Introduction to Differential Equations

1. (a) Is the point $x = 1,\ y = 3$ (i.e the point $(1, 3)$) a solution to the equation $2y + x = y + 4$? Why?

   (b) Is the point $(2, 3)$ a solution to the equation $y^2 + x = y + 8$? Why?

   (c) Is the point $(1, 1)$ a solution to the equation $x^2 + y^2 = 1$? Why?

   (d) What does it mean that a point (a, b) is a solution to an equation?

2. (a) Is the function $y = x^2$ a solution to the equation $\frac{dy}{dx} = 2x$? Why?

   (b) Is the function $y = -\frac{1}{x}$ a solution to the equation $\frac{dy}{dx} = y^2$?

   If $y = -\frac{1}{x}$, then $\frac{dy}{dx} =$ _____

   If $y = -\frac{1}{x}$, then $y^2 =$ _____

   If $y = -\frac{1}{x}$, is $\frac{dy}{dx} = y^2$?

   (c) Is the function $y = \frac{x^2}{2}$ a solution to the equation $\frac{dy}{dx} = y + x$?

   If $y = \frac{x^2}{2}$, then $\frac{dy}{dx} =$ _____

   If $y = \frac{x^2}{2}$, then $y + x =$ _____

   If $y = \frac{x^2}{2}$, is $\frac{dy}{dx} = y + x$?

3. (a) Is the function $y = e^{3x}$ a solution to the equation $\frac{dy}{dx} = 3y$?

If $y = e^{3x}$, then $\frac{dy}{dx} =$ _____

If $y = e^{3x}$, then $3y =$ _____

(b) Is the function $y = e^{3x} + 1$ a solution to the equation $\frac{dy}{dx} = 3y$?

If $y = e^{3x} + 1$, then $\frac{dy}{dx} =$ _____

If $y = e^{3x} + 1$, then $3y =$ _____

(c) Is the function $y = 2e^{3x}$ a solution to the equation $\frac{dy}{dx} = 3y$?

If $y = 2e^{3x}$, then $\frac{dy}{dx} =$ _____

If $y = 2e^{3x}$, then $3y =$ _____

Equations of the form $\frac{dy}{dx} = y^2$ or $\frac{dy}{dx} = 2x$ or $\frac{dy}{dx} = y + x$ are called differential equations. In fact, any equation of the form $\frac{dy}{dt} = g(y, t)$ is a differential equation. $y = f(t)$ is a solution if when $f(t)$ is substituted for y in the expression $g(y, t)$, the result is $\frac{dy}{dt}$. In other words, like any other equation, when you substitute your answer into both sides of the equation you get a true statement.

4. Find a function that satisfies the equation $\frac{dy}{dx} = 3x$:

Can you name another such function?

5. Find a function that satisfies the equation $\frac{dy}{dx} = ky$:

Can you name another such function?

6. Which of the following is a solution to the differential equation $\frac{dy}{dt} = 9y$?
(a) $f(t) = e^{9t}$ (b) $f(t) = 5e^{9t}$ (c) $f(t) = e^{3t}$ (d) $f(t) = e^{9t} + 5$

7. Which of the following is a solution to the differential equation $\frac{dy}{dt} = at$?
(a) $f(t) = \frac{a}{2}t^2$ (b) $f(t) = \frac{a}{2}t^2 + 3$ (c) $f(t) = e^{at}$ (d) $f(t) = (at)^2$

8. Which of the following is a solution to $\frac{dy}{dt} = 2(y - 1)$?

(a) $1 - e^{2t}$

(b) $1 + e^{2t}$

(c) $e^{2t} - 2t$

(d) $2 + e^{2t}$

In general, differential equations have an infinite number of solutions, but if we are given an initial starting point, there will be only one solution. A differential equation with an initial condition, is called an initial value problem.

9. Which of the following is **the** solution to the intial value problem
$\frac{dy}{dt} = 9y$ with $y = 3$ at $t = 0$?

(a) $f(t) = e^{9t}$ (b) $f(t) = 3e^{9t}$ (c) $f(t) = 2e^{9t}$ (d) $f(t) = 4e^{9t}$ (e) $f(t) = 2e^{9t} + 1$

10. Which of the following is **the** solution to the intial value problem
$\frac{dy}{dt} = 5t$ with $y = 4.5$ at $t = 1$?

(a) $f(t) = \frac{5}{2}t^2$ (b) $f(t) = \frac{5}{2}t^2 + 2$ (c) $f(t) = 5t^2 + \frac{1}{2}$ (d) $f(t) = \frac{5}{2}t^2 + .5$

11. Solve the the intial value problem $\frac{dy}{dt} = 6y$ with
(a) $y = 2$ at $t = 0$? (b) $y = 2$ at $t = 1$?

12. Find a function of the form $y = x^n$ that is a solution to the differential equation: $\frac{1}{2}x\frac{dy}{dx} = y$.