FINC460 - Midterm Exam

NAME: ______________________________ SECTION: ______________

1. Please do not open this exam until directed to do so.
2. Please write your name and section number on the front of this exam, and on any examination books you use.
3. Please show all work required to obtain each answer. Answers without justification will receive no credit.
4. State clearly any assumptions you are making.
5. This is a closed book exam. No books or notes are permitted, except for a formula sheet. Calculators are permitted. Laptops are permitted but you are only allowed to use Excel and a blank worksheet.
6. Brevity is strongly encouraged on all questions.
7. The exam is worth 115 points.
8. Relax, and good luck!

Hints:

1. *Think through problems before you start working. Draw pictures.*
2. *If you get stuck on part of a problem, go on to the next part. You may need to use answers from earlier parts of the question to calculate answers to the later parts. If you weren't able to solve the earlier part, assume something.*
3. *Remember, setting up the problem correctly will get you most of the points.*

Short questions (40pts)

Assess the validity of the following statements (True, False or Uncertain) and explain your answers. Each question is worth 8pts.

1. (8 points) An investor can increase her Sharpe Ratio by leveraging the mean-variance efficient portfolio with the risk-free asset.

   False.

   Call $R^p = wR^{mve} + (1-w)R_f$ the leveraged portfolio. (Note: $w > 1$ implies leverage since you borrow and invest.)

   Notice R_f doesn't contribute to the variance of R^p. Thus,

   $Sharpe(R^p) = \frac{E[R^p]-R_f}{\sigma(R^p)} = \frac{E[wR^{mve}-wR_f]}{\sigma(wR^{mve})} = \frac{w(E[R^{mve}]-R_f)}{w\sigma(R^{mve})} = Sharpe(R^{mve})$

   So the Sharpe ratio is unchanged.

2. (8 points) If the CAPM holds, then the expected return on any risky asset must be higher than the risk-free return.

   False. A portfolio could have a negative beta. Then $E[R] - R_f < 0$ so $E[R] < R_f$

3. (8 points) Value firms have higher average returns than growth firms. This represents the possibility of a risk-free profit at zero cost (an arbitrage).

   False. First, value firms may just have higher betas in the CAPM framework. Second, even if they don't, it doesn't mean there is "zero risk" (an arbitrage). Value stocks still have volatility, so they are not risk-free.

4. (8 points) Even if the CAPM is wrong, all assets should have zero alpha with respect to the mean-variance efficient portfolio.

   True. When we derived the CAPM, we went from the market being mean-variance efficient to the equation $E[R] - R_f = \beta(E[R_{mkt}] - R_f)$. Those arguments only relied on R_{mkt} being on the mean-variance frontier. So those arguments hold for "whichever" portfolio is mean variance efficient.

5. (8 points) The Black Litterman model assumes that the CAPM prices

all assets.

False/Uncertain. It assumes the CAPM prices all assets *in the absence of additional information*.

Question 2 (75pts)

Assume the CAPM properly prices all assets. There are only 2 stocks in this economy: A and B. You have the following data available to you:

Security	Expected Return	Variance	Market Capitalization
Risk-Free Asset	1%	-	-
Market Portfolio			100b
Stock A	6%	4.5%	50b
Stock B	11%	9.0%	50b

The blank entries in the table are intentional! You should assume that the risk-free rate is the same for borrowing or lending.

Recall that portfolios on the **minimum-variance frontier** are those consisting *only* of risky assets which, for a given level of expected return, achieve the lowest possible level of volatility.

1. (5 points) Which of these assets (if any) lie on the capital allocation line?

   The market and the risk-free rate both do. A and B don't. You can verify that with the following equation for the CAL:

   $y = mx + b$

   where $b = R_f$, $m = E[R_{mkt} - R_f]/\sigma(R_{mkt})$, $x = \sigma(R)$, $y = E[R]$. It turns out A and B don't satisfy that equation so are not on the line.

   Alternatively, we know the market is the tangency portfolio. And the CAL is the tangent line to the mean-variance frontier. Only one portfolio can be on both the CAL and MVE frontier, and clearly that is the market.

2. (10 points) Draw the minimum-variance frontier (approximately). Which assets lie on the frontier?

   We know where to plot $(\sigma_A, E[R_A])$ and $(\sigma_B, E[R_B])$. The minimum-variance frontier is a curve connecting these. And we know the market is the tangency portfolio on the curve. The tangency line – which goes through R_f and the market – is the CAL.

3. (10 points) What are the weights w_A and w_B of the mean-variance efficient portfolio on assets A and B?

   The CAPM holds so the market is mean-variance efficient. The weights are the percentages of market cap of A and B. Remember, the market is the value-weighted average of all stocks.

   $R_{mkt} = \frac{50}{100}R_A + \frac{50}{100}R_B = \frac{1}{2}R_A + \frac{1}{2}R_B$

   The weights are 1/2, 1/2.

4. (10 points) Find the correlation between assets A and B.

   First let's find $cov(A, B)$. We can use that and the standard deviations to find the correlation. We have to ask ourselves: what is a formula where we know everything except $cov(A, B)$? One formula is the "optimal weight" formula that gives the efficient frontier (tangency) portfolio weight. Notice that we know everything in that equation except the covariance.

   $x_A = \frac{(E[R_A]-R_f)\sigma_B^2-(E[R_b]-R_f)cov(A,B)}{(E[R_A]-R_f)\sigma_B^2+(E[R_b]-R_f)\sigma_A^2-(E[R_A]-R_f+E[R_b]-R_f)cov(A,B)}$

   We already know x_A=1/2, $E[R_A] - R_f = 5\%$, $E[R_b] - R_f = 10\%$, $\sigma_A^2 = 4.5\%$, $\sigma_b^2 = 9\%$. That leaves only 1 unknown, the covariance!

   Solving, we get $cov(A, B) = 0$. Thus, $corr(A, B)$ is also 0.

5. (10 points) Find the expected return and standard deviation of the mean-variance efficient portfolio.

   Since the CAPM holds, the mve portfolio is the market. We know $R_{mkt} = \frac{1}{2}R_A + \frac{1}{2}R_B$

   So $E[R_{mkt}] = 8.5\%$

   And $var(R_{mkt}) = var(\frac{1}{2}R_A + \frac{1}{2}R_B) = \frac{1}{4}var(R_A) + \frac{1}{4}var(R_B) = \frac{1}{4}(13.5\%) = 3.375\%$

   Where I have used the formulas $var(x+y) = var(x) + var(y) + 2cov(x,y)$ and $var(cx) = c^2 var(x)$ and also the fact that $cov(A,B) = 0$.

6. (10 points) An investor would like to invest 60% of her wealth into the mean-variance efficient portfolio. What is his implied risk aversion?

   The formula is (using the market as the mve portfolio)

   $w = \frac{E[R_{mkt}] - R_f}{A var(R_{mkt})}$

   So $.6 = 7.5\%/(3.375\% A)$ which means $A = 3.7$

7. (10 points) Now, let's find a portfolio of A and B that has *zero* correlation with the mean-variance efficient portfolio. What are the weights w_A^0 and w_B^0?

   Lets call this portfolio R^p, which is some combination of A and B. So $R^p = wR_A + (1-w)R_b$

   Zero correlation is the same thing as zero covariance or zero beta.

   Use:

   $\beta_p = w\beta_A + (1-w)\beta_b$

   Hence, if we set $\beta_p = 0$,

   $0 = \beta_p = w\beta_A + (1-w)\beta_b$

And rearranging

$w(\beta_b - \beta_A) = \beta_b$

So

$w = \beta_b/(\beta_b - \beta_A)$

Now we can use the CAPM equation to get the betas

$E[R] - R_f = \beta(E[R_{mkt}] - R_f)$

Which gives,

$\beta_A = 7.5/5$ and $\beta_b = 7.5/10$

Plugging in,

$w = 2$. So the weights are 2 and (1-2)=-1.

8. (10 points) We will call this the *zero-beta* portfolio. What is the expected return of the zero-beta portfolio? Plot the location of the portfolio on the minimum-variance frontier.

   Since we know what the beta is, we can just use the CAPM. Beta=0 implies $E[R] - R_f = 0$, so the expected return is 1%, the risk-free rate.

   To plot the return on the mve, we also need to know the variance.

   $var(2R_A + (-1)R_b) = 4var(R_A) + var(R_b) = 27\%$.

   We could plot the point on the frontier as the ordered pair $(1\%, \sqrt{27\%})$.

   Graphically, if you draw a horizontal line from R_f, where that line intersects the minimum variance curve is where this portfolio should be.