FINC460/FE312 - Midterm Exam

NAME: _____ SECTION: _____

1. Please do not open this exam until directed to do so.
2. This exam is 90 minutes long.
3. Please write your name and section number on the front of this exam, and on any examination books you use.
4. Please show all work required to obtain each answer. Answers without justification will receive no credit.
5. State clearly any assumptions you are making.
6. This is a closed book exam. No books or notes are permitted. Calculators are permitted. Laptops are permitted but you are only allowed to use Excel and a blank worksheet.
7. Brevity is strongly encouraged on all questions.
8. The exam is worth 115 points.
9. Relax, and good luck!

Hints:

1. *Think through problems before you start working. Draw pictures.*
2. *If you get stuck on part of a problem, go on to the next part. You may need to use answers from earlier parts of the question to calculate answers to the later parts. If you weren't able to solve the earlier part, assume something.*
3. *Remember, setting up the problem correctly will get you most of the points.*

Question 1 (40pts)

1. (8 points) You are working as a manager in a fund of funds. Your job is to identify promising hedge fund managers that outperform the market, and invest part of the funds assets with them. The rest of the fund's assets will be invested in an index fund that closely tracks the S&P 500. Which criterion would you use to select fund managers?

   Ideally, you would use the infomation ratio – the α they earn scaled by the extra volatility they are taking. This tells me how much I can increase my Sharpe ratio if I optimally invest in the fund.

2. (8 points) Suppose that stocks with low price-to-earnings ratios had higher average returns than large stocks. This pattern violates the CAPM.

   False. They may also have higher betas that justify their high expected returns.

3. (8 points) Investors can easily increase their Sharpe ratio by borrowing money to invest in risky securities.

   False. Changing the weight on the risk-free rate (i.e. changing leverage in this case) will not affect the Sharpe ratio

4. (8 points) If the market portfolio is efficient, investors care only about means and variances, and the average investor neither borrows nor lends, then the risk premium on the market portfolio is proportional to its variance and to the average coefficient of risk aversion across investors.

   This is true in general. To see why, consider the case where investors are identical. For each investor i, each agents fraction of wealth (over total wealth) invested in the stock market equals
   $w_i = \frac{E[r_{mkt} - r_f]}{A\sigma^2_{mkt}}$
   Market clearing implies: $\sum w_i = 1$
   Solving, $E[r_{mkt} - r_f] = \sigma^2_{mkt} A$
   In the more general case where investors have different risk aversion, we would need to do a bit more work, replacing A with the (harmonic) average risk aversion coefficient in the economy.

5. (8 points) Adding individual stocks to an already diversified portfolio will always increase the variance of the portfolio.

   False. Doing so could lower the overall variance if the stocks we add covary negatively with our portfolio.

Question 2 (75pts)

You have the following information

Security	Expected Return	Beta	Standard Deviation	Market Capitalization
Risk-Free Asset	3%	-	-	
Market Portfolio	8%		10%	100b
Stock A		1.2	25%	20b
Stock B		1.4	40%	
Stock C		0.5	10%	

Answer the following set of questions. **For all parts of this question, assume the CAPM properly prices all assets.** You should assume that the risk-free rate is the same for borrowing or lending. For some of the questions you may need the answer to previous parts to solve them. If you do not have it, assume something and move on.

1. (10 points) What are the expected returns of Stocks A, B and C in equilibrium?

   Just use the CAPM.
   $E[R_A] = 3\% + 1.2(8\% - 3\%) = 9\%$
   $E[R_B] = 3\% + 1.4(8\% - 3\%) = 10\%$
   $E[R_C] = 3\% + 0.5(8\% - 3\%) = 5.5\%$

2. (10 points) What fraction of the total variance of stocks A, B and C is due to systematic – as opposed to idiosyncratic – risk?

   The total variance for each asset is given. The systematic variance is $\beta^2\sigma^2_{mkt}$*. So we can calculate the fractions as:*
   A: $\frac{1.2^2 0.1^2}{0.25^2} = 0.23$
   B: $\frac{1.4^2 0.1^2}{0.4^2} = 0.1225$
   C: $\frac{0.5^2 0.1^2}{0.1^2} = 0.25$

3. (5 points) Assuming you have a risk aversion coefficient of 8, which combination of the five assets above should you hold? Specify the fraction of your wealth that you will put in each of the five assets.

   The CAPM holds so the market is the mean variance efficient portfolio. Thus we only need to consider investing in the market and risk free asset. Using our formula for the optimal weight:

   $w = \frac{E[r_{mkt} - r_f]}{A\sigma^2_{mkt}}$

   $w = \frac{0.05}{8(0.01)} = \frac{5}{8}$

4. Again assume that you have a risk aversion coefficient of 8. Now assume that you can only hold **one** of the three risky assets (A, B or C) in combination with the risk-free asset.

   (a) (5 points) Which of the three should you hold, and why? (Just specify A, B or C and explain).

   We will choose the one with the highest Sharpe ratio.
   $SR_A = \frac{9\%-3\%}{25\%} = 0.24$
   $SR_B = \frac{10\%-3\%}{40\%} = 0.175$
   $SR_C = \frac{5.5\%-3\%}{10\%} = 0.25$
   So asset C.

   (b) (5 points) How would your answer change if your risk aversion coefficient were very large? (Specify A, B or C and explain)

   Risk aversion is irrelevant for the above question. We only care about the Sharpe ratio. Risk aversion will dictate how we combine this asset with the risk free asset

   (c) (10 points) How much more would you be willing to pay – as a fraction of your wealth – to have access to all securities?

   To answer this, we can compare the Sharpe Ratio of the market portfolio, after paying the management fee, to the Sharpe ratio of investing in C. That is, the maximum fee that we would pay to invest in the market f would solve
   $\frac{8\%-3\%-f}{10\%} = 0.25$
   or $f = 2.5\%$.

5. (15 points) What is the market capitalization of stocks B and C?

Let's first compute the fraction of the total market cap of B and C. We know:

$E[R_{mkt}] = w_A E[R_A] + w_B E[R_B] + (1 - w_A - w_B)E[R_C]$

Plugging in $w_A = 0.2$ *which we were given, plus the expected return of each asset which we know, we only have to solve for* w_B*.*

Solving this out $w_B = 0.4$ *and hence the weight on C is* $w_C = 0.4$ *as well.*

Therefore the market caps of B and C are 40b each.

6. Assume that there are only two types of investors in the economy, types I and II, and that the total financial wealth of each of these two types is equal to 50b (and they have no other sources of wealth). The risk-aversion coefficient of the Type I investors is A_I, and of the Type II investors is A_{II}. Assume the type I investors have 25% of their wealth invested in the risk-free asset. Find:

   (a) (5 points) The fraction of their wealth that the Type I investors place in the market portfolio

   100%-25%=75%

   (b) (5 points) The fraction of their wealth that the Type II investors place in the market portfolio

   I and II have equal wealth, so their fractions must average out to 1. Therefore type II must invest 125% in the market.

   (c) (5 points) The coefficients A_I and A_{II}.

   Just use the (rearranged) formula for the optimal weight: $A = \frac{E[R]-r_f}{w\sigma^2_{mkt}}$

   $A_I = \frac{5\%}{(0.75)1\%} = 6.67$

   $A_{II} = \frac{5\%}{(1.25)1\%} = 4$

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