio.

  The intercept in the CAL should be at 3% (the risk-free rate). The slope should be the market Sharpe ratio = (9% - 3%)/15% = 2/5. We know the market lies on both the CAL and the MVE and that the CAL is the tangency portfolio to the MVE curve. A is not on the MVE since it has higher standard deviation than the market, but the same expected return.

- Suppose that you run a regression of portfolio A's excess return on excess return on the market portfolio

$$R_{A,t} - r_f = \alpha + \beta(R_{m,t} - r_f) + \epsilon_t$$

What will be the values of

1. (10 points) α

   $\alpha = 0$ since the CAPM holds (and thus prices all assets)

2. (10 points) β

   $\beta = 1$ since the expected return of asset A equals the expected return on the market.

3. (10 points) σ_ϵ

   We know that $\sigma_A^2 = \beta^2\sigma_m^2 + \sigma_\epsilon^2$

   $.2^2 = .15^2 + \sigma_\epsilon^2$

   $\sigma_\epsilon = 13.23\%$

4. (10 points) R^2

   $R^2 = \frac{var(\beta(R_{m,t}-r_f))}{var(R_{A,t}-r_f)} = \frac{.15^2}{.2^2} = .5625$

Question 2 (125 points)

The data below applies to all questions:

- A three-factor APT describes the returns of all well-diversified portfolios. The three factors are unexpected changes in production (factor 1), an inflation factor (factor 2), and an oil price change factor (factor 3).

- Over the next year, the market expects production to grow at 5%, inflation to be 2%, and oil prices to grow by 7%.

- The prices of all well diversified portfolios are set so that their expected returns over the next year are given by:

$$E(\tilde{r}_i) = 0.05 + 0.08\ b_{i,1} - 0.06\ b_{i,2},$$

  where $b_{i,k}$ denotes portfolio i's loading on the k'th factor. Notice that the coefficient on $b_{i,3}$ is zero in this equation.

- The market believes that the standard deviations of $\tilde{f}_1$, $\tilde{f}_2$, and $\tilde{f}_3$ over the next year are all 0.10 (10%), and that the three factors are uncorrelated.

- All investors in this economy (including you) can borrow and lend at a risk-free rate of 5%/year.

- The return generating process for portfolio A over the next year is:

$$\tilde{r}_A = E(\tilde{r}_A) + 0.7\tilde{f}_1 - 0.5\tilde{f}_2 - 0.8\tilde{f}_3.$$

Based on this scenario, answer the following questions:

1. (5 points) Find the expected return of portfolio A.

   We know expected returns are given by $E(\tilde{r}_i) = 0.05 + 0.08\ b_{i,1} - 0.06\ b_{i,2}$

   Using $b_{A,1} = 0.7$, $b_{A,2} = -0.5$, $b_{A,3} = -0.8$ we have

   $E(\tilde{r}_i) = 0.05 + 0.08(0.7) - 0.06(-0.5) = 13.6\%$

2. (5 points) Find the return standard deviation of portfolio A.

   Using $b_{A,1} = 0.7$, $b_{A,2} = -0.5$, $b_{A,3} = -0.8$ and $\sigma_f = 0.1$ for all the factors, we have

   $\sigma_A^2 = (0.7)^2(0.1)^2 + (0.5)^2(0.1)^2 + (0.8)^2(0.1)^2$

   $\sigma_A = 11.75\%$

3. (10 points) If both production and oil prices grow by 10% over the next year, and inflation is exactly what the market expects, what will the return on portfolio A be?

   We need to calculate the realization of the factors over the next year. Since we know how A responds to the factors, we can then calculate what happends to A.

   Remember, the industrial production factor is the "surprise" in industrial production, so it is only the unexpected part. Likewise, since inflation is as expected the inflation factor is zero. Thus,

   $f_1 = 10\% - 5\% = 5\%$

   $f_2 = 0$

   $f_3 = 10\% - 7\% = 3\%$

   Now just plug in

   $$\tilde{r}_A = E(\tilde{r}_A) + 0.7\tilde{f}_1 - 0.5\tilde{f}_2 - 0.8\tilde{f}_3$$

   $\tilde{r}_A = .136 + 0.7(0.05) - 0.5(0) - 0.8(0.03) = 14.7\%$

4. (25 points) The first factor here is an industrial production factor. Give an economic rationale for why the factor risk premium for this production factor should be positive or negative. Specifically, answer the following questions. *All explanations should* very *brief.*

   (a) Assume that a portfolio B has a positive loading on this factor (*i.e.*, $b_{B,1} > 0$). Will the return on B be unexpectedly high or low when production is higher or lower than expected?

   It will be high when production is higher than expected.

   (b) Based on this, would you think that B would have a higher or lower expected return than a portfolio C with $b_{C,1} < 0$? Explain.

   Think of production as a measure of the business cycle. Since B pays off in booms and does poorly in busts it is less valuable to hold than C, which hedges against busts. So the return on A must be higher for you to be willing to hold it.

   (c) Based on this, explain why λ_1 should be positive or negative.

   λ_1 should be positive for the expected return on B to be higher than C. $E[r_B] > E[r_C]$ plus $b_B > b_C$ implies this.

5. (25 points) Assume that you have no more information than the market. You have \$1 million to invest. Your goal in investing is to maximize the expected return of your portfolio, for a given level of return variance (our usual assumption) Also, your coefficient of risk aversion is $A = 5$. Finally, suppose that you can create factor-mimicking portfolios for factors 1, 2 and 3.

   (a) How much should you invest in the three factor mimicking portfolios, and in the risk-free asset? (*i.e.*, what are w_{FM1}, w_{FM2}, w_{FM3}, and w_{Rf}?)

   Most importantly, factor 3 is not "priced", meaning it does not give you extra return for bearing risk. Since it doesn't help your risk-return tradeoff, you should hold none of this factor. Once we realize this, the problem is simple.

   Define the mimicking portfolios as follows:

   $r_{FM1} = E[r_{FM1}] + 1 * f_1$

   $r_{FM2} = E[r_{FM2}] + 1 * f_2$

   Now we can see that

   $E[r_{FM1}] = 13\%$, $E[r_{FM2}] = -1\%$, $\sigma_{FM1} = 0.1$, $\sigma_{FM2} = 0.1$.

   We also know $cov(r_{FM1}, r_{FM2}) = 0$ since the factors are uncorrelated.

   Let's first find the tangency portfolio, then decide our leverage based on risk aversion. Using the MVE portfolio weight equation,

   $w^* = \frac{0.08*0.01}{0.08*0.01+(-0.06)*0.01} = 4$

   The MVE is therefore $R_{MVE} = 4r_{FM1} - 3r_{FM2}$.

   Finally, we can find the weight in the risk free by

   $w = \frac{E[R_{MVE}]-r_f}{5\sigma^2_{MVE}}$

$E[R_{MVE}] = 4 * 13\% - 3 * (-1\%) = 55\%$

$\sigma_{MVE} = \sqrt{4^2(0.01) + 3^2 * (0.01)} = .5 = 50\%$

$w = \frac{.5}{5 * .25} = 2/5$

Ok... so we can calculate how much we invest of our \$1 million as follows.

First, put 3/5 (\$600,000) into the risk-free asset – the rest (\$400,000) goes into the MVE portfolio. To find the amounts for F1 and F2, use the MVE weights. Put 4 times the remaining amount into r_{FM1} or $4 * 400,000 = \$1.6 million$. Put -3 times into r_{FM2} or $-\$1.2 million$.

(b) What is the Sharpe-Ratio of your portfolio?

We already have this info from (a). The Sharpe ratio is 1 – the same as the Sharpe ratio of the MVE portfolio. Remember, how much we put in the risk-free doesn't change our Sharpe ratio.

6. (30 points) Now suppose that all assets in the economy can either be classified as "large" or "small" stocks. A value-weighted portfolio of all large stocks will have a return over the next year given by:

$$\tilde{r}_L = E(\tilde{r}_L) + 1.0\tilde{f}_1 - 0.4\tilde{f}_2 - 0.4\tilde{f}_3$$

while the value-weighted portfolio of all small stocks will have a return over the next year given by:

$$\tilde{r}_S = E(\tilde{r}_S) + 1.1\tilde{f}_1 - \tilde{f}_2 + 0.2\tilde{f}_3.$$

Finally, the sum of the market capitalizations of all of the small stocks is currently \$2 trillion, and the sum of the market cap of all large stocks is currently \$4 trillion.

Based on this information:

(a) What is the expected excess return and standard deviation of the market portfolio?

The market portfolio is a weighted average of the stocks in the economy – where the weights are the market capitalizations. In this case, those are 2/(2+4) for small and 4/(2+4) for big. Therefore,
$r_m = \frac{1}{3}r_s + \frac{2}{3}r_L$
To answer the question, it is easiest to find the factor loadings of the market portfolio, then use those to calculate what we want. We can substitute the definitions of small and big stocks to get
$r_m = \frac{1}{3}(E(\tilde{r}_S) + 1.1\tilde{f}_1 - \tilde{f}_2 + 0.2\tilde{f}_3) + \frac{2}{3}(E(\tilde{r}_L) + 1.0\tilde{f}_1 - 0.4\tilde{f}_2 - 0.4\tilde{f}_3)$
$r_m = (\frac{1}{3}E(r_S) + \frac{2}{3}E(r_L)) + \frac{1.3}{3}f_1 - 0.6f_2 - 0.2f_3$
We can compute the expected return on large and small by
$E(r_S) = 0.05 + 1.1 * 0.08 + 0.06 = 19.8\%$

$E(r_L) = 0.05 + 0.08 - 0.06(-0.4) = 15.4\%$

so $E(r_m) = 16.87\%$

And $\sigma_m = \sqrt{(\frac{1.3}{3})^2(0.01) + 0.6^2(0.01) + (0.2)^2(0.01)} = 12.12\%$

(b) Does the CAPM hold for all well diversified portfolios?

Remember, the CAPM holds when the market is MVE. Let's restate the question as: is the market the MVE portfolio?

No. We need to show the CAPM is not the MVE portfolio. There are multiple ways to see this. First of all, the weights on the underlying factors are different (they are not 4 and -3 like the MVE should be). Also, the fact that there is *any* non-zero loading on the third factor already tells us the market is not MVE (since this factor just adds risk with no return). Finally, we could compare the Sharpe ratio of the market to the Sharpe ratio of the MVE we calculated before and see that it is lower.

7. (25 points) For this part only, assume the APT properly prices all assets based on publicly available information only. However, your analyst has chosen three fund managers for you who, he assures you, are very capable at stock picking, and thus might be able to generate abnormal returns according to the APT. Your analyst supplies you with the following data on the three funds, and on the risk-free asset. The expected returns are all post-expense returns.

Security	Expected Return	b_1	b_2	b_3	Standard Deviation
Risk-Free Asset	5%	-	-	-	-
Fund A	9%	0.5	-0.2	0.0	11.4%
Fund B	13%	1.0	0.5	-0.2	18.8%
Fund C	15%	1.2	-0.5	0.7	15.6%

(a) If you could hold one of the funds in combination with the risk-free asset, which would you choose? Explain.

Our goal (as always) is to find the best risk-return tradeoff, or to maximize our Sharpe ratio. If we could hold only one fund we would pick C since it has the largest Sharpe ratio

$Sharpe(A) = \frac{9-5}{11.4} = .35$

$Sharpe(B) = \frac{13-5}{18.8} = .43$

$Sharpe(C) = \frac{15-5}{15.6} = .64$

(b) Now assume that you can combine just one of the funds with the portfolio you constructed in part 5. Which should you choose? What is the highest Sharpe ratio you could achieve?

Again, we should choose the one that gives us the largest (overall) Sharpe ratio. This is equivalent to finding the one with the largest appraisal ratio. In that case our new Sharpe will be:

$Sharpe_{new} = \sqrt{Sharpe_{old}^2 + (\frac{\alpha}{\sigma_\epsilon})^2}$

Where $Sharpe_{old}$ is the MVE Sharpe from before. Let's get the appraisal ratio for each stock.

First, the alpha of a stock is the "extra return" you get above what is implied by the model (APT) or the difference between its actual expected return and the model-implied expected return:

$\alpha_r = ActualExpectedReturn - ModelPredictedReturn$

$\alpha_a = 9\% - (0.5 + 0.5 * 0.08 - 0.2 * (-0.06)) = 9 - 10.2 = -1.2\%$

$\alpha_b = 13\% - (0.5 + 1 * 0.08 - 0.5 * (-0.06)) = 13 - 10 = 3\%$

$\alpha_c = 15\% - (0.5 + 1.2 * 0.08 - 0.5 * (-0.06)) = 15 - 17.6 = -2.6\%$

Now let's calculate the additional risk we take for each of these, defined as the residual risk in addition to the factors. For a return r, that is

$\sigma_r^2 = b_1^2\sigma_{f,1}^2 + b_2^2\sigma_{f,2}^2 + b_3^2\sigma_{f,3}^2 + \sigma_\epsilon^2$

We want to get σ_ϵ for each return, which is given by

$\sigma_\epsilon = \sqrt{\sigma_r^2 - (b_1^2\sigma_{f,1}^2 + b_2^2\sigma_{f,2}^2 + b_3^2\sigma_{f,3}^2)}$

$\sigma_{\epsilon,A} = \sqrt{(0.114)^2 - (0.5^2 * (0.01) + 0.2^2 * (0.01) + 0^2 * (0.01))} = 10\%$

$\sigma_{\epsilon,B} = \sqrt{(0.188)^2 - (1^2 * (0.01) + 0.5^2 * (0.01) + 0.2^2 * (0.01))} = 15\%$

$\sigma_{\epsilon,C} = \sqrt{(0.156)^2 - (1.2^2 * (0.01) + 0.5^2 * (0.01) + 0.7^2 * (0.01))} = 5\%$

Call $app = \frac{\alpha}{\sigma_\epsilon}$ the appraisal ratio. Notice, we actually only care about the absolute value of the appraisal ratio, since shorting a stock will just reverse the sign of the alpha.

$app_a = 1.2/10 = 0.12$

$app_b = 3/15 = 0.2$

$app_c = 2.6/5 = 0.52$

C has the highest appraisal ratio, so we will take C.

Our new Sharpe ratio is

$Sharpe_{new} = \sqrt{1^2 + (0.52)^2} = 1.13$

(c) Now suppose that the managers of fund B decides to increase the fee that is charged to investors. It is currently 2%/year. How much can the fee be raised?

The fee can be raised until the funds alpha is zero. Currently, the funds *post* fee alpha is 3% so the fund could raise the fee 3 more percent, to a total of 5%.

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