SOME EMENDATIONS OF GÖDEL'S ONTOLOGICAL PROOF

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Kurt Gödel's version of the ontological argument was shown by J. Howard Sobel to be defective, but some plausible modifications in the argument result in a version which is immune to Sobel's objection. A definition is suggested which permits the proof of some of Gödel's axioms.

A new version of the ontological argument for the existence of God was outlined by Kurt Gödel and elaborated by Dana Scott. J. Howard Sobel has given a careful explication of the details and has provided a powerful critique[1]. I believe that Sobel's main objection is conclusive against the argument as sketched by Gödel. But it is possible to correct the argument, making changes which can be independently motivated, and in such a way that the revised argument is immune to the objection. And a definition of one of Gödel's primitive concepts enables the proof of some of his axioms. For the sake of those who do not enjoy symbolism, I give a statement of Gödel's argument and the suggested revisions in the vernacular. Some corollaries and a lemma have been separated off in order to clarify the proof and to isolate the difficulty. A brief statement of the formalities is given in the appendix. To see a full formalization of Gödel's original version, consult Sobel.

I. *Gödel's Axioms, Definitions, and Theorems*

Axiom 1. A property is positive if and only if its negation is not positive.

The notion of a positive property is taken as a primitive. Gödel suggests two readings—"positive in the moral-aesthetic sense" and positive as involving only "pure attribution." The only further comment in the notes on the first interpretation is to the effect that positiveness in this sense is independent of the "accidental structure of the world." The second notion is said to be "opposed to 'privation'" and to pertain to properties which do not contain privation. (The explanations in Gödel's notes are extremely terse and sometimes cryptic). Even the sympathetic reader still may not find Axiom 1 intuitively evident. I discuss this below.

Axiom 2. Any property entailed by a positive property is itself positive.

"Entailed" is understood to mean "strictly implied"—in this case, that it is impossible for something to have the one property and not the other. Let us say that a property is consistent if it is possibly exemplified, i.e., if it is possible that there exists an x such that x has that property. And let us say that a property is *necessary* if it is necessary that everything has the property. Then:

Theorem 1. If a property is positive, then it is consistent.

Proof: Let Φ be a positive property. Then Φ entails the property of self-identity—since every property entails the necessary property of self-identity. Hence, self-identity is positive by Axiom 2. So, by Axiom 1, the negation of self-identity, self-difference, is not positive. But if Φ is inconsistent, it entails self-difference—since an inconsistent property entails everything. This contradicts Axiom 2. So every positive property is consistent.[2]

Q.E.D.

The alleged modal facts used in proving Theorem 1—that a necessary property is entailed by every property and that an inconsistent property entails every property—may strike the modally naive as unintuitive. Indeed, it strikes some of the modally sophisticated thus. But given the explained meaning of "entails," these "paradoxes of strict implication" (as they have been called) are entirely unproblematic. If it is not possible that x lack Φ, then it is not possible that x have Ψ and lack Φ—so any property entails such a property Φ. And if it is not possible that x have Σ, then it is not possible that x has Σ and lacks Ψ. So such a property Σ entails every property.

Definition 1. x is *God-like* if and only if x has every positive property.

Axiom 3. The property of being God-like is positive.

It's worth noticing that there is here an implicit assumption: if we have defined a predicate, then we can straight-away form a name of the property which it expresses. (The technically minded will thus wish to note that it is in effect assumed that anything is counted as a property which can be defined by "abstraction on a formula.")

Corollary 1. The property of being God-like is self-consistent, i.e. possibly exemplified.

Proof: By Axiom 3 and Theorem 1.

Lemma. If something is God-like, then each of its properties is positive.

Proof. Suppose that something x is God-like. Let Ψ be any property of x. If Ψ is not positive, then its negation is (by Axiom 1). By definition, x, being God-like, has every positive property. But then x would exemplify the negation of Ψ—contrary to our assumption that x has Ψ. Hence Ψ is positive.

Q.E.D.

Definition 2. A property Φ is an *essence* of entity x if and only if x has Φ and Φ entails every property x has.

Gödel v as a great admirer of Leibniz[3] and this definition shows that influence. I suggest below that a more conservative characterization of essence better serves the purpose at hand.

Axiom 4. If a property is positive, then it is necessarily positive.[4]

Theorem 2. If something is God-like, then the property of being God-like is an essence of that thing.

Proof: Suppose that something x is God-like and let Ψ be any property of x. Then Ψ is positive by the lemma. Now by definition (of "God-like"), necessarily if Ψ is positive, anything which is God-like has Ψ. Hence, if necessarily Ψ is positive, then necessarily anything which is God-like has Ψ (by modal logic). But by Axiom 4, if Ψ is positive, necessarily Ψ is positive. Therefore, necessarily Ψ is positive. So necessarily anything which is God-like has Ψ—i.e., the property of being God-like entails Ψ. Thus we have shown that any property of x is entailed by the property of being God-like. So, by the definition of "essence," the property of being God-like is an essence of anything which has that property.

Q.E.D.

The modal principle used in the proof of Theorem 2 is that if it is necessary that if P, then Q, then if it is necessary that P, it is necessary that Q.

Corollary 2. If x is God-like and has a property, then that property is entailed by the property of being God-like.

The corollary is immediate by the definition of "essence" and Theorem 2. This consequence of the axioms is at the heart of Sobel's objection, to be explained below.

Definition 3. x *necessarily exists* if and only if every essence of x is necessarily exemplified (i.e., for every Φ, if Φ is an essence of x, then necessarily there exists a y such that y has Φ).

Note that while necessary existence is taken to be a property, it seems perfectly well-defined: it is defined as the property attributed to anything x when it is asserted that every essence Φ of x is such that necessarily $(\exists x)\Phi x$. (Technically, it is definable by abstraction on a second order formula). Actual existence may be taken to be expressed by the natural language counterpart of a quantifier, although one could also define a corresponding property of a thing x: every essence Φ of x is such that something is a Φ.

Axiom 5. The property of necessarily existing is a positive property.

Theorem 3. Necessarily the property of being God-like is exemplified.

Proof: If something x is God-like, then it has every positive property (by definition) and hence (by Axiom 5) it has the property of necessarily existing. That is, if x is God-like, then any essence of x is necessarily exemplified (definition of "necessary existence"). But if x is God-like, then the property of being God-like is an essence of x, by Theorem 2. Therefore, if anything x is God-like, then necessarily the property of being God-like is exemplified. Hence, if something is God-like, then necessarily something is God-like.

Since this last has been proved using only necessary truths, it is itself a necessary truth. Therefore, if it is possible that something is God-like, then it is possible that necessarily something is God-like (by modal logic). But by Corollary 1, it is possible that something is God-like. Therefore, it is possible that necessarily something is God-like. So, necessarily something is God-like (by the modal logic S5)[5].

Q.E.D.

The principles of modal logic used in this proof are: (1) if it is necessary that if P, then Q, then if it is possible that P, then it is possible that Q, and the principle of S5, (2) if it is possible that it is necessary that P, then it is necessary that P.

II. *Sobel's Objections*

The reasoning is entirely cogent. Unfortunately, too much follows from these axioms. Sobel shows that the axioms engender "modal collapse"—it follows from them that every proposition which is true at all is necessary. Suppose x is God-like and the proposition P is true. Then x has the property of being such that P is true. So by Corollary 2, this property is entailed by the property of being God-like—which latter is necessarily exemplified, by Theorem 3. Hence the property of being such that P is true is necessarily exemplified. Therefore, it is necessary that P. Again, the reasoning seems correct. (In his formalized version of this argument, Sobel uses the property which anything has when it is self-identical and P is true. Some may find this version slightly more intuitive.) Arguing along similar lines, Sobel concludes that it follows further that everything necessarily exists. Simplifying just a bit, the argument is this. Let x be the necessarily existing God-like being and consider any y distinct from x and having essence Φ. Then the necessarily existing God-like being x has the property of being such that there is something y, distinct from x and having essence Φ. This complex property, being entailed by the necessarily exemplified property of God-likeness, is itself necessarily exemplified and thus it is necessary there is such a y with essence Φ. This last is tantamount to y's necessary existence. I see no reasonable escape from Sobel's conclusions here.

III. *Analysis of the Difficulty*

Sobel suggests that a natural reaction might be to reject Axiom 5 and to give up on the ontological argument. (Sobel himself believes that ontological arguments have more serious and fundamental difficulties—we do not discuss these here). But Axiom 5 is certainly not the least plausible of the axioms. And one might agree with David Lewis[6] when he says that the ontological arguer is entitled to whatever standards of greatness (or positiveness, in the present case) he wants. Of course one can't then just *stipulate* that Axioms 1 and 2 are true—there might be a clash with the standards or with one another. And positiveness should be theologically significant (as again Lewis notes). But given this, it would be difficult to find fault with Axiom 5. Even without this, Axiom 5 has considerable intrinsic plausibility.

Consider the puzzling Axiom 1. If we separate it into the two conditionals:

(1a) If a property is positive, then its negation is not positive,

(1b) If the negation of a property is not positive, then the property is positive,

we find principles of rather different character. Chisholm and Sosa[7] have developed the logic of intrinsic value as attributed to states of affairs and there are analogies with the idea of a positive property (if we take this latter in the "moral aesthetic" sense). In particular, we can deduce (1a) from the two plausible principles about intrinsic preferability:

(B1) If a property is positive, then it is preferable to (or better than) its negation.

(B2) If a property Φ is preferable to property Ψ, then Ψ is not preferable to Φ.

These are analogues of certain theorems of Chisholm and Sosa[8].

Principle (1b), on the other hand, seems to overlook a possibility: that both a property and its negation should be *indifferent*. For example, *being such that there are stones* does not seem to be intrinsically preferable to its negation nor does its negation seem to be preferable to it—hence neither it nor its negation is positive (according to (B1)). So we should reject Axiom 1, delete the dubious part (1b), and adopt (1a) as our new axiom; call it now "Axiom 1*." Notice that (1b) is used in the proof of the lemma which, by way of Theorem 2, is involved in the proof of the troublesome Corollary 2.[9]

IV. *New Definitions and Corresponding Axioms*

Another change which seems advisable is this: a property should be defined to be an essence of an entity x when it is a property which entails all and only the *essential* properties of x—those properties which x has necessarily. There's nothing to argue about—here is a different conception of the essence of something, call it "essence*":

*Definition 2**. Φ is an *essence** of x if and only if for every property Ψ, x has Ψ necessarily (or essentially) if and only if Φ entails Ψ.[10]

Finally, I advocate that the property of being God-like, call it now "God-likeness*," be defined as follows:

*Definition 1**. x is *God-like** if and only if x has as essential properties those and only those properties which are positive (i.e., for every Φ, x has Φ necessarily if and only if Φ is positive).

Having only positive properties is, I think, too much to ask. Of an indifferent property and its negation God must have one. But having all and only the positive properties as essential properties is plausibly definitive of divinity.

These changes are theologically very pleasant: the proof of Theorem 1 still goes through (using Axiom 1*, i.e. (1a), in place of the rejected Axiom 1), the proof of the despised Corollary 2 is blocked (depending as it does on the old definition of "essence"), and we can still prove a theorem corresponding to Theorem 2—but now using our new definitions.

Note that the definition of necessary existence now to be used is of the same form as the original definition but has "essence*" in place of "essence." If we call this new notion "necessary existence*," then our new axioms are Axioms 1*, 2, and 4 and, in place of Axioms 3 and 5, respectively,:

*Axiom 3**. The property of being God-like* is positive, and

*Axiom 5**. Necessary existence* is positive. And we prove:

*Theorem 2**. If something is God-like*, then the property of being God-like* is an essence* of that thing.

Proof: Suppose that x is God-like* and necessarily has a property Ψ. Then by definition (of "God-like*"), that property is positive. But necessarily, if Ψ is positive, then if anything is God-like*, then it has Ψ—again by the definition of "God-like*," together with the fact that if something has a property necessarily, then it has the property. But if a property is positive, then it is necessarily positive (Axiom 4). Hence, if Ψ is positive, then it is entailed by being God-like* (by modal logic—as in the original Theorem 2). But Ψ is positive and hence is entailed by being God-like*. Thus we have proved that if an entity is God-like* and has a property essentially, then that property is entailed by the property of being God-like*.

Suppose a property Φ is entailed by the property of being God-like*. Then Φ is positive by Axioms 2 and 3* and therefore, since x is God-like*, x has Φ necessarily (by the definition of "God-like*"). Hence, if something is God-like*, it has a property essentially if and only if that property is entailed by being God-like—i.e., God-likeness* is an essence* of that thing.

Q.E.D.

That the property of being God-like* is necessarily exemplified follows much as before (except that one must use the modal principle "If necessarily P, then P" also in the proof of Theorem 3*). That there is at most one God-like* being also follows: if x and y are both God-like*, then y has the same essential properties as x, including identity with x.

Someone might worry that perhaps the emended axioms still lead to modal collapse. On at least one reasonable way of formalizing the proof, they do not. Symbolic versions of these axioms are satisfiable in a "possible worlds" model of second-order S5 of the sort explained by Nino Cocchiarella[11] and such that "for all P, if P, then necessarily P" is false therein. Take a model containing just two possible worlds w_1 and w_2 and just two (possible) entities *a* and *b*. Let *a* exist at both w_1 and w_2 and let *b* exist only at w_1. (The contingent entity *b* is merely possible at w_2). A property is a function which picks out a set of individuals (the extension of the property) at each possible world.[12] Since one cannot represent directly in second-order logic propositions of the form "Φ is positive" (with the property expression in the argument place)[13], let us temporarily define this as "the negation (or complement) of the property Φ entails property Δ" (intuitively, a property is defined as positive if its lack entails a defect). For the purpose of the example, identify property Δ with the property of being a contingent being—the function that picks out the set containing *b* alone at w_1 and at w_2. It is tedious, but not difficult, to show that being God-like* and necessary existence* are both positive—indeed, they are both identified in the model with that function which picks out *a* at every possible world (this is also the essence of *a*), a property whose complement entails (in the model) contingent existence. All the other axioms (using the definition) come out true in this (S5) structure as well. Taking w_1 to be the actual world, there are true but contingent propositions—for example, that there are at least two things.[14]

The idea used in the model suggests a simplification of the axioms. (Actually it was the idea of the simplification that suggested the model). Take as a new primitive the idea of something's being imperfect (or, what is not quite the same, being defective). Then just *define* a property to be positive if its absence in an entity entails that the entity is imperfect *and* its presence does not entail that the entity in question is imperfect. A little more formally, we may say that a property Φ is positive if and only if (1) necessarily for every x, if x does not have Φ, then x is imperfect and (2) it is not necessary that for every x, if x has Φ, then x is imperfect.[15] A little less formally, we explain that a property is positive if and only if it is necessary for, and compatible with, perfection. Axioms 1*, 2, and 4 are then provable as theorems in modal logic. (The second conjunct in the definition is needed to prove Axiom 1*). But note that, on this definition, to assert that being God-like* is positive is already to assert that being God-like* does not entail any imperfection. And if being God-like* were not self-consistent, then it would entail everything. So asserting that the property is positive is quite close to the outright assertion of self-consistency. Theorem 1 does not then seem to give us much new assurance about the possible exemplification of God-likeness. But there seems to be no epistemic loss and the logical economy gained is of some independent interest.[16]

It is hoped that the suggested changes preserve at least some of the essentials of Gödel's proof. One may doubt that an ontological argument will ultimately succeed and yet still hold that reason demands consideration of the best arguments that can be constructed—on both sides of the question of God's existence. If Kurt Gödel thought that the matter can be settled in the affirmative by proof, perhaps those of us who are interested in the question ought to see what merit we can find in his line of reasoning.[17]

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NOTES

1. Jordan Howard Sobel, "Gödel's Ontological Proof," in *On Being and Saying. Essays for Richard Cartwright*, ed. Judith Jarvis Thomson (Cambridge, Mass. & London, England: The MIT Press, 1987). Sobel provides detailed renderings of both Gödel's handwritten note and Scott's elaboration. Apparently the contents of these have not appeared in print before.

2. One of the anonymous referees pointed out that there is a simpler proof of this which does not require the use of self-identity and self-difference. Suppose Φ were positive and inconsistent. Then it would entail its own negation, which would have to be positive by Axiom 2. But this contradicts Axiom 1.

3. According to Hao Wang, *Reflections on Kurt Gödel* (Cambridge, Mass. & London: The MIT Press, 1987).

4. Strictly speaking, the proof requires the necessitation of this axiom—that a property is positive *entails* that it is necessarily positive. In the proof of Theorem 2 below it is asserted that a conditional (namely, "If something is God-like, then necessarily something is God-like.") has been proved using only necessary truths. Other than definitions, the only thing required is that Axiom 4 be necessary. But probably no one who accepts the axioms will shrink from asserting all their necessitations.

5. Because of the arguments of Hugh Chandler ("Plantinga and the Contingently Possible," *Analysis* 36 (1976): 106-109) elaborated by Nathan Salmon (*Reference and Essence* (Princeton University Press and Basil Blackwell, 1981), section 28, pp. 229-52; "Impossible Worlds," *Analysis* 44 (1984); Modal Paradox: Parts and Counterparts, Points and Counterpoints," in French, Uehling, and Wettstein, eds., *Midwest Studies in Philosophy XI: Studies in Essentialism* (Minneapolis: University of Minnesota Press, 1986): 75-120; and "On the Logic of What Might Have Been," *Philosophical Review* (forthcoming)), I have begun to worry that S5 may not be the appropriate modal logic for de re modality. For the purpose of proving Theorem 3, it would actually suffice to use the weaker modal logic B, but Salmon (in the last mentioned article) casts doubt on this as well—and in any case it does not appear that the logical weakening corresponds to any epistemic advance. (That the modal principle of B, "If it is possibly necessary that P, then P," will work just as well as S5 for certain modal ontological arguments was to my knowledge first noticed in print by Robert Merrihew Adams, "The Logical Structure of Anselm's Arguments," *The Philosophical Review* 80 (1971): 28-54). These criticisms cause no difficulty in the present case since the only uses of the characteristic principles of S5 are applications to de dicto modalities. Perhaps it would be better to isolate the de

re by introducing a new primitive 'Essentl(Φ,x)' meaning that Φ is an essential property of x and then to adopt the axiom: if Essentl(Φ,x), then Φ(x). The proofs go through as before using 'Essentl(Φ,x)' in place of 'necessarily x has Φ.' Also this would focus (but presumably not alleviate) any worries someone may have about "quantifying in."

6. David Lewis, "Anselm and Actuality," *Noûs* 4 (1970): 175-88. Reprinted in *Readings in the Philosophy of Religion: An Analytic Approach*, ed. Baruch A. Brody (Englewood Cliffs, New Jersey: Prentice Hall, 1974).

7. Roderick M. Chisholm and Ernest Sosa, "On the Logic of 'Intrinsically Better,'" *American Philosophical Quarterly* 3 (1966): 244-249.

8. But notice that the Chisholm-Sosa definition of an "intrinsically good state of affairs" as a state of affairs which is preferable to some indifferent state of affairs is *not* the appropriate analogue of the idea of a positive property. A positive property, in Gödel's sense, is *purely* positive—it entails only properties which are themselves positive. A good state of affairs, in the Chisholm-Sosa sense, may entail indifferent or even bad states of affairs. A definition which might do is this: a property is positive if and only if every property Φ it entails is such that Φ is preferable to the negation of Φ. This definition has some of the same advantages and drawbacks as the definition considered below. One additional advantage is that it is then possible to prove Axiom 1 from the principle that preferability is asymmetric. The analogue of this for states of affairs is an axiom of the Chisholm-Sosa calculus. But which definition is to be used depends largely on theological considerations. Alternatively, one might adopt no definition at all.

9. There may be something to be said for (1b) from the point of view of Gödel's other interpretation of positiveness as involving only "pure attribution." I do not fully understand this alternative but one might suppose that (1b) is a principle of "fullness of being." But, as noted, (1b) entails the lemma (that if something is God-like, then each of its properties is positive). Below we prove a theorem corresponding to Theorem 2 (which seems crucial to the idea of the main proof), but with new definitions of "God-like" and "essence." There does not follow from this anything corresponding to Corollary 2 (because of the new definition of "essence"), but in the presence of (1b), a correlate of the lemma can still be proved. This and the new version of Theorem 2 again permit an argument like Sobel's to the conclusion that everything which is true is necessary. We assume that most will find this consequence unacceptable.

10. This definition is equivalent to a definition of essence which is now quite standard: an essence is an essential property of something which only it has in any possible world. Proof sketch: Suppose Φ is a property which entails all and only x's essential properties. Then since Φ entails itself, it is an essential property of x. Further, if something else had Φ (in some possible world), then it would have x's essential property of being identical with x and so would itself be identical with x.

Suppose now that Φ is an essential property of x which only x has in any possible world. Let Ψ be an essential property of x. Then if, in some possible world, something y has Φ, y is identical with x and thus has Ψ in that world. So Φ entails Ψ. For the converse, assume that Φ entails some property Ψ. Then x will have Ψ in every possible world in which it has Φ. Therefore, since Φ is an essential property of x, Ψ is also an essential property of x. (The reader who wants to formalize this reasoning in the system of Note 11 is advised to take all quantifiers to be "subsistential" or "possibilist"—see further Note 14).

The use of the idea of an essential property in dealing with the difficulty corresponds to an observation Sobel makes that one might only require of a God-like being that its "intrinsic" properties be positive.

11. Nino B. Cocchiarella, "A Completeness Theorem in Second Order Modal Logic," *Theoria* 35 (1969): 81-103.

12. I do not accept this identification in general. Nothing really turns on it in connection with the use of the model to show that there is no modal collapse. One can think of these functions as the "spectra" of properties without any damage to purpose at hand.

13. We could of course extend the logic to third order but this would require some elaboration of Cocchiarella's semantics.

14. One should consult Cocchiarella's article for a detailed and precise account of his semantics for second-order modal logic. But, at least for those having some familiarity with the usual "possible world" approach, the following outline of the semantics may make it possible to see that there is no modal collapse. We are to imagine given a set I of "possible worlds" where each such world has associated with it an "L-model," a triple consisting of a set A (the individuals existing in the world), a set B (the possible individuals; the set of actual individuals must be a subset of this set) and an interpretation function R which assigns appropriate extensions to the constants of the language—possible individuals to individual constants, sets of possible individuals to one-place predicates and so on. It is worth emphasizing that the denotation of a constant (as given by R) at a world and the extension of a predicate (also given by R) at a world need not consist of individuals which are "actual" at the world. And Cocchiarella's logic has two kinds of quantifiers: what we might call "subsistential quantifiers" which (in the semantics) are construed as ranging over all *possible* individuals (the set B), and "e-quantifiers," "existence quantifiers," which are interpreted at a world as ranging only over the entities which exist in that world (the set A). The logic is interpreted by giving a "world system"—a collection of such L-models, one for each possible world i belonging to I, it being required that the possible individuals of each L-model be the same as those of any other (the possible individuals are the same no matter what world you are in). (Technically the L-models are "indexed"; the set I is the domain of a function which yields an L-model for each i belonging to I). And the existing individuals of all the worlds, taken together, must be a subset of the set of possible individuals. A singular attribute is as usual a function which takes as arguments possible worlds and yields as value in each case a set of possible individuals. And we may take the one-place second-order variables as ranging over all the attributes which correspond to a given world system (Cocchiarella has two kinds of second-order quantifiers, but only one is relevant to our present concern). The n-ary attributes are defined analogously. Ignoring for simplicity some not-immediately-relevant complications of Cocchiarella's actual construction, our desired world system may be taken to contain two possible worlds 1 and 2 and corresponding L-models $A_1 = <\{a,b\}, \{a,b\}, R_1>$ and $A_2 = <\{a\}, \{a,b\}, R_2>$. The first set listed in each case contains the actual individuals of the world, the second contains the possible individuals, and R_i assigns extensions (from the second set) at the world to the constants and predicates of the language. The singularly attributes of the world system are therefore all the functions whose ranges consists of just the two worlds and whose values are in each case sets of possible individuals. The n-ary attributes ($n > 1$) of the world system are defined analo-

gously (although we do not actually care about them for the present purpose). Now use the notation 'Φ_{aV}' to stand for the attribute that picks out singleton a at the world 1 (corresponding to A_1) and picks out the set of all possible individuals $V = \{a,b\}$ at world 2 (corresponding to A_2), with analogous notation for the other fifteen attributes which exist in this world system. Then take R_1('Δ') = R_2('Δ') = Φ_{bb}. It is easy to check that the only positive attributes (those whose negations entail Φ_{bb}) are Φ_{aa}, Φ_{VV}, Φ_{Va}, and Φ_{aV}, and that the first of these is the attribute corresponding to 'G' and 'NE' and is the unique essence of a. Note well that the appropriate translation of the argument into Cocchiarella's notation will take the existential quantifier in the definition of necessary existence to be an e-quantifier (an "existence," rather than "subsistence," quantifier) and so too the quantifier in the conclusion of the argument. All others individual quantifiers may be taken to be subsistential and hence to range over all possibles. It is a purely combinatorial task to show that the modified Gödelian axioms all come out true in this model and that there is no modal collapse. And not everything is a necessary existent. Cocchiarella proves that his axioms for second-order S5 are complete in the "Henkin-sense." Given the definition of "positive property," one can completely formalize the present ontological proof in that system. Thus, there is at least one formalization of the present proof using a reasonably adequate logic in which no modal collapse is demonstrable.

15. The idea of this simplification is based on that of Alan R. Anderson, "A Reduction of Deontic Logic to Alethic Modal Logic," *Mind* 67 (1958): 100-103. The model outlined still shows that there is no modal collapse even if we regard positiveness as thus defined. The positive properties of the model turn out to be the same as before.

16. I personally do not find Gödel's proof of possibility very reassuring. Consideration of the axioms, especially Axiom 2, may tend to dampen one's confidence in Axioms 3 and 5—that is, if one harbors any real doubt about self-consistency. I don't say that the argument begs the question of possibility; the charge is too difficult to establish. But observe that one cannot just tell by scrutinizing a property what it entails; one might be surprised at a consequence. Thus it may not be *so* obvious that being God-like is positive, given that positiveness obeys Axiom 2. The best course for ontological arguers may just be to take the possibility as an axiom and rebut attempts to show inconsistency. Of course the model for the modified axioms shows that there is in that case no danger of *formal* inconsistency.

In some respects Axiom 3 does not seem to do full justice to Gödel's intentions. (Axiom 3 appears in Dana Scott's notes and is presumably his explication of Gödel's sketch). In his notes Gödel assumes instead an axiom that the conjunction of any two positive properties is positive and adds that this is so for any number of conjuncts ("summands")—presumably meaning to include an infinite number as well (See Sobel). If we think of the property of having all positive properties as such a conjunction (instead of what it is—a property involving universal quantification), then the positiveness of being God-like (as defined by Gödel) is included. It may be that a principle along these general lines can be used to give a plausible argument for the new version of Axiom 3 (with the modified definition of "God-like").

17. I am grateful to an anonymous referee for substantial improvements in both form and content.

APPENDIX

I. *Gödel's Axioms, Definitions, and Theorems*

$Pos(\Phi)$: Φ is positive

$\sim S$: It is not the case that S

$\sim\Phi$: The property attributed to anything x when it is asserted that x is not a Φ

$\Box S$: It is necessary that S

$\Diamond S$: It is possible that S

Axiom 1. $Pos(\Phi) \equiv \sim Pos(\sim\Phi)$

We write $(\Phi \Rightarrow \Psi)$ for $\Box(x)(\Phi(x) \supset \Psi(x))$.

Axiom 2. $Pos(\Phi) \supset [\,(\Phi \Rightarrow \Psi) \supset Pos(\Psi)]$

Definition 1. $G(x) = df\ (\Phi)(Pos(\Phi) \supset \Phi(x))$ (God-like)

Axiom 3. $Pos(G)$

Theorem 1. $Pos(\Phi) \supset \Diamond(\exists x)\Phi(x)$

Corollary 1. $\Diamond(\exists x)G(x)$

Lemma. $G(x) \supset (\Phi)(\Phi(x) \supset Pos(\Phi))$

Definition 2. Φ Ess x =df $\Phi(x)$. $(\Psi)[\Psi(x) \supset (\Phi \Rightarrow \Psi)]$ (Essence)

Axiom 4. $Pos(\Phi) \supset \Box Pos(\Phi)$

Theorem 2. $G(x) \supset$ (G Ess x)

Corollary 2. $G(x) \supset [\Phi(x) \supset (G \Rightarrow \Phi)]$

Definition 3. NE(x) =df $(\Phi)[\Phi$ Ess x $\supset \Box(\exists x)\Phi(x)]$ (Necessary Existence)

Axiom 5. Pos(NE)

II. *Sobel's Objections*

It follows that $P \supset \Box P$ and that (y)NE(y)

III. *Analysis of the Difficulty*

(1a) $Pos(\Phi) \supset \sim Pos(\sim\Phi)$

(1b) $\sim Pos(\sim\Phi) \supset Pos(\Phi)$

$\Phi B \Psi$: Φ is better than (or preferable to) Ψ

(B1) $Pos(\Phi) \supset (\Phi B \sim\Phi)$

(B2) $(\Phi B \Psi) \supset \sim (\Psi B \Phi)$

IV. *New Definitions and Corresponding Axioms*

*Definition 1**. G*(x) =df $(\Phi)[\,\Box\Phi(x) \equiv Pos\ (\Phi)]$ (God-like*)

*Definition 2**. Φ Ess* x =df $(\Psi)\ [\Box\ \Psi(x) \equiv\ (\Phi \Rightarrow \Psi)]$ (Essence*)

*Definition 3**. NE*(x) =df $(\Phi)[\Phi$ Ess* x $\supset \Box\,(\exists x)\ \Phi\ (x)]$ (ecessary existence*)

(If this symbolism is transcribed into Cocchiarella's logic for the purpose of constructing a formal proof, the existential quantifier in the definition of "NE*" should be taken to be an "e-quantifier").

*Axiom 3** Pos(G*)

*Axiom 5** Pos(NE*)

$\Delta(x)$: x is imperfect (defective)

Definition. $\mathrm{Pos}\,(\Phi) =_{\mathrm{df}} (\sim\Phi \Rightarrow \Delta)\,.\,\sim(\Phi \Rightarrow \Delta)$ (Positive property)