Section 3.8: Implicit Differentiation

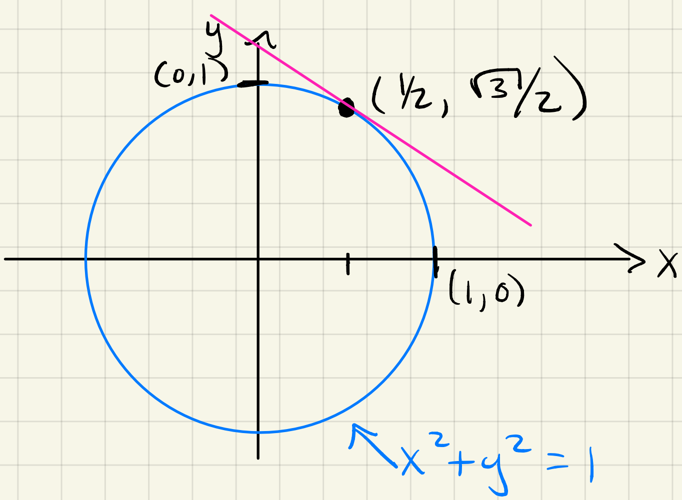
So far, we have techniques for calculating derivatives of explicit functions of the form $y = f(x)$.

Examples of implicit functions:

$$3x + 4y = 10$$
$$2x - 5y = e^{x^2}$$
$$x^2 + y^2 = 1$$
$$y^5 + x^2y^3 = e^{2y}$$

How do we find $\frac{dy}{dx}$ when y is defined implicity as a function of x?

Example 1: Find an equation of the tangent line to the unit circle $x^2+y^2=1$ at the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.



To use Implicit Differentiation, think of y as a function of x, then differentiate both sides of the equation w.r.t. x:

Example 2: Find $'$ if $3\cos(x)\sin(y) = 10$. $\Rightarrow \cos(x)\sin(y) = \frac{10}{3}$

Example 3: Find the slope of tangent line to $2(x+y)^{1/3} = y$ at the point $(4,4)$.

Example 4: Find $\frac{d^2y}{dx^2}$ (or y'') if $x^2+y^2=1$.

Using the result of Ex. 1, $\frac{dy}{dx} = \frac{-x}{y}$

Derivatives of Inverse Trigonometric Functions:

$$\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\csc^{-1}(x)\right] = \frac{-1}{|x|\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\cos^{-1}(x)\right] = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\sec^{-1}(x)\right] = \frac{1}{|x|\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\tan^{-1}(x)\right] = \frac{1}{1+x^2} \qquad \frac{d}{dx}\left[\cot^{-1}(x)\right] = \frac{-1}{1+x^2}$$

We can derive all of these derivatives using implicit differentiation.

Example: Show $\frac{d}{dx}\left[\tan^{-1}(x)\right] = \frac{1}{1+x^2}$.

Example:

a.) $\frac{d}{dx}\left[\sqrt{\tan^{-1}(x)}\right]$

b.) $\frac{d}{dx}\left[\sin^{-1}(x^4)\right]$

c.) $\frac{d}{dx}\left[x\, e^{\sec^{-1} x}\right]$