

## Foreword

I recall a moment, perhaps ten or fifteen years ago, of prodigious scientific activity. To give our reader a sense of this blessed time, consider a series of regular scientific workshops, each involving at most forty participants. Despite the small size and almost intimate nature of these workshops, they brought together an energized and enthusiastic mix of people from an array of disciplines, including mathematics, computer science, engineering, and the life sciences. What a privilege to be in a room with mathematicians such as Terence Tao and Roman Vershynin and learn about high-dimensional geometry; with applied mathematicians and engineers such as David Donoho, Joel Tropp, Thomas Ströhmer, Michael Elad, and Freddy Bruckstein and learn about the power of algorithms; with statistical physicists such as Andrea Montanari and learn about phase transitions in large stochastic systems. What a privilege to learn about fast numerical methods for large-scale optimization from computer scientists such as Stephen Wright and Stanley Osher. What a privilege to learn about compressive optical systems from David Brady and Richard Baraniuk and Kevin Kelly (of single-pixel camera fame); about compressive analog-to-digital conversion and wideband spectrum sensing from Dennis Healy, Yonina Eldar, and Azita Emami Neyestanak; about breakthroughs in computer vision from Yi Ma, John Wright, and René Vidal; and about dramatically faster scan times in magnetic resonance imaging from Michael Lustig and Leon Axel. Bringing all these people—and others I regretfully cannot name for lack of space—together, with their different perspectives and interests, sparked spirited discussions. Excitement was in the air and progress quickly followed.

Yi Ma and John Wright were frequent participants to these workshops and their book magically captures their spirit and richness. It exposes readers to (1) a variety of real-world applications including medical and scientific imaging, computer vision, wideband spectrum sensing, and so on, (2) the mathematical ideas powering algorithms in use in these fields, and (3) the algorithmic ideas needed to implement them. Let me illustrate with an example. On the one hand, this is a book in which we learn about the principles of magnetic resonance (MR) imaging. There is a chapter in which we learn how an MR scan excites the nucleus of atoms by means of a magnetic field. These nuclei have a magnetic spin, and will respond to this excitation, and it is precisely this response that gets recorded. As for other imaging modalities, such as computed tomography,

there is a mathematical transformation, which relates the object we wish to infer and the data we collect. In this case, after performing a few approximations, this mathematical transformation is given by the Fourier transform. On the other hand, this is a book in which we learn that most of the mass of a high-dimensional sphere is concentrated not just around the equator—this is already sufficiently surprising—but around any equator! Or that the intersection between two identical high-dimensional cubes, one being randomly oriented vis-à-vis the other, is essentially a sphere! These are fascinating subjects, but what is the connection? There is one, of course, and explaining it is the most wonderful strength of the book. In a nutshell, ideas and tools from probability theory, high-dimensional geometry, and convex analysis inform concrete applied problems and explain why algorithms actually work. Returning to our MR imaging problem, we learn how to leverage mathematical models of sparsity to recover exquisite images of body tissues from what appear to be far too few data points. Such a feat allows us to scan patients ten times faster today.

Through three fairly distinct parts — roughly: theory, computations, and applications — the book proposes a scientific vision concerned by the development of insightful mathematics to create models for data, to create processing algorithms, and to ultimately inspire real concrete improvements; for instance, in human health as in the example above.

The first part of the book explores data models around two main themes, namely, sparsity, and low-rankedness. Sparsity expresses the idea that most of the entries of an  $n$ -dimensional signal vanish or nearly vanish so that the information can be effectively summarized using fewer than  $n$  data bits. Low-rankedness expresses the idea that the columns of a data matrix ‘live’ near a linear subspace of lower dimension, thereby also suggesting the possibility of an effective summary. We then find out how to use these data models to create data processing algorithms, for instance, to find solutions of underdetermined systems of linear equations. The emphasis is on algorithms formulated as solutions to well-formulated convex optimization problems. That said, we are also introduced to nonconvex methods in Chapter 7 to learn effective empirical representations from data in which signals exhibit enhanced sparsity. All along, the authors use their rich experiences to communicate insights and to explain why some things work while others do not.

The second part reviews effective methods for solving optimization problems—convex or not—at scale; that is, involving possibly millions of decision variables and a possibly equally large number of constraints. This is an area that has seen tremendous progress in the last fifteen years and the book provides readers with a valuable point of entry to the key ideas and vast literature.

The last part is a deep dive into applications. In addition to the imaging challenges I already mentioned, we find a chapter on wireless radio communication, where we see how ideas from sparse signal processing and compressed sensing allow cognitive radios to efficiently identify the available spectrum. We also find three chapters on crucial problems in computer vision, a field in which

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the authors have brought and developed formidable tools, enabling major advances and opening new perspectives. Exposition starts with a special contribution, which also exploits ideas from compressed sensing, to the crucial problem of face recognition in the presence of occlusions and other nonidealities. (I recall an exciting *Wired* article about this work when it came out.) The book then introduces methods for inferring 3D structure from a series of 2D photographs, and to identify structured textures from a single photograph; solving the latter problem is often the starting point to recover the appearance, pose, and shape of multiple objects in a scene. Finally, at the time of this writing, deep learning (DL) is all the rage. The book contains an epilogue which establishes connections between all the better understood data models reviewed in the book and DL: the one hundred million dollar question is whether they will shed significant insights on deep learning and influence or improve its practice.

Who would enjoy this book? First and foremost, students in mathematics, applied mathematics, statistics, computer science, electrical engineering, and related disciplines. Students will learn a lot from reading this book because it is so much more than a text about a tool being applied with minor variations. They will learn about mathematical reasoning, they will learn about data models and about connecting those to reality, and they will learn about algorithms. The book also contains computer scripts so that we can see ideas in action and carefully crafted exercises making it perfect for upper-level undergraduate or graduate-level instruction. The breadth and depth makes this a reference for anyone interested in the mathematical foundations of data science. I also believe that members of the applied mathematical sciences community at large would enjoy this book. They will be reminded of the power of mathematical reasoning and of the all-around positive impact it can have.

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