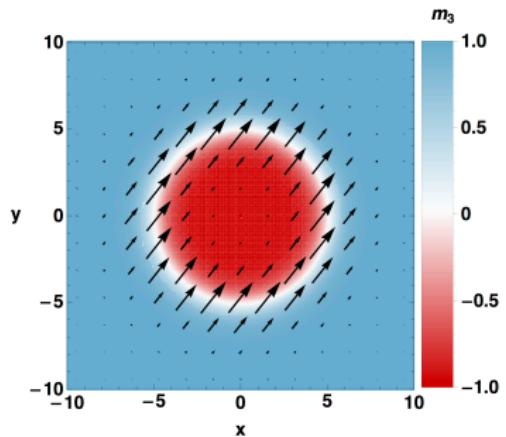


Soliton-soliton interactions in thin ferromagnetic films



Lake Bookman



Overview

1 What are solitons?

- Examples of Solitons
- Mathematics of Solitons

2 Torque Equation

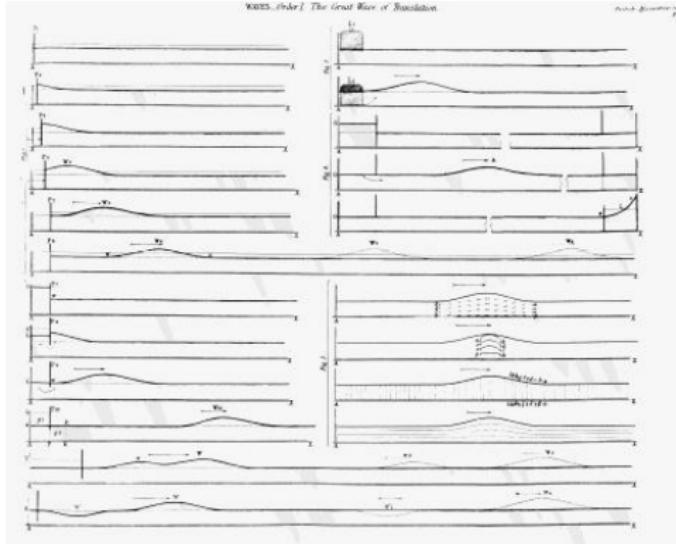
- Physical Model
- Droplet Solitons

3 Interacting Droplets

4 Applications for Droplets

What are solitons?

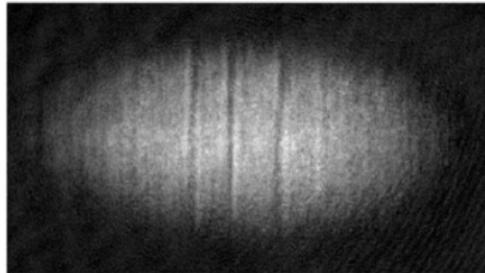
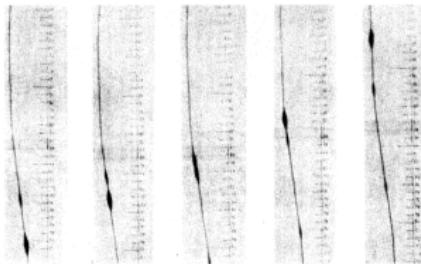
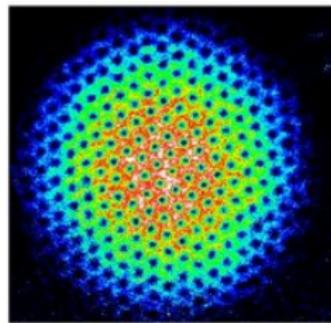
Solitons



"I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation." - John Scott Russel, "Report on Waves." 1844

Solitons

Examples from fluid mechanics, Bose-Einstein condensates and granular media



Sources: Ablowitz and Baldwin (2012)
Engels and Atherton (2007)
Zabusky (2014), Whitehead (1987)

Math of Solitons

Solitons

Inviscid Burgers
Equation

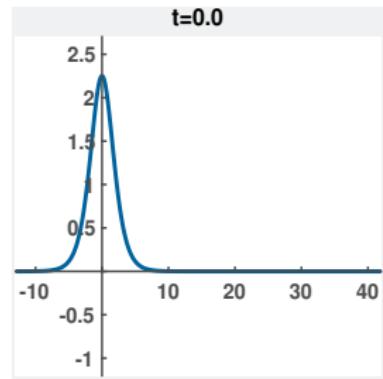
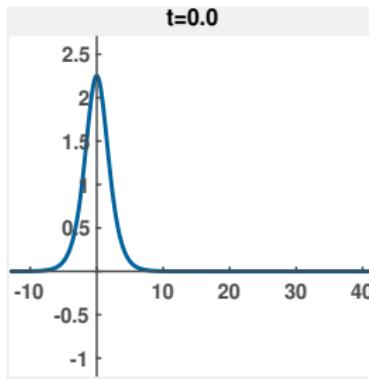
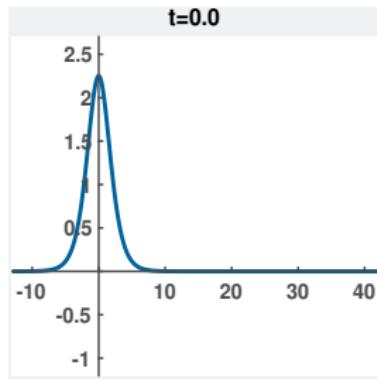
$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0$$

Linear
Korteweg de-Vries
Equation

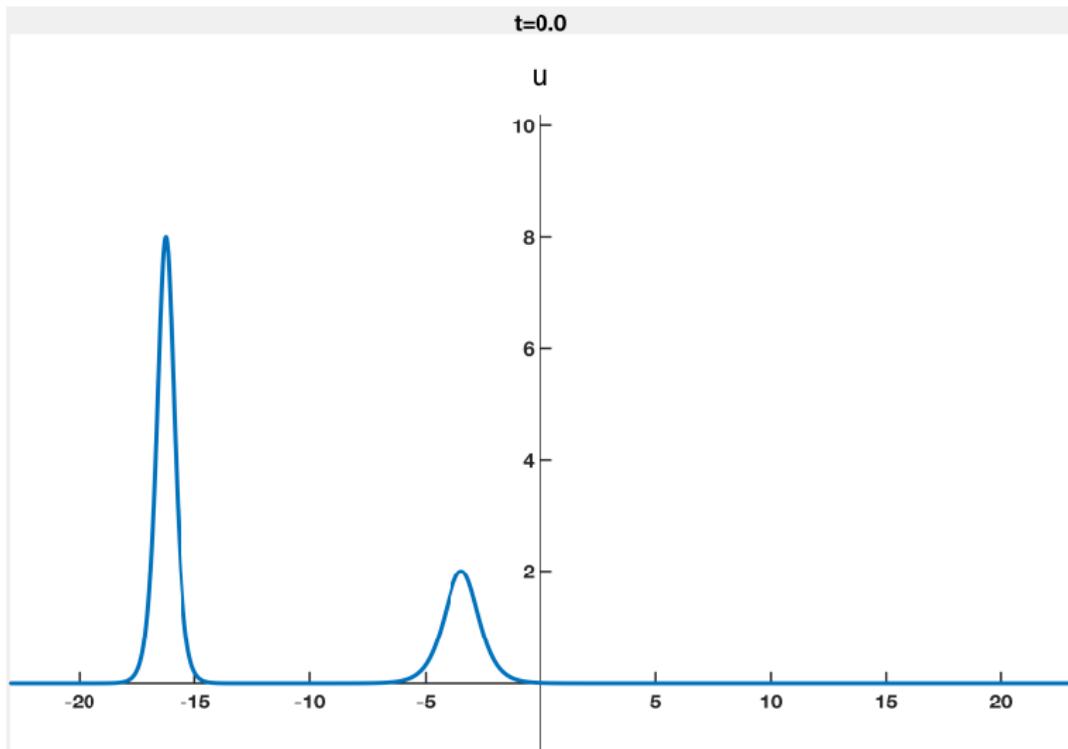
$$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} = 0$$

Korteweg de-Vries
Equation

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0$$



2-Soliton Interaction in KdV

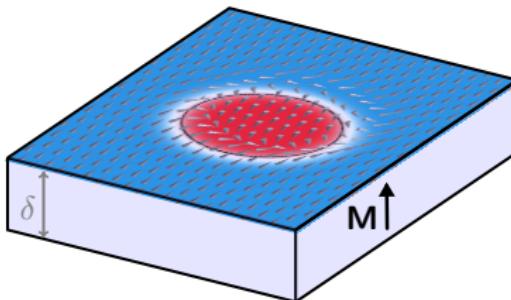


Torque Equation

Physical Model

Torque Equation

$$\left\{ \begin{array}{l} \frac{\partial \mathbf{M}}{\partial t} = -|\gamma| \mu_0 \mathbf{M} \times (\mathbf{H}_0 + \mathbf{H}_{\text{ex}} + \mathbf{H}_k + \mathbf{H}_d) \\ \mathbf{M} : \mathbb{R}^2 \times [-\delta/2, \delta/2] \times \mathbb{R}^+ \rightarrow M_s \mathbb{S}^2 \\ \lim_{x^2+y^2 \rightarrow \infty} \mathbf{M} \rightarrow M_s \hat{\mathbf{z}} \quad \frac{\partial \mathbf{M}}{\partial z} \Big|_{z=\pm\delta/2} = 0 \end{array} \right.$$



Parameters: $\left\{ \begin{array}{ll} M_s & - \text{saturation magnetization} \\ \gamma & - \text{gyromagnetic ratio} \\ \mu_0 & - \text{permeability of free space} \end{array} \right.$

Physical Model

Torque Equation

$$\left\{ \begin{array}{l} \frac{\partial \mathbf{M}}{\partial t} = -|\gamma| \mu_0 \mathbf{M} \times (\overbrace{\mathbf{H}_0 + \mathbf{H}_{\text{ex}} + \mathbf{H}_k + \mathbf{H}_d}^{\mathbf{H}_{\text{eff}}}) \\ \mathbf{M} : \mathbb{R}^2 \times [-\delta/2, \delta/2] \times \mathbb{R}^+ \rightarrow M_s \mathbb{S}^2 \\ \lim_{x^2+y^2 \rightarrow \infty} \mathbf{M} \rightarrow M_s \hat{\mathbf{z}} \quad \frac{\partial \mathbf{M}}{\partial z} \Big|_{z=\pm\delta/2} = 0 \end{array} \right.$$

Contributions to \mathbf{H}_{eff} :

\mathbf{H}_0 - External, Applied Field

\mathbf{H}_{ex} - Exchange Field, (short-range interaction), $\sim \nabla^2 \mathbf{M}$

\mathbf{H}_k - Field Due to Crystalline Structure

\mathbf{H}_d - Dipolar Field (nonlocal effect): $\nabla \cdot \mathbf{H}_d = -\nabla \cdot \mathbf{M}$

Parameters: $\left\{ \begin{array}{ll} M_s & - \text{saturation magnetization} \\ \gamma & - \text{gyromagnetic ratio} \\ \mu_0 & - \text{permeability of free space} \end{array} \right.$

Idealized Model

2D Nondimensional Torque Equation

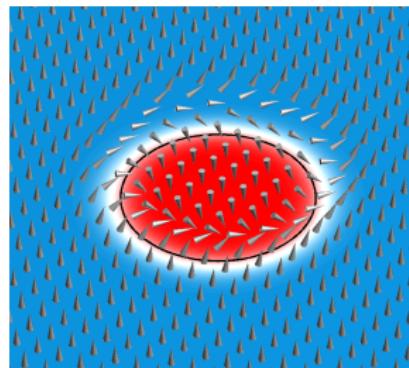
$$\begin{cases} \frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times (\nabla^2 \mathbf{m} + (m_z + h_0)\mathbf{z}) \\ \mathbf{m} : \mathbb{R}^2 \times \mathbb{R}^+ \rightarrow \mathbb{S}^2, \lim_{|\mathbf{x}| \rightarrow \infty} \mathbf{m} \rightarrow \hat{\mathbf{z}} \end{cases}$$

Physical Assumptions

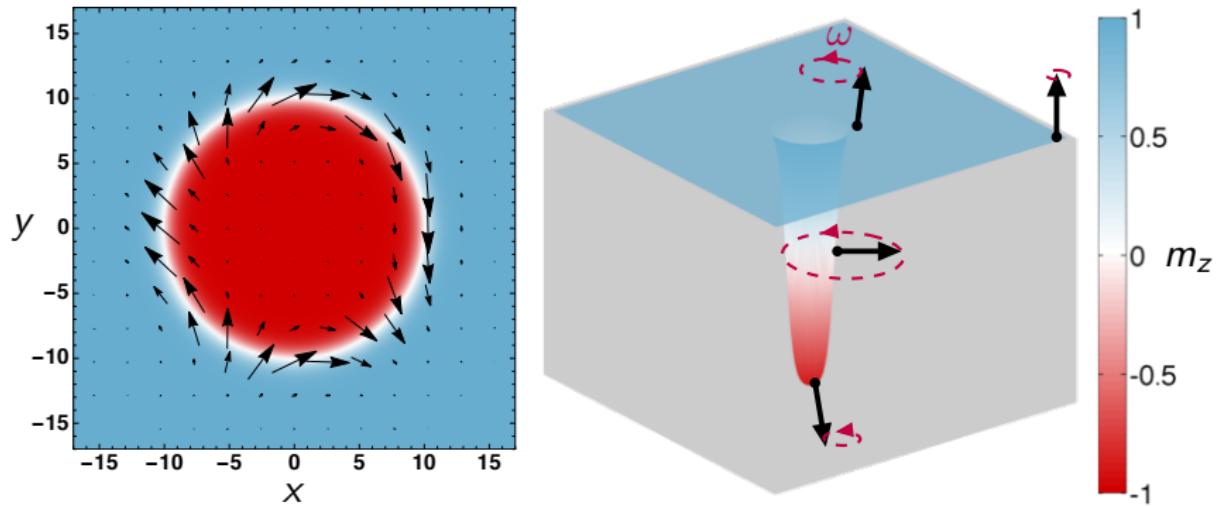
- ▶ Perpendicular anisotropy $\Rightarrow \mathbf{H}_k \sim M_z \hat{\mathbf{z}}$
- ▶ Perpendicular external field $\Rightarrow \mathbf{H}_0 \sim H_0 \hat{\mathbf{z}}$
- ▶ Thin ferromagnetic layer $\Rightarrow \mathbf{H}_d \sim M_z \hat{\mathbf{z}}$

Units:

- x - scaled exchange length $\sim 10^{-9} m$
 - t - scaled Larmor frequency $\sim 10^{-9} s$
- \Rightarrow (See Hoefer and Sommacal (2012) for nondimensionalization)

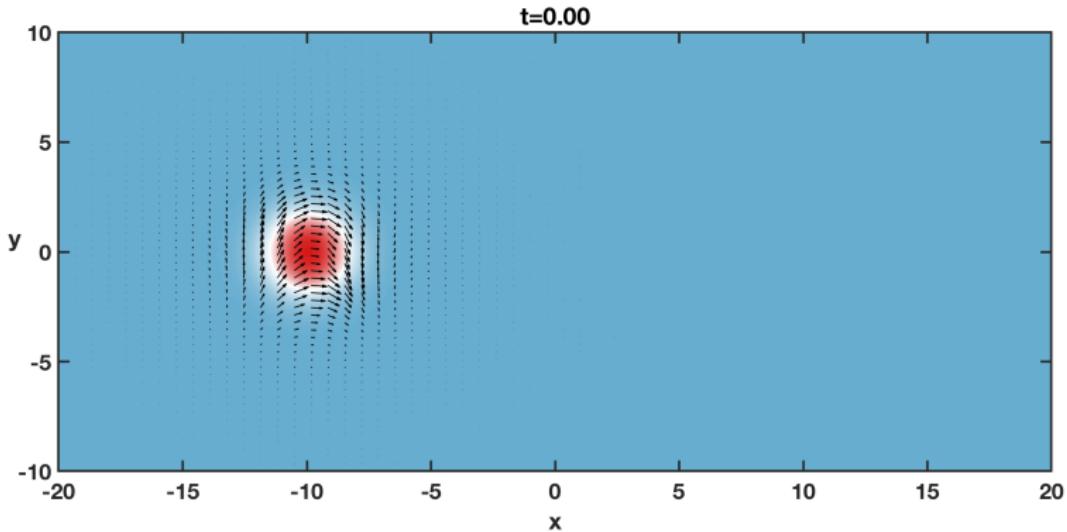


Droplet Solitons



Review article: “Magnetic Solitons” Phys. Reports. 194 Kosevich et al. (1990)

Droplet Solitons



Review article: “Magnetic Solitons” Phys. Reports. 194 Kosevich et al. (1990)

Droplet Solitons

Soliton Parameters:

x_0 - initial position

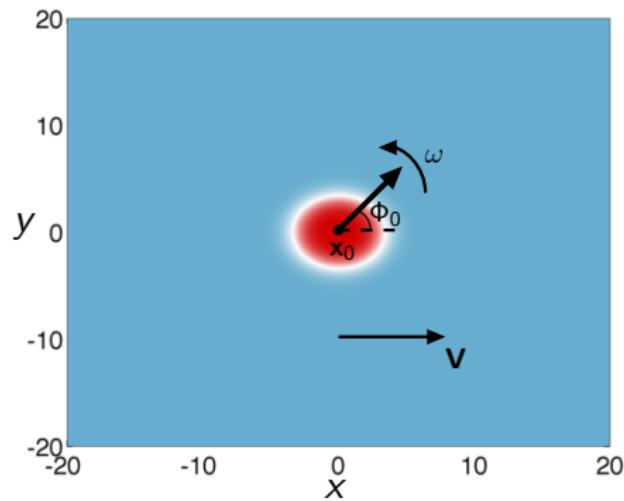
ϕ_0 - initial phase

v - propagation velocity

ω - frequency

Analytical solution for small V, ω :

- ▶ Slowly precessing, radially symmetric solution
- ▶ Radius inversely proportional to frequency



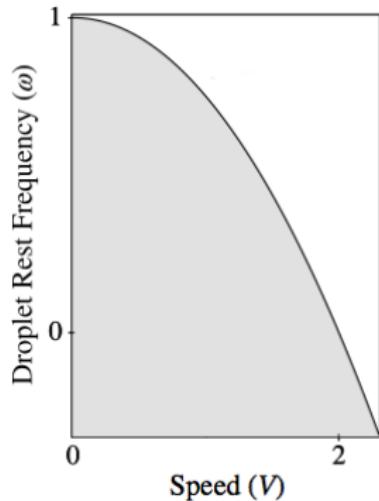
Droplet Solitons

Soliton Parameters:

- x_0 - initial position
- Φ_0 - initial phase
- V - propagation velocity
- ω - frequency

Analytical solution for small V, ω :

- ▶ Slowly precessing, radially symmetric solution
- ▶ Radius inversely proportional to frequency

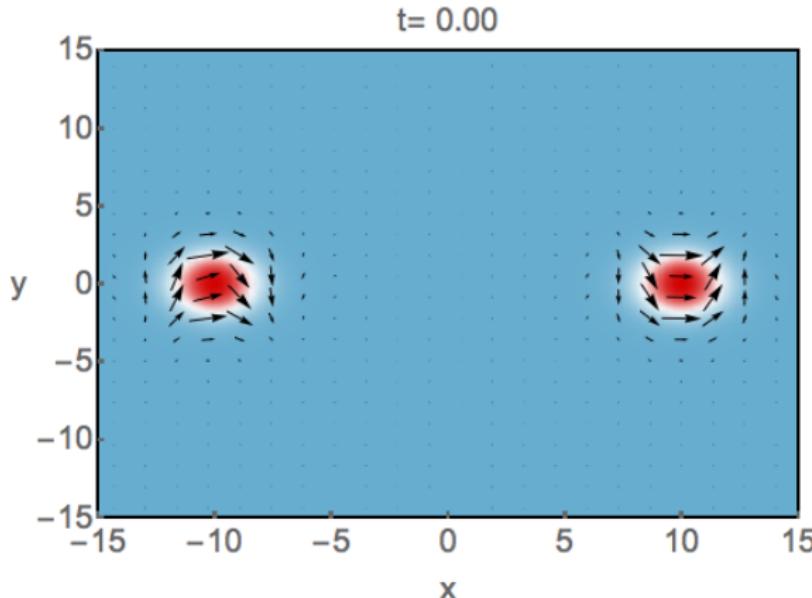


[From Hoefer and Sommacal (2012)]

Interacting Droplets

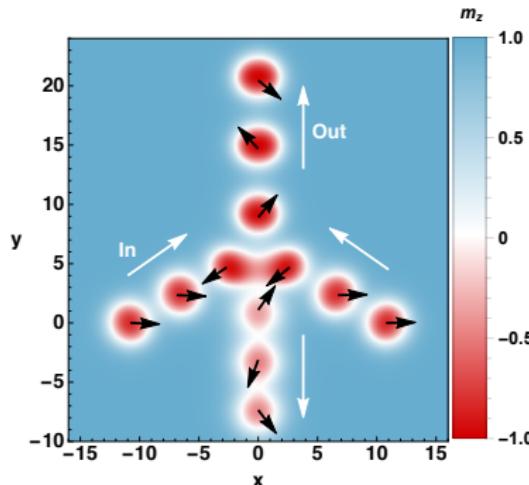
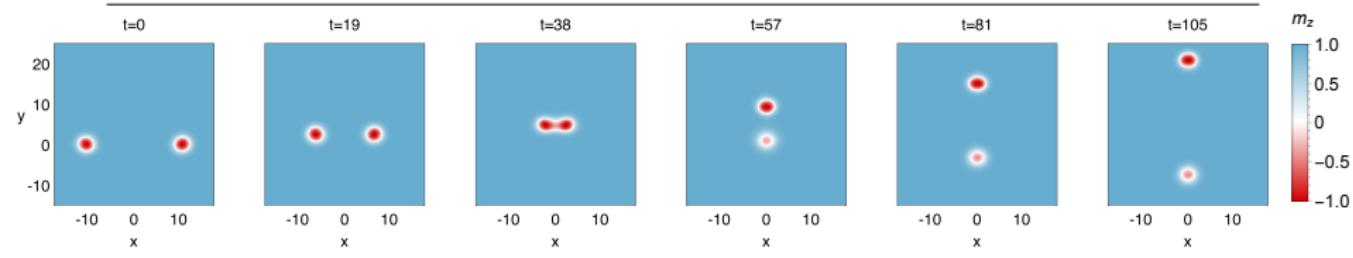
Droplet Interactions

$$\Delta\Phi = 0$$



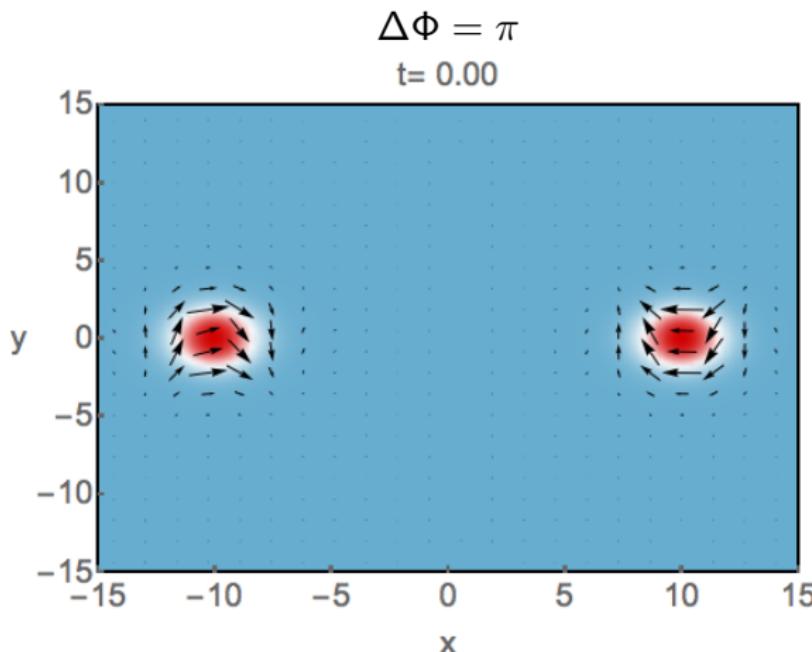
Droplet Interactions

Attraction \Rightarrow scattering along line of symmetry



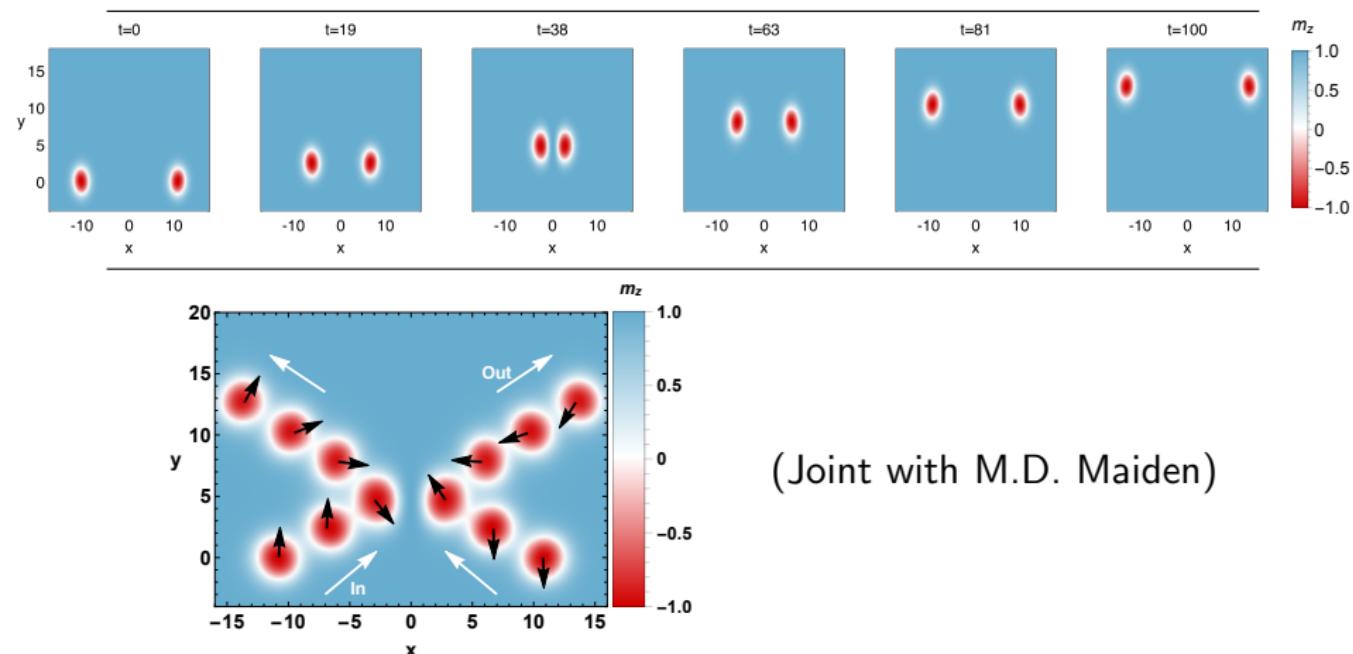
(Joint with M.D. Maiden)

Droplet Interactions



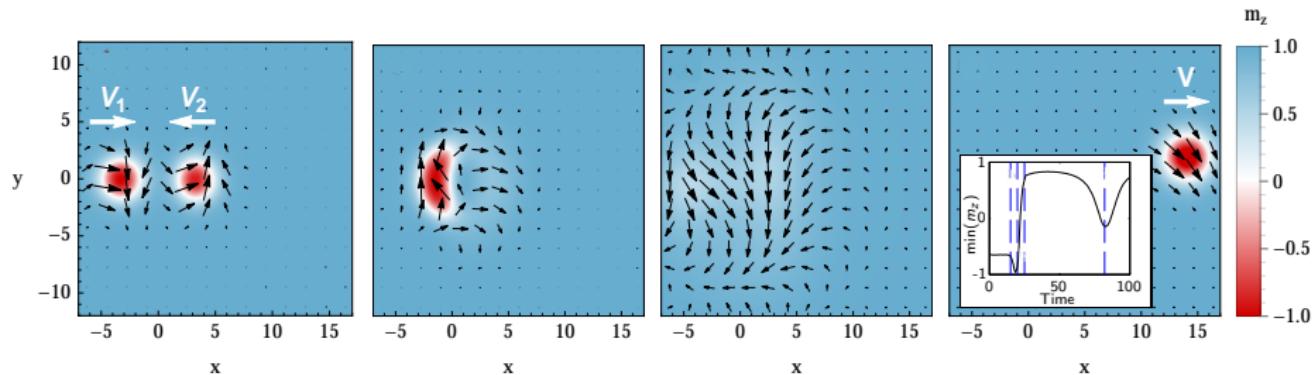
Droplet Interactions

Repulsion



Droplet Interactions

Annihilation



[Figure from Maiden et al. (2014)]

Application: Spintronics

Damping Contribution

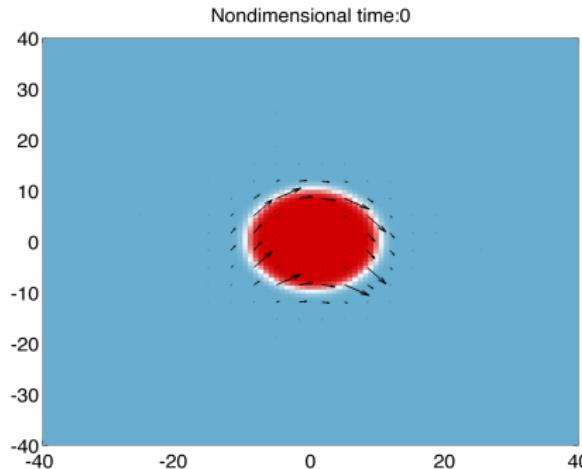
[Landau and Lifshitz (1935)]:

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathbf{h}_{\text{eff}} - \alpha \mathbf{m} \times (\mathbf{m} \times \mathbf{h}_{\text{eff}})$$

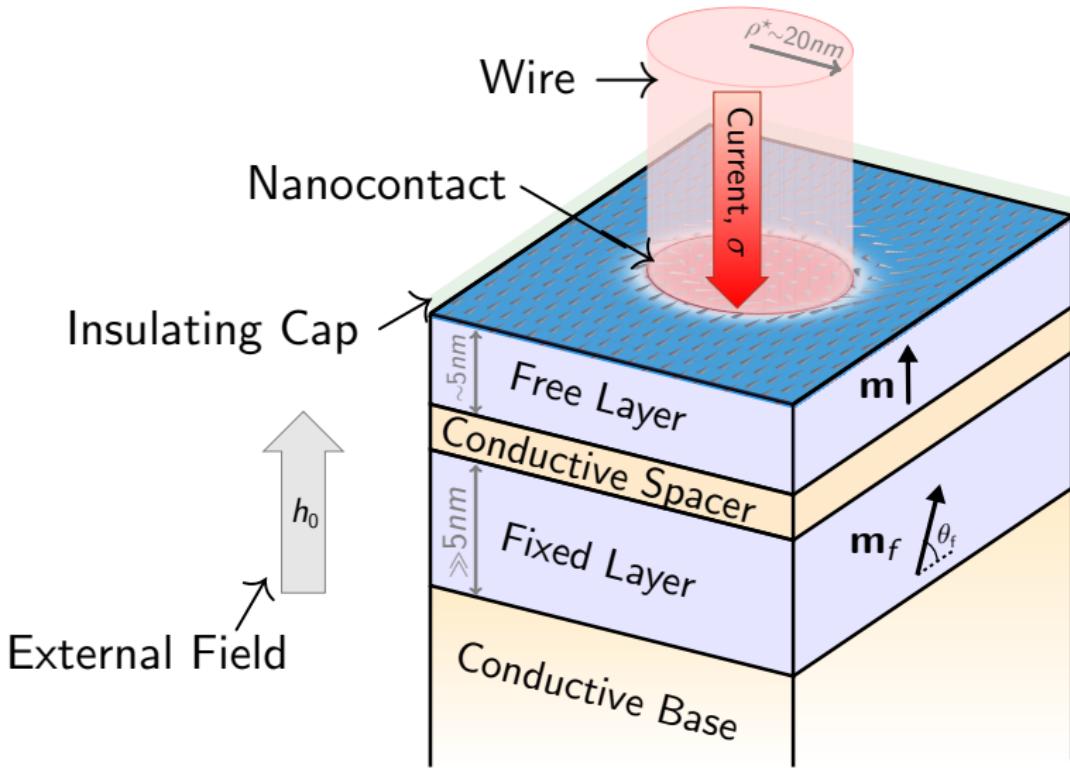
α - nondimensional damping parameter

$$\omega_0 = 0.1$$

$$|\mathbf{V}_0| = 0.01$$

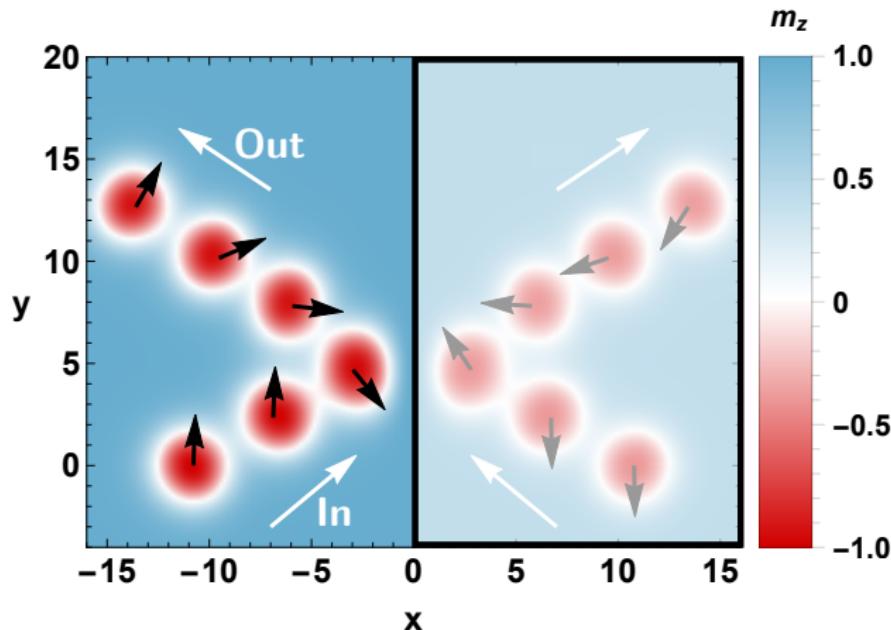


Droplets in Experiment



Droplet Interactions

Repulsion (from pinned boundary)



Future Work

Extensions of this work:

- ▶ Computation of numerically exact breathers, dissipative droplets etc.
- ▶ Impact of other parameters on soliton-soliton interaction
- ▶ Similar experiments with skyrmions

Other projects:

- ▶ Models of rogue waves on the deep ocean (breathers in a nonlinear Schrodinger-type equations)
- ▶ (Dynamic) Causal Inference (inference from data, text)
- ▶ Agent-based decision models (general game playing)

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- Norman J Zabusky. Soliton (scholarpedia.org), April 2014. URL <http://www.scholarpedia.org/article/Soliton>.

Index of Slides

1.

Overview

- What are solitons?
 - Mathematics of Solitons
- Torque Equation
 - Physical Model
 - Droplet Solitons
- Interacting Solitons
- Applications for Droplets

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2.

What are solitons?

Olin College 2 Tuesday, January 16, 2018

3.

Solitons

Olin College 3 Tuesday, January 16, 2018

4.

Solitons

Examples from fluid mechanics, Bénard-Crosson concretes, and granular media
Source: Albiez and Rubins (2005)
Engels and Schermann (2007)
Garcia-Garcia et al. (2007)
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5.

Math of Solitons

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6.

Solitons

Initial Burgers Equation	Lions-Kompane-Esien Equation	Korteweg-de-Vries Equation
$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0$	$\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} = 0$	$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} = 0$

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7.

2-Soliton Interaction in KdV

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8.

Torque Equation

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9.

Physical Model

Torque Equation

$$\begin{cases} \frac{\partial \mathbf{M}}{\partial t} = -[\gamma]\mu_0(\mathbf{M} \times (\mathbf{H}_0 - \mathbf{H}_0 + \mathbf{H}_d)) \\ \mathbf{M} : \mathbb{R}^2 \times [-\gamma, \gamma] \times \mathbb{R}^2 \rightarrow \mathbb{M}_2^{2x2} \\ \lim_{t \rightarrow \infty} \mathbf{M}(t) = \mathbf{M}_d \quad \forall t \in [-\gamma, \gamma] \end{cases}$$

Parameters: M_d - saturation magnetization, γ - gyromagnetic ratio, μ_0 - permeability of free space

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11.

Idealized Model

2D Non-dimensional Torque Equation

$$\begin{cases} \frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times (\nabla^2 \mathbf{m} + (\alpha_1 + \alpha_2)\mathbf{z}) \\ \mathbf{m} : \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{M}_{2x2}, \mathbf{m} \in \mathbb{R}^2 \end{cases}$$

Physical Assumptions:

- Perpendicular anisotropy $\Rightarrow \mathbf{H}_0 = M_d \mathbf{z}$
- Dependence external field $\propto \mathbf{H}_d = M_d \mathbf{z}$
- This ferromagnetic layer $\Rightarrow \mathbf{H}_d = M_d \mathbf{z}$

Units:

- α - scaled exchange length $\sim 10^{-8} \text{ m}$
- τ - scaled Larmor frequency $\sim 10^{-8} \text{ s}$

(See Hofer and Semperlari (2022) for non-dimensionalization)

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12.

Droplet Solitons

Review article: "Magnetic Solitons" Phys. Reports., 194 Kusnicz et al. (1990)

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14.

Droplet Solitons

Soliton Parameters:

- α_0 - saturation magnetization
- θ_0 - initial phase
- \mathbf{V} - propagation velocity
- ω - frequency

Analytical solution for small ω, ω :

- Stably precessing, rotatory symmetric
- Radius inversely proportional to frequency

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16.

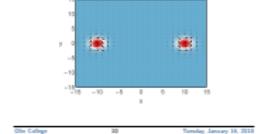
Interacting Droplets

17.

Droplet Interactions

$$\Delta\phi = 0$$

$$k_0 = 0.01$$



22.

Application: Spintronics

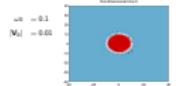
23.

Damping Contribution

[Landau and Lifshitz (1935)]

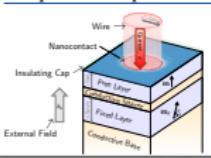
$$\frac{\partial \mathbf{u}}{\partial t} = -\alpha \times \mathbf{h}_0 + \mathbf{h}_0 \times \mathbf{u} + (\mathbf{u} \times \mathbf{h}_0)$$

α - nondimensional damping parameter



24.

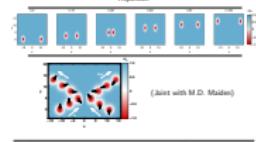
Droplets in Experiment



25.

Droplet Interactions

Repulsion



26.

Future Work

Extension of this work:

- Computation of numerically exact breathers, dissipative droplets etc.
- Impact of other parameters on sultan-silicon interaction
- Similar experiments with skyrmions

Other projects:

- Models of rogue waves on the deep ocean (breathers in a non-linear Schrödinger-type equation)
- [Dynamic] Causal Inference (inference from data, not)
- Agent-based decision models (general game playing)

27.

References

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