101

SHORT CUTS IN MATH ANYONE CAN DO

By

GORDON ROCKMAKER

A World of Books That Fill a Need

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PREFACE

101 Short Cuts in Math Anyone Can Do will unlock the secrets of the art of calculation. It will increase your power of computation and thereby enable you to get more out of the mathematics you now know. You will soon be amazed at your ability to solve once complex problems quickly.

Mathematics is perhaps the most important basic science today. It is a powerful and indispensable tool in every phase of science and engineering. The world of business and finance could not survive without it. From law and medicine to the fine arts, from atomic physics to shopping at the supermarket, mathematics plays an essential role in our daily lives.

Many people never get farther than grade-school mathematics simply because they become bogged down in the elementary arithmetic operations. For them mathematics is something mysterious and beyond understanding. They read about electronic computers performing complicated arithmetic operations at speeds measured in microseconds. (a microsecond is a millionth part of a second) and wonder why it is still important to know how to perform these operations themselves.

The reason is obvious. For most people in their offices, shops, classrooms, stores, or homes, use of such electronic brains is impractical or impossible. The simple fact is that engineers and scientists have yet to develop a computer as compact and efficient as the human brain.

The short cuts in this book cover the basic arithmetic operations of addition, subtraction, multiplication, and division. They are used with whole numbers, decimals, fractions, mixed numbers, and percentage. In a word, they range across the whole field of calculation one is likely to use.

In compiling the short cuts to be included in this book, only authentic ones were chosen. An authentic short cut is one that will produce an answer quickly and easily without the necessity of going through the usual intermediate steps, and it is usually very specific. By cutting through the time-consuming mechanical operations and going straight to the heart of the answer, a tremendous amount of needless work is avoided.

All computations in this book are performed from left to right. This is the first time this approach has been applied to short-cut methods. It permits you to write the answer to a problem immediately in the same sequence in which it is read—from left to right.

Emphasis has been deliberately placed on general uses rather than on the specific uses of a particular short cut. The practical value of a short cut is that it can be used in a wide variety of applications. By demonstrating only one or at best a few specific applications, the danger exists that the reader will not venture beyond the ones described.

No book can contain every short cut in math, but this book does include some of the most useful modern methods devised.

Perhaps the most important function of this book is to introduce you to the wide practical application of mathematical short cuts. Using your own creative spirit and the curiosity to experiment, there is no limit to the number of short cuts you can devise for your own special needs.

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INTRODUCTION

CUTTING CORNERS

Whether due to curiosity or sheer laziness, man has always been experimenting, searching for and stumbling upon ways of making work easier for himself. That anonymous caveman who chipped the corners off a flat rock and invented the wheel started this tradition.

Most of man's efforts in the past were directed at conserving or increasing his muscle power, but as time went on some were aimed at saving wear and tear on another vital organ: his brain. It followed naturally that his attention turned to reducing such laborious tasks as calculating.

WHAT SHORT CUTS ARE

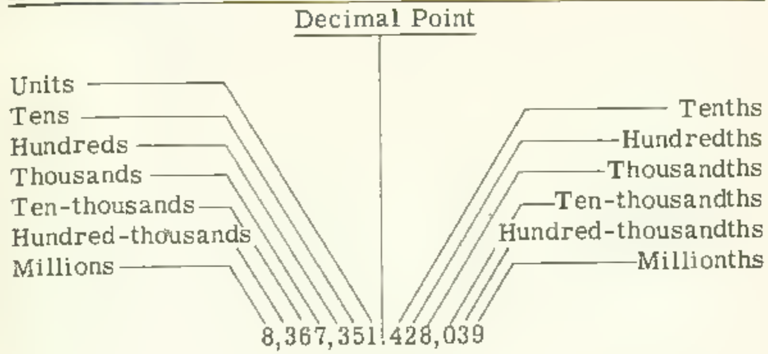
Short cuts in mathematics are ingenious little tricks in calculating that can save enormous amounts of time and labor — not to mention paper — in solving otherwise complicated problems. There are no magical powers connected with these tricks; each is based on sound mathematical principles growing out of the very properties of numbers themselves. The results they produce are absolutely accurate and infallible when applied correctly. Short-cut methods are by no means of recent origin; they were known even to the ancient Greeks. The supply of short cuts is unlimited. Many are known, and many are yet to be discovered. The 101 short cuts included in this book have been selected because they are easy to learn, simple to use, and can be applied to the widest range of calculating problems.

PUTTING NUMBERS IN THEIR PLACE

The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 are called digits. Integers are numbers consisting of one or more digits. For example, 72,958 is an integer consisting of five digits, 7, 2, 9, 5, and 8. In practice, the word number is applied to many different combinations of digits ranging from whole numbers, to fractions, mixed numbers, and decimals. The word integer, however, applies only to whole numbers.

Each digit in a number has a name based on its position in the number. The number system we are accustomed to dealing with is based on the number 10. Each number position in this system is named for a power of 10. The position immediately to the left of the decimal point of a number is called the units position. In the number 1.4 the digit 1 is in the units position and is called the units digit. In fact, any digit that occupies that position is called the units digit. The next position to the left of the units position is called the tens position, and any digit occupying that space is called the tens digit. In the number 51.4 the 5 is the tens digit. Continuing to the left, in order, are the hundreds, thousands, ten-thousands, hundred-thousands, millions positions, and so on.

The positions of the digits to the right of the decimal point also have names similar to those to the left. The position immediately to the right of the decimal point is called the tenths position. Notice that the name is tenths and not tens. In fact, all positions to the right of the decimal point end in ths. The next position to the right of the tenths position is the hundredths position, then the thousandths position, and, in order, the ten-thousandths, the hundred-thousandths, the millionths.



Remember, the position names never change. The position to the left of the decimal point is always the units position; the one to the right is always the tenths position, no matter what digit occupies the space.

In addition to the names of the positions as given above, the letters A, B, C, . . . will be used in this book to help explain the various short-cut methods. Thus, in some short cuts the digits will be arranged as given below:

A B C D E F G H I J K L M

8 3 6 7 3 5 1.4 2 8 0 3 9

The letters themselves have no significance beyond helping identify and locate a particular digit under discussion in the short cut. For that reason it is important not only to learn the various position names but also to gain familiarity with the letter notation just mentioned. Both will be used frequently throughout this book.

GETTING THE POINT

All numbers may be considered to have a decimal point. The point is used to separate those numbers that are equal to or greater than 1 from those numbers that are less than 1. Even if we write a number without a decimal point, it is understood that there is one to the right of the units digit. For example, we can write seven dollars and forty-nine cents as

$7.49

Clearly the decimal point separates the dollars figure (1 or more) from the cents figure (the part that is less than one dollar). But when we speak of seven dollars alone we may write it as $7 or $7. or $7.00. These three forms are exactly equal. In the first case the decimal is omitted but nevertheless is understood to be to the right of the 7. It is also understood that the only digits that can be placed to the right of the decimal point without changing the value of the number are zeros. And as many zeros may be placed to the right of the decimal point as we wish. Later in the application of many short-cut methods you will see why this is an important property of decimals.

LEARNING TO TAKE THE SHORT CUT

The preceding sections dealt with the language of mathematics. Before studying any of the methods that follow, make certain that you are thoroughly familiar with the terms that will be used. When you read about the "hundreds digit," you must immediately recognize that this refers to a position in an integer and not the number 100. Also, never confuse the hundreds digit with the hundredths digit.

Once you have familiarized yourself with the language, the next step is to develop a routine for learning and memorizing the short cuts. Maximum efficiency can be achieved only through constant practice. You will soon discover that short cuts fall into logical groups or classifications. Short cuts involving numbers ending in 5 are an example of such a group. Learn to recognize a problem in terms of its group. It would be pointless to have to refer to this book each time you wanted to apply a short cut.

TAKING THE SHORT CUT FROM LEFT TO RIGHT

Most of us were taught the arithmetic operations of multiplication, addition, and subtraction from right to left. We always started from the units digit and worked to the left. After we got our answer, we reversed the number in our mind and read it from left to right. Not only was the process awkward, but the mental gymnastics wasted time. Take this simple example:

364×7

The product was obtained in the following order:

8, 4, 5, 2

Then to read the answer, it became

2,548

Why cannot answers be obtained in their natural reading order? There is no reason at all why we cannot solve problems just as easily from left to right as we do from right to left.

In this book all work will be performed in the natural order in which we write and read numbers — from left to right. Initially this method may seem strange; but once mastered, its advantages will become evident and the time-saving ease with which it can be used will prove its worth.

<u>In this book, the term "first digit" refers to the left-handmost digit.</u>

FOUR TO GO

Here are a few hints to get you started on the right foot.

<u>First</u>, read and reread the Rules as many times as necessary (at least twice) until a general idea of the short-cut method is established in your thought. Keep in mind that

you are studying and not reading a novel. Try to follow the method in general terms without thinking of specific numbers at this stage.

Next, follow the sample problems carefully, step by step. Do not skip steps just because you feel they involve some trivial operation, such as adding 1. After you have read the sample problem a few times, try to do the same problem yourself, writing the numbers as you go along. Do not refer to the book at this point. If you don't get all the steps correct, go back over them again. You may have to reread the Rule.

Finally, when you are completely satisfied that you have mastered the short cut, try the Practice Exercises. The answers should be written directly in the space provided. Try doing intermediate steps mentally. Very soon you'll find that you can solve most problems without paper and pencil.

Remember, systematic study and concentration on what you are doing are vital to the mastery of each of the 101 short cuts in mathematics.

Chapter 1

SHORT CUTS IN ADDITION

Addition is probably the first arithmetic operation most of us learned after we found out what numbers were. Do you remember the admonition, never to add dissimilar objects? One must not add 2 oranges to 2 apples (unless one were making fruit salad). Different methods of adding were usually taught to help speed the process. However, strictly speaking, there are no short cuts to adding random groups of numbers. No matter what method of addition is used, eventually they all require adding digit by digit until the final sum is obtained.

In adding regular sequences of numbers, short cuts are possible. These sequences can be groups of consecutive numbers, series of numbers that differ by some constant amount, or series of numbers where each term differs from the preceding term by some common ratio. An example of the first group would be the numbers

73, 74, 75, 76, 77, 78, 79, 80, 81

This is a series of consecutive numbers from 73 to 81. An example of the second series would be the numbers

5, 12, 19, 26, 33

In this series each number is always 7 more than the preceding number. An example of the third group would be the series

7, 21, 63, 189, 567

Here each number is 3 times more than the preceding number.

In each case, of course, the sum of the terms in the series can be found by simply adding digit by digit, but briefer, less laborious ways of finding these sums are presented in the short cuts that follow.

1

ADDING CONSECUTIVE NUMBERS

Rule:	Add the smallest number in the group to the largest number in the group, multiply the result by the amount of numbers in the group, and divide the resulting product by 2.

Suppose we want to find the sum of all numbers from 33 to 41. First, add the smallest number to the largest number.

$33 + 41 = 74$

Since there are nine numbers from 33 to 41, the next step is

$74 \times 9 = 666$ (see Short Cut 15)

Finally, divide the result by 2.

$666 \div 2 = 333$ *Answer*

The sum of all numbers from 33 to 41 is therefore 333.

2

ADDING CONSECUTIVE NUMBERS STARTING FROM 1

Consider the problem of adding a group of consective numbers such as: 1, 2, 3, 4, 5, 6, 7, 8, and 9. How would you go about finding their sum? This group is certainly easy enough to add the usual way. But if you're really clever you might notice that the first number, 1, added to the last number, 9, totals 10 and the second number, 2, plus the next to last number, 8, also totals 10. In fact, starting from both ends and adding pairs, the total in each case is 10. We find there are four pairs, each adding to 10; there is no pair for the number 5. Thus $4 \times 10 = 40$; $40 + 5 = 45$. Going a step further, we can develop a method for finding the sum of as many numbers in a row as we please.

> Rule: Multiply the amount of numbers in the group by one more than their number, and divide by 2.

As an example, suppose we are asked to find the sum of all the numbers from 1 to 99. There are 99 integers in this series; one more than this is 100. Thus

$$99 \times 100 = 9{,}900$$

$$9{,}900 \div 2 = 4{,}950 \quad \textit{Answer}$$

The sum of all numbers from 1 to 99 is therefore 4,950.

3

FINDING THE SUM OF ALL ODD NUMBERS STARTING FROM 1

> Rule: Square the amount of numbers in the series.

To show this, the sum of all numbers from 1 to 100 will be calculated. There are 50 odd numbers in this group. Therefore

$$50 \times 50 = 2{,}500 \quad \textit{Answer}$$

This is the sum of all odd numbers from 1 to 100. As a check, we can compare this answer with the answers found in Short Cuts 2 and 4.

4

FINDING THE SUM OF ALL EVEN NUMBERS STARTING FROM 2

Rule: Multiply the amount of numbers in the group by one more than their number

We shall use this rule to find the sum of all even numbers from 1 to 100. Half of the numbers will be even and half will be odd, which means there are 50 even numbers from 1 to 100. Applying the rule,

$50 \times 51 = 2,550$

Thus the sum of all even numbers from 1 to 100 is 2,550. In Short Cut 2 the sum of all the numbers from 1 to 99 is found to be 4,950; consequently the sum of all numbers from 1 to 100 is 5,050. In Short Cut 3 the sum of all odd numbers from 1 to 100 is found to be 2,500. Our answer for the sum of all the even numbers from 1 to 100 is therefore in agreement.

Sum of all numbers		Sum of all odd numbers		Sum of all even numbers
5,050	−	2,500	=	2,550

5

ADDING A SERIES OF NUMBERS WITH A COMMON DIFFERENCE

Sometimes it is necessary to add a group of numbers that have a common difference. No matter what the common difference is and no matter how many numbers are being added, only one addition, multiplication, and division will be necessary to obtain the answer.

Rule: Add the smallest number to the largest number, multiply the sum by the amount of numbers in the group, and divide by 2.

As an example, let us find the sum of the following numbers:

87, 91, 95, 99, and 103

Notice that the difference between adjacent numbers is always 4. This short-cut method can therefore be used. Add the smallest number, 87, to the largest number, 103. Multiply the sum, 190, by 5, since there are five numbers in the group.

$190 \times 5 = 950$ (Short Cut 11)

Divide by 2 to obtain the answer.

$950 \div 2 = 475$ Answer

Thus $87 + 91 + 95 + 99 + 103 = 475$.

(Naturally, this is exactly the same as the rule in Short Cut 1, because there we were simply adding a series of numbers with a common difference of one. So, for ease of remembering, you can combine Short Cuts 1 and 5.)

6

ADDING A SERIES OF NUMBERS HAVING A COMMON RATIO

> Rule: Multiply the ratio by itself as many times as there are numbers in the series. Subtract 1 from the product and multiply by the first number in the series. Divide the result by one less than the ratio.

This rule is best applied when the common ratio is a small number or when there are few numbers in the series. If there are many numbers and the ratio is large, the necessity of multiplying the ratio by itself many times diminishes the ease with which this short cut can be applied. But suppose we are given the series:

53, 106, 212, 424

Here each term is twice the preceding term, and there are four terms in the series. The ratio, 2, is therefore multiplied four times.

$2 \times 2 \times 2 \times 2 = 16$

Subtract 1 and multiply by the first number.

$16 - 1 = 15; \quad 15 \times 53 = 795$ (Short Cut 27)

The next step is to divide by one less than the ratio; however, since the ratio is 2, we need divide only by 1.

Thus the sum of our series is

53 + 106 + 212 + 424 = 795 Answer

Practice Exercises for Short Cuts 1 through 6

Find the sum in each case.

1) All odd numbers from 1 to 23 =

2) 3 + 6 + 12 + 24 + 48 + 96 =

3) All numbers from 84 to 105 =

4) 56 + 59 + 62 + 65 =

5) 24 + 72 + 216 =

6) 14 + 15 + 16 + 17 + 18 + 19 + 20 + 21 =

7) All numbers from 1 to 1,000 =

8) All even numbers from 1 to 50 =

9) 132 + 137 + 142 + 147 =

10) 197 + 198 + 199 + 200 + 201 + 202 + 203 =

Chapter 2

SHORT CUTS IN MULTIPLICATION

Multiplication is itself a short-cut process. For example, a problem in repeated addition,

$$3 + 3 + 3 + 3 + 3 + 3 + 3 = 21$$

is quickly recognized as nothing more than

$$7 \times 3 = 21$$

This shorthand notation led us directly to the answer, eliminating the necessity of six additions along the way.

For most of us, the multiplication table, drummed into our minds early in our mathematical training, provided the reference source for obtaining the answer. But, happily, proficiency in multiplication does not depend on memorizing tables. The short-cut methods described in this section employ addition, subtraction, division, and, of course, elementary multiplication. But if you can add two numbers quickly and halve or double a number with ease, you should have no trouble at all.

THE DIGITS

The basic calculating unit is the digit. When two numbers are multiplied, every combination of their individual digits is multiplied, and by correctly adding the results (with proper regard to their position) the product of the two numbers is obtained.

Consider the following example:

432 × 678

The nine possible combinations of digits of the two numbers are

4 × 6;	3 × 6;	2 × 6
4 × 7;	3 × 7;	2 × 7
4 × 8;	3 × 8;	2 × 8

By arranging the products according to number position, we can obtain the product desired.

24	18	12	2,712
28	21	14	2,034
32	24	16	1,356
2,712	2,034	1,356	292,896

432 × 678 = 292,896 Answer

Thus, by memorizing only the multiplication tables for all digits from 1 to 9 we are able to multiply one number by another, regardless of how many digits each of them contains.

But memorizing the eighty-one products in the multiplication table is not essential for multiplying by the digits. The methods for multiplying by the digits described in this section involve only addition, subtraction, and doubling or halving.

The rules are given in detail intentionally. For some digits, the rule may appear unusually long. This is only because the presentation must consider all exigencies. Don't be discouraged by what seems like a complicated

way of multiplying a simple digit. After the second or third reading of the rule a pattern will emerge and the process will become a mere routine.

A rule for multiplication by 1 has been omitted, since the product obtained by multiplying any number by 1 is the original number.

7

MULTIPLYING BY NUMBERS ENDING IN ZEROS

Numbers ending in zeros may be thought of as the product of the nonzero part multiplied by a power of 10. For example, 37,000 is really $37 \times 1{,}000$. Since multiplying by zero results in zero, multiplying by numbers ending in zeros may be shortened by ignoring the zeros and then affixing the required amount after the nonzero part has been multiplied.

> Rule: Multiply the two numbers as if they did not end in zeros. Then affix an amount of zeros equal to the sum of all the zeros ignored in the multiplication.

A simple case will be chosen. Let us find the product of

$37{,}000 \times 6{,}000{,}000$

By ignoring the zeros, we have

37×6

Using Short Cut 12, we find $37 \times 6 = 222$. A total of nine zeros was ignored before the multiplication; therefore nine zeros are affixed to the product.

222,000,000,000 Answer

8

MULTIPLYING BY 2

Multiplying by 2 is another way of saying we are doubling a number or simply that we are adding a number to itself. Doubling a number may be accomplished quickly without carrying by applying the following simple rule.

> Rule: Starting from the first digit of the given number, double the digit if it is 4 or less and put the answer under the respective digits of the given number. For digits 5 to 9, subtract 5 and double the result. Place the answer under the respective digits of the given number. Now inspect the tentative answer. Each digit of the answer to the immediate left of a digit in the given number 5 or greater should be increased by 1. The result is the final answer.

At first reading, this rule may sound more complicated than simply adding digit by digit. The beauty of this shortcut method is, however, that the answer is obtained immediately from left to right and we are never bothered by having to remember to carry over any digits. As an example, suppose we were asked to multiply 5,377 by 2. First let us write the given number, using our alphabetic identification:

A B C D

5 3 7 7

Starting from A, double each number less than 5 (but not equal to 5); if the number is greater than 5, subtract 5 from it and double the result, placing a small line under each digit of the answer that is to the immediate left of a digit in the given number that is 5 or more. The reason for this small line will be explained shortly. In our given number, the first digit is 5; subtract 5 from this and double the result.

$$5 - 5 = 0; \qquad 0 + 0 = 0$$

Place 0 under the 5 and a small line under the space to the left of the 0 (since there is no number in that space). Our first result will look like this:

A B C D	
5 3 7 7	Given number
$\_\ 0$	Tentative answer after first step

The next digit is less than 5, so we merely double it, and our answer begins to look like this now:

A B C D	
5 3 7 7	Given number
$\_\ 0\ 6$	Tentative answer after second step

The C digit is a 7; subtract 5 from this and double the result.

$$7 - 5 = 2; \qquad 2 + 2 = 4$$

This is the C digit of the answer; but remember, a small line must be placed under the next digit to the left in the answer (the 6). We have now come this far in our answer:

A B C D	
5 3 7 7	Given number
$\_\ 0\ \underline{6}\ 4$	Tentative answer after third step

Finally, the D digit is more than 5, so once again we obtain 4 and place a small line under the previous 4 in the answer. Our answer now looks like this:

A B C D	
5 3 7 7	Given number
$\_\ 0\ \underline{6}\ \underline{4}\ 4$	Tentative answer after fourth step

Each underlined digit is increased by 1 to obtain the final answer.

10,754 Answer

9

MULTIPLYING BY 3

> Rule: The first tentative digit of the answer is obtained by taking one-half the first digit of the given number.
>
> Next, in turn, each digit of the given number is subtracted from 9, the result doubled, then added to one-half the digit to the right in the given number to obtain each digit of the answer. If the original digit in the given number is odd, add an extra 5. Ignore any fraction that occurs when taking one-half a number.
>
> To find the units digit of the answer, subtract the units digit of the given number from 10 and double the result. Add an extra 5 if the units digit of the given number is odd.
>
> In each of the steps above, record only the units digit in the answer. Any tens digit should be carried and added to the answer digit immediately to the left.
>
> To obtain the final answer from the tentative answer digits obtained above, subtract 2 from the first digit recorded.

Naturally, when multiplying a small number by 3, the "long" way would probably be as quick, though maybe not as simple to use; but when long numbers are multiplied, the short cut explained above is an excellent time and labor saver.

For example: $4{,}635{,}117 \times 3$.

One-half of 4 is the first tentative digit.

$\frac{1}{2}(4) = 2$

The next digit of the answer is found by subtracting 4 from 9, doubling the result, and adding one-half the digit to the right, 6. Since 4 is even, the additional 5 is <u>not</u> added.

$9 - 4 = 5; \quad 5 \times 2 = 10; \quad 10 + \frac{1}{2}(6) = 13$

Record the 3, carry the 1, and add it to the 2 previously determined.

The next digit in the given number is 6.

$9 - 6 = 3; \quad 3 \times 2 = 6; \quad 6 + \frac{1}{2}(3) = 7$

(The fraction $\frac{1}{2}$ is ignored.) The next digit in the given number is 3.

$9 - 3 = 6; \quad 6 \times 2 = 12; \quad 12 + \frac{1}{2}(5) = 14$

$14 + 5 = 19$

(The 5 was added because 3 is odd.) Record the 9; carry the 1 to the left. The four digits thus far obtained in the answer are

3 3 8 9

Continue with the other digits of the given number.

$9 - 5 = 4; \quad 4 \times 2 = 8; \quad 8 + \frac{1}{2}(1) = 8$

$8 + 5 = 13$

Record the 3; carry the 1 to the left.

$9 - 1 = 8; \quad 8 \times 2 = 16; \quad 16 + \frac{1}{2}(1) = 16$

$16 + 5 = 21$

Record the 1; carry 2.

$9 - 1 = 8; \quad 8 \times 2 = 16; \quad 16 + \frac{1}{2}(7) = 19$

$19 + 5 = 24$

Record 4; carry 2.

The units digit is next.

$10 - 7 = 3; \quad 3 \times 2 = 6; \quad 6 + 5 = 11$

Record 1; carry 1. The digits obtained are

33,905,351

The final step involves subtracting 2 from the first digit.

13,905,351 Answer

10

MULTIPLYING BY 4

> Rule: The first tentative digit of the answer will be one-half the first digit of the given number. Ignore any fraction in this and other steps. The other answer digits are found by subtracting each of the digits of the given number from 9 and adding one-half the digit to the right. If the digit of the given number is odd, add an extra 5.
>
> To find the units digit of the answer, subtract the units digit of the given number from 10. Add 5 if the units digit of the given number is odd. To obtain the final answer, subtract 1 from the first digit recorded.
>
> In each of the cases, above, if the result of one of the steps is a two-digit number, record the units digit and carry any tens digit left to the preceding answer digit.

As an example: Multiply 37,485,109 by 4.

The first tentative digit is one-half the first digit of the given number, 3.

$\frac{1}{2}(3) = 1$

(Ignore the fraction.)

In each of the next steps, subtract the digit of the given number from 9, add one-half the digit to the right, and add 5 more if the digit in the given number is odd.

$9 - 3 = 6; \quad 6 + \frac{1}{2}(7) + 5 = 14$

(Here 5 is added because 3 is odd.)

Record the 4 and add 1 to the 1 previously determined.

The next digit in the given number is 7.

$9 - 7 = 2; \quad 2 + \frac{1}{2}(4) + 5 = 9$

(Again, 5 is added because 7 is odd.) Continue in turn with 4, 8, 5, 1, and 0.

$9 - 4 = 5; \quad 5 + \frac{1}{2}(8) = 9$

$9 - 8 = 1; \quad 1 + \frac{1}{2}(5) = 3$

$9 - 5 = 4; \quad 4 + \frac{1}{2}(1) + 5 = 9$

$9 - 1 = 8; \quad 8 + \frac{1}{2}(0) + 5 = 13$

Record 3; carry 1 to the left.

$9 - 0 = 9; \quad 9 + \frac{1}{2}(9) = 13$

(The zero is considered even.) Record 3; carry 1 forward.

We have now reached the units digit of the given number. To obtain the units digit of the answer, subtract the units digit of the given number from 10. Add 5, since it is odd.

$10 - 9 = 1; \quad 1 + 5 = 6$

We have now obtained the following tentative answer:

249,940,436

The final answer is obtained by subtracting 1 from the first digit, 2.

149,940,436 Answer

11

MULTIPLYING BY 5

When any digit is multiplied by 5, the units digit of the product is always either 5 or 0 and the tens digit is always equal to one-half the given digit (ignoring the fraction $\frac{1}{2}$). This interesting property of 5 leads us to the first of two short-cut methods for multiplying by 5.

First Method

Rule: The first digit of the answer is equal to one-half the first digit of the given number. Each succeeding answer digit is equal to 5, if the corresponding digit in the given number is odd; or 0, if the corresponding digit in the given number is even; plus one-half of the digit to the right in the given number. The units digit of the answer is 5, if the given number is odd; and 0, if the given number is even. Ignore any fraction resulting from the halving process.

Second Method

Rule: Move the decimal point of the given number one place to the right and divide the resulting number by 2.

Although the second method seems simpler at first reading, both methods are equally easy to employ and both will find applications, depending on the problem. Usually for small even numbers, the second method would probably be used more often. However, both methods will be demonstrated, using the same given number.

Multiply 78,439 by 5.

A B C D E F	
7 8 4 3 9	Given number

First Method. The first digit of the product (the A digit) will be equal to one-half of 7 (ignoring the $\frac{1}{2}$).

A	
3	First digit of product

Since the B digit of the given number is odd, the B digit of the product will be 5 plus one-half the C digit of the given number (5 + 4 = 9). The C digit of the given number is even, so that the C digit of the product will be 0 plus one-half the D digit (0 + 2 = 2). The D digit of the product is 0 + 1 = 1. The E digit of the product is 5 + 4 = 9. The F digit of the product is the units digit in this case, and since the units digit of the given number is odd, the units digit of the product will be 5. The final product is

A B C D E F

3 9 2, 1 9 5 Answer

Second Method. Move the decimal point of the given number one place to the right.

7 8, 4 3 9 . 0 becomes 7 8 4, 3 9 0.

Divide the new number by 2.

7 8 4, 3 9 0 ÷ 2 = 3 9 2, 1 9 5 Answer

The same result was obtained with the first method.

12

MULTIPLYING BY 6

> Rule: The first digit of the answer is one-half the first digit of the given number.
>
> The other answer digits are obtained by adding each of the digits of the given number to one-half the digit to its right. An extra 5 is added if the given digit is odd.
>
> Ignore any fraction that occurs when halving a number.
>
> The units digit of the answer is the units digit of the given number, if even. If odd, add 5 to the units digit of the given number to obtain the units digit of the answer.
>
> In each case, if the result is a two-digit number, record only the units digit. Carry any tens digit left and add it to the preceding answer digit.

This short cut may seem like a roundabout way to multiply by 6, but the opposite is actually true. In fact, the beauty of this method is the simplicity and ease with which an answer may be written directly. You will soon find yourself able to multiply any number by 6, using only a little quick mental addition without bothering to write any intermediate steps.

As an example of the procedure, we shall multiply

714,098 × 6

The first digit (tentatively) will be one-half of 7, or 3 (neglecting the $\frac{1}{2}$, of course). The answer digits that follow will depend on whether the corresponding digits in the given number are odd or even. Since the first digit is odd, add 5 and one-half the next digit to the right.

$7 + 5 + \frac{1}{2}(1) = 7 + 5 + 0 = 12$

(Remember, $\frac{1}{2}$ is ignored.) Record the units digit 2 and carry the tens digit to the left to be added to the 3 previously written. The first two answer digits are

4 2

The next digit of the given number is 1, which is also odd.

$1 + 5 + \frac{1}{2}(4) = 1 + 5 + 2 = 8$

Record the 8 and move on to the next digit in the given number, 4. Since this is even, merely add to it one-half the next digit to the right in the given number.

$4 + \frac{1}{2}(0) = 4$

Record this in the answer. Thus far we have determined the following digits in the answer:

4 2 8 4

The next digit is 0 (which is considered even). Therefore, add one-half the next digit to the right.

$0 + \frac{1}{2}(9) = 4$ (ignoring $\frac{1}{2}$)

The next digit is 9, which is odd.

$9 + 5 + \frac{1}{2}(8) = 9 + 5 + 4 = 18$

Record the 8 in the answer and carry the tens digit, 1, left to be added to the preceding digit, 4. The units digit of the given number is next; since it is even, it is also the units digit of the answer.

The product is therefore

4, 2 8 4, 5 8 8 Answer

13

MULTIPLYING BY 7

Rule:	The first tentative digit of the answer is one-half the first digit of the given number. The rest of the answer digits are obtained by doubling the digit of the given number and adding one-half the digit to its right. Add an extra 5 if the given digit is odd. The units digit of the answer is twice the given units digit. Add 5 if the given units digit is odd. Ignore any fraction that may occur. Record only the units digit in each case. Any tens digit should be carried and added to the answer digit immediately to the left.

Example: 97,841 × 7.

The first digit is one-half 9.

$\frac{1}{2}(9) = 4$

(Ignore the fraction.) Next, in turn, double each digit of the given number, add one-half the digit to the right, and add an extra 5 if the given digit is odd.

$9 \times 2 = 18; \quad 18 + \frac{1}{2}(7) = 21 \quad 21 + 5 = 26$

Record 6 and add 2 to the preceding answer digit, 4.

$7 \times 2 = 14; \quad 14 + \frac{1}{2}(8) = 18 \quad 18 + 5 = 23$

Record 3; carry 2 to the left.

$8 \times 2 = 16; \quad 16 + \frac{1}{2}(4) = 18$

Record 8; carry 1.

$4 \times 2 = 8; \quad 8 + \frac{1}{2}(1) = 8$

Finally, the units digit of the answer is determined.

$1 \times 2 = 2; \quad 2 + 5 = 7$

The digits obtained are

6 8 4, 8 8 7 Answer

14

MULTIPLYING BY 8

> Rule: Write the first digit of the given number as the first tentative digit of the answer. The next answer digit is obtained by subtracting the first digit of the given number from 9, doubling the result, and adding the second digit of the given number. Continue the process by subtracting each digit of the given number from 9, doubling the result, and adding the next digit to its right. To obtain the units digit of the answer, simply subtract the units digit of the given number from 10 and double the result. In each of the steps above, record only the units digit of the sum; any tens digit should be carried and added to the preceding answer digit. To obtain the final answer, subtract 2 from the first digit obtained.

A typical example is sufficient to show how this short cut works.

Example: $379{,}146 \times 8$.

First, write the 3 as the tentative first digit of the answer. Next subtract 3 from 9, double the result, and add the next digit to its right, 7.

$9 - 3 = 6; \quad 6 \times 2 = 12; \quad 12 + 7 = 19$

Record the 9, carry the 1, and add it to the 3 previously recorded. The first two tentative digits of the answer are

4 9

Proceed with the next digit, 7.

$9 - 7 = 2; \quad 2 \times 2 = 4; \quad 4 + 9 = 13$

Record the 3, carry the 1, and add it to the previously determined 9. But 9 + 1 = 10. Therefore record the 0 and carry the 1 another digit to the left, adding it to the 4. The first three digits are now

5 0 3

Continue this procedure.

9 - 9 = 0; 0 × 2 = 0; 0 + 1 = 1

Record 1.

9 - 1 = 8; 8 × 2 = 16; 16 + 4 = 20

Record 0 and add the 2 to the 1 preceding. The answer digits obtained to this point are

5 0 3 3 0

Continue with the process.

9 - 4 = 5; 5 × 2 = 10; 10 + 6 = 16

Record the 6; carry the 1. The next digit is the units digit of the given number. Subtract this from 10 and double the result. Record the result as the units digit of the answer.

10 - 6 = 4; 4 × 2 = 8

The tentative answer to the problem is

5, 0 3 3, 1 6 8

To obtain the final answer, we must subtract 2 from the first digit, 5.

3, 0 3 3, 1 6 8 Answer

15

MULTIPLYING BY 9

> Rule: The first digit of the given number minus 1 is the first digit of the answer. The second digit of the answer is obtained by subtracting the first digit of the given number from 9 and adding it to the second digit of the given number. Continue this process by subtracting each digit in the given number from 9 and adding to the result the next digit to its right. Stop this procedure after the tens digit of the answer is obtained. The units digit of the answer is obtained by subtracting the units digit of the given number from 10. In each case, if the sum is a two-digit number, record the units digit and carry the tens digit to the preceding answer digit.

Multiply 7,149 by 9.

The first digit of the given number minus 1 is the first digit of the answer.

7 - 1 = 6

The second digit of the answer is 9 minus the first digit of the given number plus the second digit of the given number.

9 - 7 = 2; 2 + 1 = 3

We now have the first two digits of the answer (at least tentatively).

6 3

To obtain the third digit of the answer, subtract the second digit of the given number from 9 and add the result to the third digit of the given number.

9 - 1 = 8; 8 + 4 = 12

Here the result is a two-digit number. The units digit is recorded as part of the answer, and the tens digit is carried and added to the 3 previously determined. The first three digits of the answer are now

6 4 2

The tens digit and the units digit of the given number are used to obtain the tens digit of the answer.

$9 - 4 = 5; \quad 5 + 9 = 14$

Record the 4, carry the 1, and add it to the 2 previously determined. The units digit of the answer is merely 10 minus the units digit of the given number.

$10 - 9 = 1$

The product is therefore

7,149 × 9 = 64,341 Answer

Practice Exercises for Short Cuts 7 through 15

1) 47,821 × 5 =

2) 8,300 × 2,000,000 =

3) 7,914 × 8 =

4) 682 × 9 =

5) 1,356 × 7 =

6) 51,007 × 2 =

7) 6,045 × 6 =

8) 497 × 3 =

9) 12,760,195 × 4 =

10) 1,116 × 9 =

11) 436 × 5 =

12) 31,875 × 3 =

13) 613,767 × 7 =

14) 44,060 × 6 =

15) 831,615 × 8 =

NUMBERS BEGINNING OR ENDING IN 1

When a given number is multiplied by 1, the product is the same given number. This unique property of 1 is used to good advantage in numerous short cuts. When a multiplier containing 1 is used, somewhere in the answer is the number being multiplied. This fact forms the basis of many of the short cuts that follow.

16

MULTIPLYING BY 11

> Rule: The first digit of the given number is the first digit of the answer. Add the first digit to the second digit of the given number to obtain the second digit of the answer. Next, add the second digit of the given number to the third digit of the given number to obtain the third digit of the answer. Continue adding adjacent digits until the tens digit of the given number is added to the units digit of the given number to obtain the tens digit of the answer. The units digit of the answer will be the units digit of the given number. If any of the sums are two-digit numbers, record only the units digit and add the tens digit to the preceding answer digit.

Two examples will best show how to use this short cut.

Example No. 1: Multiply 81,263 by 11.

The first digit of the answer will be 8, the first digit of the given number. The second digit of the answer will be the sum of the first and second digits of the given number, 8 + 1 = 9. Continuing from left to right, the sum of adjacent digits in the given number will produce digits of the answer. The result is shown below:

8 1 2 6 3 Given number

8 9 3 8 9 3 *Answer*

In the example above, each sum was less than 10. But what would happen if the sum was 10 or more?

Example No. 2: Multiply 67,295 by 11.

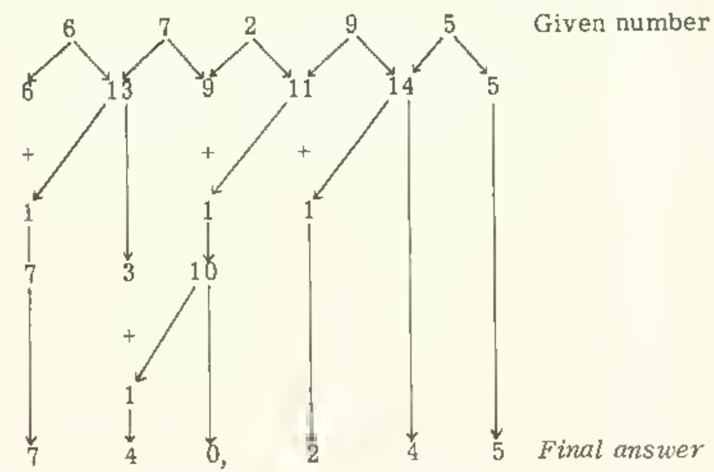
The 6 is the tentative first digit. The second digit is the sum of 6 and 7, or 13. Here the sum is greater than 10. The 3 becomes the tentative second digit of the answer, but the 1 is carried left and added to the first digit.

6 + 1 = 7

This is the new first digit of the answer. The third digit of the answer is found by adding the second digit of the given number to the third digit, 7 + 2 = 9. The next digit in the answer is 2 + 9 = 11. Again the units digit becomes part of the answer, and the tens digit is carried left to the previously determined answer digit.

9 + 1 = 10.

The 0 is the new third digit of the answer, and the 1 is carried still further left to the second digit, 3 + 1 = 4. This is the new second digit. Continue in this fashion until all adjacent digits have been added. The final digit in the answer is 5. This process is shown pictorially thus:



17

MULTIPLYING BY 12

> **Rule:** Precede the given number with a zero. Starting from this zero, double each digit and add to it the next digit to its right. Record the sum. When the units digit of the given number is reached, simply double it and record the sum. In each step, if the doubling process results in a two-digit number, record only the units digit and add the tens digit to the preceding answer digit.

This simple short cut is particularly handy when we want to project some monthly event over the entire year. Suppose we are asked to find the total rent paid during the year if the monthly rental is \$132.50. To do this we multiply the monthly rental by 12. Our problem then becomes

\$132.50 × 12

First, place a zero in front of the number.

0 1 3 2. 5 0

Next, double each digit and add to it the digit to the right. Adding 1 to 0 gives the first digit of the answer, 1. Adding twice 1 (the second digit of the given number) to 3 (its neighbor to the right) gives the second digit of the answer, 5. Continuing in this manner, we obtain the answer.

0 1 3 2. 5 0 Given number

\$1, 5 9 0. 0 0 Answer

Notice that in doubling the 5 in the given number the result was 10. The 0 was recorded and the 1 was added to the preceding digit. The preceding digit, however, was a 9, which when increased by 1 became 10. Again the 0 was recorded and the 1 again carried another step to the left. This time it increased the previously determined 8 to a 9. Until the 5 was doubled, the answer digits were 1,589.

18

MULTIPLYING BY 111

Rule: Imagine a number whose digits are

A B C D E F G H I J K L M

The first digit of the answer will be A. The second digit will be A + B. The third digit will be A + B + C. The fourth digit will be B + C + D. The fifth digit will be C + D + E. This procedure is followed, adding three adjacent digits together, until the final three digits are reached. The hundreds digit of the answer will be K + L + M. The tens digit of the answer will be L + M. The units digit of the answer will always be the units digit of the given number, in this case, M. Remember that whenever the sum is a two-digit number, the units digit is the answer portion and the tens digit is added to the preceding answer digit. Thus, if I + J + K is a two-digit number, the tens digit will be added to the sum of H + I + J previously determined.

Follow the next example step by step.

6 5 9, 8 4 5 × 1 1 1

The first digit of the answer will be the first digit of the given number, 6. The second digit of the answer is

6 + 5 = 11

Write the 1 and carry the tens digit (also 1) left.

6 + 1 = 7

The next digit is

6 + 5 + 9 = 20

Write the 0 and carry the 2 left.

$1 + 2 = 3$

The three digits we have found thus far are

730

Now begin adding the digits of the given number in groups of three.

$5 + 9 + 8 = 22$

Write 2; carry 2.

$9 + 8 + 4 = 21$

Write 1; carry 2. Continue this process until the last three digits, 845, are reached.

$8 + 4 + 5 = 17$

Write 7; carry 1.

$4 + 5 = 9$

Write 9; no carry.

The final digit of the answer will be the units digit of the given number, 5.

In pictorial form, the entire example looks like this:

6 5 9, 8 4 5

6	6+5	6+5+9	5+9+8	9+8+4	8+4+5	4+5	5
6	11	20	22	21	17	9	5
+	+	+	+	+			
1	2	2	2	1			
7	3	2	4	2	7	9	5

73,242,795 Answer

19

MULTIPLYING BY A MULTIPLE OF 11

> Rule: Multiply by the units digit of the multiple of 11 (using the appropriate short cut). Then multiply by 11 (Short Cut 16).

Although the beginner will usually apply this short cut in two separate operations, as he becomes more expert in its use the final answer will be obtained in only one operation. The explanation given below is in two distinct steps, since this presentation is easier to follow.

Multiply 84,756 by 66.

Here we are multiplying by the sixth multiple of 11, since $6 \times 11 = 66$. First, apply Short Cut 12 for multiplying by 6. Next, multiply the result by 11, using Short Cut 16.

$6 \times 84{,}756 = 508{,}536$

$11 \times 508{,}536 = 5{,}593{,}896$

Therefore

84,756 × 66 = 5,593,896 Answer

20

MULTIPLYING BY 21

> Rule: The first digit (or digits) of the answer will be twice the first digit of the given number. The second digit of the answer will be the first digit of the given number plus twice the second digit of the given number. The third digit of the answer will be the second digit of the given number plus twice the third digit of the given number. Continue the process until the tens digit of the given number is added to twice the units digit of the given number. This sum is the tens digit of the answer. The units digit of the answer is the units digit of the given number. Whenever a sum is a two-digit number, record its units digit and add the tens digit to the preceding answer digit.

This rule is very much like the one for multiplying by 11. In fact, since 21 is the sum of 11 and 10, it does belong to the same family of short cuts.

As an example, we shall multiply 5,392 by 21.

The first digits of the answer will be equal to twice the first digit of the given number.

$5 \times 2 = 10$

Next, add the first digit of the given number, 5, to twice the second digit, 3.

$5 + (2 \times 3) = 11$

The units digit becomes the next answer digit, and the tens digit is added to the 10 previously determined. The first three digits up to this point are

1 1 1

The next digit is obtained by adding 3 to twice 9.

$3 + (2 \times 9) = 21$

Record the 1 and carry the 2 to the left. The first four digits of the answer are now

1 1 3 1

The tens digit of the answer is obtained by adding the tens digit of the given number to twice the units digit of the given number.

$9 + (2 \times 2) = 13$

Record the 3; carry the 1 to the left. The units digit of the answer is the units digit of the given number, 2.

The product is therefore

5,392 × 21 = 113,232 Answer

21

MULTIPLYING BY 121

> Rule: Multiply the given number by 11, using Short Cut 16. Multiply the product obtained by 11 again.

The ease with which Short Cut 16 can be used permits even a two-step method such as this to be applied with rapidity. When used with small numbers, say, two- or three-digit numbers, the numbers obtained in the first step may be retained in the mind and the second step performed by writing the answer immediately. In the sample problem, the two steps will be shown.

Multiply 8,591 by 121.

Multiply the given number by 11, using Short Cut 16.

8,591 × 11 = 94,501

Multiply the result by 11.

94,501 × 11 = 1,039,511 Answer

22

MULTIPLYING BY 101

> Rule: First, write the first two digits of the given number as the first two answer digits. Then, starting from the third digit of the given number, add each of the digits of the given number in turn, adding the third digit to the first digit, the fourth digit to the second digit, and so on. When the last digit of the original number is reached, continue writing the remaining digits of the given number.

For example:

164,759 × 101.

The first two answer digits are

1 6

Starting from the third digit, 4, add in turn the digits of the given number, 1-6-4-7-5-9.

1 + 4 = 5; 6 + 7 = 13; 4 + 5 = 9; 7 + 9 = 16

The 9 is the last digit of the original given number. Thereafter merely record the balance of the digits of the given number not added: in this case, 5 and 9. Naturally, in the additions performed above, the units digit is recorded as the answer digit; any tens digit is added to the preceding answer digit.

Therefore

164,759 × 101 = 16,640,659 Answer

23

MULTIPLYING BY 1,001

> Rule: First, write the first three digits of the given number as the first three answer digits. Then, starting from the fourth digit of the given number, add each of the digits of the given number in turn, adding the fourth digit to the first digit, the fifth digit to the second digit, and so on. When the last digit of the original given number is reached, continue writing the remaining digits of the given number.

For example: 23,107 × 1,001.
The first three answer digits are

2 3 1

Starting from the fourth digit of the given number, 0, add the digits of the given number in turn.

2 + 0 = 2; 3 + 7 = 10

The 7 is the last digit of the original given number; therefore the digits of the given number not yet added, 1, 0, and 7, are merely written as the answer digits.

Whenever a sum is greater than 9, record the units digit and add the tens digit to the preceding answer digit.

23,107 × 1,001 = 23,130,107 Answer

24

MULTIPLYING BY ONE MORE THAN A POWER OF 10

> Rule: Write as many digits of the given number as there are digits in the multiplier less one. Then, starting from the digit whose place is equal to the number of digits in the multiplier, add, digit by digit, the given number to the original given number.

What this rule means is that if the multiplier has seven digits, the addition should start from the seventh digit. The first digit of the given number is to be added to the seventh digit of the given number, the second digit added to the eighth, and so on. Naturally, the first six digits of the answer will be the same as the first six digits of the given number unless they are changed by some digit that is carried forward.

For example: 66,809,542 × 100,001.

There are six digits in the multiplier; therefore write the first five digits of the given number as the first five answer digits. Starting at the sixth digit of the given number, add the digits of the given number.

6 + 5 = 11; 6 + 4 = 10; 8 + 2 = 10

The balance of the digits not added are merely written as given in the original number. When the sums are two-digit numbers, record the units digit as part of the answer and add the tens digit to the preceding answer digit. Thus, in the three sums shown above, 1 is recorded and 1 is carried forward; 0 is recorded and 1 is carried forward; 0 is recorded and 1 is carried forward. The rest of the digits are recorded as they appear in the original given number.

0 9 5 4 2

The product is therefore

66,809,542 × 100,001 = 6,681,021,009,542 Answer

Which, in case you are interested, can be read as: Six trillion; six hundred eighty-one billion; twenty-one million; nine thousand; five hundred and forty-two.

25

MULTIPLYING "TEEN" NUMBERS

> Rule: To one of the numbers, add the units digit of the other number. To the result, affix the units digit of the product obtained by multiplying the units digits of the given numbers. Any tens digit should be added to the sum found in the first step.

The teen numbers include all numbers from 10 to 19.

Example: 13 × 17.

The units digit of the first number may be added to the second number, or the units digit of the second number may be added to the first number. In either case the result is the same.

$$13 + 7 = 20 \qquad \text{or} \qquad 17 + 3 = 20$$

Affix the units digit of the product obtained by multiplying the units digits of the given number.

$$7 \times 3 = 21$$

Affix the units digit, 1; the tens digit, 2, is added to the sum found in the first step.

$$20 + 2 = 22$$

Thus

13 × 17 = 221 Answer

26

MULTIPLYING BY ANY TWO-DIGIT NUMBER ENDING IN 1

> Rule: Multiply the first digit of the given number by the tens digit of the multiplier. The product is the first digit (or digits) of the answer. The next digit is obtained by adding the first digit of the given number to the product of the second digit of the given number and the tens digit of the multiplier. Continue this process until the tens digit of the given number is added to the product of the units digit of the given number and the tens digit of the multiplier. This will be the tens digit of the answer. The units digit of the answer will always be the units digit of the given number. Notice that only the tens digit of the multiplier is used in the various steps. Keep in mind that whenever a two-digit sum is obtained, the units digit is recorded while the tens digit is added to the preceding answer digit.

The beauty of these general short cuts is that they permit the choice of many different methods for obtaining an answer, depending on the tens digit. If we were called upon to multiply by 91, the rule above might not be as easy to use as some other rule. for example. Short Cut 60.

Multiply: 843×31.

The first digits of the answer will be three times the first digit of the given number.

$$8 \times 3 = 24$$

The next digit is the sum of the first digit of the given number, 8, and three times the second digit of the given number,

4. The three, of course, comes from the tens digit of the multiplier, 31.

$$8 + (3 \times 4) = 20$$

Record the 0 and carry the 2 to the left. Next, add 4 to three times 3 to obtain the tens digit of the answer.

$$4 + (3 \times 3) = 13$$

Record the 3 and carry the 1. So far, our answer looks like this:

2 6 1 3

Only the units digit is yet to be determined.

The units digit of the answer is the units digit of the given number, 3.

Therefore

$$843 \times 31 = 26{,}133 \quad \textit{Answer}$$

Practice Exercises for Short Cuts 16 through 26

1) 6,528 × 33 =

2) 172,645 × 11 =

3) 956 × 121 =

4) 13 × 18 =

5) 2,742 × 1,001 =

6) 24,863 × 21 =

7) 726 × 111 =

8) 2,665 × 12 =

9) 547 × 10,001 =

10) 42 × 111 =

11) 23,316 × 11 =

12) 167 × 101 =

13) 74,155 × 41 =

14) 89 × 12 =

15) 1,038 × 121 =

NUMBERS BEGINNING OR ENDING IN 5

The number 5 is perhaps the most interesting one to work with as well as one of the simplest. When we multiply a number ending in 5 by any other number, the units digit of the product is always either 0 or 5, depending on whether the given number is even or odd. In fact, the ease with which 5 is multiplied permits us to adapt short cuts to numbers having 5 in a position other than at either end. Short Cut 36, for example, can be applied even when 5 appears in the middle of a number. Thus, although this section concerns itself particularly with numbers having 5 at either end, the methods are by no means restricted to such numbers.

27

MULTIPLYING BY 15

> Rule: Add one-half the first digit to itself to obtain the first answer digit (or digits).
>
> Continue this process until the units digit is reached. Add an extra 5 if the digit to the left of the given digit is odd.
>
> If any of the sums is more than 9, record the units digit and add the tens digit to the preceding answer digit.
>
> Ignore any fractions that may occur.
>
> If the units digit of the given number is even, the units digit of the answer is 0. If the units digit of the given number is odd, the units digit of the answer will be 5.

For instance, multiply 738 by 15.

Add 7 to one-half itself, ignoring the fraction.

$7 + \frac{1}{2}(7) = 10$

These are the first two answer digits. Next, add 3 to one-half itself.

$3 + \frac{1}{2}(3) = 4$

The number to its left is 7, which is odd. Therefore add 5.

$4 + 5 = 9$

This is the third answer digit.

1 0 9

The next digit is 8, and there is an odd digit to its left.

$8 + \frac{1}{2}(8) + 5 = 8 + 4 + 5 = 17$

Record the 7 and add the 1 to the preceding 9, which becomes 10. Record the 0 and add 1 to the answer digit preceding it.

$0 + 1 = 1$

The answer digits are now

1 1 0 7

The units digit of the given number is even; therefore, the units digit of the answer is 0.

1 1, 0 7 0 Answer

28

MULTIPLYING BY 25

> Rule: Move the decimal point of the given number two places to the right and divide by 4.

What is being done here is to substitute multiplication of a two-digit number with division by a single digit, 4.

Multiply 649,372 by 25.

First, move the decimal point two places to the right.

649,372.00 becomes 64,937,200.

Next, divide the result by 4.

64,937,200 ÷ 4 = 16,234,300 Answer

29

MULTIPLYING BY 52

> Rule: Move the decimal point of the given number two places to the right and divide by 2. Add twice the original number to the result.

Suppose we wanted to find the yearly salary of someone earning $117 per week. Since there are 52 weeks in the year, the problem becomes

117 × 52

Move the decimal point of the given number two places to the right.

117.00 becomes 11,700

Divide by 2.

11,700 ÷ 2 = 5,850.

To this add twice the original number.

2 × 117 = 234;

```
  5,850
+   234
  6,084
```

Thus the yearly salary of someone earning $117 a week is $6,084.

MULTIPLYING A TWO-DIGIT NUMBER BY 95

Here is a case where a series of short-cut methods, each capable of being done mentally, are strung together into one unified short-cut method.

> Rule: Subtract 5 from the given number and affix two zeros to the result. This will be called the partial product. Next, subtract the given number from 100 and multiply the result by 5. Add this product to the partial product to obtain the final answer.

This can be best demonstrated by trying the example: 95×73.

First subtract 5 from the given number and affix two zeros to the result.

6,800

(This is the partial product.) Next, subtract the given number from 100 (Short Cut 66).

$100 - 73 = 27$

Multiply by 5 (Short Cut 11).

$27 \times 5 = 135$

(Note that the tens digit and the units digit of this product are always the tens digit and the units digit of the final answer.)

Finally, add this product to the previously determined partial product.

6,800 + 135 = 6,935 Answer

MULTIPLYING BY 125

> Rule: Move the decimal point of the given number three places to the right and divide by 8.

Dividing by 8 may not seem to be much of a short cut at first, but a simple application of the method will prove its worth.

Multiply 1,483 by 125.

The usual multiplication process would require twelve multiplication steps plus many steps in addition. The short-cut method uses one step in division. First, move the decimal point of the given number three places to the right.

1,483.000 becomes 1,483,000.

Next, divide by 8. Division by 8 can be simplified by dividing the given number by 2, then dividing the quotient by 2, and finally dividing the second quotient by 2. This third quotient is the final answer. Thus, 1,483,000 can be mentally divided by 2, giving us 741,500. Inspection shows that 741,500 can once again be easily divided by 2, giving 370,750. Each time we halve the given number, the divisor 8 must also be halved.

$8/2 = 4; \quad 4/2 = 2$

To obtain the product we are looking for, we need merely divide 370,750 by 2.

370,750 ÷ 2 = 185,375 Answer

Naturally, the same answer would have been obtained by dividing by 8 directly.

1,483,000 ÷ 8 = 185,375 Answer

32

MULTIPLYING TWO TWO-DIGIT NUMBERS WHEN BOTH END IN 5 AND ONE TENS DIGIT IS ODD WHILE THE OTHER IS EVEN

Rule:	To the product of the tens digit add one-half their sum (ignoring the fraction $\frac{1}{2}$). Affix 75 to the result.

This short cut will be tried with the numbers 75 and 45. The product of the tens digits is

$7 \times 4 = 28$

One-half the sum of the tens digits (neglecting $\frac{1}{2}$) is

$\frac{1}{2}(7 + 4) = 5$

The sum of these two numbers is 33. Affix 75.

3,375

Thus

$75 \times 45 = 3{,}375$

A word of caution about "affixing a number." This merely means the number is attached or tagged on at the beginning or end of a group of numbers; it does not mean the number is to be added to another number.

33

MULTIPLYING TWO TWO-DIGIT NUMBERS WHEN BOTH END IN 5 AND THEIR TENS DIGITS ARE EITHER BOTH ODD OR BOTH EVEN

Rule:	To the product of the tens digits add one-half their sum. Affix 25 to the result.

Although to use this short-cut method both tens digits must be either odd or even, they need not be equal.

If we are asked to multiply 65 by 45, we observe, first, that both tens digits are even and this method may be used. The product of the tens digits is $6 \times 4 = 24$. To this, one-half the sum of the tens digits is added.

$6 + 4 = 10; \quad \frac{1}{2} \times 10 = 5 \quad 24 + 5 = 29$

Affix 25.

2,925

Thus

$65 \times 45 = 2{,}925$ *Answer*

34

MULTIPLYING TWO TWO-DIGIT NUMBERS WHOSE TENS DIGITS ARE BOTH 5 AND WHOSE UNITS DIGITS ARE BOTH ODD OR BOTH EVEN

> Rule: Add one-half the sum of the units digits to 25. Affix the product of the units digits to the result. If the product is less than 10, precede it with a zero.

If we are asked to multiply 52 by 58, we see that the units digits are both even and therefore this short cut can be used. The sum of the units digits is 10, and one-half this is 5.

$25 + 5 = 30$

Multiply the units digits.

$2 \times 8 = 16$

Affix this to the 30 obtained above.

3,016 Answer

Suppose we are asked to multiply 51 by 57. This time the units digits are both odd. Again the short cut is applicable. One-half the sum of the digits is 4; with this added to 25, the result is 29. However, in this case the product of the units digits is 7, which is less than 10; therefore a zero precedes the product before it is affixed to the 29.

2,907 Answer

35

MULTIPLYING TWO TWO-DIGIT NUMBERS WHOSE TENS DIGITS ARE BOTH 5 AND ONE UNITS DIGIT IS ODD WHILE THE OTHER IS EVEN

> Rule: Add one-half the sum of the units digits to 25, ignoring the fraction $\frac{1}{2}$. Add the product of the units digits to 50 and affix the result to the sum obtained in the first step.

In this case we need not worry whether the product of the units digits is greater or less than 10 since it is eventually added to 50.

Let us find the product of 54 and 59. One units digit is odd, while the other one is even. One-half the sum of the units digits is $6\frac{1}{2}$. Ignoring the fraction and adding this to 25, we obtain

$25 + 6 = 31$

The product of the units digits is 36. This is added to 50 and the sum affixed to 31.

$50 + 36 = 86$

3,186 Answer

36

MULTIPLYING TWO TWO-DIGIT NUMBERS WHOSE TENS DIGITS ARE BOTH 5 AND WHOSE UNITS DIGITS ADD TO 10

Rule:	Affix the product of the units digits to 30. If the product is less than 10, precede it with a zero.

Multiply 53 by 57.

The units digits, 3 and 7, total 10 so that this short-cut method can be used. The first two digits of the answer are 30. The product of the units digits is

$3 \times 7 = 21$

Affix this to 30, resulting in the product:

3,021

Through an interesting property of numbers, this same short cut can be applied to numbers of more than two digits. The short cut for multiplying numbers in their teens will be used as an example.

Multiply 152 by 158.

Imagine just for this example that 52 and 58 may each be considered as if they were units digits. In actuality, only the 2 of the first number and only the 8 of the second number are the units digits. But what happens if we treat 52 and 58 as units digits? The rule for multiplying teen numbers (Short Cut 25) requires adding the units digit of one number to the other number. This provides the first two digits of the answer. The product of the units digits gives the units digit of the answer with any tens digit being added to the previously determined sum. Now our "teen"

number is 152, and the units digit of the other number is 58. Therefore their sum is

$152 + 58 = 210$

The product of the "units" digits is

$52 \times 58 = 3{,}016$

Remember, we are treating the last two digits as the units digits in this example. Therefore 1 and 6 are the final two digits in the answer, and the 30 is added to the previous sum.

$210 + 30 = 240$

We now have the result.

152 × 158 = 24,016 Answer

This is just one way in which a short cut of apparently limited application may have its usefulness enhanced. By redefining our terms and following through correctly, almost any short-cut's area of application may be broadened. Careful practice and a working knowledge of the intricacies of numbers as discussed throughout this book are all that is necessary.

Practice Exercises for Short Cuts 27 through 36

1) $713 \times 52 =$

2) $29{,}621 \times 125 =$

3) $6{,}104 \times 15 =$

4) $51 \times 59 =$

5) $53 \times 56 =$

6) $8{,}298 \times 25 =$

7) $65 \times 75 =$

8) $64 \times 95 =$

9) $3{,}871 \times 125 =$

10) $52 \times 54 =$

11) $81{,}927 \times 25 =$

12) $25 \times 65 =$

13) $144 \times 52 =$

14) $92 \times 95 =$

15) $54 \times 56 =$

NUMBERS BEGINNING OR ENDING IN 9

All numbers ending in 9 are one less than a multiple of 10. All numbers beginning with 9 are some power of 10 less than a number beginning with 10. These two characteristics of numbers beginning or ending in 9 are used to good advantage in the short cuts that follow.

For example, if we increase a number ending in 9 by one, the units digit of the new number is zero. Therefore, we have one less digit to multiply, and a simple subtraction restores the original multiplier. When a number begins with 9, it can also be increased easily to a simpler form. Thus 942 can be changed to 1,042 by adding 100 (which is 10×10). Although the new number has four digits, one of them is zero and the other is one; both are much simpler multipliers than 9.

37

MULTIPLYING BY 19

Rule:	Double the given number and affix a zero to the result. Subtract the given number.

Example: 7,390,241 × 19.
Double the given number.

$2 \times 7{,}390{,}241 = 14{,}780{,}482$

Affix a zero and subtract the given number.

$$\begin{array}{r} 147{,}804{,}820 \\ -\quad 7{,}390{,}241 \\ \hline 140{,}414{,}579 \end{array}$$ *Answer*

38

MULTIPLYING BY 99

Rule:	Move the decimal point of the given number two places to the right and subtract the given number.

Multiply 1,152 by 99.
Move the decimal point two places to the right and subtract the given number from the result.

1,152.00 becomes 115,200.

$$\begin{array}{r} 115{,}200 \\ -\quad 1{,}152 \\ \hline 114{,}048 \end{array}$$ *Answer*

39

MULTIPLYING BY 999

Rule: Move the decimal point of the given number three places to the right and subtract the given number.

Example: $1,152 \times 999$.
Move the decimal point three places to the right.

1,152.000 becomes 1,152,000.

Subtract the given number.

$$\begin{array}{r} 1,152,000 \\ -\quad 1,152 \\ \hline \end{array}$$
1,150,848 *Answer*

40

MULTIPLYING BY A NUMBER CONSISTING ONLY OF NINES

Rule: Move the decimal point of the given number to the right as many places as there are nines in the multiplier. Then subtract the given number.

Multiply 73 by 9,999,999.

There are seven nines in the multiplier; therefore the decimal point in the given number will be moved seven places to the right.

73.0000000 becomes 730,000,000.

Subtract the given number.

$$\begin{array}{r} 730,000,000 \\ -\quad 73 \\ \hline \end{array}$$
729,999,927 *Answer*

41

MULTIPLYING TWO TWO-DIGIT NUMBERS ENDING IN 9 AND WHOSE TENS DIGITS ADD TO 10

Rule:	Add 9 to the product of the tens digits and affix 81 to the result.

Note that the number 81 is merely attached to the end of the previously determined sum; 81 is not added to the sum.

For example: Multiply 39 by 79.

Since the sum of the tens digits, 3 and 7, is 10 this short-cut method can be used. The product of the tens digits is

$$3 \times 7 = 21$$

To which 9 is added.

$$21 + 9 = 30$$

Affix 81 to this sum and obtain the product.

3,081 Answer

42

MULTIPLYING BY A TWO-DIGIT MULTIPLE OF 9

Rule:	Multiply the given number by one more than the tens digit of the multiplier. Move the decimal point of the product one place to the right and subtract the original product.

Of course, the usefulness of this short cut is increased if the short cuts for multiplying by each of the digits is known.

As an example, multiply 87 by 63.

63 is a multiple of 9 (that is, $9 \times 7 = 63$). One more than the tens digit of the multiplier is 7.

Multiply the given number by 7, using Short Cut 13.

$$87 \times 7 = 609$$

Move the decimal point one place to the right.

$$609.0 \text{ becomes } 6{,}090.$$

Now subtract the original product, 609.

$$\begin{array}{r} 6{,}090. \\ -\ \ 609. \\ \hline 5{,}481. \end{array}$$

Answer

43

MULTIPLYING BY ANY TWO-DIGIT NUMBER ENDING IN 9

Rule:	Move the decimal point of the given number one place to the right and multiply by one more than the tens digit of the multiplier. Subtract the given number from the result.

Multiply 713 by 39.

Move the decimal point of the given number one place to the right.

713.0 becomes 7,130.

One more than the tens digit of the multiplier is 4. Multiply 7,130 by 4, using Short Cut 10.

7,130 × 4 = 28,520

Subtract the given number.

$$\begin{array}{r} 28{,}520 \\ -\ \ 713 \\ \hline 27{,}807 \end{array}$$ *Answer*

Practice Exercises for Short Cuts 37 through 43

1) 5,803 × 999 =

2) 437 × 39 =

3) 598,974 × 36 =

4) 1,325 × 19 =

5) 710 × 99 =

6) 423 × 99,999 =

7) 29 × 89 =

8) 53,161 × 19 =

9) 1,524 × 59 =

10) 69 × 49 =

SQUARING NUMBERS

When we speak of "squaring" a number, we mean multiplying the number by itself. To square 23 we write

23×23 (or commonly 23^2)

The process of multiplying a number by itself follows a systematic pattern which lends itself readily to short-cut methods. The simple rules explained in this section cover an amazingly wide range of numbers. Most of the short cuts included here involve two-digit numbers, but a few involve three- and four-digit numbers. With a little ingenuity, numbers of any size can be squared easily, using the short cuts that follow as the basis for many others. But there is a law of diminishing returns in using larger numbers; then, instead of saving time and labor, the short cut becomes merely a "stunt."

The squares of numbers play an important role in many other short-cut methods. By means of the few very basic methods in this section, the range of the multiplication problems which may be performed by short-cut methods becomes practically unlimited.

44

SQUARING ANY NUMBER ENDING IN 1

Rule:	First, square the number to the left of the units digit. Then double the number to the left of the units digit. Affix the units digit of this result to the square found in the first step. If the result is more than 9, add the part to the left of the units digit to the square found in the first step. The units digit of the answer is always 1.

Consider the following example: Square 251.

The number to the left of the 1 is 25. Using Short Cut 45, we find the square of 25 is 625. Next, twice 25 is 50. Affix the zero in 50 to 625 and add the 5 to 625.

$625 + 5 = 630$

To which are affixed the 0 and the units digit (which is always 1).

63,001 Answer

45

SQUARING ANY TWO-DIGIT NUMBER ENDING IN 5

Squaring a two-digit number ending in 5 is a special case of the short cut for multiplying any two-digit numbers ending in 5. In this particular case, the tens digits are equal.

Rule: Multiply one more than the tens digit by the original tens digit and affix 25 to the result.

For example, we shall square 45. First, add 1 to the tens digit.

$4 + 1 = 5$

Next, multiply by the original tens digit.

$4 \times 5 = 20$

To this affix 25.

2,025

and we have the answer.

$45 \times 45 = 2{,}025$

Remember to merely attach the 25 to the product: do not add it to the product.

From this rule we see that the square of any two-digit number ending in 5 always has 5 as its units digit and 2 as its tens digit.

46

SQUARING ANY NUMBER ENDING IN 5

Rule: Multiply the complete number to the left of the 5 by one more than itself and affix 25 to the result.

To demonstrate, we shall find the square of 195. The complete number to the left of the 5 is 19. Raising this one number higher gives us 20.

$20 \times 19 = 380$

To which 25 is affixed.

38,025 Answer

47

SQUARING ANY THREE-DIGIT NUMBER ENDING IN 25

The square of 25 is 625. Oddly enough, these are the last three digits in the square of any three-digit number ending in 25. Since squaring a three-digit number results in at most six digits, the problem here is merely to find the first three digits of the answer.

> Rule: The first two digits (that is, the hundred-thousands digit and the ten-thousands digit) are found by squaring the hundreds digit of the given number and adding to the result one-half the hundreds digit of the given number (ignoring the fraction $\frac{1}{2}$ if it occurs). If the result is a one-digit number, then there is no hundred-thousands digit in the answer and the result is the ten-thousands digit of the answer. The thousands digit of the answer is 5 if the hundreds digit of the given number is odd and 0 if the hundreds digit of the given number is even. Affix 625 to obtain the final answer.

Two illustrative examples will be used to demonstrate the ease with which this short cut may be used.

Example No. 1: Square 225

First, square the hundreds digit of the given number, to obtain

4

To this add one-half the hundreds digit of the given number.

$4 + 1 = 5$

Since the answer is a one-digit number, 5 is the ten-

thousands digit of the answer. The thousands digit of the answer will be 0, since the hundreds digit of the given number, 2, is even. To this we affix 625 to obtain the final answer.

50,625

Example No. 2: Square 725

First, square the hundreds digit of the given number.

$7 \times 7 = 49$

To this add one-half of 7 (ignoring the $\frac{1}{2}$).

$49 + 3 = 52$

The first digit, 5, is the hundred-thousands digit of the answer; the second digit, 2, is the ten-thousands digit of the answer. The thousands digit of the answer is 5, since the hundreds digit of the given number is odd. Affix 625 to obtain the final answer.

$725 \times 725 = 525,625$ Answer

48

SQUARING ANY FOUR-DIGIT NUMBER ENDING IN 25

The explanation for this short-cut method will be made a little clearer if the digits of the given number are assigned letters. The thousands, hundreds, tens, and units digits will be designated A, B, C, and D respectively.

> Rule: Square digit A of the given number to obtain the tentative ten-millions and millions answer digits (if there is only one digit, it is the millions answer digit).
>
> Double the product of A and B to obtain the hundred-thousands digit of the answer. If the result in this and subsequent steps is a two-digit number, the units digit is the answer digit; the tens digit should be added to the preceding answer digit.
>
> To 5 times A add the square of B. The sum is the ten-thousands digit of the answer.
>
> Multiply B by 5. This product is the thousands digit of the answer.
>
> Affix 625 to the answer digits found above to obtain the final answer.

This short-cut method will be tried in two illustrative examples.

Example No. 1: Square 2,825. Line the numbers up with their respective letters.

A B C D

2 8 2 5

Square A.

$2 \times 2 = 4$

The square has only one digit: therefore this is our tentative millions digit. Multiply A by B, and double the result.

$2 \times 8 \times 2 = 32$

The 2 is the hundred-thousands digit. The 3 is added to the 4 obtained in the first step.

$3 + 4 = 7$

Thus 7 is now the millions digit.

Add 5 times A to the square of B.

$(5 \times 2) + (8 \times 8) = 74$

The 4 is the ten-thousands digit. Add the 7 to the preceding answer digit.

$2 + 7 = 9$

The 9 becomes the new hundred-thousands digit. Stop and recapitulate what we have:

Millions digit

Hundred-thousands digit

Ten-thousands digit

7 9 4

Multiply B by 5.

$8 \times 5 = 40$

The 0 is the thousands digit. Add the 4 to the preceding answer digit.

$4 + 4 = 8$

The final ten-thousands digit is 8. The previous digits, 7 and 9, now are also final. Affix 625 to obtain the answer.

The square of 2,825 is therefore

7,980,625 Answer

Example No. 2: Square 7,325. Again the digits will be lined up with their respective letters.

A B C D

7 3 2 5

Square A.

$7 \times 7 = 49$

Since the answer is a two-digit number, the first digit, 4, is the ten-millions digit and the 9 is the millions digit.

Multiply A and B and double the result.

$7 \times 3 \times 2 = 42$

The 2 is the hundred-thousands digit. The 4 is carried to the left and added to the previous millions digit, 9.

$4 + 9 = 13$

The 3 becomes our new millions digit, and the 1 is added to the ten-millions digit.

$4 + 1 = 5$

This is the new ten-millions digit. At this point let us write the digits we have determined:

Ten-millions digit

Millions digit

Hundred-thousands digit

5 3 2

Add five times A to the square of B.

$(5 \times 7) + (3 \times 3) = 44$

One 4 (the units digit) is the ten-thousands digit; the other

4 (the tens digit) is added to the hundred-thousands digit, 2.

$4 + 2 = 6$

This is the new hundred-thousands digit.

Multiply B by 5.

$3 \times 5 = 15$

The 5 is the final thousands digit. The 1 is added to the ten-thousands digit.

$4 + 1 = 5$

The 5 is the final ten-thousands digit. Since there is no digit to carry, the previous digits become final.

Affix 625 to obtain the final answer.

53,655,625 Answer

49

SQUARING ANY TWO-DIGIT NUMBER WHOSE TENS DIGIT IS 5

Rule:	Add the units digit to 25 and affix the square of the units digit to the result. If the square of the units digit is a one-digit number, precede it with a 0.

Find the square of 53, using this method.
First, add the units digit, 3, to 25.

$25 + 3 = 28$

Next, affix the square of the units digit to the result.

$3 \times 3 = 9$

Since the answer is a one-digit number, place a zero in front of the 9 before affixing it to the 28.

2,809 Answer

As another example, find the square of 57.
Again, the units digit is added to 25.

$25 + 7 = 32$

Next, square the units digit

$7 \times 7 = 49$

and affix to the previous result.

3,249 Ansuer

This time the square of the units digit was a two-digit number, and therefore it was not necessary to precede it with a zero.

50

SQUARING ANY NUMBER ENDING IN 9

Rule:	Multiply the number to the left of the 9 by two more than itself. Affix an 8 to the result and subtract twice the number to the left of the 9. Affix a 1 to the result.

This short cut can be applied to any number, no matter how many digits it has, so long as the units digit is 9. Of course, as the number gets larger, multiplying the two numbers of the first step will become cumbersome unless a short cut can be used. However, most two- and three-digit numbers ending in 9 can be readily squared, once a facility with the other short-cut methods has been achieved.

Example: Square 149.

The number to the left of the 9 is 14. Two more than this is 16. Multiply 14 by 16. (Short Cut 53 can be used here).

$14 \times 16 = 225 - 1 = 224$

Affix 8.

2,248

Subtract twice the number to the left of the 9.

$2{,}248 - (2 \times 14) = 2{,}248 - 28 = 2{,}220$

Affix a 1 to obtain the final answer.

22,201 Answer

51

SQUARING ANY NUMBER CONSISTING ONLY OF NINES

Rule: Write one less 9 than there is in the given number. Follow this with an 8. Then write as many zeros as the nines previously written. Finally, write a 1 as the units digit.

This method is purely mechanical and requires nothing more than being able to count the nines in the given number.

Square 9,999.

There are four nines; therefore write three nines as the first part of the answer and follow with an 8. Next, write three zeros and end with a 1.

99,980,001 Answer

52

SQUARING ANY TWO-DIGIT NUMBER

Rule: Square the tens digit and affix the square of the units digit to the result. If the square of the units digit is less than 10, precede it with a zero before affixing it to the square of the tens digit. Double the product of the digits of the given number. Add the units digit of this product to the tens digit of the previous number and add the tens digit of the product to the hundreds digit of the previous number.

Example: Square 63.

Square the tens digit.

$6 \times 6 = 36$

Square the units digit.

$3 \times 3 = 9$

Precede this with a zero, since it is less than 10, and affix it to the result above.

3,609

Double the product of the digits.

$(6 \times 3) \times 2 = 36$

Add the units digit, 6, to the tens digit of 3,609 and add the tens digit, 3, to the hundreds digit of this number.

$$\begin{array}{r} 3,609 \\ +\underline{\quad 36} \\ \textit{3,969} \end{array}$$ *Answer*

Practice Exercises for Short Cuts 44 through 52

Square each of the following numbers.

1) 59

2) 73

3) 425

4) 621

5) 99,999

6) 65

7) 175

8) 47

9) 119

10) 1,925

11) 52

12) 81

13) 535

14) 1,425

15) 34

MULTIPLYING TWO NUMBERS THAT DIFFER ONLY SLIGHTLY

Once you have mastered the art of squaring a number, you have in your possession a powerful tool applicable to more general problems in multiplication. The product of two numbers that differ from each other only slightly is nearly equal to the square of the number midway between the given numbers. (This is particularly true of differences of up to about 20). The definite mathematical relationship which exists between the product and the square will be employed in the short cuts that follow. Since the squaring process is so important in all these methods, a review of the short cuts used in squaring numbers is recommended at this time.

53

MULTIPLYING TWO NUMBERS WHOSE DIFFERENCE IS 2

> Rule: Square the number between the two given numbers and subtract 1.

This short cut is simple to apply when the square can be found easily. For example, suppose we have to multiply 24 by 26. The number between the two given numbers is 25. The square of 25 is quickly found to be 625 (Short Cut 45).

$$24 \times 26 = 625 - 1 = 624 \quad \textit{Answer}$$

Multiply 67 by 69.

Here the number between the given numbers is 68. Its square is

4,624.

Therefore

$$67 \times 69 = 4,623 \quad \textit{Answer}$$

54

MULTIPLYING TWO NUMBERS WHOSE DIFFERENCE IS 3

> Rule: Square one more than the smaller number and add one less than the smaller number to the result.

Example: Multiply 34 by 37.

One more than the smaller number, 34, is 35. Square 35 (Short Cut 45).

$$35 \times 35 = 1{,}225$$

Add one less than 34 to the result.

$$1{,}225 + 33 = 1{,}258$$

Therefore

$$34 \times 37 = 1{,}258 \quad \textit{Answer}$$

55

MULTIPLYING TWO NUMBERS WHOSE DIFFERENCE IS 4

> Rule: Square the number midway between the two given numbers and subtract 4.

Naturally the success of this short cut will depend on how easy it is to square a number. For example, if the number midway between the two given numbers ends in 5, Short Cut 45 can be applied very simply and the answer obtained as quickly as it takes to write the digits. There are numerous other short cuts which can also be used in conjunction with this one. Take the following example:

Multiply 69 by 73.

The number midway between the two numbers is 71. Short Cut 26 shows how to square any two-digit number ending in 1. Applying this short cut, we find

$$71 \times 71 = 5{,}041 \qquad 5{,}041 - 4 = 5{,}037$$

Therefore

$$69 \times 73 = 5{,}037 \quad \textit{Answer}$$

56

MULTIPLYING TWO NUMBERS WHOSE DIFFERENCE IS 6

> Rule: Square the number midway between the two given numbers and subtract 9.

Example: 48×54.

The number midway between the two given numbers is 51. Short Cut 26 can be used to square 51, since it ends in 1, or Short Cut 49 can be used, since its tens digit is 5. In either case, the square of 51 is found to be 2,601.

Next, subtract 9.

2,601 - 9 = 2,592 Answer

57

MULTIPLYING TWO NUMBERS WHOSE DIFFERENCE IS ANY SMALL EVEN NUMBER

> Rule: Square the number midway between the two given numbers; then square one-half the difference between the two given numbers and subtract the result from the square obtained in the first step.

This rule may be applied to numbers of any size. The limiting factor, however, is the calculation of the square. Take, for example, the following problem:

Multiply 109 by 121.

The number midway between the given numbers is 115. It can be squared easily by using Short Cut 46.

$115 \times 115 = 13{,}225$

Square one-half the difference between the two given numbers.

$121 - 109 = 12$

$\frac{1}{2}(12) = 6$

$6 \times 6 = 36$

Subtract the result from the square obtained in the first step.

$13{,}225 - 36 = 13{,}189$

Thus

109 × 121 = 13,189 Answer

Practice Exercises for Short Cuts 53 through 57

1) $31 \times 43 =$

2) $113 \times 121 =$

3) $21 \times 23 =$

4) $88 \times 92 =$

5) $79 \times 81 =$

6) $98 \times 101 =$

7) $62 \times 66 =$

8) $322 \times 326 =$

9) $102 \times 108 =$

10) $51 \times 54 =$

MORE SHORT CUTS IN MULTIPLICATION

The short cuts that follow do not fall within any special category. They do not involve a common number or a common factor. As a matter of fact, some are generalizations of specific short cuts discussed elsewhere in this book. Because they are so easy to use, they are often applied to problems in place of the more specific methods. Remember that although there may be a number of short cuts which can be used to solve a particular problem, the important thing is to be able to choose the most effective method possible.

58

MULTIPLYING TWO TWO-DIGIT NUMBERS WHOSE TENS DIGITS ARE THE SAME

> Rule: Add the units digit of one number to the other number. Multiply the result by the tens digit and affix a zero to the product. Add the product of the units digits to the result.

Since multiplication by the digits is the key step in this short cut, a review of Short Cuts 8 to 15 is recommended.

Example: Multiply 72 by 79.

Add the units digit of one number to the other number. It does not matter whether 72 is added to 9 or 79 is added to 1; the result is the same.

$72 + 9 = 81$ or $79 + 2 = 81$

Multiply by the tens digit, 7.

$81 \times 7 = 567$ (Short Cut 13)

Affix a zero

5,670

and add the product of the units digits.

$2 \times 9 = 18$

$5{,}670 + 18 = 5{,}688$

Therefore

72 × 79 = 5,688 Answer

59

MULTIPLYING TWO TWO-DIGIT NUMBERS WHOSE UNITS DIGITS ARE THE SAME

> Rule: Multiply the tens digits. Next, add the tens digits and multiply the sum by the units digit. Add any tens or hundreds digit of the result of this step to the product obtained in the first step and affix the units digit obtained in this step to the result. Square the units digit and add any tens digit to the preceding number. The units digit of the answer will be the units digit of the square.

As an example, we shall multiply 76 by 46.

The product of the tens digits is

$7 \times 4 = 28$

Add the tens digits and multiply the sum by the units digit.

$7 + 4 = 11$ $11 \times 6 = 66$

Add the tens digit, 6, to 28 and affix the units digit, also 6 in this case, to the result.

$28 + 6 = 34$

346

Square the units digit of the given number.

$6 \times 6 = 36$

Add the tens digit, 3, to the answer digits already obtained

$346 + 3 = 349$

and affix the units digit of the square.

3,496

Therefore

76 × 46 = 3,496 Answer

60

MULTIPLYING TWO NUMBERS THAT ARE JUST A LITTLE LESS THAN 100

> Rule: Subtract each number from 100. Subtract one of the differences from the other given number. The result is the first two answer digits. Affix the product of the differences, which then becomes the last two digits of the answer. If the product of the differences is a single digit, precede it with a zero before affixing it to the first two answer digits. If the product of the differences has a hundreds digit, add it to the preceding answer digit.

Two examples will be used to demonstrate this short-cut method.

First, we shall multiply 86 by 78.

The difference between each number and 100 is obtained.

100 - 86 = 14

100 - 78 = 22

Subtract either difference from the other given number; in each case the result is the same.

78 - 14 = 64 or 86 - 22 = 64

The first two digits of the answer are therefore 64.

Multiply the differences.

22 × 14 = 308 (Short Cut 19 can be used here)

Affix 08 to the previously determined answer digits and add the 3 to the hundreds digit of the new number.

6 (4 + 3) 0 8

6,708

Therefore

78 × 86 = 6,708 Answer

Next, we shall try this short cut on the example:

97 × 98

First, find the difference between each number and 100.

100 - 97 = 3   100 - 98 = 2

Subtract either difference from the other given number. Notice again that the result is the same, no matter which difference is used.

98 - 3 = 95   or   97 - 2 = 95

Next, multiply the differences and affix to 95.

3 × 2 = 6

In this case the product is a one-digit number, which means we must precede the 6 with a zero before affixing it to the 95.

9,506 Answer

61

MULTIPLYING TWO NUMBERS THAT ARE JUST A LITTLE LESS THAN 1,000

Rule:	Subtract each of the given numbers from 1,000. Subtract one of the differences from the other given number. The result is the first three answer digits. Affix the product of the differences to obtain the final answer.

The question one might ask is, "What constitutes a little less than 1,000?" Actually, the answer depends on the particular problem. The ease and rapidity with which the differences can be multiplied will often be the determining factor.

With this short-cut method it is possible to multiply 998 by 996 as quickly as the answer can be written.

The difference between each of the given numbers and 1,000 is 2 and 4 respectively. Subtracting 4 from 998, or 2 from 996 (the result is the same), produces the first three answer digits. Using the A B C notation, the answer digits are

A	B	C	D	E	F
9	9	4			

Only the letters A to F are necessary, since the product of two numbers just a little less than 1,000 is a six-digit number.

The last three digits are obtained by multiplying 4 by 2. Since the product has less than three digits, the D and E digits will be zero.

A	B	C	D	E	F
9	9	4	0	0	8

Therefore

998 × 996 = 994,008 Answer

The preceding answer was obtained with the simplest mental arithmetic. The given numbers were only two and four less than 1,000. But suppose the two given numbers were almost a hundred less than 1,000; can this short cut still be used? The answer is that this method is applicable no matter how much less than 1,000 the given numbers are. However, it becomes a "short cut" only when the necessary steps can be handled quickly.

The following example will demonstrate this point.

966 × 964

The operations are:

1,000 - 966 = 34

1,000 - 964 = 36

964 - 34 = 930 or 966 - 36 = 930

34 × 36 = 1,224

Short Cut 58 made the last step simple. The first three digits are:

A B C D E F

9 3 0

Since the product of the differences is a four-digit number, record only 224 in D, E, and F respectively. Carry the 1 left and add it to the C digit.

The final product is therefore

931,224 Answer

62

MULTIPLYING TWO NUMBERS THAT ARE JUST A LITTLE MORE THAN 100

> Rule: Subtract 100 from each of the given numbers. Add one of the differences to the other given number. The three digits of the sum are the first three digits of the answer. Multiply the differences and affix the product to the first part of the answer to obtain the final answer.

When multiplying two numbers that are just a little more than 100, the product will be somewhere between 10,000 and 15,000 or perhaps a little larger. But it will be a five-digit number. We can therefore denote the answer digits by

A B C D E

The following example will be used to show how this short cut works.

117 × 109

Subtract 100 from each of the given numbers.

117 - 100 = 17   109 - 100 = 9

Add one of the differences to the other given number. It does not matter whether we add 9 to 117 or 17 to 109; the result is the same.

117 + 9 = 126 or 109 + 17 = 126

The digits, 1, 2, 6, are the A, B, C digits.

A B C D E

1 2 6

Multiply the differences to obtain the D and E digits.

17 × 9 = 153

Since the product is a three-digit number, the last two digits are recorded as D and E. The 1 is carried left and added to the previously determined C digit.

A	B	C	D	E
1	2	7	5	3

Therefore

$$117 \times 109 = 12,753 \quad \textit{Answer}$$

If the product of the differences was a one-digit number, it would be recorded as the units digit of the answer (the E portion) and a zero would be placed in the D position.

63

MULTIPLYING TWO NUMBERS THAT ARE JUST A LITTLE MORE THAN 1,000

> Rule: Subtract 1,000 from each of the given numbers. Add one of the differences to the other given number. The four digits of the sum are the first four digits of the answer. Affix the product of the differences to obtain the final answer.

The product of two numbers just a little more than 1,000 is a seven-digit number. The answer can therefore be shown using the letter notation,

A B C D E F G

Multiply 1,078 by 1,015.

Subtract 1,000 from each of the given numbers.

$$1,078 - 1,000 = 78 \qquad 1,015 - 1,000 = 15$$

Add one of the differences to the other given number.

$$1,078 + 15 = 1,093 \qquad \text{or} \qquad 1,015 + 78 = 1,093$$

These are the first four answer digits.

A	B	C	D	E	F	G
1	0	9	3			

Multiply the differences (Short Cut 27 can be used here).

$$78 \times 15 = 1,170$$

The last three digits, 1, 7, 0, are the last three digits of the answer. The thousands digit, 1, is added to the preceding answer digit.

A	B	C	D	E	F	G
1	0	9	4	1	7	0

Therefore

1,078 × 1,015 = 1,094,170 Answer

Note that if the product of the differences was less than a three-digit number, zeros would occupy the E or E and F positions, depending on whether the product was a two- or one-digit number.

64

MULTIPLYING TWO NUMBERS WHOSE UNITS DIGITS ADD TO 10 AND THE OTHER CORRESPONDING DIGITS ARE EQUAL

Rule: Multiply the number to the left of the units digit by one more than itself. Affix the product of the units digits to the result.

This short cut can be used with numbers of any size providing the corresponding digits to the left of the units digits are the same and the units digits add to 10.

For example, multiply 324 by 326.

The units digits add to 10, and the other digits are the same in each number. Multiply 32 by one more than 32.

32 × 33 = 1,056 (Short Cut 19 or 58)

Multiply the units digits.

4 × 6 = 24

Affix this product to the result of the first step. If the product of the units digit was a one-digit number, a zero would precede it before it was affixed to the product obtained in the first step.

105,624

Therefore

324 × 326 = 105,624 Answer

Practice Exercises for Short Cuts 58 through 64

1) 92 × 97 =

2) 73 × 75 =

3) 975 × 997 =

4) 43 × 73 =

5) 81 × 83 =

6) 12 × 32 =

7) 987 × 991 =

8) 1,042 × 1,011 =

9) 106 × 121 =

10) 89 × 93 =

11) 103 × 108 =

12) 356 × 354 =

13) 1,213 × 1,217 =

14) 998 × 998 =

15) 95 × 91 =

Chapter 3

SHORT CUTS IN SUBTRACTION

Like addition, subtraction does not lend itself to true short cuts. Unlike addition, there are no sequences of subtraction for which short cuts may be used. For this reason, only a few simple problems in subtraction can be aided through the use of short-cut methods. Two of the best are given here.

65

SUBTRACTING A NUMBER FROM THE NEXT HIGHEST POWER OF 10

> Rule: Starting from the first given digit, record the difference between the digit and 9. Continue this process through the tens digit of the given number. The units digit of the answer is obtained by subtracting the given units digit from 10.

The nearest power of 10 which is greater than a given number is a 1 followed by as many zeros as there are digits in the given number. The nearest power of 10 to a number in the "teens" is therefore 100; the nearest power of 10 to a number in the tens of thousands is 100,000, and so on.

The following table will help you with the differences required in this short cut. It is recommended that you memorize them until they become second nature.

Given digit:	0	1	2	3	4	5	6	7	8	9
Difference with 9:	9	8	7	6	5	4	3	2	1	0

Subtract 762 from 1,000.

One thousand, of course, is the next highest power of 10 to 762.

Subtract each of the digits from 9, starting from the first. Stop at the units digit. The first two answer digits are therefore

23

Subtract the given units digit, 2, from 10 to obtain the units answer digit.

238 Answer

66

SUBTRACTING A NUMBER FROM ANY POWER OF 10

Short Cut 65 can be extended to include subtraction from any power of 10 by affixing zeros in front of the given number and proceeding as before. What we are doing is increasing the number of digits (even though the new digits are all zeros) so that we are, in effect, subtracting from the next highest power of 10.

> Rule: Affix as many zeros in front of the given number as are necessary to give the given number one less digit than the power of 10. Proceed as in Short Cut 65.

Thus, if our given number has three digits and we want to subtract it from 1,000,000, we must affix three zeros in front of the given number. It will then have six digits, while the power of 10 from which the number is being subtracted has seven digits.

As an example, let us subtract 78,215 from 10,000,000. There are eight digits in the power of 10 and only five digits in the given number; therefore four zeros (5 - 1) must be attached to the given number.

0 0 0 0 7 8 2 1 5

Now record the difference with 9 for each digit under the corresponding digits of the given number except the units digit, where the difference with 10 is used.

0 0 0 0 7 8 2 1 5 Given number
9 9 9, 9 2 1, 7 8 5 Answer

Practice Exercises for Short Cuts 65 and 66

1) 1,000 − 42 =

2) 1,000,000 − 23,680 =

3) 100 − 83 =

4) 10,000 − 9,014 =

5) 1,000,000 − 103,855 =

Chapter 4

SHORT CUTS IN DIVISION

Division can be considered a short cut for repeated subtraction.

For example, when we say 8 goes into 72 nine times, we mean 8 can be subtracted from 72 nine times. We can obtain this answer by subtracting 8 from 72 over and over until nothing remains, then counting the number of times we subtracted. The same result is obtained by dividing 72 by 8.

Like multiplication, division lends itself to many interesting and useful short cuts. Many of them, in fact, involve multiplication. Consequently short cuts 7 to 64 will find frequent use in the methods that follow.

DETERMINING A NUMBER'S DIVISORS

Many times it is useful to know whether or not a number can divide another number evenly. One method of making such a determination is to actually carry out the division. Unfortunately, when the given number is very large this process can be quite laborious. The short-cut methods that follow eliminate the necessity of such work. Rules are included for all single digits and 11 and 13. These rules also form the foundation for determining many other divisors. For example, if a number is found to be evenly divisible by 2 and 7, it is evenly divisible by 14 (that is, 2 × 7); if a number is evenly divisible by 3 and 5, it is evenly divisible by 15 (that is, 3 × 5). The rule for divisors is therefore clear.

> Rule: If a given number is evenly divisible by each of several digits, which themselves do not have common divisors, then the given number is evenly divisible by the product of these digits.

Short Cuts 67 to 76 are among the most useful ones you will learn.

67

DIVISIBILITY BY 2

> Rule: If the units digit of the number is even, the number is evenly divisible by 2.

This rule is clear and straightforward, so that no example is necessary.

DIVISIBILITY BY 3

> Rule: Add the digits of the given number. If the sum contains more than one digit, continue adding subsequent sums until a one-digit answer is obtained. If the answer is 3, 6, or 9, the given number is evenly divisible by 3.

Determine whether or not 9,781,052,214 is evenly divisible by 3.

Add the digits.

$$9 + 7 + 8 + 1 + 0 + 5 + 2 + 2 + 1 + 4 = 39$$

Since the sum has two digits, add again.

$$3 + 9 = 12$$

Again the sum has two digits. Add once more.

$$1 + 2 = 3$$

This time the result is a single digit. Since the sum is 3, the given number is evenly divisible by 3.

DIVISIBILITY BY 4

> Rule: If the number formed by the last two digits is evenly divisible by 4, the entire number is evenly divisible by 4.

Determine whether or not 763,052 is evenly divisible by 4.

The last two digits are 52. Divide by 4.

$$52 \div 4 = 13$$

Therefore the entire number is evenly divisible by 4.

Is 614 evenly divisible by 4?

The last two digits, 14, are not evenly divisible by 4.

$$14 \div 4 = 3\tfrac{1}{2}$$

Therefore 614 is not evenly divisible by 4.

70

DIVISIBILITY BY 5

> Rule: If the number ends in 0 or 5 it is evenly divisible by 5.

No example is necessary, since the meaning of this rule is very clear.

71

DIVISIBILITY BY 6

> Rule: If the sum of the digits of an even number is 3, 6, or 9, the entire number is divisible by 6.

As in Short Cut 68, the digits of the sums should continue to be added until a one-digit answer is obtained.

Determine if 866,125 is evenly divisible by 6.

Since this an odd number, it is not evenly divisible by 6.

Is 1,274 divisible by 6?

The digits ultimately add to 5, so that the number is not evenly divisible by 6. If 1 were added to any of the digits so that the given number became 2,274 or 1,374 or 1,284 or 1,275, the digits of these numbers would ultimately add to 6. The first three are evenly divisible by 6 but the fourth, 1,275, is an odd number and therefore is not.

72

DIVISIBILITY BY 7

> Rule: Mark off groups of three digits, starting from the right. Add alternate groups. Find the difference between the two sums thus obtained. If the difference is zero or a multiple of 7, the given number is evenly divisible by 7.

This method works only if the given number has four or more digits.

Determine whether or not 58,556,344 is evenly divisible by 7.

Divide the number into groups of three digits, starting from the right.

58 556 344

Add the first and third groups. Find the difference between this sum and the second group.

$344 + 58 = 402$

$$\begin{array}{r} 556 \\ -402 \\ \hline 154 \end{array}$$

To determine whether or not 154 is a multiple of 7, divide it by any apparent divisor. In this case it is obvious that 2 is a divisor.

$154 \div 2 = 77$

Since 77 is a multiple of 7, the given number 58,556,344 is evenly divisible by 7.

It does not follow that as the given number becomes larger, the difference will become larger. Consider the following.

Is 1,636,871,900,629 divisible by 7?

The alternate groups are

1 871 629 and 636 900

The two sums are

$1 + 871 + 629 = 1{,}501$ and $636 + 900 = 1{,}536$

Their difference is 35, which is a multiple of 7. Consequently the given number is evenly divisible by 7.

73

DIVISIBILITY BY 8

> Rule: If the number formed by the last three digits of the given number is evenly divisible by 8, the entire number is evenly divisible by 8.

Of course, the first requirement is that the given number is even. A further test that can be made by inspection is to see whether or not the given number is evenly divisible by 4 (Short Cut 69). If it is not, then it cannot be evenly divided by 8.

For instance, which of the following numbers is evenly divisible by 8?

8,241
537,104
9,468,188
12,726

The first number is ruled out because it is not even.

The fourth number is ruled out because it is not divisible by 4.

The second and third numbers are possibilities. It is therefore necessary to apply the rule for divisibility by 8.

$104 \div 8 = 13$

$188 \div 8 = 23\frac{1}{2}$

Thus, 537,104 is evenly divisible by 8, but 9,468,188 is not.

74

DIVISIBILITY BY 9

> Rule: If the sum of the digits of the given number is equal to 9 or a multiple of 9, the given number is evenly divisible by 9.

The digits of the given number should be added and the digits of the resulting sums re-added until a number is obtained which is clearly recognized as a multiple of 9 or not.

The sum of the digits of

34,762,195

is 37, which is not divisible by 9. But the digits of

24,762,195 or 34,762,194 or 34,662,195

each total 36, and therefore each of the numbers is evenly divisible by 9.

75

DIVISIBILITY BY 11

> Rule: Add alternate digits to obtain two sums. If the difference between the two sums is equal to zero or a multiple of 11, the given number is evenly divisible by 11.

To determine if

4,372,258

is divisible by 11, add alternate digits.

$$4 + 7 + 2 + 8 = 21$$

$$3 + 2 + 5 = 10$$

Find the difference between the two sums.

$$21 - 10 = 11$$

Therefore the given number is evenly divisible by 11.

The number 3,289 is evenly divisible by 11 because the difference of the sums of alternate digits is equal to zero.

$$\begin{array}{r} 3 \\ +\ 8 \\ \hline 11 \end{array} \qquad \begin{array}{r} 9 \\ +\ 2 \\ \hline 11 \end{array}$$

$$11 - 11 = 0$$

Similarly 93,819 is divisible by 11 because

$$\begin{array}{r} 9 \\ 8 \\ 9 \\ \hline 26 \end{array} \qquad \begin{array}{r} 3 \\ +1 \\ \hline 4 \end{array}$$

$$26 - 4 = 22$$

and 22 is a multiple of 11.

76

DIVISIBILITY BY 13

> Rule: Mark off groups of three digits, starting from the right. Add alternate groups and find the difference between the two sums thus obtained. If the difference is zero or a multiple of 13, the given number is evenly divisible by 13.

It is interesting to note the similarity between this short cut and Short Cut 72. In either case, when the difference is zero, the given number is evenly divisible by both 7 and 13.

Determine whether or not 82,108 is evenly divisible by 13.

Mark off alternate groups of three digits, starting from the right.

82 108

In this case we merely find the difference between the two groups.

$$108 - 82 = 26$$

Since 26 is a multiple of 13, the given number is evenly divisible by 13.

The following example shows how this short cut works with larger numbers.

Is 1,116,248,953,781 evenly divisible by 13?

Add alternate groups of three digits, starting from the right.

$$1 + 248 + 781 = 1{,}030$$

$$116 + 953 = 1{,}069$$

The difference between these two sums is 39. Since it is a multiple of 13, the given number is evenly divisible by 13.

Practice Exercises for Short Cuts 67 through 76

Determine whether the following numbers are evenly divisible by 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13.

1) 690

2) 1,309

3) 216

4) 1,001

5) 1,079

NUMBERS ENDING IN 5

When dividing by a number ending in 5, the number 2, oddly enough, plays a very useful role. This is because 5 can be written in this form

$\frac{1}{2} \times 10$

Now if we divide by this new number, which you must remember is only 5 in a slightly different dress, we are in effect dividing by $\frac{1}{2}$ and by 10. Division by $\frac{1}{2}$ is the same as multiplication by 2. Division by 10 simply involves moving the decimal point of the given number one place to the left.

77

DIVIDING BY 5

> **Rule:** Double the number and move the decimal point one place to the left.

By using Short Cut 8 the answer can be written directly, except for the position of the decimal point. If the number is not evenly divisible by 5, there will always be digits to the right of the decimal point in the final answer.

Divide 8,327 by 5.

From Short Cut 70 we know that the given number is not evenly divisible by 5.

Double 8,327. The result may be written directly.

16,654.

Move the decimal point one place to the left.

16,654. ← becomes 1,665.4

Thus

$$8{,}327 \div 5 = 1{,}665.4$$

The decimal remainder, .4, is the same as 4/10 or 2/5. If 8,327 had been divided the usual way, the remainder would have been 2.

8,327 ÷ 5 = 1,665-2/5 Quotient

The decimal equivalent of 2/5 is 0.4.

78

DIVIDING BY 15

> **Rule:** Move the decimal point of the given number one place to the left. Double the result and divide by 3.

Essentially this short cut involves three separate steps, and the beginner would do well to approach it that way. But once mastered, the entire process can be condensed into one step. The three separate steps will be outlined in the first explanation given below. After that the shorter one-step method will be examined.

Divide 8,371 by 15.

Move the decimal point one place to the left.

8,371. ← becomes 837.1

Double the number.

$$\begin{array}{r} 837.1 \\ +\ 837.1 \\ \hline 1{,}674.2 \end{array}$$

Divide by 3.

$$1{,}674.2 \div 3 = 558.06\text{-}2/3$$

The fact that the sum of the digits of 1,674.2 is equal to 20 indicates that the number is not evenly divisible by 3.

The same result may be obtained in one step as follows. Start with the doubling process, keeping the result in your mind instead of writing it down; then mentally divide by 3. Do this digit by digit, starting from the left.

Using the same number as in the example above, follow each operation carefully. Double the 8 but do not record the result. Instead, divide by 3 and record the quotient.

8 + 8 = 16;   16 ÷ 3 = 5 + 1 remainder

Record the 5 as the first answer digit. Next, double the second digit in the given number.

3 + 3 = 6

Precede it with the remainder, 1, obtained in the previous step.

16

Do not record this number, but divide it by 3 and record the result.

16 ÷ 3 = 5 + 1 remainder

Record the 5 again, so that the first two answer digits are

55

Double the third digit of the given number.

7 + 7 = 14

The remainder, 1, obtained in the previous step precedes the 4, but since a 1 already precedes the 4 the two ones must be added to obtain 24.

Divide the resulting number by 3.

24 ÷ 3 = 8

This is the third answer digit.

558

Double the next number and divide by 3.

1 + 1 = 2   2 ÷ 3 = 2/3

This time the result is less than 3. Therefore record a zero in the answer.

5,580

Affix a zero to the 2 and divide again by 3.

20 ÷ 3 = 6-2/3

Record this as the final part of the answer.

55,806-2/3

The decimal point is located by counting one less digit to the left of the decimal point than appeared in the given number. The given number in this case had four digits to the left of the decimal point; therefore the answer will have three digits to the left of the decimal point.

558.06-2/3 Answer

Actually the answer could have been extended by continuously obtaining a remainder of 2. Since in this case all digits to the right of the decimal point in the given number are zeros, this would amount to repeatedly dividing 20 by 3 to obtain 6 and a remainder of 2. If the given number was evenly divisible by 15, the last division by 3 would be exact without any remainder.

79

DIVIDING BY 25

> Rule: Move the decimal point two places to the left and multiply by 4.

Multiplication by 4 is discussed in Short Cut 10. Division by 25 then becomes merely a problem of simple addition and multiplication. But watch the decimal point carefully.

Divide 1,387.76 by 25.

Move the decimal point two places to the left.

1,387.76 becomes 13.8776

Multiply by 4.

$13.8776 \times 4 = 55.5104$

Thus

1,387.76 ÷ 25 = 55.5104 Answer

80

DIVIDING BY 125

> Rule: Move the decimal point three places to the left and multiply by 8.

Multiplication by 8 is discussed in Short Cut 14.

Divide 8,639.705 by 125.

Move the decimal point three places to the left.

8,639.705 becomes 8.639705

Multiply by 8.

$8.639705 \times 8 = 69.117640$

Thus

8,639.705 ÷ 125 = 69.117640 Answer

Practice Exercises for Short Cuts 77 through 80

1) 87 ÷ 25 =

2) 1,427,006 ÷ 5 =

3) 192.38 ÷ 125 =

4) 58 ÷ 15 =

5) 239 ÷ 25 =

MORE SHORT CUTS IN DIVISION

The first of the two short cuts that follow is interesting and unique because it requires only the use of addition. It is based on an unusual property of 9. If any digit from 1 to 8 is divided by 9, the result is a decimal point followed by the numerator continuing indefinitely. For instance,

1/9 = .111111 . . .

(The three dots indicate that the ones continue on without end.)

Reducing a number to its various factors helps us to learn many things about the number. By using factors, we are able to find the highest common multiple as well as the lowest common denominator of a group of numbers. We can also cancel common factors and thereby simplify otherwise complex problems in division. The latter property is the basis of the second short cut in this section.

81

DIVIDING BY 9

> Rule: The first digit of the answer is equal to the first digit of the given number. The second answer digit is equal to the sum of the first and second digits of the given number. The third answer digit is equal to the sum of the first, second, and third digits of the given number. Continue this process until all digits of the given number are added except the units digits. This sum is the units digit of the answer. The sum of all digits of the given number is the tentative remainder. Repeat this sum three times. If any of the sums are two-digit numbers, record only the units digit and add the tens digit to the preceding answer digit. The first digit to the right of the units digit of the answer is the remainder. If the remainder is 9, cross it out and add 1 to the units digit of the answer. The rest of the digits to the right of the remainder may be discarded.

The letters A, B, C, and so forth will be used to designate the digits of the given number; the letters a, b, c, and so forth will designate the answer digits. The first digit of the answer is equal to A and should be placed over the B digit of the given number in this fashion:

a	Answer digits
A B C D . . .	Given number

A + B = b, which goes over C and is the second digit of the answer. A + B + C = c, which goes over D. The units digit of the answer is the sum of all digits in the given number except the units digit. The sum of all digits

in the given number is the tentative remainder digit. Repeat the process of adding all digits in the given number and record each result until there are three digits after the units digit of the answer. The digits appearing over the digits of the given number constitute the quotient. The first digit to the right of the units digit in the quotient is the remainder. If the remainder is 9, cross it out and add 1 to the units digit of the quotient. The balance of the digits to the right of the remainder digit may be discarded. Remember, if any of the sums is a two-digit number, carry and add any tens digits to the preceding answer digits.

As an example we shall divide 639,125 by 9.

We shall use the A, B, C notation to make it easier to follow the explanation.

A	B	C	D	E	F	
6	3	9	1	2	5	Given number

The A digit, 6, goes over B as the first digit in the answer.

	6					Answer digits
A	B	C	D	E	F	
6	3	9	1	2	5	Given number

Next, we add 6 + 3 and put the answer over C; 6 + 3 + 9, and put the answer over D (carrying left the tens digit of the sum); 6 + 3 + 9 + 1 goes over E (remembering to carry the tens digit again). The answer thus far looks like this (taking into account all tens digits that were carried, of course):

	7	0	9	9		Answer digits
A	B	C	D	E	F	
6	3	9	1	2	5	Given number

The sum 6 + 3 + 9 + 1 + 2 will give the units digit; the sum of all digits 6 + 3 + 9 + 1 + 2 + 5 will give the tentative remainder. Repeat this sum until three digits are given after the units digit of the quotient. The answer will now look like this:

7 1 0 1 3 8 8 6	Answer digits
A B C D E F	
6 3 9 1 2 5	Given number

The quotient is 71,013; the remainder, the first digit after the units digit of the quotient, is 8. The complete answer is therefore

71,013-8/9 Answer

82

DIVIDING BY FACTORS

> Rule: Determine the factors of the given number and the divisor. Cross out the factors that are common to both. Divide the resulting given number by the resulting divisor.

A number that divides another number evenly is called a factor of the larger number. Thus 2, 3, 4, 6 are all factors of 12 since each of them divides 12 evenly.

Short Cuts 67 to 76 will be used to determine the factors in the example given below, but sometimes simple trial and error is just as easy.

For example: Divide 435,240 by 14,040.

Start from the highest factor. Using Short Cut 76, we see that both the given number and the divisor are evenly divisible by 13. The digits of the given number add to 18 and the digits of the divisor add to 9. Therefore both numbers also have a factor of 9. Likewise they both have a factor of 8 and 5. We can also show that 7 is not a factor of either of the numbers.

At this point we can stop determining the factors for a while. It is apparent that the digits 2, 3, 4, and 6 are factors of both numbers since each of these digits have common factors with the numbers 8 and 9 previously determined.

Now divide by the factors 13, 9, 8, and 5.

$$435{,}240 \div 13 = 33{,}480 \qquad 14{,}040 \div 13 = 1{,}080$$
$$33{,}480 \div 9 = 3{,}720 \qquad 1{,}080 \div 9 = 120$$
$$3{,}720 \div 8 = 465 \qquad 120 \div 8 = 15$$
$$465 \div 5 = 93 \qquad 15 \div 5 = 3$$

Now we can easily divide 93 by 3.

$93 \div 3 = 31$

Thus

$$435{,}240 \div 14{,}040 = 31 \quad \textit{Answer}$$

Practice Exercises for Short Cuts 81 and 82

1) $387 \div 9 =$

2) $4{,}100.4 \div 9 =$

3) $1{,}218 \div 210 =$

Chapter 5

SHORT CUTS WITH FRACTIONS, MIXED NUMBERS, AND PERCENTAGE

Fractions and mixed numbers are not as difficult to work with as many people believe. Since they follow the rules used with whole numbers, no new operation needs to be learned. However, special care is necessary to avoid overlooking intermediate steps and seeing that numbers are put in their proper place. A number of the short cuts are merely extensions of short cuts used elsewhere in this book for whole numbers and decimals.

Percentages represent fractional parts of 100. When we say "50%" we mean "fifty-hundredths" or 50/100, which is merely $\frac{1}{2}$. Since percentages are fractions, a "percent" of a given number is less than the given number, provided the "percentage" is less than 100. Percentages greater than 100 are really mixed numbers, with the digits to the left of the tens digit being the whole number and the tens and units digits making up the fractional part of 100. Thus 215% is actually the mixed number 2-15/100.

83

ADDING TWO FRACTIONS WHOSE NUMERATORS ARE BOTH 1

> Rule: Write the sum of the denominators over the product of the denominators.

The numerator is the number over the fraction line and, of course, the denominator is the number under the fraction line.

To find the sum of

$$\frac{1}{7} + \frac{1}{12}$$

merely add the denominators,

$$7 + 12 = 19$$

and put the sum over the product of the denominators.

$$7 \times 12 = 84$$

Thus

1/7 + 1/12 = 19/84 Answer

It is always a good idea to reduce any fractional answer to its simplest form by canceling any common factors. In the example above, 19 and 84 have no common factors; therefore $\frac{19}{84}$ is the simplest form of the fraction.

84

FINDING THE DIFFERENCE BETWEEN TWO FRACTIONS WHOSE NUMERATORS ARE BOTH 1

> Rule: Write the difference between the denominators over the product of the denominators.

Watch how a couple of simple short cuts can save a lot of work and time in solving an otherwise difficult problem.

Subtract $\frac{1}{87}$ from $\frac{1}{83}$.

The difference between the denominators is 4.

The product of the denominators is

$$83 \times 87 = 7{,}221 \text{ (Short Cut 55)}$$

The answer is

$$\frac{4}{7{,}221}$$

A fractional answer should always be tested to see if it can be reduced to a simpler form by canceling common factors in the numerator and denominator. In the case above, it is obvious that the only factors of 4 are 2 and 4. But since 7,221 is an odd number, it does not contain either a 2 or 4, so that $\frac{4}{7{,}221}$ is the simplest form of the fraction.

85

MULTIPLYING BY 3/4

Rule:	Divide the given number by 4 and subtract the result from the given number.

Multiply 8,924 by 3/4.

Divide 8,924 by 4 and subtract the result from the original number.

4) 8,924	Given number
−2,231	Quotient
6,693	Answer

Therefore

8,924 × 3/4 = 6,693 Answer

86

MULTIPLYING BY $2\frac{1}{2}$

Rule:	Starting from the first digit of the given number, double each digit and add one-half the given digit. Ignore any fractions. Add any tens digits to the previous answer digit. Add 5 if the preceding digit of the given number is odd. Affix $\frac{1}{2}$ to the answer if the given number is odd.

Use this short cut when the given number is an integer or a decimal number. Mixed numbers and fractions are difficult to handle with this method.

This short cut will be used in the example:

$517,849 \times 2\frac{1}{2}$

Double the 5 and add one-half of itself to obtain the first answer digit.

$5 + 5 = 10; \quad 10 + \frac{1}{2}(5) = 10 + 2 = 12$

(The fraction $\frac{1}{2}$ is ignored.)

Record 12 as the first two answer digits.

Double 1. One-half of 1 is $\frac{1}{2}$, which is ignored. But since the preceding given digit is odd, add 5 to the result of this step.

$1 + 1 = 2; \quad 2 + \frac{1}{2}(1) = 2 + 0 = 2 \quad 2 + 5 = 7$

Record the 7 as the next answer digit.

127

The next given digit is 7.

$7 + 7 = 14; \quad 14 + \frac{1}{2}(7) = 14 + 3 = 17$

The previous given digit, 1, is odd; therefore add 5.

$17 + 5 = 22$

Record the units digit, 2, as an answer digit. Carry the tens digit, also 2, and add it to the previous answer digit. The answer thus far is

1292

Continue this process with the balance of the given digits.

$8 + 8 = 16; \quad 16 + 4 = 20; \quad 20 + 5 = 25$

Record 5; add the 2 to the previous answer digit. The answer at this point is

12945

The 4 is next.

$4 + 4 = 8; \quad 8 + 2 = 10$

Record 0; add 1 to previous answer digit.

129460

Next is 9.

$9 + 9 = 18; \quad 18 + \frac{1}{2}(9) = 18 + 4 = 22$

Record 2; add 2 to the previous answer digit.

1,294,622

This is the last digit of the given number. Since the given number is odd, affix $\frac{1}{2}$ to the answer.

1,294,622-1/2 Answer

87

MULTIPLYING BY $7\frac{1}{2}$

> Rule: Move the decimal point one place to the right, divide by 4, and subtract the quotient from the number first divided.

Since difficulties are sometimes encountered when moving the decimal point of a fraction or mixed number, limit the use of this short cut to integers and decimal numbers.

Multiply 63 by $7\frac{1}{2}$.

Move the decimal point one place to the right.

63.0 becomes 630.

Divide by 4 and subtract the quotient from 630.

$630 \div 4 = 157\frac{1}{2}$

$$\begin{array}{r} 630 \\ -\,157\frac{1}{2} \\ \hline 472\frac{1}{2} \end{array}$$

Thus

63 × 7-1/2 = 472-1/2 Answer

88

MULTIPLYING BY $12\frac{1}{2}$

> Rule: Move the decimal point two places to the right and divide by 8.

Use this short cut on whole numbers and decimal numbers only. Odd whole numbers will always end in $\frac{1}{2}$; i.e., the remainder after dividing by 8 will be 4.

As an example: Multiply 631 by $12\frac{1}{2}$.

Move the decimal point two places to the right.

631.00 becomes 63,100.

Divide by 8.

$63,100 \div 8 = 7,887\frac{1}{2}$

Therefore

631 × 12-1/2 = 7,887-1/2 Answer

89

MULTIPLYING TWO MIXED NUMBERS WHOSE WHOLE NUMBERS ARE THE SAME AND WHOSE FRACTIONS ADD TO 1

> Rule: Multiply the whole number by one more than itself. Affix the product of the fractions.

Multiply 9-4/9 by 9-5/9.

Multiply the whole number by one more than itself.

$9 \times 10 = 90$

Affix the product of the fractions.

$4/9 \times 5/9 = 20/81$

9-4/9 × 9-5/9 = 90-20/81 Answer

90

MULTIPLYING TWO MIXED NUMBERS WHEN THE DIFFERENCE BETWEEN THE WHOLE NUMBERS IS 1 AND THE SUM OF THE FRACTIONS IS 1

> Rule: Increase the larger of the whole numbers by one and multiply by the other whole number. Square the fraction of the larger number and subtract the square from 1. Affix the result to the product obtained in the first step.

For the sum of two fractions to be equal to 1, their denominators must be the same (at least when both fractions are written in their simplest form) and the sum of the numerators must equal the denominator.

Multiply 15-3/4 by 14-1/4.

Increase the larger whole number by one and multiply by the smaller whole number.

$15 + 1 = 16; \quad 16 \times 14 = 224$ (Short Cut 53 or 25)

Square the fraction of the larger number

$3/4 \times 3/4 = 9/16$

Subtract the result from 1.

$1 - 9/16 = 7/16$

Affix the result to the product obtained in the first step.

224-7/16 Answer

91

SQUARING A NUMBER ENDING IN $\frac{1}{2}$

> Rule: Multiply the whole-number part of the given number by one more than itself and affix 1/4.

Naturally the ease with which the whole number is multiplied by one more than itself will determine when this short cut is used. Often other short-cut methods can be applied to reduce the job of multiplying the whole numbers.

Square $87\frac{1}{2}$

Multiply the whole number, 87, by one more than itself (Short Cut 58 or 62 can be employed here).

$87 \times 88 = 7{,}656$

Affix 1/4.

7,656-1/4 Answer

92

DIVIDING BY $2\frac{1}{2}$

> Rule: Move the decimal point one place to the left and multiply by 4.

If enough zeros are added to the right of the decimal point of the given number, there will never be a fractional remainder left after dividing by 4.

Divide 87.6 by $2\frac{1}{2}$.

Move the decimal point one place to the left.

87.6 becomes 8.76

Multiply by 4 (Short Cut 10).

$8.76 \times 4 = 35.04$

Therefore

87.6 ÷ 2-1/2 = 35.04 Answer

93

DIVIDING BY $12\frac{1}{2}$

> Rule: Move the decimal point two places to the left and multiply by 8.

Use this short cut on integers and decimal numbers. Fractions and mixed numbers sometimes present problems in moving their decimal point.

The use of Short Cut 14 will naturally facilitate multiplication by 8.

Example: 57,813 ÷ $12\frac{1}{2}$.

Move the decimal point two places to the left.

57,813. becomes 578.13

Multiply by 8.

$578.13 \times 8 = 4{,}625.04$

Therefore

57,813 ÷ 12-1/2 = 4,625.04 Answer

The answer is a decimal number. This will always be true (although, of course, the decimal portion can be zero at times).

94

DIVIDING BY 33-1/3

> Rule: Multiply the given number by 3 and move the decimal point two places to the left.

Divide 83 by 33-1/3.

Multiply by 3 (use Short Cut 9 if necessary).

$83 \times 3 = 249$

Move the decimal point two places to the left.

249.0   becomes   2.49

Thus

83 ÷ 33-1/3 = 2.49 Answer

95

FINDING 16-2/3% OF A NUMBER

> Rule: Divide the given number by 6.

If the given number is odd, the answer will contain a fraction. See Short Cut 71 for the rule for divisibility by 6.

Find 16-2/3% of 132.00

Short Cut 71 shows that this number is evenly divisible by 6.

$132 \div 6 = 22$

Therefore

16-2/3% of 132 = 22.00 Answer

96

FINDING 33-1/3% OF A NUMBER

Rule: Divide the given number by 3.

Short Cut 68 will indicate whether or not the given number is evenly divisible by 3. If it is not, the answer will have a fraction.

What is 33-1/3% of 12?

Divide by 3.

$$12 \div 3 = 4$$

Therefore 4 is 33-1/3% of 12.

97

FINDING $37\frac{1}{2}\%$ OF A NUMBER

Rule: Multiply the given number by 3 and divide the result by 8.

Either operation may be performed first; the answer will be the same.

For example: Find $37\frac{1}{2}\%$ of 7,216.

Multiply by 3 (Short Cut 9).

$$7{,}216 \times 3 = 21{,}648$$

Divide by 8.

$$21{,}648 \div 8 = 2{,}706$$

Therefore

37-1/2% of 7,216 = 2,706 Answer

If the order of operation were changed, 2,706 would again be obtained. This time, divide by 8 first.

$$7{,}216 \div 8 = 902$$

(This can be obtained almost by inspection.)

Next, multiply by 3.

$$902 \times 3 = 2{,}706$$

It is evident that in this particular example the second order of operation is the one that produces the answer with the least mental effort.

98

FINDING $62\frac{1}{2}\%$ OF A NUMBER

Rule: Move the decimal point one place to the right and divide by 16.

Dividing by 16 is not as formidable as it may seem at first. As a matter of fact, it can usually be done easier in two steps. Dividing the given number by 2 and then dividing the result by 8 is one way of simplifying this division. Another way is to divide the given number by 4 and then divide the result by 4 again.

Both methods will be demonstrated in the example below; but first, division by 16 will be shown.

Find $62\frac{1}{2}\%$ of 512.

Move the decimal point one place to the right.

512.0 becomes 5,120.

Divide by 16.

5,120 ÷ 16 = 320 Answer

The same number, 5,120, will be divided by 2 and the result divided by 8.

5,120 ÷ 2 = 2,560

2,560 ÷ 8 = 320 Answer

Next, the number will be divided by 4 and the result again divided by 4.

5,120 ÷ 4 = 1,280

1,280 ÷ 4 = 320 Answer

In a given problem, at least three choices are open, depending on what the given number is and how easily it can be divided by 2, 4, or 8.

99

FINDING 66-2/3% OF A NUMBER

Rule: Divide by 3 and subtract the result from the given number.

Find 66-2/3% of 75.

Divide by 3 and subtract the result from the given number.

75 ÷ 3 = 25

75 - 25 = 50

Therefore 50 is 66-2/3% of 75.

100

FINDING $87\frac{1}{2}\%$ OF A NUMBER

Rule: Divide the given number by 8 and subtract the result from the given number.

By carrying on the division under the given number, the subtraction can easily follow.

Find $87\frac{1}{2}\%$ of 37.52.

Divide the given number by 8.

$$\begin{array}{rrl} 8\,) & 37.52 & \text{Given number} \\ - & 4.69 & \text{Quotient} \\ \hline & 32.83 & \text{Difference} \end{array}$$

Thus

87-1/2% of 37.52 = 32.83 Answer

Practice Exercises for Short Cuts 83 through 100

1) $57 \times 2\frac{1}{2} =$
2) $14\text{-}1/7 \times 14\text{-}6/7 =$
3) $8\frac{1}{2} \times 8\frac{1}{2} =$
4) $7{,}018 \times 3/4 =$
5) $27\text{-}1/3 \times 26\text{-}2/3 =$
6) $1/23 + 1/27 =$
7) $13 \times 12\frac{1}{2} =$
8) $37\frac{1}{2}\%$ of $1{,}250 =$
9) $1/11 - 1/17 =$
10) $87\frac{1}{2}\%$ of $43 =$
11) $382 \times 7\frac{1}{2} =$
12) $63\frac{1}{2} \times 62\frac{1}{2} =$
13) $2{,}408 \div 12\frac{1}{2} =$
14) 16-2/3% of 12 =
15) $1{,}659 \div 12\frac{1}{2} =$
16) 33-1/3% of 57 =
17) 66-2/3% of 9,072 =
18) $42 \div 2\frac{1}{2} =$
19) $62\frac{1}{2}\%$ of 888 =
20) $104 \div 33\text{-}1/3 =$

Chapter 6

POSTSCRIPT

One hundred is a nice number, and it is always pleasant to conclude things in a nice fashion. But the 100 short cuts you have just learned do not constitute the conclusion of this book. The previous 100 short cuts were only an introduction—something to whet your appetite while at the same time serving a useful purpose. The one hundred and first short cut is therefore not a summary but rather a "foreword." Here, then, is Short Cut 101, with the hope that very shortly you will be the one to make it 102, 103, 104 . . .

101

DO-IT-YOURSELF SHORT CUTS

The first one hundred short cuts in this book are merely the prelude to the fascinating art of "do-it-yourself short cuts." There is an old proverb that says. "Necessity is the mother of invention," and in mathematics we find some of the best examples of this.

The salesman who has to figure the price of an article if it is sold at a discount of 33-1/3% off list soon discovers a quick way of performing that particular calculation in his head. The mechanic, the engineer, the housewife—each of them meets with dozens of problems in mathematics each day. Many of the problems are repetitious or else involve specific numbers or groups of numbers. Before long a method is evolved for reducing paper work, and so another short cut is born.

To be able to invent short cuts, one must first have a good, solid familiarity with numbers and the various forms in which they appear — fractions, mixed numbers, decimals, and integers. One must recognize that a number's value may be changed by the position of a decimal point while its appearance as a group of digits can remain the same.

For example, if we know a short cut for multiplying by 25 we can easily extend it to include 2.5, 0.25, 250, or even 25,000,000! The important thing to remember is the position of the decimal point after the answer is obtained. It is also very important to be able to convert from a decimal to a fraction and from a fraction to a decimal.

For instance

$\frac{1}{2}$ 0.5 4/8

have exactly the same value. Thus if we are given the following problem:

$2.5 \times 2\text{-}7/14$

we should be able to see it as

2.5×2.5

or

$25/10 \times 25/10$

or

$2\frac{1}{2} \times 2\frac{1}{2}$

Some of these are forms for which short-cut methods have already been discussed.

Suppose we are given the problem of finding a short cut for squaring mixed numbers ending in 1/4. In decimal form, 1/4 is equal to 0.25. Short Cut 47 can be used for squaring numbers ending in 25. With a little ingenuity it can also be applied to numbers ending in 1/4.

Take the example: Square 9-1/4.

This can be written as 9.25 or 92.5/10 or even 925/100. In this last form the example becomes

$$925/100 \times 925/100 \quad \text{or} \quad \frac{925 \times 925}{10{,}000}$$

Short Cut 47 tells us to square the hundreds digit (remember, this was the whole-number portion of our original given number). Then add one-half the hundreds digit. Follow this with a 5 or a zero, depending on whether the hundreds digit is odd or even. The last three digits of the answer will be 625. Thus the last four digits are always either 5625 or 0625.

Let us go back to our original number, 9-1/4. From the procedure above we see that only the whole number is involved in the short cut. It is squared.

$9 \times 9 = 81$

One-half itself is added to the result (ignoring any fraction).

$\frac{1}{2}$ of $9 = 4$ (ignore $\frac{1}{2}$)

$81 + 4 = 85$

The next step is to affix 5625, since 9 is odd. But 5625 actually represents 0.5625, and 0.5625 is 9/16 in fractional form. Thus we need affix only the fraction, 9/16, to obtain the answer. If the whole number were even, we would affix 0.0625, which is equal to 1/16. From what has just been explained, see if you can formulate a short cut for squaring any mixed number ending in 1/4.

By simply interchanging fractions and decimals and manipulating the decimal point, an entirely new area of short-cut methods is opened up.

Anyone can be a "do-it-yourself short cutter." All he needs is a good working knowledge of numbers and an ability to visualize a problem in a variety of different forms. Given the job of performing routine arithmetic operations and enough laziness to want to reduce the amount of work as much as possible, there is no limit to the number of short-cut methods one can invent.