FINC460 - Final Exam

Winter 2008

NAME: ______________________________ SECTION: __________

1. Please do not open this exam until directed to do so.
2. This exam is 3 hours long.
3. Please write your name and section number on the front of this exam, and on any examination books you use.
4. Please show all work required to obtain each answer. Answers without justification will receive no credit.
5. This is a closed book exam. No books, notes or laptops are permitted. Calculators are permitted.
6. Brevity is strongly encouraged on all questions.
7. The exam is worth 110 points.
8. Relax, and good luck!

Hints:

1. *Think through problems before you start working*
2. *Draw pictures*
3. *If you get stuck on part of a problem, go on to the next part. You may need to use answers from earlier parts of the question to calculate answers to the later parts. If you werent able to solve the earlier part, assume something.*
4. *Remember, setting up the problem correctly will get you most of the points.*

Short questions

Assess the validity of the following statements (True, False or Uncertain) and explain your answers.

1. (5 points) Since its variance is always positive, a risky asset can *never* have a negative rate of expected return.

   FALSE. An asset can have a negative expected return, for example if the CAPM holds and the beta is negative.

2. (5 points) A fund manager can increase his Sharpe Ratio by leveraging his portfolio with the risk-free asset.

   FALSE. The Sharpe Ratio is immune to changes in leverage because both the numerator and the denominator scale proportionally.

3. (5 points) A factor model is a statistical model for returns that assumes that individual firms have no idiosyncratic risk.

   FALSE. A factor model is a model for returns that assumes that idiosyncratic risk is uncorrelated across assets.

4. (5 points) The stock price of acquisition targets increases before takeover announcements. This is a clear violation of efficient markets.

   FALSE. If anything it is consistent with efficient markets, especially strong-form efficiency which states that prices incorporate all information, public and private.

5. (5 points) The CAPM states that the market portfolio is the single source of systematic risk and β is its price, i.e. the slope of the SML.

   FALSE. Even though the first part is correct, the slope of the SML is $ER_M - r_f$*, not beta.*

Question 1

A two-factor APT describes the returns of all well-diversified portfolios. The two factors are unexpected changes in production (factor 1) and an inflation factor (factor 2).

- Over the next year, the market expects production to grow at 5% and inflation to be 2%
- The prices of all well diversified portfolios are set so that their expected returns over the next year are given by:

$$E(\tilde{r}_i) = 0.05 + 0.08\ b_{i,1} - 0.06\ b_{i,2}.$$

  where $b_{i,k}$ denotes portfolio i's loading on the k'th factor.

- The market believes that the standard deviations of $\tilde{f}_1$ and $\tilde{f}_2$, over the next year are all 0.10 (10%), and that the two factors are uncorrelated.
- The return generating process for portfolios A, B and C over the next year are:

$$\begin{aligned}
\tilde{r}_A &= E(\tilde{r}_A) + 0.7\tilde{f}_1 - 0.5\tilde{f}_2 \\
\tilde{r}_B &= E(\tilde{r}_B) + 1.2\tilde{f}_1 + 0.3\tilde{f}_2 \\
\tilde{r}_C &= E(\tilde{r}_C) + 1.1\tilde{f}_1 - 1.2\tilde{f}_2
\end{aligned}$$

Based on this scenario, answer the following questions:

1. (5 points) Find the expected return of portfolio A

$$\begin{aligned} E\tilde{r}_A &= 0.05 + 0.08\ b_{A,1} - 0.06\ b_{A,2} \\ &= 0.05 + 0.08 \times 0.7 + 0.06 \times 0.5 \\ &= 13.6\% \end{aligned}$$

2. (5 points) Find the return standard deviation of portfolio A.

$$\begin{aligned} var(\tilde{r}_A) &= b_{A,1}^2 var(f_1) + b_{A,2}^2 var(f_2) \\ &= 0.7^2 \times 0.1^2 + 0.5^2 \times 0.1^2 \\ &= 0.0074 \end{aligned}$$

and $\sigma(r_A) = \sqrt{0.0074} = 0.086$

3. (5 points) What is the risk-free rate implied by the absence of arbitrage?

It's 5%, i.e. the return of an asset with zero systematic risk ($b_1 = b_2 = 0$).

4. (10 points) If production grows by 10% over the next year, and inflation is exactly what the market expects, what will the return on portfolio A be?

$$\begin{aligned} \tilde{r}_A &= E(\tilde{r}_A) + 0.7\tilde{f}_1 - 0.5\tilde{f}_2 \\ &= 0.136 + 0.7(0.1 - 0.05) - 0.5(0.02 - 0.02) \\ &= 0.136 + 0.7(0.1 - 0.05) - 0.5(0.02 - 0.02) \\ &= 0.171 \end{aligned}$$

5. (10 points) The second factor here is an inflation factor. Give an economic rationale for why the factor risk premium for this inflation factor should be positive or negative. Specifically, answer the following questions. *All explanations should* very *brief, but a complete answer should include an economic story, and not only be based on numbers.*

   (a) Consider portfolio B which has a positive loading on this factor. Will the return on B be unexpectedly high or low in when inflation is higher or lower than expected?

   Portfolio B's return will be higher when inflation is higher than expected.

   (b) Based on this, would you think that B would have a higher or lower expected return than portfolio C? Explain.

   Assuming that we dislike inflation, we prefer having a dollar when inflation is high to a dollar when inflation is low (in real terms). As a result, assets that pay off when inflation is high, like asset B, should be more expensive, or equivalently have lower expected returns. This implies that the price of the inflation factor is negative.

6. (10 points) Assume that you believe very strongly that inflation will be a great deal lower than the market expects. Your estimate for inflation for the next year is zero, while, as discussed above, the market expects that it will be 2%. Assume that you wish to construct a portfolio (using A, B and C) which will take advantage of this. You do not wish to have any exposure to risk related to production, but you want your portfolio to earn an extra 4% /year (above what the market expects) if your conjecture about inflation is correct.

   (a) What would the factor loadings of your portfolio be?

   (b) What fraction of your portfolio should be invested in A, B, and C?

   (c) What does the market think the expected return on your portfolio is (per year)?

   (d) What do you think the expected return on your portfolio is

If you want the portfolio to have zero production risk, it must have a loading of $b_{p,1} = 0$ *with respect to the first factor, whereas if you want your portfolio to yield a 4% higher return if the realization on factor 2 is 2% lower than expected, you need a loading of* $b_{p,2} - 2$ *on the second factor. This answers part (1).*

As a result, the portfolio weights (w_1, w_2, w_3) *need to satisfy the following system of equations*

$$\begin{aligned} w_1 + w_2 + w_3 &= 1 \\ 0.7w_1 + 1.2w_2 + 1.1w_3 &= 0 \\ -0.5w_1 + 0.3w_2 - 1.2w_3 &= -2 \end{aligned}$$

The solution to this system is $[2.34, -1.62, 0.28]$*, which answers part (2).*

The market thinks that our portfolio has an expected return of

$$E\tilde{r}_p = 0.05 + 0.08 \times 0 - 0.06 \times (-2) = 17\%$$

whereas we think it is 4% higher, i.e. 21%. This answers parts (3) and (4).

7. (10 points) Assume that you estimate that the market portfolio is an equally weighted average of the returns of portfolios A, B and C. This means that returns on the market portfolio are given by

$$R_M = \frac{1}{3}R_A + \frac{1}{3}R_B + \frac{1}{3}R_C$$

Does the CAPM hold in this economy?

Hint 1: All portfolios on the frontier are diversified, which means that they are linear combinations of the two factors (more precisely the factor-mimicking portfolios).

Hint 2: The MVE portfolio weights when there are two risky assets A and B ($x_B = (1 - x_A)$) are:

$$x_A = \frac{E(\tilde{r}_A^e)\sigma_B^2 - E(\tilde{r}_B^e)cov(\tilde{r}_A^e, \tilde{r}_B^e)}{E(\tilde{r}_A^e)\sigma_B^2 + E(\tilde{r}_B^e)\sigma_A^2 - [E(\tilde{r}_A^e) + E(\tilde{r}_B^e)]\, cov(r_A^e, r_B^e)}$$

The market portfolio is

$$\begin{aligned} r_M &= \frac{1}{3}r_A + \frac{1}{3}r_B + \frac{1}{3}r_C \\ &= E(r_M) + (\frac{1}{3}0.7 + \frac{1}{3}1.2 + \frac{1}{3}1.1)f_1 + (-\frac{1}{3}0.5 + \frac{1}{3}0.3 - \frac{1}{3}1.2)f_2 \\ &= E(r_M) + 1f_1 - 0.46f_2 \end{aligned}$$

Now consider two portfolios: the first has a loading of 1 on the first factor and 0 on the second factor, and the other has a loading of 0 on the first factor and 1 on the second. The two factor mimicking portfolios will then have

$$Er_1 = 0.05 + 0.08 = 0.13 \qquad \sigma(r_1) = 10\%$$
$$Er_2 = 0.05 - 0.06 = -0.01 \qquad \sigma(r_2) = 10\%$$

and the two portfolios are uncorrelated. Now, all portfolios on the frontier will be linear combinations of these two factor mimicking portfolios. The MVE efficient portfolio will have weights

$$x_1 = \frac{E(\tilde{r}_1^e)\sigma_1^2}{E(\tilde{r}_A^e)\sigma_B^2 + E(\tilde{r}_B^e)\sigma_A^2} = \frac{0.08 \times 0.1^2}{0.08 \times 0.1^2 - 0.06 \times 0.1^2} = 4$$

and $x_2 = 1 - x_1 = -3$.

Now, if the CAPM holds, the market portfolio is *the* mean variance efficient portfolio. Here it is not. If we scale the market by 4 so it has a loading of $4 \times 1 = 4$ on the first factor, it will have a loading of $-4 \times 0.46 = -1.84$ on the second factor, so the market portfolio is *not* MVE, so the CAPM does not hold.

Question 2

You are managing the endowment of the Minnesota Institute of Technocracy (MIT). You are currently 100% invested into the market portfolio and this is your only source of income. Assume that the risk-free rate is 4%.

Your analyst has gathered data on the following two mutual funds:

	μ	σ	β_{MKT}	β_{HML}	β_{SMB}	β_{MOM}
A	18%	19%	0.4	1.1	0.1	1.2
B	18%	11%	0.2	-0.3	-0.9	0.1

where β_i refers to the beta of fund returns with factor i, i.e. the market portfolio (MKT), the two Fama-French factors (SMB, HML) and the momentum strategy (MOM). Also, μ refers to the expected return on the fund and σ its standard deviation.

In addition, you know that expected returns on the market is 7%, on the HML factor is 5%, on the SMB factor its 3% and on the momentum strategy is 9%. The standard deviation of all 4 factors is 10% and assume that they are uncorrelated with each other.

a) (10 points) Based on the table above, what can you infer about each of the two funds (A, B) investment strategy?

The first fund is investing in Value firms and Momentum, whereas the second find is investing mostly in growth stocks.

b) (10 points) You are thinking of switching 10% of your assets out of the market portfolio and into one of these funds. In other words, before you were 100% invested in the market portfolio, now you will be 90% invested in the market and 10% invested in one of these two funds. Which one would you pick?

you can answer this question in a couple of ways:

- If I invest 10% in fund A and the remaining on the market portfolio, my Sharpe Ratio will be

  $$SR = \frac{0.1 \times 0.18 + 0.9 \times 0.07 - 0.04}{\sqrt{0.9^2 \times 0.1^2 + 0.1^2 \times 0.19^2 + 2 \times 0.9 \times 0.1 \times 0.4 \times 0.1^2}} = 0.427$$

  where the last term in the denominator comes from the covariance of fund A with the market portfolio which equals $cov(R_A, R_m) = 0.4 \times var(R_m)$.

  Similarly, if I invest in fund B, a similar calculation yields that my Sharpe ratio will be 0.443.

  Based on this I prefer B.

- Alternatively, you could have calculated the Appraisal Ratio versus the market portfolio, and picked the fund with the highest appraisal ratio.

  The CAPM alpha of fund A equals $0.18 - 0.04 - 0.4 \times (0.07 - 0.04) = 0.128$, and its idiosyncratic standard deviation *with respect to the CAPM* equals $\sqrt{0.19^2 - 0.4^2 \times 0.1^2} = 0.185$. The Appraisal Ratio for the first fund is equal to 0.689 and for the second is 1.24. Based on this, we prefer fund B.

c) (10 points) Now you remember that, being a technology school, a lot of the alumni donations come from technology firms, which tend to be growth firms. Suppose your *overall* endowment portfolio can be described as

$$R_E = 1 \times MKT - 0.5 \times HML.$$

Does your answer to the previous question change?

Now we need to take that into account when calculating the Sharpe ratio (or the Appraisal Ratio). In other words, the covariance of these funds with the market portfolio will now be adjusted to account for the correlation with HML, that is

$$cov(R_A, R_E) = 1 \times 0.4 \times 0.1^2 - 0.5 \times 1.1 \times 0.1^2 = -0.0015$$

and

$$cov(R_B, R_E) = 1 \times 0.2 \times 0.1^2 + 0.5 \times 0.3 \times 0.1^2 = 0.0035$$

and if we calculate the Sharpe ratios for the ex-post combinations we get 0.453 and 0.436 respectively, which means that now we prefer fund A.