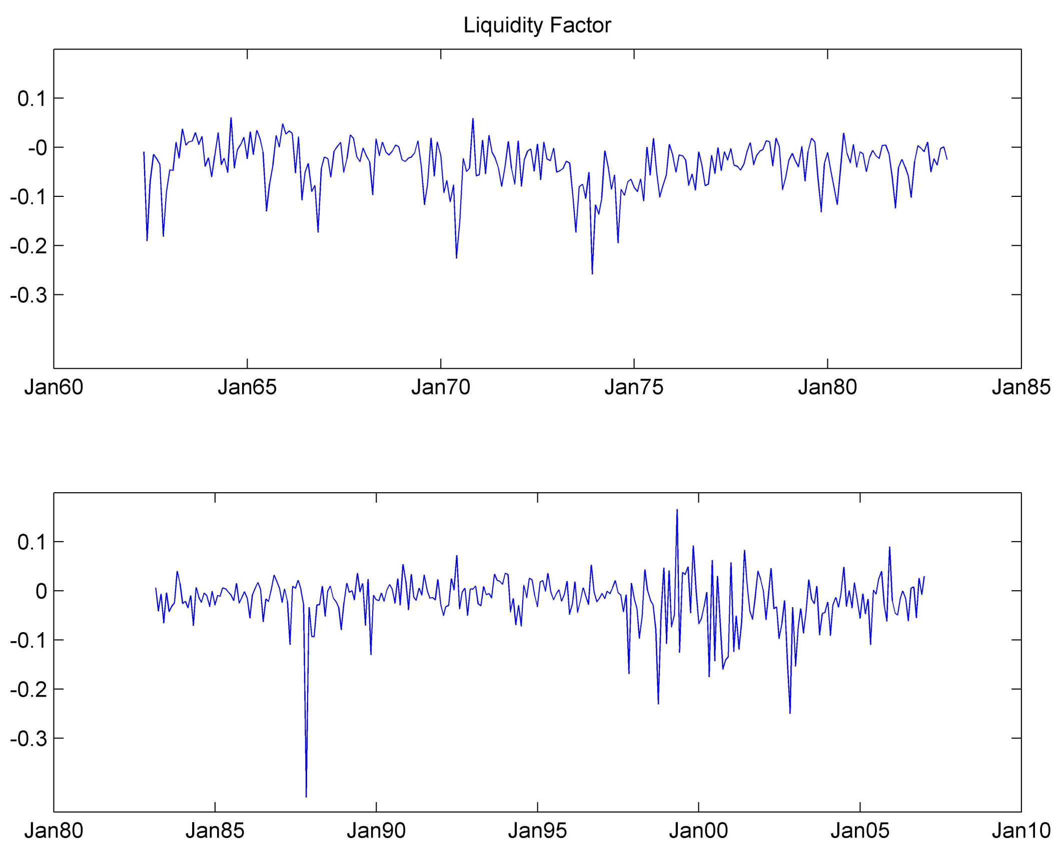
FINC312: Homework 6

Solution

1 Testing a Liquidity Factor Model

1. The liquidity factor had its lowest levels on November 1973 and October 1987. November 1973 was the first full month of the Mideast oil embargo, and October 1987 included the Black Monday stock market crash.



2. The innovation series is the fitted residual from a regression of the differences in the level series on the lagged differenced level series and the lagged level series (with scaling adjustments). The comovement in the innovation series and returns can be used as a measure of liquidity risk.

3. The historical average return for the market is 0.46%.

4. The factor loadings are:

Momentum Portfolio	b_{liq}	b_{mkt}	$\hat{\mu}$
$Losers$	-0.020	1.358	0.0016
R_2	-0.014	1.136	0.0071
R_3	-0.003	0.985	0.0083
R_4	-0.004	0.938	0.0088
R_5	0.023	0.887	0.0077
R_6	0.060	0.902	0.0088
R_7	0.053	0.887	0.0091
R_8	0.029	0.910	0.0114
R_9	0.039	0.969	0.0123
$Winners$	0.000	1.193	0.0162
$Winners - Losers$	0.020	-0.165	0.0145

5. The liquidity factor loadings are lowest for the losing portfolios. The loadings describe the sensitivity of the portfolio's returns to a shock to the liquidity factor. When liquidity is high (and positive), the losers portfolio returns are less than the winners portfolio returns. However, the standard error on b_{liq} for the Winners-Losers portfolio is 0.063, so the difference is not statistically significant.

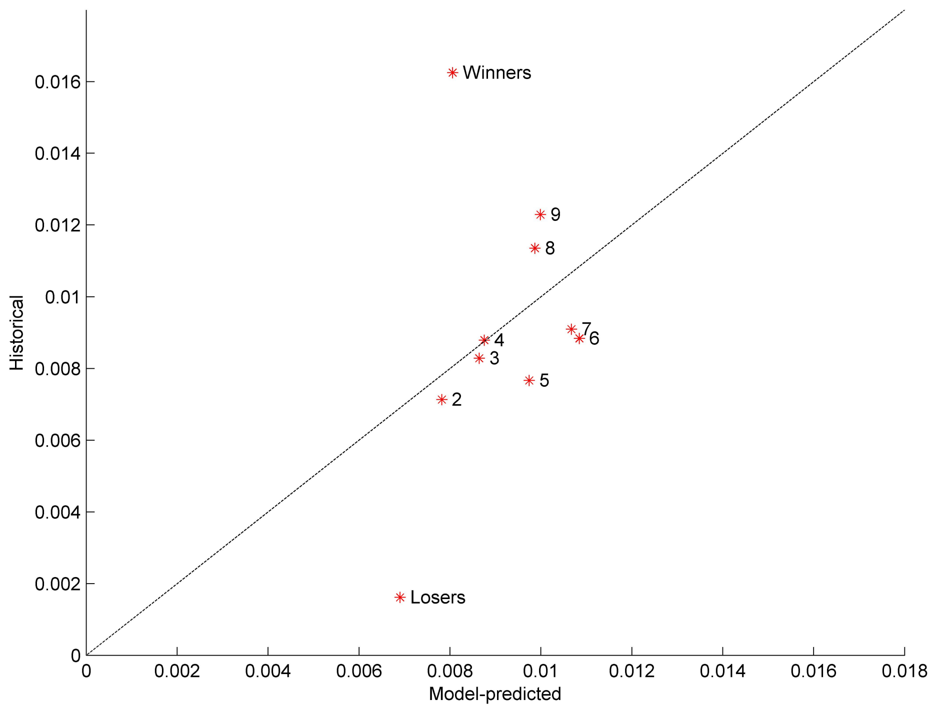
6. Historical averages shown in the table above.

7. The estimated market prices of risk are:

	λ	t-statistic
Constant	0.012	0.89
Market	-0.0033	-0.27
Liquidity	0.031	0.44
R^2	0.12	

8. The prices of risk λ_{mkt} and λ_{liq} are the expected return premium for bearing market and liquidity risk respectively. Neither are statistically significant from zero and the price of market risk is negative. The risk-free rate, λ_0, is 1.2% a month, which is very high.

9. Given the statistical insignificance of the prices of risk, it appears that liquidity risk is not priced in the cross-section of momentum portfolios. Also, liquidity does not seem to fully explain the momentum results since the liquidity model-predicted returns fail to capture the dispersion of the historical momentum returns.



2 Factor Models

1) Using the formula for covariances for a 2-factor model:

$$cov(R_i, R_j) = b_{i,1}\, b_{j,1} var(f_1) + b_{i,2}\, b_{j,2} var(f_2)$$

we obtain the following table

Asset	E(R)	$var(\epsilon)$	var(R)	R^2	Loadings		Covariance		
					b_1	b_2	A	B	C
A	0.36	0.15	2.15	93.0%	2	4	2	1.4	0.6
B	0.225	0.28	1.58	82.3%	3	2	1.4	1.3	0.5
C	0.12	0.05	0.25	80.0%	1	1	0.6	0.5	0.2

2) To answer this question we need to construct a portfolio that has zero loadings on factors 1 and 2. Denote the weights on each asset as $[w_A, w_B, w_C]$, we have three equations in three unknowns

$$\begin{aligned} w_A + w_B + w_C &= 1 \\ 2\,w_A + 3\,w_B + w_C &= 0 \\ 4\,w_A + 2\,w_B + w_C &= 0 \end{aligned}$$

The solution to these equations is $[-20.0\%, -40.0\%, 160.0\%]$ and the return on this portfolio equals 3%, which according to the APT must be the risk-free rate.

3) We use the formula for total variance:

$$var(R_i) = b_{i,1}^2 var(f_1) + b_{i,2}^2 var(f_2) + var(\epsilon_i)$$

and the formula for R^2:

$$R^2 = \frac{b_{i,1}^2 var(f_1) + b_{i,2}^2 var(f_2)}{var(R_i)}.$$

See the previous table for the results.

4) To answer this question we need to construct 2 portfolios that has zero loadings on factors 1 or 2 respectively. Denote the weights on each asset as $[w_A^k, w_B^k, w_C^k]$ for the k-factor mimicking portfolio. We have two sets of three equations in three unknowns

$$\begin{aligned} w_A^1 + w_B^1 + w_C^1 &= 1 \\ 2\,w_A^1 + 3\,w_B^1 + w_C^1 &= 1 \\ 4\,w_A^1 + 2\,w_B^1 + w_C^1 &= 0 \end{aligned}$$

yielding $w_A^1 = -0.4$, $w_B^1 = 0.2$ and $w_C^1 = 1.2$.

Similarly

$$\begin{aligned} w_A^2 + w_B^2 + w_C^2 &= 1 \\ 2\,w_A^2 + 3\,w_B^2 + w_C^2 &= 0 \\ 4\,w_A^2 + 2\,w_B^2 + w_C^2 &= 1 \end{aligned}$$

yielding $w_A^2 = 0.2$, $w_B^2 = -0.6$ and $w_C^2 = 1.4$.

Using these weights, the expected return of the 1-st factor-mimicking portfolio is 4.5% and the expected return on the 2-nd factor mimicking portfolio is 10.5%. Hence, their risk premia are $\lambda_1 = 1.5\%$ and $\lambda_1 = 7.5\%$

Last, to find their variances, I can use the fact that

$$var(R_1) = var(f_1) + (w_A^1)^2 var(\epsilon_A) + (w_B^1)^2 var(\epsilon_B) + (w_C^1)^2 var(\epsilon_C) = 0.2072$$

$$var(R_2) = var(f_2) + (w_A^2)^2 var(\epsilon_A) + (w_B^2)^2 var(\epsilon_B) + (w_C^2)^2 var(\epsilon_C) = 0.3048$$

The presence of idiosyncratic risk ϵ introduces additional variance into the factor mimicking portfolio than just the underlying factor.

5) We will use the markowitz spreadsheet in this exercise

   (a) Plugging everything into Markowitz, yields

Number of securities: 3

No	Name	Fraction	Expected Return	Standard Deviation
1	A	110%	0.36	147%
2	B	-21%	0.225	126%
3	C	12%	0.12	50%
		1.00		

Correlations		2	3
1		0.7596	0.8184
2		1	0.7956
		YES	

Loadings	1	2
A	2	4
B	3	2
C	1	1
P	1.673527	4.074899

Portfolio's Expected Return	0.3607
Portfolio's Standard Deviation	1.4607

Risk Free Rate	0.0300	**Risk Aversion Coefficient: A=**	2.00
Slope of CAL	0.2264	**Weight on optimal risky portfolio: x*=**	7.75%

(b) Introducing an additional constraint into Markowitz, namely that the loading of the portfolio on factor 2 is zero we get

Number of securities: 3

No	Name	Fraction	Expected Return	Standard Deviation
1	A	-58%	0.36	147%
2	B	75%	0.225	126%
3	C	83%	0.12	50%
		1.00		

Correlations		2	3
1		0.7596	0.8184
2		1	0.7956
		YES	

Loadings	1	2
A	2	4
B	3	2
C	1	1
P	1.91866	0

Portfolio's Expected Return	0.0588
Portfolio's Standard Deviation	0.7822

Risk Free Rate	0.0300	Risk Aversion Coefficient: A=	2.00
Slope of CAL	0.0368	Weight on optimal risky portfolio: x*=	2.35%

(c) Choosing between the two factor mimicking portfolios, I get

$$x_1 = \frac{\lambda_1 var(R_2)}{\lambda_1 var(R_2) + \lambda_2 var(R_1)} = \frac{0.015 \times 0.3048}{0.015 \times 0.3048 + 0.075 \times 0.2072} = 0.2273$$

and $x_2 = 1 - 0.2273 = 0.7727$

When we choose between the two factor mimicking portfolios only, we get a different answer. Why? The reason is that there is idiosyncratic risk ϵ that is not diversified away

6) The only way that the CAPM holds in this economy is if the market cap of A, B and C is proportional to the optimal portfolio weights in part (a) above