Thermodynamics & Statistical Physics

Chapter 11. Statistical mechanics for non-equilibrium processes

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- The number moving out through surface $x+\mathrm{d}x$: $(fv_x)_{x+\mathrm{d}x}\mathrm{d}t\mathrm{d}y\mathrm{d}z\mathrm{d}\omega =$ $[(fv_x)_x + \frac{\partial}{\partial x}(fv_x)\mathrm{d}x]\mathrm{d}t\mathrm{d}y\mathrm{d}z\mathrm{d}\omega$.

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- The net increase: $-\frac{\partial}{\partial x}(fv_x)\mathrm{d}x\mathrm{d}t\mathrm{d}y\mathrm{d}z\mathrm{d}\omega$ $= -\frac{\partial}{\partial x}(fv_x)\mathrm{d}t\mathrm{d}\tau\mathrm{d}\omega = -\frac{\partial}{\partial x}(f\dot{x})\mathrm{d}t\mathrm{d}\tau\mathrm{d}\omega$.

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- In total, the number for the whole "volume": $-[\frac{\partial}{\partial x}(fv_x)+\frac{\partial}{\partial y}(fv_y)+\frac{\partial}{\partial z}(fv_z)+\frac{\partial}{\partial v_x}(f\dot{v_x})+\frac{\partial}{\partial v_y}(f\dot{v_y})+\frac{\partial}{\partial v_z}(f\dot{v_z})]\mathrm{d}t\mathrm{d}\tau\mathrm{d}\omega$.

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- $\therefore -(v_x\frac{\partial f}{\partial x}+v_y\frac{\partial f}{\partial y}+v_z\frac{\partial f}{\partial z}+\dot{v_x}\frac{\partial f}{\partial v_x}+\dot{v_y}\frac{\partial f}{\partial v_y}+\dot{v_z}\frac{\partial f}{\partial v_z})\mathrm{d}t\mathrm{d}\tau\mathrm{d}\omega$.

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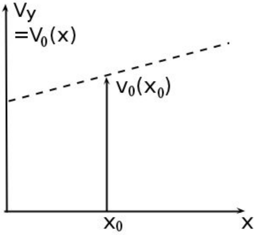

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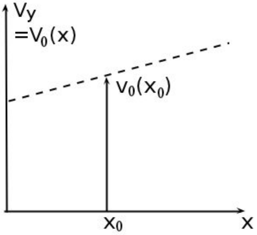

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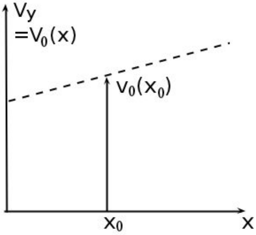

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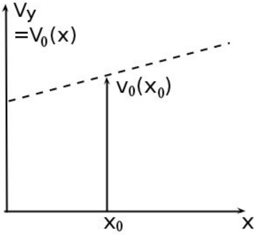

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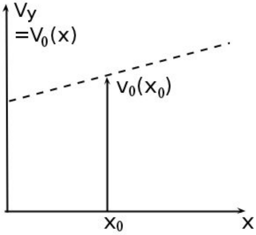

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  - From (7.3.16), in unit time, crossing the unit area at x_0, the number of molecules: $\mathrm{d}\Gamma = v_x f \mathrm{d}\omega$.
  - Each molecule has momentum mv_y. The total momentum from left to the right: $\int_0^\infty \mathrm{d}v_x \int_{-\infty}^\infty \mathrm{d}v_y \int_{-\infty}^\infty \mathrm{d}v_z (mv_y \cdot v_x f)$...(a)

Viscous phenomenon of gas

$$\boxed{v_x\frac{\partial f}{\partial x}+v_y\frac{\partial f}{\partial y}+v_z\frac{\partial f}{\partial z}+X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z}=-\frac{f-f^{(0)}}{\tau_0}}$$

- Similarly, from right to the left:
  $-\int_{-\infty}^{0}\mathrm{d}v_x\int_{-\infty}^{\infty}\mathrm{d}v_y\int_{-\infty}^{\infty}\mathrm{d}v_z(mv_y\cdot v_xf)$...(b)

Viscous phenomenon of gas

$$\boxed{v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} + v_z \frac{\partial f}{\partial z} + X \frac{\partial f}{\partial v_x} + Y \frac{\partial f}{\partial v_y} + Z \frac{\partial f}{\partial v_z} = -\frac{f - f^{(0)}}{\tau_0}}$$

- Similarly, from right to the left:
  $-\int_{-\infty}^{0} \mathrm{d}v_x \int_{-\infty}^{\infty} \mathrm{d}v_y \int_{-\infty}^{\infty} \mathrm{d}v_z (m v_y \cdot v_x f)$...(b)
- The net momentum from right to the left (b)-(a):

Viscous phenomenon of gas

$$\boxed{v_x\frac{\partial f}{\partial x}+v_y\frac{\partial f}{\partial y}+v_z\frac{\partial f}{\partial z}+X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z}=-\frac{f-f^{(0)}}{\tau_0}}$$

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Viscous phenomenon of gas

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Viscous phenomenon of gas

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Viscous phenomenon of gas

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Viscous phenomenon of gas

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Viscous phenomenon of gas

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- If $v_y=v_0=\mathrm{const.}$, uniform flowing. Gas is in equilibrium, $f^{(0)}=n(\frac{m}{2\pi kT})^{\frac{3}{2}}e^{-\frac{m}{2kT}[v_x^2+(v_y-v_0)^2+v_z^2]}$.
- Notice f is only function of x, without external force ($X=0$, $Y=0$, $Z=0$), (11.1.13): $v_x\frac{\partial f}{\partial x}=-\frac{f-f^{(0)}}{\tau_0}$.

$$\boxed{v_x \frac{\partial f}{\partial x} = -\frac{f - f^{(0)}}{\tau_0}}, \quad \boxed{f^{(0)} = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m}{2kT}\left[v_x^2 + (v_y - v_0)^2 + v_z^2\right]}}$$

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- Expand f as: $f = f^{(0)} + f^{(1)}$. $(f^{(1)} \ll f^{(0)})$

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- $\Rightarrow\frac{\partial f^{(0)}}{\partial x}=-\frac{\partial f^{(0)}}{\partial v_y}\frac{\mathrm{d}v_0(x)}{\mathrm{d}x}$.

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- $\Rightarrow f^{(1)} = \tau_0 v_x \frac{\partial f^{(0)}}{\partial v_y} \frac{\mathrm{d} v_0(x)}{\mathrm{d} x}$.

Viscous phenomenon of gas $\boxed{P_{xy} = \eta \frac{\mathrm{d}v_0(x)}{\mathrm{d}x}}$

$\boxed{f^{(0)} = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m}{2kT}[v_x^2+(v_y-v_0)^2+v_z^2]}}$ $\boxed{f^{(1)} = \tau_0 v_x \frac{\partial f^{(0)}}{\partial v_y} \frac{\mathrm{d}v_0(x)}{\mathrm{d}x}}$

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Viscous phenomenon of gas $\boxed{P_{xy} = \eta \frac{\mathrm{d}v_0(x)}{\mathrm{d}x}}$

$$\boxed{f^{(0)} = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m}{2kT}[v_x^2+(v_y-v_0)^2+v_z^2]}} \qquad \boxed{f^{(1)} = \tau_0 v_x \frac{\partial f^{(0)}}{\partial v_y}\frac{\mathrm{d}v_0(x)}{\mathrm{d}x}}$$

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- $$\begin{aligned} \Rightarrow \eta &= -\iiint_{-\infty}^{\infty} m v_x v_y \tau_0 v_x \frac{\partial f^{(0)}}{\partial v_y} \mathrm{d}\omega \\ &= -m\overline{\tau}_0 \iiint_{-\infty}^{\infty} v_x^2 v_y \frac{\partial f^{(0)}}{\partial v_y} \mathrm{d}\omega \end{aligned}$$

Viscous phenomenon of gas $\boxed{P_{xy} = \eta\frac{\mathrm{d}v_0(x)}{\mathrm{d}x}}$

$\boxed{f^{(0)} = n(\frac{m}{2\pi kT})^{\frac{3}{2}}e^{-\frac{m}{2kT}[v_x^2+(v_y-v_0)^2+v_z^2]}}$ $\boxed{f^{(1)} = \tau_0 v_x\frac{\partial f^{(0)}}{\partial v_y}\frac{\mathrm{d}v_0(x)}{\mathrm{d}x}}$

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Viscous phenomenon of gas $\boxed{P_{xy} = \eta \frac{\mathrm{d}v_0(x)}{\mathrm{d}x}}$

$$\boxed{f^{(0)} = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m}{2kT}[v_x^2+(v_y-v_0)^2+v_z^2]}} \quad \boxed{f^{(1)} = \tau_0 v_x \frac{\partial f^{(0)}}{\partial v_y}\frac{\mathrm{d}v_0(x)}{\mathrm{d}x}}$$

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Viscous phenomenon of gas

$$\boxed{f^{(0)} = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m}{2kT}[v_x^2+(v_y-v_0)^2+v_z^2]}}$$

- $\eta = -m\overline{\tau}_0 \int_{-\infty}^{\infty} \mathrm{d}v_z \int_{-\infty}^{\infty} \mathrm{d}v_x v_x^2 [f^{(0)} v_y|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f^{(0)} \mathrm{d}v_y]$

Viscous phenomenon of gas

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  $= m\overline{\tau}_0 \int_{-\infty}^{\infty} \mathrm{d}v_z \int_{-\infty}^{\infty} \mathrm{d}v_x v_x^2 \int_{-\infty}^{\infty} f^{(0)} \mathrm{d}v_y$

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- $\Rightarrow \sigma = \dfrac{ne^2\tau_F}{m}$.

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§11.4 Boltzmann integro-differential equation

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- Before and after the collision, momentum and energy are conserved.

Two-body collision

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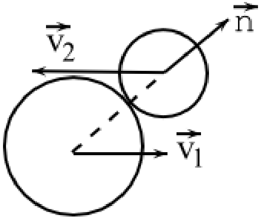

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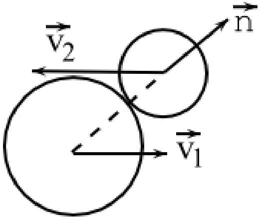

- The solution:
  $\vec{v}_1' = \vec{v}_1 + \frac{2m_2}{m_1+m_2}[(\vec{v}_2 - \vec{v}_1)\cdot\vec{n}]\vec{n}$
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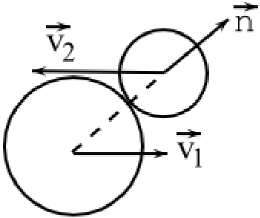

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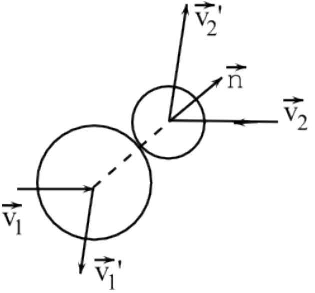
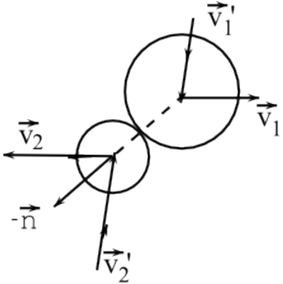


elementary direct collision

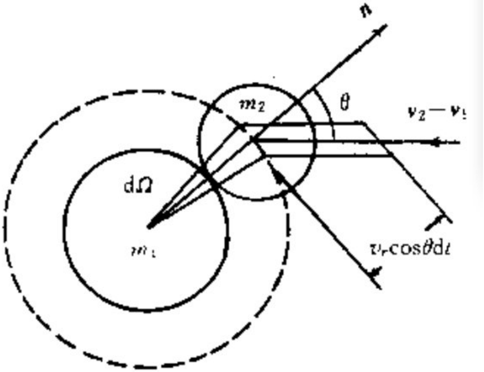


elementary inverse collision

Collision frequency

Collision frequency

- To calculate the frequency of collisions.
- Build coordinate at the center of molecule 1.



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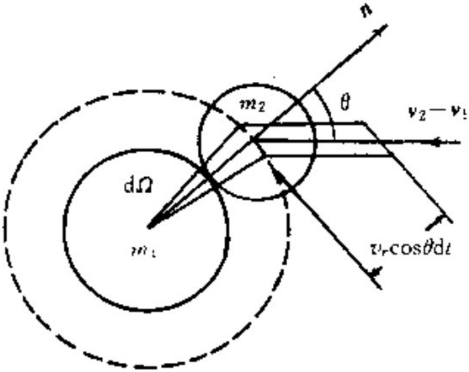

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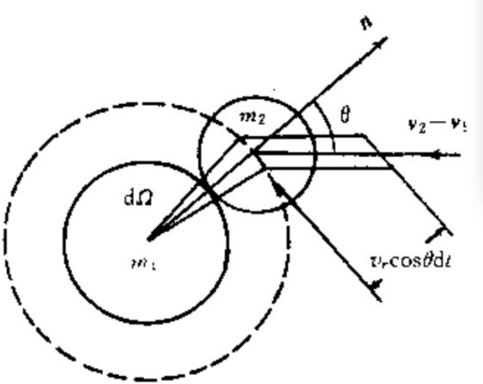

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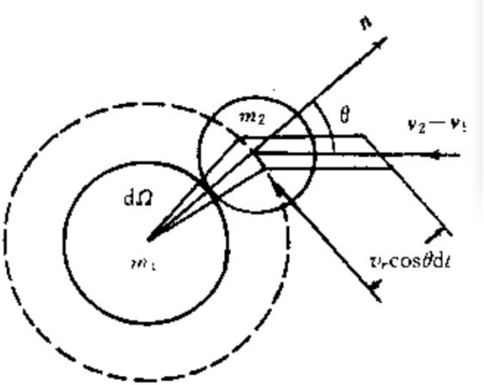
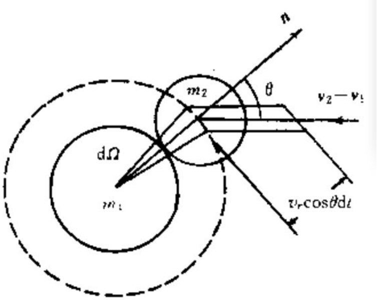


- d_{12} is the distance of two centers, i.e., radius of the dashed circle.
- The collisions on the surface of m_1 in $\mathrm{d}\Omega$, should be on the surface of $d_{12}^2 \mathrm{d}\Omega$ for m_2's center.

Collision frequency

- Length in $\mathrm{d}t$: $v_r \mathrm{d}t$.



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- Length in $\mathrm{d}t$: $v_r \mathrm{d}t$.
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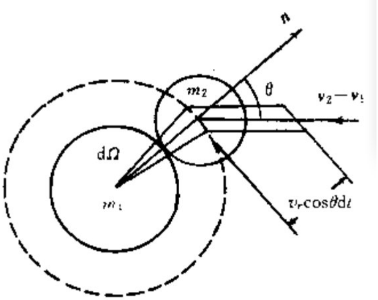

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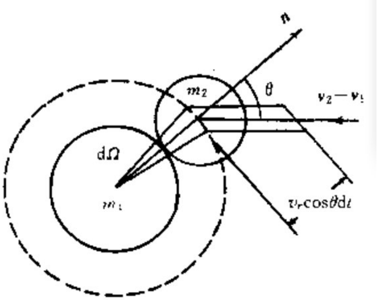

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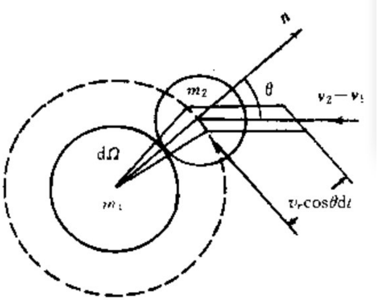
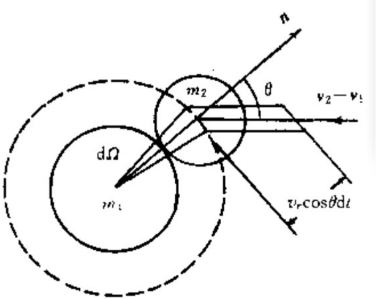


- Setting $\Lambda \mathrm{d}\Omega \equiv d_{12}^2 v_r \cos\theta \mathrm{d}\Omega$, the number: $f_2 \Lambda \mathrm{d}\omega_2 \mathrm{d}\Omega \mathrm{d}t$, also means the number of collisions for a given molecule 1.

Collision frequency



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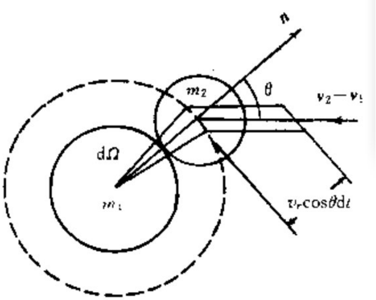
Collision frequency



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- Number of m_1: $f_1\mathrm{d}\tau\mathrm{d}\omega_1$.
- Total number of collisions: $f_1 f_2\mathrm{d}\omega_1\mathrm{d}\omega_2\Lambda\mathrm{d}\Omega\mathrm{d}t\mathrm{d}\tau$.

Boltzmann integro-differential equation

- Those are elementary direct collisions, which makes the molecules in $\mathrm{d}\omega_1$ decrease $(f_1 f_2 \mathrm{d}\omega_1 \mathrm{d}\omega_2 \Lambda \mathrm{d}\Omega \mathrm{d}t \mathrm{d}\tau)$.

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- The elementary inverse collisions make the molecules in $\mathrm{d}\omega_1$ increase. Number of the inverse collisions: $f_1' f_2' \mathrm{d}\omega_1' \mathrm{d}\omega_2' \Lambda' \mathrm{d}\Omega \mathrm{d}t \mathrm{d}\tau$.

Boltzmann integro-differential equation

- Those are elementary direct collisions, which makes the molecules in $\mathrm{d}\omega_1$ decrease $(f_1 f_2 \mathrm{d}\omega_1 \mathrm{d}\omega_2 \Lambda \mathrm{d}\Omega \mathrm{d}t \mathrm{d}\tau)$.
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- Coordinate transferring shows $\mathrm{d}\omega_1 \mathrm{d}\omega_2 = \mathrm{d}\omega_1' \mathrm{d}\omega_2'$, and $\Lambda = d_{12}^2 v_r \cos\theta = d_{12}^2 (\vec{v}_2 - \vec{v}_1) \cdot \vec{n} = d_{12}^2 (\vec{v}_2' - \vec{v}_1') \cdot \vec{n}' = \Lambda'$

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Boltzmann integro-differential equation

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- Then the number: $f_1' f_2' \mathrm{d}\omega_1 \mathrm{d}\omega_2 \Lambda \mathrm{d}\Omega \mathrm{d}t \mathrm{d}\tau$.
- $\therefore$ the number change because of the collisions: $(\frac{\partial f_1}{\partial t})_{\mathrm{c}} \mathrm{d}t \mathrm{d}\tau \mathrm{d}\omega_1 = \mathrm{d}t \mathrm{d}\tau \mathrm{d}\omega_1 \iint (f_1' f_2' - f_1 f_2) \mathrm{d}\omega_2 \Lambda \mathrm{d}\Omega$.

Boltzmann integro-differential equation

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- $1 \to \ , 2 \to 1$, (11.1.12):

Boltzmann integro-differential equation

- Those are elementary direct collisions, which makes the molecules in $\mathrm{d}\omega_1$ decrease $(f_1 f_2 \mathrm{d}\omega_1 \mathrm{d}\omega_2 \Lambda \mathrm{d}\Omega \mathrm{d}t \mathrm{d}\tau)$.
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- Coordinate transferring shows $\mathrm{d}\omega_1 \mathrm{d}\omega_2 = \mathrm{d}\omega_1' \mathrm{d}\omega_2'$, and $\Lambda = d_{12}^2 v_r \cos\theta = d_{12}^2 (\vec{v}_2 - \vec{v}_1) \cdot \vec{n} = d_{12}^2 (\vec{v}_2' - \vec{v}_1') \cdot \vec{n}' = \Lambda'$
- Then the number: $f_1' f_2' \mathrm{d}\omega_1 \mathrm{d}\omega_2 \Lambda \mathrm{d}\Omega \mathrm{d}t \mathrm{d}\tau$.
- $\therefore$ the number change because of the collisions: $(\frac{\partial f_1}{\partial t})_{\mathrm{c}} \mathrm{d}t \mathrm{d}\tau \mathrm{d}\omega_1 = \mathrm{d}t \mathrm{d}\tau \mathrm{d}\omega_1 \iint (f_1' f_2' - f_1 f_2) \mathrm{d}\omega_2 \Lambda \mathrm{d}\Omega$.
- $1 \to\ , 2 \to 1$, (11.1.12): $\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} + v_z \frac{\partial f}{\partial z}$ $+X \frac{\partial f}{\partial v_x} + Y \frac{\partial f}{\partial v_y} + Z \frac{\partial f}{\partial v_z} = \iint (f' f_1' - f f_1) \mathrm{d}\omega_1 \Lambda \mathrm{d}\Omega$.

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  $= \iint (f' f_1' - f f_1) \mathrm{d}\omega_1 \Lambda \mathrm{d}\Omega.$

§11.5 H theorem

- $\frac{\partial f}{\partial t}+v_x\frac{\partial f}{\partial x}+v_y\frac{\partial f}{\partial y}+v_z\frac{\partial f}{\partial z}+X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z}$
  $=\iint(f'f_1'-ff_1)\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega.$
- Define $H\equiv\iint f\ln f\mathrm{d}\tau\mathrm{d}\omega.$

§11.5 H theorem

- $\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} + v_z \frac{\partial f}{\partial z} + X \frac{\partial f}{\partial v_x} + Y \frac{\partial f}{\partial v_y} + Z \frac{\partial f}{\partial v_z} = \iint (f' f_1' - f f_1) \mathrm{d}\omega_1 \Lambda \mathrm{d}\Omega.$
- Define $H \equiv \iint f \ln f \mathrm{d}\tau \mathrm{d}\omega.$
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§11.5 H theorem

- $\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} + v_z \frac{\partial f}{\partial z} + X \frac{\partial f}{\partial v_x} + Y \frac{\partial f}{\partial v_y} + Z \frac{\partial f}{\partial v_z}$
  $= \iint (f' f'_1 - f f_1) \mathrm{d}\omega_1 \Lambda \mathrm{d}\Omega.$
- Define $H \equiv \iint f \ln f \mathrm{d}\tau \mathrm{d}\omega$.
- $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} \iint f \ln f \mathrm{d}\tau \mathrm{d}\omega = \iint \frac{\partial}{\partial t} (f \ln f) \mathrm{d}\tau \mathrm{d}\omega$

§11.5 H theorem

- $\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} + v_z \frac{\partial f}{\partial z} + X \frac{\partial f}{\partial v_x} + Y \frac{\partial f}{\partial v_y} + Z \frac{\partial f}{\partial v_z}$ $= \iint (f' f_1' - f f_1) \mathrm{d}\omega_1 \Lambda \mathrm{d}\Omega$.
- Define $H \equiv \iint f \ln f \mathrm{d}\tau \mathrm{d}\omega$.
- $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} \iint f \ln f \mathrm{d}\tau \mathrm{d}\omega = \iint \frac{\partial}{\partial t}(f \ln f) \mathrm{d}\tau \mathrm{d}\omega$ $= \iint (1 + \ln f) \frac{\partial f}{\partial t} \mathrm{d}\tau \mathrm{d}\omega$

§11.5 H theorem

- $\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} + v_y\frac{\partial f}{\partial y} + v_z\frac{\partial f}{\partial z} + X\frac{\partial f}{\partial v_x} + Y\frac{\partial f}{\partial v_y} + Z\frac{\partial f}{\partial v_z}$
  $= \iint (f'f_1' - ff_1)\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega$.
- Define $H \equiv \iint f\ln f\mathrm{d}\tau\mathrm{d}\omega$.
- $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\iint f\ln f\mathrm{d}\tau\mathrm{d}\omega = \iint \frac{\partial}{\partial t}(f\ln f)\mathrm{d}\tau\mathrm{d}\omega$
  $= \iint (1+\ln f)\frac{\partial f}{\partial t}\mathrm{d}\tau\mathrm{d}\omega$
  $= -\iint (1+\ln f)(v_x\frac{\partial f}{\partial x} + v_y\frac{\partial f}{\partial y} + v_z\frac{\partial f}{\partial z})\mathrm{d}\tau\mathrm{d}\omega$ (1)
  $-\iint (1+\ln f)(X\frac{\partial f}{\partial v_x} + Y\frac{\partial f}{\partial v_y} + Z\frac{\partial f}{\partial v_z})\mathrm{d}\tau\mathrm{d}\omega$ (2)
  $-\iiiint (1+\ln f)(ff_1 - f'f_1')\mathrm{d}\tau\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega$ (3).

§11.5 H theorem

- $\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} + v_y\frac{\partial f}{\partial y} + v_z\frac{\partial f}{\partial z} + X\frac{\partial f}{\partial v_x} + Y\frac{\partial f}{\partial v_y} + Z\frac{\partial f}{\partial v_z}$
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  $-\iint (1+\ln f)(X\frac{\partial f}{\partial v_x} + Y\frac{\partial f}{\partial v_y} + Z\frac{\partial f}{\partial v_z})\mathrm{d}\tau\mathrm{d}\omega$ (2)
  $-\iiiint (1+\ln f)(ff_1 - f'f_1')\mathrm{d}\tau\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega$ (3).
- Inside (1), $\int (1+\ln f)(\vec{v}\cdot\nabla f)\mathrm{d}\tau$

§11.5 H theorem

- $\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} + v_y\frac{\partial f}{\partial y} + v_z\frac{\partial f}{\partial z} + X\frac{\partial f}{\partial v_x} + Y\frac{\partial f}{\partial v_y} + Z\frac{\partial f}{\partial v_z}$
$= \iint (f'f_1' - ff_1)\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega.$
- Define $H \equiv \iint f\ln f\mathrm{d}\tau\mathrm{d}\omega.$
- $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\iint f\ln f\mathrm{d}\tau\mathrm{d}\omega = \iint \frac{\partial}{\partial t}(f\ln f)\mathrm{d}\tau\mathrm{d}\omega$
$= \iint (1+\ln f)\frac{\partial f}{\partial t}\mathrm{d}\tau\mathrm{d}\omega$
$= -\iint (1+\ln f)(v_x\frac{\partial f}{\partial x} + v_y\frac{\partial f}{\partial y} + v_z\frac{\partial f}{\partial z})\mathrm{d}\tau\mathrm{d}\omega \quad (1)$
$-\iint (1+\ln f)(X\frac{\partial f}{\partial v_x} + Y\frac{\partial f}{\partial v_y} + Z\frac{\partial f}{\partial v_z})\mathrm{d}\tau\mathrm{d}\omega \quad (2)$
$-\iiiint (1+\ln f)(ff_1 - f'f_1')\mathrm{d}\tau\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega \quad (3).$
- Inside (1), $\int (1+\ln f)(\vec{v}\cdot\bigtriangledown f)\mathrm{d}\tau = \int \bigtriangledown\cdot(\vec{v}f\ln f)\mathrm{d}\tau$

§11.5 H theorem

- $\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} + v_y\frac{\partial f}{\partial y} + v_z\frac{\partial f}{\partial z} + X\frac{\partial f}{\partial v_x} + Y\frac{\partial f}{\partial v_y} + Z\frac{\partial f}{\partial v_z}$
  $= \iint (f'f_1' - ff_1)\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega.$
- Define $H \equiv \iint f\ln f\mathrm{d}\tau\mathrm{d}\omega.$
- $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\iint f\ln f\mathrm{d}\tau\mathrm{d}\omega = \iint \frac{\partial}{\partial t}(f\ln f)\mathrm{d}\tau\mathrm{d}\omega$
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  $-\iiiint (1+\ln f)(ff_1 - f'f_1')\mathrm{d}\tau\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega \quad (3).$
- Inside (1), $\int(1+\ln f)(\vec{v}\cdot\nabla f)\mathrm{d}\tau = \int\nabla\cdot(\vec{v}f\ln f)\mathrm{d}\tau$
  $= \oint \mathrm{d}\vec{\Sigma}\cdot\vec{v}f\ln f$

§11.5 H theorem

- $\frac{\partial f}{\partial t}+v_x\frac{\partial f}{\partial x}+v_y\frac{\partial f}{\partial y}+v_z\frac{\partial f}{\partial z}+X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z}$
  $=\iint(f'f_1'-ff_1)\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega.$
- Define $H\equiv\iint f\ln f\mathrm{d}\tau\mathrm{d}\omega.$
- $\frac{\mathrm{d}H}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}\iint f\ln f\mathrm{d}\tau\mathrm{d}\omega=\iint\frac{\partial}{\partial t}(f\ln f)\mathrm{d}\tau\mathrm{d}\omega$
  $=\iint(1+\ln f)\frac{\partial f}{\partial t}\mathrm{d}\tau\mathrm{d}\omega$
  $=-\iint(1+\ln f)(v_x\frac{\partial f}{\partial x}+v_y\frac{\partial f}{\partial y}+v_z\frac{\partial f}{\partial z})\mathrm{d}\tau\mathrm{d}\omega$ (1)
  $-\iint(1+\ln f)(X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z})\mathrm{d}\tau\mathrm{d}\omega$ (2)
  $-\iiiint(1+\ln f)(ff_1-f'f_1')\mathrm{d}\tau\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega$ (3).
- Inside (1), $\int(1+\ln f)(\vec{v}\cdot\nabla f)\mathrm{d}\tau=\int\nabla\cdot(\vec{v}f\ln f)\mathrm{d}\tau$
  $=\oint\mathrm{d}\vec{\Sigma}\cdot\vec{v}f\ln f=0$, as $\oint$ represents the integration along the surface of the container.

H theorem

- Inside (2), $\int(1+\ln f)(X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z})\mathrm{d}\omega$

H theorem

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  $=\int(1+\ln f)(\frac{\partial(Xf)}{\partial v_x}+\frac{\partial(Yf)}{\partial v_y}+\frac{\partial(Zf)}{\partial v_z})\mathrm{d}\omega$

H theorem

- Inside (2), $\int(1+\ln f)(X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z})\mathrm{d}\omega$
$=\int(1+\ln f)(\frac{\partial(Xf)}{\partial v_x}+\frac{\partial(Yf)}{\partial v_y}+\frac{\partial(Zf)}{\partial v_z})\mathrm{d}\omega$
$=\int[\frac{\partial}{\partial v_x}(Xf\ln f)+\frac{\partial}{\partial v_y}(Yf\ln f)+\frac{\partial}{\partial v_z}(Zf\ln f)]\mathrm{d}\omega$

H theorem

- Inside (2), $\int(1+\ln f)(X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z})\mathrm{d}\omega$
$= \int(1+\ln f)(\frac{\partial(Xf)}{\partial v_x}+\frac{\partial(Yf)}{\partial v_y}+\frac{\partial(Zf)}{\partial v_z})\mathrm{d}\omega$
$= \int[\frac{\partial}{\partial v_x}(Xf\ln f)+\frac{\partial}{\partial v_y}(Yf\ln f)+\frac{\partial}{\partial v_z}(Zf\ln f)]\mathrm{d}\omega$
$= \int \mathrm{d}v_y\mathrm{d}v_z(Xf\ln f)|_{v_x=-\infty}^{v_x=\infty}+..+..$

H theorem

- Inside (2), $\int(1+\ln f)(X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z})\mathrm{d}\omega$
$=\int(1+\ln f)(\frac{\partial(Xf)}{\partial v_x}+\frac{\partial(Yf)}{\partial v_y}+\frac{\partial(Zf)}{\partial v_z})\mathrm{d}\omega$
$=\int[\frac{\partial}{\partial v_x}(Xf\ln f)+\frac{\partial}{\partial v_y}(Yf\ln f)+\frac{\partial}{\partial v_z}(Zf\ln f)]\mathrm{d}\omega$
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H theorem

- Inside (2), $\int(1+\ln f)(X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z})\mathrm{d}\omega$
  $=\int(1+\ln f)(\frac{\partial(Xf)}{\partial v_x}+\frac{\partial(Yf)}{\partial v_y}+\frac{\partial(Zf)}{\partial v_z})\mathrm{d}\omega$
  $=\int[\frac{\partial}{\partial v_x}(Xf\ln f)+\frac{\partial}{\partial v_y}(Yf\ln f)+\frac{\partial}{\partial v_z}(Zf\ln f)]\mathrm{d}\omega$
  $=\int\mathrm{d}v_y\mathrm{d}v_z(Xf\ln f)|_{v_x=-\infty}^{v_x=\infty}+..+..=0.$
- $\frac{\mathrm{d}H}{\mathrm{d}t}=-\int(1+\ln f)(ff_1-f'f_1')\mathrm{d}\tau\mathrm{d}\omega\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega$

H theorem

- Inside (2), $\int(1+\ln f)(X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z})\mathrm{d}\omega$
  $=\int(1+\ln f)(\frac{\partial(Xf)}{\partial v_x}+\frac{\partial(Yf)}{\partial v_y}+\frac{\partial(Zf)}{\partial v_z})\mathrm{d}\omega$
  $=\int[\frac{\partial}{\partial v_x}(Xf\ln f)+\frac{\partial}{\partial v_y}(Yf\ln f)+\frac{\partial}{\partial v_z}(Zf\ln f)]\mathrm{d}\omega$
  $=\int \mathrm{d}v_y\mathrm{d}v_z(Xf\ln f)|_{v_x=-\infty}^{v_x=\infty}+..+..=0.$
- $\frac{\mathrm{d}H}{\mathrm{d}t}=-\int(1+\ln f)(ff_1-f'f_1')\mathrm{d}\tau\mathrm{d}\omega\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega$ ...

H theorem

- Inside (2), $\int(1+\ln f)(X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z})\mathrm{d}\omega$
  $=\int(1+\ln f)(\frac{\partial(Xf)}{\partial v_x}+\frac{\partial(Yf)}{\partial v_y}+\frac{\partial(Zf)}{\partial v_z})\mathrm{d}\omega$
  $=\int[\frac{\partial}{\partial v_x}(Xf\ln f)+\frac{\partial}{\partial v_y}(Yf\ln f)+\frac{\partial}{\partial v_z}(Zf\ln f)]\mathrm{d}\omega$
  $=\int \mathrm{d}v_y\mathrm{d}v_z(Xf\ln f)|_{v_x=-\infty}^{v_x=\infty}+..+..=0.$
- $\frac{\mathrm{d}H}{\mathrm{d}t}=-\int(1+\ln f)(ff_1-f'f'_1)\mathrm{d}\tau\mathrm{d}\omega\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega\ \ldots =$
  $-\frac{1}{4}\iiiint[\ln(ff_1)-\ln(f'f'_1)](ff_1-f'f'_1)\mathrm{d}\omega\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega\mathrm{d}\tau.$

H theorem

- Inside (2), $\int(1+\ln f)(X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z})\mathrm{d}\omega$
$=\int(1+\ln f)(\frac{\partial(Xf)}{\partial v_x}+\frac{\partial(Yf)}{\partial v_y}+\frac{\partial(Zf)}{\partial v_z})\mathrm{d}\omega$
$=\int[\frac{\partial}{\partial v_x}(Xf\ln f)+\frac{\partial}{\partial v_y}(Yf\ln f)+\frac{\partial}{\partial v_z}(Zf\ln f)]\mathrm{d}\omega$
$=\int \mathrm{d}v_y\mathrm{d}v_z(Xf\ln f)|_{v_x=-\infty}^{v_x=\infty}+..+..=0.$
- $\frac{\mathrm{d}H}{\mathrm{d}t}=-\int(1+\ln f)(ff_1-f'f_1')\mathrm{d}\tau\mathrm{d}\omega\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega\ ...\ =$
$-\frac{1}{4}\iiiint[\ln(ff_1)-\ln(f'f_1')](ff_1-f'f_1')\mathrm{d}\omega\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega\mathrm{d}\tau.$
- The integrand is like $(x-y)(e^x-e^y)$

H theorem

- Inside (2), $\int(1+\ln f)(X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z})\mathrm{d}\omega$
$=\int(1+\ln f)(\frac{\partial(Xf)}{\partial v_x}+\frac{\partial(Yf)}{\partial v_y}+\frac{\partial(Zf)}{\partial v_z})\mathrm{d}\omega$
$=\int[\frac{\partial}{\partial v_x}(Xf\ln f)+\frac{\partial}{\partial v_y}(Yf\ln f)+\frac{\partial}{\partial v_z}(Zf\ln f)]\mathrm{d}\omega$
$=\int \mathrm{d}v_y\mathrm{d}v_z(Xf\ln f)|_{v_x=-\infty}^{v_x=\infty}+..+..=0.$
- $\frac{\mathrm{d}H}{\mathrm{d}t}=-\int(1+\ln f)(ff_1-f'f_1')\mathrm{d}\tau\mathrm{d}\omega\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega\ ...\ =$
$-\frac{1}{4}\iiiint[\ln(ff_1)-\ln(f'f_1')](ff_1-f'f_1')\mathrm{d}\omega\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega\mathrm{d}\tau.$
- The integrand is like $(x-y)(e^x-e^y)\geq 0.$

H theorem

- Inside (2), $\int(1+\ln f)(X\frac{\partial f}{\partial v_x}+Y\frac{\partial f}{\partial v_y}+Z\frac{\partial f}{\partial v_z})\mathrm{d}\omega$
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  $=\int \mathrm{d}v_y\mathrm{d}v_z(Xf\ln f)|_{v_x=-\infty}^{v_x=\infty}+..+..=0.$
- $\frac{\mathrm{d}H}{\mathrm{d}t}=-\int(1+\ln f)(ff_1-f'f_1')\mathrm{d}\tau\mathrm{d}\omega\mathrm{d}\omega_1\Lambda\mathrm{d}\Omega$ ... $=$
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- Means before and after the collision, something is conserved. Number, momentum, energy.

The distribution function in equilibrium

- Particular solutions: $\ln f = 1, mv_x, mv_y, mv_z, \frac{1}{2}mv^2$.

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- $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ can be solved by: $n = \int f \mathrm{d}\omega$,
  $v_{0x} = \frac{1}{n}\int v_x f \mathrm{d}\omega$, $v_{0y} = \frac{1}{n}\int v_y f \mathrm{d}\omega$, $v_{0z} = \frac{1}{n}\int v_z f \mathrm{d}\omega$,
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  and $\frac{3}{2}kT = \frac{1}{n}\int \frac{1}{2}m(\vec{v} - \vec{v}_0)^2 f \mathrm{d}\omega$.
- The result:
  $f = n(\frac{m}{2\pi kT})^{\frac{3}{2}} e^{-\frac{m}{2kT}[(v_x - v_{0x})^2 + (v_y - v_{0y})^2 + (v_z - v_{0z})^2]}$.

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- Take f into (11.6.2): $v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} + v_z \frac{\partial f}{\partial z}$
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- $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ can be solved by: $n = \int f \mathrm{d}\omega$,
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  $\Rightarrow \frac{1}{f}\vec{v} \cdot \bigtriangledown f + \frac{1}{f}\vec{F} \cdot (\frac{\partial f}{\partial v_x}\vec{i} + \frac{\partial f}{\partial v_y}\vec{j} + \frac{\partial f}{\partial v_z}\vec{k}) = 0$

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$$\boxed{f = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m}{2kT}[(v_x - v_{0x})^2 + (v_y - v_{0y})^2 + (v_z - v_{0z})^2]}}$$

- $\Rightarrow \vec{v} \cdot \bigtriangledown \ln f + \vec{F} \cdot \left(\frac{\partial \ln f}{\partial v_x}\vec{i} + \frac{\partial \ln f}{\partial v_y}\vec{j} + \frac{\partial \ln f}{\partial v_z}\vec{k}\right) = 0.$

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  then $\frac{\partial \ln f}{\partial v_x}\vec{i} + \frac{\partial \ln f}{\partial v_y}\vec{j} + \frac{\partial \ln f}{\partial v_z}\vec{k} = -\frac{m}{kT}(\vec{v} - \vec{v}_0)$.

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- $\Rightarrow \vec{v}\cdot\nabla \ln f + \vec{F}\cdot\left(\frac{\partial \ln f}{\partial v_x}\vec{i} + \frac{\partial \ln f}{\partial v_y}\vec{j} + \frac{\partial \ln f}{\partial v_z}\vec{k}\right) = 0.$
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- $\Rightarrow$
  $\vec{v} \cdot \bigtriangledown [\ln n + \frac{3}{2} \ln \frac{m}{2\pi kT} - \frac{m}{2kT}(\vec{v} - \vec{v}_0)^2] - \frac{m}{kT}\vec{F} \cdot (\vec{v} - \vec{v}_0) = 0.$
  Equation about $a_0 + a_1\vec{v} + a_2\vec{v}^2 + a_3\vec{v}^3 = 0$,
  where $\vec{v}$ is arbitrary.

The distribution function in equilibrium

$$\boxed{f = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m}{2kT}\left[(v_x - v_{0x})^2 + (v_y - v_{0y})^2 + (v_z - v_{0z})^2\right]}}$$

- $\Rightarrow \vec{v} \cdot \bigtriangledown \ln f + \vec{F} \cdot \left(\frac{\partial \ln f}{\partial v_x}\vec{i} + \frac{\partial \ln f}{\partial v_y}\vec{j} + \frac{\partial \ln f}{\partial v_z}\vec{k}\right) = 0.$
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- $\Rightarrow$
  $\vec{v} \cdot \bigtriangledown\left[\ln n + \frac{3}{2}\ln\frac{m}{2\pi kT} - \frac{m}{2kT}(\vec{v} - \vec{v}_0)^2\right] - \frac{m}{kT}\vec{F} \cdot (\vec{v} - \vec{v}_0) = 0.$
  Equation about $a_0 + a_1\vec{v} + a_2\vec{v}^2 + a_3\vec{v}^3 = 0$,
  where $\vec{v}$ is arbitrary. So $a_0 = 0, a_1 = 0, a_2 = 0, a_3 = 0$.

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$$\boxed{f = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m}{2kT}[(v_x - v_{0x})^2 + (v_y - v_{0y})^2 + (v_z - v_{0z})^2]}}$$

- $\Rightarrow \vec{v} \cdot \bigtriangledown \ln f + \vec{F} \cdot \left(\frac{\partial \ln f}{\partial v_x}\vec{i} + \frac{\partial \ln f}{\partial v_y}\vec{j} + \frac{\partial \ln f}{\partial v_z}\vec{k}\right) = 0.$
  Notice $\ln f = \ln n + \frac{3}{2}\ln\frac{m}{2\pi kT} - \frac{m}{2kT}(\vec{v} - \vec{v}_0)^2$,
  and $\frac{\partial \ln f}{\partial v_x} = -\frac{m}{2kT}2(v_x - v_{0x})$,
  then $\frac{\partial \ln f}{\partial v_x}\vec{i} + \frac{\partial \ln f}{\partial v_y}\vec{j} + \frac{\partial \ln f}{\partial v_z}\vec{k} = -\frac{m}{kT}(\vec{v} - \vec{v}_0)$.
- $\Rightarrow$
  $\vec{v} \cdot \bigtriangledown[\ln n + \frac{3}{2}\ln\frac{m}{2\pi kT} - \frac{m}{2kT}(\vec{v} - \vec{v}_0)^2] - \frac{m}{kT}\vec{F} \cdot (\vec{v} - \vec{v}_0) = 0.$
  Equation about $a_0 + a_1\vec{v} + a_2\vec{v}^2 + a_3\vec{v}^3 = 0$,
  where $\vec{v}$ is arbitrary. So $a_0 = 0, a_1 = 0, a_2 = 0, a_3 = 0.$
- $a_3 = -\bigtriangledown\frac{m}{2kT}$,

The distribution function in equilibrium

$$\boxed{f = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m}{2kT}[(v_x-v_{0x})^2+(v_y-v_{0y})^2+(v_z-v_{0z})^2]}}$$

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- $\Rightarrow$
  $\vec{v}\cdot\bigtriangledown[\ln n + \frac{3}{2}\ln\frac{m}{2\pi kT} - \frac{m}{2kT}(\vec{v}-\vec{v}_0)^2] - \frac{m}{kT}\vec{F}\cdot(\vec{v}-\vec{v}_0) = 0.$
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- $a_3 = -\bigtriangledown\frac{m}{2kT}$, $\Rightarrow \frac{\partial T}{\partial x} = \frac{\partial T}{\partial y} = \frac{\partial T}{\partial z} = 0.$

The distribution function in equilibrium

$$\boxed{f = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m}{2kT}[(v_x - v_{0x})^2 + (v_y - v_{0y})^2 + (v_z - v_{0z})^2]}}$$

- $\Rightarrow \vec{v} \cdot \bigtriangledown \ln f + \vec{F} \cdot \left(\frac{\partial \ln f}{\partial v_x}\vec{i} + \frac{\partial \ln f}{\partial v_y}\vec{j} + \frac{\partial \ln f}{\partial v_z}\vec{k}\right) = 0.$
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  where $\vec{v}$ is arbitrary. So $a_0 = 0, a_1 = 0, a_2 = 0, a_3 = 0.$
- $a_3 = -\bigtriangledown\frac{m}{2kT}$, $\Rightarrow \frac{\partial T}{\partial x} = \frac{\partial T}{\partial y} = \frac{\partial T}{\partial z} = 0.$
  Meaning: Temperature is uniform in equilibrium.

The distribution function in equilibrium

$$\boxed{\vec{v}\cdot\bigtriangledown[\ln n+\frac{3}{2}\ln\frac{m}{2\pi kT}-\frac{m}{2kT}(\vec{v}-\vec{v}_0)^2]-\frac{m}{kT}\vec{F}\cdot(\vec{v}-\vec{v}_0)=0}$$

- For a_2, i.e., $\vec{v}\cdot\bigtriangledown[\frac{m}{kT}(\vec{v}\cdot\vec{v}_0)]=0$.

The distribution function in equilibrium

$$\boxed{\vec{v}\cdot\bigtriangledown[\ln n+\frac{3}{2}\ln\frac{m}{2\pi kT}-\frac{m}{2kT}(\vec{v}-\vec{v}_0)^2]-\frac{m}{kT}\vec{F}\cdot(\vec{v}-\vec{v}_0)=0}$$

- For a_2, i.e., $\vec{v}\cdot\bigtriangledown[\frac{m}{kT}(\vec{v}\cdot\vec{v}_0)]=0$. Solution is $\vec{v}_0=\vec{a}+\vec{\omega}\times\vec{r}$, where $\vec{a}$ and $\vec{\omega}$ are constant vectors.

The distribution function in equilibrium

$$\boxed{\vec{v}\cdot\bigtriangledown[\ln n+\frac{3}{2}\ln\frac{m}{2\pi kT}-\frac{m}{2kT}(\vec{v}-\vec{v}_0)^2]-\frac{m}{kT}\vec{F}\cdot(\vec{v}-\vec{v}_0)=0}$$

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The distribution function in equilibrium

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- For a_1, i.e., $\bigtriangledown(\ln n-\frac{m}{2kT}v_0^2)-\frac{m}{kT}\vec{F}=0$.

The distribution function in equilibrium

$$\boxed{\vec{v}\cdot\bigtriangledown[\ln n+\frac{3}{2}\ln\frac{m}{2\pi kT}-\frac{m}{2kT}(\vec{v}-\vec{v}_0)^2]-\frac{m}{kT}\vec{F}\cdot(\vec{v}-\vec{v}_0)=0}$$

- For a_2, i.e., $\vec{v}\cdot\bigtriangledown[\frac{m}{kT}(\vec{v}\cdot\vec{v}_0)]=0$. Solution is $\vec{v}_0=\vec{a}+\vec{\omega}\times\vec{r}$, where $\vec{a}$ and $\vec{\omega}$ are constant vectors. Meaning: To be in equilibrium, the whole motion can only be uniformly moving or/and rotating with constant angular velocity.
- For a_1, i.e., $\bigtriangledown(\ln n-\frac{m}{2kT}v_0^2)-\frac{m}{kT}\vec{F}=0$. Notice $\vec{F}=-\bigtriangledown\varphi$, where φ is kind of potential energy.

The distribution function in equilibrium

$$\boxed{\vec{v}\cdot\nabla[\ln n+\frac{3}{2}\ln\frac{m}{2\pi kT}-\frac{m}{2kT}(\vec{v}-\vec{v}_0)^2]-\frac{m}{kT}\vec{F}\cdot(\vec{v}-\vec{v}_0)=0}$$

- For a_2, i.e., $\vec{v}\cdot\nabla[\frac{m}{kT}(\vec{v}\cdot\vec{v}_0)]=0$. Solution is $\vec{v}_0=\vec{a}+\vec{\omega}\times\vec{r}$, where $\vec{a}$ and $\vec{\omega}$ are constant vectors. Meaning: To be in equilibrium, the whole motion can only be uniformly moving or/and rotating with constant angular velocity.
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  Notice $\vec{F}=-\nabla\varphi$, where φ is kind of potential energy.
  $\Rightarrow\nabla(\ln n-\frac{m}{2kT}v_0^2+\frac{m}{kT}\varphi)=0$.

The distribution function in equilibrium

$$\boxed{\vec{v}\cdot\nabla[\ln n+\frac{3}{2}\ln\frac{m}{2\pi kT}-\frac{m}{2kT}(\vec{v}-\vec{v}_0)^2]-\frac{m}{kT}\vec{F}\cdot(\vec{v}-\vec{v}_0)=0}$$

- For a_2, i.e., $\vec{v}\cdot\nabla[\frac{m}{kT}(\vec{v}\cdot\vec{v}_0)]=0$. Solution is $\vec{v}_0=\vec{a}+\vec{\omega}\times\vec{r}$, where $\vec{a}$ and $\vec{\omega}$ are constant vectors. Meaning: To be in equilibrium, the whole motion can only be uniformly moving or/and rotating with constant angular velocity.
- For a_1, i.e., $\nabla(\ln n-\frac{m}{2kT}v_0^2)-\frac{m}{kT}\vec{F}=0$. Notice $\vec{F}=-\nabla\varphi$, where φ is kind of potential energy.
  $\Rightarrow\nabla(\ln n-\frac{m}{2kT}v_0^2+\frac{m}{kT}\varphi)=0$.
  $\Rightarrow\ln n-\frac{m}{2kT}v_0^2+\frac{m}{kT}\varphi=\ln n_0$, where $\ln n_0$ is the integration constant.

The distribution function in equilibrium

$$\boxed{\vec{v}\cdot\bigtriangledown[\ln n+\frac{3}{2}\ln\frac{m}{2\pi kT}-\frac{m}{2kT}(\vec{v}-\vec{v}_0)^2]-\frac{m}{kT}\vec{F}\cdot(\vec{v}-\vec{v}_0)=0}$$

- $\Rightarrow n=n_0e^{\frac{m}{2kT}v_0^2-\frac{m}{kT}\varphi}$.

The distribution function in equilibrium

$$\boxed{\vec{v}\cdot\bigtriangledown[\ln n+\frac{3}{2}\ln\frac{m}{2\pi kT}-\frac{m}{2kT}(\vec{v}-\vec{v}_0)^2]-\frac{m}{kT}\vec{F}\cdot(\vec{v}-\vec{v}_0)=0}$$

- $\Rightarrow n = n_0 e^{\frac{m}{2kT}v_0^2-\frac{m}{kT}\varphi}$.
  Meaning: number of density can change with place. ($\vec{v}_0$ and φ can vary with coordinate.)

The distribution function in equilibrium

$$\boxed{\vec{v}\cdot\nabla[\ln n+\frac{3}{2}\ln\frac{m}{2\pi kT}-\frac{m}{2kT}(\vec{v}-\vec{v}_0)^2]-\frac{m}{kT}\vec{F}\cdot(\vec{v}-\vec{v}_0)=0}$$

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The distribution function in equilibrium

$$\boxed{\vec{v} \cdot \bigtriangledown[\ln n + \frac{3}{2}\ln\frac{m}{2\pi kT} - \frac{m}{2kT}(\vec{v}-\vec{v}_0)^2] - \frac{m}{kT}\vec{F}\cdot(\vec{v}-\vec{v}_0) = 0}$$

- $\Rightarrow n = n_0 e^{\frac{m}{2kT}v_0^2 - \frac{m}{kT}\varphi}$.
  Meaning: number of density can change with place. ($\vec{v}_0$ and φ can vary with coordinate.)
- For a_0, $\vec{v}_0 \cdot \vec{F} = 0$.
  Meaning: the whole motion must be perpendicular to the external force.

The distribution function in equilibrium

$$\boxed{\vec{v}\cdot\nabla[\ln n+\frac{3}{2}\ln\frac{m}{2\pi kT}-\frac{m}{2kT}(\vec{v}-\vec{v}_0)^2]-\frac{m}{kT}\vec{F}\cdot(\vec{v}-\vec{v}_0)=0}$$

- $\Rightarrow n=n_0e^{\frac{m}{2kT}v_0^2-\frac{m}{kT}\varphi}$.
  Meaning: number of density can change with place. ($\vec{v}_0$ and φ can vary with coordinate.)
- For a_0, $\vec{v_0}\cdot\vec{F}=0$.
  Meaning: the whole motion must be perpendicular to the external force.
- To have equilibrium, the 4 conditions (properties) above should all be satisfied. $\frac{\partial T}{\partial x}=\frac{\partial T}{\partial y}=\frac{\partial T}{\partial z}=0$; $\vec{v}_0=\vec{a}+\vec{\omega}\times\vec{r}$; $n=n_0e^{\frac{m}{2kT}v_0^2-\frac{m}{kT}\varphi}$ and $\vec{v_0}\cdot\vec{F}=0$.

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