FINC460: Homework 5

Solution

1 Factor Models

1) Using the formula for covariances for a 2-factor model:

$$cov(R_i, R_j) = b_{i,1}\, b_{j,1} var(f_1) + b_{i,2}\, b_{j,2} var(f_2)$$

we obtain the following table

Asset	E(R)	$var(\epsilon)$	var(R)	R^2	Loadings		Covariance		
					b_1	b_2	A	B	C
A	0.36	0.15	2.15	93.0%	2	4	2	1.4	0.6
B	0.225	0.28	1.58	82.3%	3	2	1.4	1.3	0.5
C	0.12	0.05	0.25	80.0%	1	1	0.6	0.5	0.2

2) To answer this question we need to construct a portfolio that has zero loadings on factors 1 and 2. Denote the weights on each asset as $[w_A, w_B, w_C]$, we have three equations in three unknowns

$$\begin{aligned} w_A + w_B + w_C &= 1 \\ 2\,w_A + 3\,w_B + w_C &= 0 \\ 4\,w_A + 2\,w_B + w_C &= 0 \end{aligned}$$

The solution to these equations is $[-20.0\%, -40.0\%, 160.0\%]$ and the return on this portfolio equals 3%, which according to the APT must be the risk-free rate.

3) We use the formula for total variance:

$$var(R_i) = b_{i,1}^2 var(f_1) + b_{i,2}^2 var(f_2) + var(\epsilon_i)$$

and the formula for R^2:

$$R^2 = \frac{b_{i,1}^2 var(f_1) + b_{i,2}^2 var(f_2)}{var(R_i)}.$$

See the previous table for the results.

4) To answer this question we need to construct 2 portfolios that has zero loadings on factors 1 or 2 respectively. Denote the weights on each asset as $[w_A^k, w_B^k, w_C^k]$ for the k-factor mimicking portfolio. We have two sets of three equations in three unknowns

$$\begin{aligned} w_A^1 + w_B^1 + w_C^1 &= 1 \\ 2\, w_A^1 + 3\, w_B^1 + w_C^1 &= 1 \\ 4\, w_A^1 + 2\, w_B^1 + w_C^1 &= 0 \end{aligned}$$

yielding $w_A^1 = -0.4$, $w_B^1 = 0.2$ and $w_C^1 = 1.2$.

Similarly

$$\begin{aligned} w_A^2 + w_B^2 + w_C^2 &= 1 \\ 2\, w_A^2 + 3\, w_B^2 + w_C^2 &= 0 \\ 4\, w_A^2 + 2\, w_B^2 + w_C^2 &= 1 \end{aligned}$$

yielding $w_A^2 = 0.2$, $w_B^2 = -0.6$ and $w_C^2 = 1.4$.

Using these weights, the expected return of the 1-st factor-mimicking portfolio is 4.5% and the expected return on the 2-nd factor mimicking portfolio is 10.5%. Hence, their risk premia are $\lambda_1 = 1.5\%$ and $\lambda_1 = 7.5\%$

Last, to find their variances, I can use the fact that

$$var(R_1) = var(f_1) + (w_A^1)^2 var(\epsilon_A) + (w_B^1)^2 var(\epsilon_B) + (w_C^1)^2 var(\epsilon_C) = 0.2072$$

$$var(R_2) = var(f_2) + (w_A^2)^2 var(\epsilon_A) + (w_B^2)^2 var(\epsilon_B) + (w_C^2)^2 var(\epsilon_C) = 0.3048$$

The presence of idiosyncratic risk ϵ introduces additional variance into the factor mimicking portfolio than just the underlying factor.

5) We will use the markowitz spreadsheet in this exercise

(a) Plugging everything into Markowitz, yields

Number of securities: 3

No	Name	Fraction	Expected Return	Standard Deviation
1	A	110%	0.36	147%
2	B	-21%	0.225	126%
3	C	12%	0.12	50%
		1.00		

Correlations		2	3
1		0.7596	0.8184
2		1	0.7956
		YES	

Loadings	1	2
A	2	4
B	3	2
C	1	1
P	1.673527	4.074899

Portfolio's Expected Return	0.3607
Portfolio's Standard Deviation	1.4607

Risk Free Rate	0.0300	**Risk Aversion Coefficient: A=**	2.00
Slope of CAL	0.2264	**Weight on optimal risky portfolio: x*=**	7.75%

(b) Introducing an additional constraint into Markowitz, namely that the loading of the portfolio on factor 2 is zero we get

Number of securities: 3

No	Name	Fraction	Expected Return	Standard Deviation
1	A	-58%	0.36	147%
2	B	75%	0.225	126%
3	C	83%	0.12	50%
		1.00		

Correlations		2	3
1		0.7596	0.8184
2		1	0.7956
		YES	

Loadings	1	2
A	2	4
B	3	2
C	1	1
P	1.91866	0

Portfolio's Expected Return	0.0588
Portfolio's Standard Deviation	0.7822

Risk Free Rate	0.0300	**Risk Aversion Coefficient: A=**	2.00
Slope of CAL	0.0368	**Weight on optimal risky portfolio: x*=**	2.35%

(c) Choosing between the two factor mimicking portfolios, I get

$$x_1 = \frac{\lambda_1 var(R_2)}{\lambda_1 var(R_2) + \lambda_2 var(R_1)} = \frac{0.015 \times 0.3048}{0.015 \times 0.3048 + 0.075 \times 0.2072} = 0.2273$$

and $x_2 = 1 - 0.2273 = 0.7727$

When we choose between the two factor mimicking portfolios only, we get a different answer. Why? The reason is that there is idiosyncratic risk ϵ that is not diversified away

6) The only way that the CAPM holds in this economy is if the market cap of A, B and C is proportional to the optimal portfolio weights in part (a) above