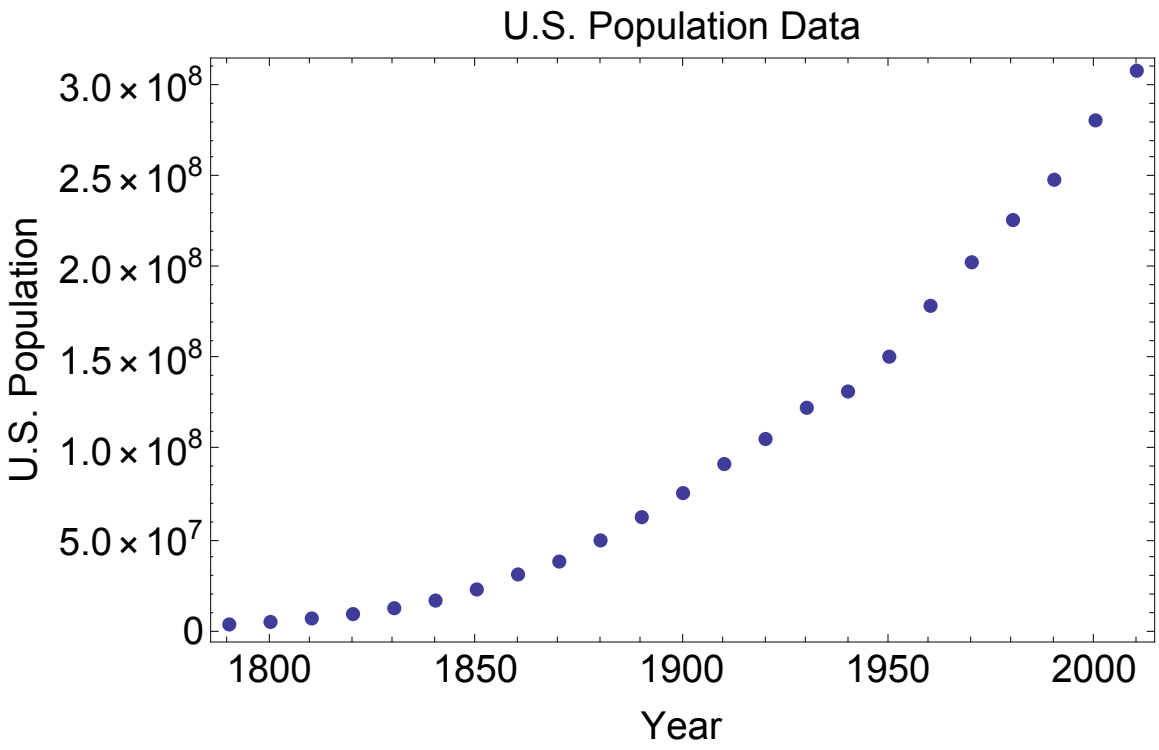
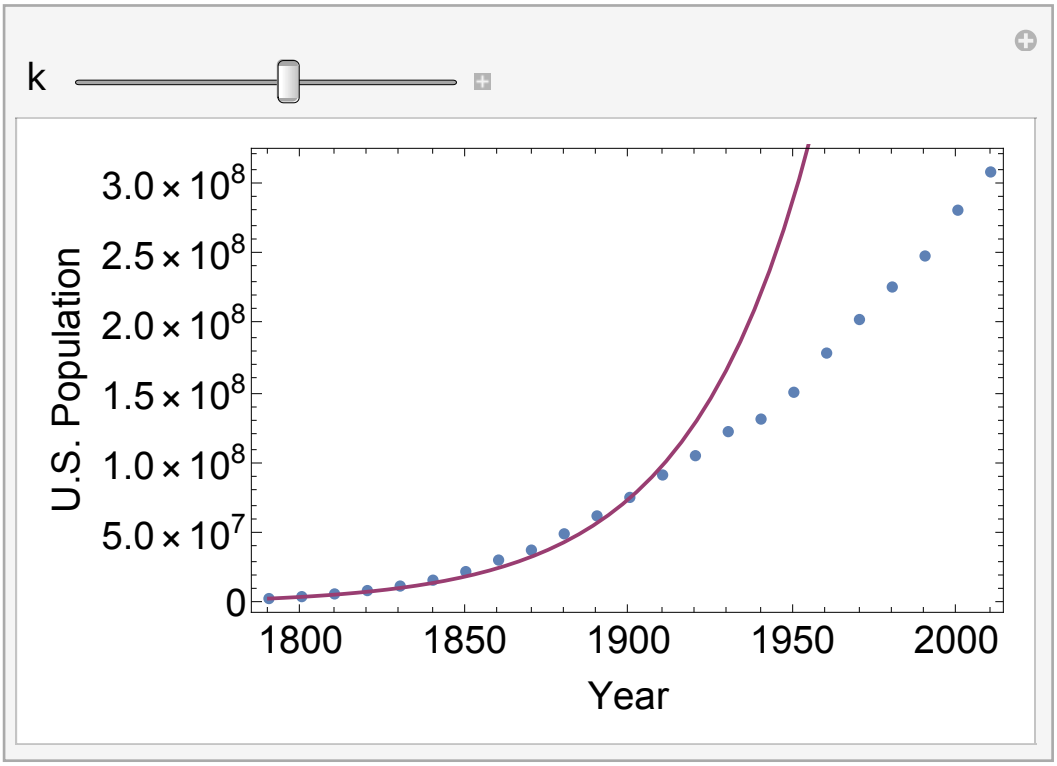
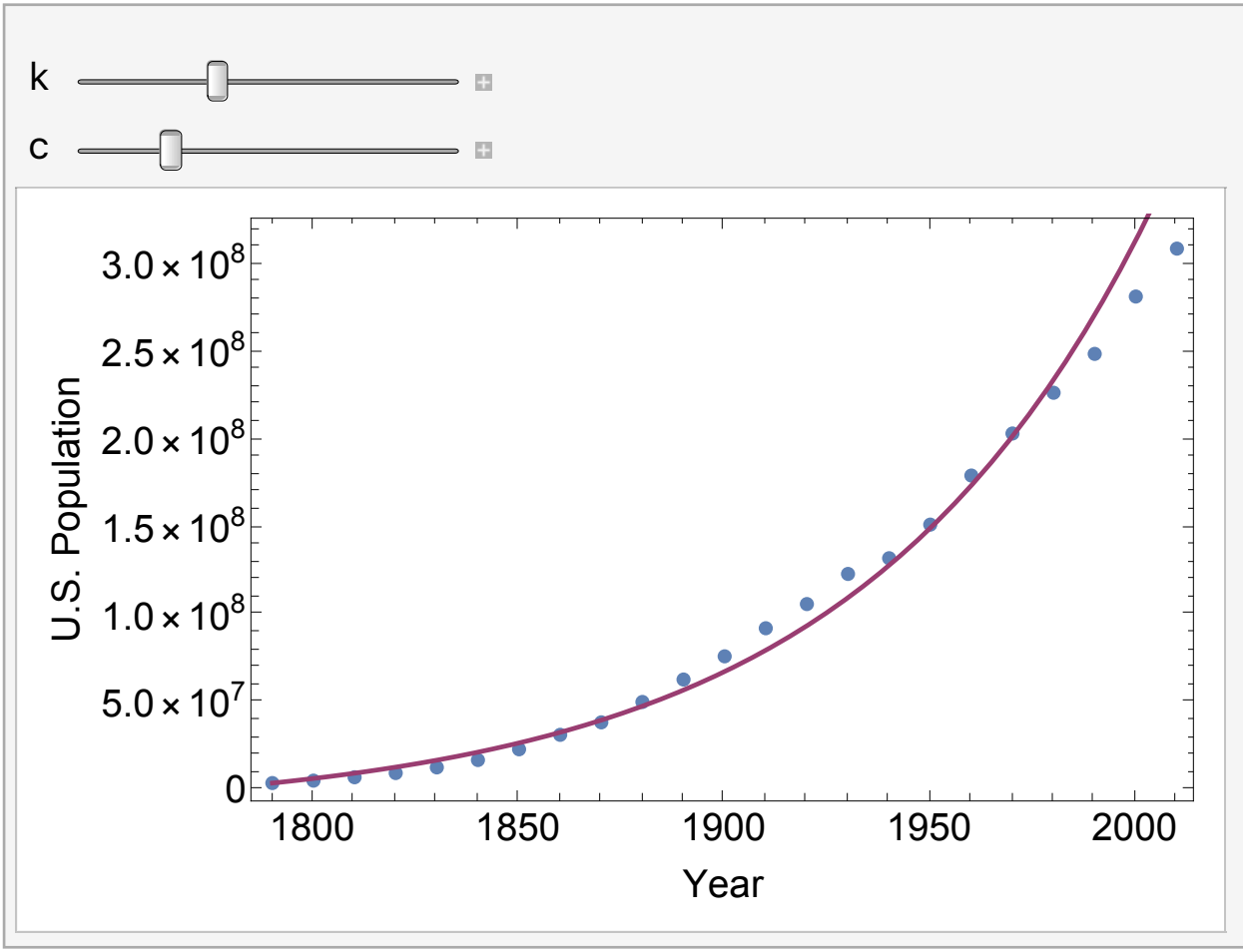
```
(*Data from http://www.census.gov/population/www/censusdata/files/table-2.pdf,
http://www.census.gov/main/www/cen2000.html,
http://quickfacts.census.gov/qfd/states/00000.html,retrieved February 15 2015 *)
HistoricalPopulationDataUS = {
   {1790, 3 929 214},
   {1800, 5 308 438},
   {1810, 7 239 881},
   {1820, 9 638 453},
   {1830, 12 866 020},
   {1840, 17 069 453},
   {1850, 23 191 876},
   {1860, 31 443 321},
   {1870, 38 558 371},
   {1880, 50 189 209},
   {1890, 62 979 776},
   {1900, 76 212 168},
   {1910, 92 228 496},
   {1920, 106 021 537},
   {1930, 123 202 624},
   {1940, 132 164 569},
   {1950, 151 325 798},
   {1960, 179 323 175},
   {1970, 203 302 031},
   {1980, 226 542 199},
   {1990, 248 709 873},
   {2000, 281 421 906},
   {2010, 308 745 538}};

ListPlot[HistoricalPopulationDataUS, PlotStyle → PointSize[.015], Frame → True,
 FrameLabel → {"Year", "U.S. Population", "U.S. Population Data"},
 FrameStyle → Directive[FontSize → 20], ImageSize → 600]
```


U.S. Population Data
U.S. Population
3.0 × 10^8
2.5 × 10^8
2.0 × 10^8
1.5 × 10^8
1.0 × 10^8
5.0 × 10^7
0
1800
1850
1900
1950
2000
Year

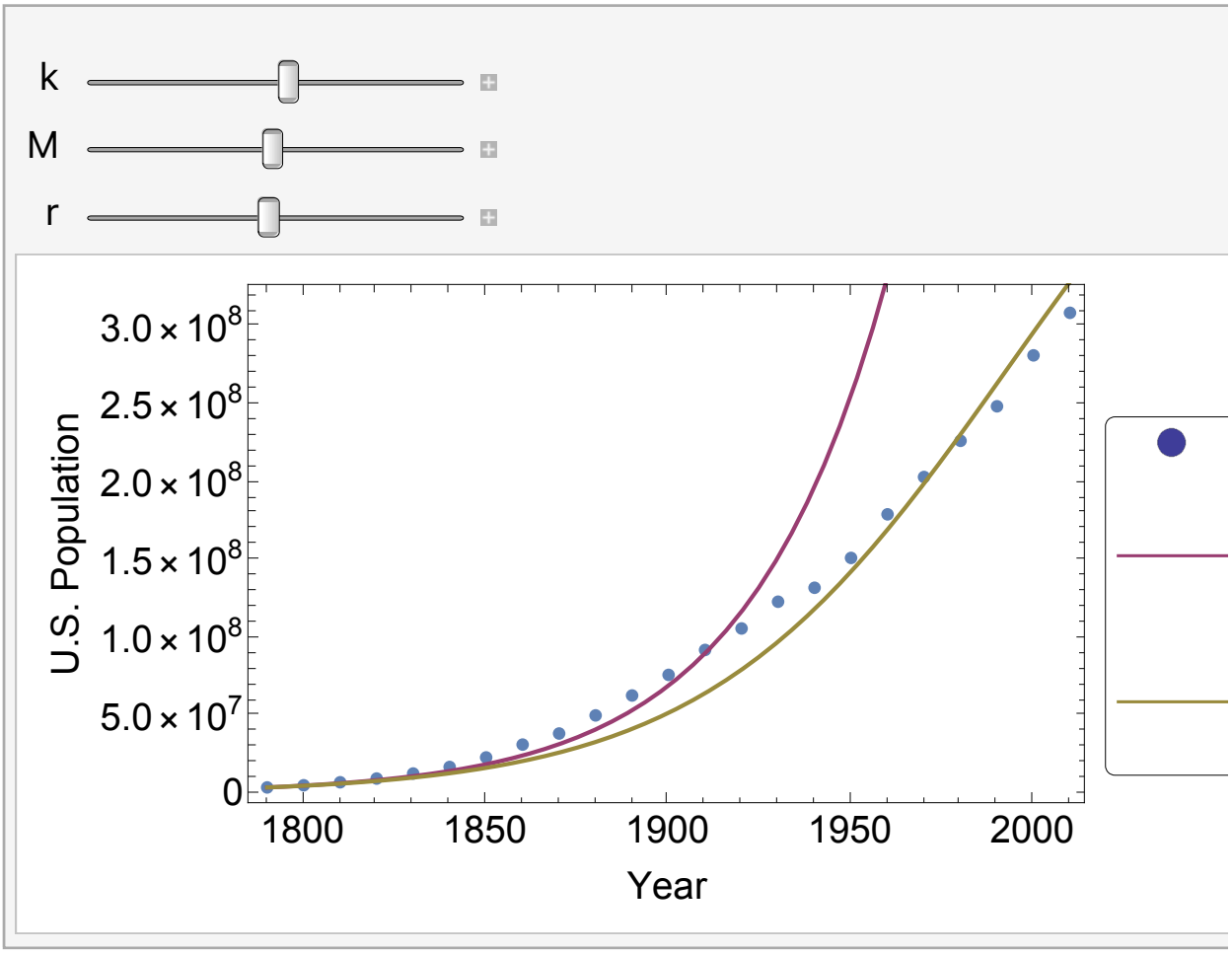
```
Manipulate[Show[ListPlot[HistoricalPopulationDataUS,
    PlotStyle → PointSize[.015], Frame → True, FrameLabel → {"Year",
      "U.S. Population", "U.S. Population Data and Model for k=" <> ToString[k]},
    FrameStyle → Directive[FontSize → 20]], Plot[3 929 214 Exp[k (x - 1790)],
    {x, 1790, 2010}, PlotStyle → {Thickness[0.005], ColorData[1, 2]}],
  ImageSize → 500], {k, .01, .04}, LabelStyle → Directive[FontSize → 20]]
```



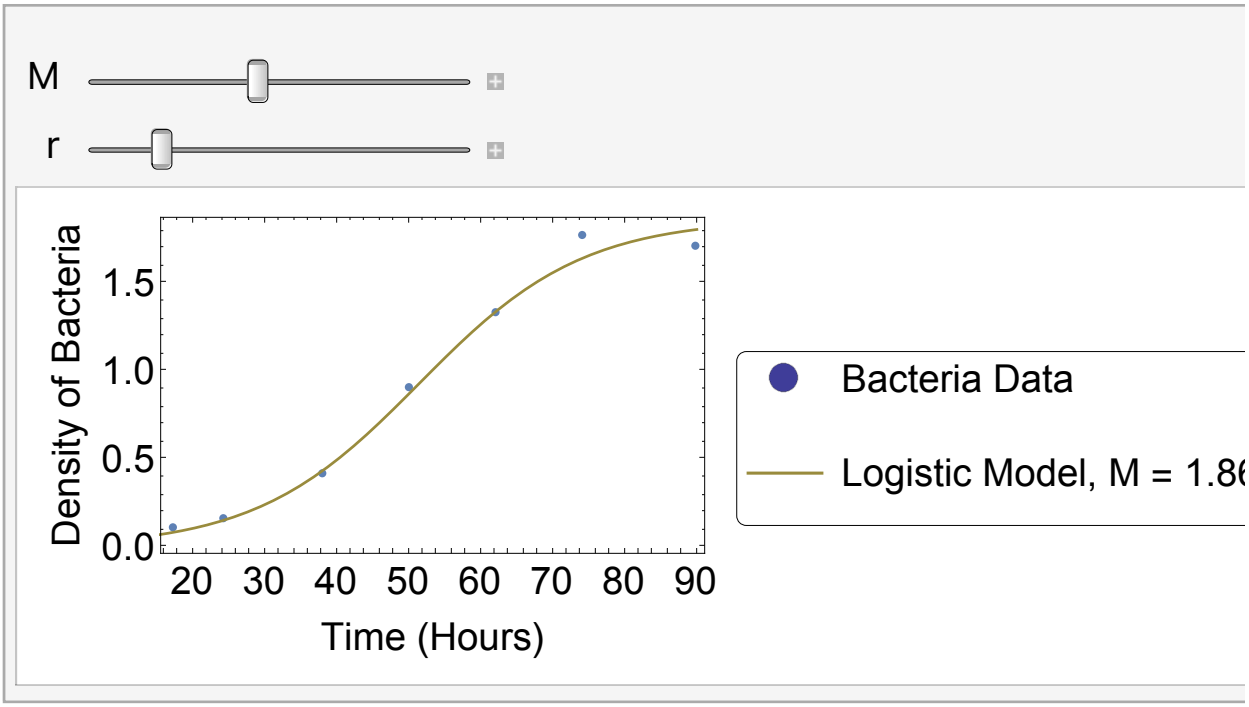
```
Manipulate[Show[ListPlot[HistoricalPopulationDataUS, PlotStyle → PointSize[.015],
    Frame → True, FrameLabel → {"Year", "U.S. Population",
      "U.S. Population Data and Model With Immigration for k=" <> ToString[k] <>
       "and c=" <> ToString[c]}, FrameStyle → Directive[FontSize → 20]],
   Plot[(-c + e^(k (-1790+t)) (c + 3 929 214 k))/k, {t, 1790, 2010},
    PlotStyle → {Thickness[0.005], ColorData[1, 2]}], ImageSize → 600],
  {k, 0.0001, .04}, {c, 0, 900 000}, LabelStyle → Directive[FontSize → 20]]
```



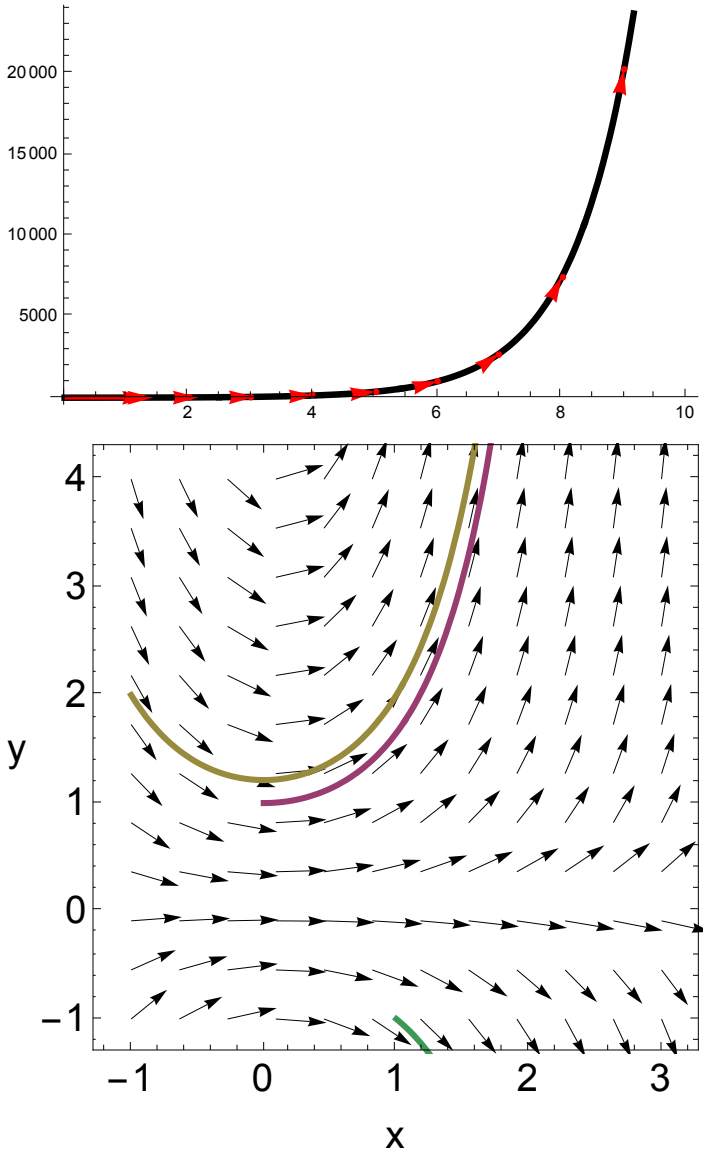
```
Manipulate[Show[ListPlot[HistoricalPopulationDataUS, PlotStyle → PointSize[0.015],
    Frame → True, FrameLabel → {"Year", "U.S. Population",
      "U.S. Population Data with Exponential and Logistic Models"},
    FrameStyle → Directive[FontSize → 20],
    PlotLegends → PointLegend[{ColorData[1, 1], ColorData[1, 2], ColorData[1, 3]},
      {"Historical Data                                                    ",
       "Exponential Model k =" <> ToString[k] ,
       "Logistic Model, M = " <> ToString[M/10^6] <> " million, r=" <> ToString[r]},
      LegendMarkers → {{Graphics[{Disk[]}], 15}, {Graphics[{Line[{{0, 0}, {2, 0}}]}],
         60}, {Graphics[{Line[{{0, 0}, {2, 0}}]}], 60}},
      LegendFunction → "Frame", LabelStyle → 20]],
  Plot[3 929 214 Exp[k (t - 1790)], {t, 1790, 2010},
   PlotStyle → {ColorData[1, 2], Thickness[0.005]}],
  Plot[3 929 214 e^(r (-1790+t)) M / (-3 929 214 + 3 929 214 e^(r (-1790+t)) + M), {t, 1790, 2010},
   PlotStyle → {Thickness[0.005], ColorData[1, 3]}]
  , ImageSize → 550], {k, .01, .04}, {M, 300 000 000, 800 000 000},
 {r, .01, .04}, LabelStyle → Directive[FontSize → 20]]
```


k
M
r
U.S. Population
3.0 × 10^8
2.5 × 10^8
2.0 × 10^8
1.5 × 10^8
1.0 × 10^8
5.0 × 10^7
0
1800
1850
1900
1950
2000
Year

```
(*Bacteria Culture: http://
 www.nature.com/srep/2014/140527/srep05057/full/srep05057.html,
from Saikin,Semion K.,
et al."Chromatic acclimation and population dynamics of green sulfur bacteria
    grown with spectrally tailored light." Scientific reports 4 (2014). *)
BacteriaData = {
   {17.1633, 0.1163},
   {24.1732, 0.168},
   {37.9327, 0.4231},
   {49.9703, 0.9121},
   {62.0156, 1.3378},
   {74.0593, 1.7762},
   {89.7925, 1.7151}
  };
Manipulate[Show[ListPlot[BacteriaData, Frame → True,
   PlotStyle → PointSize[0.015], FrameLabel → {"Time (Hours)",
     "Density of Bacteria", "Experimental Bacteria Data and Logistic Model"},
   FrameStyle → Directive[FontSize → 20], ImageSize → 350,
   PlotLegends → PointLegend[{ColorData[1, 1], ColorData[1, 3]},
     {"Bacteria Data                                    ",
      "Logistic Model, M = " <> ToString[M] <> ", r=" <> ToString[r]},
     LegendMarkers → {{Graphics[{Disk[]}], 15}, {Graphics[{Line[{{0, 0}, {2, 0}}]}],
        40}, {Graphics[{Line[{{0, 0}, {2, 0}}]}], 40}},
     LegendFunction → "Frame", LabelStyle → 20]], Plot[ 0.02 e^(r t) M / (0.02 (-1 + e^(r t)) + M) ,
   {t, 0, 90}, PlotStyle → {Thickness[0.005], ColorData[1, 3]}]],
 {M, 1, 3}, {r, .05, .3}, LabelStyle → Directive[FontSize → 20]]
```


M
r
Density of Bacteria
0.0
0.5
1.0
1.5
20
30
40
50
60
70
80
90
Time (Hours)
Bacteria Data
Logistic Model, M = 1.8

```
DirectionField[f_, xrange_, yrange_] :=
 VectorPlot[{1/Sqrt[1 + f^2], f/Sqrt[1 + f^2]}, xrange, yrange, VectorPoints → 12,
   VectorScale → Small, VectorStyle → Black, Frame → True, FrameLabel → {"x", "y"},
   FrameStyle → Directive[FontSize → 20], RotateLabel → False] /.
  Arrow[{p_, q_}] :→ Arrow[{p + (q - p) / 2, q + (q - p) / 2}]
DirectionFieldWithSolutions[f_, InitialData_, xrange_, yrange_, xmax_] :=
 Module[{init, sol, kk, result},
  result = DirectionField[f, xrange, yrange];
  For[kk = 1, kk ≤ Length[InitialData], kk++,
   sol =
    NDSolve[{y'[x] == (f /. y → y[x]), y[InitialData[[kk, 1]]] == InitialData[[kk, 2]]},
     y[x], {x, InitialData[[kk, 1]], xmax}];
   result = Show[result, Plot[y[x] /. sol, {x, InitialData[[kk, 1]], xmax},
      PlotStyle → {ColorData[1, kk + 1], Thickness[0.01]}]];
  ];
  result
 ]

SingleArrow[p_, q_, f_, len_] := Graphics[{Red, Thick,
   Arrow[{{p, q}, {p + len/Sqrt[1 + (f /. {x → p, y → q})^2], q + len (f /. {x → p, y → q})/Sqrt[1 + (f /. {x → p, y → q})^2]}}]}]
DirectionFieldAlongSolutionCurve[f_, xmin_, xmax_, init_, numarrows_, len_] :=
 Module[{sol, kk, result, arrowlocs},
  sol = NDSolve[{y'[x] == (f /. y → y[x]), y[xmin] == init}, y[x], {x, xmin, xmax}];
  result = Plot[y[x] /. sol, {x, xmin, xmax}, PlotStyle → {Black, Thickness[0.01]}];
  arrowlocs =
   ({x, y[x]} /. sol[[1]] /. x → #) & /@ (xmax - xmin) Range[0, numarrows]/numarrows + xmin;
  For[kk = 1, kk ≤ numarrows, kk++,
   result = Show[result, SingleArrow[arrowlocs[[kk, 1]], arrowlocs[[kk, 2]], f, len]]
  ];
  result
 ]
DirectionFieldAlongSolutionCurve[y + Sin[x], 0, 10, 2, 10, 3]
DirectionFieldWithSolutions[x y,
 {{0, 1}, {-1, 2}, {1, -1}}, {x, -1, 3}, {y, -1, 4}, 10]
```




```
? Arrow
```

Arrow[{pt_1, pt_2}] is a graphics primitive that represents an arrow from pt_1 to pt_2.
Arrow[{pt_1, pt_2}, s] represents an arrow with its ends set back from pt_1 and pt_2 by a distance s.
Arrow[{pt_1, pt_2}, {s_1, s_2}] sets back by s_1 from pt_1 and s_2 from pt_2.
Arrow[$curve$, ...] represents an arrow following the specified $curve$. ≫

```
DirectionField[f_, xrange_, yrange_] :=
 VectorPlot[{1/Sqrt[1 + f^2], f/Sqrt[1 + f^2]}, xrange, yrange, VectorPoints → 12,
   VectorScale → Small, VectorStyle → Black, Frame → True,
   FrameLabel → {"x", "y", " ", " "}, FrameStyle → Directive[FontSize → 20],
   RotateLabel → False] /. Arrow[{p_, q_}] :⇒ Arrow[{p + (q - p) / 2, q + (q - p) / 2}]
Export["simple_slope_field.pdf",
 Show[DirectionField[y (1 - y/10), {x, 0, 20}, {y, -3, 21}], ImageSize → 300]]
```

```
simple_slope_field.pdf
```