Adiabatic Coefficient

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Introduction

A thermodynamic system is said to undergo an adiabatic process when no heat energy is transferred between the system and its environment. In the event that the process is reversible, an ideal gas undergoing an adiabatic process must satisfy the equation:

$$PV^{\chi} = c$$

Where P is the pressure of the gas, V is its volume, c is a real constant and χ is called the adiabatic coefficient of the gas. In this lab, we measured the adiabatic coefficient of three different gasses; air, nitrogen and carbon dioxide. We did this by repeating the Ruchardt experiment. The principle of the experiment is as follows: A typical experiment consists of a glass tube of volume V which is open on one of its end. A mass body m with the same cross-section as the tube (creating an air-tight seal), is allowed to fall under gravity causing the entrapped gas to compress under the weight of the piston, which leads to an increase in temperature. Consequently, a gas cushion is created and the piston bounces back up. This induces a rapid sequence of expansion and compression of the gas in the tube, creating harmonic oscillation. The equations and physics necessary are developed more in Appendix 1.

Collecting Data

In order to reduce error, we measured each data point for approximately 300 oscillations. We divided the total time elapsed by the number of oscillations to get the period of oscillations τ, and we then computed the χ and f value associated to each period τ, and finally we computed the average and standard deviation of the χ and f. See Appendix 2 for our raw data and the computations.

We first collected data using air. We determined an adiabatic coefficient and degrees of freedom of air using an air pump.

$$\chi_{air} = 1.31 \pm 0.01$$

$$f_{air} = 6.43 \pm 0.22$$

Since air is comprised mainly of nitrogen, we knew that adiabatic coefficient and degrees of freedom of air should be close to that of nitrogen. However, our data is much closer to that of carbon dioxide than nitrogen (based on the data in the lab instructions document). One possible explanation for this is that the previous group may have used carbon dioxide last. We assumed that the system was already full of air and so did not let the pump run for 15 minutes to cleanse the system. However, if the previous group used carbon dioxide, since carbon dioxide is more dense than air, there may have been carbon dioxide left in the system and so we were, without knowing it, measuring the adiabatic coefficient of a mixture largely containing carbon dioxide. At the end of the experiment we repeated the measurement with air to test our hypothesis. However, even with waiting longer than the recommended 15 minutes to cleanse the system and confirming that the oscillation periods were constant, we still got values which were very similar to our first measurements which suggests that the gas in the system was the same for both sets of measurements

$$\chi_{air,2} = 1.30 \pm 0.01$$

$$f_{air,2} = 6.62 \pm 0.09$$

Our second data set was collected for nitrogen gas. We observed that the counter was approximately aligned with the equilibrium point of the body, and so each oscillation the body passed the counter twice. As such, we waited until the counter was at approximately 600 before taking the time measurement. At the end, we got values of

$$\chi_{N_2} = 1.36 \pm 0.06$$

$$f_{N_2} = 5.61 \pm 0.80$$

The third data set was for carbon dioxide. For this data set, we thought to change the height of the counter so that it was aligned with the top of the oscillations, and so each count corresponded to one oscillation. After approximately 300 oscillations, we collected the data. Our final results are

$$\chi_{CO_2} = 1.30 \pm 0.01$$

$$f_{CO_2} = 6.56 \pm 0.21$$

Further Analysis

Comparing our results to the accepted theory (as presented in the lab instructions document), we obtain the following table:

Table 1: Comparison with accepted theory

		Experimental data	**Scientific consensus**
Air 1	χ	1.31 ± 0.01	1.67
	f	6.43 ± 0.22	3
Air 2	χ	1.30 ± 0.01	1.67
	f	6.62 ± 0.09	3
N_2	χ	1.36 ± 0.06	1.4
	f	5.61 ± 0.80	5
CO_2	χ	1.30 ± 0.01	1.33
	f	6.56 ± 0.21	6

Comparing our experimental values to the accepted values, we see a number of concerns. For our measurements for air, our experimental values were much more than three standard deviations away from the scienfitic consensus. We knew that the values for air should be close to nitrogen since air is approximately 70% nitrogen, however our values are much closer to that of carbon dioxide. We also noted that although the accepted values for nitrogen are within one standard deviation of our experimental value, the accepted values for carbon dioxide are also within one standard deviation of the experimental values of nitrogen. This means that we can not experimentally identify if the gas that we measured is nitrogen or carbon dioxide.

The uncertainty in each of our measurements was obtained by computing the standard deviation of our sample, thus giving us a measure of the spread of our data around the center point. However, they do not take into consideration the error that comes from each measurement. In order to incorporate the error that comes from each measured data type, we used the formula for the propagation of error (developed in detail in Appendix 3), and we got the following results.

Table 2: Considering the propagation of error

		Experimental data	**Scientific consensus**
Air 1	χ	1.31 ± 0.15	1.67
	f	6.43 ± 0.32	3
Air 2	χ	1.30 ± 0.15	1.67
	f	6.62 ± 0.34	3
N_2	χ	1.36 ± 0.16	1.4
	f	5.61 ± 0.30	5
CO_2	χ	1.30 ± 0.15	1.33
	f	6.56 ± 0.33	6

There is clear indication that there was large experimental error. In particular, a large portion of uncertainty came from measuring the mass of the body. We are not sure why this is so, as we expected the majority of the error to come from the measurement of the time. Even with the large error accounted for, we still see evidence for intrinsic issues. Our results are still very inaccurate. The accepted values for the adiabatic coefficient and degrees of freedom of air are still not within our margin of error. Furthermore, we still have the same problem with nitrogen and carbon dioxide; effectively, our experiment cannot distinguish between the two gasses.

Appendix 1: Theoretical principle

By Newton's law, the dynamics of this mass is represented by the differential equation:

$$m\frac{d^2x}{dt^2} = \pi r^2 \Delta P$$

where x denotes the displacement of the body from its equilibrium position, r is the radius of the body, $P = P_L + \frac{mg}{\pi r^2}$ is the internal gas pressure as a function of the external atmospheric pressure P_L, and ΔP is the variation of the internal gas pressure P due to its movement. Denoting $\Delta V = \pi r^2 x$ to be the variation of the volume displaced due to the movement of the body in the tube, and using the equation $PV^\chi = c$, the differential equation becomes:

$$\frac{d^2x}{dt^2} + \frac{\chi^2\pi^2 r^4 x}{mV} = 0$$

which is known to have solutions with angular velocity ω as:

$$\omega = \sqrt{\frac{\chi^2\pi^2 r^4 x}{mV}}$$

The adiabatic coefficient χ is then given by the equation:

$$\chi = \frac{4mV}{\tau^2 P r^4}$$

where, $\tau = 2\pi/\omega$ is the period of oscillations. Theoretically, the adiabatic coefficient is defined as:

$$\chi = \frac{f+2}{f}$$

where f is the number of degrees of freedom of the gas. Once we have χ, we can thus calculate f as:

$$f = \frac{2}{\chi - 1}$$

Appendix 2: Raw data

We present the data sets we recorded in the lab. The first table shows some of the constants of the experiment. The next four tables show the data we acquired in each of the four experiments.

Ambient pressure (pascal)	1.03E+05
Mass (kg)	4.42E-03
Volume (m^3)	1.14E-03
Radius (m)	5.93E-03

Figure 1: Constant quantities

As explained in Appendix 3, we consider the ambient pressure and volume to have no error. Based on our measurements, the uncertainty in the mass is $\delta m = 10. \times 10^{-6} kg$ and the uncertainty in the radius is $\delta r = 50 \times 10^{-6} m$. We also considered that each measurement of the time elapsed had an error of $\delta T = 0.1s$.

Air				
Number of oscillations	Time elapsed (s)	Period (s)	Chi	f
302	105.3	0.3487	1.3036	6.587793
300	104.4	0.3480	1.3087	6.479674
300	104.6	0.3487	1.3037	6.58636
300	104.8	0.3493	1.2987	6.695977
300	103.8	0.3460	1.3238	6.176076
300	104.1	0.3470	1.3162	6.324889
300	104.0	0.3467	1.3187	6.274639
308	106.2	0.3448	1.3330	6.005633
300	104.6	0.3487	1.3037	6.58636
300	104.5	0.3483	1.3062	6.532658
			Average	Std Dev
		Chi	1.31	± 0.01
		f	6.43	± 0.22

Figure 2: First data set of Air

Air 2				
Number of oscillations	Time elapsed (s)	Period (s)	Chi	f
300	104.5	0.3483	1.3062	6.532658
300	104.9	0.3497	1.2962	6.751922
330	115.0	0.3485	1.3050	6.556978
300	104.8	0.3493	1.2987	6.695977
300	104.6	0.3487	1.3037	6.58636
			Average	Std Dev
		Chi	1.30 ± 0.00	
		f	6.62 ± 0.09	

Figure 3: Second data set of Air

Nitrogen				
Number of oscillations	Time elapsed (s)	Period (s)	Chi	f
600	103.8	0.3460	1.3238	6.176076
600	102.8	0.3427	1.3497	5.719009
650	110.0	0.3385	1.3835	5.215701
600	104.1	0.3470	1.3162	6.324889
750	122.1	0.3256	1.4949	4.041121
650	109.0	0.3354	1.4090	4.890471
600	104.0	0.3467	1.3187	6.274639
600	104.0	0.3467	1.3187	6.274639
600	104.0	0.3467	1.3187	6.274639
700	117.5	0.3357	1.4062	4.923771
			Average	Std Dev
		Chi	1.36 ± 0.06	
		f	5.61 ± 0.80	

Figure 4: Data for N_2

Carbon Dioxide				
Number of oscillations	Time elapsed (s)	Period (s)	Chi	f
300	104.4	0.3480	1.3087	6.479674
300	104.8	0.3493	1.2987	6.695977
300	104.0	0.3467	1.3187	6.274639
300	104.4	0.3480	1.3087	6.479674
350	121.9	0.3483	1.3065	6.525045
300	104.0	0.3467	1.3187	6.274639
350	122.5	0.3500	1.2937	6.808646
300	105.2	0.3507	1.2888	6.924497
300	104.8	0.3493	1.2987	6.695977
300	104.4	0.3480	1.3087	6.479674
			Average	Std Dev
		Chi	1.30	± 0.01
		f	6.56	± 0.21

Figure 5: Data for CO_2

Appendix 3: Error propagation

We first restate the relevant equations:

$$\chi = \frac{4mV}{\tau^2 P r^4} \qquad f = \frac{2}{\chi - 1} \qquad \tau = \frac{T}{N} \qquad P = P_L + \frac{mg}{\pi r^2}$$

Since N was counted on a counter and we were able to see each number clearly, we consider it to have no uncertainty, and all of the uncertainty in the measurement of the period comes from the time measurement T. Additionally, since we were given that $V = 1.14 \times 10^{-3} m^3$ by our TA and we found that $P_L = 1.028 \times 10^5 Pa$ in Palaiseau on March 21^{st} using www.worldweatheronline.com (as instructed by our TA), we will consider that both of those values have zero uncertainty as well. As mentioned in Appendix 2, we took the following values for the absolute uncertainties.

$$\delta m = 10. \times 10^{-6} kg \qquad \delta r = 50 \times 10^{-6} m \qquad \delta T = 0.1 s$$

Now we apply the formula for the propagation of error, which states

$$\delta\chi = \sqrt{\left(\frac{\partial\chi}{\partial r}\delta r\right)^2 + \left(\frac{\partial\chi}{\partial m}\delta m\right)^2 + \left(\frac{\partial\chi}{\partial T}\delta T\right)^2}$$

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial r}\delta r\right)^2 + \left(\frac{\partial f}{\partial m}\delta m\right)^2 + \left(\frac{\partial f}{\partial T}\delta T\right)^2}$$

Note that $f = \dfrac{2}{\chi - 1}$ implies that $\frac{\partial f}{\partial i} = \frac{-2}{(\chi-1)^2}\frac{\partial\chi}{\partial i}$ for some variable i of the system, so

$$\begin{aligned}
\delta f &= \sqrt{\left(\frac{-2}{(\chi-1)^2}\frac{\partial\chi}{\partial r}\delta r\right)^2 + \left(\frac{-2}{(\chi-1)^2}\frac{\partial\chi}{\partial m}\delta m\right)^2 + \left(\frac{-2}{(\chi-1)^2}\frac{\partial\chi}{\partial T}\delta T\right)^2} \\
&= \frac{2}{(\chi-1)^2}\sqrt{\left(\frac{\partial\chi}{\partial r}\delta r\right)^2 + \left(\frac{\partial\chi}{\partial m}\delta m\right)^2 + \left(\frac{\partial\chi}{\partial T}\delta T\right)^2} \\
\delta f &= \frac{2}{(\chi-1)^2}\delta\chi
\end{aligned}$$

We then compute each partial derivative:

$$\begin{aligned}
\frac{\partial \chi}{\partial r} &= \frac{4mV}{\tau} \frac{\partial}{\partial r} \left(\frac{1}{r^4 P} \right) \\
&= \frac{4mV}{\tau} \left(\frac{-4}{r^5} \frac{1}{P} + \frac{1}{r^4} \frac{-1}{P^2} \frac{\partial P}{\partial r} \right) \\
&= \frac{4mV}{\tau} \left(\frac{-4}{r^5} \frac{1}{P} + \frac{1}{r^4} \frac{-1}{P^2} \frac{-2mg}{\pi r^3} \right) \\
\frac{\partial \chi}{\partial r} &= \frac{4mV}{\tau} \left(\frac{-4}{r^5 P} + \frac{1}{\pi r^7 P^2} \right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \chi}{\partial m} &= \frac{4V}{\tau r^4} \frac{\partial}{\partial m} \left(m \frac{1}{P} \right) \\
&= \frac{4V}{\tau r^4} \left(\frac{1}{P} + \frac{-1}{P^2} \frac{\partial P}{\partial m} \right) \\
&= \frac{4V}{\tau r^4} \left(\frac{1}{P} - \frac{1}{P^2} \frac{g}{\pi r^2} \right) \\
\frac{\partial \chi}{\partial m} &= \frac{4V}{\tau r^4} \left(\frac{1}{P} - \frac{g}{\pi r^2 P^2} \right)
\end{aligned}$$

$$\frac{\partial \chi}{\partial T} = \frac{-8mVN^2}{T^3 r^4}$$

And by inputting the uncertainties of each variable, we obtain the uncertainties listed in Table 2.