Thermodynamics & Statistical Physics

Exercises

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- $S = \frac{3}{2}Nk\ln T + Nk\ln\frac{V}{N} + \frac{3}{2}Nk[\frac{5}{3} + \ln\frac{2\pi mk}{h^2}]$. (7.6.2)

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- Compare with the black-body:
  $\rho(\omega)\mathrm{d}\omega = \frac{V}{\pi^2 c^3} \frac{\omega^2}{e^{\frac{\hbar\omega}{kT}} - 1} \mathrm{d}\omega$. (8.4.6)

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  $\simeq 4.5 \times 10^{-5} \ll 1.$

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  $\Rightarrow T = \frac{\varepsilon}{k \ln(\frac{2N + 6\omega_2}{2N + 3\omega_1})}$.

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A9.14 For the free electrons in metal, prove: the probability to find an electron at state with energy $\mu - \Delta$ is equal to not to find an electron at state with energy $\mu + \Delta$.

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$\mathcal{Z}(\alpha, \beta, V) = e^{a[\frac{4}{15}(-\alpha)^{5/2}+\frac{\pi^2}{6}(-\alpha)^{1/2}-\frac{7\pi^4}{1440}(-\alpha)^{-3/2}]\beta^{-3/2}}$, where $a = \frac{2\pi g V(2m)^{3/2}}{h^3}$, g is the inner degeneracy. Then calculate the grand potential $J(T, V, \mu)$. (non-relativistic)

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  ..(A grand dic of phys problems & solutions 5, P_{322}.)

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$\mathcal{Z}(\alpha,\beta,V)=e^{a[\frac{4}{15}(-\alpha)^{5/2}+\frac{\pi^2}{6}(-\alpha)^{1/2}-\frac{7\pi^4}{1440}(-\alpha)^{-3/2}]\beta^{-3/2}}$, where $a=\frac{2\pi gV(2m)^{3/2}}{h^3}$, g is the inner degeneracy. Then calculate the grand potential $J(T,V,\mu)$. (non-relativistic)

- $D(\varepsilon)\mathrm{d}\varepsilon=g\cdot\frac{V4\pi p^2\mathrm{d}p}{h^3}=\frac{2\pi gV(2m)^{3/2}}{h^3}\varepsilon^{1/2}\mathrm{d}\varepsilon=a\varepsilon^{1/2}\mathrm{d}\varepsilon.$
  $\ln\mathcal{Z}=\sum\omega_l\ln(1+e^{-\alpha-\beta\varepsilon_l})$
  $=\int_0^\infty D(\varepsilon)\ln(1+e^{-\alpha-\beta\varepsilon})\mathrm{d}\varepsilon$
  $=a\int_0^\infty\varepsilon^{1/2}\ln(1+e^{-\alpha-\beta\varepsilon})\mathrm{d}\varepsilon$
  $=a\{\frac{2}{3}\varepsilon^{3/2}\ln(1+e^{-\alpha-\beta\varepsilon})|_0^\infty+\frac{2}{3}\int_0^\infty\varepsilon^{3/2}\frac{\beta e^{-\alpha-\beta\varepsilon}}{1+e^{-\alpha-\beta\varepsilon}}\mathrm{d}\varepsilon\}$
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  $=a[\frac{4}{15}(-\alpha)^{5/2}+\frac{\pi^2}{6}(-\alpha)^{1/2}-\frac{7\pi^4}{1440}(-\alpha)^{-3/2}]\beta^{-3/2}.$

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  $= a[\frac{4}{15}(-\alpha)^{5/2}+\frac{\pi^2}{6}(-\alpha)^{1/2}-\frac{7\pi^4}{1440}(-\alpha)^{-3/2}]\beta^{-3/2}$.
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- Enthalpy: $H = U + pV = \frac{5}{2}pV \simeq N\mu_0(1 + \frac{5}{12}\theta^2 - \frac{1}{16}\theta^4)$.

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- (Convert to (p, V, T)) We have $p = \frac{2}{5V} N\mu_0(1 + \frac{5}{12}\theta^2)$, where $\theta = \frac{\pi}{\beta\mu_0} = \frac{\pi kT}{\mu_0}$, $\mu_0 = (\frac{3N}{2a})^{\frac{2}{3}}$, $a = 2\pi gV(\frac{2m}{h^2})^{\frac{3}{2}}$. Define: $b = \frac{h^2}{2m}(\frac{3N}{4\pi g})^{\frac{2}{3}}$.

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  $\beta \equiv \frac{1}{p}(\frac{\partial p}{\partial T})_V = \frac{5\pi^2 k^2}{6\mu_0^2}(1 - \frac{5}{12}\theta^2)T$.

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- (Convert to (p, V, T)) We have $p = \frac{2}{5V} N\mu_0 (1 + \frac{5}{12}\theta^2)$, where $\theta = \frac{\pi}{\beta\mu_0} = \frac{\pi kT}{\mu_0}$, $\mu_0 = (\frac{3N}{2a})^{\frac{2}{3}}$, $a = 2\pi g V (\frac{2m}{h^2})^{\frac{3}{2}}$.
  Define: $b = \frac{h^2}{2m}(\frac{3N}{4\pi g})^{\frac{2}{3}}$.
  $\Rightarrow p = \frac{2bN}{5} V^{-\frac{5}{3}} + \frac{\pi^2 k^2}{6b} N V^{-\frac{1}{3}} T^2$.
- $\alpha \equiv \frac{1}{V}(\frac{\partial V}{\partial T})_p = \frac{\pi^2 k^2}{2\mu_0^2}(1 - \frac{1}{12}\theta^2)T$.
  $\beta \equiv \frac{1}{p}(\frac{\partial p}{\partial T})_V = \frac{5\pi^2 k^2}{6\mu_0^2}(1 - \frac{5}{12}\theta^2)T$.
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  $\simeq 6.6\times10^{35}\rm m^{-3}(\frac{6.626\times10^{-34}J\cdot s\cdot3\times10^8m/s}{1.38\times10^{-23}J/K\cdot10^7K})^3$
  $\simeq 2\times10^9 \gg 1$. (Only estimation if using $kT_{\rm WD}$).

A9.55 He white dwarf in $T_{\mathrm{WD}} = 10^7$K at center. $M_{\mathrm{WD}} \sim 1 M_{\odot} \simeq 2 \times 10^{30}\mathrm{kg}$, $R_{\mathrm{WD}} \sim 1 R_{\oplus} \simeq 6 \times 10^6 \mathrm{m}$.

- $kT \simeq 862.5$eV, ionized.
- Number density: $n_e = 2 \cdot \frac{M_{\mathrm{WD}}}{\frac{4}{3}\pi R_{\mathrm{WD}}^3 \cdot m_{\mathrm{He}}}$
  $\simeq \frac{2\cdot 2\times 10^{30}\mathrm{kg}}{\frac{4}{3}\pi(6\times 10^6\mathrm{m})^3\cdot 4\times 1.67\times 10^{-27}\mathrm{kg}} \simeq 6.6\times 10^{35}\mathrm{m}^{-3}$.
- i. Non-relativistic: $n\lambda^3 = n\left(\frac{h^2}{2\pi mkT}\right)^{\frac{3}{2}}$
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  $\simeq 8.5\times10^{-14}\mathrm{J} \simeq 0.53\mathrm{MeV}$. (This is kinetic energy, mildly relativistic.)

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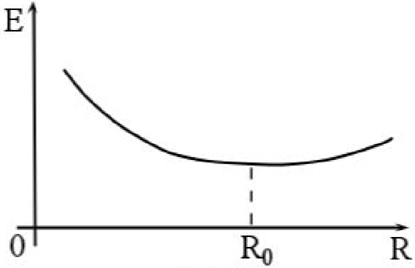
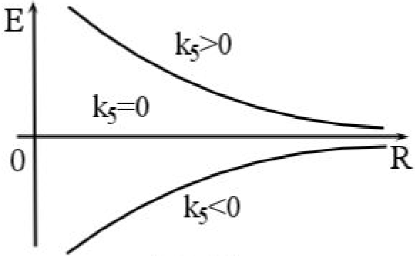
  $E_{\text{tot}} = k_4 \frac{M^{4/3}}{R} - k_2 \frac{M^2}{R} = \frac{k_5}{R}$. No minimum! Not stable.



non-relativistic



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References

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