Thermodynamics & Statistical Physics

Chapter 1. Basic Laws in Thermodynamics

Yuan-Chuan Zou
zouyc@hust.edu.cn

School of Physics, Huazhong University of Science and Technology

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- Thermal SP (additional): temperature (T).

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- Thermometric scale T (K) (Kelvin).
- Ideal gas scaling: $T = 273.16\mathrm{K} \times \lim_{p_t \to 0} (p/p_t)$.
- Daily use, Celsius scale $(T - 273.15)$ °C.

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- Using $\left(\frac{\partial T}{\partial V}\right)_p \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial p}\right)_T = -1$,

  $\therefore \alpha = \kappa_T \beta p$.

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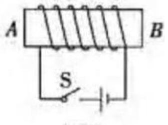
- Summary: $đW = \sum_i Y_i \mathrm{d}y_i$,
  where Y_i is the generalized force, and y_i is the generalized displacement.

§1.4 Work – Magnetic medium



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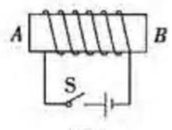

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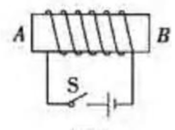

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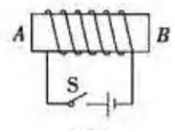

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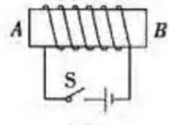

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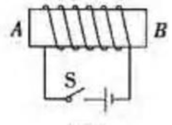

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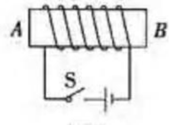

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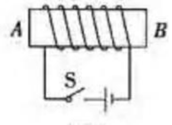

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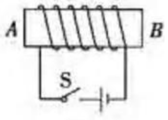

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  where $\frac{\mu_0 H^2}{2}$ is the magnetic energy density (work by increasing the magnetic field); M is the magnetization (work done by magnetize the material).

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- $\therefore \boxed{C_p - C_V = nR}$, define $\gamma \equiv \frac{C_p}{C_V}$,
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§1.8 Adiabatic process of the ideal gas

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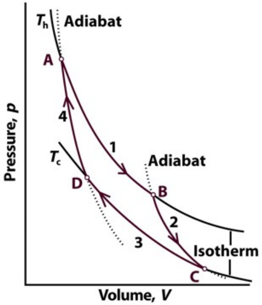

Figure 3-6
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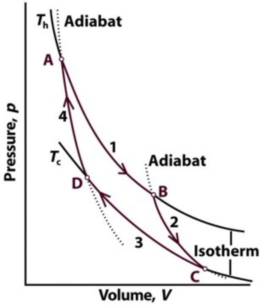

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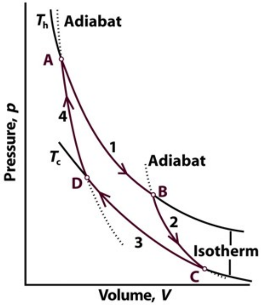

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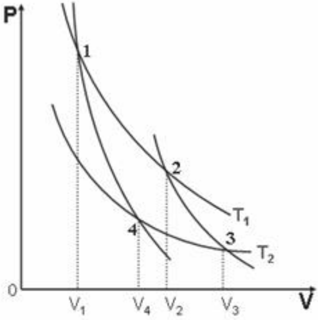
- Velocity of sound: $a^2 = \gamma\frac{p}{\rho}$.

§1.9 Carnot cycle in the ideal gas



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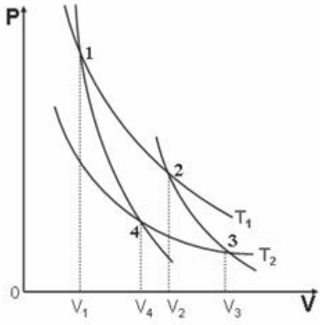

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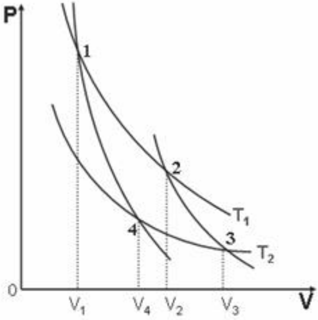

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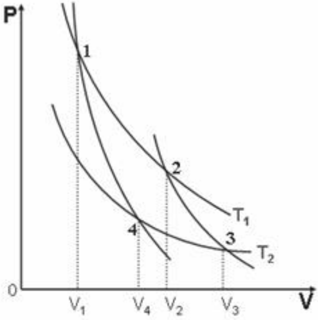

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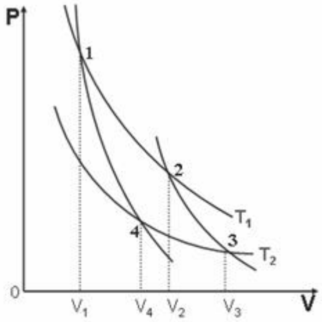

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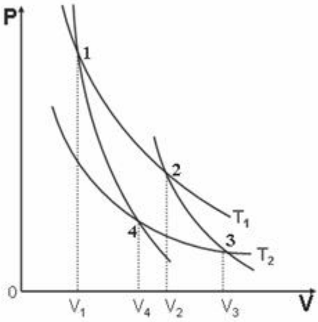

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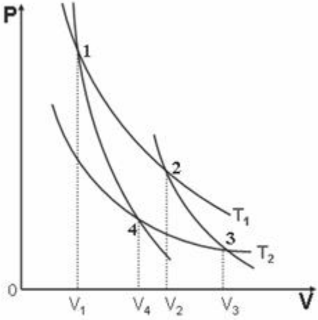

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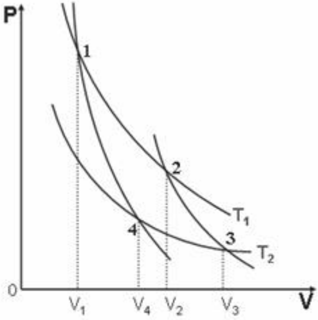

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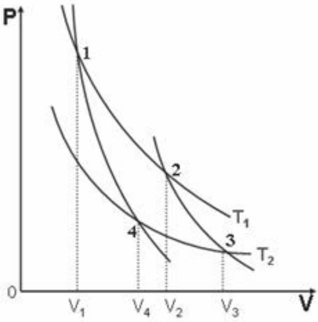

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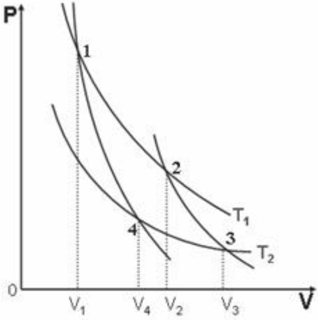

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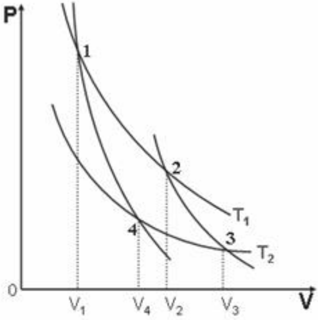

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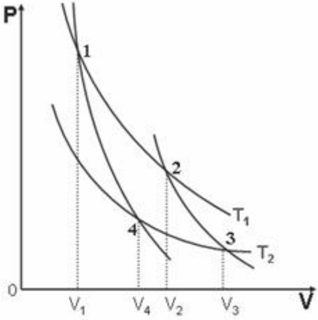

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Carnot cycle in the ideal gas

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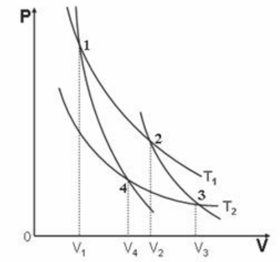

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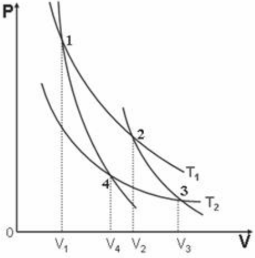

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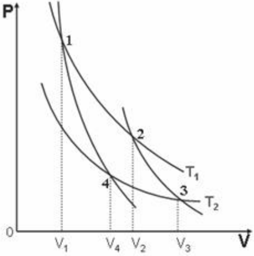

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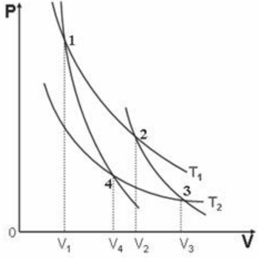

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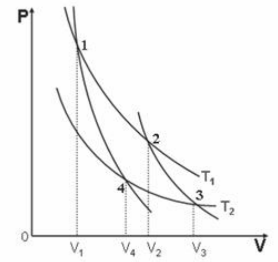

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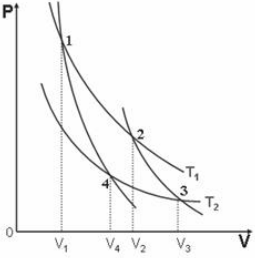

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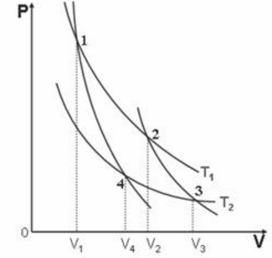

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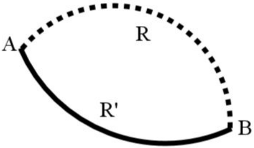

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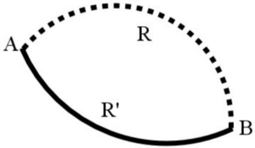

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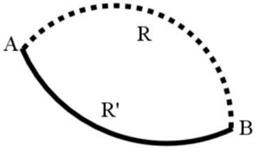

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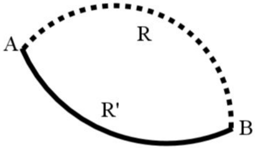

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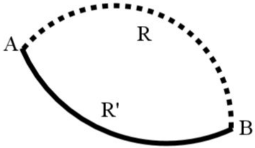

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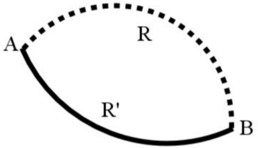

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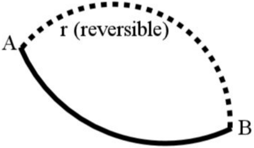

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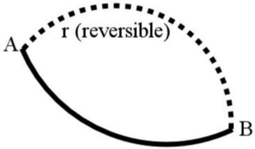

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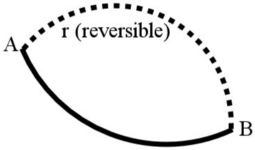

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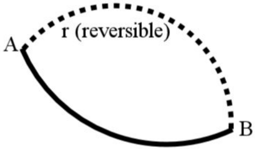

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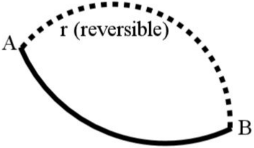

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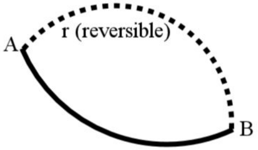

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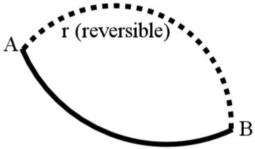

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