Population Models

Tuesday, January 16, 2018

Today's main questions:

- *What makes a model good?*
- *Once I have a model, what should I do with it?*

Year	U.S. Population
1790	3,929,214
1800	5,308,438
1810	7,239,881
1820	9,638,453
1830	12,866,020
1840	17,069,453
1850	23,191,876
1860	31,443,321
1870	38,558,371
1880	50,189,209
1890	62,979,776
1900	76,212,168
1910	92,228,496
1920	106,021,537
1930	123,202,624
1940	132,164,569
1950	151,325,798
1960	179,323,175
1970	203,302,031
1980	226,542,199
1990	248,709,873
2000	281,421,906
2010	308,745,538

Today, the main focus will be on population models. On the left, is U.S. population data gathered from the U.S. Census website. Over the next several questions you will investigate various models of the population and compare them to this data. If you have downloaded, the associated Matlab code, the data in that table is stored in "`us_population.mat`". We'll also be using bacteria the bacteria population data (below) from Semion K., et al. "Chromatic acclimation and population dynamics of green sulfur bacteria grown with spectrally tailored light." Scientific reports 4 (2014).

Hours	Bacteria Density
17.1633	0.1163
24.1732	0.1680
37.9327	0.4231
49.9703	0.9121
62.0156	1.3378
74.0593	1.7762
89.7925	1.7151

One standard population model is given by

$$\frac{dP}{dt} = kP.$$

This model will be our starting point for the questions below.

1. As stated, the population model, $\frac{dP}{dt} = kP$ is in some sense incomplete. What information would you need in order to make a quantitative comparison to actual data? Use the US population data (and Matlab) to infer the missing information to the best of your ability. Be prepared to explain your approach.

2. Suppose instead someone proposed the model

$$\frac{dP}{dt} = kP + c.$$

Provide a practical meaning for the value of c. Again using the US population model, determine to the best of your ability a reasonable numerical value for c. Did your method from the previous question generalize?

3. Next suppose that you know that if the population ever exceeded a certain value, the population would tend to decrease. Write down a differential equation that appropriately models this situation.

4. Which of the above models is best for US Data? Which of these models is best for the bacteria data? In which sense is it best? What evidence do you have that it's best? Do you think you can do better? What do you think is missing?

5. Consider the model from question 3.

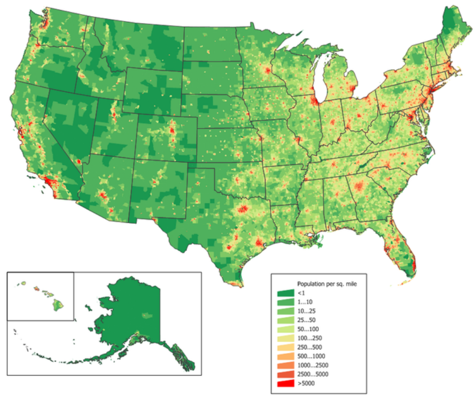
   (a) Intuitively (and without appealing to an explicit solution to differential equation), what do you expect to have happen in the long run if the population began above that maximum value.

   (b) ...or below that maximum value?

   (c) Draw a graphic that illustrates the flow of the system as time evolves. What I'm after here is a phase diagram (or phase line) if those words mean anything to you.

6. What would the phase diagram say if the population started off negative? Is this a problem in your mind?

Here's a map of population density of the US (shamelessly stolen from wikipedia)



Our models from the previous section are not going to be adequate to capture this degree of complexity. We're clearly missing something–some geographic regions seem to be more desirable to live than others.

7. Suppose we have the model
$$\frac{\partial P}{\partial t} = kP(A(x) - P)$$
where $A(x)$ is a known spatially varying function. What situation does this model describe?

8. Assuming that $A(x) = 300 + 100\cos\left(\frac{2*\pi}{50}x\right)$ and $k = 0.05$. Further suppose we know the population is initially given by $100 + 50\cos\left(\frac{2*\pi}{50}x\right)$. What do you expect to happen to the population in the long run?

9. Did the initial data really matter in that last question? Did the specific value of $A(x)$?

10. Consider the model
$$\frac{\partial P}{\partial t} = -c\frac{\partial P}{\partial x}$$
What does this model say about how the population changes? `Problem10.m` solves problem via a numerical approximation that we'll discuss in a later class. After you've thought about the structure of this model for a while, run `Problem10.m` for a couple of different initial conditions. Does it output roughly what you expect?

11. Consider the model
$$\frac{\partial P}{\partial t} = -Ac\cos\left(ct\right)\frac{\partial P}{\partial x}$$
What does this model say about how the population changes? Can you think of a situation in nature that this might be an appropriate model for? (Run `Problem11.m` to see a solution)

12. Consider the following scenario. Suppose you have two populations of different bacteria growing in a dish. The two bacterial strains grow with different relative growth rates. While neither bacteria “preys” on the other, the two are competing for resources. Write the simplest model you can for this system.

13. What factors do you think are missing from your two-bacteria model from the previous question? Are modeling considerations that would make you want to use a partial differential equation?