Investments - Spring 2013 Final Exam

NAME: ______________________________ SECTION: ____________

1. Please do not open this exam until directed to do so.
2. This exam is 3 hours long.
3. Please write your name and section number on the front of this exam, and on any examination books you use.
4. Please show all work required to obtain each answer. Answers without justification will receive no credit.
5. State clearly any assumptions you are making.
6. This is a closed book exam. No books or notes are permitted. Calculators are permitted. Laptops are permitted but you are only allowed to use Excel and only a blank worksheet. You are not allowed to use other spreadsheets with pre-entered formulas.
7. Brevity is strongly encouraged on all questions.
8. The exam is worth 160 points.
9. Relax, and good luck!

Hints:

1. *Think through problems before you start working. Draw pictures.*
2. *If you get stuck on part of a problem, go on to the next part. You may need to use answers from earlier parts of the question to calculate answers to the later parts. If you weren't able to solve the earlier part, assume something and move on.*
3. *Remember, setting up the problem correctly will get you most of the points.*

Short questions (50 points)

Assess the validity of the following statements (True, False or Uncertain) and explain your answers.

1. (10 points) All assets have zero alpha with respect to the mean variance efficient portfolio.

2. (10 points) Loss averse agents have higher risk aversion for losses than gains.

3. (10 points) It is not possible for the APT and the CAPM to both hold at the same time.

4. (10 points) In the APT, absence of arbitrage implies that all factors have a positive risk premium.

5. (10 points) If market makers were perfectly competitive, bid-ask spreads would be equal to zero.

Question 1 (60 points)

You are a managing a fixed income portfolio for your clients. You believe that the following factor model holds for bond returns:

$$
\begin{aligned}
r_t^1 &= 1.1\% + 1\,\tilde{f}_{L,t} - 1\,\tilde{f}_{S,t} \\
r_t^5 &= 2.0\% + 5\,\tilde{f}_{L,t} \\
r_t^{10} &= 4.0\% + 10\,\tilde{f}_{L,t} + 10\,\tilde{f}_{S,t},
\end{aligned}
$$

where r_t^n represents the return on a zero-coupon bond with maturity n, and $\tilde{f}_{i,t}$ represents factor i's surprise realizations. The two term structure factors, (f_L) and (f_S) are uncorrelated, i.e. $cov(f_L, f_S) = 0$, and $var(f_L) = 0.01$ and $var(f_S) = 0.005$.

Based on how bond returns load on these factors, we can characterize (f_L) and (f_S) as a 'level' and 'slope' factor respectively.

1. (10 points) Construct two portfolios that exactly mimic the two term structure factors, f_L and f_S. What are the weights on these portfolios?

2. (10 points) Construct a portfolio that has zero factor L and factor S risk.

3. (10 points) Find the risk free rate and the factor risk premia λ_S and λ_L implied by the absence of arbitrage opportunities.

4. (15 points) Find the portfolio that has the maximum Sharpe Ratio.

5. Assume now that you believe that the Federal Reserve is likely to raise short term interest rates by 100bps but because this move signals a tougher inflation regime, long term rates will actually fall by 100 bps. Thus you believe that the yield curve is likely to flatten with the level being unaffected. You believe, like everyone else, $\tilde{f}_L = 0$ over the next year, but that $\tilde{f}_S = 1\%$. Your share the same beliefs about variances as the rest of the market.

   (a) (5 points) Construct a portfolio that takes advantage of this view but has no L factor risk.

   (b) (5 points) What is the Sharpe Ratio of this portfolio according to the market?

   (c) (5 points) What is the Sharpe Ratio on this portfolio according to you?

Question 2 (50 points)

You are a partner in Rock and Roll Asset Management. Your firm uses the APT as a tool for managing money. Your team of analysts has determined that there are three factors in the economy: industrial production (IP), inflation (IN), and consumer confidence (CF). Furthermore, your analysts have determined that, over the next year, the risk-permia (the λ's) of the three factors will be as follows:

Factor	Risk Premium
Industrial Production (IP)	0.02
Inflation (IN)	-0.01
Consumer Confidence (CF)	0.00

In addition, they have determined that the loadings of the S&P 500 index on the three factors are as follows:

$$b_{IP} = 1.7 \qquad b_{IN} = 0.0 \qquad b_{CF} = 0.5$$

(Assume that the S&P 500 is a well-diversified portfolio, and therefore has only factor risk) They have also forecast the covariance matrix for the three factors for the coming year to be:

	f_{IP}	f_{IN}	f_{CF}
f_{IP}	0.01	0.00	0.00
f_{IN}	0.00	0.01	0.00
f_{CF}	0.00	0.00	0.04

Furthermore, assume that you can borrow or lend at the a rate of 3%/year.

1. (10 points) The second factor here is an inflation factor. Give an economic rationale for why the factor risk premium for this inflation factor should be positive or negative. Specifically, answer the following questions. *All explanations should* very *brief.*

   (a) Assume that a portfolio B has a positive loading on this factor (*i.e.*, $b_{B,IN} > 0$). Will the return on B be unexpectedly high or low when inflation is higher or lower than expected?

   (b) Based on this, would you think that B would have a higher or lower expected return than a portfolio C with $b_{C,IN} < 0$? Explain.

   (c) Based on this, explain why λ_{IN} should be positive or negative.

2. (10 points) What do your analysts expect the return and the standard deviation of the S&P 500 to be over the next year?

3. (10 points) Is the S&P 500 a mean-variance efficient portfolio?

4. (10 points) Assume you were to test the CAPM using the S&P 500 as a proxy for the “market.” Would you be likely to find that the CAPM held? Explain.

5. (10 points) Assuming the CAPM holds, what are the loadings of the market portfolio (the real one, not the S&P 500) on the three factors?

Finance 460 Final Equation Sheet

- Covariance Relations:

$$cov(a\tilde{x}, b\tilde{y}) = a \cdot b \cdot cov(\tilde{x}\tilde{y}) \qquad cov(\tilde{x}, \tilde{y} + \tilde{z}) = cov(\tilde{x}, \tilde{y}) + cov(\tilde{x}, \tilde{z}) \qquad var(a\tilde{x}) = a^2 var(\tilde{x})$$

- Covariance of two securities when their residuals are uncorrelated:

$$cov(\tilde{r}_i, \tilde{r}_j) = \beta_i \beta_j \sigma_m^2 \quad \textbf{if } cov(\epsilon_i, \epsilon_j) = 0$$

- Statistical Functions:

$$var(\tilde{r}_A) = E[(\tilde{r}_A - \overline{r_A})^2] \qquad cov(\tilde{r}_A, \tilde{r}_B) = E[(\tilde{r}_A - \overline{r_A})(\tilde{r}_B - \overline{r_B})]$$

- Fraction of the your wealth you put in the risky asset 1, given a risk aversion coefficient of A:

$$w_1^* = \frac{E(\tilde{r}_1) - r_f}{A\sigma_1^2}$$

- The MVE portfolio weights when there are two risky assets A and B ($x_B = (1 - x_A)$):

$$x_A = \frac{E(\tilde{r}_A^e)\sigma_B^2 - E(\tilde{r}_B^e) cov(\tilde{r}_A^e, \tilde{r}_B^e)}{E(\tilde{r}_A^e)\sigma_B^2 + E(\tilde{r}_B^e)\sigma_A^2 - [E(\tilde{r}_A^e) + E(\tilde{r}_B^e)]\, cov(r_A^e, r_B^e)}$$

- The CAPM equation

$$E(\tilde{r}_i) = r_f + \beta_i \left[E(\tilde{r}_m) - r_f\right]$$

- The CAPM Beta:

$$\beta_i = \frac{cov(\tilde{r}_i, \tilde{r}_m)}{\sigma_m^2}$$

- The security market line

$$\tilde{r}_{i,t} - r_{f,t} = \alpha_i + \beta_i \cdot (\tilde{r}_{m,t} - r_{f,t}) + \tilde{\epsilon}_{i,t}$$

- The Systematic Variance of an Asset with a beta of β_i, assuming a single factor model:

$$\sigma_{sys,i}^2 = \beta_i^2 \cdot \sigma_m^2$$

- the R^2 is then $\frac{\sigma_{sys,i}^2}{\sigma_i^2}$
- Equation for *Merrill Lynch* adjusted β's:

$$\beta_i^{Adj} = 1/3 + (2/3) \cdot \beta_i$$

- Equation for the variance of portfolio a; and for the covariance of portfolios a and b:

$$var(\tilde{r}_a) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i^a w_j^a \sigma_{i,j} \qquad\qquad cov(\tilde{r}_a, \tilde{r}_b) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i^a w_j^b \sigma_{i,j}$$

  - For two assets(1 and 2):

$$var(\tilde{r}_a) = (w_1^a)^2 \cdot (\sigma_1)^2 + (w_2^a)^2 \cdot (\sigma_2)^2 + 2 \cdot w_1^a \cdot w_2^a \cdot cov(\tilde{r}_1, \tilde{r}_2)$$

$$cov(\tilde{r}_a, \tilde{r}_b) = w_1^a \cdot w_1^b \cdot (\sigma_1)^2 + w_2^a \cdot w_2^b \cdot (\sigma_2)^2 + (w_1^a \cdot w_2^b + w_2^a \cdot w_1^b) \cdot cov(\tilde{r}_1, \tilde{r}_2)$$

- The APT return generating process:

$$\tilde{r}_{i,t} = E(\tilde{r}_{i,t}) + b_{i,1}\tilde{f}_{1,t} + b_{i,2}\tilde{f}_{2,t} + \cdots + b_{i,k}\tilde{f}_{k,t} + \tilde{\epsilon}_{i,t}$$

- The APT pricing equation:

$$E(r_i) = \lambda_0 + \lambda_1 \cdot b_{i,1} + \lambda_2 \cdot b_{i,2} + \cdots + \lambda_k \cdot b_{i,k}$$

- The covariance between two stocks, i and j under a 2-factor model

$$cov(R_i, R_j) = b_{i,1}\, b_{j,1} var(f_1) + b_{i,2}\, b_{j,2} var(f_2) + (b_{i,1}\, b_{j,2} + b_{j,1}\, b_{i,2})\, cov(f_1, f_2)$$

- Managed Fund Performance Measures:

  1. The **Sharpe Measure** of Portfolio p:
$$S_p = \frac{r_p^e}{\sigma_p}$$

  2. The **Jensen Measure** is the α_p from the regression:
$$r_{p,t}^e = \alpha_p + \beta_p \cdot r_{m,t}^e + \epsilon_{p,t} \text{ (CAPM)}$$
$$r_{p,t}^e = \alpha_p + b_{1,p} \cdot r_{1,t}^e + b_{2,p} \cdot r_{2,t}^e + \cdots + b_{k,p} \cdot r_{k,t}^e + e_{p,t} \text{ (APT)}$$
     - Where $r_{k,t}^e$ is the excess return on the k'th factor mimicking portfolio.

  3. The **Appraisal Ratio** of portfolio p (CAPM and APT):
$$AR_p = \frac{\alpha_p}{\sigma(\epsilon_p)}$$
     - Sharpe Ratio of optimal portfolio C of mkt and p is: $SR_C = \sqrt{SR_m^2 + AR_p^2}$