FINC460: Homework 4

Solution

1 CAPM and portfolio selection

a) see below

b) expected returns and standard deviations are

	DELL	EXXON	GM	IBM	TGT	SONY	CITI
E(R)	4.15%	1.27%	0.72%	1.23%	1.62%	0.76%	2.14%
σ	15.32%	4.65%	9.51%	8.90%	8.19%	10.06%	8.89%

The correlation matrix of returns is

	DELL	*EXXON*	*GM*	*IBM*	*TGT*	*SONY*	*CITI*
DELL	1.00						
EXXON	0.07	1.00					
GM	0.18	0.21	1.00				
IBM	0.35	0.26	0.26	1.00			
TGT	0.34	0.15	0.29	0.23	1.00		
SONY	0.20	0.14	0.24	0.21	0.10	1.00	
CITI	0.24	0.29	0.33	0.38	0.41	0.25	1.00

and if you plugged everything into the optimizer, you should get

Number of securiti	7

No	Name	Fraction	Expected Return	Standard Deviation
1	DELL	28%	4.2%	15.3%
2	EXXON	69%	1.3%	4.7%
3	GM	-13%	0.7%	9.5%
4	IBM	-13%	1.2%	8.9%
5	TARGET	7%	1.6%	8.2%
6	SONY	-7%	0.8%	10.1%
7	CITIGROU	29%	2.1%	8.9%
		1.00		

Correlations		2 EXXON	3 GM	4 IBM	5 TARGET	6 SONY	7 CITIGROUP
1	DELL	0.07	0.18	0.35	0.34	0.20	0.24
2	EXXON	1.00	0.21	0.26	0.15	0.14	0.29
3	GM		1.00	0.26	0.29	0.24	0.33
4	IBM			1.00	0.23	0.21	0.38
5	TARGET				1.00	0.10	0.41
6	SONY					1.00	0.25

Portfolio's Expected Return	0.0247
Portfolio's Standard Deviatio	0.0627

Risk Free Rate	0.0034	Risk Aversion Coefficient: A=	2.00
Slope of CAL	0.3398	Weight on optimal risky portfolio: x*=	271%

c) All, right. First thing is we need to estimate β_i for every security:

	DELL	EXXON	GM	IBM	TGT	SONY	CITI
beta	1.68	0.48	1.02	1.11	1.05	1.08	1.46

Using these betas, the standard deviation of the market portfolio $\sigma_m = 4.13\%$, and the standard deviations we computed above, we use the formula for the correlation in the 1-factor model:

$$\rho_{i,j} = \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j}$$

we should get

	DELL	EXXON	GM	IBM	TGT	SONY	CITI
DELL	1.00	0.19	0.20	0.23	0.24	0.20	0.31
EXXON		1.00	0.19	0.22	0.22	0.19	0.29
GM			1.00	0.23	0.23	0.20	0.30
IBM				1.00	0.27	0.23	0.35
TGT					1.00	0.23	0.36
SONY						1.00	0.30
CITI							1.00

and if you plugged everything into the optimizer, you should get

Number of securities: 7

No	Name	Fraction	Expected Return	Standard Deviation
1	DELL	25%	4.2%	15.3%
2	EXXON	59%	1.3%	4.7%
3	GM	-13%	0.7%	9.5%
4	IBM	-2%	1.2%	8.9%
5	TARGET	16%	1.6%	8.2%
6	SONY	-12%	0.8%	10.1%
7	CITI	26%	2.1%	8.9%
		1.00		

Correlations		2 EXXON	3 GM	4 IBM	5 TARGET	6 SONY	7 CITI
1	DELL	0.19	0.20	0.23	0.24	0.20	0.31
2	EXXON	1.00	0.19	0.22	0.22	0.19	0.29
3	GM		1.00	0.23	0.23	0.20	0.30
4	IBM			1.00	0.27	0.23	0.35
5	TARGET				1.00	0.23	0.36
6	SONY					1.00	0.30

Portfolio's Expected Return	0.0242
Portfolio's Standard Deviation	0.0642

Risk Free Rate 0.0034 **Risk Aversion Coefficient: A=** 2.00

Slope of CAL 0.3239 **Weight on optimal risky portfolio: x*** 252%

d) Let's use the β_is we estimated above, and the CAPM formula

$$E(R_i) - r_f = \beta_i(E(R_m) - r_f)$$

to get the expected returns implied by the CAPM:

	DELL	EXXON	GM	IBM	TGT	SONY	CITI
CAPM E(R)	1.38%	0.63%	0.97%	1.02%	0.98%	1.00%	1.24%

and using the historical correlation matrix and the CAPM returns, the MV optimizer gives us

Number of securities	7

No	Name	Fraction	Expected Return	Standard Deviation
1	DELL	7%	1.4%	15.3%
2	EXXON	24%	0.6%	4.7%
3	GM	7%	1.0%	9.5%
4	IBM	10%	1.0%	8.9%
5	TGT	16%	1.0%	8.2%
6	SONY	13%	1.0%	10.1%
7	CITI	24%	1.2%	8.9%
		1.00		

Correlations		2 EXXON	3 GM	4 IBM	5 TGT	6 SONY	7 CITI
1	DELL	0.07	0.18	0.35	0.34	0.20	0.24
2	EXXON	1.00	0.21	0.26	0.15	0.14	0.29
3	GM		1.00	0.26	0.29	0.24	0.33
4	IBM			1.00	0.23	0.21	0.38
5	TARGET				1.00	0.10	0.41
6	SONY					1.00	0.25

Portfolio's Expected Return	0.0099
Portfolio's Standard Deviation	0.0514

Risk Free Rate	0.0034	Risk Aversion Coefficient: A=	2.00
Slope of CAL	0.1268	Weight on optimal risky portfolio: x	123%

e) Let's adjust the expected return on Dell by 1% and plug everything in the optimizer

Number of securities	7

No	Name	Fraction	Expected Return	Standard Deviation
1	DELL	29%	2.4%	15.3%
2	EXXON	28%	0.6%	4.7%
3	GM	6%	1.0%	9.5%
4	IBM	0%	1.0%	8.9%
5	TGT	5%	1.0%	8.2%
6	SONY	9%	1.0%	10.1%
7	CITI	24%	1.2%	8.9%
		1.00		

Correlations		2 EXXON	3 GM	4 IBM	5 TGT	6 SONY	7 CITI
1	DELL	0.07	0.18	0.35	0.34	0.20	0.24
2	EXXON	1.00	0.21	0.26	0.15	0.14	0.29
3	GM		1.00	0.26	0.29	0.24	0.33
4	IBM			1.00	0.23	0.21	0.38
5	TARGET				1.00	0.10	0.41
6	SONY					1.00	0.25

Portfolio's Expected Return	0.0134
Portfolio's Standard Deviation	0.0637

Risk Free Rate	0.0034	Risk Aversion Coefficient: A=	2.00
Slope of CAL	0.1574	Weight on optimal risky portfolio: x	124%