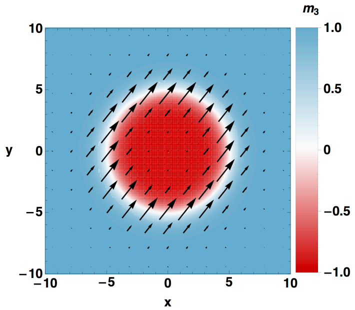
Soliton-soliton interactions in thin ferromagnetic films



Lake Bookman

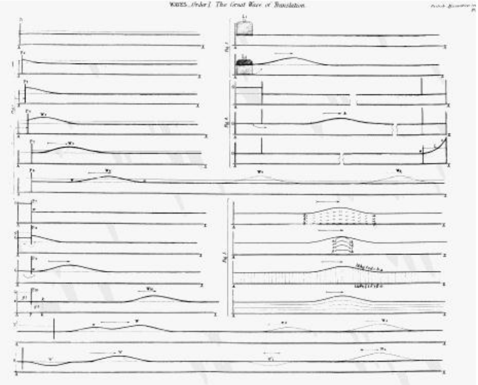
BAE SYSTEMS

Olin College, January 2018

Overview

What are solitons?

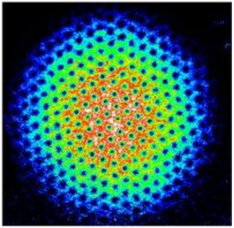
Solitons

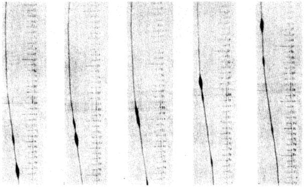


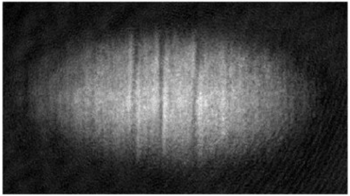
"I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation." - John Scott Russel, "Report on Waves." 1844

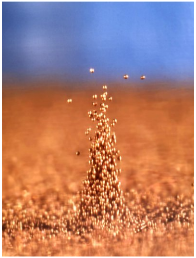
Solitons

Examples from fluid mechanics, Bose-Einstein condensates and granular media







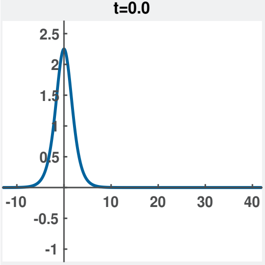


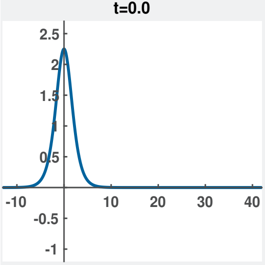
Sources: Ablowitz and Baldwin (2012)
Engels and Atherton (2007)
Zabusky (2014), Whitehead (1987)

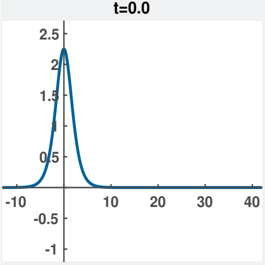
Math of Solitons

Solitons

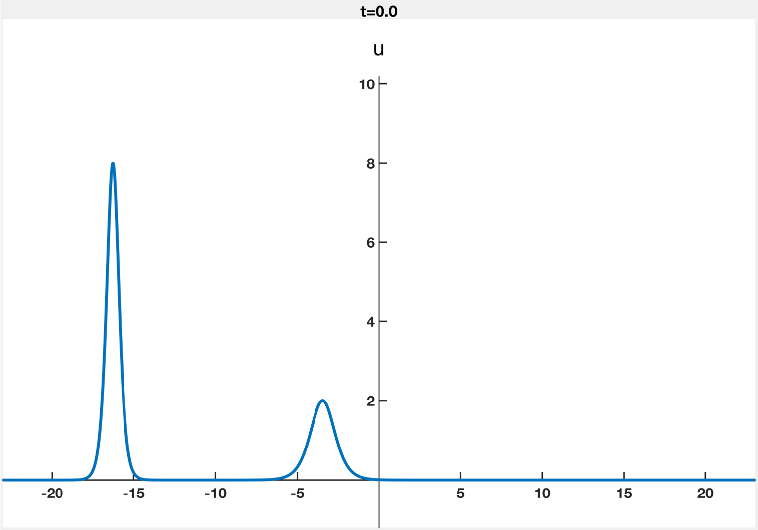
Inviscid Burgers Equation	Linear Korteweg de-Vries Equation	Korteweg de-Vries Equation
$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$	$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} = 0$	$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0$







2-Soliton Interaction in KdV

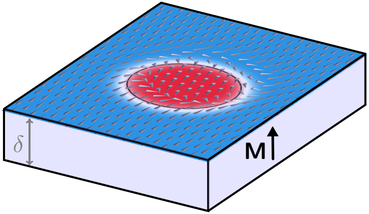


Torque Equation

Physical Model

Torque Equation

$$\begin{cases} \frac{\partial \mathbf{M}}{\partial t} = -\left|\gamma\right| \mu_0 \mathbf{M} \times \left(\mathbf{H}_0 + \mathbf{H}_{\mathrm{ex}} + \mathbf{H}_{\mathrm{k}} + \mathbf{H}_{\mathrm{d}}\right) \\ \mathbf{M} : \mathbb{R}^2 \times [-\delta/2, \delta/2] \times \mathbb{R}^+ \to M_s \mathbb{S}^2 \\ \lim_{x^2+y^2\to\infty} \mathbf{M} \to M_s \hat{\mathbf{z}} \qquad \left.\frac{\partial \mathbf{M}}{\partial z}\right|_{z=\pm\delta/2} = 0 \end{cases}$$



Parameters:

$$\begin{cases} M_s & - & \text{saturation magnetization} \\ \gamma & - & \text{gyromagnetic ratio} \\ \mu_0 & - & \text{permeability of free space} \end{cases}$$

Physical Model

Torque Equation

$$\begin{cases} \frac{\partial \mathbf{M}}{\partial t} = -\left|\gamma\right| \mu_0 \mathbf{M} \times (\overbrace{\mathbf{H}_0 + \mathbf{H}_{\mathrm{ex}} + \mathbf{H}_{\mathrm{k}} + \mathbf{H}_{\mathrm{d}}}^{\mathbf{H}_{\mathrm{eff}}}) \\ \mathbf{M} : \mathbb{R}^2 \times [-\delta/2, \delta/2] \times \mathbb{R}^+ \to M_s \mathbb{S}^2 \\ \lim_{x^2+y^2 \to \infty} \mathbf{M} \to M_s \hat{\mathbf{z}} \qquad \left.\frac{\partial \mathbf{M}}{\partial z}\right|_{z=\pm\delta/2} = 0 \end{cases}$$

<u>Contributions to $\mathbf{H}_{\mathrm{eff}}$:</u>

$\mathbf{H}_0$	-	External, Applied Field
$\mathbf{H}_{\mathrm{ex}}$	-	Exchange Field, (short-range interaction), $\sim \nabla^2 \mathbf{M}$
$\mathbf{H}_{\mathrm{k}}$	-	Field Due to Crystalline Structure
$\mathbf{H}_{\mathrm{d}}$	-	Dipolar Field (nonlocal effect): $\nabla \cdot \mathbf{H}_{\mathrm{d}} = -\nabla \cdot \mathbf{M}$

<u>Parameters:</u>

M_s	-	saturation magnetization
γ	-	gyromagnetic ratio
μ_0	-	permeability of free space

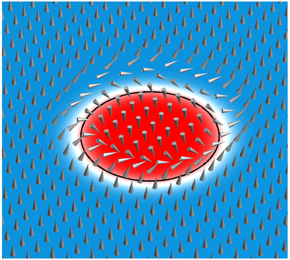
Idealized Model

2D Nondimensional Torque Equation

$$\begin{cases} \frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \left(\nabla^2 \mathbf{m} + (m_z + h_0)\mathbf{z}\right) \\ \mathbf{m} : \mathbb{R}^2 \times \mathbb{R}^+ \to \mathbb{S}^2, \lim_{|\mathbf{x}| \to \infty} \mathbf{m} \to \hat{\mathbf{z}} \end{cases}$$

Physical Assumptions

- Perpendicular anisotropy $\Rightarrow \mathbf{H}_k \sim M_z \hat{\mathbf{z}}$
- Perpendicular external field $\Rightarrow \mathbf{H}_0 \sim H_0 \hat{\mathbf{z}}$
- Thin ferromagnetic layer $\Rightarrow \mathbf{H}_d \sim M_z \hat{\mathbf{z}}$



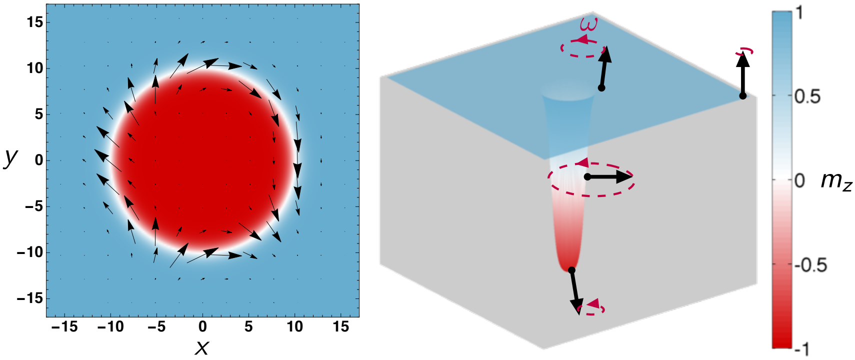
Units:

$\mathbf{x}$ - scaled exchange length $\sim 10^{-9} m$

t - scaled Larmor frequency $\sim 10^{-9} s$

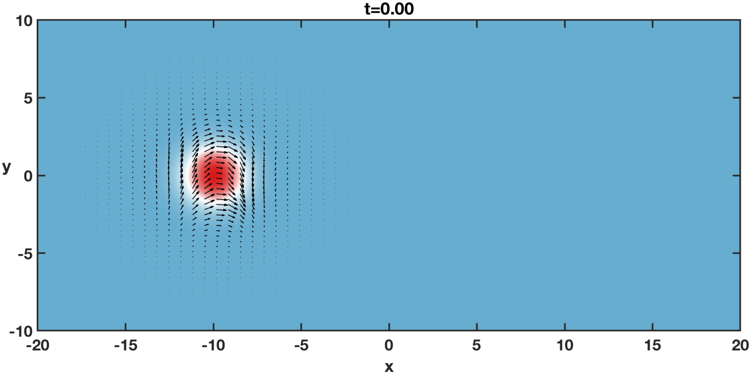
$\Rightarrow$ (See Hoefer and Sommacal (2012) for nondimensionalizaton)

Droplet Solitons



Review article: “Magnetic Solitons” Phys. Reports. 194 Kosevich et al. (1990)

Droplet Solitons

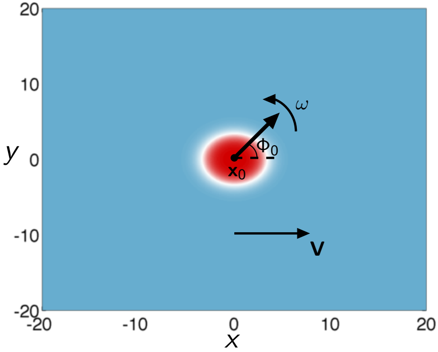


Review article: "Magnetic Solitons" Phys. Reports. 194 Kosevich et al. (1990)

Droplet Solitons

Soliton Parameters:

$\mathbf{x}_0$	-	initial position
Φ_0	-	initial phase
$\mathbf{V}$	-	propagation velocity
ω	-	frequency



Analytical solution for small V, ω:

- Slowly precessing, radially symmetric solution
- Radius inversely proportional to frequency

Droplet Solitons

Soliton Parameters:

$\mathbf{x}_0$	-	initial position
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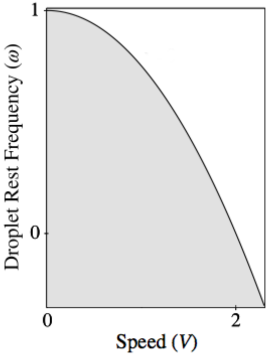
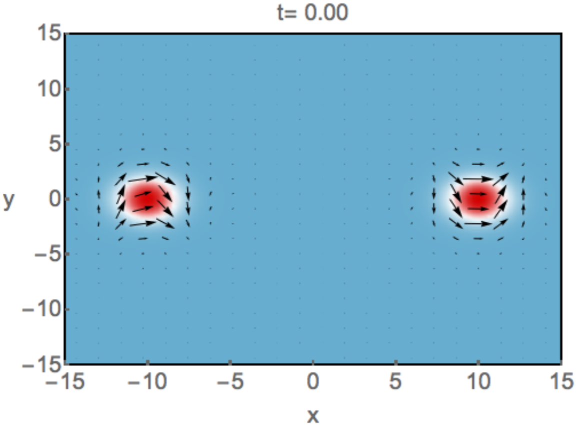


[From Hoefer and Sommacal (2012)]

Interacting Droplets

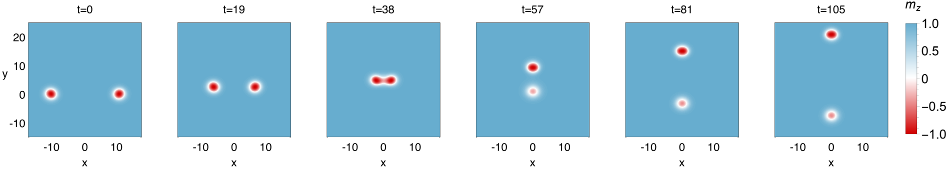
Droplet Interactions

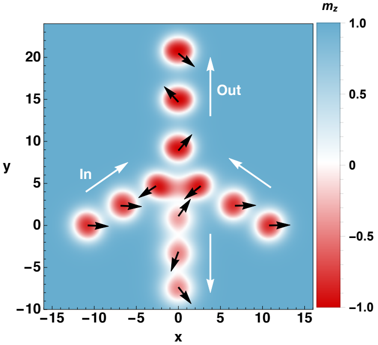
$$\Delta\Phi = 0$$



Droplet Interactions

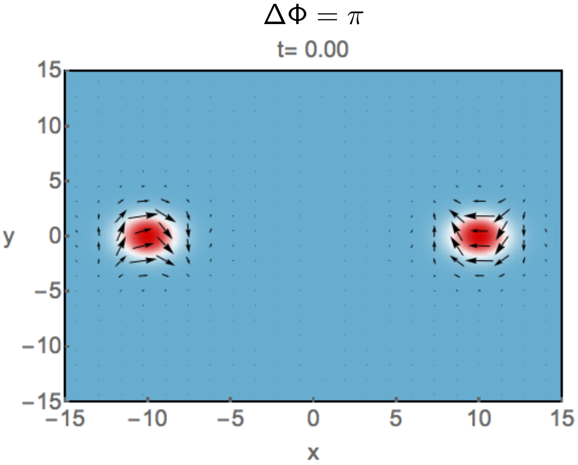
Attraction $\Rightarrow$ scattering along line of symmetry





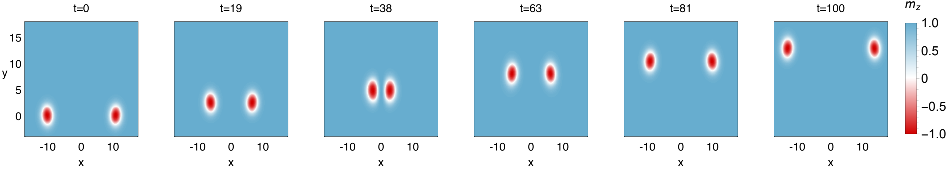
(Joint with M.D. Maiden)

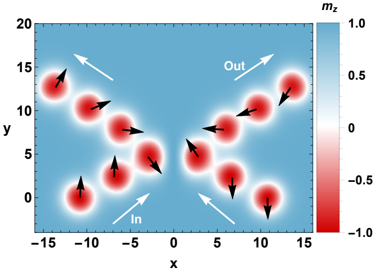
Droplet Interactions



Droplet Interactions

Repulsion

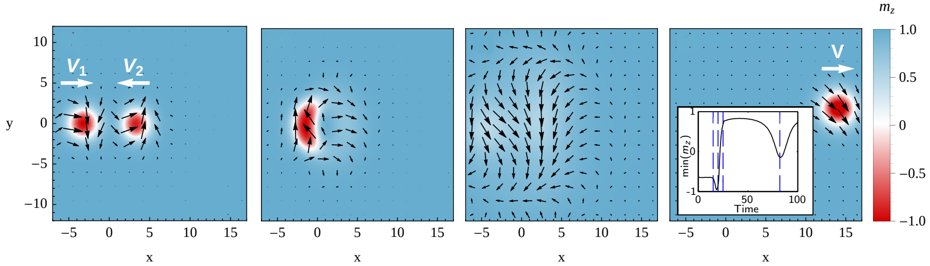




(Joint with M.D. Maiden)

Droplet Interactions

Annihilation



[Figure from Maiden et al. (2014)]

Application: Spintronics

Damping Contribution

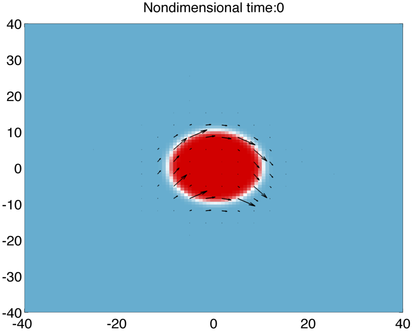
[Landau and Lifshitz (1935)]:

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathbf{h}_{\text{eff}} - \alpha \mathbf{m} \times (\mathbf{m} \times \mathbf{h}_{\text{eff}})$$

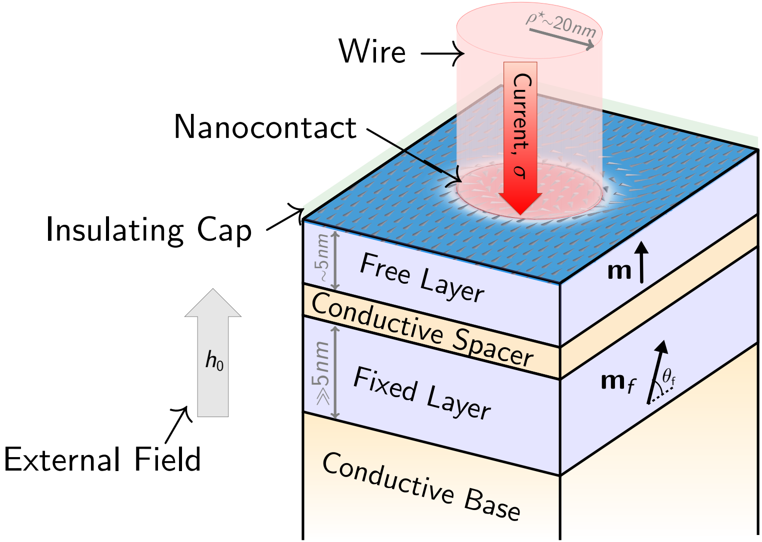
α - nondimensional damping parameter

$$\omega_0 = 0.1$$

$$|\mathbf{V}_0| = 0.01$$

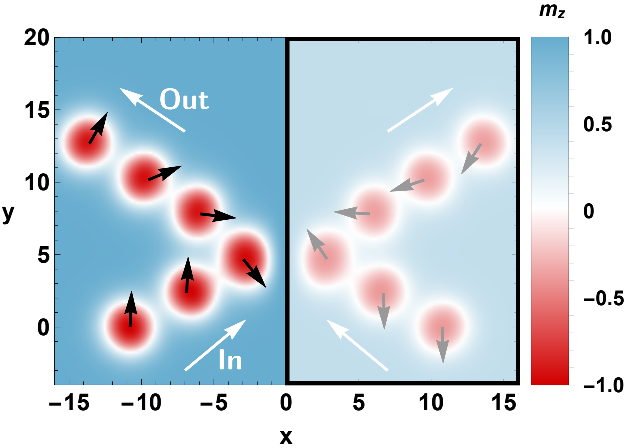


Droplets in Experiment



Droplet Interactions

Repulsion (from pinned boundary)



Future Work

Extensions of this work:

- Computation of numerically exact breathers, dissipative droplets etc.
- Impact of other parameters on soliton-soliton interaction
- Similar experiments with skyrmions

Other projects:

- Models of rogue waves on the deep ocean (breathers in a nonlinear Schrodinger-type equations)
- (Dynamic) Causal Inference (inference from data, text)
- Agent-based decision models (general game playing)

References

Mark J Ablowitz and Douglas E Baldwin. Nonlinear shallow ocean-wave soliton interactions on flat beaches. *Physical Review E*, 86(3):036305, 2012.

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Lev D Landau and Es Lifshitz. On the theory of the dispersion of magnetic permeability in ferromagnetic bodies. *Phys. Z. Sowjetunion*, 8(153):101–114, 1935.

MD Maiden, LD Bookman, and MA Hoefer. Attraction, merger, reflection, and annihilation in magnetic droplet soliton scattering. *Physical Review B*, 89(18):180409, 2014.

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Norman J Zabusky. Soliton (scholarpedia.org), April 2014. URL `http://www.scholarpedia.org/article/Soliton`.

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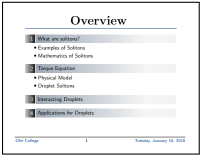

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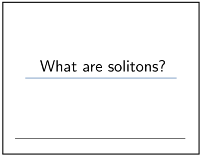

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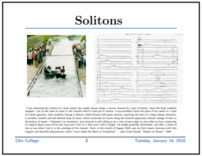

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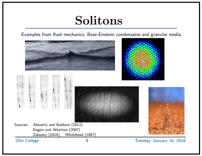

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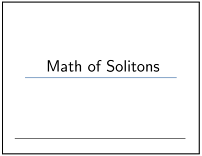

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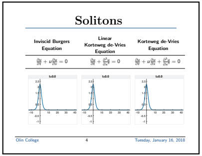

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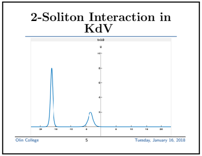

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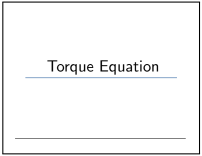

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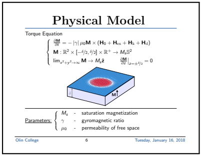

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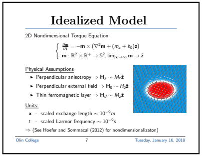

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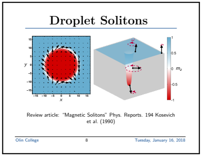

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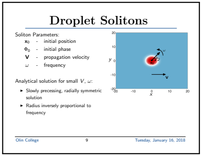

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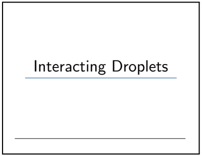

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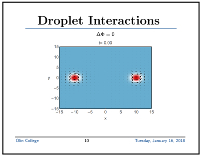

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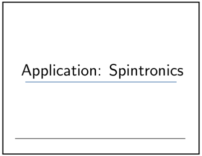

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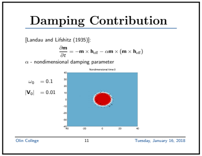

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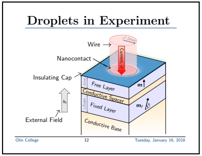

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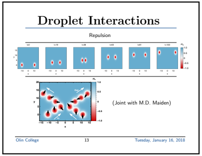

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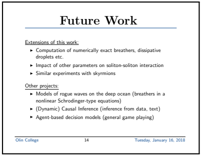

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