Thermodynamics & Statistical Physics

Exercises

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Table of contents

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Table of contents

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- $S = \frac{3}{2}Nk\ln T + Nk\ln\frac{V}{N} + \frac{3}{2}Nk[\frac{5}{3} + \ln\frac{2\pi mk}{h^2}]$. (7.6.2)

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- Compare with the black-body:
  $\rho(\omega)\mathrm{d}\omega = \frac{V}{\pi^2 c^3} \frac{\omega^2}{e^{\frac{\hbar\omega}{kT}} - 1} \mathrm{d}\omega$. (8.4.6)

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Table of contents

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Table of contents

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  ..(A grand dic of phys problems & solutions 5, P_{322}.)

A9.32 Prove $\mathcal{Z}$ for strong degenerated Fermi **gas**: $\mathcal{Z}(\alpha,\beta,V)=e^{a[\frac{4}{15}(-\alpha)^{5/2}+\frac{\pi^2}{6}(-\alpha)^{1/2}-\frac{7\pi^4}{1440}(-\alpha)^{-3/2}]\beta^{-3/2}}$, where $a=\frac{2\pi gV(2m)^{3/2}}{h^3}$, g is the inner degeneracy. Then calculate the grand potential $J(T,V,\mu)$. (non-relativistic)

- $D(\varepsilon)\mathrm{d}\varepsilon=g\cdot\frac{V4\pi p^2\mathrm{d}p}{h^3}=\frac{2\pi gV(2m)^{3/2}}{h^3}\varepsilon^{1/2}\mathrm{d}\varepsilon=a\varepsilon^{1/2}\mathrm{d}\varepsilon.$
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- Enthalpy: $H = U + pV = \frac{5}{2}pV \simeq N\mu_0(1 + \frac{5}{12}\theta^2 - \frac{1}{16}\theta^4)$.

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A9.35 Calculate expansion coefficient α, pressure coefficient β, and isothermal compressibility κ_T, for the strong degenerated Fermi gas.

- (Convert to (p, V, T)) We have $p = \frac{2}{5V} N\mu_0(1 + \frac{5}{12}\theta^2)$, where $\theta = \frac{\pi}{\beta\mu_0} = \frac{\pi kT}{\mu_0}$, $\mu_0 = (\frac{3N}{2a})^{\frac{2}{3}}$, $a = 2\pi gV(\frac{2m}{h^2})^{\frac{3}{2}}$.
  Define: $b = \frac{h^2}{2m}(\frac{3N}{4\pi g})^{\frac{2}{3}}$.
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- $J = -kT\ln\mathcal{Z} = -a(\frac{1}{12}\mu^4 + \frac{\pi^2}{6\beta^2}\mu^2 + \frac{7\pi^2}{180\beta^4})$.

Table of contents

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  $\simeq 8660 \gg 1$.
- ii. Relativistic: $n\lambda^3 = n(\frac{h}{p})^3 = n(\frac{hc}{\varepsilon})^3 = n(\frac{hc}{kT_{\rm WD}})^3$
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A9.55 He white dwarf in $T_{\rm WD} = 10^7$K at center. $M_{\rm WD} \sim 1M_{\odot} \simeq 2\times 10^{30}\rm kg$, $R_{\rm WD} \sim 1R_{\oplus} \simeq 6\times 10^6\rm m$.

- $kT \simeq 862.5$eV, ionized.
- Number density: $n_e = 2\cdot \frac{M_{\rm WD}}{\frac{4}{3}\pi R_{\rm WD}^3 \cdot m_{\rm He}}$
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  $\simeq 8.5\times10^{-14}\mathrm{J} \simeq 0.53\mathrm{MeV}$. (This is kinetic energy, mildly relativistic.)

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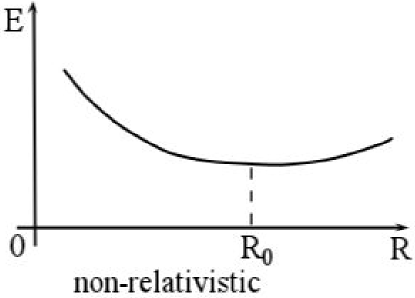
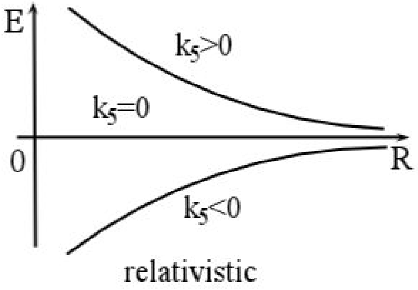
  $\delta E_{\text{tot}} = 0 \Rightarrow (-2k_1 \frac{M^{5/3}}{R^3} + k_2 \frac{M^2}{R^2})\delta R = 0$
  $\Rightarrow R = k_3 M^{-1/3}$; Minimum.
- Relativistic: $U = \frac{3}{4} Nhc(\frac{3N}{8\pi V})^{\frac{1}{3}} \propto \frac{M^{4/3}}{R}$
  $E_{\text{tot}} = k_4 \frac{M^{4/3}}{R} - k_2 \frac{M^2}{R} = \frac{k_5}{R}$. No minimum! Not stable.





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References

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Table of contents