Thermodynamics & Statistical Physics

Chapter 1. Basic Laws in Thermodynamics

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- Thermal SP (additional): temperature (T).

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- Thermometric scale T (K) (Kelvin).
- Ideal gas scaling: $T = 273.16\mathrm{K} \times \lim_{p_t \to 0} (p/p_t)$.
- Daily use, Celsius scale $(T - 273.15)$ °C.

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  Isothermal compressibility: $\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$.
- Using $\left(\frac{\partial T}{\partial V}\right)_p \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial p}\right)_T = -1$,
  $\therefore \alpha = \kappa_T \beta p$.

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- Extensive quantity: $m, n, V, \Sigma M$.
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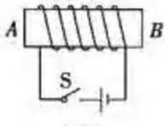
- Summary: $đW = \sum_i Y_i \mathrm{d}y_i$,
  where Y_i is the generalized force, and y_i is the generalized displacement.

§1.4 Work – Magnetic medium



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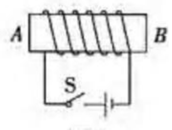

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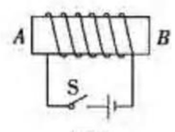

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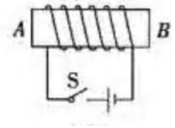

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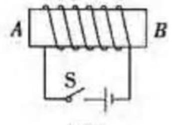

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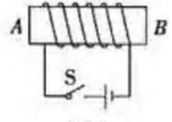

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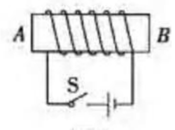

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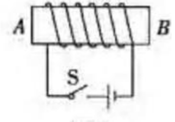

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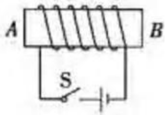

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  where $\frac{\mu_0 H^2}{2}$ is the magnetic energy density (work by increasing the magnetic field); M is the magnetization (work done by magnetize the material).

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  $\therefore \left(\frac{\partial U}{\partial V}\right)_T = 0 \Rightarrow U(T,V) \to U(T)$.

§1.7 Internal energy of the ideal gas

- General: $f(U,T,V)=0$. Why no p?
- $\left(\frac{\partial U}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_U\left(\frac{\partial T}{\partial U}\right)_V=-1$
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- $\therefore \boxed{C_p - C_V = nR}$, define $\gamma \equiv \frac{C_p}{C_V}$,
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§1.8 Adiabatic process of the ideal gas

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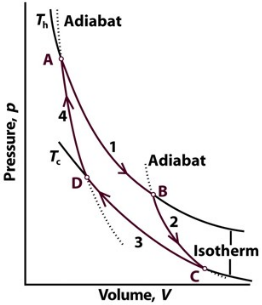

Figure 3-6
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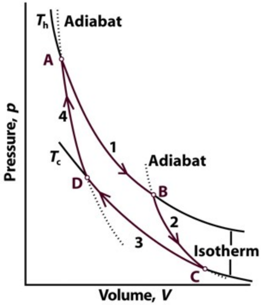

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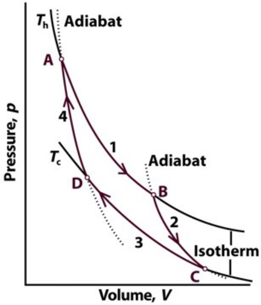

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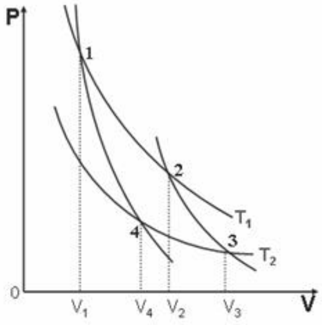
- Velocity of sound: $a^2 = \gamma\frac{p}{\rho}$.

§1.9 Carnot cycle in the ideal gas



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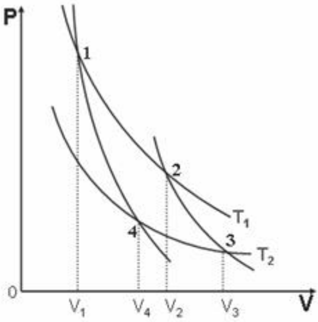

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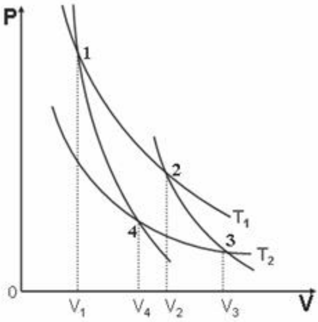

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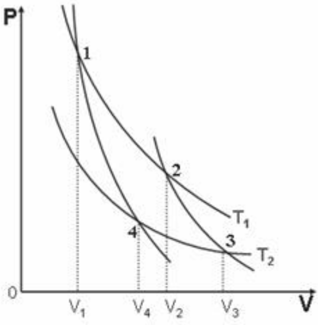

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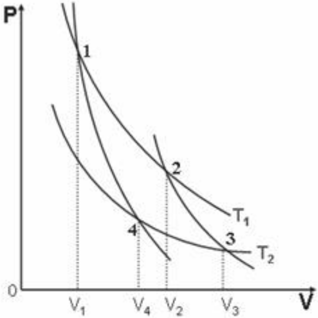

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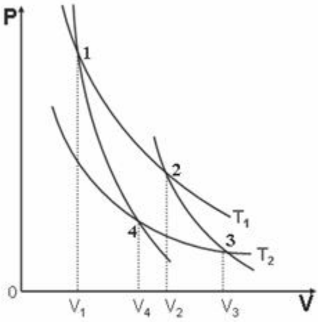

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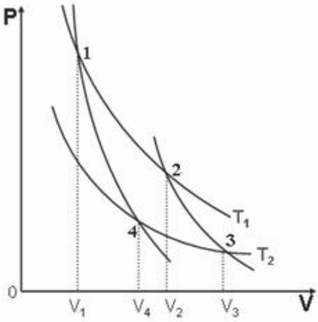

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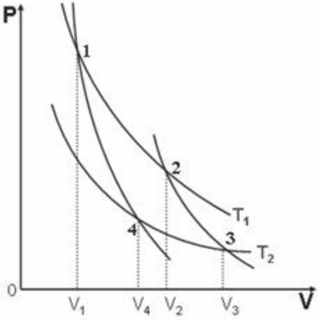

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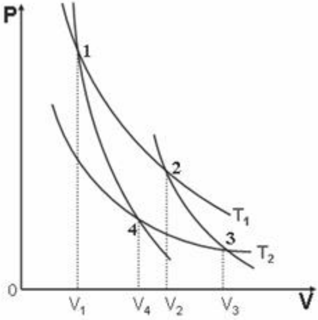

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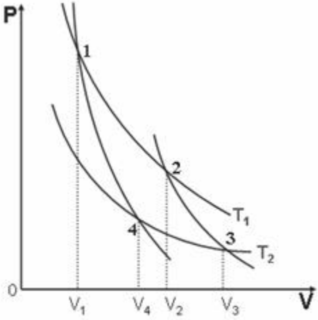

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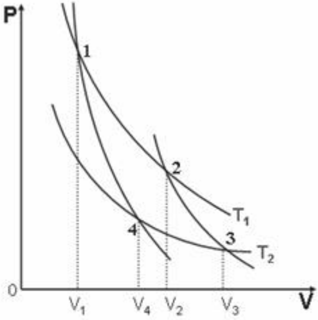

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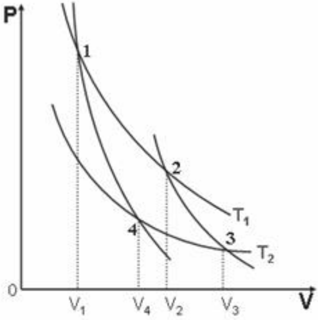

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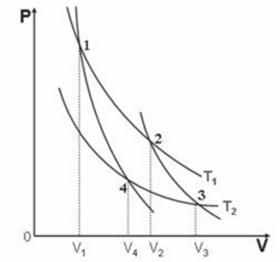
- Total work: $W = Q_1 - Q_2 = RT_1\ln\frac{V_2}{V_1} - RT_2\ln\frac{V_3}{V_4}$.

Carnot cycle in the ideal gas

- Total work of a cycle: $W = Q_1 - Q_2 = RT_1 \ln \frac{V_2}{V_1} - RT_2 \ln \frac{V_3}{V_4}$.



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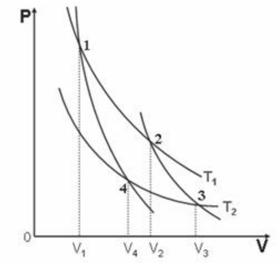

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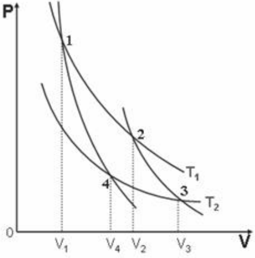

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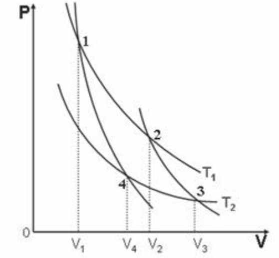

$$\therefore W = R(T_1 - T_2) \ln \frac{V_2}{V_1}.$$

Carnot cycle in the ideal gas

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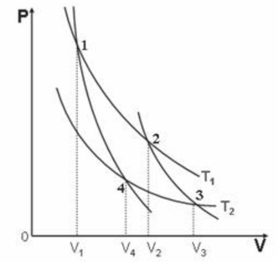

$$\therefore W = R(T_1 - T_2) \ln \frac{V_2}{V_1}.$$

- Efficiency: $\eta = \frac{W}{Q_1} = \frac{R(T_1 - T_2) \ln \frac{V_2}{V_1}}{RT_1 \ln \frac{V_2}{V_1}} = 1 - \frac{T_2}{T_1}$.

Carnot cycle in the ideal gas

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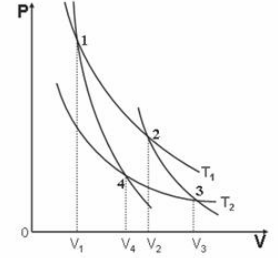

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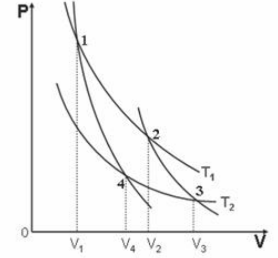

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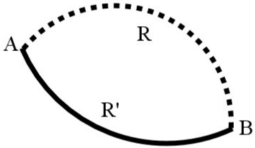

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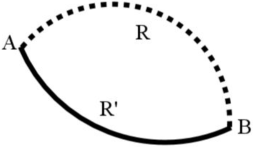

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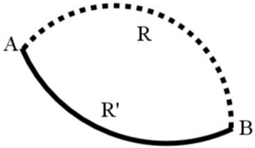

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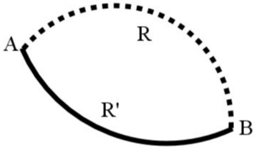

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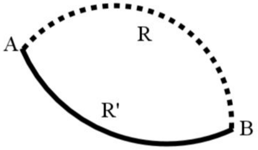

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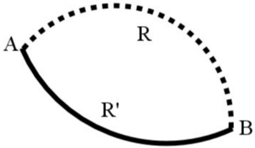

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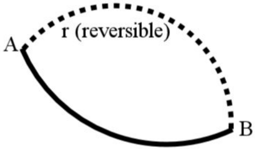

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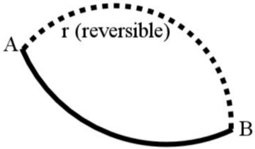

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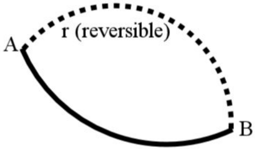

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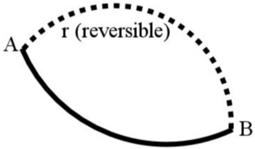

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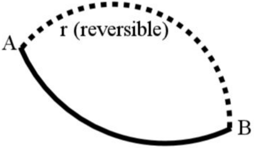

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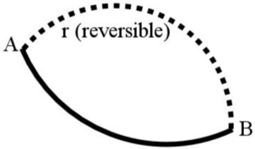

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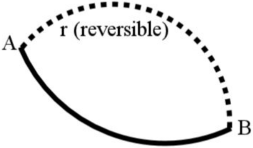

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