Lecture 8: Liquidity and Limits to Arbitrage

Investments

Liquidity

- Liquidity is often an ill-defined concept.
- Market liquidity: The ease of trading a security
  - ↪ A security is considered liquid if it is easy to trade: Low bid-ask spread, small price impact, high resilience, easy search (in OTC markets)
  - ↪ Market liquidity risk: The risk that the market liquidity worsens
- Funding liquidity: The available of funding to the arbitrageur
  - ↪ A trader's available funding from own capital and (collateralized) loans
  - ↪ Funding liquidity risk: The risk that a trader cannot fund his position and is forced to unwind

Market Liquidity

- Direct Costs
  1. Bid-Ask spreads

     Dealers are willing to buy at a lower price (bid) than the price they are selling (ask).

  2. Cost of short-selling

     Short-selling involves borrowing the underlying security from someone at an additional cost (rebate).

- Indirect Costs
  1. Price impact

     Trading, especially in large sizes or in illiquid markets can move the market against you, incurring additional costs.

Transaction Costs: Bid/Ask Spread

STOCK	BID	ASK	BID Size	ASK Size	LAST	CHG
MSFT	32.87	32.90	100	1000	32.87	-2.02%
INTC	26.03	26.05	100	900	26.05	-0.76%
GOOG	681.53	681.90	500	1900	681.9	0.05%
FORD	2.26	2.45	200	200	2.428	-0.08%
GRMN	103.60	104.00	100	700	104	-0.03%
NVDA	32.00	32.20	2000	1100	32.2	0.63%
FSLR	230.50	230.75	1000	200	230.5	0.56%
DELL	23.90	23.93	400	3700	23.94	0.00%
ETFC	4.06	4.07	400	4400	4.06	-1.22%

- The size of the Bid/Ask varies for different securities. The average Bid/Ask spread for the smallest stocks (< 25 million) is around 4%, for the largest stocks (> 1.5 billion) it is closer to 0.5%.

Economic Reasons for Bid/Ask Spreads

Why does not competition eliminate spreads?

- Inventory costs
  - ↪ Dealers carry inventory – hence, they deviate from MVE portfolio
  - ↪ Bid/Ask spread compensates for holding undiversified positions
  - ↪ Bid/Ask spreads are positively correlated with price volatility and negatively correlated with volume

- Adverse selection / market for lemons
  - ↪ Dealers may trade against better-informed market participants
  - ↪ Set bid and ask price such that

$$Bid = E[\text{Value of security} \mid \text{Sell order}]$$
$$Ask = E[\text{Value of security} \mid \text{Buy order}]$$

  - ↪ Bid/Ask spreads are negatively correlated with size

Kellogg School of Management

Transaction Costs: Price Impact

Bid/Ask spreads apply for a given size (in terms of 100 shares).

GRAB Equity MQ

At 11:02 Vol 11,949,800 ValTrd 811.762m

Market Quote Monitor

MSFT US MICROSOFT CORP page 1 of 4

Time	Mmkr	Size	BID	ASK	Size	Mmkr	Time
11:02	ISLD	5	68.75	68.81	71	INCA	11:02
11:02	INCA	3	68.75	68.81	47	ISLD	11:02
11:01	FBCOp	1	68.75	68.81	30	BTRD	11:02
11:02	SBSHp	1	68.74	68.81	20	BRUT	11:02
11:02	ABNAp	1	68.73	68.81	20	ARCA	11:02
11:02	NDBCp	10	68.70	68.81	16	REDI	11:02
11:02	LEHMp	10	68.70	68.81	1	NITEp	11:02
11:02	PRUSp	10	68.70	68.82	10	GSCOp	11:02
10:53	MLCOp	10	68.70	68.82	10	MSCOp	11:02
11:02	ARCA	3	68.69	68.82	1	HRZGp	11:02
11:02	MSCOp	10	68.68	68.83	8	NDBCp	11:02
11:02	HRZGp	1	68.67	68.83	8	MASHp	11:02
10:57	JEFFp	1	68.60	68.83	1	SLKCp	11:02
10:52	COWNp	1	68.58	68.92	1	FBCOp	11:01
10:55	MONTp	10	68.55	68.93	1	SBSHp	11:02
10:54	DBABp	10	68.53	68.93	1	RAMSp	11:01
11:02	REDI	9	68.50	68.95	10	MONTp	10:55
10:56	PERTp	5	68.50	68.95	1	ABNAp	11:02
11:02	MWSE	2	68.50	68.95	1	SMCOp	11:01
11:01	GKMCp	1	68.50	68.97	1	CANTp	9:43

A sell (market) order for 1.500 shares will be executed at four different prices (900 at 68.75, 100 at 68.75; 100 at 68.73, and 400 at 68.70)

Transaction Costs: Price Impact

- Quoted prices apply to moderate size orders only, a large order may not be executed at the same price.
- Market impact must be estimated by the investor based on the current perceived liquidity
- Price impact often used as a measure of illiquidity:
  - ↪ In a deep and liquid market, large trades should have minimal effect on the price.
  - ↪ Price impact is measured as

$$\Delta P = \lambda \Delta Q$$

  - ↪ λ measures the effect on the price of an increase in (signed) volume, i.e. Q measures the net volume of buy orders.

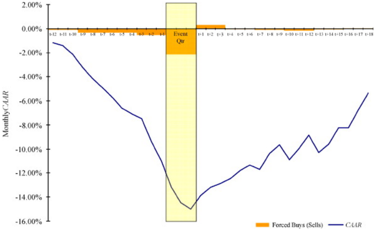
Reasons for Price Impact

- Information.
  - ↪ A buy order contains information: informed agents with good news are more likely to buy.
  - ↪ The effect should be larger the more information asymmetry in the market. Trading in opaque or difficult to value assets makes it more likely that we are trading against an insider, as returns to insider trading are large.
  - ↪ The effect on the price should be *permanent*, as long as markets are efficient.

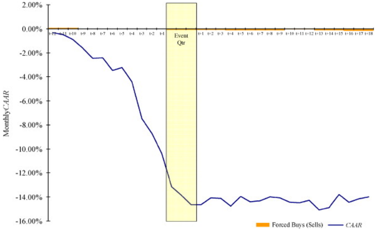
Reasons for Price Impact

- Limited Risk-Bearing.
  - ↪ Market makers or specialists who absorb the other side of a large trade are temporarily undiversified and bearing more risk. Thus they may be only willing to buy at a discount, to compensate them for the risk that the price may move against them while they search for someone to sell the security.
  - ↪ The effect should be *temporary*, as risk is eventually absorbed by many other agents step in to pick up
  - ↪ The effect should be larger in markets trading specialized assets with large costs to entry that are segmented.

- Coval and Stafford (2008) look at the price impact of selling by “distressed” funds.
- Such sell orders carry no information: funds are forced to liquidate their holdings to meet outflows.



Distressed



Non-distressed

Strategies for minimizing price impact

- Break up large trades
  - ↪ By breaking up a large trade into smaller pieces, it can lead to lower price impact, especially if the effect is nonlinear, i.e. larger trades have a disproportionate large impact on the price.
  - ↪ Cost of breaking up is execution risk: the price at which the total trade is executed is uncertain. This can be problematic in highly volatile markets.
  - ↪ This opportunity cost of waiting will in general also depend on the horizon of the strategy, the longevity of information and competition among market participants.
- Several hedge funds develop relationships with prime brokers that enable them to trade larger quantities for non-informational reasons.
  - ↪ See the DFA case for a related example.

- Round-trip transaction costs depend on
  - ↪ The size of the trade
  - ↪ The market cap (or float) of the security

Transaction Costs:

	Dollar Value of Block ($ Thousands)								
Sector	**5**	**25**	**250**	**500**	**1,000**	**2,500**	**5,000**	**10,000**	**20,000**
1 (Small)	17.3%	27.3%	43.8%						
2	8.9%	12.0%	23.8%	33.4%					
3	5.0%	7.6%	18.8%	25.9%	30.0%				
4	4.3%	5.8%	9.6%	16.9%	25.4%	31.5%			
5	2.8%	3.9%	5.9%	8.1%	11.5%	15.7%	25.7%		
6	1.8%	2.1%	3.2%	4.4%	5.6%	7.9%	11.0%	16.2%	
7	1.9%	2.0%	3.1%	4.0%	5.6%	7.7%	10.4%	14.3%	20.0%
8	1.9%	1.9%	2.7%	3.3%	4.6%	6.2%	8.9%	13.6%	18.1%
9 (large)	1.1%	1.2%	1.3%	1.7%	2.1%	2.8%	4.1%	5.9%	8.0%

Table 5

Performance Under Proportionate Transaction Costs

We evaluate the performance of momentum trading strategies according to the trading model developed here, using proportionate transaction costs. We form a portfolio in the beginning of February 1967 according to a chosen momentum strategy (see Table 1 for a description of momentum strategies) with a certain initial monetary amount of investment. The portfolio is rebalanced on a monthly basis, following the trading rule of the chosen strategy, until the end of December 1999. The proportionate costs considered here include effective and quoted spreads. Effective spreads are measured as the absolute price improvement relative to mid-point of quoted bid and ask. Quoted spread is measured as the ratio between the quoted bid-ask spread and the mid-point (half the quoted spread is considered as trading cost). Transaction costs are estimated on a monthly basis, using NYSE-listed stocks for the period January 1993 to May 1997. Then, using cross-sectional relation between the different liquidity measures and pre-determined firm-characteristics (see Table 2), the spreads are re-estimated for the entire sample period, February 1967 to December 1999. Assuming that the estimated price spreads are perfectly foreseeable, we rebalance the portfolio every month while keeping it self-financing, after considering the execution costs of trades. For every momentum-based trading strategy we calculate the time series of monthly returns, net of transaction costs. Three performance measures are reported: (1) The intercept (Alpha) of the conditional Fama and French (1993) regressions (as explained in this paper), (2) The *t*-statistic associated with Alpha, and (3) the slope of the investment frontier of a set consisting four assets: the three Fama and French (1993) portfolios and the momentum portfolio (this is calculated as the maximum attainable Sharpe ratio of a combination of the four assets). Since we use proportionate transaction costs, all performance measures are invariant to the initial investment. The analysis uses monthly returns of all NYSE stocks available on CRSP.

Panel A: 11/1/3 Strategy

	Equal-Weighted		
	Alpha	T-Stat of Alpha	Slope of Investment Frontier
Raw Return	0.0080	8.92	0.44
Return net of Effective	0.0061	6.86	0.38
Return net of Quoted	0.0054	6.08	0.35

	Value-Weighted		
	Alpha	T-Stat of Alpha	Slope of Investment Frontier
Raw Return	0.0057	4.54	0.32
Return net of Effective	0.0045	3.59	0.29
Return net of Quoted	0.0040	3.17	0.28

Panel B: 5/1/6 Strategy

	Equal-Weighted		
	Alpha	T-Stat of Alpha	Slope of Investment Frontier
Raw Return	0.0059	8.07	0.41
Return net of Effective	0.0041	5.60	0.34
Return net of Quoted	0.0035	4.72	0.32

	Value-Weighted		
	Alpha	T-Stat of Alpha	Slope of Investment Frontier
Raw Return	0.0033	3.46	0.29
Return net of Effective	0.0022	2.31	0.26
Return net of Quoted	0.0017	1.82	0.25

Transaction costs and trading strategies

- Estimates of the net of transaction cost profitability of various trading strategies vary across studies
  - ↪ Korajczyk and Sadka (2004) find that, after taking into account the price impact induced by trades, as much as 5 billion dollars (relative to December 1999 market capitalization) may be invested in some momentum-based strategies before the apparent profit opportunities vanish.
  - ↪ Frazzini, Israel and Moskowitz (2013) use live trading data from a large institutional money manager (i.e. AQR) and find that actual trading costs are less than a tenth as large – and therefore the potential scale of these strategies is 10$\times$ as larger.
    Results vary across styles, with value and momentum being more scalable than size, and short-term reversals being the most constrained by trading costs.

Liquidity due to search frictions

- Sometimes counter-parties are not available immediately to trade
- This can be a problem in specialized assets
  - ↪ Physical commodities
  - ↪ Housing. You need to find a buyer for **this particular** house
  - ↪ Art
  - ↪ Structured Credit products
- Should this inability to trade frequently affect portfolio allocation?

Harvard in 2009

- Harvard's endowment is large ($43.0B net assets in June 2008) and is used to fund 34% of university operations. Spending of out of the endowment is smooth and has stayed around 4.5%.
- Investment is in a leveraged, diversified portfolio with many "alternative investments":
  - ↪ Real assets (e.g. timber) make up 29% of assets. Hedge Funds and private equity are an additional 32%.
  - ↪ Foreign and domestic equity make up 19% of assets.
  - ↪ Derivatives, fixed income, and emerging markets are 30%.
  - ↪ Cash position is -10% of assets.
- From June 2008 through June 2009, the endowment lost 27% of its value.

Harvard in 2009, Continued

What happened?

- Harvard's theory: Endowments are long-term investors and so they can better absorb liquidity shocks.
- In reality, Harvard has immediate institutional cash flow obligations (e.g. salaries, maintenance).
- Faced with the choice of selling endowment assets at a large loss or cutting university funding, Harvard chose to reduce its operating budget by 20%.

Harvard's problem was the result of the *interaction* between the need for smooth cash flows and risk in trading conditions.

Portfolio allocation with illiquidity risk

- If certain assets are illiquid – they can be traded infrequently, or only at a cost – it may make sense to reduce our portfolio allocation
  - ↪ Problem more acute if we have intermediate funding needs
  - ↪ To answer this question fully, we need a dynamic model, which for the most part is outside the scope of this class.
- We can obtain some intuition by examining the output of the model of Ang, Papanikolaou and Westerfield, 2013
- Asset allocation problem with
  - ↪ Two risky assets, L and I, and a risk-free asset
  - ↪ Investor has long-horizon, cares about smoothing payout c
  - ↪ Risky asset I can only be traded with probability λ per period

Illiquidity and asset allocation – Example

Need to smooth payout c

Average Turnover	Optimal	Illiquidity	Average policies		
$E(T)=1/\lambda$	Rebalance (w_I^*)	cost	w_I	c	w_L
0	0.593	-	0.593	0.089	0.593
1/50 years	0.535	0.018	0.541	0.088	0.583
1/10 years	0.493	0.028	0.511	0.087	0.578
1/4	0.475	0.036	0.485	0.086	0.571
1/2	0.442	0.045	0.461	0.083	0.568
1	0.373	0.067	0.409	0.081	0.558
2	0.251	0.103	0.299	0.075	0.546
4	0.132	0.165	0.212	0.069	0.536
10	0.048	0.222	0.214	0.059	0.489
∞	0.593	-	0.593	0.070	0.593

- The table is computed using the following parameter values: $A=6$, $\mu_L=\mu_I=.12$, $r=.04$, $\sigma_L=\sigma_I=.15$, and $\rho_{LI}=0$.
- Compare to frictionless benchmark $w^*=\frac{\mu-r_f}{A\sigma^2}=0.593$

Turnover	Optimal	Illiquidity	Average policies	
$E[T]=1/\lambda$	Rebalance (ξ^*)	Utility cost	w_I	w_L
0	0.593	-	0.593	0.593
1/50 years	0.557	0.002	0.555	0.580
1/10 years	0.547	0.002	0.546	0.578
1/4	0.544	0.002	0.543	0.576
1/2	0.535	0.002	0.534	0.574
1	0.527	0.003	0.526	0.571
2	0.523	0.003	0.523	0.567
4	0.520	0.003	0.518	0.563
10	0.518	0.004	0.516	0.555
∞	0.593	-	0.593	0.593

- The table is computed using the following parameter values: $A=6$, $\mu_L=\mu_I=.12$, $r=.04$, $\sigma_L=\sigma_I=.15$, and $\rho_{LI}=0$.
- Compare to frictionless benchmark $w^*=\frac{\mu-r_f}{A\sigma^2}=0.593$

Liquidity and asset prices

- Transaction costs on financial markets reduce the return on investments
- Rational investors will require a compensation for expected transaction costs
- This affects the price an investor is willing to pay for an asset
- In equilibrium, transaction costs lead to lower prices for assets
- As a result, expected returns (before transaction costs) will be higher

Example

- Suppose you buy an asset for \$ 100 now (ask price) and you hold the asset for one year
- After one year, the ask price is \$ 104, the bid price is \$ 102 (bid-ask spread is \$ 2)
- You sell the asset at the bid price, i.e. \$ 102
- Return on the asset (ask-ask) is 4%, but your realized return is only 2%!

Liquidity-adjusted CAPM

- Amihud and Mendelson (1986) incorporate transaction costs into the CAPM framework

$$E[R_i] = r_f + \beta_{Mi}(E[R_M] - r_f) + \mu S_i$$

- Expected returns are a sum of three components
  - $\hookrightarrow$ the risk free rate of return (r_f).
  - $\hookrightarrow$ a risk premium, determined by the beta of the asset β_{Mi}).
  - $\hookrightarrow$ a liquidity premium, determined by the relative bid-ask spread (S).
- Here μ is the expected trading frequency (no. times per period)

Liquidity-adjusted CAPM

- Amihud and Mendelson (1986) test the relation between expected return and spreads
  - ↪ Use data from NYSE stocks, 1960-1979
  - ↪ Control for other determinants of expected returns such as beta and firm size
- Their analysis follows the (by now) familiar steps
  1. for every stock, calculate relative spread and estimate its beta
  2. sort all assets into portfolio's based on beta and spread
  3. cross sectional regression of average monthly portfolio return $\bar{\mu}_i$ on the spread S_p and portfolio beta β_{Mp}

$$\bar{\mu}_p = \gamma_1 + \gamma_2 \beta_{Mp} + \gamma_3 S_p$$

- AM estimate $\gamma_3 = 0.211$; it implies that each stock is traded approximately once every 5 months

Summary

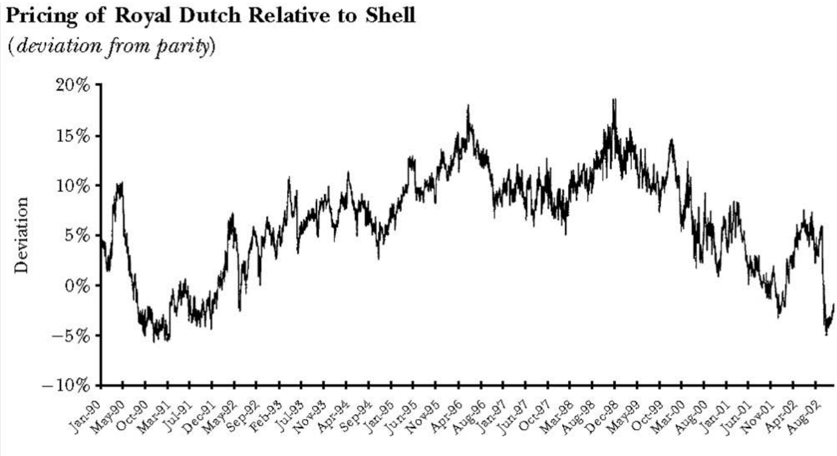
- The existing framework that we have seen so far can be extended to allow for transaction costs and illiquidity
- All else equal, investors will invest less in illiquid securities, or securities that are costly to trade
- So far, we have implicitly assume that the amount of liquidity is constant
- In the next section, we will see what happens if the amount of market liquidity varies over time

- Perhaps the most damning evidence against the assumptions of modern finance are the evidence on the existence and *persistence* of apparent arbitrage opportunities.
- Several researchers have found evidence that
  - ↪ Identical firms often trade at different prices
  - ↪ Parts of a firm often trade for *negative* prices
- The issue here is not that these apparently mis-priced assets exist, because as we saw investors may possess different biases.
- The puzzle is that the mis-pricing seems to persist long after it is discovered. Surely, a rational investor would have eliminated the profitability of these strategies.

- In 1907 Royal Dutch and Shell Transport merged.
  - ↪ However, they remained separate entities, even though they have claims to *the exact same cashflows.*
  - ↪ Royal Dutch shares are traded in Amsterdam and Shell shares are traded in London.
  - ↪ All cash flows are split so that Royal Dutch shares receive 60 percent and Shell shares receive 40 percent.
- As a result, one would expect

$$P_{RoyalDutch} = 1.5 P_{Shell}$$

Royal Dutch / Shell



Limits to Arbitrage

1. To benefit from the arbitrage, investors would have to wait forever.
   - ↪ Unlike a bond, stocks have no liquidation date, so there is no pre-specified date at which the arbitrage will be corrected.
   - ↪ Also, if the securities are mis-priced today, the mis-pricing may get worse tomorrow, leading to capital losses. Potential arbitrageurs may be worried about this.
2. Costs of implementing the arbitrage may be to high.
   - ↪ The arbitrage trade involves short-selling some securities, which may be expensive.
   - ↪ Arbitrageurs may be worried about this, especially if they expect to wait for a long time!
3. (1) and (2) imply that arbitrageurs may face margin calls, forcing them to liquidate their positions at a loss.

Palm/3COM example

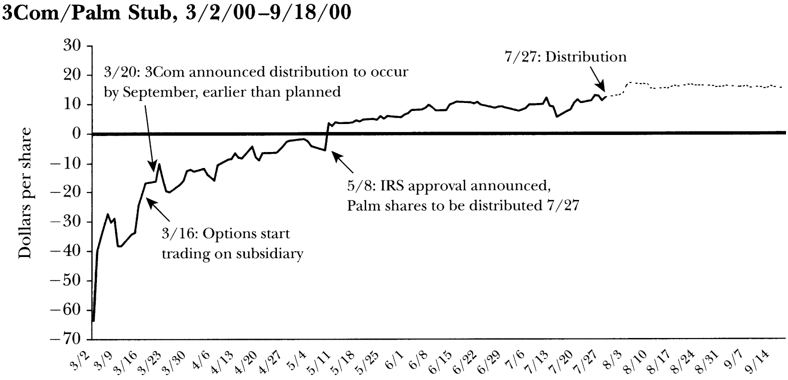
In March 2000 3Com sold 5% of Palm Inc. in an IPO. After 9 months the remaining shares were to be distributed among the 3Com shareholders in a ratio of 1.5 Palm Inc shares per 3Com share. The Price of Palm at the end of the first day of trading was $95 while, on the same day, the price of 3Com was $81.

- What's wrong with these prices?

Palm/3COM example

- Each share of 3Com is entitled to 1.5 shares of Palm.
- What is the stub value of 3Com, i.e. the value of 3Com, *excluding* the value of Palm?
  - ↪ 3Com is worth $\$81$ per share.
  - ↪ each share of 3Com entitles to $1.5 \times \$95 = \$45$
  - ↪ Stub value of 3Com equals $\$81 - \$145 = -\$64$ per share!
- In theory this is a riskless arbitrage:
  - ↪ Buy 1 share of 3Com
  - ↪ Short 1.5 shares of Palm
  - ↪ Pocket the difference: $\$145 - \$81 = \$64$.
  - ↪ Wait for 9 months, receive the 1.5 shares of Palm and close the short position.
  - ↪ Net gain: $\$64$ plus the residual value of the 3Com shares after 9 months.

- Mis-pricing eventually got corrected, although not immediately.

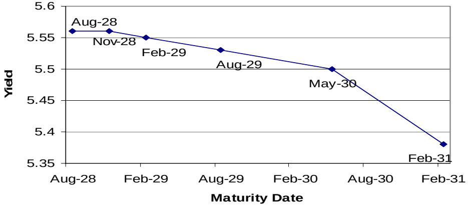


Near-arbitrage opportunities

- True arbitrage opportunities are rare.
  - $\hookrightarrow$ It is difficult to find two identical securities.
- However, it is sometimes possible to find two 'similar' securities that trade at different prices.
- While there is no guarantee that the spread in prices will converge, sometimes they do in a predictable fashion
- A classic example is the new bond-old bond spread.

Old Bond - New Bond Spread

- 30 year US Treasury bonds are among the most actively traded bonds in the world, serving as benchmark for long-term interest rates in the US.
- The US Treasury has, until recently, auctioned 30 year bonds on roughly a six-month cycle.
- Upon issuance, a bond is dubbed the “new bond” and acquires benchmark status, replacing the bond that was issued six months prior, now the “old bond”.
- The yield curve as of 2/9/01



Old Bond - New Bond Spread

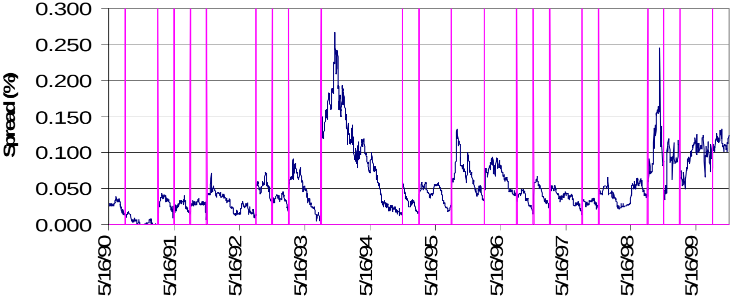
- The new bond is more expensive than the old bond. However, after 6 months, the new bond becomes just like the old bond.
- Some traders believe this represents an arbitrage opportunity
  - $\hookrightarrow$ The 29.5-yr and the 30-yr bond are almost identical securities, yet they trade at different prices.
  - $\hookrightarrow$ If we wait for 30 years we could generate almost riskless profits.
  - $\hookrightarrow$ Given the historical behavior of the spread, we need only wait until the next auction, where with high probability the spread will converge to zero.

Old Bond - New Bond Spread

- Consider the following convergence trade:
  a) Short sell the new bond, at a spread of 12 bps to the old bond
  b) Purchase the old bond in order to lock in the spread.
  c) Hold the spread position until the next auction date, and then unwind the position at the smaller spread of potentially 3bps.
- Since the spread between new and old bond will with high probability converge towards zero as time passes, one can generate large trading profits.

Old Bond - New Bond Spread

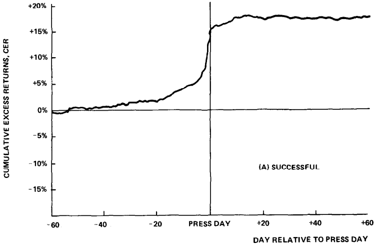
- Purple lines show Treasury auctions
- Spread does seem to converge at auction, though not always.
- LTCM lost a lot of money on this trade in August 1998.

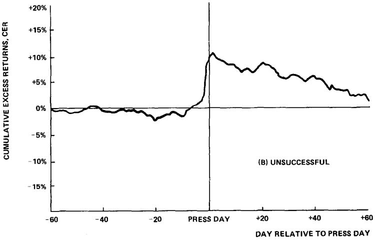


Other examples

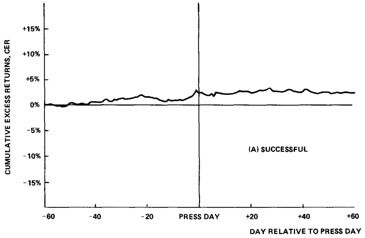
- Convertible bond arbitrage
  - Convertible bonds can be converted into shares of common stock if the stock price appreciates to a predetermined level
  - Convertible bond $\approx$ Corporate bond + call option
  - Buy convertible bond if 'cheap' relative to cost of replicating strategy
- Merger "arbitrage"
  - In a merger, target is bought at a premium, say 20-30%.
  - At announcement, the price of the target firm increases to a value close to the offer value.
  - But, there remains a "deal spread", typically around 3%
  - Buy target, if stock deal hedge by shorting acquirer
  - Not a true arbitrage since it is risky

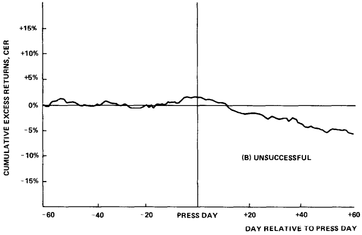
Target firms





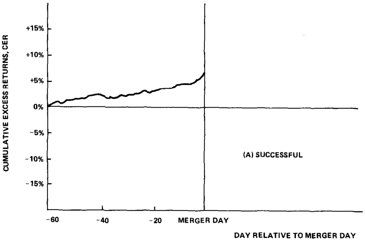
Acquirer firms

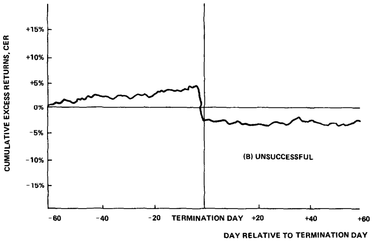




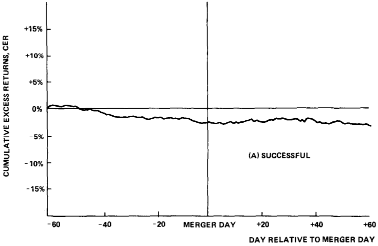
Market reaction until merger day

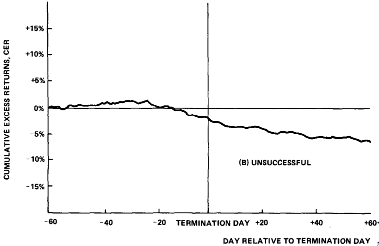
Target firms



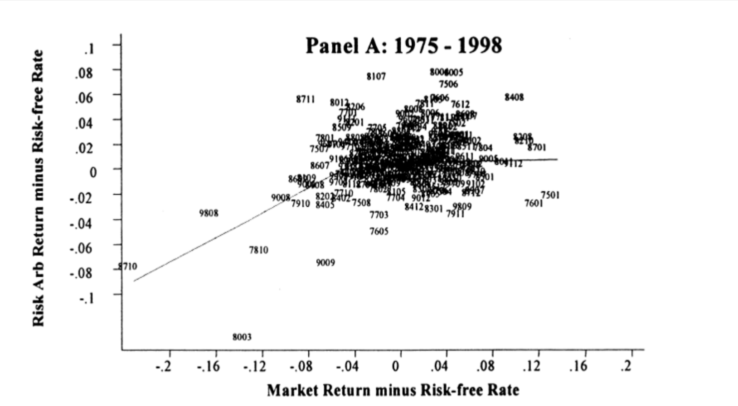


Acquirer firms





Merger arbitrage



- Source: Mark Mitchell and Todd Pulvino, Journal of Finance
- Small consistent payoff; but a small probability of disaster. Just like the payoff of writing index puts!

Kellogg School of Management

Merger arbitrage

- Even though it's an all-equity strategy (no option positions) dynamic trading gives an option-like payoff.
- Writing index puts earns a premium. It provides "disaster insurance" to the market. But no need to pay 2+20 to write index puts!
  - ↪ "Alpha", "beta", benchmark, performance evaluation should be relative to the strategy of writing index puts!
  - ↪ (Mitchell and Pulvino are now running a merger-arb hedge fund, so at least they think such alpha is there.)

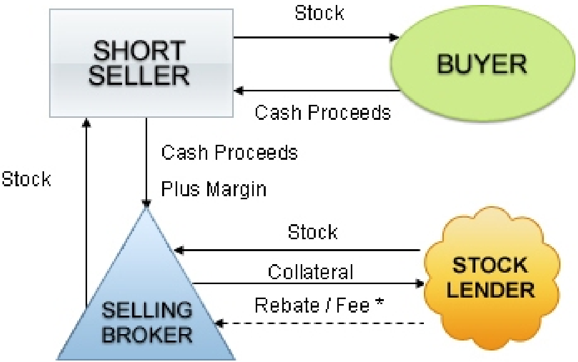
Potential explanations

1. Short-selling is costly
   - ↪ In the presence of short-selling constraints, prices can deviate from fundamental values even if all investors are rational
   - ↪ Harrison and Kreps (1978) provided an example in which heterogeneous beliefs can lead to a persis- tent speculative trade and what they call a "speculative premium" i.e. the difference between the price of an asset and the expected present value the stream of the future dividends.
2. Funding liquidity
   - ↪ These strategies are not riskless **in the short term**. Margin constraints may force arbitrageurs to exit prematurely
   - ↪ Arbitrageurs wary of this risk underinvest in arbitrage opportunities

Costs of Short-selling

- Short-selling a security involves two transactions, a *sale* and a *reverse repo* transaction.
    1. In most markets, the sale occurs today, with delivery of the security tomorrow.
    2. Tomorrow, when it is time to settle the trade, we need to *borrow* the security from an investor who owns it, agreeing to return the security in the future. In return, we post cash as collateral.
    3. When the short is reversed, we need to purchase the security in the market. We return the security to the investor and receive in return our cash collateral plus the *repo rate*.
- The repo-rate depends on the underlying security and is determined by supply and demand.
- The difference between the interest rate we would normally get on our collateral and the repo rate represents *the cost of shorting this particular security.*

Mechanics of Short-selling



- To short- a stock we need to find someone who is willing to lend it.
  - ↪ The short-seller borrows the stock using cash as collateral.
  - ↪ The lender of the security borrows cash (at below market rates) using the stock as collateral

Costs of Short-selling

Company Name	Transaction Date	Minimum Short Rebate
Stratos Lightwave	July 6, 2000	−40.0%
Palm	July 28, 2000	−30.0%
Net2Phone	October 1, 1999	−8.0%
Retek	June 26, 2000	−4.0%
Plug Power	September 18, 2000	−4.0%
PFSWeb	January 20, 2000	−3.0%
MIPS Technology	September 28, 2000	−2.0%
Williams Communications	September 19, 2000	0.0%
Xpedior	December 30, 1999	0.0%
Iturf	September 24, 1999	0.0%
Ubid	January 26, 1999	0.0%
Marketwatch.Com	May 25, 1999	0.5%
Intimate Brands	February 23, 1999	0.5%
IXNet	September 7, 1999	1.0%
Interspeed	October 6, 1999	2.0%
Digex	August 16, 1999	2.0%
NetSilicon	September 30, 1999	2.3%
XM Satellite Radio	January 24, 2000	3.0%
US Search	March 28, 2000	3.0%
Veritas Software	June 7, 1999	3.0%
Barnes & Noble	August 23, 1999	4.0%
Kaiser Aluminum	December 22, 1998	4.4%
Nabisco Brands	July 19, 1999	4.7%
Keebler Foods	August 8, 2000	5.0%
CareInSite	December 16, 1999	5.0%
Superior Telecom	September 27, 2000	5.0%
Deltathree.Com	July 31, 2000	6.0%
Mean		−1.5%
Median		1.0%

- Costs of shortselling can be substantial if securities are 'special', i.e. there is a lot of demand for shorting

Costs of Short-selling

- Short-Selling costs can be quite important and can eliminate a large fraction of the apparent profitability of “arbitrage” strategies.
- Example: profitability of the new bond/old bond strategy after shorting costs

Table 2: Profits by auction cycle

Cycle	Profits Bps. per annum	Average Bond spread	Average Repo spread
1	187	3.3	6
2	98	7.0	16
3	-8	6.8	20
4	115	3.7	9
5	128	4.1	-5
6	145	4.0	-23
7	-48	3.0	70
8	-41	4.1	45
9	-286	11.1	158
10	-22	8.9	-130
11	-39	9.0	59

Funding Liquidity

- Short-selling is an example of a levered transaction. Leverage requires collateral.
- Speculators face margin requirements on their positions, both long x^+ and short x^-

$$\sum_j \left(x_j^+ m_j^+ + x_j^- m_j^- \right) \leq W_t$$

- The presence of margin requirements leads to the possibility of lack of funding liquidity, depending on
  - $\hookrightarrow$ Cost of Intermediary Capital (e.g. Federal Funds Rate)
  - $\hookrightarrow$ Margin Requirements (m^- and m^+)
  - $\hookrightarrow$ Capital Adequacy Requirements (Risk Management)

Kellogg School of Management

- When you take a levered position or a short position in a security you are required to keep money in a margin account.
- The amount of money required depends on the mark-to-market value of your position.
- This can create *limits to arbitrage*, where arbitrage opportunities that on paper appear safe are in fact risky

Limits to Arbitrage - Example

- There is an arbitrage opportunity
  - $\hookrightarrow$ P_t is the date-t deviation of the market price, which could be positive or negative, from its “fundamental value”, which here is 0
- There are three dates in this example:
  - $\hookrightarrow$ At $t = 2$ the mispricing disappears and $P_2 = 0$.
  - $\hookrightarrow$ An intermediate date $t = 1$.
  - $\hookrightarrow$ Today, $t = 0$ where $P_0 = \$10$
- You exploit this opportunity by taking a short position at time 0.
- Your initial wealth at time 0 is $W_0 = \$100m$
  - $\hookrightarrow$ you don't have any other source of credit or collateral.
- Your broker has a 50% margin requirement on short sales.

Margin Calls - Example

- You decide to short-sell as much as possible at time 0.
- Since you need an initial margin requirement of 50%, the value of securities that you short, X, needs to satisfy

$$\frac{100}{X} = 0.5$$

  so, given your 100 starting wealth, you can short up to $200m$ worth of securities.
- Since the price at time 0 is 10, you can short up to 20 units.
- If we could make it till $t = 2$, we would make a profit of $20 \times 10 = 200$, or a return of 200%.
- Unfortunately, we have to pass time 1 before reaching time 2.

Margin Calls - Example

- When time 1 arrives, your broker will mark your position to the market and recalculate the margin requirement of your short position. Consider the following scenarios:
  a) $P_1 = 10$
  b) $P_1 = 12.5$
  c) $P_1 = 15$
- Your margin ratio, if the price of the security is P, is

$$MR = \frac{300 - 20P}{20P}$$

  a) if $P_1 = 10$ your margin ratio is 50%. You cannot increase your short position, so your total return is

$$\frac{200}{100} = 200\%.$$

Margin Calls - Example

b) if $P_1 = 12.5$ your margin ratio is 20%. This means that you must liquidate part of your position, i.e. you need to buy back Δ units of the security, such that

$$\frac{300 - 12.5\Delta - (20 - \Delta)12.5}{(20 - \Delta)12.5} = 0.5$$

which gives you $\Delta = 12$ and you are left with 8 units. So you sold 20 units at 10 and bought 12 units at 12.5 giving you a return of

$$\frac{20 \times 10 - 12 \times 12.5}{100} = 50\%$$

b) if $P_1 = 15$ your margin ratio is 0. You are totally wiped out and must liquidate your position, i.e. buy back 20 units now at a price of 15. Your return is

$$\frac{20 \times 10 - 20 \times 15}{100} = -100\%$$

Margin Calls - Example

Suppose that the three scenarios were equally likely:

a) The expected return of this strategy is

$$\frac{1}{3} \times 200\% + \frac{1}{3} \times 50\% + \frac{1}{3} \times (-100\%) = 50\%$$

b) The variance of this strategy is

$$\frac{1}{3} \times (200\% - 50\%)^2 + \frac{1}{3} \times (50\% - 50\%)^2 + \frac{1}{3} \times (-100\% - 50\%)^2 = 150\%$$

and the standard deviation is $\sqrt{150\%} = 122\%$.

c) The Sharpe ratio of this strategy is $0.50/1.22 = 0.408$. Given that the Sharpe ratio of the "HML" strategy is around 0.4, this doesn't sound like such an amazing deal anymore!

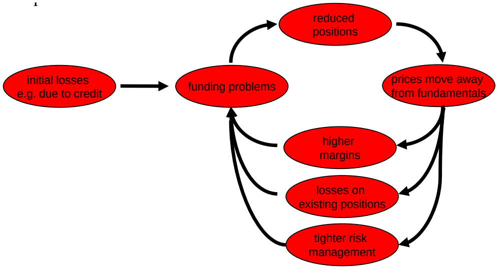
- The previous example highlights the idea that there are *limits to arbitrage*. This idea is advanced by Shleifer and Vishny (1996).
- Margin calls are one way that this could happen.
- Fund managers may also have to deal with fund outflows, which may increase if the fund has under-performed in the short run.
- Funds are aware of this issue and take steps to prevent this. Remember DFAs redemption policy?
- Nevertheless, sometimes things just go wrong (LTCM in 1998, several funds in 2007/08).

Liquidity spirals

- Liquidity is provided by market makers, hedge funds, speculators
- If speculators are well funded (large capital W and/or low margins m), then they can trade more (larger x^+ and x^-), which enhances market liquidity
  - ↪ Funding liquidity is a driver of market liquidity
- There is also feedback in the opposite direction:
  - ↪ Better market liquidity can lower margins because financiers more willing to lend when they can more easily and quickly sell the collateral
  - ↪ market liquidity can lower volatility, easing funding restriction
- This mutual feedback can give rise to liquidity spirals

Liquidity Spirals

- Some traders hit or near margin constraints (or risk limits)
- These traders reduce positions, which
  - ↪ moves prices against them (and others with similar positions) leading to further losses
  - ↪ increases volatility and reduces market liquidity, leading to increased margins and tightened risk management
- Increase in volatility leads to tighter margin constraints, so now more traders are affected

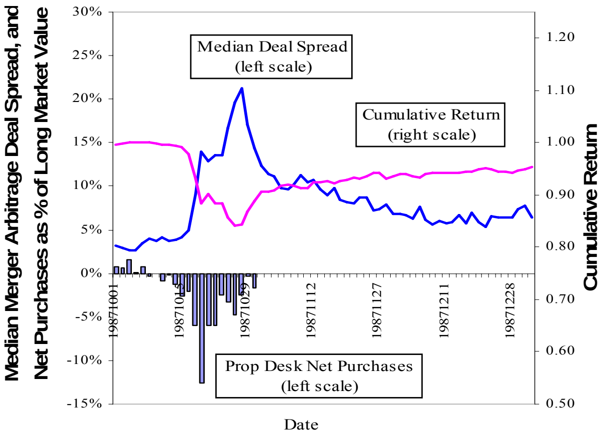


Merger Arbitrage and the 1987 Crash

Oct. 14-16	House Ways and Means Committee proposed legislation containing anti-takeover provisions
Oct. 19-20	Stock market crash
Oct: 21-31	Stock market rebounds

- Eventually, Congress backs off proposed legislation
- Merger-arbitrage traders had lost a significant amount of capital
- What happened to merger spreads?

Merger Arbitrage and the 1987 Crash

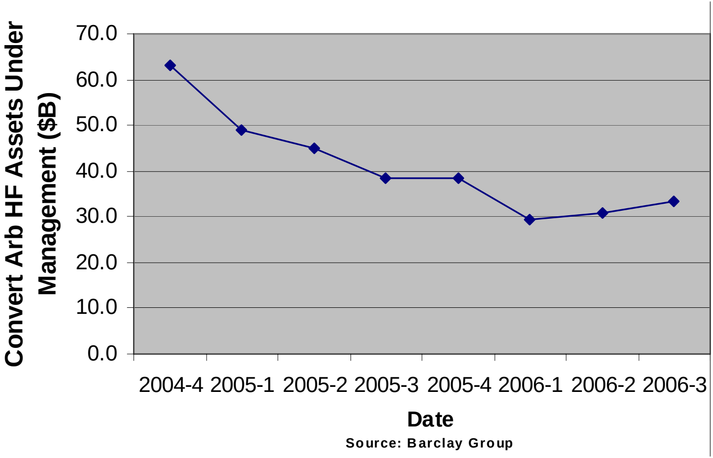


- Berkshire Hathaway Annual Report (Warren Buffett): During 1988 we made unusually large profits from [risk] arbitrage...the trick, a la Peter Sellers in the movie, has simply been ‘Being There.’

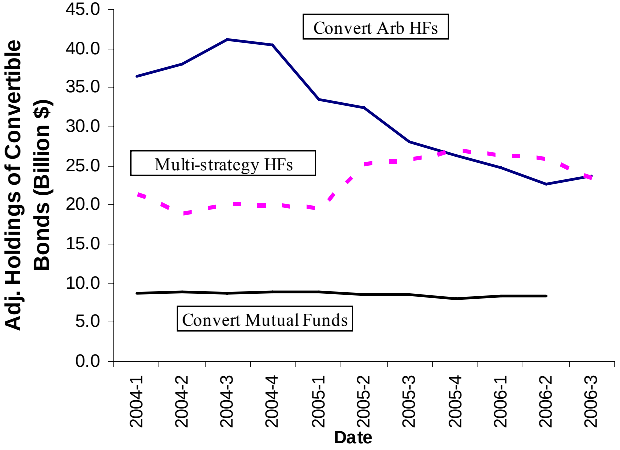
2005 - Convertible Bond Event

- 2005: Convertible bond hedge funds experience outflows
  - ↪ 2005Q1: 20% capital redeemed
  - ↪ 2005Q1 - 2006Q1: assets fell by half
- Single-strategy hedge funds:
  - ↪ forced to sell convertible bonds
- Multi-strategy hedge funds had a choice:
  - ↪ what do you think that they did?
- What happened to the price of convertible bonds?

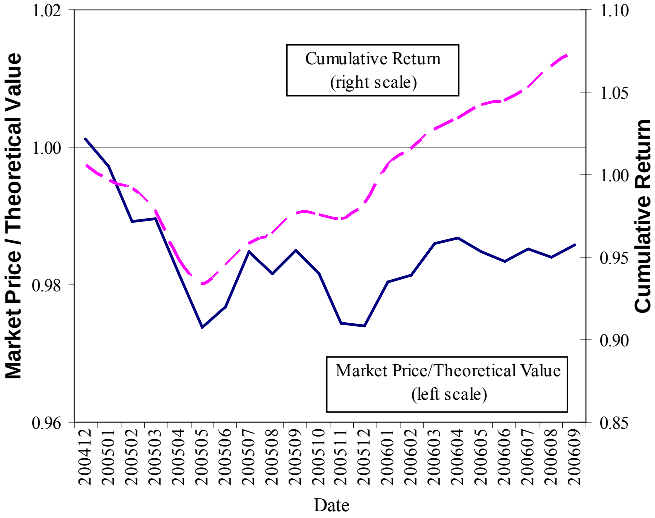
Redemptions in 2005



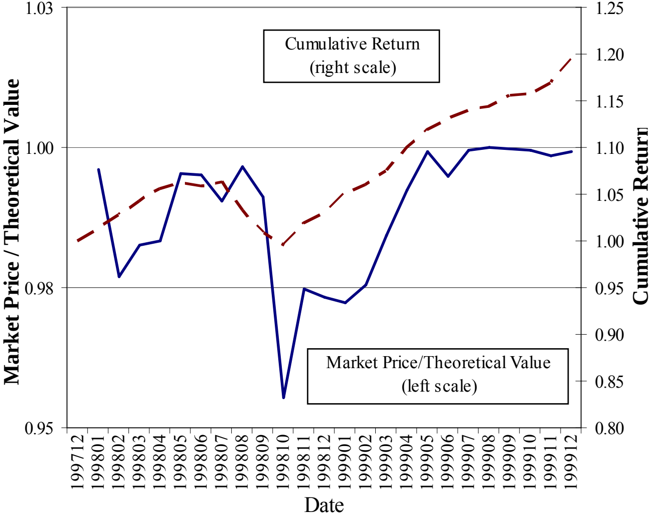
Adjusted Holdings of Convertible Bonds



Market Price / Theoretical Value



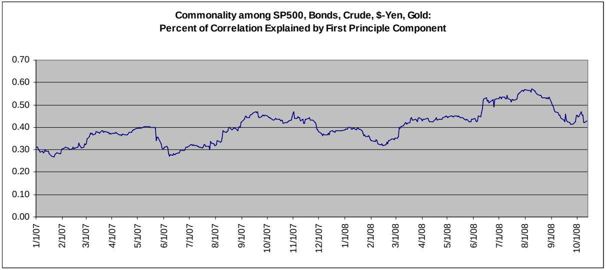
Market Price / Theoretical Value during LTCM crisis

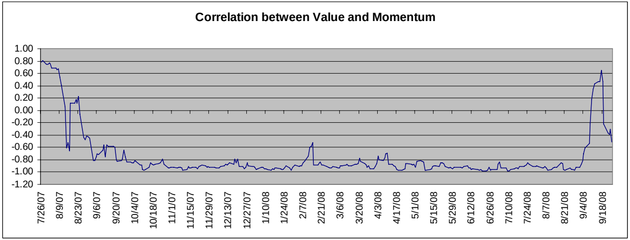


Limits to Arbitrage and Liquidity

- Arbitrage opportunities may look good on paper.
- Most of these opportunities are present in illiquid securities.
- Everyone seeks the highest alpha portfolio $\Rightarrow$ most arbitrageurs follow similar strategies
  - $\hookrightarrow$ If arbitrageurs liquidate in a hurry, they might have to do it in fire-sale prices.
  - $\hookrightarrow$ Forced sales impose externality to other investors
  - $\hookrightarrow$ In times of crisis, correlations between strategies may increase due to forced liquidation

Correlations Increase in times of Illiquidity





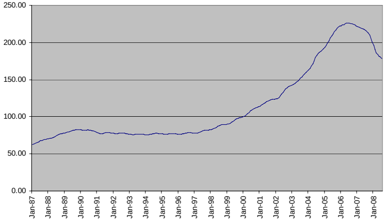
Contagion – Example: 2008/09 Crisis

- Sometimes these liquidity events can spill over to other asset classes
- 2008/09 Housing crisis
  - $\hookrightarrow$ Housing bubble and burst
  - $\hookrightarrow$ Large losses in the levered financial sector spill over to the rest of the economy
- Liquidity spirals as
  1. Banks' balance sheets deteriorate $\Rightarrow$ banks de-lever, selling assets across the board
  2. risk management tighten, lending reduced, counterparty exposures minimized
  3. margins increase, liquidity vanishes
  4. prices drop, back to step 1.

Housing Bubble Crisis

The trigger

Case-Shiller CSXR

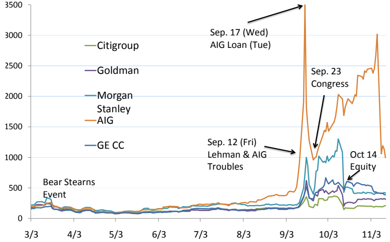


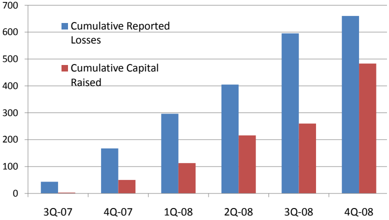
Real Estate Exposure

	Loans	HELOC / 2nd mort.	Agency MBS	Non-agency AAA	CDOs - subord.	Non-CDO - subord.	Total
US banks & thrifts	2020	869	852	383	90	0	4212
GSEs & FHLB	444	0	741	308	0	0	1493
Broker/Dealers	0	0	49	100	130	24	303
Financial guarantors	0	62	0	0	100	0	162
Insurance companies	0	0	856	125	65	24	1070
Overseas	0	0	689	413	45	24	1172
Other	461	185	1175	307	46	49	2268
Total	2925	1116	4362	1636	476	121	10680

Figures in $ billion
Source: Lehman Brothers (April 2008)

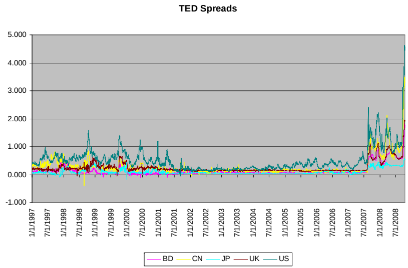
Losses and Funding Liquidity Problems for Banks

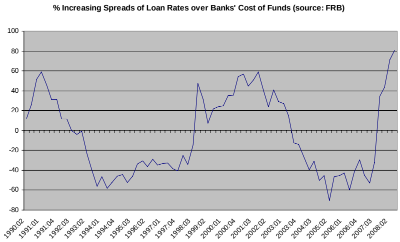




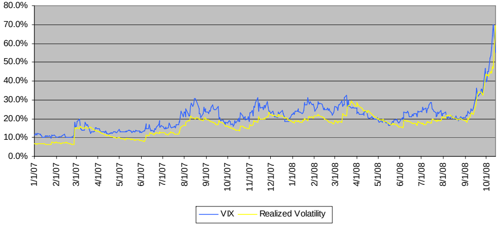
Source: Bloomberg

Banks tighten risk management and reduce Inter-bank Lending: Funding Spreads Rise



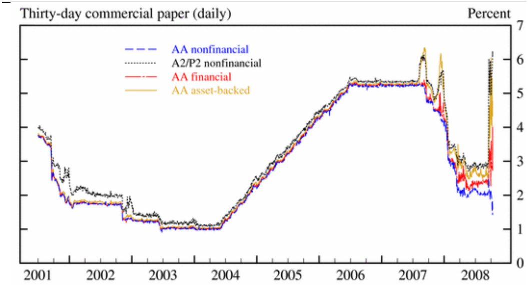


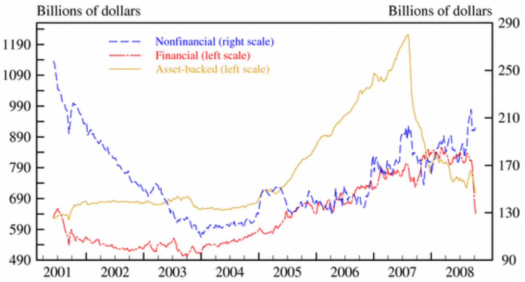
Volatility spikes, further increasing margin requirements



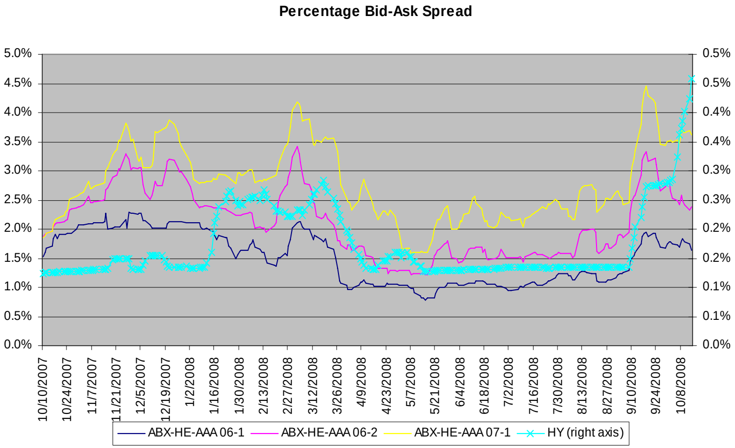
Kellogg School of Management

Further Funding Problems: Commercial Paper Market





Market Liquidity Deteriorates: Bid-Ask Spreads



Anatomy of a typical 'banking' crisis

- Banks or other speculators take levered-up positions in "arbitrage opportunities", i.e. high return, low (apparent) risk assets.
- A small trigger (the collapse of the bubble, the Fed raising rates) can lead to large losses to speculators
- If speculators get in trouble, they have to delever, further driving down market prices in the process
- As prices drop, more speculators get into trouble and reduce their holdings of risky securities
- Typical outcome
  - ↪ Funding illiquidity (limits to arbitrage) lead to market illiquidity.
  - ↪ Flight to quality, preference for holding Treasuries.
  - ↪ If speculators lucky or "too big to fail", government bails them out.

Speculator health and 'risk aversion'

- A common narrative during the crisis is that 'risk aversion increases'. Why?
- There are two types of investors in the economy, speculators S and retail investors R, each with wealth X_S and X_R. Both invest in risky assets

$$w_S = \frac{E(R_m) - r_f}{A_S \sigma_m^2} \quad \text{and} \qquad w_R = \frac{E(R_m) - r_f}{A_R \sigma_m^2}$$

- Suppose that S have higher risk tolerance than R, so $A_S < A_R$. In this case, speculators lever up, borrowing money from retail investors.
- In a crisis, since speculators have levered positions, they suffer much more than retail investors, so X_S relative to X_R drops
- How do market prices depend on the financial health X_S of speculators?

Speculator health and ‘risk aversion’

- Clearing the market for the risk-free asset implies that

$$E(R_m) - r_f = \bar{A}\,\sigma_m^2$$

where $\bar{A}$ is the (harmonic) average risk-aversion, weighted by relative wealth of the two groups

$$\bar{A} = \left(\frac{X_S}{X_R + X_S} A_S^{-1} + \frac{X_R}{X_R + X_S} A_R^{-1} \right)^{-1}$$

- What happens to $\bar{A}$ – and therefore $E(R_m) - r_f$ – as X_S decreases relative to X_R?
  - ↪ Since $A_S < A_R$, the ‘average investor’ ’s risk aversion becomes closer to A_R, expected returns rise – and prices fall today
  - ↪ As X_S falls relative to X_R, the speculators reduce their demand for risky securities. Retail investors are more risk averse and demand a higher risk premium – lower prices – to hold these risky securities

A factor model with liquidity **risk**

- The Amihud and Mendelson model assumes that liquidity is *constant*
  - $\hookrightarrow$ You are only compensated for the *expected* transaction costs.
- However, liquidity fluctuates over time.
- Several studies have documented commonality in liquidity
- If liquidity varies over time, then liquidity *risk* should command an additional risk premium.
- Such a common liquidity shock cannot be diversified and may be a priced risk factor

A factor model with liquidity risk

- Consider unexpected shocks to measure of illiquidity L_t

$$U_t = L_t - E_{t-1}[L_t]$$

1. Estimate exposures of stock returns on the market return and the unexpected changes in liquidity:

$$R_i t = a_i + \beta_{Mi} R_{Mt} + \gamma_i U_t + e_{it}$$

2. Expected returns as function of market and liquidity exposures

$$E(R_i t) = r_f + \beta_{Mi}(E[R_M] - r_f) + \lambda\gamma_i,$$

where λ is the liquidity risk premium.

- Pastor and Stambaugh (2003) and Sadka (2006) find positive and significant estimates of λ.

A factor model with liquidity risk

TABLE 7
PROPERTIES OF PORTFOLIOS SORTED ON HISTORICAL LIQUIDITY BETAS

	Decile Portfolio										
	1	2	3	4	5	6	7	8	9	10	10–1
	A. Liquidity Betas										
Jan. 1968–Dec. 1999	−6.02	−.65	−.62	−.54	1.12	−1.58	1.37	2.00	3.04	−.04	5.99
	(−2.57)	(−.37)	(−.48)	(−.41)	(.96)	(−1.24)	(1.00)	(1.49)	(1.99)	(−.02)	(1.88)
Jan. 1968–Dec. 1983	−7.59	−1.17	3.87	−1.54	−.48	1.65	−1.18	.02	1.26	.41	7.99
	(−1.84)	(−.44)	(1.86)	(−.68)	(−.25)	(.71)	(−.55)	(.01)	(.54)	(.14)	(1.60)
Jan. 1984–Dec. 1999	−4.17	−1.49	−4.10	−.30	2.55	−2.75	2.80	3.79	4.38	1.18	5.35
	(−1.52)	(−.63)	(−2.46)	(−.18)	(1.72)	(−2.00)	(1.56)	(2.08)	(2.07)	(.39)	(1.26)
	B. Additional Properties, January 1968–December 1999										
Market cap	7.11	7.69	10.44	17.65	16.76	22.18	16.26	11.64	9.89	6.97	
Liquidity	−.52	−.19	−.06	−.04	−.02	−.05	−.05	−.05	−.05	−.12	
MKT beta	1.12	1.09	1.02	.96	.98	.99	1.02	1.01	1.02	1.09	−.03
	(37.25)	(48.37)	(61.23)	(56.63)	(65.92)	(59.99)	(58.01)	(58.52)	(51.53)	(40.84)	(−.74)
SMB beta	.37	−.00	−.13	−.16	−.09	−.15	−.11	−.00	.04	.16	−.20
	(8.02)	(−.02)	(−5.11)	(−6.03)	(−4.21)	(−6.10)	(−4.19)	(−.02)	(1.20)	(4.06)	(−3.25)
HML beta	−.20	−.05	.02	−.02	.10	.12	.07	.09	−.01	−.15	.05
	(−4.04)	(−1.31)	(.87)	(−.80)	(4.22)	(4.40)	(2.60)	(3.27)	(−.38)	(−3.39)	(.76)
MOM beta	.04	−.00	.02	.01	−.02	−.00	−.01	.01	−.02	−.01	−.05
	(1.64)	(−.18)	(1.25)	(1.13)	(−1.91)	(−.17)	(−.76)	(.65)	(−1.11)	(−.46)	(−1.51)

NOTE.—At each year end between 1967 and 1998, eligible stocks are sorted into 10 portfolios according to historical liquidity betas. The betas are estimated as the slope coefficients on the aggregate liquidity innovation in regressions of excess stock returns on that innovation and the three Fama-French factors. The regressions are estimated using the most recent five years of data, and eligible stocks are defined as ordinary common shares traded on the NYSE, AMEX, or NASDAQ with five years of monthly returns continuing through the current year end and with stock prices between \$5 and \$1,000. The portfolio returns for the 12 postranking months are linked across years to form one series of postranking returns for each decile. Panel A reports the decile portfolios' postranking liquidity betas, estimated by regressing value-weighted portfolio excess returns on the liquidity innovation and the Fama-French factors. Panel B reports the time-series averages of each decile's market capitalization and liquidity, obtained as value-weighted averages of the corresponding measures across the stocks within each decile. Market capitalization is reported in billions of dollars. A stock's liquidity in any given month is the slope coefficient $\gamma_{i,t}$ from eq. (1), multiplied by 100. Also reported are postranking betas with respect to the Fama-French and momentum factors, estimated by regressing value-weighted portfolio excess returns on the four factors. The *t*-statistics are in parentheses.

TABLE 8

Alphas of Value-Weighted Portfolios Sorted on Historical Liquidity Betas

	Decile Portfolio										
	1	2	3	4	5	6	7	8	9	10	10–1
	A. January 1968–December 1999										
CAPM alpha	−2.06	−.36	.63	.49	.07	.49	1.42	1.36	−.02	2.60	4.66
	(−1.30)	(−.34)	(.76)	(.57)	(.10)	(.58)	(1.64)	(1.63)	(−.02)	(1.96)	(2.36)
Fama-French alpha	−.62	−.09	.46	.57	−.62	−.28	.90	.84	.03	3.53	4.15
	(−.42)	(−.08)	(.57)	(.68)	(−.86)	(−.35)	(1.06)	(1.00)	(.03)	(2.71)	(2.08)
Four-factor alpha	−1.20	−.04	.22	.34	−.29	−.25	1.05	.71	.29	3.67	4.87
	(−.79)	(−.04)	(.26)	(.40)	(−.40)	(−.31)	(1.20)	(.82)	(.29)	(2.74)	(2.38)
	B. January 1968–December 1983										
CAPM alpha	−1.10	1.04	.94	.35	−.28	.46	.09	.83	.33	2.51	3.62
	(−.46)	(.70)	(.79)	(.27)	(−.26)	(.34)	(.08)	(.72)	(.25)	(1.51)	(1.32)
Fama-French alpha	−1.24	2.32	1.66	1.53	−1.05	−.49	−.06	−.07	.17	1.61	2.85
	(−.53)	(1.56)	(1.41)	(1.21)	(−.98)	(−.38)	(−.05)	(−.06)	(.13)	(1.01)	(1.01)
Four-factor alpha	−3.74	1.50	.87	.86	−.20	.21	.59	−.18	.59	1.64	5.38
	(−1.58)	(.96)	(.71)	(.66)	(−.18)	(.16)	(.47)	(−.15)	(.43)	(.98)	(1.86)
	C. January 1984–December 1999										
CAPM alpha	−2.79	−1.63	.21	.40	.37	.23	3.12	1.70	−.11	2.70	5.49
	(−1.31)	(−1.04)	(.18)	(.36)	(.36)	(.23)	(2.51)	(1.40)	(−.08)	(1.28)	(1.90)
Fama-French alpha	.03	−2.04	−.60	−.33	−.40	−.55	2.21	1.50	−.11	4.41	4.38
	(.02)	(−1.29)	(−.53)	(−.30)	(−.40)	(−.59)	(1.83)	(1.22)	(−.07)	(2.20)	(1.54)
Four-factor alpha	.57	−1.50	−.50	−.28	−.39	−.87	2.06	1.35	.02	4.55	3.98
	(.30)	(−.94)	(−.44)	(−.25)	(−.38)	(−.93)	(1.68)	(1.08)	(.01)	(2.23)	(1.38)

Note.—See the note to table 7. The table reports the decile portfolios' postranking alphas, in percentage per year. The alphas are estimated as intercepts from the regressions of excess portfolio postranking returns on excess market returns (CAPM alpha), on the Fama-French factor returns (Fama-French alpha), and on the Fama-French and momentum factor returns (four-factor alphas). The t-statistics are in parentheses.

A factor model with liquidity risk

TABLE 11

LIQUIDITY RISK SPREADS AND INVESTMENT OPPORTUNITIES: ALPHAS FROM THE REGRESSION OF MOMENTUM ON PORTFOLIOS LISTED

	January 1966–December 1999	January 1966–December 1982	January 1983–December 1999
MKT, SMB, HML	16.30	21.65	11.10
	(4.85)	(4.53)	(2.29)
MKT, SMB, HML, LIQ^V	13.89	19.46	8.03
	(4.09)	(4.04)	(1.63)
MKT, SMB, HML, LIQ^E	8.41	16.11	−1.29
	(2.55)	(3.35)	(−.28)

NOTE.—The table reports the alphas (percent per year) of the momentum portfolio MOM with respect to the factors listed in each row. These factors include the Fama-French factors MKT, SMB, and HML and two liquidity risk spreads, both of which go long decile 10, containing the stocks with the highest predicted liquidity betas, and short decile 1, containing the stocks with the lowest betas. Each leg of the spread is value-weighted in LIQ^V and equally weighted in LIQ^E. The t-statistics are in parentheses.

Conclusion

- Market frictions can reconcile two apparently conflicting pieces of evidence:
  i) We have seen a number of market 'anomalies' or arbitrage opportunities that have persisted over time.
  ii) On the other hand, there is little evidence that professional money managers outperform the market.
- Arbitrage opportunities may look good on paper. However, once transaction costs have been factored in, their profitability is diminished.
- Also, most of these opportunities are present in illiquid securities. If the arbitrageur needs to liquidate in a hurry he might have to do it in fire-sale prices.
- **Bottom line:** when you start your hedge fund, make sure you have ample lines of credit