Growth Opportunities and Technology Shocks*

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Abstract

The market value of a firm can be decomposed into two fundamental parts: the value of assets in place and the value of future growth opportunities. We propose a theoretically-motivated procedure for measuring heterogeneity in growth opportunities across firms. We identify firms with high growth opportunities based on the covariance of their stock returns with the investment-specific productivity shock. We find that, empirically, our procedure is able to identify economically significant and theoretically consistent differences in firms' investment behavior, as well as risk and risk premia in their stock returns. Our empirical findings are quantitatively consistent with a calibrated structural model of firms' growth.

1 Introduction

The market value of a firm can be decomposed into two fundamental parts: the value of assets in place and the value of future growth opportunities. If the systematic risk of growth opportunities differs from that of assets in place, heterogeneity in firms' growth option shares could help explain observed cross-sectional differences in stock returns. This basic observation underpins many of the theoretical models connecting firms' characteristics to the properties of their stock returns. Successful applications of this idea depend on the

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quality of empirical measures of growth opportunities. We propose a theoretically-motivated procedure for measuring growth option heterogeneity and document its empirical properties. We find that, empirically, our procedure is able to identify economically significant and theoretically consistent differences in firms' investment behavior, as well as risk and risk premia in their stock returns.

The literature on real determinants of economic growth has documented that a significant fraction of observed growth variability can be attributed to productivity shocks in the capital goods sector [Greenwood, Hercowitz and Krusell, 1997; Fisher, 2006]. Under certain assumptions, one can identify such shocks with the price of investment equipment. Greenwood et al. (1997) show that the historical series of investment-goods prices is negatively correlated with aggregate investment, both at business-cycle and lower frequencies. Our theoretical model predicts (see Proposition 2 below) that stock returns of firms for which growth options account for a relatively large fraction of their market value (high-growth firms) respond more to the investment-specific productivity shocks (z-shocks). Our empirical procedure is based on this intuition, relating unobservable asset composition (growth options relative to assets in place) to observed differences in stock price sensitivity to the z-shocks.

We sort firms on their stock return sensitivity to the z-shocks. The macroeconomic literature has focused on the price of new equipment, where a positive z-shock refers to a decline in the price of investment goods. However, since the data on investment-goods prices is available only at the annual frequency, we instead use a portfolio mimicking z-shocks, constructed according to our theoretical model. Specifically, we use a zero-investment portfolio long the stocks of investment-good producers and short the stocks of consumption-good producers (IMC).

Since growth opportunities are not directly observable, we use indirect metrics to assess the success of our procedure. In particular, the key metric is the response of firms' investment to the z-shock. Intuitively, firms with more growth opportunities should invest relatively more in response to a favorable z-shock, since they have more potential projects to invest in. In addition, high-growth firms have other observable characteristics. In most standard models, such firms tend to have higher Tobin's Q, higher average investment rates, and higher market betas.

We find that the mimicking portfolio IMC betas (β^{imc}) are able to identify heterogeneity in firms' investment responses to the z-shocks. High-β^{imc} firms not only invest more on average, but their investment increases more in response to a positive investment shock, as measured by high returns on the IMC portfolio or a decline in investment-goods prices. Economically, these effects are significant. The difference in investment-goods price sensitivity between the high-beta and the low-beta firms is two to three times larger than the sensitivity of an average firm. The average investment rate of low-beta firms is twenty percent less of that of the high-beta firms. High-β^{imc} firms tend to have higher Tobin's Q and higher market beta, however, the investment-shock betas contain information about the firms' asset mix which is not reflected their Tobin's Q or market beta. Moreover, consistent with our economic intuition, high β^{imc} firms hold more cash, pay less in dividends and invest more in R&D.

We explore whether our measure for growth opportunities can capture heterogenous firm response to other aggregate shocks that should affect investment. We find that high β^{imc} firms respond significantly more than low β^{imc} firms to innovations in credit spreads, implying that a tightening of credit conditions is more likely to affect high-growth firms. In addition, we show that when the aggregate investment rate increases, the investment rate of high-β^{imc} firms increases significantly more than that of low β^{imc} firms. This suggests that, on average, a macroeconomic shock affecting aggregate investment impacts firms with richer growth opportunities relatively more. These findings support our conjecture that β^{imc} is a valid empirical proxy for the z-shock beta and that the latter captures cross-sectional differences in growth opportunities across firms.

We show that our empirical findings are quantitatively consistent with a parsimonious structural model of investment. In our partial-equilibrium model, firms derive value from implementing positive-NPV projects, which arrive randomly. The price of capital goods varies stochastically, affecting firms' investment choices. Randomness in project arrival and expiration leads to cross-sectional heterogeneity in the firms' mix of growth opportunities and assets in place. Our model matches the key qualitative and quantitative features of the empirical data, including cross-sectional differences in firms' response to investment-specific shocks and their risk premia. In particular, we find that the beta of stock returns with respect to the investment-specific shock positively predicts the sensitivity of firm investment to such shocks. These effects are consistent in magnitude with the corresponding empirical estimates. We also find that the cross-sectional differences in stock returns between portfolios sorted by investment-shock betas and market-to-book ratios are quantitatively similar to the data.

The rest of the paper is organized as follows. Section 2 relates our paper to existing work. In Section 3 we present the structural model of investment. Section 4 presents empirical results. In Section 6 we evaluate our model quantitatively using calibration.

2 Relation to the Literature

Our paper bridges and complements two distinct strands of the macroeconomic and finance literature. The first argues for the importance of investment-specific shocks for aggregate quantities and the second argues that differences in firm's mix between growth options and assets are important in understanding the cross-section of risk premia.

In macroeconomics, a number of studies have shown that investment-specific technological shocks can account for a large fraction of the variability output and employment, both in the long-run, as well as at business cycle frequencies [Greenwood et al., 1997; Greenwood, Hercowitz and Krusell, 2000; Boldrin, Christiano and Fisher, 2001; Fisher, 2006; Justiniano, Giorgio and Tambalotti, 2008]. Investment shocks can be modelled as either shocks to the marginal cost of capital as in Solow (1960) or as shocks to the productivity of a sector producing capital goods as in Rebelo (1991) or Boldrin et al. (2001). Given that investment shocks lead to an improvement in the real investment opportunity set in the economy, they are a natural place to start to understand the heterogeneity in the risk of growth options versus assets in place.

In financial economics, the idea that growth options may have different risk characteristics than assets in place is not new. [Berk, Green and Naik, 1999; Gomes, Kogan and Zhang, 2003; Carlson, Fisher and Giammarino, 2004; Zhang, 2005]. These studies have argued that decomposing value into assets in place versus growth opportunities may be useful in understanding the cross-section of risk premia. In these models, assets in place are riskier than growth options in bad times. A counter-cyclical price of risk may lead to value firms having higher returns on average than growth firms. Our work complements this literature by illustrating how a different mechanism can generate differences in risk premia between assets in place and growth options. Papanikolaou (2008) shows that in a two-sector general equilibrium model, investment shocks can generate a value premium. On the other hand, there is no other source of firm heterogeneity in his model, whereas we explicitly model firm heterogeneity in terms of the mix between growth options and assets in place.

Our work is also connected to the investment literature that links Tobin's Q, a measure of growth opportunities to firm investment. In order to generate a non-zero value for growth opportunities, some investment friction is often assumed such as convex or fixed adjustment costs, or investment irreversibility [Hayashi, 1982; Abel, 1985; Abel and Eberly, 1994; Abel and Eberly, 1996; Abel and Eberly, 1998; Eberly, Rebelo and Vincent, 2008]. In these models, marginal Q measures the valuation of an additional unit of capital invested in the firm, which in the finance literature is closely linked to the notion of growth options. Tobin's Q is often proxied by the market value of capital divided by it's historical cost. We contribute to this literature by introducing a new empirical measure of growth opportunities that relies on stock price changes rather than levels.

3 The Model

In this section we develop a structural model of investment. We show that the value of assets in place and the value of growth opportunities have different exposure to the investment-specific productivity shocks. Thus, the relative weight of growth opportunities in a firm's value can be identified by measuring the sensitivity of its stock returns to investment-specific shocks.

There are two sectors in our model, the consumption-good sector, and the investment-good sector. Investment-specific shocks enter the production function of the investment-good sector. We focus on heterogeneity in growth opportunities among consumption-good producers.

3.1 Consumption-Good Producers

There is a continuum of measure one of infinitely lived firms producing a homogeneous consumption good. Firms behave competitively and there is no explicit entry or exit in this sector.

Assets in Place

Each firm owns a finite number of individual projects. Firms create projects over time through investment, and projects expire randomly.[1] Let $\mathcal{F}$ denote the set of firms and $\mathcal{J}^{(f)}$ the set of projects owned by firm f.

Project j managed by firm f produces a flow of output equal to

$$y_{fjt} = \varepsilon_{ft} u_{jt} x_t K_j^{\alpha}, \tag{1}$$

where K_j is physical capital chosen irreversibly at the project j's inception date, u_{jt} is the

[1] Firms with no current projects may be seen as firms that temporarily left the sector. Likewise, idle firms that begin operating a new project can be viewed as new entrants. Thus, our model implicitly captures entry and exit by firms.

project-specific component of productivity, ε_{ft} is the firm-specific component of productivity, such as managerial skill of the parent firm, and x_t is the economy-wide productivity shock affecting output of all existing projects. We assume decreasing returns to scale at the project level, $\alpha \in (0,1)$. Projects expire according to independent Poisson processes with the same arrival rate δ.

The three components of projects' productivity evolve according to

$$\begin{aligned} d\varepsilon_{ft} &= -\theta_\epsilon(\varepsilon_{ft}-1)\,dt + \sigma_e\sqrt{\varepsilon_{ft}}\,dB_{ft} \\ du_{jt} &= -\theta_u(u_{jt}-1)\,dt + \sigma_u\sqrt{u_{jt}}\,dB_{jt} \\ dx_t &= \mu_x\,x_t\,dt + \sigma_x x_t\,dB_{xt}, \end{aligned}$$

where dB_{ft}, dB_{jt} and dB_{xt} are independent standard Brownian motions. All idiosyncratic shocks are independent of the aggregate shock, $dB_{ft}\cdot dB_{xt}=0$ and $dB_{jt}\cdot dB_{xt}=0$. The firm and project-specific components of productivity are stationary processes, while the process for aggregate productivity follows a Geometric Brownian motion, generating long-run growth.

Investment

Firms acquire new projects exogenously according to a Poisson process with a firm-specific arrival rate λ_{ft}. The firm-specific arrival rate of new projects is

$$\lambda_{ft} = \lambda_f \cdot \tilde{\lambda}_{f,t} \tag{2}$$

where $\tilde{\lambda}_{ft}$ follows a two-state, continuous time Markov process with transition probability matrix between time t and $t+dt$ given by

$$P = \begin{pmatrix} 1-\mu_L\,dt & \mu_L\,dt \\ \mu_H\,dt & 1-\mu_H\,dt \end{pmatrix}. \tag{3}$$

We label the two states as $[\lambda_H, \lambda_L]$, with $\lambda_H > \lambda_L$. Thus, at any point in time, a firm can be either in the high-growth $(\lambda_f \cdot \lambda_H)$ or in the low-growth state $(\lambda_f \cdot \lambda_L)$, and $\mu_H\, dt$ and $\mu_L\, dt$ denote the instantaneous probability of entering each state respectively. We impose that $E[\tilde{\lambda}_{f,t}] = 1$, which translates to the restriction

$$1 = \lambda_L + \frac{\mu_H}{\mu_H + \mu_L}(\lambda_H - \lambda_L) \tag{4}$$

When presented with a new project at time t, a firm must make a take-it-or-leave-it decision. If the firm decides to invest in a project, it chooses the associated amount of capital K_j and pays the investment cost $z_t x_t K_j$. The cost of capital relative to it's average productivity, z_t, is assumed to follow a Geometric Brownian motion

$$dz_t = \mu_z z_t\, dt + \sigma_z z_t\, dB_{zt}, \tag{5}$$

where $dB_{zt} \cdot dB_{xt} = 0$. The z shock represents the component of the price of capital that is unrelated to it's current level of average productivity, x, and is the investment-specific shock in our model. Finally, at the time of investment, the project-specific component of productivity is at its long-run average value, $u_{jt} = 1$.

Valuation

Let π_t denote the stochastic discount factor. The time-0 market value of a cash flow stream C_t is then given by $\mathrm{E}\left[\int_0^\infty (\pi_t/\pi_0) C_t\, dt\right]$. For simplicity, we assume that the aggregate productivity shocks x_t and z_t have constant prices of risk β_x, β_z, and the risk-free interest rate r is also constant. Then,

$$\frac{d\pi_t}{\pi_t} = -r\, dt - \beta_x\, d\, B_{x,t} - \beta_z\, d\, B_{z,t}. \tag{6}$$

This form of the stochastic discount factor is motivated by a general equilibrium model with with investment-specific technological shocks in Papanikolaou (2008). In Papanikolaou

(2008), states with low cost of new capital are high marginal valuation states because of improved investment opportunities. This is analogous to a positive value of β_z. Our analysis below shows that empirical properties of stock returns imply a positive value of β_z. Finally, we choose a price of risk of the aggregate productivity shock x is positive, which is consistent with most equilibrium models and empirical evidence.

Firms' investment decisions are based on a tradeoff between the market value of a new project and the cost of physical capital. The time-t market value of an existing project j, $p(\varepsilon_{ft}, u_{jt}, x_t, K_j)$, is computed using the discounted cash flow formula:

$$p(\varepsilon_{ft}, u_{jt}, x_t, K_j) = \mathrm{E}_t \left[\int_t^{\infty} e^{-\delta(s-t)} \frac{\pi_s}{\pi_t} \varepsilon_{fs} u_{js} x_s K_j^{\alpha} \, ds \right] = A(\varepsilon_{ft}, u_{jt}) x_t K_j^{\alpha}, \tag{7}$$

where

$$\begin{aligned} A(\varepsilon, u) \quad = \quad & \frac{1}{r + \delta - \mu_X} + \frac{1}{r + \delta - \mu_X + \theta_e}(\varepsilon - 1) + \frac{1}{r + \delta - \mu_X + \theta_u}(u - 1) \\ & + \frac{1}{r + \delta - \mu_X + \theta_e + \theta_u}(\varepsilon - 1)(u - 1) \end{aligned}$$

Firms' investment decisions are straightforward because the arrival rate of new projects is exogenous and does not depend on their previous decisions. Thus, optimal investment decisions are based on the NPV rule. Firm f chooses the amount of capital K_j to invest in project j to maximize

$$p(\varepsilon_{ft}, u_{jt}, x_t, K_j) - z_t x_t K_j$$

Lemma 1 *The optimal investment K_j in project j, undertaken by firm f at time t is*

$$K^*(\varepsilon_{ft}, z_t) = \left(\frac{\alpha A(\varepsilon_{ft}, 1)}{z_t} \right)^{\frac{1}{1-\alpha}}.$$

The scale of firm's investment depends on firm-specific productivity, ε_{ft}, and the price of investment goods relative to average productivity, z_t. Because our economy features decreasing returns to scale at the project level, it is always optimal to invest a positive and finite amount.

The value of the firm can be computed as a sum of market values of its existing projects and the present value of its growth opportunities. The former equals the present value of cash flows generated by existing projects. The latter equals the expected discounted NPV of future investments. Following the standard convention, we call the first component of firm value *the value of assets in place*, VAP_{ft}, and the second component *the present value of growth opportunities*, $PVGO_{ft}$. The value of the firm then equals

$$V_{ft} = VAP_{ft} + PVGO_{ft}$$

The value of a firm's assets in place is simply the value of its existing projects:

$$VAP_{ft} = \sum_{j \in \mathcal{J}_f} p(e_{ft}, u_{jt}, x_t, K_j) = x_t \sum_{j \in \mathcal{J}_f} A(\varepsilon_{ft}, u_{j,t}) K_j^{\alpha}.$$

The present value of growth options is given by the following lemma.

Lemma 2 *The value of growth opportunities for firm i*

$$PVGO_{ft} = z_t^{\frac{\alpha}{\alpha-1}} \, x_t \, G(\varepsilon_{ft}, \lambda_{ft})$$

$$\begin{aligned} G(\varepsilon_{ft}, \lambda_{ft}) &= C \cdot E_t \left[\int_t^{\infty} e^{-\rho(s-t)} \, \lambda_{fs} \, A(\varepsilon_{fs})^{\frac{1}{1-\alpha}} \, ds \right] \\ &= \lambda_f \left(G_1(\varepsilon_{ft}) + \frac{\mu_L}{\mu_L + \mu_H} (\lambda_H - \lambda_L) \, G_2(\varepsilon_{ft}) \right), \qquad \tilde{\lambda}_{ft} = \lambda_H \\ & \quad \lambda_f \left(G_1(\varepsilon_{ft}) - \frac{\mu_H}{\mu_L + \mu_H} (\lambda_H - \lambda_L) \, G_2(\varepsilon_{ft}) \right), \qquad \tilde{\lambda}_{ft} = \lambda_L, \end{aligned}$$

where

$$\rho = r + \frac{\alpha}{1-\alpha}(\mu_z - \sigma_z^2/2) - \mu_x - \frac{\alpha^2 \sigma_z^2}{2(1-\alpha)^2},$$

and

$$C = \alpha^{\frac{1}{1-\alpha}} \left(\alpha^{-1} - 1 \right).$$

The functions $G_1(\varepsilon)$ and $G_2(\varepsilon)$ solve

$$\begin{aligned} C\cdot A(\varepsilon,1)^{\frac{1}{1-\alpha}} - \rho\, G_1(\varepsilon) - \theta_\epsilon(\varepsilon-1)\frac{d}{d\,\varepsilon}\,G_1(\varepsilon) + \frac{1}{2}\sigma_e^2\,\varepsilon\,\frac{d^2}{d\,\varepsilon^2}\,G_1(\varepsilon) &= 0 \\ C\cdot A(\varepsilon,1)^{\frac{1}{1-\alpha}} - (\rho+\mu_H+\mu_L)\,G_2(\varepsilon) - \theta_\epsilon(\varepsilon-1)\frac{d}{d\,\varepsilon}\,G_2(\varepsilon) + \frac{1}{2}\sigma_e^2\,\varepsilon\,\frac{d^2}{d\,\varepsilon^2}\,G_2(\varepsilon) &= 0. \end{aligned}$$

In addition to the aggregate and firm-specific productivity, the present value of growth opportunities depends on the investment-specific shock, z, because the net present value of future projects depends on the cost of new investment. In summary, the firm value in our model is

$$V_{ft} = VAP_{ft} + PVGO_{ft} = x_t \sum_j A(\varepsilon_{ft}, u_{jt}) K_j^\alpha + z_t^{\frac{\alpha}{\alpha-1}} x_t\, G(\varepsilon_{ft}, \lambda_{ft}) \tag{8}$$

Risk and Expected Returns

Both assets in place and growth opportunities have constant exposure to systematic shocks dB_{xt} and dB_{zt}. However, their betas with respect to the productivity shocks are different. The value of assets in place is independent of the investment-specific shock and loads only on the aggregate productivity shock. The present value of growth option depends positively on aggregate productivity, and negatively on the unit cost of new capital. Thus, firm's betas with respect to the aggregate shocks are time-varying, and depend linearly on the fraction of firm value accounted for by growth opportunities. Since, by assumption, the price of risk of aggregate shocks is constant, expected excess return on a firm is an affine function of the weight of growth opportunities in firm value, as shown in the following proposition.

Proposition 1 *The expected excess return on firm f is*

$$ER_{ft} - r_f = \beta_x \sigma_x - \frac{\alpha}{1-\alpha}\beta_z \sigma_z \frac{PVGO_{ft}}{V_{ft}} \tag{9}$$

Many existing models of the cross-section of stock returns generate an affine relationship between expected stock return and firms' asset composition similar to (9). It is easy to see, in the context of our model, how the relationship (9) can give rise to a value premium. Assume that both prices of risk β_x and β_z are positive, which we justify in the following sections. Then growth firms, which derive a relatively large fraction of their value from growth opportunities, have relatively low expected excess returns because of their exposure to investment-specific shocks. To the extent that firms' book-to-market (B/M) ratios are partially driven by the value of firms' growth opportunities, firms with high B/M ratios tend to have higher average returns than firms with low B/M ratios.

3.2 Investment-Good Producers

There is a continuum of firms producing new capital goods. We assume that these firms produce the demanded quantity of capital goods at the current unit price z_t. We assume that profits of investment firms are a fraction ϕ of total sales of new capital goods.[2] Consequently, profits accrue to investment firms at a rate of $\Pi_t = \phi z_t\, x_t\, \overline{\lambda} \int_{\mathcal{F}} K_{ft} df$, where $\overline{\lambda} = \int_{\mathcal{F}} \lambda_{ft}$ is the average arrival rate of new projects among consumption-good producers. Even though λ_{ft} is stochastic, it has a stationary distribution, so $\overline{\lambda}$ is a constant.

Lemma 3 *The price of the investment firm satisfies*

$$V_{I,t} = \Gamma\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} \frac{1}{\rho_I} \tag{10}$$

where we assume

$$\rho_I \equiv r - \mu_X + \frac{\alpha}{1-\alpha}\,\mu_Z - \frac{1}{2}\frac{\alpha}{1-\alpha}\,\sigma_Z^2 - \frac{1}{2}\frac{\alpha^2\,\sigma_Z^2}{(1-\alpha)^2} > 0$$

and

$$\Gamma \equiv \phi\,\overline{\lambda}\,\alpha^{\frac{1}{1-\alpha}} \left(\int A(e_f, 1)^{\frac{1}{1-\alpha}} df \right).$$

The value of the investment firms will equal the present value of their cashflows. If we assume that these firms incur proportional costs of producing their output, and given that

[2] Alternatively, one can specify a production function of investment firms so that z_t is a market clearing price and their profit is a fraction ϕ of sales.

the market price of risk is constant for the two shocks, their value will be proportional to cashflows or the aggregate investment expenditures in the economy. The stock returns of the investment firms will then load on the investment shock (z) as well as the common productivity shock (x).

We define an IMC portfolio in the model as a portfolio that is long the investment sector and short the consumption sector. The beta of firm f with respect to the IMC portfolio return is given by

$$\beta_{ft}^{imc} = \frac{cov_t(R_{ft}, R_t^I - R_t^C)}{var_t(R_t^I - R_t^C)}$$

where $R_t^I - R_t^C$ is the return on the IMC portfolio.

Proposition 2 *The beta of firm i with respect to the IMC portfolio return is given by*

$$\beta_{ft}^{imc} = \beta_{0t} \left(\frac{PVGO_{ft}}{V_{ft}} \right) \tag{11}$$

where

$$\beta_{0t} = \frac{\overline{V}_t}{\overline{VAP}_t}$$

Proposition 2 is the basis of our empirical approach to measuring growth opportunities. The covariance of firm f's return with respect to the IMC portfolio return is proportional to the fraction of firm f's value represented by its growth opportunities. Firms that have few active projects but expect to create many projects in the future derive most of their value from their future growth opportunities. These firms are anticipated to increase their investment in the future, and their stock price reflects that. There is also an aggregate term in (11) that depends on the fraction of aggregate value that is due to growth opportunities, which affects the IMC portfolio's correlation with the z-shock.

4 Data and Empirical Procedures

Our analysis in Section 3 suggests that the firm-specific return-based measure of z-shock sensitivity could be used to measure growth opportunities as a fraction of firm value. Our theoretical model also predicts that returns on the IMC portfolio, which is long the stocks of investment-good producers and short the stocks of consumption-good producers, should be a valid proxy for the investment-specific shocks. In this section we investigate these predictions empirically.

4.1 Investment-specific shocks

Based on the model developed in Section 3, we use the IMC portfolio as a mimicking portfolio for the investment-specific shocks.[3] We first classify industries as producing either investment or consumption goods according to the NIPA Input-Output Tables. We then and match firms to industries according to their NAICS codes. Gomes, Kogan and Yogo (2008) and Papanikolaou (2008) describe the details of this classification procedure.

4.2 Estimation of β^{imc}

We use the firm's stock return beta with respect to the IMC portfolio returns as a measure of this firm's investment-specific shock sensitivity. For every firm in Compustat with sufficient stock return data, we estimate a time-series of (β_{ft}^{imc}) from the following regression

$$r_{ftw} = \alpha_{ft} + \beta_{ft}^{imc} r_{tw}^{imc} + \varepsilon_{ftw}, \qquad w = 1 \ldots 52. \tag{12}$$

Here r_{ftw} refers to the (log) return of firm f in week w of year t, and r_{ftw}^{imc} refers to the log return of the IMC portfolio in week w of year t. Thus, β_{ft}^{imc} is constructed using information only in year t.

We omit firms with fewer than 50 weekly stock-return observations per year, firms in their

[3] Papanikolaou (2008) also uses IMC returns as a factor-mimicking portfolio for investment-specific shocks.

first three years following the first appearance in COMPUSTAT, firms in the investment sector, financial firms (SIC codes 6000-6799), utilities (SIC codes 4900-4949), firms with missing values of CAPEX (Compustat item capx), PPE (Compustat item ppent), Tobin's Q, CRSP market capitalization, firms whose investment rate exceeds 1 in absolute value, firms with Tobin's Q greater than 100, firms with negative book values and firms where the ratio of cashflows to capital exceeds 5 in absolute value. Our final sample contains 6,831 firms and 62,495 firm-year observations and covers the 1965-2007 period.

5 Empirical Findings

In this section we test the qualitative predictions of our model for the response of firm-level investment to investment-specific shocks. Since the model implications for stock returns depend on the quantitative assumptions, we postpone that discussion until Section 6, where we compare our empirical findings to the output of the calibrated model.

5.1 Main Results

Summary statistics

We focus our analysis on firms in the consumption-good sector, following our theoretical analysis above. Every year we split the universe of consumption-good producers into 10 portfolios based on their estimate of β^{imc}. Table 2 reports the summary statistics for firms in different β^{imc}-deciles. The patterns across the deciles are consistent with our interpretation of β^{imc} as measuring heterogeneity in growth opportunities. High-β^{imc} firms tend to have higher investment rates (25.2% vs 20.1% for the low-β^{imc} firms), higher Tobin's Q (1.38 vs 1.13), higher R&D expenditures (6.0% vs 1.4%), and pay less in dividends (2.8% vs 9.0%), although the latter relationship is hump-shaped. Furthermore, high β^{imc} firms tend to be smaller, both in terms of market capitalization as well as book value of capital. The highest β^{imc} portfolio accounts for a fraction of 3.9% and 2.8% of the total market capitalization and book value of capital versus 9.8% and 8.8% for the low β^{imc} portfolio. Moreover, high-β^{imc}

firms have higher market betas, which, as we show in Section 6 to be consistent with them having more growth opportunities.

Response of firm-level investment to IMC returns

Since growth opportunities are not observable directly, we base our empirical tests on observable differences between firms with high and low growth opportunities. In particular, our model makes an intuitive prediction that firms with high growth opportunities, being better positioned to take advantage of positive investment-specific shocks, should increase investment more in response to a positive investment shock than firms with low growth opportunities. While this prediction is easy to verify given the simple structure of our model, one would expect it to hold much more generally.

We estimate the sensitivity of firms' investment to z-shocks using the following econometric specification:

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d \, D(\beta^{imc}_{f,t-1})_d + b_1 \, \tilde{R}^{imc}_t + \sum_{d=2}^{5} b_d \, D(\beta^{imc}_{f,t-1})_d \times \tilde{R}^{imc}_t + cX_{f,t-1} + \gamma_f + u_t. \quad (13)$$

where $i_t \equiv \frac{I_t}{K_{t-1}}$ is the firm's investment rate, defined as capital Expenditures (Compustat item capx) over Property Plant and Equipment (Compustat item ppent), $\tilde{R}^{imc}_t = R^{imc}_t + R^{imc}_{t-1}$ refers to accumulated log returns on the factor-mimicking portfolio (IMC) and $D(x)_d$ is a β^{imc}-quintile dummy variable ($D(\beta^{imc}_{i,t-1})_n = 1$ if the firm's β^{imc} belongs to the quintile n in year $t-1$). $X_{f,t-1}$ is a vector of controls, which includes the firm's Tobin's Q, its lagged investment, leverage, cash flows and log of its capital stock relative to the aggregate capital stock. Definitions of these variables are standard and are summarized in Table 1. We standardize all independent variables to zero mean and unit standard deviation using unconditional moments. The sample covers the 1962-2007 period.

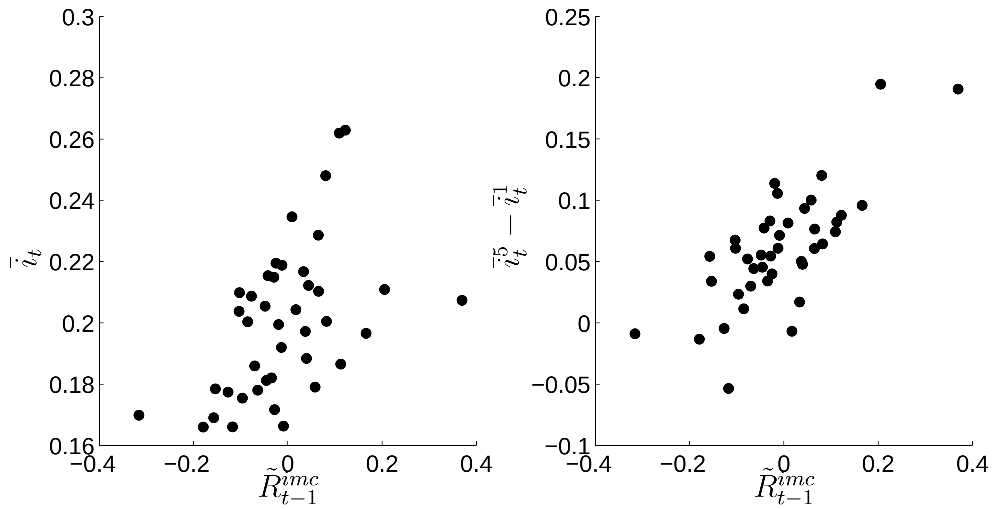
The coefficients $(a_1, \ldots, a_5)$ and $(b_1, \ldots, b_5)$ on the dummy variables measure differences in the level of investment and response of investment to z-shocks respectively. We estimate the investment response both with and without firm- and industry-level fixed effects, and both with and without controlling for commonly used predictors of firm-level investment. Our estimates of β^{imc} are persistent, but exhibit sufficient variation over time to separate the effect of β^{imc} from firm-level fixed effects. Table 3 reports transition probabilities among the β^{imc} quintiles.

When computing standard errors we account for the fact that investment may contain an unobservable firm and time component. Following Petersen (2009), we cluster standard errors both by firm and time.[4]

We summarize the results in Table 4. The results show that for all specifications, firms with high β^{imc} invest more on average and their investment rate responds more to an investment-specific shock. A single-standard-deviation IMC return shock changes firm-level investment by 0.096 standard deviations on average. This number varies between 0.053 for the low-β^{imc} firms and 0.176 for the high-β^{imc} firms. The spread between quintiles is economically significant, and equal to 0.123 standard deviations, which is larger than the average sensitivity of investment rate to z-shocks. In response to a single-standard-deviation IMC return shock, the level of the investment rate of low-β^{imc} firms changes by 0.9%, compared to the 3.1% response by the high-β^{imc} firms. Fluctuations of this magnitude are substantial compared to the unconditional volatility of the aggregate investment rate changes in our sample, which is 2.4%. Figure 5.1 illustrates the magnitude of the effects by contrasting the scatter plot of the average investment rate versus the lagged return on the IMC portfolio with the analogous plot for the average difference in investment rates between the extreme β^{imc}-quintiles.

[4]Petersen (2009) suggests following Cameron, Gelbach and Miller (2006) and Thomson (2006) who estimate the variance-covariance matrix by combining the matrices obtained by separately clustering by firm and by time.

Figure 1: Portfolio investment rates vs $\tilde{R}^{imc}$. The left panel shows the scatter plot of the aggregate investment rate, defined as the total investment by firms in our sample normalized by their total capital stock, $\bar{i}_t = \sum_{f\in F_t} I_{ft} / \sum_{f\in F_t} K_{ft-1}$, versus the lagged return on the IMC portfolio, $\tilde{R}^{imc}_{t-1} \equiv \sum_{l=1}^{2} R^{imc}_{t-1}$. In the right panel, we replace the aggregate investment rate with the difference in investment rates between the highes and the lowest β^{imc}-quintile portfolios.



When controlling for industry fixed effects and Tobin's Q, lagged investment rate, leverage, cash flows and log capital, the difference in coefficients on the z-shock between the extreme β^{imc} quintiles of firms diminishes somewhat to 0.086, and it is at 0.089 once firm fixed effects are included in the specification.

5.2 Additional Results and Robustness Checks

Investment response to price of equipment shocks

As our first robustness check, we consider the quality-adjusted price series of new equipment as an alternative proxy for investment-specific shocks. This proxy has been used in the literature to measure the economic impact of investment-specific shocks on aggregate growth (e.g., [Greenwood et al., 1997; Greenwood et al., 2000; Fisher, 2006]), and is therefore intrinsically interesting.

The quality-adjusted price series of new equipment has been constructed by Gordon

(1990), Cummins and Violante (2002) and Israelsen (2008).[5] We define investment-specific technological changes as changes in the log relative price of new equipment goods.[6] As Fisher (2006) points out, the real equipment price experiences an abrupt increase in its average rate of decline in 1982, which is likely due to the effect of more accurate quality adjustment in more recent data [Moulton, 2001]. To address this issue, we remove the time trend from the series of equipment prices. Specifically, we construct z by regressing the logarithm of the quality-adjusted price of new equipment relative to the NIPA personal consumption deflator on a piece-wise linear time trend:

$$p_t = a_0 + b_0\mathbf{1}_{1982} + (a_1 + b_1\mathbf{1}_{1982}) \cdot t + z_t \tag{14}$$

where $\mathbf{1}_{1982}$ is an indicator function that takes the value 1 post 1982. We then define investment-specific technology shocks as increments of the de-trended series:

$$\Delta z_t = z_t - z_{t-1}, \tag{15}$$

Innovations in investment technology lead to a decline in the quality-adjusted price of new equipment, therefore we refer to a negative realization of Δz_t as a positive investment-specific shock. The resulting series is weakly positively correlated with the series of returns on the IMC portfolio. The historical correlation between the two series is 22.3% with a HAC-t-statistic of 2.31.

Using the new measure of investment-specific technological changes, we estimate equation (13) with Δz_t replacing $\tilde{R}^{imc}$. We present the results in Table 6. A one-standard deviation shock to Δz_t increases firm-level investment on average by 0.035 standard deviations, but the response differs in the cross-section and ranges from 0.007 to 0.069 for the low- and high-β^{imc} quintiles respectively. Thus, high-β^{imc} firms invest more in response to

[5] Cummins and Violante (2002) extrapolate the quality adjustment of Gordon (1990) to construct a price series for the period 1943-2000. Israelsen (2008) extends the price series through 2006.

[6] To compute relative prices, we normalize the price of new equipment by the NIPA consumption deflator.

a decline in equipment prices, which further supports our interpretation of these firms as having more growth opportunities.

Investment response to credit shocks

We show that IMC portfolio returns predict cross-sectional dispersion in investment rates between firms with high and low β^{imc}. Using the same methodology, one can assess investment response to a variety of economic shocks affecting the willingness of firms to investment. Here we consider one important example: investment response to unexpected changes in aggregate credit or liquidity conditions. Tightening credit conditions should have a similar effect on investment as a negative investment-specific shock, effectively leading to increased cost of investment. Thus, states with with tight credit are effectively states with low real investment opportunities.

We consider the innovation in the spread between Baa and Treasury bonds as a measure of innovation in the aggregate credit environment.Specifically, we use an AR(1) model of credit spread dynamics to define innovations (Δs_t) in credit spreads:

$$\Delta s_t = cr_t - 0.784\, cr_{t-1}, \tag{16}$$

where cr_t is the yield spread between Baa and Treasury bonds. The correlation between Δs_t and our two measures of investment shocks, $\tilde{R}^{imc}$ and Δz is equal to -0.36 and 0.14 respectively in the 1965-2007 sample.

We estimate cross-sectional differences in the firm-level investment response to changes in credit spreads across the β^{imc}-quintiles. Specifically, we estimate equation (13) with Δs_t replacing $\tilde{R}^{imc}$. Table 8 reports the results. On average, firms increase investment when credit spreads fall, and the sensitivity of investment rate to credit shocks increases across the β^{imc}-quintiles. A single-standard-deviation positive credit shock increases the average firm-level investment rate by 0.078 standard deviations. The difference in investment rate responses between high- and low-β^{imc} quintiles of firms is statistically significant and equal to

0.064 standard deviations. With various additional controls, the latter estimate falls between 0.049 and 0.061.

Firm investment and aggregate investment shocks

In contrast to the stylized setting of the model, investment-specific shocks are not be the only driver of investment in the data. Nevertheless, β^{imc} may be informative regarding the response of investment to aggregate shocks that do not necessarily originate in the investment-goods sector. Thus, we explore how the investment rates of firms with different values of β^{imc} respond differently to shocks to the aggregate investment rate that is uncorrelated with $\tilde{R}^{imc}$. First, we estimate the part of aggregate investment that is not captured by IMC returns. Then, we allow the response of firm-level investment to this component to vary with β^{imc} . The intuition is similar to the previous tests: firms with many growth opportunities are likely to invest relatively more in response to an aggregate shock that increases economy-wide investment, even if this shock does not originate in the investment-goods sector.

We first define the aggregate investment rate as $\bar{i}_t = \sum_{f=1}^{F_t} I_{ft} / \left(\sum_{f=1}^{F_t} K_{f,t-1} \right)$, where F_t refers to the set of firms in our sample in date t. We define the shock to the aggregate investment rate $\bar{i}_t^{\epsilon}$ as the residual in the regression of aggregate investment rate on lagged IMC return:

$$\bar{i}_t = a + b\tilde{R}_{t-1}^{imc} + \bar{i}_t^{\epsilon}. \tag{17}$$

IMC returns predict aggregate investment with a significant coefficient and the R^2 of 29%. We are interested in the relationship between the residual, unexplained by IMC returns, and firm-level investment. Table 7 summarizes the results. The magnitude of the effects is smaller than the investment response to IMC returns (Table 4), but β^{imc} quintiles show statistically significant differences in their response to aggregate investment rate shocks.

Tobin's Q, market β, and growth opportunities

Next, we investigate how well the Tobin's Q or market β perform as alternative measures of growth opportunities. Tobin's Q, defined as the market value of the firm divided by the replacement cost of its capital, is commonly used as an empirical proxy for growth opportunities. The underlying intuition is well known: firms with abundant growth opportunities have relatively high market value compared to their physical assets, and thus tend to have high Tobin's Q.

We consider the firm's market beta as the second alternative measure of growth opportunities. This is motivated by the lessons from real options literature. Typically, the part of the firm's value that is due to growth opportunities behaves as a levered claim on assets in place, and therefore it has higher volatility and is more sensitive to aggregate shocks than assets in place. Thus, real options models predict that high-growth-opportunity firms have relatively high market betas. As we document in Table 2, high-β^{imc} firms tend to have higher market beta in the data, and we show below in Table 16 that our model shares this property.

We estimate equation (13) using either Tobin's Q or the market beta instead of β^{imc}. In the first case, we also drop Tobin's Q as a control.

Using Tobin's Q as an alternative measure of growth options leads to results that are qualitatively similar but noticeably weaker than those obtained with β^{imc}, as we show in Table 9. The difference in the response of the investment rate to the IMC return between high- and low- Tobin's Q firms is 0.056. Heterogeneity in Tobin's Q does not lead to differential response to credit shocks, as we show in Table 10.

Cross-sectional differences in market betas predict a statistically significant response of investment to IMC returns, as can be seen in Table 11. Absent any controls, the difference in response between the high and low β^{mkt} portfolio to $\tilde{R}^{imc}$ is equal to 0.068, which is roughly half of the effect for β^{imc} deciles. Differences between β^{mkt} quintiles decline when additional controls are introduced, but remain statistically significant. Thus, we conclude that market betas to have some ability to predict differential response of firms to investment-specific shocks, but are less informative than betas with respect to IMC returns.

Additional robustness checks

We perform a number of additional robustness checks. First, it is possible that β^{imc} captures firms' financial constraints and not the differences in their real production opportunities. This possibility is consistent with our approach, since financially constrained firms, defined as firms with insufficient cash holdings and limited access to external funds, cannot take advantage of investment opportunities and as such have effectively low growth opportunities. Thus, future growth opportunities depend both on the firm's financial constraints and its real investment opportunities. To sharpen the interpretation of our empirical results, we attempt to distinguish financial constraints from real effects. We replicate our empirical analysis on a sample of firms relatively less likely to be constrained, namely, firms that have been assigned a credit rating by Standard and Poor's. This restricts our sample to $1,336$ firms and $13,456$ firm-year observations. We find that our results hold in this sample, with the difference in the response of investment to $\tilde{R}^{imc}$ between the extreme β^{imc}-quintiles of 0.205. This estimate is in fact greater than the one obtained for the entire sample of firms, indicating that our findings are unlikely to be explained by financial constraints alone.

Second, we estimate β^{imc} using stock return return data, while the theory suggests using returns on the total firm value. Our findings could be explained by investment of highly levered firms being relatively sensitive to investment shocks. The results in Table 2 suggest that this is not likely to be the case, as there does not seem to be systematic differences in leverage across portfolios. To address this discrepancy, we approximate β^{imc} at the *asset* level (de-lever the equity-based estimates) under the assumption that firms' debt is risk-free. We re-estimate Equation 13 using de-levered β^{imc}. We find that the difference in investment responses between the high- and low-β^{imc} firms is statistically significant and equal to 0.097 and 0.118, depending on whether we use book or market leverage.

Finally, we consider whether β^{imc} may be capturing inter-industry differences in technology instead of capturing meaningful differences in growth opportunities.[7] We investigate this possibility by defining β^{imc}-quintiles based on the firms' intra-industry β^{imc} ranking, where we use the 30 industry classification of Fama and French (1997). We find that our results are driven by intra- rather than inter-industry variation. The difference in investment responses between the firms in high- and low-β^{imc}-quintiles relative to their industry peers is statistically significant and equal to 0.121. In contrast, intra-industry ranking of firms on market betas leads to largely insignificant response differences between the β^{mkt} quintiles. Thus, cross-sectional differences in market betas may reflect heterogeneity in cyclicality across industries and not the heterogeneity in growth opportunities that we aim to capture. This shows that IMC betas are superior to market betas at identifying cross-sectional differences in growth opportunities.

To conserve space, we do not report the full details of the above robustness checks and refer the reader to the web Appendix.

6 Calibration

We calibrate our model to approximately match moments of aggregate dividend growth and investment growth, accounting ratios, and asset returns. Thus, most of the parameters are chosen jointly based on the behavior of financial and real variables.

We pick $\alpha = 0.85$, the parameters governing the projects' cash flows ($\sigma_\varepsilon = 0.2, \theta_e = 0.35, \sigma_u = 1.5, \theta_u = 0.5$) and the parameters of the distribution of λ_f jointly, to match the average values and the cross-sectional distribution of the investment rate, the market-to-book ratio, and the return to capital (ROE).

[7] This possibility is not addressed by the controls we use in estimation, since we do not allow the loadings on quintile dummies to interact with the industry fixed effects.

We model the distribution of mean project arrival rates $\lambda_f = E[\lambda_{ft}]$ across firms as

$$\lambda_f = \mu_\lambda \delta - \sigma_\lambda \delta \log(X_f) \quad X_f \sim U[0,1], \tag{18}$$

We pick $\sigma_\lambda = \mu_\lambda = 2$. Regarding the dynamics of the stochastic component of the firm-specific arrival rate, $\tilde{\lambda}_{ft}$, we pick $\mu_H = 0.075$ and $\mu_L = 0.16$. We pick $\lambda_H = 2.35$, which according to (4) implies $\lambda_L = 0.35$. These parameter values ensure that the firm grows about twice than average in its high growth phase and about a third as fast in the low growth phase.

We set the project expiration rate δ to 10%, to be consistent with commonly used values for the depreciation rate. We set the interest rate r to 2.5%, which is close to the historical average risk-free rate (2.9%). We choose the parameters governing the dynamics of the shocks x_t and z_t to match the first two moments of the aggregate dividend growth and investment growth. We choose $\phi = 0.07$ to match the relative size of the consumption and investment sectors in the data.

Finally, the parameters of the pricing kernel, $\beta_x = 0.69$ and $\beta_z = 0.35$ are picked to match approximately the average excess returns on the market portfolio and the IMC portfolio. Given our calibration, the model produces a somewhat lower average return on the IMC portfolio -3.9% vs -1.9% in the $1963 - 2008$ sample. However, investment firms tend to be quite a bit smaller than consumption firms, so the size effect may the estimated return of the IMC portfolio upwards. In fact, when excluding the month of January, which is when the size effect is strongest, the average return on the IMC portfolio is -3.5%, whereas it's α with respect to the Small-minus-Big (SMB)) portfolio of Fama and French (1993) is -3.7%.

We simulate the model at a weekly frequency ($dt = 1/52$) and time-aggregate the data to form annual observations. Each simulation sample contains 2,500 firms for 100 years. We use the first half of each simulated sample for burn-in. We simulate 1,000 samples and report averages of parameter estimates and t-statistics across simulations.

6.1 Investment

We first evaluate how well our model accounts for the empirical properties of firms' investment. We estimate equation 13 using the simulated data.

We define firm-level investment during year t as a sum of the investment expenses incurred throughout that year, i.e. $I_{ft} = \sum_{s \in t} x_s z_s K^*_{fs}$, where K^*_{fs} refers to the capital of project acquired by firm f at time s.

We define the book value of the firm as the replacement cost of its capital, $B_{ft} = z_t \, x_t \sum_{j \in \mathcal{J}_{ft}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{ft}$ denotes the set of projects owned by firm f at the end of year t.[8]

In the simulated data, we estimate firm-level β^{imc} using the same methodology as in our empirical results, namely by estimating equation (12) using weekly data every year. In simulated data, the estimated β^{imc} have similar if a bit higher persistence than in actual data, as we show in Table 13.

We normalize all variables to zero mean and unit standard deviation and compute standard errors clustered by firm and time. We report the median coefficient estimates and t-statistics across 1,000 simulations.

Table 14 shows that in simulated data, a single-standard-deviation investment shock leads to an increase in firm-level investment of 0.056 standard deviations. However, as in the actual data, the impact of investment shocks varies in the cross-section of firms from 0.026 to 0.110 between the low- and high-β^{imc} firms respectively. The difference in coefficients between the high- and low-β^{imc} firms drops to 0.029 when we include Tobin's Q and cash flows in the specification. Thus, the magnitude of investment response to z-shocks in the model is very similar to the empirical estimates in Table 4.

In section 5.2 we showed that the sensitivity of a firm's investment rate to the aggregate

[8]As a robustness check, we also perform simulations with the book value of the firm defined as the cumulative historical investment cost of its current portfolio of projects. Our results are essentially the same under the two definitions.

investment rate, the firm's "investment beta", was monotonically related to β^{imc} . Firms with more growth opportunities should exhibit higher sensitivity to aggregate shocks that affect the aggregate investment rate. We verify that the same relationship holds true in the model, by estimating equation (13) in the simulated data using the average investment rate, $\bar{i}_t$, in place of $\tilde{R}^{imc}$. We show the results in Table **??**. A one standard deviation shock in the average investment rate is associated with a 0.1 standard deviation increase in the average firm's investment rate. However, as in the data, this response varies in the cross-section from 0.049 to 0.208.

In the model, Tobin's Q, or the market-to-book ratio, also contains information about growth opportunities. To verify this, we estimate equation (13) in simulated data. We report simulation averages of coefficients and t-statistics in Table 15. In simulated data, the effect of a single-standard-deviation investment shock on firm investment varies from 0.12 for the top Q-quintile to 0.02 for the bottom quintile. The difference in coefficients between the high- and low-Q firms drops to 0.04 when we control directly for Tobin's Q. From this, we conclude that in the model, Tobin's Q is a good proxy for growth opportunities. Of course, this is partly because it is measured with accurately in simulations, whereas in the data it might be contaminated by measurement error.

We conclude that our model is able to replicate the key empirical properties of firms' investment, both qualitatively and quantitatively. Next, we verify that the model also captures the properties asset returns reported in Section 4.

6.2 Stock Returns

As we show in Proposition 1, cross-sectional differences in the relative value of growth opportunities of firms lead to cross-sectional differences in their risk premia. Furthermore, we show in Proposition 2 that unobservable growth opportunities can be measured empirically using the firms' betas with respect to the IMC portfolio returns. We now verify that our model implies empirically realistic behavior of stock returns in relation to the differences in growth opportunities (captured by β^{imc}) across firms.

We sort firms annually into 10 value-weighted portfolios based on the past value of β^{imc}. Both in actual and simulated data, we estimate β^{imc} using weekly returns, and rebalance the portfolios at the end of every year. In each simulation, and for each portfolio p, we estimate average excess returns $E[R_{pt}] - r_f$, return standard deviations $\sigma(R_{pt})$, and regressions

$$R_{pt} - r_f = \alpha_p + \beta_{m,p}(R_{Mt} - r_f) + \epsilon_{pt} \tag{19}$$

and

$$R_{pt} - r_f = \alpha_p + \beta_{m,p}(R_{Mt} - r_f) + \beta_p^{imc}(R_{It} - R_{Ct}) + \epsilon_{p,t}, \tag{20}$$

where R_{It} and R_{Ct} denote returns on the portfolios of investment-good producers and consumption-good producers respectively.

Table 16 compares the properties of returns in historical and simulated data. The top panel replicates the findings of Papanikolaou (2008), who shows that sorting firms into portfolios based on β^{imc} results in i) a declining pattern in average returns; ii) an increasing pattern of return volatility and market betas; and iii) a declining patter of CAPM alphas. The difference in average returns and CAPM alphas between the high and low β^{imc} portfolios is -3.2% and -7.1% respectively. The high-β^{imc} portfolio has a standard deviation of 29.7% and a market beta of 1.6 versus 15.8% and 0.75 respectively for the low-β^{imc} portfolio. The bottom panel of Table 16 contains the corresponding simulation-based estimates. The difference in average returns and CAPM alphas between the high- and low-β^{imc} portfolios is -3.5% and -5.7% respectively. Moreover, the high-β^{imc} portfolio has both a higher standard deviation (20%)and market beta (1.2) than the low-β^{imc} portfolio (14% and 0.8 respectively). Furthermore, in simulated data, the estimates of α_p in (20) are close to zero for the spread portfolio, whereas in actual data the point estimate is -3.00% with a t-statistic of -1.25. Thus, returns on the β^{imc}-sorted portfolios are well described by a two-factor pricing model that includes market returns and returns on the IMC portfolio.

As we discuss in the introduction, many papers use the decomposition of firm's value into asset in place and growth options in an attempt to explain the value premium puzzleIt is therefore useful to assess the ability of our model to replicate the empirical relationship between stock returns and the book-to-market ratio. As we show in Section 3, as long as book to market is a good proxy for $PVGO/V$, our model will exhibit a positive value premium. We now investigate the model's implications quantitatively.

The top panel in Table 17 replicates the empirical findings of Fama and French (1993). The difference in average returns and CAPM alphas between value firms and growth firms is 6.5% and 6.8% respectively. Moreover, with the exception of the extreme value portfolio, the CAPM beta tend to be negatively related to the book-to-market ratio. The bottom panel presents corresponding simulation results. The difference in average returns and CAPM alphas between the two extreme book-to-market portfolios is 4.3% and 6.3% respectively. Moreover, as in the data, the CAPM betas decline across the book-to-market deciles. Thus, our model replicates the failure of the CAPM to price the cross-section of book-to-market portfolios. We also report the estimates of equation (20), where we use both the market and the IMC portfolio as risk factors. The two-factor unconditional pricing model fails to price the cross-section of book-to-market portfolios empirically. The difference in estimated alphas between the extreme book-to-market deciles is 6.0%. The two-factor unconditional specification works rather well in simulated data, where the difference in alphas between the extreme book-to-market deciles is 0.8%.

7 Conclusion

In this paper we propose a novel measure of growth opportunities available to firms. Our measure relies on the idea that firms with abundant growth opportunities benefit more from investment-specific technological improvements than firms with few growth opportunities, and therefore, stock returns of high-growth firms have higher exposure to investment-specific technological shocks. Our empirical tests support this conjecture. Investment rates of high-growth firms, as identified by our measure, are relatively high on average and more sensitive to investment-specific shocks than investment rates of low-growth firms. Our measure of growth opportunities also captures cross-sectional differences in risk premia. Empirically, high-growth firms have lower average returns than low-growth firms. Such return differences are not explained by differences in market risk (CAPM), since, as discussed in Papanikolaou (2008), investment-specific shocks represent a distinct risk factor. The return premium on low-growth firms is distinct from the well-known value premium, and accounts for a fraction of the latter. We use calibration to show that the observed empirical patterns are quantitatively consistent with a stylized structural model of investment.

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Tables

Table 1: Data Definitions

Variable	Source
Investment (I)	Compustat item capx
capital (K)	Compustat item ppent
Book Assets (A)	Compustat item at
Book Debt (D)	Compustat item dltt
Book Preferred Equity (EP)	Compustat item pstkrv
Book Common Equity (EC)	Compustat item ceq
Operating cashflows (CF)	Compustat item dp + item ib
Inventories (INV)	Compustat item invt
Deferred Taxes (T)	Compustat item txdb
Market capitalization (MKCAP)	CRSP December market cap
R&D Expenditures (R&D)	Compustat item xrd
Cash Holdings (CASH)	Compustat item che
Dividends (DIV)	Compustat item dvc +item dvp
Share Repurchases (REP)	Compustat item prstkc
Tobin's Q (Q)	(MKCAP + EP + D-INV)/(EC+EP + D)
Quality Adjusted Price of Investment Goods	Israelsen (2008)
Consumption Deflator	NIPA

Table 2: Summary Statistics: β^{imc} sorted portfolios

β^{imc} sort	I/K	CASH/A	D/A	Tobin's Q	CF/K	k/K	m/M	R&D/S	DIV/CF	β^{mkt}
Low	20.1%	6.6%	16.1%	1.130	21.4%	9.8%	8.8%	1.4%	9.0%	0.75
2	19.1%	6.0%	17.2%	1.077	25.1%	16.0%	15.7%	1.2%	16.6%	0.77
3	19.4%	6.0%	17.5%	1.089	25.5%	15.9%	14.4%	1.2%	18.4%	0.79
4	19.5%	6.1%	17.5%	1.102	26.1%	13.0%	12.6%	1.3%	18.1%	0.85
5	19.9%	6.0%	17.6%	1.104	26.4%	11.9%	10.8%	1.5%	17.4%	0.92
6	20.4%	6.3%	17.7%	1.112	27.0%	10.0%	11.0%	1.5%	16.9%	1.02
7	21.0%	6.6%	17.7%	1.140	25.9%	8.9%	9.2%	1.8%	13.7%	1.06
8	22.3%	7.3%	17.3%	1.193	25.1%	7.0%	7.6%	2.4%	10.3%	1.20
9	23.4%	8.9%	17.2%	1.228	21.9%	4.9%	6.0%	3.7%	7.3%	1.40
High	25.2%	11.4%	14.6%	1.376	17.3%	2.8%	3.9%	6.0%	2.8%	1.61

Table 2 shows summary statistics for 10 portfolios of firms sorted by β^{imc}_{t-1}. β^{imc}_{t} refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. I/K is investment over capital, CASH/A refers to Cash Holdings over Assets, D/A is Debt over Assets, Q refers to Tobin's Q, CF/K refers to cashflows over lagged property, plant and equipment, $k/\ K$ refers to the sum of property plant and equipment (PPE) within each portfolio as a fraction of the total PPE, m/M refers to each portfolio's market capitalization as a fraction of total market capitalization, R&D/A refers to Research and Development over Sales, DIV/CF refers to Dividends plus Share Repurchases over cashflows, and β^{mkt} refers to the portfolio's market beta estimated using monthly returns. When computing DIV/CF, we drop firms with negative cashflows. For the firm-level variables we report the time series averages of portfolio's median characteristic. Sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 3: Portfolio Transition Probabilities: 5 Portfolios sorted on β^{imc}

		Sort(t-1)				
		Lo	2	3	4	Hi
	Lo	30.4%	23.1%	18.8%	15.0%	12.5%
	2	24.2%	25.2%	23.1%	18.7%	11.7%
Sort(t)	3	18.7%	23.3%	22.6%	22.3%	14.7%
	4	15.1%	17.9%	21.9%	24.3%	21.7%
	Hi	11.7%	10.5%	13.6%	19.7%	39.5%

Table 3 plots the transition probabilities across portfolio quintiles. Stocks are sorted into 5 portfolios based on β_{t-1}^{imc}. β_t^{imc} refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. Sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 4: Response of I/K to R^{imc}: firms sorted by β^{imc}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.1190	-0.1039	-0.0909	-0.0815	-0.0440
		(-6.18)	(-5.52)	(-4.34)	(-4.06)	(-2.30)
$D(\beta^{imc})_2$		0.0380	0.0361	0.0361	0.0339	0.0275
		(2.85)	(2.84)	(3.00)	(2.83)	(2.59)
$D(\beta^{imc})_3$		0.0870	0.0818	0.0691	0.0658	0.0411
		(5.26)	(5.28)	(5.02)	(4.99)	(3.15)
$D(\beta^{imc})_4$		0.1794	0.1572	0.1385	0.1247	0.0676
		(8.66)	(7.69)	(8.02)	(7.35)	(4.21)
$D(\beta^{imc})_5$		0.2908	0.2448	0.2113	0.1833	0.0841
		(11.00)	(9.45)	(9.26)	(8.41)	(4.10)
$\tilde{R}^{imc}_{t-1}$	0.0959	0.0532	0.0491	0.0634	0.0592	0.0571
	(4.90)	(4.52)	(4.01)	(4.10)	(3.88)	(4.13)
$D(\beta^{imc})_2 \times \tilde{R}^{imc}_{t-1}$		0.0014	0.0023	-0.0008	0.0004	0.0027
		(0.12)	(0.21)	(-0.07)	(0.04)	(0.28)
$D(\beta^{imc})_3 \times \tilde{R}^{imc}_{t-1}$		0.0256	0.0247	0.0125	0.0132	0.0144
		(1.68)	(1.79)	(0.83)	(0.96)	(1.09)
$D(\beta^{imc})_4 \times \tilde{R}^{imc}_{t-1}$		0.0641	0.0610	0.0411	0.0413	0.0394
		(2.76)	(2.85)	(2.10)	(2.25)	(2.48)
$D(\beta^{imc})_5 \times \tilde{R}^{imc}_{t-1}$		0.1226	0.1169	0.0862	0.0855	0.0885
		(4.88)	(5.59)	(4.38)	(5.13)	(6.20)
Observations	62495	62495	62495	62495	62495	62495
R^2	0.009	0.022	0.077	0.162	0.192	0.438
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 4 shows estimates of

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d \, D(\beta^{imc}_{f,t-1})_d + b_1 \, \tilde{R}^{imc}_{t-1} + \sum_{d=2}^{5} b_d \, D(\beta^{imc}_{f,t-1})_d \times \tilde{R}^{imc}_{t-1} + cX_{f,t-1} + \gamma_f + u_{ft},$$

where $i_t \equiv I_t/K_{t-1}$ is firm investment over the lagged capital stock, on cumulative log returns on the IMC portfolio, $\tilde{R}^{imc}_{t-1} \equiv \sum_{l=1}^{2} R^{imc}_{t-1}$, and a vector of controls X_t which includes lagged values of log Tobin's Q, cashflows over lagged capital, log book equity over book assets, and log capital. $D(\beta^{imc}_{i,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in term of β^{imc}_{t-1}. β^{imc}_t refers to the firm's beta with respect to the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by firm and year. Sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 5: Response of I/K to R^{imc}: portfolios sorted by β^{imc}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)
Constant		-0.588	-0.418	-0.515	-0.411
		(-6.86)	(-4.29)	(-5.98)	(-4.92)
$D(\beta^{imc})_2$		0.157	0.138	0.100	0.165
		(2.23)	(2.20)	(0.99)	(2.05)
$D(\beta^{imc})_3$		0.427	0.285	0.306	0.270
		(5.51)	(3.24)	(3.04)	(2.77)
$D(\beta^{imc})_4$		0.862	0.576	0.733	0.552
		(8.94)	(5.18)	(6.34)	(5.01)
$D(\beta^{imc})_5$		1.493	1.008	1.437	0.968
		(12.35)	(6.05)	(10.70)	(6.10)
$\tilde{R}^{imc}_{t-1}$	0.492	0.271	0.159	0.279	0.188
	(5.34)	(4.43)	(3.13)	(5.01)	(3.86)
$D(\beta^{imc})_2 \times \tilde{R}^{imc}_{t-1}$		-0.0377	0.0239	-0.0718	-0.00913
		(-0.58)	(0.43)	(-1.14)	(-0.14)
$D(\beta^{imc})_3 \times \tilde{R}^{imc}_{t-1}$		0.133	0.122	0.0357	0.0402
		(1.78)	(1.25)	(0.35)	(0.31)
$D(\beta^{imc})_4 \times \tilde{R}^{imc}_{t-1}$		0.316	0.264	0.138	0.131
		(2.91)	(2.40)	(1.12)	(0.98)
$D(\beta^{imc})_5 \times \tilde{R}^{imc}_{t-1}$		0.690	0.625	0.385	0.377
		(4.86)	(5.32)	(2.55)	(3.02)
Observations	205	205	205	205	205
R^2	0.242	0.604	0.654	0.671	0.710
Controls (i_{t-1})	N	N	Y	N	Y
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y

Table 5 shows estimates of

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d\, D(\beta^{imc}_{f,t-1})_d + b_1\, \tilde{R}^{imc}_{t-1} + \sum_{d=2}^{5} b_d\, D(\beta^{imc}_{f,t-1})_d \times \tilde{R}^{imc}_{t-1} + cX_{f,t-1} + \gamma_f + u_{ft},$$

where $i_{ft} \equiv i_{ft}/K_{it-1}$ is firm investment over the lagged capital stock, on cumulative log returns on the IMC portfolio, $\tilde{R}^{imc}_{t-1} \equiv \sum_{l=1}^{2} R^{imc}_{t-1}$, and a vector of controls X_t which includes lagged values of log Tobin's Q, cashflows over lagged capital, log book equity over book assets, and log capital. β^{imc}_t refers to the firm's beta with respect to the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. Every year, we sort firms into 5 portfolios based on $\beta^{imc}_{i,t-1}$. Portfolio-level variables are constructed by averaging across firms within the portfolio. $D(\beta^{imc}_{i,t-1})_d$ is a portfolio indicator variable, denoting the portfolio containing firms in the d-th quintile of β^{imc}_{t-1}. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by year. Sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 6: Response of I/K to z-shocks: firms sorted by β^{imc}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.1190	-0.1038	-0.0908	-0.0813	-0.0442
		(-5.63)	(-5.11)	(-3.84)	(-3.62)	(-2.05)
$D(\beta^{imc})_2$		0.0380	0.0361	0.0353	0.0332	0.0274
		(2.91)	(2.91)	(3.03)	(2.85)	(2.56)
$D(\beta^{imc})_3$		0.0870	0.0818	0.0683	0.0650	0.0406
		(5.23)	(5.26)	(5.11)	(5.09)	(3.15)
$D(\beta^{imc})_4$		0.1794	0.1570	0.1383	0.1244	0.0673
		(8.02)	(7.15)	(7.88)	(7.22)	(4.18)
$D(\beta^{imc})_5$		0.2908	0.2445	0.2124	0.1843	0.0856
		(9.27)	(7.87)	(8.72)	(7.82)	(4.20)
$-\Delta z_{t-1}$	0.0355	0.0070	0.0029	0.0170	0.0131	0.0156
	(2.13)	(0.55)	(0.24)	(0.76)	(0.64)	(0.60)
$D(\beta^{imc})_2 \times (-\Delta z_{t-1})$		0.0169	0.0155	0.0173	0.0162	0.0099
		(1.66)	(1.78)	(2.22)	(2.32)	(0.99)
$D(\beta^{imc})_3 \times (-\Delta z_{t-1})$		0.0249	0.0241	0.0240	0.0236	0.0241
		(2.40)	(2.42)	(2.85)	(2.89)	(2.46)
$D(\beta^{imc})_4 \times (-\Delta z_{t-1})$		0.0382	0.0360	0.0354	0.0340	0.0361
		(2.66)	(2.57)	(3.74)	(3.59)	(2.96)
$D(\beta^{imc})_5 \times (-\Delta z_{t-1})$		0.0623	0.0541	0.0432	0.0386	0.0367
		(3.13)	(2.85)	(2.61)	(2.33)	(2.24)
Observations	62495	62495	62495	62495	62495	62495
R^2	0.001	0.013	0.068	0.155	0.184	0.432
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 6 shows estimates of

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d \, D(\beta^{imc}_{i,t-1})_d + b_1 \, (-\Delta z_{t-1}) + \sum_{d=2}^{5} b_d \, D(\beta^{imc}_{i,t-1})_d \times (-\Delta z_{t-1}) + cX_{it-1} + \gamma_i + u_{it},$$

where $i_t \equiv I_t/K_{t-1}$ is firm investment over the lagged capital stock, on the innovation in the quality-adjusted price of new equipment Δz_t, and a vector of controls X_t which includes lagged values of log Tobin's Q, cashflows over lagged capital, log book equity over book assets, and log capital. β^{imc}_t refers to the firm's beta with respect to the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. $D(\beta^{imc}_{i,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in terms of β^{imc}_{t-1}. The innovation Δz_t is the first difference of the detrended quality-adjusted price of investment goods divided by the consumption deflator from Cummins and Violante (2002) and extended by @@@@@RI. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by firm and year. Sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 7: Firm investment and aggregate investment shocks: firms sorted by β^{imc}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.1190	-0.1040	-0.0902	-0.0811	-0.0436
		(-6.47)	(-5.71)	(-4.81)	(-4.41)	(-2.44)
$D(\beta^{imc})_2$		0.0380	0.0361	0.0378	0.0356	0.0289
		(3.29)	(3.24)	(3.51)	(3.26)	(2.85)
$D(\beta^{imc})_3$		0.0870	0.0819	0.0702	0.0669	0.0415
		(5.50)	(5.53)	(5.38)	(5.35)	(3.27)
$D(\beta^{imc})_4$		0.1795	0.1574	0.1378	0.1244	0.0673
		(9.37)	(8.33)	(9.22)	(8.48)	(4.65)
$D(\beta^{imc})_5$		0.2908	0.2452	0.2057	0.1788	0.0803
		(11.59)	(9.88)	(9.99)	(9.05)	(4.13)
$\bar{i}^e_t$	0.0667	0.0336	0.0295	0.0706	0.0642	0.0798
	(5.42)	(2.26)	(2.04)	(5.09)	(4.75)	(5.14)
$D(\beta^{imc})_2 \times \bar{i}^e_t$		0.0367	0.0329	0.0305	0.0283	0.0185
		(2.93)	(2.76)	(2.53)	(2.38)	(1.76)
$D(\beta^{imc})_3 \times \bar{i}^e_t$		0.0288	0.0273	0.0209	0.0204	0.0157
		(2.61)	(2.57)	(1.78)	(1.81)	(1.30)
$D(\beta^{imc})_4 \times \bar{i}^e_t$		0.0487	0.0464	0.0418	0.0408	0.0351
		(2.94)	(2.87)	(3.16)	(3.12)	(2.98)
$D(\beta^{imc})_5 \times \bar{i}^e_t$		0.0512	0.0418	0.0417	0.0362	0.0245
		(3.02)	(2.68)	(2.65)	(2.42)	(1.65)
Observations	62495	62495	62495	62495	62495	62495
R^2	0.014	0.027	0.081	0.172	0.199	0.446
Industry/Firm FE	N	N	N	I	I	F
Controls ($\tilde{R}^{imc}$)	Y	Y	Y	Y	Y	Y
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 7 shows estimates of

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d\, D(\beta^{imc}_{f,t-1})_d + b_1\, \bar{i}^\epsilon_t + \sum_{d=2}^{5} b_d\, D(\beta^{imc}_{f,t-1})_d \times \bar{i}^\epsilon_t + cX_{f,t-1} + \gamma_f + u_{ft}.$$

$i_t \equiv I_t/K_{t-1}$ is firm investment over the lagged capital stock. $\bar{i}^\epsilon_t$ is the residual in the regression of aggregate average investment on lagged IMC returns, defined in (17). X_t is a vector of controls which includes lagged values of log Tobin's Q, cashflows over lagged capital, log book equity over book assets, and log capital. $D(\beta^{imc}_{i,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in term of β^{imc}_{t-1}. β^{imc}_t refers to the firm's beta with respect to the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by firm and year. Sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 8: Response of I/K to Credit Spreads: firms sorted by β^{imc}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.1190	-0.1038	-0.0911	-0.0816	-0.0911
		(-6.27)	(-5.74)	(-4.07)	(-3.88)	(-4.07)
$D(\beta^{imc})_2$		0.0380	0.0360	0.0356	0.0334	0.0356
		(2.85)	(2.85)	(3.00)	(2.83)	(3.00)
$D(\beta^{imc})_3$		0.0870	0.0818	0.0687	0.0654	0.0687
		(5.15)	(5.15)	(5.02)	(4.98)	(5.02)
$D(\beta^{imc})_4$		0.1794	0.1570	0.1387	0.1248	0.1387
		(7.85)	(6.92)	(7.64)	(6.91)	(7.64)
$D(\beta^{imc})_5$		0.2908	0.2444	0.2128	0.1844	0.2128
		(9.34)	(7.78)	(8.82)	(7.77)	(8.82)
$-\Delta s_{t-1}$	0.0781	0.0565	0.0540	0.0476	0.0464	0.0476
	(3.56)	(3.27)	(3.33)	(1.96)	(2.03)	(1.96)
$D(\beta^{imc})_2 \times (-\Delta s_{t-1})$		0.0046	0.0052	0.0121	0.0120	0.0121
		(0.43)	(0.51)	(1.18)	(1.20)	(1.18)
$D(\beta^{imc})_3 \times (-\Delta s_{t-1})$		0.0145	0.0157	0.0163	0.0170	0.0163
		(0.99)	(1.15)	(1.52)	(1.67)	(1.52)
$D(\beta^{imc})_4 \times (-\Delta s_{t-1})$		0.0247	0.0241	0.0192	0.0194	0.0192
		(0.94)	(0.95)	(0.97)	(0.98)	(0.97)
$D(\beta^{imc})_5 \times (-\Delta s_{t-1})$		0.0643	0.0612	0.0503	0.0491	0.0503
		(2.12)	(2.27)	(2.33)	(2.48)	(2.33)
Observations	62495	62495	62495	62495	62495	62495
R^2	0.006	0.018	0.073	0.158	0.188	0.158
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 8 shows estimates of the regression of the ratio of firm investment to its capital stock, $i_t \equiv I_t/K_{t-1}$, on the innovation in the spread between Baa and Aaa bonds, ξ_t, and a vector of controls X_{it} which includes lagged values of log Tobin's Q, cashflows over lagged capital, log Book Equity over Book Assets, and log capital:

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d \, D(\beta^{imc}_{f,t-1})_d + b_1 \, (-\Delta s_{t-1}) + \sum_{d=2}^{5} b_d \, D(\beta^{imc}_{f,t-1})_d \times (-\Delta s_{t-1}) + cX_{f,t-1} + \gamma_f + u_{ft},$$

The innovation Δs_t is computed as the innovation of an AR(1) model on the difference between Baa and Treasury bond yields. The data on bond yields are from the St. Louis Federal Reserve. β^{imc}_t refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. $D(\beta^{imc}_{f,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in terms of β^{imc}_{t-1}. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by firm and year. Sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 9: Response of I/K to R^{imc}: firms sorted by Tobin's Q

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.2491	-0.2217	-0.2588	-0.2371	-0.3211
		(-12.01)	(-11.26)	(-12.02)	(-11.47)	(-13.85)
$D(Q)_2$		0.0202	0.0107	0.0666	0.0560	0.1320
		(1.54)	(0.85)	(4.66)	(4.10)	(7.92)
$D(Q)_3$		0.1495	0.1295	0.2036	0.1847	0.2841
		(8.42)	(8.22)	(11.73)	(11.53)	(13.08)
$D(Q)_4$		0.3607	0.3246	0.3675	0.3394	0.4430
		(16.88)	(14.41)	(18.18)	(15.89)	(15.59)
$D(Q)_5$		0.7158	0.6448	0.6573	0.6064	0.7477
		(25.81)	(21.72)	(25.93)	(21.91)	(22.72)
$\tilde{R}^{imc}_{t-1}$	0.0959	0.0688	0.0635	0.0621	0.0584	0.0611
	(4.90)	(3.16)	(3.01)	(2.86)	(2.78)	(3.10)
$D(Q)_2 \times \tilde{R}^{imc}_{t-1}$		0.0147	0.0139	0.0162	0.0153	0.0239
		(0.99)	(0.99)	(0.95)	(0.93)	(1.70)
$D(Q)_3 \times \tilde{R}^{imc}_{t-1}$		0.0364	0.0346	0.0338	0.0325	0.0321
		(3.69)	(3.78)	(3.33)	(3.37)	(2.84)
$D(Q)_4 \times \tilde{R}^{imc}_{t-1}$		0.0280	0.0302	0.0338	0.0351	0.0267
		(1.36)	(1.59)	(1.64)	(1.81)	(1.53)
$D(Q)_5 \times \tilde{R}^{imc}_{t-1}$		0.0563	0.0567	0.0677	0.0670	0.0536
		(2.02)	(2.05)	(2.30)	(2.30)	(2.30)
Observations	62495	62495	62495	62495	62495	62495
R^2	0.009	0.080	0.125	0.174	0.202	0.447
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 9 shows estimates of

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d \, D(Q_{f,t-1})_d + b_1 \, \tilde{R}^{imc}_{t-1} + \sum_{d=2}^{5} b_d \, D(Q_{f,t-1})_d \times \tilde{R}^{imc}_{t-1} + cX_{f,t-1} + \gamma_f + u_{ft},$$

where $i_t \equiv I_t/K_{t-1}$ is firm investment over lagged capital, on cumulative log returns on the IMC portfolio, $\tilde{R}^{imc}_{t-1} \equiv \sum_{l=1}^{2} R^{imc}_{t-1}$, and a vector of controls X_t which includes lagged values of log Tobin's Q, cashflows over lagged capital, log book equity over book assets, and log capital. $D(Q_{i,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in terms of Tobin's Q. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by firm and year. Sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 10: **Response of I/K to Credit Spreads: firms sorted by Tobin's Q**

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.2490	-0.2215	-0.2592	-0.2373	-0.2592
		(-12.41)	(-12.28)	(-12.67)	(-12.64)	(-12.67)
$D(Q)_2$		0.0202	0.0106	0.0666	0.0559	0.0666
		(1.52)	(0.84)	(4.60)	(4.03)	(4.60)
$D(Q)_3$		0.1495	0.1294	0.2037	0.1846	0.2037
		(8.10)	(7.76)	(11.23)	(10.90)	(11.23)
$D(Q)_4$		0.3607	0.3243	0.3681	0.3398	0.3681
		(16.69)	(14.07)	(17.85)	(15.46)	(17.85)
$D(Q)_5$		0.7158	0.6443	0.6584	0.6070	0.6584
		(24.78)	(20.65)	(23.84)	(20.26)	(23.84)
$-\Delta s_{t-1}$	0.0781	0.0763	0.0732	0.0687	0.0670	0.0687
	(3.56)	(3.75)	(3.90)	(3.54)	(3.69)	(3.54)
$D(Q)_2 \times (-\Delta s_{t-1})$		0.0038	0.0047	0.0029	0.0035	0.0029
		(0.37)	(0.46)	(0.26)	(0.31)	(0.26)
$D(Q)_3 \times (-\Delta s_{t-1})$		0.0162	0.0162	0.0123	0.0123	0.0123
		(1.24)	(1.33)	(1.01)	(1.06)	(1.01)
$D(Q)_4 \times (-\Delta s_{t-1})$		0.0069	0.0065	0.0071	0.0065	0.0071
		(0.41)	(0.41)	(0.52)	(0.50)	(0.52)
$D(Q)_5 \times (-\Delta s_{t-1})$		-0.0180	-0.0160	-0.0169	-0.0156	-0.0169
		(-0.65)	(-0.60)	(-0.63)	(-0.59)	(-0.63)
Observations	62495	62495	62495	62495	62495	62495
R^2	0.006	0.077	0.123	0.170	0.199	0.170
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 10 shows estimates of the regression of the ratio of firm investment to its lagged capital stock, $i_t \equiv I_t/K_{t-1}$, on the innovation in the spread between Baa and Treasury bonds, Δs_t, and a vector of controls X_{it} which includes lagged values of log Tobin's Q, cashflows over lagged capital, log book equity over book assets, and log capital:

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d \, D(Q_{f,t-1})_d + b_1 \, (-\xi_{t-1}) + \sum_{d=2}^{5} b_d \, D(Q_{f,t-1})_d \times (-\xi_{t-1}) + cX_{f,t-1} + \gamma_f + u_{ft},$$

The innovation ξ_t is computed as the innovation of an AR(1) model on the difference between Baa and Treasury bond yields. The data on bond yields are from the St. Louis Federal Reserve. β_t^{imc} refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. $D(Q_{f,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in terms of Tobin's Q. Industries are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by firm and year. Sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 11: Response of I/K to R^{imc}: firms sorted by β^{mkt}

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)	(6)
Constant		-0.1283	-0.1168	-0.1307	-0.1211	-0.0958
		(-6.77)	(-6.20)	(-5.69)	(-5.43)	(-5.53)
$D(\beta_{mkt})_2$		0.0499	0.0495	0.0593	0.0572	0.0431
		(3.49)	(3.59)	(4.67)	(4.51)	(3.85)
$D(\beta_{mkt})_3$		0.1175	0.1110	0.1197	0.1133	0.0655
		(6.60)	(6.61)	(7.63)	(7.36)	(4.94)
$D(\beta_{mkt})_4$		0.1885	0.1734	0.1814	0.1696	0.0965
		(9.52)	(8.63)	(10.48)	(9.40)	(7.26)
$D(\beta_{mkt})_5$		0.3485	0.3060	0.2951	0.2671	0.1543
		(12.95)	(10.58)	(13.95)	(11.62)	(6.86)
$\tilde{R}^{imc}_{t-1}$	0.0959	0.0841	0.0800	0.0873	0.0836	0.0731
	(4.90)	(6.09)	(6.10)	(4.82)	(4.94)	(5.24)
$D(\beta_{mkt})_2 \times \tilde{R}^{imc}_{t-1}$		-0.0122	-0.0138	-0.0118	-0.0128	-0.0028
		(-0.95)	(-1.09)	(-1.19)	(-1.30)	(-0.30)
$D(\beta_{mkt})_3 \times \tilde{R}^{imc}_{t-1}$		0.0067	0.0042	0.0013	0.0001	0.0134
		(0.41)	(0.28)	(0.10)	(0.01)	(1.03)
$D(\beta_{mkt})_4 \times \tilde{R}^{imc}_{t-1}$		0.0162	0.0125	0.0134	0.0115	0.0229
		(0.75)	(0.59)	(0.63)	(0.54)	(1.11)
$D(\beta_{mkt})_5 \times \tilde{R}^{imc}_{t-1}$		0.0676	0.0665	0.0451	0.0469	0.0514
		(2.87)	(3.05)	(2.29)	(2.45)	(2.35)
Observations	62495	62495	62495	62495	62495	62495
R^2	0.009	0.025	0.077	0.166	0.193	0.434
Industry/Firm FE	N	N	N	I	I	F
Controls (i_{t-1})	N	N	Y	N	Y	N
Controls ($Q_{t-1}, CF_{t-1}, K_{t-1}, E_{t-1}/A_{t-1}$)	N	N	N	Y	Y	Y

Table 11 shows estimates of

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d \, D(\beta^{mkt}_{f,t-1})_d + b_1 \, \tilde{R}^{imc}_{t-1} + \sum_{d=2}^{5} b_d \, D(\beta^{mkt}_{f,t-1})_d \times \tilde{R}^{imc}_{t-1} + cX_{f,t-1} + \gamma_i + u_{ft},$$

where $i_t \equiv I_t/K_{t-1}$ is firm investment over the lagged capital stock, on cumulative log returns on the IMC portfolio, $\tilde{R}^{imc}_{t-1} \equiv \sum_{l=2}^{2} R^{imc}_{t-1}$, and a vector of controls X_t which includes lagged values of log Tobin's Q, cashflows over lagged capital, log book equity over book assets, and log capital. $D(\beta^{mkt}_{f,t-1})_d$ is a dummy variable which takes the value of 1 if the firm falls in the d-th quintile in term of β^{mkt}_{t-1}, where firms are sorted within industry, following the Fama and French (1997) 30-industry classifications. β^{mkt}_t refers to the firm's beta with respect to the market portfolio in year t, estimated using non-overlapping weekly returns within year t. Industry fixed effects are defined at the 2-digit SIC code level. All variables have been standardized to zero mean and unit standard deviation. We report t statistics in parenthesis using standard errors clustered by firm and year. Sample period is 1965-2007 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 12: Parameter values and Calibration

parameter	r	μ_x	σ_x	β_x	μ_z	σ_z	β_z	ϕ	α	δ
value	0.025	0.010	0.13	0.69	-0.005	0.036	0.40	0.07	0.85	0.10
parameter	θ_ϵ	σ_e	θ_u	σ_u	μ_λ	σ_λ	μ_H	μ_L	λ_H	λ_L
value	0.35	0.20	0.50	1.50	2.00	2.00	0.075	0.16	2.35	0.35

Moment	Data	Model
$\mu(D_t)$	0.025 (D) 0.038 (P)	0.017
$\sigma(D_t)$	0.118 (D) 0.384 (P)	0.150
$\mu(I_t)$	0.047	0.035
$\sigma(I_t)$	0.157	0.243
$E(R_M) - r_f$	0.059	0.056
$\sigma(R_M)$	0.161	0.165
$E(R_{IMC})$	-0.019	-0.039
$\sigma(R_{IMC})$	0.112	0.115
$\rho(R_{IMC}, R_M - r_f)$	0.267	0.522
Market Cap of I rel to C	0.149	0.140
Investment over Capital (mean)	0.202	0.128
Investment over Capital (IQR)	0.187	0.168
Cashflows over Capital (mean)	0.284	0.248
Cashflows over Capital (IQR)	0.359	0.223
Market-to-Book (median)	1.569 (E) 1.287 (A)	1.988
Market-to-Book (IQR)	1.437 (E) 0.967 (A)	1.564
$\hat{\beta}^{imc}$ (median)	0.683	0.731
$\hat{\beta}^{imc}$ (IQR)	0.990	0.639

The top panel of Table 12 shows the parameters in our calibration. The bottom panel shows sample moments. We report mean and standard deviation of dividend growth [$\mu(D_t)$, $\sigma(D_t)$], mean and standard deviation of investment growth [$\mu(I_t), \sigma(I_t)$], mean and standard deviation of excess returns on the market portfolio [$E(R_M) - r_f$, $\sigma(R_M)$], mean and standard deviation of the investment minus consumption portfolio [$E(R^{imc})$, $\sigma(R^{imc})$], and the ratio of the market capitalization of the investment sector relative to the consumption sector. We report separate moments for dividends (D) and net payout (P). Investment is real private nonresidential investment in equipment and software. We report time series averages of the mean and inter-quintile range (IQR) of the investment rate and cashflows over capital, and the median and inter-quintile range of the market to book ratio. For market to book, we report moments separately for equity (E) and assets (A). Stock return moments are estimated over the sample 1963-2008. The moments of investment growth are estimated over the sample 1927-2008. Moments of firm-specific variables are estimated using Compustat data over the 1963-2007 period. Moments of dividend growth are from the long sample in Campbell and Cochrane (1999). Moments of net payout are from Larrain and Yogo (2008).

Table 13: Model: Portfolio Transition Probabilities: 5 Portfolios sorted on β^{imc}

		Sort(t-1)				
		Lo	2	3	4	Hi
	Lo	49.1%	28.3%	14.4%	6.2%	1.9%
	2	27.6%	32.6%	24.4%	12.0%	3.4%
Sort(t)	3	14.0%	23.8%	30.7%	23.7%	8.0%
	4	6.4%	11.4%	22.8%	36.6%	22.9%
	Hi	2.7%	3.7%	7.7%	21.4%	63.6%

Table 3 plots the estimated transition probabilities across β^{imc} portfolio quintiles in the model. We simulate 2,500 firms for 50 years and repeat the procedure 1,000 times. We report median estimates of the transition probabilities across simulations. Data are simulated at weekly frequency ($dt = 1/52$) and then aggregated to form annual values. β_t^{imc} refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t.

Table 14: **Model: Response to R^{imc}: sorted by β^{imc}**

Dependent variable i_t	(1)	(2)	(3)	(4)	(5)
Constant		-0.122	-0.119	-0.063	0.072
		(-18.34)	(-19.01)	(-5.35)	(4.14)
$D(\beta^{imc})_2$		0.026	0.024	0.015	-0.032
		(7.41)	(7.23)	(3.84)	(-4.63)
$D(\beta^{imc})_3$		0.058	0.055	0.019	-0.071
		(12.31)	(11.84)	(3.16)	(-5.65)
$D(\beta^{imc})_4$		0.113	0.109	0.036	-0.125
		(15.85)	(15.12)	(3.63)	(-5.87)
$D(\beta^{imc})_H$		0.384	0.375	0.221	-0.152
		(15.65)	(14.86)	(11.53)	(-4.18)
$\tilde{R}^{imc}_{t-1}$	0.053	0.026	0.025	0.017	-0.022
	(4.40)	(4.07)	(4.05)	(2.25)	(-3.02)
$D(\beta^{imc})_2 \times \tilde{R}^{imc}_{t-1}$		0.006	0.006	0.004	-0.005
		(2.21)	(2.18)	(1.40)	(-1.42)
$D(\beta^{imc})_3 \times \tilde{R}^{imc}_{t-1}$		0.014	0.014	0.011	-0.008
		(3.16)	(3.13)	(2.74)	(-1.48)
$D(\beta^{imc})_4 \times \tilde{R}^{imc}_{t-1}$		0.026	0.025	0.023	-0.008
		(3.86)	(3.81)	(3.55)	(-1.13)
$D(\beta^{imc})_H \times \tilde{R}^{imc}_{t-1}$		0.084	0.083	0.082	0.029
		(3.74)	(3.71)	(3.73)	(1.91)
R^2	0.003	0.025	0.026	0.037	0.074
Controls (i_{t-1})	N	N	Y	Y	Y
Controls (CF_{t-1}, K_{t-1})	N	N	N	Y	Y
Controls (Q_{t-1})	N	N	N	N	Y

Table 14 shows median coefficients and t-statistics across 1,000 simulations. We estimate a regression of the ratio of the firm investment to its book value, $i_t \equiv I_{ft}/B_{f,t-1}$, on cumulative log returns on the IMC portfolio, $\tilde{R}^{imc}_{t-1} \equiv \sum_{l=2}^{2} R^{imc}_{t-1}$ and a vector of controls X_t, which includes lagged values of log Tobin's Q, cash flows over lagged capital, and log capital:

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d\, D(\beta^{imc}_{f,t-1})_d + b_1\, \tilde{R}^{imc}_{t-1} + \sum_{d=2}^{5} b_d\, D(\beta^{imc}_{f,t-1})_d \times \tilde{R}^{imc}_{t-1} + cX_{f,t-1} + u_{ft},$$

Data are simulated at weekly frequency ($dt = 1/52$) and then aggregated to form annual values. β^{imc}_t refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. $D(\beta^{imc}_{f,t-1})_d$ is a dummy variable which takes the value of 1 if the firm f falls in the d-th quintile in terms of β^{imc}_{t-1}. Investment by firm is computed as the sum of the market value of new investment, i.e. $I_{ft} = \sum_{s\in t} x_s z_s K^*_{fs}$, where K^*_{fs} is the capital of project acquired by firm f at time s. Book Value is computed as the replacement cost of capital, $B_{ft} = z_t\, x_t \sum_{j\in \mathcal{J}_{ft}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{ft}$ denotes the set of projects owned by firm f at the end of year t.All variables have been standardized to zero mean and unit standard deviation. We report averages across simulations of coefficients and t statistics (in parenthesis). Standard errors are clustered by firm and time. Each simulation sample contains 2,500 firms for 50 years. We simulate 1,000 samples.

Table 15: Model: Response to R^{imc}: sorted by Tobin's Q

	(1)	(2)	(3)	(4)	(5)
Constant		-0.162	-0.159	-0.096	0.203
		(-28.15)	(-30.37)	(-7.19)	(5.99)
$D(Q)_2$		0.052	0.049	0.026	-0.122
		(16.97)	(16.03)	(5.37)	(-7.35)
$D(Q)_3$		0.096	0.091	0.041	-0.206
		(21.61)	(20.46)	(5.02)	(-7.09)
$D(Q)_4$		0.154	0.149	0.066	-0.309
		(24.07)	(22.56)	(-0.01)	(-6.85)
$D(Q)_H$		0.483	0.477	0.321	-0.408
		(17.64)	(17.01)	(14.04)	(-5.33)
$\tilde{R}^{imc}_{t-1}$	0.053	0.020	0.018	0.013	-0.047
	(4.40)	(3.63)	(3.64)	(2.19)	(-3.60)
$D(Q)_2 \times \tilde{R}^{imc}_{t-1}$		0.011	0.011	0.009	-0.000
		(3.75)	(3.75)	(3.14)	(-0.00)
$D(Q)_3 \times \tilde{R}^{imc}_{t-1}$		0.018	0.018	0.016	-0.002
		(4.36)	(4.36)	(3.77)	(-1.27)
$D(Q)_4 \times \tilde{R}^{imc}_{t-1}$		0.028	0.029	0.026	-0.004
		(4.76)	(4.76)	(4.34)	(1.31)
$D(Q)_H \times \tilde{R}^{imc}_{t-1}$		0.100	0.102	0.100	0.041
		(4.00)	(3.98)	(3.95)	(2.35)
R^2	0.003	0.035	0.037	0.041	0.079
Controls (i_{t-1})	N	N	Y	Y	Y
Controls (CF_{t-1}, K_{t-1})	N	N	N	Y	Y
Controls (Q_{t-1})	N	N	N	N	Y

Table 15 shows median coefficients and t-statistics across 1,000 simulations. We estimate a regression of the ratio of the firm investment to its book value, $i_t \equiv I_{ft}/B_{f,t-1}$, on cumulative log returns on the IMC portfolio, $\tilde{R}^{imc}_{t-1} \equiv \sum_{l=2}^{2} R^{imc}_{t-1}$ and a vector of controls X_t which includes cash flows over lagged capital, and log capital:

$$i_{ft} = a_1 + \sum_{d=2}^{5} a_d \, D(Q_{f,t-1})_d + b_1 \, \tilde{R}^{imc}_{t-1} + \sum_{d=2}^{5} b_d \, D(Q_{f,t-1})_d \times \tilde{R}^{imc}_{t-1} + cX_{f,t-1} + u_{ft},$$

Data are simulated at weekly frequency ($dt = 1/52$) and then aggregated to form annual values. Tobin's Q is computed as the ratio of the market value of the firm divided by Book Value. $D(Q_{f,t-1})_d$ is a dummy variable which takes the value of 1 if the firm f falls in the d-th quintile in terms of Tobin's Q. Investment by firm is computed as the sum of the market value of new investment, i.e. $I_{ft} = \sum_{s \in t} x_s z_s K_{fs}$, where K_{fs} denotes the capital of project acquired by firm f at time s. Book value is computed as the replacement cost of capital, $B_{ft} = z_t \, x_t \, \sum_{j \in \mathcal{J}_{ft}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{ft}$ denotes the set of projects owned by firm f at the end of year t. All variables have been standardized to zero mean and unit standard deviation. We report averages across simulations of coefficients and t statistics (in parenthesis). Standard errors are robust to heteroscedasticity and clustered at the firm level. Each simulation sample contains 2,500 firms for 50 years.

Table 16: 10 portfolios sorted on IMC beta

	Data										
β^{imc}	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
$E(R) - r_f$ (%)	5.62	5.51	6.36	6.72	5.43	5.15	4.84	4.83	4.15	2.42	-3.20
	(2.31)	(2.51)	(2.89)	(2.96)	(2.26)	(1.98)	(1.76)	(1.52)	(1.10)	(0.53)	(-0.80)
σ (%)	15.78	14.23	14.27	14.74	15.61	16.86	17.80	20.56	24.36	29.70	25.88
β_{MKT}	0.75	0.77	0.79	0.85	0.92	1.02	1.06	1.20	1.40	1.61	0.86
	(17.74)	(27.77)	(29.86)	(36.37)	(41.10)	(59.01)	(54.44)	(50.65)	(34.57)	(27.40)	(9.81)
α(%)	2.22	2.01	2.78	2.88	1.26	0.55	0.04	-0.61	-2.19	-4.88	-7.10
	(1.40)	(1.74)	(2.56)	(2.96)	(1.48)	(0.68)	(0.04)	(-0.53)	(-1.37)	(-2.10)	(-2.13)
R^2(%)	56.75	73.75	77.31	83.30	87.62	91.44	89.05	85.77	82.99	74.00	27.87
β_{MKT}	0.86	0.86	0.88	0.92	0.99	1.04	1.06	1.14	1.27	1.39	0.53
	(21.17)	(34.96)	(42.91)	(54.68)	(56.58)	(58.23)	(56.46)	(62.21)	(44.73)	(36.52)	(8.28)
β_{IMC}	-0.48	-0.39	-0.41	-0.33	-0.29	-0.08	-0.01	0.28	0.59	1.00	1.48
	(-9.71)	(-10.67)	(-14.66)	(-7.16)	(-11.05)	(-2.66)	(-0.17)	(4.42)	(10.86)	(10.99)	(17.40)
α(%)	0.88	0.92	1.63	1.97	0.45	0.31	0.02	0.16	-0.55	-2.11	-2.99
	(0.61)	(0.97)	(2.03)	(2.56)	(0.64)	(0.40)	(0.02)	(0.13)	(-0.45)	(-1.26)	(-1.25)
R^2(%)	67.56	82.62	87.02	89.03	91.65	91.73	89.05	87.87	89.82	87.06	65.73

	Model										
β^{imc}	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
$E(R) - r_f$(%)	7.50	7.29	7.03	6.78	6.50	6.20	5.83	5.41	4.84	3.99	-3.51
	(3.72)	(3.50)	(3.30)	(3.10)	(2.88)	(2.67)	(2.42)	(2.15)	(1.81)	(1.34)	(-2.50)
σ(%)	14.35	14.80	15.16	15.54	15.98	16.47	17.04	17.75	18.71	20.38	10.51
β_{MKT}	0.82	0.87	0.89	0.92	0.95	0.98	1.02	1.06	1.11	1.19	0.36
	(22.83)	(29.91)	(37.78)	(48.46)	(66.69)	(91.27)	(102.76)	(75.40)	(48.45)	(30.98)	(5.02)
α(%)	2.70	2.24	1.81	1.38	0.92	0.44	-0.13	-0.79	-1.65	-2.98	-5.67
	(4.63)	(4.80)	(4.73)	(4.48)	(3.90)	(2.40)	(-0.82)	(-3.33)	(-4.39)	(-4.70)	(-4.85)
R^2(%)	91.28	94.71	96.57	97.81	98.70	99.21	99.30	98.92	97.64	94.51	34.79
β_{MKT}	0.96	0.98	0.99	0.99	1.00	1.01	1.01	1.02	1.02	1.03	0.06
	(52.55)	(68.83)	(80.04)	(86.59)	(93.31)	(92.06)	(89.09)	(87.59)	(84.15)	(83.47)	(2.43)
β_{IMC}	-0.33	-0.27	-0.22	-0.17	-0.11	-0.05	0.02	0.10	0.21	0.38	0.71
	(-11.57)	(-12.57)	(-12.16)	(-10.76)	(-8.04)	(-3.82)	(1.04)	(6.03)	(11.85)	(21.23)	(18.19)
α(%)	0.29	0.27	0.21	0.14	0.08	0.03	-0.04	-0.08	-0.11	-0.07	-0.36
	(0.92)	(1.07)	(0.92)	(0.64)	(0.40)	(0.10)	(-0.21)	(-0.36)	(-0.45)	(-0.23)	(-0.79)
R^2(%)	97.82	98.80	99.15	99.31	99.37	99.38	99.35	99.32	99.22	99.20	91.86

The top panel of Table 16 reports asset-pricing tests on 10 portfolios sorted on β^{imc}_{t-1}. β^{imc}_t refers to the firm's beta with the investment minus consumption portfolio (IMC) in year t, estimated using non-overlapping weekly returns within year t. The construction of the IMC portfolio is detailed in Papanikolaou (2008). Sample period is 1965-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949). Standard errors are computed using Newey-West with 1 lag, to adjust for autocorrelation in returns. t-statistics are computed in parenthesis. We report annualized estimates of mean returns and alphas by multiplying the monthly estimates by 12. The bottom panel reports the corresponding estimates for simulated data. We report medians across 1,000 simulations of coefficients and t-statistics. Each simulation sample contains 2,500 firms and has a length of 50 years. Returns of the market portfolio are computed as average return of the investment and consumption sectors, weighted by their market capitalization at time.

Table 17: 10 portfolios sorted on BE/ME

	Data										
BE/ME	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
$E(R)-r_f$ (%)	2.99	4.81	5.72	5.73	5.05	5.73	6.75	7.36	8.84	9.43	6.45
	(1.04)	(1.83)	(2.22)	(2.22)	(2.09)	(2.36)	(2.83)	(3.07)	(3.46)	(3.17)	(2.65)
σ (%)	18.55	17.01	16.67	16.77	15.67	15.74	15.43	15.54	16.54	19.30	15.77
β_{MKT}	1.09	1.03	1.00	0.98	0.90	0.91	0.85	0.85	0.90	1.00	-0.09
	(44.44)	(55.69)	(41.65)	(36.21)	(33.18)	(34.54)	(26.59)	(24.06)	(24.47)	(20.27)	(-1.30)
α(%)	-1.93	0.16	1.21	1.29	1.00	1.60	2.90	3.51	4.76	4.91	6.84
	(-1.72)	(0.20)	(1.39)	(1.27)	(0.92)	(1.60)	(2.41)	(2.79)	(3.61)	(2.67)	(2.60)
R^2(%)	86.21	91.59	89.90	86.22	82.12	84.15	76.06	75.32	74.61	67.37	0.77
β_{MKT}	1.01	1.04	1.05	1.07	1.00	1.00	0.97	0.98	1.00	1.06	0.05
	(36.88)	(55.57)	(40.63)	(42.44)	(38.21)	(41.50)	(32.57)	(30.21)	(29.52)	(20.57)	(0.70)
β_{IMC}	0.18	-0.03	-0.13	-0.22	-0.25	-0.21	-0.29	-0.30	-0.24	-0.15	-0.32
	(6.49)	(-1.17)	(-3.96)	(-5.69)	(-8.45)	(-7.59)	(-7.95)	(-7.96)	(-6.09)	(-2.71)	(-4.96)
α(%)	-1.49	0.08	0.88	0.75	0.37	1.09	2.18	2.77	4.17	4.55	6.04
	(-1.40)	(0.10)	(1.03)	(0.83)	(0.39)	(1.22)	(2.11)	(2.55)	(3.41)	(2.48)	(2.36)
R^2(%)	87.59	91.65	90.85	88.72	86.03	86.69	81.27	80.78	77.68	68.20	7.05
	Model										
BE/ME	Lo	2	3	4	5	6	7	8	9	Hi	Hi - Lo
$E(R)-r_f$(%)	3.62	4.65	5.26	5.72	6.12	6.46	6.78	7.06	7.40	7.90	4.28
	(1.21)	(1.76)	(2.12)	(2.40)	(2.66)	(2.89)	(3.11)	(3.31)	(3.53)	(3.83)	(2.98)
σ(%)	20.49	18.49	17.48	16.83	16.30	15.87	15.50	15.18	14.91	14.67	10.65
β_{MKT}	1.19	1.09	1.04	1.00	0.97	0.94	0.92	0.90	0.87	0.84	-0.34
	(29.75)	(48.67)	(75.39)	(98.94)	(87.67)	(64.14)	(48.75)	(38.70)	(31.12)	(24.01)	(-4.71)
α(%)	-3.35	-1.76	-0.85	-0.17	0.42	0.92	1.40	1.83	2.31	2.98	6.34
	(-5.16)	(-4.88)	(-3.70)	(-1.01)	(2.22)	(3.85)	(4.60)	(4.94)	(5.18)	(5.33)	(5.41)
R^2(%)	93.81	97.65	98.93	99.29	99.14	98.59	97.71	96.56	94.90	91.60	31.02
β_{MKT}	1.02	1.01	1.01	1.00	1.00	1.00	0.99	0.99	0.98	0.98	-0.04
	(78.45)	(76.73)	(81.33)	(88.22)	(92.69)	(93.95)	(90.01)	(81.18)	(69.01)	(54.13)	(-1.30)
β_{IMC}	0.41	0.20	0.09	0.00	-0.06	-0.12	-0.17	-0.22	-0.26	-0.33	-0.74
	(20.54)	(9.92)	(4.84)	(0.11)	(-4.58)	(-8.78)	(-11.19)	(-11.97)	(-12.06)	(-11.42)	(-17.24)
α(%)	-0.23	-0.30	-0.23	-0.17	-0.07	0.01	0.13	0.23	0.38	0.57	0.80
	(-0.92)	(-1.22)	(-1.09)	(-0.89)	(-0.43)	(0.01)	(0.63)	(1.06)	(1.47)	(1.83)	(1.69)
R^2(%)	99.22	99.13	99.27	99.34	99.39	99.37	99.27	99.12	98.77	97.88	91.17

The top panel of Table 17 reports asset-pricing tests on 10 portfolios sorted on Book to Market Equity. The data come from Kenneth French's website. The construction of the IMC portfolio is detailed in Papanikolaou (2008). We use monthly data from January 1965 through December 2008. Standard errors are computed using NW with 1 lag, to adjust for autocorrelation in returns. t-statistics are computed in parenthesis. We report annualized estimates of mean returns and alphas. The bottom panel reports the corresponding estimates for simulated data. Market Equity equals the value of the firm, V_{ft}, and book to market equals Book Value divided by Market Equity. Book Value is computed as the replacement cost of capital, $B_{ft} = z_t\, x_t \sum_{j \in \mathcal{J}_{ft}} K_{jt}$, where K_j refers to capital employed by project j, and $\mathcal{J}_{ft}$ denotes the set of projects owned by firm f at the end of year t. We report medians across 1,000 simulations of coefficients and t-statistics. Each simulation sample contains 2,500 firms and has a length of 50 years. Returns of the market portfolio are computed as average return of the investment and consumption sectors, weighted by their market capitalization at time.

8 Appendix

8.1 Proofs and Derivations

Proof of Lemma 1. That is, K_f is the solution to the problem:

$$\max_{K_f} A(\varepsilon_{ft}, 1) x_t K_f^{\alpha} - z_t \, x_t \, K_f. \tag{21}$$

The first order condition is

$$\alpha \, A(\varepsilon_{ft}, 1) K_f^{\alpha-1} = z_t. \tag{22}$$

■

Proof of Lemma 2. The value of growth options depends on the NPV of future projects. When a project is financed, the value added net of investment costs is

$$\left[\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right] z_t^{\frac{\alpha}{\alpha-1}} x_t \, A(\varepsilon_{ft}, 1)^{\frac{1}{1-\alpha}} = C z_t^{\frac{\alpha}{\alpha-1}} x_t \, A(\varepsilon_{ft}, 1)^{\frac{1}{1-\alpha}}.$$

The value of growth options for firm f equals the sum of the net present value of all future projects

$$\begin{aligned} PVGO_{ft} &= E_t^{\mathcal{Q}} \left[\int_t^{\infty} e^{-r(s-t)} \lambda_{fs} C z_s^{\frac{\alpha}{\alpha-1}} x_s \, A(\varepsilon_{fs}, 1)^{\frac{1}{1-\alpha}} \, ds \right] \\ &= C z_t^{\frac{\alpha}{\alpha-1}} x_t \, E_t^{\mathcal{Q}} \left[\int_t^{\infty} e^{-\rho(s-t)} \lambda_{fs} \, A(\varepsilon_{fs}, 1)^{\frac{1}{1-\alpha}} \, ds \right] \\ &= C z_t^{\frac{\alpha}{\alpha-1}} x_t \, E_t \left[\int_t^{\infty} e^{-\rho(s-t)} \lambda_{fs} \, A(\varepsilon_{fs}, 1)^{\frac{1}{1-\alpha}} \, ds \right] \\ &= z_t^{\frac{\alpha}{\alpha-1}} x_t \, G(\varepsilon_{ft}, \lambda_{ft}), \end{aligned}$$

where $E_t^{\mathcal{Q}}$ denotes expectations under the risk-neutral measure $\mathcal{Q}$, where

$$\frac{d\mathcal{Q}}{d\mathcal{P}} = \exp\left(-\beta_x B_{xt} - \beta_z B_{zt} - \frac{1}{2}\beta_x^2 \, t - \frac{1}{2}\beta_z^2 \, t \right).$$

The second to last equality follows from the fact that λ_{ft} and ε_{ft} are idiosyncratic, and thus have the same dynamics under $\mathcal{P}$ and $\mathcal{Q}$.

Let $\mathbf{M}$ be the infinitesimal matrix associated with the transition density [Karlin and Taylor, 1975] of λ_{ft}:

$$\mathbf{M} = \begin{pmatrix} -\mu_L & \mu_L \\ \mu_H & -\mu_H \end{pmatrix}$$

The eigenvalues of $\mathbf{M}$ are 0 and $-(\mu_L + \mu_H)$. Let $\mathbf{U}$ be the matrix of the associated eigenvectors, and define

$$\Lambda(u) = \begin{pmatrix} 1 & 0 \\ 0 & e^{(-\mu_L+\mu_H)\,u} \end{pmatrix}$$

Then

$$E_t[\lambda_{fs}] = \lambda_f \cdot \mathbf{U}\,\Lambda(s-t)\,\mathbf{U}^{-1} \begin{bmatrix} \lambda_H \\ \lambda_L \end{bmatrix} = \lambda_f \cdot \begin{bmatrix} 1 + \frac{\mu_L}{\mu_L+\mu_H}(\lambda_H - \lambda_L) e^{-(\mu_L+\mu_H)(s-t)} \\ 1 - \frac{\mu_H}{\mu_L+\mu_H}(\lambda_H - \lambda_L) e^{-(\mu_L+\mu_H)(s-t)} \end{bmatrix}$$

and

$$\begin{aligned} G(\varepsilon_{ft}, \lambda_{ft}) &= C \cdot E_t \left[\int_t^\infty e^{-\rho(s-t)}\, \lambda_{fs}\, A(\varepsilon_{fs}, 1)^{\frac{1}{1-\alpha}}\, ds \right] = C \cdot E_t \left[\int_t^\infty e^{-\rho(s-t)}\, E_t[\lambda_{fs}]\, A(\varepsilon_{fs}, 1)^{\frac{1}{1-\alpha}}\, ds \right] \\ &= \begin{cases} \lambda_f \left(G_1(\varepsilon_{ft}) + \frac{\mu_L}{\mu_L+\mu_H}(\lambda_H - \lambda_L)\, G_2(\varepsilon_{ft}) \right), & \tilde{\lambda}_{ft} = \lambda_H \\ \lambda_f \left(G_1(\varepsilon_{ft}) - \frac{\mu_H}{\mu_L+\mu_H}(\lambda_H - \lambda_L)\, G_2(\varepsilon_{ft}) \right), & \tilde{\lambda}_{ft} = \lambda_L \end{cases} \end{aligned}$$

The second equality uses the fact the law of iterated expectations and the fact that λ_{ft} is independent across firms. The functions $G_1(\varepsilon)$ and $G_2(\varepsilon)$ are defined as

$$\begin{aligned} G_1(\varepsilon_t) &= C \cdot E_t \int_t^\infty e^{-\rho(s-t)}\, A(\varepsilon_s, 1)^{\frac{1}{1-\alpha}}\, ds \\ G_2(\varepsilon_t) &= C \cdot E_t \int_t^\infty e^{-(\rho+\mu_L+\mu_H)(s-t)}\, A(\varepsilon_s, 1)^{\frac{1}{1-\alpha}}\, ds. \end{aligned}$$

$G_1(\varepsilon)$ and $G_1(\varepsilon)$ will satisfy the ODEs:

$$\begin{aligned} C \cdot A(\varepsilon, 1)^{\frac{1}{1-\alpha}} - \rho G_1(\varepsilon) - \theta_\epsilon(\varepsilon - 1)\frac{d}{d\,\varepsilon}\, G_1(\varepsilon) + \frac{1}{2}\sigma_e^2\, \varepsilon\, \frac{d^2}{d\,\varepsilon^2}\, G_1(\varepsilon) &= 0 \\ C \cdot A(\varepsilon, 1)^{\frac{1}{1-\alpha}} - (\rho + \mu_H + \mu_L) G_2(\varepsilon) - \theta_\epsilon(\varepsilon - 1)\frac{d}{d\,\varepsilon}\, G_2(\varepsilon) + \frac{1}{2}\sigma_e^2\, \varepsilon\, \frac{d^2}{d\,\varepsilon^2}\, G_2(\varepsilon) &= 0. \end{aligned}$$

■

Proof of Lemma 1. The risk premium on assets in place will be determined by the covariance with the pricing kernel:

$$E_t R_{ft}^{vap} - r_f = -cov\left(\frac{dVAP_{ft}}{VAP_{ft}}, \frac{d\pi_t}{\pi_t} \right) = \beta_x \sigma_x$$

Similarly for growth options:

$$E_t R_{ft}^{gro} - r_f = -cov\left(\frac{dPVGO_{ft}}{PVGO_{ft}}, \frac{d\pi_t}{\pi_t}\right) = \beta_x \sigma_x - \frac{\alpha}{1-\alpha}\beta_z \sigma_z$$

The risk premium on growth options will be lower than assets in place as long as $\beta_z > 0$. Consequently, expected returns excess returns of the firm are a weighted average of the risk premia of its components

$$E_t R_{ft} - r_f = \frac{VAP_{ft}}{V_{ft}}(ER_{ft}^{vap} - r_f) + \frac{PVGO_{ft}}{V_{ft}}(ER_{ft}^{gro} - r_f)$$

■

Proof of Lemma 3. Profits accruing to the I-sector can be written as

$$\begin{aligned} \Pi_t &= \phi\, z_t\, x_t \int K_{ft} df \\ &= \phi \left(\int A(e_{ft}, 1)^{\frac{1}{1-\alpha}} df \right) \alpha^{\frac{1}{1-\alpha}}\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} \\ &= \phi\, \Gamma \cdot x_t\, z_t^{\frac{\alpha}{\alpha-1}} \end{aligned}$$

K_{ft} is the solution to the first order condition 22. Because ε_{ft} has a stationary distribution, $\Gamma = \left(\int A(e_{ft}, 1)^{\frac{1}{1-\alpha}} df \right)$ is a constant.

The price of the investment firm satisfies

$$\begin{aligned} V_{It} &= E_t^{\mathcal{Q}} \int_t^\infty \exp\{-r(s-t)\}\ \phi \Pi_s ds \\ &= \phi\, \Gamma\, E_t^{\mathcal{Q}} \int_t^\infty \exp\{-r(s-t)\}\ x_s\, z_s^{\frac{\alpha}{\alpha-1}} ds \\ &= \phi\, \Gamma\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} E_t^{\mathcal{Q}} \int_t^\infty \exp\left\{ \left(-r + \mu_X - \frac{1}{2}\sigma_X^2 - \frac{\alpha\, \mu_Z}{1-\alpha} + \frac{1}{2}\frac{\alpha}{1-\alpha}\, \sigma_Z^2 \right)(s-t) + \right. \\ &\quad \left. + \sigma_X (B_{xs} - B_{xt}) + \frac{\alpha\, \sigma_Z}{\alpha - 1}(B_{zs} - B_{zt}) \right\} \\ &= \phi\, \Gamma\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} \int_t^\infty \exp\left\{ \left(-r + \mu_X - \frac{\alpha}{1-\alpha}\, \mu_Z + \frac{1}{2}\frac{\alpha}{1-\alpha}\, \sigma_Z^2 + \frac{1}{2}\frac{\alpha^2\, \sigma_Z^2}{(1-\alpha)^2} \right)(s-t) \right\} \\ V_{It} &= \phi\, \Gamma\, x_t\, z_t^{\frac{\alpha}{\alpha-1}} \frac{1}{D_I} \end{aligned}$$

■

Proof of Lemma 2. Returns on the IMC portfolio follow:

$$
\begin{aligned}
R_t^I - R_t^C &= (\cdot)\, dt + \sigma_X dB_{xt} + \frac{\alpha}{\alpha-1}\sigma_Z dB_{zt} - \sigma_X dB_{xt} - \frac{\overline{PVGO}_t}{\overline{V}_t}\frac{\alpha}{\alpha-1}\sigma_Z dB_{zt} \\
&= (\cdot)\, dt + \frac{\overline{VAP}_t}{\overline{V}_t}\frac{\alpha}{\alpha-1}\sigma_Z dB_{zt}
\end{aligned}
$$

whereas the return of firm i in the consumption sector is:

$$
\begin{aligned}
R_{ft} &= (\cdot)\, dt + \frac{VAP_{ft}}{V_{ft}}\sigma_X dB_t^x + \left(1 - \frac{VAP_{ft}}{V_{ft}}\right)\left(\sigma_X dB_{xt} + \frac{\alpha}{\alpha-1}\sigma_Z dB_{zt}\right) + (\cdot)dB_{ft} + \sum_j (\cdot)dB_{jt} \\
&= (\cdot)\, dt + \sigma_X dB_{xt} + \left(1 - \frac{VAP_{ft}}{V_{ft}}\right)\left(\frac{\alpha}{\alpha-1}\sigma_Z dB_{zt}\right) + (\cdot)dB_{ft} + \sum_j (\cdot)dB_{jt}
\end{aligned}
$$

so

$$
cov_t(R_{ft}, R_t^I - R_t^C) = \left(\frac{PVGO_{ft}}{V_{ft}}\right)\left(\frac{\overline{VAP}_{ft}}{\overline{V}_{ft}}\right)\frac{\alpha^2}{(1-\alpha)^2}\sigma_Z^2
$$

and

$$
var_t(R_t^I - R_t^C) = \left(\frac{\overline{VAP}_{ft}}{\overline{V}_{ft}}\right)^2 \frac{\alpha^2}{(1-\alpha)^2}\sigma_Z^2
$$

which implies that the beta of firm f with the IMC portfolio is increasing in firm f's growth options. ■