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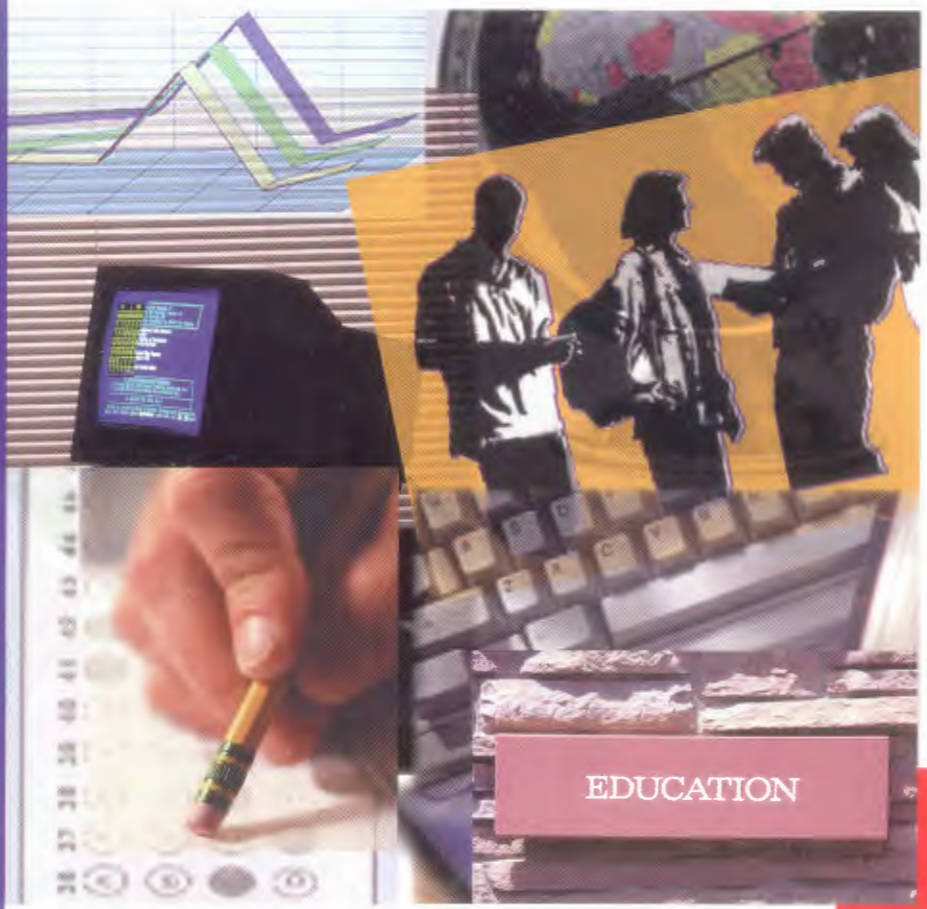
GRE®

Practicing to take the Mathematics Test



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- Two Actual Full-length GRE Mathematics Tests
- Plus Additional Practice Questions
- Strategies and Tips from the Test Maker

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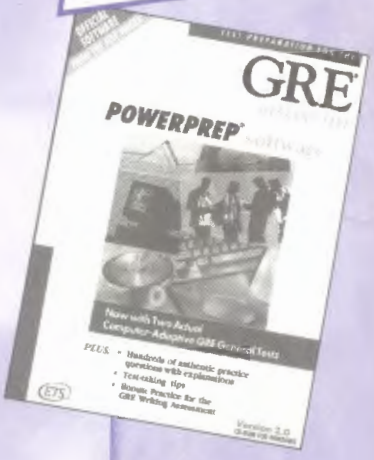

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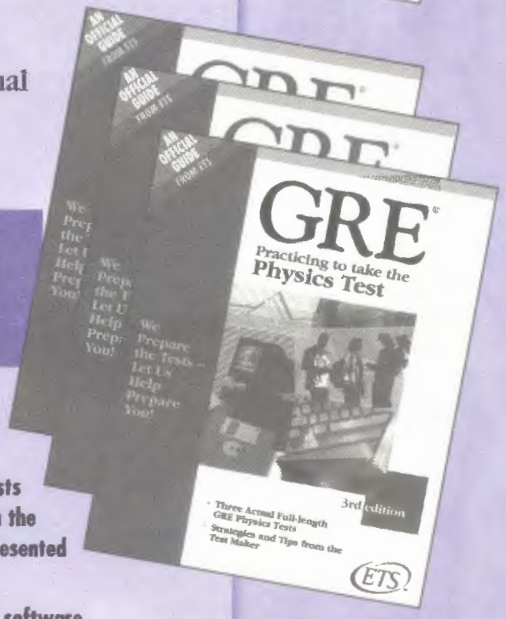

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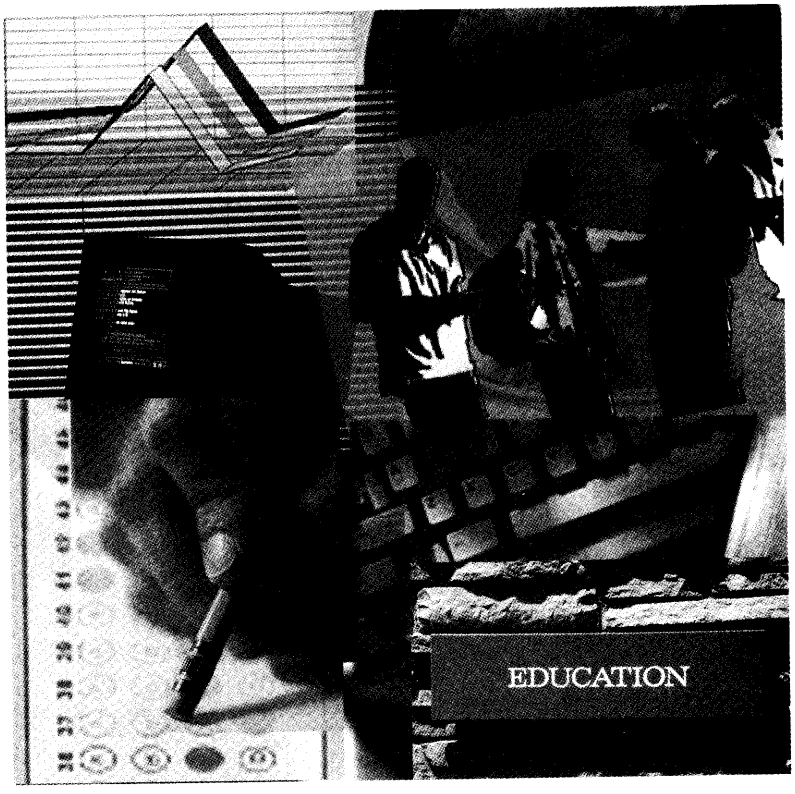

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BACKGROUND FOR THE TEST

HOW TO USE THIS PUBLICATION

This practice book has been published on behalf of the Graduate Record Examinations Board to help potential graduate students prepare to take the GRE Mathematics Test. The book contains two actual GRE Mathematics Tests and includes information about the purpose of the GRE Subject Tests, along with a section of sample questions, a detailed description of the content specifications for the GRE Mathematics Test, and a description of the procedures for developing the test. All test questions that were scored have been included in the practice test.

The sample questions included in this practice book are organized by content category and represent the types of questions included in the test. The purpose of these questions is to provide some indication of the range of topics covered in the test as well as to provide some additional questions for practice purposes. These questions do not represent either the length of the actual test or the proportion of actual test questions within each of the content categories.

Before you take a full-length test, you may want to answer the sample questions. A suggested time limit is provided to give you a rough idea of how much time you would have to complete the sample questions if you were answering them on an actual timed test. After answering the sample questions, evaluate your performance within content categories to determine whether you would benefit by reviewing certain courses.

This practice book contains two complete test books, including the general instructions printed on the back covers of the tests. When you take the test at the test center, you will be given time to read these instructions. They show you how to mark your answer sheet properly and give you advice about guessing.

Try to take these practice tests under conditions that simulate those in an actual test administration. Use the answer sheets provided on pages 131 to 136 and mark your answers with a No. 2 (soft-lead) pencil as you will do at the test center. Give yourself 2 hours and 50 minutes in a quiet place and work through the tests without interruption, focusing your attention on the questions with the same concentration you would use in taking a test to earn a score. Since you will not be permitted to use them at the test center, do not use keyboards, dictionaries or other books, compasses, pamphlets, protractors, highlighter pens, rulers, slide rules, calculators (including watch calculators), stereos or radios with headphones, watch alarms (including those with flashing lights or alarm sounds), or paper of any kind.

After you complete each practice test, use the work sheets and conversion tables on pages 25-26 and 82-83 to score your tests. The work sheets also show the estimated percent of a sample of GRE Mathematics Test examinees who answered each question correctly. This will enable you to compare your performance on the questions with theirs. Evaluating your performance on the actual test questions, as well as the sample questions, should help you determine whether you would benefit by further reviewing certain courses before taking the test at the test center.

We believe that if you use this practice book as we have suggested, you will be able to approach the testing experience with increased confidence.

ADDITIONAL INFORMATION

If you have any questions about any of the information in this book, please send an email message to the GRE Program at gre-info@ets.org, or write to:

Graduate Record Examinations
Educational Testing Service
P.O. Box 6000
Princeton, NJ 08541-6000

PURPOSE OF THE GRE SUBJECT TESTS

The GRE Subject Tests are designed to help graduate school admission committees and fellowship sponsors assess the qualifications of applicants in their subject fields. The tests also provide students with an assessment of their own qualifications.

Scores on the tests are intended to indicate knowledge of the subject matter emphasized in many undergraduate programs as preparation for graduate study. Since past achievement is usually a good indicator of future performance, the scores are helpful in predicting success in graduate study. Because the tests are standardized, the test scores permit comparison of students from different institutions with different undergraduate programs.

The Graduate Record Examinations Board recommends that scores on the Subject Tests be considered in conjunction with other relevant information about applicants. Because numerous factors influence success in graduate school, reliance on a single measure to predict success is not advisable. Other indicators of competence typically include undergraduate transcripts showing courses taken and grades earned, letters of recommendation, and GRE General Test scores.

For information about the appropriate use of GRE scores, visit the GRE Web site at www.gre.org or write to the GRE Program, Educational Testing Service, Mailstop 06-L, Princeton, NJ 08541.

DEVELOPMENT OF THE GRE MATHEMATICS TEST

Each new edition of the GRE Mathematics Test is developed by a committee of examiners composed of professors in the subject who are on undergraduate and graduate faculties in different types of institutions and in different regions of the United States. In selecting members for the committee of examiners, the GRE Program seeks the advice of the Mathematical Association of America.

The content and scope of each test are specified and reviewed periodically by the committee of examiners. Test questions are written by the committee and by other faculty who are also subject-matter specialists and by subject-matter specialists at ETS. All questions proposed for the test are reviewed by the committee and revised as necessary. The accepted questions are assembled into a test in accordance with the content specifications developed by the committee to ensure adequate coverage of the various aspects of the field and at the same time to prevent overemphasis on any single topic. The entire test is then reviewed and approved by the committee.

Subject-matter and measurement specialists on the ETS staff assist the committee, providing information and advice about methods of test construction and helping to prepare the questions and assemble the test. In addition, individual test questions and the test as a whole are reviewed to eliminate language, symbols, or content considered to be potentially offensive, inappropriate for major subgroups of the test-taking population, or serving to perpetuate any negative attitude that may be conveyed to these subgroups. The test as a whole is also reviewed to make sure that the test questions, where applicable, include an appropriate balance of people in different groups and different roles.

Because of the diversity of undergraduate curricula in mathematics, it is not possible for a single test to cover all the material an examinee may have studied. The examiners, therefore, select questions that test the basic knowledge and understanding most important for successful graduate study in the particular field. The committee keeps the test up-to-date by regularly developing new editions and revising existing editions. In this way, the test content changes steadily but gradually, much like most curricula. In addition, curriculum surveys are conducted periodically to ensure that the content of a test reflects what is currently being taught in the undergraduate curriculum.

After a new edition of a GRE Mathematics Test is first administered, examinees' responses to each test question are analyzed in a variety of ways to determine whether each question functioned as expected. These analyses may reveal that a question is ambiguous, requires knowledge beyond the scope of the test, or is inappropriate for the total group or a particular subgroup of examinees taking the test. Answers to such questions are not used in computing scores.

CONTENT OF THE GRE MATHEMATICS TEST

The test consists of 66 multiple-choice questions, drawn from courses commonly offered at the undergraduate level. Approximately 50 percent of the questions involve calculus and its applications — subject matter that can be assumed to be common to the backgrounds of almost all mathematics majors. About 25 percent of the questions in the test are in elementary algebra, linear algebra, abstract algebra, and number theory. The remaining questions deal with other areas of mathematics currently studied by undergraduates in many institutions.

The following content descriptions may assist students in preparing for the test. The percentages given are estimates; actual percentages will vary slightly from one edition of the test to another.

Calculus (50%)

The usual material of two years of calculus, including trigonometry, coordinate geometry, introductory differential equations, and applications based on the calculus.

Algebra (25%)

Elementary algebra: the kind of algebra taught in precalculus courses.

Linear algebra: matrices, linear transformations, characteristic polynomials, eigenvectors, and other standard material

Abstract algebra and number theory: topics from the elementary theory of groups, rings, and fields; elementary topics from number theory

Additional Topics (25%)

Introductory real variable theory: the elementary topology of $\mathbb{R}$ and $\mathbb{R}^n$; properties of continuous functions: differentiability and integrability

Other topics: general topology, complex variables, probability and statistics, set theory and logic, combinatorics and discrete mathematics, algorithms and numerical analysis

There may also be questions that ask the test taker to match "real-life" situations to appropriate mathematical models.

The above descriptions of topics covered in the test should not be considered exhaustive; it is necessary to understand many other related concepts. Knowledge of the material included in the descriptions is a necessary, but not sufficient, condition for answering the questions on the test. Prospective test takers should be aware that questions requiring no more than a good precalculus background may be quite challenging; some of these questions turn out to be among the most difficult questions on the test. In general, the questions are intended not only to test recall of information, but also to assess the test taker's understanding of fundamental concepts and the ability to apply these concepts in various situations.

SCRATCHWORK

SAMPLE QUESTIONS

The sample questions included in this practice book represent the types of questions included in the test. The purpose of the sample questions is to provide some indication of the range of topics covered in the test as well as to provide some additional questions for practice purposes. These questions do not represent either the length of the actual test or the proportion of actual test questions within each of the content categories. A time limit of 140 minutes is suggested to give you a rough idea of how much time you would have to complete the sample questions if you were answering them on an actual timed test. Correct answers to the sample questions are listed on page 22.

Directions: Each of the questions or incomplete statements is followed by five suggested answers or completions. Select the one that is best in each case.

CALCULUS

1. If S is a plane in Euclidean 3-space containing $(0, 0, 0)$, $(2, 0, 0)$, and $(0, 0, 1)$, then S is the

   (A) xy-plane
   (B) xz-plane
   (C) yz-plane
   (D) plane $y - z = 0$
   (E) plane $x + 2y - 2z = 0$

2. $\int_0^1 \int_0^x xy \, dy \, dx =$

   (A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{3}$ (D) 1 (E) 3

3. For $x \geq 0$, $\frac{d}{dx}(x^e \cdot e^x) =$

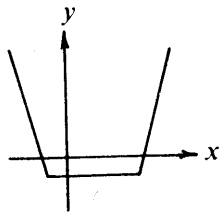
   (A) $x^e \cdot e^x + x^{e-1} \cdot e^{x+1}$ (B) $x^e \cdot e^x + x^{e+1} \cdot e^{x-1}$ (C) $x^e \cdot e^x$ (D) $x^{e-1} \cdot e^{x+1}$ (E) $x^{e+1} \cdot e^{x-1}$

4. All functions f defined on the xy-plane such that

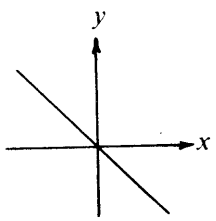
   $$\frac{\partial f}{\partial x} = 2x + y \quad \text{and} \quad \frac{\partial f}{\partial y} = x + 2y$$

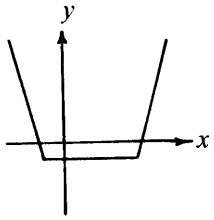
   are given by $f(x, y) =$

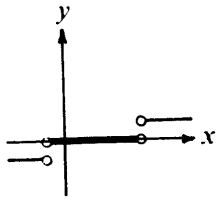
   (A) $x^2 + xy + y^2 + C$ (B) $x^2 - xy + y^2 + C$ (C) $x^2 - xy - y^2 + C$
   (D) $x^2 + 2xy + y^2 + C$ (E) $x^2 - 2xy + y^2 + C$



5. Which of the following could be the graph of the derivative of the function whose graph is shown in the figure above?

(A)


(B)


(C)


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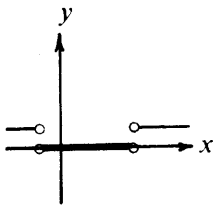
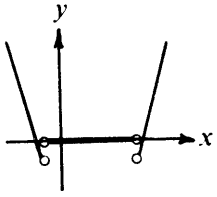
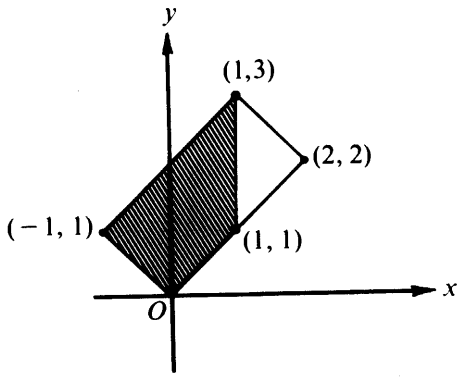


(E)




6. Which of the following integrals represents the area of the shaded portion of the rectangle shown in the figure above?

(A) $\int_{-1}^{1} (x + 2 - |x|)\, dx$

(B) $\int_{-1}^{1} (|x| + x + 2)\, dx$

(C) $\int_{-1}^{1} (x + 2)\, dx$

(D) $\int_{-1}^{1} |x|\, dx$

(E) $\int_{-1}^{1} 2\, dx$

7. $\sum_{n=1}^{\infty} \frac{n}{n+1} =$

(A) $\frac{1}{e}$

(B) $\log 2$

(C) 1

(D) e

(E) $+\infty$

8. If $\sin^{-1}x = \frac{\pi}{6}$, then the acute angle value of $\cos^{-1}x$ is

(A) $\frac{5\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\sqrt{1 - \frac{\pi^2}{6^2}}$ (D) $1 - \frac{\pi}{6}$ (E) 0

9. $\int_0^{\pi} e^{\sin^2 x} e^{\cos^2 x} dx =$

(A) π (B) $e\pi$ (C) e^{π} (D) $e^{\sin^2 \pi}$ (E) $e^{\pi} - 1$

10. Which of the following is true of the behavior of $f(x) = \frac{x^3 + 8}{x^2 - 4}$ as $x \to 2$?

(A) The limit is 0.
(B) The limit is 1.
(C) The limit is 4.
(D) The graph of the function has a vertical asymptote at 2.
(E) The function has unequal, finite left-hand and right-hand limits.

11. Suppose that an arrow is shot from a point p and lands at a point q such that at one and only one point in its flight is the arrow parallel to the line of sight between p and q. Of the following, which is the best mathematical model for the phenomenon described above?

(A) A function f differentiable on $[a, b]$ such that there is one and only one point c in $[a, b]$ with $\int_a^b f'(x)\,dx = c(b - a)$

(B) A function f whose second derivative is at all points negative such that there is one and only one point c in $[a, b]$ with $f'(c) = \frac{f(b) - f(a)}{b - a}$

(C) A function f whose first derivative is at all points positive such that there is one and only one point c in $[a, b]$ with $\int_a^b f(x)\,dx = f(c) \cdot (b - a)$

(D) A function f continuous on $[a, b]$ such that there is one and only one point c in $[a, b]$ with $\int_a^b f(x)\,dx = f(c) \cdot (b - a)$

(E) A function f continuous on $[a, b]$ and $f(a) < d < f(b)$ such that there is one and only one point c in $[a, b]$ with $f(c) = d$

12. If $c > 0$ and $f(x) = e^x - cx$ for all real numbers x, then the minimum value of f is

(A) $f(c)$ (B) $f(e^c)$ (C) $f\left(\frac{1}{c}\right)$ (D) $f(\log c)$ (E) nonexistent

13. For all $x > 0$, if $f(\log x) = \sqrt{x}$, then $f(x) =$

(A) $e^{\frac{x}{2}}$ (B) $\log\sqrt{x}$ (C) $e^{\sqrt{x}}$ (D) $\sqrt{\log x}$ (E) $\frac{\log x}{2}$

14. $\int_0^1 \left(\int_0^{\sin y} \frac{1}{\sqrt{1-x^2}}\, dx \right) dy =$

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{\pi}{4}$ (D) 1 (E) $\frac{\pi}{3}$

15. For what triples of real numbers (a, b, c) with $a \neq 0$ is the function

defined by $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ ax^2 + bx + c, & \text{if } x > 1 \end{cases}$

differentiable at all real x ?

(A) $\{(a, 1-2a, a): a$ is a nonzero real number$\}$

(B) $\{(a, 1-2a, c): a, c$ are real numbers and $a \neq 0\}$

(C) $\{(a, b, c): a, b, c$ are real numbers, $a \neq 0$, and $a + b + c = 1\}$

(D) $\left\{\left(\frac{1}{2}, 0, 0\right)\right\}$

(E) $\{(a, 1-2a, 0): a$ is a nonzero real number$\}$

Questions 16-18 are based on the following information.

Let f be a function such that the graph of f is a semicircle S with endpoints $(a, 0)$ and $(b, 0)$ where $a < b$.

16. $\left| \int_a^b f(x)dx \right| =$

(A) $f(b) - f(a)$ (B) $\frac{f(b) - f(a)}{b - a}$ (C) $(b-a)\frac{\pi}{4}$ (D) $(b-a)^2\pi$ (E) $(b-a)^2\frac{\pi}{8}$

17. The graph of $y = 3f(x)$ is a

(A) translation of S (B) semicircle with radius three times that of S (C) subset of an ellipse
(D) subset of a parabola (E) subset of a hyperbola

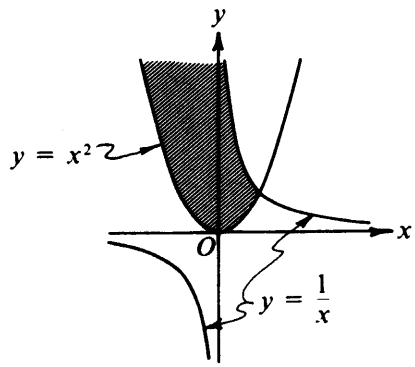
18. The improper integral $\int_a^b f(x)f'(x)dx$ is

(A) necessarily zero
(B) possibly zero but not necessarily
(C) necessarily nonexistent
(D) possibly nonexistent but not necessarily
(E) none of the above

---

19. $\lim_{x \to \pi} \frac{e^{-\pi} - e^{-x}}{\sin x} =$

(A) $-\infty$ (B) $-e^{-\pi}$ (C) 0 (D) $e^{-\pi}$ (E) 1



20. The shaded region in the figure above indicates the graph of which of the following?

(A) $x^2 < y$ and $y < \frac{1}{x}$ (B) $x^2 < y$ or $y < \frac{1}{x}$ (C) $x^2 > y$ and $y > \frac{1}{x}$

(D) $x^2 > y$ or $y > \frac{1}{x}$ (E) $x^2 < y$ and $xy < 1$

21. The shortest distance from the curve $xy = 8$ to the origin is

(A) 4 (B) 8 (C) 16 (D) $2\sqrt{2}$ (E) $4\sqrt{2}$

22. If $f(x) = \begin{cases} \frac{|x|}{x}, & \text{for } x \neq 0 \\ 0, & \text{for } x = 0, \end{cases}$ then $\int_{-1}^{1} f(x)\,dx$ is

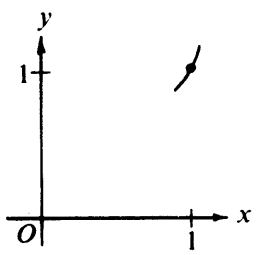
(A) -2 (B) 0 (C) 2 (D) not defined

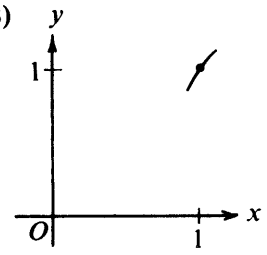
(E) none of the above

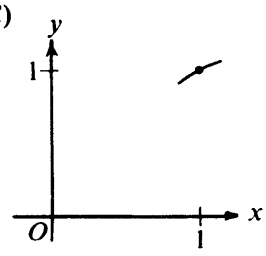
23. Let $y = f(x)$ be a solution of the differential equation $x\,dy + (y - xe^x)\,dx = 0$ such that $y = 0$ when $x = 1$. What is the value of $f(2)$?

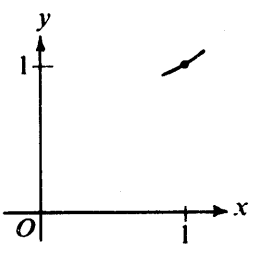
(A) $\frac{1}{2e}$ (B) $\frac{1}{e}$ (C) $\frac{e^2}{2}$ (D) $2e$ (E) $2e^2$

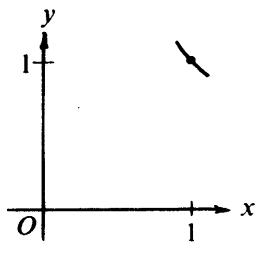
24. Of the following, which best represents a portion of the graph of $y = \frac{1}{e^x} + x - \frac{1}{e}$ near $(1, 1)$?

(A) 

(B) 

(C) 

(D) 

(E) 

25. In xyz-space, the degree measure of the angle between the rays

$z = x \geq 0,\ y = 0$

and

$z = y \geq 0,\ x = 0$ is

(A) 0° (B) 30° (C) 45° (D) 60° (E) 90°

26. Suppose f is a real function such that $f'(x_0)$ exists. Which of the following is the value of $\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0 - h)}{h}$?

(A) 0 (B) $2f'(x_0)$ (C) $f'(-x_0)$ (D) $-f'(x_0)$ (E) $-2f'(x_0)$

27. The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{e^n}{n!} x^n$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) $+\infty$

28. In the xy-plane, the graph of $x^{\log y} = y^{\log x}$ is

(A) empty (B) a single point (C) a ray in the open first quadrant
(D) a closed curve (E) the open first quadrant

29. Let $x_1 = 1$ and $x_{n+1} = \sqrt{3 + 2x_n}$ for all positive integers n. If it is assumed that $\{x_n\}$ converges, then $\lim_{n\to\infty} x_n =$

(A) -1 (B) 0 (C) $\sqrt{5}$ (D) e (E) 3

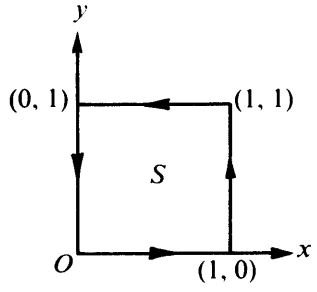
30. Let $f(x, y) = x^3 + y^3 + 3xy$ for all real x and y. Then there exist distinct points P and Q such that f has a

(A) local maximum at P and at Q
(B) saddle point at P and at Q
(C) local maximum at P and a saddle point at Q
(D) local minimum at P and a saddle point at Q
(E) local minimum at P and at Q

31. The polynomial $p(x) = 1 + \frac{1}{2}(x - 1) - \frac{1}{8}(x - 1)^2$ is used to approximate $\sqrt{1.01}$. Which of the following most closely approximates the error $\sqrt{1.01} - p(1.01)$?

(A) $\left(\frac{1}{16}\right) \times 10^{-6}$ (B) $\left(\frac{1}{48}\right) \times 10^{-8}$ (C) $\left(\frac{3}{8}\right) \times 10^{-10}$

(D) $-\left(\frac{3}{8}\right) \times 10^{-10}$ (E) $-\left(\frac{1}{16}\right) \times 10^{-6}$



32. If B is the boundary of S as indicated in the figure above, then $\int_B (3y\,dx + 4x\,dy) =$

(A) 0 (B) 1 (C) 3 (D) 4 (E) 7

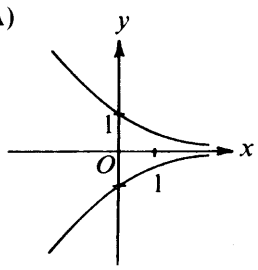
33. Let f be a continuous, strictly decreasing, real-valued function such that $\int_0^{+\infty} f(x)\,dx$ is finite and $f(0) = 1$. In terms of f^{-1} (the inverse function of f), $\int_0^{+\infty} f(x)\,dx$ is

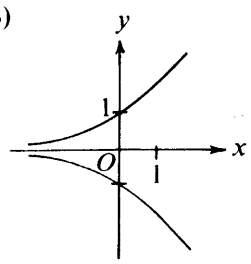
(A) less than $\int_1^{+\infty} f^{-1}(y)\,dy$ (B) greater than $\int_0^1 f^{-1}(y)\,dy$ (C) equal to $\int_1^{+\infty} f^{-1}(y)\,dy$

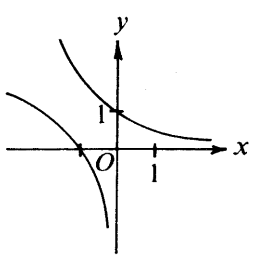
(D) equal to $\int_0^1 f^{-1}(y)\,dy$ (E) equal to $\int_0^{+\infty} f^{-1}(y)\,dy$

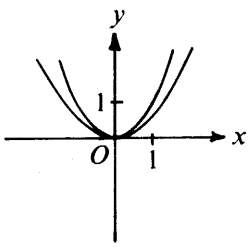
34. Which of the following indicates the graphs of two functions that satisfy the differential equation

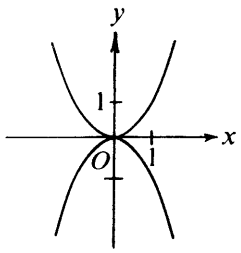
$$\left(\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx} + y^2 = 0?$$

(A) 

(B) 

(C) 

(D) 

(E) 

ALGEBRA

35. If a, b, and c are real numbers, which of the following are necessarily true?

I. If $a < b$ and $ab \neq 0$, then $\frac{1}{a} > \frac{1}{b}$.

II. If $a < b$, then $ac < bc$ for all c.

III. If $a < b$, then $a + c < b + c$ for all c.

IV. If $a < b$, then $-a > -b$.

(A) I only (B) I and III only (C) III and IV only (D) II, III, and IV only (E) I, II, III, and IV

36. In order to send an undetected message to an agent in the field, each letter in the message is replaced by the number of its position in the alphabet and that number is entered in a matrix M. Thus, for example, "DEAD" becomes the matrix $M = \begin{pmatrix} 4 & 5 \\ 1 & 4 \end{pmatrix}$. In order to further avoid detection, each message with four letters is sent to the agent encoded as MC, where $C = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$. If the agent receives the matrix $\begin{pmatrix} 51 & -3 \\ 31 & -8 \end{pmatrix}$, then the message is

(A) RUSH (B) COME (C) ROME (D) CALL
(E) not uniquely determined by the information given

37. If f is a linear transformation from the plane to the real numbers and if $f(1, 1) = 1$ and $f(-1, 0) = 2$, then $f(3, 5) =$

(A) -6 (B) -5 (C) 0 (D) 8 (E) 9

38. Let $*$ be the binary operation on the rational numbers given by $a * b = a + b + 2ab$. Which of the following are true?

I. $*$ is commutative.
II. There is a rational number that is a $*$-identity.
III. Every rational number has a $*$-inverse.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

39. A group G in which $(ab)^2 = a^2b^2$ for all a, b in G is necessarily

(A) finite
(B) cyclic
(C) of order two
(D) abelian
(E) none of the above

40. Suppose that $f(1 + x) = f(x)$ for all real x. If f is a polynomial and $f(5) = 11$, then $f\left(\frac{15}{2}\right)$ is

(A) -11 (B) 0 (C) 11 (D) $\frac{33}{2}$
(E) not uniquely determined by the information given

41. Let x and y be positive integers such that $3x + 7y$ is divisible by 11. Which of the following must also be divisible by 11 ?

(A) $4x + 6y$ (B) $x + y + 5$ (C) $9x + 4y$ (D) $4x - 9y$ (E) $x + y - 1$

42. The dimension of the subspace spanned by the real vectors

$$\begin{pmatrix}1\\1\\0\\0\end{pmatrix}, \begin{pmatrix}2\\2\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}2\\0\\0\\3\end{pmatrix}, \begin{pmatrix}1\\-2\\0\\8\end{pmatrix}, \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \quad \text{is}$$

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

43. The rank of the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{pmatrix} \quad \text{is}$$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

44. If M is the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, then M^{100} is

(A) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

(E) none of the above

45. If a polynomial $f(x)$ over the real numbers has the complex numbers $2 + i$ and $1 - i$ as roots, then $f(x)$ could be

(A) $x^4 + 6x^3 + 10$ (B) $x^4 + 7x^2 + 10$ (C) $x^3 - x^2 + 4x + 1$

(D) $x^3 + 5x^2 + 4x + 1$ (E) $x^4 - 6x^3 + 15x^2 - 18x + 10$

46. Let V be the set of all real polynomials $p(x)$. Let transformations T, S be defined on V by $T: p(x) \to xp(x)$ and $S: p(x) \to p'(x) = \frac{d}{dx}p(x)$, and interpret $(ST)(p(x))$ as $S(T(p(x)))$. Which of the following is true?

(A) $ST = 0$
(B) $ST = T$
(C) $ST = TS$
(D) $ST - TS$ is the identity map of V onto itself.
(E) $ST + TS$ is the identity map of V onto itself.

47. If the finite group G contains a subgroup of order seven but no element (other than the identity) is its own inverse, then the order of G could be

(A) 27 (B) 28 (C) 35 (D) 37 (E) 42

48. Which of the following is the larger of the eigenvalues (characteristic values) of the matrix $\begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$?

(A) 4 (B) 5 (C) 6 (D) 10 (E) 12

49. Let V be the vector space, under the usual operations, of real polynomials that are of degree at most 3. Let W be the subspace of all polynomials $p(x)$ in V such that $p(0) = p(1) = p(-1) = 0$. Then $\dim V + \dim W$ is

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

50. The map $x \to axa^2$ of a group G into itself is a homomorphism if and only if

(A) G is abelian (B) $G = \{e\}$ (C) $a = e$ (D) $a^2 = a$ (E) $a^3 = e$

51. Let $I \neq A \neq -I$, where I is the identity matrix and A is a real 2×2 matrix. If $A = A^{-1}$, then the trace of A is

(A) 2 (B) 1 (C) 0 (D) -1 (E) -2

52. Which of the following subsets are subrings of the ring of real numbers?

I. $\{a + b\sqrt{2} : a \text{ and } b \text{ are rational}\}$

II. $\left\{\frac{n}{3^m} : n \text{ is an integer and } m \text{ is a non-negative integer}\right\}$

III. $\{a + b\sqrt{5} : a \text{ and } b \text{ are real numbers and } a^2 + b^2 \leq 1\}$

(A) I only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III

ADDITIONAL TOPICS

53. k digits are to be chosen at random (with repetitions allowed) from $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the probability that 0 will not be chosen?

(A) $\frac{1}{k}$ (B) $\frac{1}{10}$ (C) $\frac{k-1}{k}$ (D) $\left(\frac{1}{10}\right)^k$ (E) $\left(\frac{9}{10}\right)^k$

54. $S(n)$ is a statement about positive integers n such that whenever $S(k)$ is true, $S(k+1)$ must also be true. Furthermore, there exists some positive integer n_0 such that $S(n_0)$ is not true. Of the following, which is the strongest conclusion that can be drawn?

(A) $S(n_0 + 1)$ is not true.
(B) $S(n_0 - 1)$ is not true.
(C) $S(n)$ is not true for any $n \leq n_0$.
(D) $S(n)$ is not true for any $n \geq n_0$.
(E) $S(n)$ is not true for any n.

55. Let f and g be functions defined on the positive integers and related in the following way:

$$f(n) = \begin{cases} 1, & \text{if } n = 1 \\ 2f(n-1), & \text{if } n \neq 1 \end{cases}$$

and

$$g(n) = \begin{cases} 3g(n+1), & \text{if } n \neq 3 \\ f(n), & \text{if } n = 3. \end{cases}$$

The value of $g(1)$ is

(A) 6 (B) 9 (C) 12 (D) 36
(E) not uniquely determined by the information given

56. If k is a real number and

$$f(x) = \begin{cases} \sin \frac{1}{x} & \text{for } x \neq 0 \\ k & \text{for } x = 0 \end{cases}$$

and if the graph of f is <u>not</u> a connected subset of the plane, then the value of k

(A) could be -1
(B) must be 0
(C) must be 1
(D) could be less than 1 and greater than -1
(E) must be less than -1 or greater than 1

57. What is wrong with the following argument?

Let R be the real numbers.

(1) "For all $x, y \in R$, $f(x) + f(y) = f(xy)$."

is equivalent to

(2) "For all $x, y \in R$, $f(-x) + f(y) = f((-x)y)$."

which is equivalent to

(3) "For all $x, y \in R$, $f(-x) + f(y) = f((-x)y) = f(x(-y)) = f(x) + f(-y)$."

From this for $y = 0$, we make the conclusion

(4) "For all $x \in R$, $f(-x) = f(x)$."

Since the steps are reversible, any function with property (4) has property (1). Therefore, for all $x, y \in R$, $\cos x + \cos y = \cos(xy)$.

(A) (2) does not imply (1).
(B) (3) does not imply (2).
(C) (3) does not imply (4).
(D) (4) does not imply (3).
(E) (4) is not true for $f = \cos$.

58. Suppose that the space S contains exactly eight points. If $\mathcal{B}$ is a collection of 250 distinct subsets of S, which of the following statements must be true?

(A) S is an element of $\mathcal{B}$.

(B) $\bigcap_{G \in \mathcal{B}} G = S$

(C) $\bigcap_{G \in \mathcal{B}} G$ is a nonempty proper subset of S.

(D) $\mathcal{B}$ has a member that contains exactly one element.

(E) The empty set is an element of $\mathcal{B}$.

59. In a game two players take turns tossing a fair coin; the winner is the first one to toss a head. The probability that the player who makes the first toss wins the game is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

60. Acceptable input for a certain pocket calculator is a finite sequence of characters each of which is either a digit or a sign. The first character must be a digit, the last character must be a digit, and any character that is a sign must be followed by a digit. There are 10 possible digits and 4 possible signs. If N_k denotes the number of such acceptable sequences having length k, then N_k is given recursively by

(A) $N_1 = 10$
$N_k = 10N_{k-1}$

(B) $N_1 = 10$
$N_k = 14N_{k-1}$

(C) $N_1 = 10$
$N_2 = 100$
$N_k = 10N_{k-1} + 40N_{k-2}$

(D) $N_1 = 10$
$N_2 = 140$
$N_k = 14N_{k-1} + 40N_{k-2}$

(E) $N_1 = 14$
$N_2 = 196$
$N_k = 10N_{k-1} + 14N_{k-2}$

61. If $f(z)$ is an analytic function that maps the entire finite complex plane into the real axis, then the imaginary axis must be mapped onto

(A) the entire real axis
(B) a point
(C) a ray
(D) an open finite interval
(E) the empty set

62. A fair die is tossed 360 times. The probability that a six comes up on 70 or more of the tosses is

(A) greater than 0.50
(B) between 0.16 and 0.50
(C) between 0.02 and 0.16
(D) between 0.01 and 0.02
(E) less than 0.01

63. Let S be a compact topological space, let T be a topological space, and let f be a function from S onto T. Of the following conditions on f, which is the weakest condition sufficient to ensure the compactness of T?

(A) f is a homeomorphism.
(B) f is continuous and one-to-one
(C) f is continuous.
(D) f is one-to-one
(E) f is bounded.

ANSWER KEY FOR SAMPLE QUESTIONS

Calculus

1. B
2. B
3. A
4. A
5. C
6. A
7. E
8. B
9. B
10. D
11. B
12. D
13. A
14. B
15. A
16. E
17. C
18. A
19. B
20. E
21. A
22. B
23. C
24. D
25. D
26. B
27. E
28. E
29. E
30. C
31. A
32. B
33. D
34. A

Algebra

35. C
36. C
37. E
38. C
39. D
40. C
41. D
42. B
43. B
44. A
45. E
46. D
47. C
48. C
49. B
50. E
51. C
52. B

Additional Topics

53. E
54. C
55. D
56. E
57. D
58. D
59. D
60. C
61. B
62. C
63. C

TAKING THE TEST

PREPARING FOR THE GRE MATHEMATICS TEST

GRE Mathematics Test questions are designed to measure skills and knowledge gained over a long period of time. Although you might increase your scores to some extent through preparation a few weeks or months before you take the test, last minute cramming is unlikely to be of further help. The following information will help guide you if you decide to spend time preparing for the test.

- A general review of your college courses is probably the best preparation for the test. However, the test covers a broad range of subject matter, and no one is expected to be familiar with the content of every question.
- Use official GRE publications, published by ETS, to familiarize yourself with questions used on the GRE Mathematics Test.
- Become familiar with the types of questions used in the test, paying special attention to the directions. If you thoroughly understand the directions before you take the test, you will have more time during the test to focus on the questions themselves.

TEST-TAKING STRATEGIES

When you take the test, you will be marking your answers on a separate machine-scorable answer sheet. Total testing time is two hours and 50 minutes; there are no separately timed sections. Following are some general test-taking strategies you may want to consider.

- Read the test directions carefully, and work as rapidly as you can without being careless. For each question, you should choose the best answer from the available options.
- All questions are of equal value; do not waste time pondering individual questions you find extremely difficult or unfamiliar.
- You may want to work through the test quite rapidly, first answering only the questions about which you feel confident, then going back and answering questions that require more thought, and concluding with the most difficult questions if there is time.
- If you decide to change an answer, make sure you completely erase it and fill in the oval corresponding to your desired answer.
- Questions for which you mark no answer or more than one answer are not counted in scoring.
- As a correction for haphazard guessing, one-fourth of the number of questions you answer incorrectly are subtracted from the number of questions you answer correctly. It is improbable that mere guessing will improve your score significantly; it may even lower your score. If, however, you are not certain of the correct answer but have some knowledge of the question and are able

to eliminate one or more of the answer choices, your chance of getting the right answer is improved, and it may be to your advantage to answer such a question.

- Record all answers on your answer sheet. Answers recorded in your test book will not be counted.
- Do not wait until the last five minutes of a testing session to record answers on your answer sheet.

HOW TO SCORE YOUR TEST (GR9367)

Total Subject Test scores are reported as three-digit scaled scores with the third digit always zero. The maximum possible range for all Subject Test total scores is from 200 to 990. The actual range of scores for a particular Subject Test, however, may be smaller. The possible range for GRE Mathematics Test scores is from 400 to 990. The range for different editions of a given test may vary because different editions are not of precisely the same difficulty. The differences in ranges among different editions of a given test, however, usually are small. This should be taken into account, especially when comparing two very high scores. In general, differences between scores at the 99th percentile should be ignored. **The score conversion table provided shows the score range for this edition of the test only.**

The work sheet on page 25 lists the correct answers to the questions. Columns are provided for you to mark whether you chose the correct (C) answer or an incorrect (I) answer to each question. Draw a line across any question you omitted, because it is not counted in the scoring. At the bottom of the page, enter the total number correct and the total number incorrect. Divide the total incorrect by 4 and subtract the resulting number from the total correct. This is the adjustment made for guessing. Then round the result to the nearest whole number. This will give you your raw total score. Use the total score conversion table to find the scaled total score that corresponds to your raw total score.

Example: Suppose you chose the correct answers to 48 questions and incorrect answers to 15. Dividing 15 by 4 yields 3.75. Subtracting 3.75 from 48 equals 44.25, which is rounded to 44. The raw score of 44 corresponds to a scaled score of 870.

WORK SHEET for the GRE Mathematics Test, Form GR9367
Answer Key and Percentages* of Examinees Answering Each Question Correctly

QUESTION Number	Answer	P+	TOTAL C	I
1	D	96		
2	C	95		
3	A	84		
4	C	76		
5	A	70		
6	B	86		
7	D	80		
8	B	76		
9	E	74		
10	A	94		
11	C	82		
12	E	69		
13	A	65		
14	D	62		
15	D	56		
16	A	63		
17	E	69		
18	D	48		
19	A	57		
20	B	52		
21	D	79		
22	E	73		
23	C	61		
24	A	51		
25	D	33		
26	C	69		
27	A	59		
28	B	45		
29	E	68		
30	E	56		
31	C	64		
32	B	36		
33	C	88		
34	E	60		
35	B	45		

QUESTION Number	Answer	P+	TOTAL C	I
36	D	70		
37	B	66		
38	D	23		
39	D	69		
40	A	40		
41	B	75		
42	D	31		
43	E	17		
44	D	31		
45	C	53		
46	E	83		
47	C	18		
48	D	58		
49	D	28		
50	A	38		
51	B	37		
52	D	72		
53	A	49		
54	C	57		
55	E	28		
56	B	17		
57	C	72		
58	C	65		
59	A	49		
60	D	22		
61	C	52		
62	B	10		
63	D	49		
64	C	50		
65	A	60		
66	B	53		

Correct (C) _____

Incorrect (I) _____

Total Score:

C – I/4 = _____

Scaled Score (SS) = _____

*The P+ column indicates the percent of GRE Mathematics Test examinees who answered each question correctly; it is based on a sample of February 1993 examinees selected to represent all GRE Mathematics Test examinees tested between October 1, 1996 and September 30, 1999.

Score Conversions and Percents Below* for GRE Mathematics Test, Form GR9367

TOTAL SCORE					
Raw Score	**Scaled Score**	**%**	**Raw Score**	**Scaled Score**	**%**
55-66	990	82	26	690	33
54	980	81	25	680	31
53	970	80	24	670	30
52	960	79	23	660	28
51	950	77	22	650	26
50	930	74	21	640	24
49	920	72	20	630	23
48	910	71	19	620	22
47	900	69	18	600	18
46	890	67	17	590	17
45	880	66	16	580	15
44	870	64	15	570	13
43	860	62	14	560	12
42	850	61	13	550	11
41	840	59	12	540	10
40	830	57	11	530	8
39	820	55	10	520	7
38	810	54	9	510	6
37	800	52	8	500	5
36	790	51	7	490	4
35	780	49	6	480	2
34	770	47	5	470	2
33	760	45	4	460	1
32	750	43	3	450	1
31	740	41	2	440	0
30	730	39	1	430	0
29	720	38	0	420	0
28	710	37			
27	700	35			

*Percent scoring below the scaled score is based on the performance of 7,092 examinees who took the GRE Mathematics Test between October 1, 1996 and September 30, 1999. This percent below information was used for score reports during the 2000-01 testing year.

EVALUATING YOUR PERFORMANCE (GR9367)

Now that you have scored your test, you may wish to compare your performance with the performance of others who took this test. A worksheet and table are provided, both using performance data from GRE Mathematics Test examinees.

The worksheet (on page 25) is based on the performance of a sample of the examinees who took this particular test in February1993. This sample was selected to represent the total population of GRE Mathematics Test examinees tested between October 1996 and September 1999. On the work sheet you used to determine your score is a column labeled "P+." The numbers in this column indicate the percent of the examinees in this sample who answered each question correctly. You may use these numbers as a guide for evaluating your performance on each test question.

The table on page 26 contains, for each scaled score, the percentage of examinees tested between October 1996 and September 1999 who received lower scores. Interpretive data based on the scores earned by examinees tested in this three-year period were used by admissions officers in 2000-2001. These percentages appear in the score conversion table in a column to the right of the scaled scores. For example, in the percent column opposite the scaled score of 870 is the number 64. This means that 64 percent of the GRE Mathematics Test examinees tested between October 1996 and September 1999 scored lower than 870. To compare yourself with this population, look at the percent next to the scaled score you earned on the practice test. This number is a reasonable indication of your rank among GRE Mathematics Test examinees if you followed the test-taking suggestions in this practice book.

It is important to realize that the conditions under which you tested yourself were not exactly the same as those you will encounter at a test center. It is impossible to predict how different test-taking conditions will affect test performance, and this is only one factor that may account for differences between your practice test scores and your actual test scores. By comparing your performance on this practice test with the performance of other GRE Mathematics Test examinees, however, you will be able to determine your strengths and weaknesses and can then plan a program of study to prepare yourself for taking the GRE Mathematics Test under standard conditions.

Before you start timing yourself on the test that follows, we suggest that you remove an answer sheet (pages 131 to 136) and turn first to the back cover of the test book (page 130), as you will do at the test center, and follow the instructions for completing the identification areas of the answer sheet. When you are ready to begin the test, note the time and start marking your answers to the questions on the answer sheet.

FORM GR9367

67

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MATHEMATICS TEST

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Princeton, N.J. 08541

MATHEMATICS TEST

Time—170 minutes

66 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is the best of the choices offered and then mark the corresponding space on the answer sheet.

Computation and scratchwork may be done in this examination book.

Note: In this examination:

(1) All logarithms are to the base e unless otherwise specified.
(2) The set of all x such that $a \leq x \leq b$ is denoted by $[a, b]$.

1. If $f(g(x)) = 5$ and $f(x) = x + 3$ for all real x, then $g(x) =$

(A) $x - 3$ (B) $3 - x$ (C) $\frac{5}{x+3}$ (D) 2 (E) 8

---

2. $\lim_{x \to 0} \frac{\tan x}{\cos x} =$

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) $+\infty$

---

3. $\int_0^{\log 4} e^{2x} dx =$

(A) $\frac{15}{2}$ (B) 8 (C) $\frac{17}{2}$ (D) $\frac{\log 16}{2} - 1$ (E) $\log 4 - \frac{1}{2}$

---

4. Let $A - B$ denote $\{x \in A: x \notin B\}$. If $(A - B) \cup B = A$, which of the following must be true?

(A) B is empty.
(B) $A \subseteq B$
(C) $B \subseteq A$
(D) $(B - A) \cup A = B$
(E) None of the above

GO ON TO THE NEXT PAGE.

SCRATCHWORK

5. If $f(x) = |x| + 3x^2$ for all real x, then $f'(-1)$ is

(A) -7 (B) -5 (C) 5 (D) 7 (E) nonexistent

---

6. For what value of b is the value of $\int_b^{b+1} (x^2 + x)\,dx$ a minimum?

(A) 0 (B) -1 (C) -2 (D) -3 (E) -4

---

7. In how many of the eight standard octants of xyz-space does the graph of $z = e^{x+y}$ appear?

(A) One (B) Two (C) Three (D) Four (E) Eight

---

8. Suppose that the function f is defined on an interval by the formula $f(x) = \sqrt{\tan^2 x - 1}$. If f is continuous, which of the following intervals could be its domain?

(A) $\left(\frac{3\pi}{4}, \pi\right)$

(B) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(C) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

(D) $\left(-\frac{\pi}{4}, 0\right)$

(E) $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right)$

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SCRATCHWORK

9. $\int_0^1 \frac{x}{2 - x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\frac{5}{3}$ (C) $\frac{\log 2 - e}{2}$ (D) $-\frac{\log 2}{2}$ (E) $\frac{\log 2}{2}$

---

10. If $f''(x) = f'(x)$ for all real x, and if $f(0) = 0$ and $f'(0) = -1$, then $f(x) =$

(A) $1 - e^x$ (B) $e^x - 1$ (C) $e^{-x} - 1$ (D) e^{-x} (E) $-e^x$

---

11. If $\phi(x, y, z) = x^2 + 2xy + xz^{\frac{3}{2}}$, which of the following partial derivatives are identically zero?

I. $\frac{\partial^2 \phi}{\partial y^2}$

II. $\frac{\partial^2 \phi}{\partial x \partial y}$

III. $\frac{\partial^2 \phi}{\partial z \partial y}$

(A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

GO ON TO THE NEXT PAGE.

SCRATCHWORK

12. $\lim_{x \to 0} \frac{\sin 2x}{(1 + x)\log(1 + x)} =$

(A) -2 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 2

---

13. $\lim_{n \to \infty} \int_1^n \frac{1}{x^n}\, dx =$

(A) 0 (B) 1 (C) e (D) π (E) $+\infty$

---

14. At a 15 percent annual inflation rate, the value of the dollar would decrease by approximately one-half every 5 years. At this inflation rate, in approximately how many years would the dollar be worth $\frac{1}{1{,}000{,}000}$ of its present value?

(A) 25 (B) 50 (C) 75 (D) 100 (E) 125

GO ON TO THE NEXT PAGE.

SCRATCHWORK

15. Let $f(x) = \int_1^x \frac{1}{1+t^2}\,dt$ for all real x. An equation of the line tangent to the graph of f at the point $(2, f(2))$ is

(A) $y - 1 = \frac{1}{5}(x - 2)$ (B) $y - \text{Arctan } 2 = \frac{1}{5}(x - 2)$ (C) $y - 1 = (\text{Arctan } 2)(x - 2)$

(D) $y - \text{Arctan } 2 + \frac{\pi}{4} = \frac{1}{5}(x - 2)$ (E) $y - \frac{\pi}{2} = (\text{Arctan } 2)(x - 2)$

---

16. Let $f(x) = e^{g(x)}h(x)$ and $h'(x) = -g'(x)h(x)$ for all real x. Which of the following must be true?

(A) f is a constant function.
(B) f is a linear nonconstant function.
(C) g is a constant function.
(D) g is a linear nonconstant function.
(E) None of the above

---

17. $1 - \sin^2\left(\text{Arccos } \frac{\pi}{12}\right) =$

(A) $\sqrt{\frac{1 - \cos\frac{\pi}{24}}{2}}$ (B) $\sqrt{\frac{1 - \cos\frac{\pi}{6}}{2}}$ (C) $\sqrt{\frac{1 + \cos\frac{\pi}{24}}{2}}$ (D) $\frac{\pi}{6}$ (E) $\frac{\pi^2}{144}$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

18. If $f(x) = \sum_{n=0}^{\infty} (-1)^n x^{2n}$ for all $x \in (0, 1)$, then $f'(x) =$

(A) $\sin x$ (B) $\cos x$ (C) $\frac{1}{1 + x^2}$ (D) $\frac{-2x}{(1 + x^2)^2}$ (E) $\frac{2x}{(1 - 2x)^2}$

---

19. Which of the following is the general solution of the differential equation

$$\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - y = 0?$$

(A) $c_1e^t + c_2te^t + c_3t^2e^t$

(B) $c_1e^{-t} + c_2te^{-t} + c_3t^2e^{-t}$

(C) $c_1e^t - c_2e^{-t} + c_3te^{t^2}$

(D) $c_1e^t + c_2e^{2t} + c_3e^{3t}$

(E) $c_1e^{2t} + c_2te^{-2t}$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

20. Which of the following double integrals represents the volume of the solid bounded above by the graph of $z = 6 - x^2 - 2y^2$ and bounded below by the graph of $z = -2 + x^2 + 2y^2$?

(A) $4\int_{x=0}^{x=2} \int_{y=0}^{y=\sqrt{2}} (8 - 2x^2 - 4y^2)\, dy\, dx$

(B) $\int_{x=-2}^{x=2} \int_{y=-\sqrt{(4-x^2)/2}}^{y=\sqrt{(4-x^2)/2}} (8 - 2x^2 - 4y^2)\, dy\, dx$

(C) $4\int_{y=0}^{y=\sqrt{2}} \int_{x=-\sqrt{4-2y^2}}^{x=\sqrt{4-2y^2}} dx\, dy$

(D) $\int_{y=-\sqrt{2}}^{y=\sqrt{2}} \int_{x=-2}^{x=2} (8 - 2x^2 - 4y^2)\, dx\, dy$

(E) $2\int_{y=0}^{y=\sqrt{2}} \int_{x=0}^{x=\sqrt{4-2y^2}} (8 - 2x^2 - 4y^2)\, dx\, dy$

---

21. Let a be a number in the interval [0, 1] and let f be a function defined on [0, 1] by

$$f(x) = \begin{cases} a^2 & \text{if } 0 \le x \le a, \\ ax & \text{otherwise.} \end{cases}$$

Then the value of a for which $\int_0^1 f(x)\, dx = 1$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1 (E) nonexistent

---

GO ON TO THE NEXT PAGE.

SCRATCHWORK

22. If b and c are elements in a group G, and if $b^5 = c^3 = e$, where e is the unit element of G, then the inverse of $b^2cb^4c^2$ must be

(A) b^3c^2bc (B) $b^4c^2b^2c$ (C) $c^2b^4cb^2$ (D) $cb^2c^2b^4$ (E) cbc^2b^3

---

23. Let f be a real-valued function continuous on the closed interval [0, 1] and differentiable on the open interval (0, 1) with $f(0) = 1$ and $f(1) = 0$. Which of the following must be true?

I. There exists $x \in (0, 1)$ such that $f(x) = x$.
II. There exists $x \in (0, 1)$ such that $f'(x) = -1$.
III. $f(x) > 0$ for all $x \in [0, 1)$.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

---

24. If A and B are events in a probability space such that $0 < P(A) = P(B) = P(A \cap B) < 1$, which of the following CANNOT be true?

(A) A and B are independent. (B) A is a proper subset of B. (C) $A \neq B$
(D) $A \cap B = A \cup B$ (E) $P(A)P(B) < P(A \cap B)$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

25. Let f be a real-valued function with domain [0, 1]. If there is some $K > 0$ such that $f(x) - f(y) \leq K|x - y|$ for all x and y in [0, 1], which of the following must be true?

(A) f is discontinuous at each point of (0, 1).
(B) f is not continuous on (0, 1), but is discontinuous at only countably many points of (0, 1).
(C) f is continuous on (0, 1), but is differentiable at only countably many points of (0, 1).
(D) f is continuous on (0, 1), but may not be differentiable on (0, 1).
(E) f is differentiable on (0, 1).

---

26. Let $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$, and $\mathbf{k} = (0, 0, 1)$. The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal if $\mathbf{v}_1 = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{v}_2 =$

(A) $\mathbf{i} + \mathbf{j} - \mathbf{k}$ (B) $\mathbf{i} - \mathbf{j} + \mathbf{k}$ (C) $\mathbf{i} + \mathbf{k}$ (D) $\mathbf{j} - \mathbf{k}$ (E) $\mathbf{i} + \mathbf{j}$

---

27. If the curve in the yz-plane with equation $z = f(y)$ is rotated around the y-axis, an equation of the resulting surface of revolution is

(A) $x^2 + z^2 = [f(y)]^2$
(B) $x^2 + z^2 = f(y)$
(C) $x^2 + z^2 = |f(y)|$
(D) $y^2 + z^2 = |f(y)|$
(E) $y^2 + z^2 = [f(x)]^2$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

28. Let A and B be subspaces of a vector space V. Which of the following must be subspaces of V?

I. $A + B = \{a + b:\ a \in A \text{ and } b \in B\}$
II. $A \cup B$
III. $A \cap B$
IV. $\{x \in V:\ x \notin A\}$

(A) I and II only
(B) I and III only
(C) III and IV only
(D) I, II, and III only
(E) I, II, III, and IV

---

29. $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{2^k}\right) =$

(A) 0 (B) 1 (C) 2 (D) 4 (E) $+\infty$

---

30. If $f(x_1, \ldots, x_n) = \sum_{1 \le i < j \le n} x_i x_j$, then $\frac{\partial f}{\partial x_n} =$

(A) $n!$ (B) $\sum_{1 \le i < j < n} x_i x_j$ (C) $\sum_{1 \le i < j < n} (x_i + x_j)$ (D) $\sum_{j=1}^{n} x_j$ (E) $\sum_{j=1}^{n-1} x_j$

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SCRATCHWORK

31. If $f(x) = \begin{cases} \sqrt{1 - x^2} & \text{for } 0 \le x \le 1 \\ x - 1 & \text{for } 1 < x \le 2, \end{cases}$

then $\int_0^2 f(x)\,dx$ is

(A) $\frac{\pi}{2}$

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{1}{2} + \frac{\pi}{4}$

(D) $\frac{1}{2} + \frac{\pi}{2}$

(E) undefined

---

32. Let R denote the field of real numbers, Q the field of rational numbers, and Z the ring of integers. Which of the following subsets F_i of R, $1 \le i \le 4$, are subfields of R?

$$F_1 = \{a/b:\ a, b \in Z \text{ and } b \text{ is odd}\}$$
$$F_2 = \{a + b\sqrt{2}:\ a, b \in Z\}$$
$$F_3 = \{a + b\sqrt{2}:\ a, b \in Q\}$$
$$F_4 = \{a + b\sqrt[4]{2}:\ a, b \in Q\}$$

(A) No F_i is a subfield of R.

(B) F_3 only

(C) F_2 and F_3 only

(D) F_1, F_2, and F_3 only

(E) F_1, F_2, F_3, and F_4

GO ON TO THE NEXT PAGE.

SCRATCHWORK

33. If n apples, no two of the same weight, are lined up at random on a table, what is the probability that they are lined up in order of increasing weight from left to right?

(A) $\frac{1}{2}$ (B) $\frac{1}{n}$ (C) $\frac{1}{n!}$ (D) $\frac{1}{2^n}$ (E) $\left(\frac{1}{n}\right)^n$

---

34. $\frac{d}{dx}\int_0^{x^2} e^{-t^2}\,dt =$

(A) e^{-x^2} (B) $2e^{-x^2}$ (C) $2e^{-x^4}$ (D) $x^2e^{-x^2}$ (E) $2xe^{-x^4}$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

35. Let f be a real-valued function defined on the set of integers and satisfying $f(x) = \frac{1}{2}f(x-1) + \frac{1}{2}f(x+1)$. Which of the following must be true?

I. The graph of f is a subset of a line.
II. f is strictly increasing.
III. f is a constant function.

(A) None
(B) I only
(C) II only
(D) I and II
(E) I and III

---

36. If F is a function such that, for all positive integers x and y, $F(x, 1) = x + 1$, $F(1, y) = 2y$, and $F(x+1, y+1) = F(F(x, y+1), y)$, then $F(2, 2) =$

(A) 8 (B) 7 (C) 6 (D) 5 (E) 4

---

37. If $\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix} = 9$, then $\det \begin{pmatrix} 3a & 3b & 3c \\ g-4a & h-4b & k-4c \\ d & e & f \end{pmatrix} =$

(A) -108 (B) -27 (C) 3 (D) 12 (E) 27

GO ON TO THE NEXT PAGE.

SCRATCHWORK

38. $\lim_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n}\left[\left(\frac{3i}{n}\right)^2 - \left(\frac{3i}{n}\right)\right] =$

(A) $-\frac{1}{6}$ (B) 0 (C) 3 (D) $\frac{9}{2}$ (E) $\frac{31}{6}$

---

39. For a real number x, $\log(1 + \sin 2\pi x)$ is not a real number if and only if x is

(A) an integer

(B) nonpositive

(C) equal to $\frac{2n - 1}{2}$ for some integer n

(D) equal to $\frac{4n - 1}{4}$ for some integer n

(E) any real number

---

40. If x, y, and z are selected independently and at random from the interval [0, 1], then the probability that $x \geq yz$ is

(A) $\frac{3}{4}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E) $\frac{1}{4}$

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SCRATCHWORK

41. If $A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$, then the set of all vectors X for which $AX = X$ is

(A) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| a = 0 \text{ and } b \text{ is arbitrary} \right\}$

(B) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| a \text{ is arbitrary and } b = 0 \right\}$

(C) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| a = -b \text{ and } b \text{ is arbitrary} \right\}$

(D) $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

(E) the empty set

---

42. What is the greatest value of b for which any real-valued function f that satisfies the following properties must also satisfy $f(1) < 5$?

(i) f is infinitely differentiable on the real numbers;
(ii) $f(0) = 1$, $f'(0) = 1$, and $f''(0) = 2$; and
(iii) $|f'''(x)| < b$ for all x in $[0, 1]$.

(A) 1 (B) 2 (C) 6 (D) 12 (E) 24

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SCRATCHWORK

43. Let n be an integer greater than 1. Which of the following conditions guarantee that the equation $x^n = \sum_{i=0}^{n-1} a_i x^i$ has at least one root in the interval (0, 1)?

I. $a_0 > 0$ and $\sum_{i=0}^{n-1} a_i < 1$

II. $a_0 > 0$ and $\sum_{i=0}^{n-1} a_i > 1$

III. $a_0 < 0$ and $\sum_{i=0}^{n-1} a_i > 1$

(A) None
(B) I only
(C) II only
(D) III only
(E) I and III

---

44. If x is a real number and P is a polynomial function, then $\lim_{h \to 0} \frac{P(x + 3h) + P(x - 3h) - 2P(x)}{h^2} =$

(A) 0 (B) $6P'(x)$ (C) $3P''(x)$ (D) $9P''(x)$ (E) ∞

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SCRATCHWORK

45. Consider the system of equations

$$ax^2 + by^3 = c$$
$$dx^2 + ey^3 = f$$

where a, b, c, d, e, and f are real constants and $ae \neq bd$. The maximum possible number of real solutions (x, y) of the system is

(A) none (B) one (C) two (D) three (E) five

---

46. If $x^3 - x + 1 = a_0 + a_1(x - 2) + a_2(x - 2)^2 + a_3(x - 2)^3$ for all real numbers x, then (a_0, a_1, a_2, a_3) is

(A) $\left(1, \frac{1}{2}, 0, -\frac{1}{8}\right)$

(B) $(1, -1, 0, 1)$

(C) $(7, 6, 10, 1)$

(D) $(7, 11, 12, 6)$

(E) $(7, 11, 6, 1)$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

47. Let C be the ellipse with center (0, 0), major axis of length $2a$, and minor axis of length $2b$. The value of $\oint_C x\,dy - y\,dx$ is

(A) $\pi\sqrt{a^2 + b^2}$

(B) $2\pi\sqrt{a^2 + b^2}$

(C) $2\pi ab$

(D) πab

(E) $\frac{\pi ab}{2}$

---

48. Let G_n denote the cyclic group of order n. Which of the following direct products is NOT cyclic?

(A) $G_{17} \times G_{11}$

(B) $G_{17} \times G_{11} \times G_5$

(C) $G_{17} \times G_{33}$

(D) $G_{22} \times G_{33}$

(E) $G_{49} \times G_{121}$

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SCRATCHWORK

49. Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{3}{4}(1 - x^2) & \text{if } -1 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

What is the standard deviation of X?

(A) 0 (B) $\frac{1}{5}$ (C) $\frac{\sqrt{30}}{15}$ (D) $\frac{1}{\sqrt{5}}$ (E) 1

---

50. The set of all points (x, y, z) in Euclidean 3-space such that

$$\begin{vmatrix} 1 & x & y & z \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{vmatrix} = 0$$

is

(A) a plane containing the points (1, 0, 0), (0, 1, 0), and (0, 0, 1)
(B) a sphere with center at the origin and radius 1
(C) a surface containing the point (1, 1, 1)
(D) a vector space with basis {(1, 0, 0), (0, 1, 0), (0, 0, 1)}
(E) none of the above

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SCRATCHWORK

51. An automorphism ϕ of a field F is a one-to-one mapping of F onto itself such that $\phi(a + b) = \phi(a) + \phi(b)$ and $\phi(ab) = \phi(a)\phi(b)$ for all $a, b \in F$. If F is the field of rational numbers, then the number of distinct automorphisms of F is

(A) 0 (B) 1 (C) 2 (D) 4 (E) infinite

---

52. Let T be the transformation of the xy-plane that reflects each vector through the x-axis and then doubles the vector's length.

If A is the 2×2 matrix such that $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = A\begin{bmatrix} x \\ y \end{bmatrix}$ for each vector $\begin{bmatrix} x \\ y \end{bmatrix}$, then $A =$

(A) $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} \frac{\sqrt{2}}{2} & 1 \\ 1 & \frac{-\sqrt{2}}{2} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$

(E) $\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

53. Let $r > 0$ and let C be the circle $|z| = r$ in the complex plane. If P is a polynomial function, then $\int_C P(z)\,dz =$

(A) 0
(B) πr^2
(C) $2\pi i$
(D) $2\pi P(0)i$
(E) $P(r)$

---

54. If f and g are real-valued differentiable functions and if $f'(x) \geq g'(x)$ for all x in the closed interval [0, 1], which of the following must be true?

(A) $f(0) \geq g(0)$

(B) $f(1) \geq g(1)$

(C) $f(1) - g(1) \geq f(0) - g(0)$

(D) $f - g$ has no maximum on [0, 1].

(E) $\frac{f}{g}$ is a nondecreasing function on [0, 1].

---

55. Let p and q be distinct primes. There is a proper subgroup J of the additive group of integers which contains exactly three elements of the set $\{p, p + q, pq, p^q, q^p\}$. Which three elements are in J?

(A) pq, p^q, q^p
(B) $p + q, pq, p^q$
(C) $p, p + q, pq$
(D) p, p^q, q^p
(E) p, pq, p^q

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SCRATCHWORK

56. For a subset S of a topological space X, let $\text{cl}(S)$ denote the closure of S in X, and let $S' = \{x: x \in \text{cl}(S - \{x\})\}$ denote the derived set of S. If A and B are subsets of X, which of the following statements are true?

I. $(A \cup B)' = A' \cup B'$
II. $(A \cap B)' = A' \cap B'$
III. If A' is empty, then A is closed in X.
IV. If A is open in X, then A' is not empty.

(A) I and II only
(B) I and III only
(C) II and IV only
(D) I, II, and III only
(E) I, II, III, and IV

---

57. Consider the following procedure for determining whether a given name appears in an alphabetized list of n names.

Step 1. Choose the name at the middle of the list (if $n = 2k$, choose the kth name); if that is the given name, you are done; if the list is only one name long, you are done. If you are not done, go to Step 2.

Step 2. If the given name comes alphabetically before the name at the middle of the list, apply Step 1 to the first half of the list; otherwise, apply Step 1 to the second half of the list.

If n is very large, the maximum number of steps required by this procedure is close to

(A) n
(B) n^2
(C) $\log_2 n$
(D) $n \log_2 n$
(E) $n^2 \log_2 n$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

58. Which of the following is an eigenvalue of the matrix

$$\begin{pmatrix} 2 & 1-i \\ 1+i & -2 \end{pmatrix}$$

over the complex numbers?

(A) 0 (B) 1 (C) $\sqrt{6}$ (D) i (E) $1+i$

---

59. Two subgroups H and K of a group G have orders 12 and 30, respectively. Which of the following could NOT be the order of the subgroup of G generated by H and K?

(A) 30 (B) 60 (C) 120 (D) 360 (E) Countable infinity

---

60. Let A and B be subsets of a set M and let $S_0 = \{A, B\}$. For $i \geq 0$, define S_{i+1} inductively to be the collection of subsets X of M that are of the form $C \cup D$, $C \cap D$, or $M - C$ (the complement of C in M), where $C, D \in S_i$. Let $S = \bigcup_{i=0}^{\infty} S_i$. What is the largest possible number of elements of S?

(A) 4
(B) 8
(C) 15
(D) 16
(E) S may be infinite.

GO ON TO THE NEXT PAGE.

SCRATCHWORK

61. A city has square city blocks formed by a grid of north-south and east-west streets. One automobile route from City Hall to the main firehouse is to go exactly 5 blocks east and 7 blocks north. How many different routes from City Hall to the main firehouse traverse exactly 12 city blocks?

(A) $5 \cdot 7$

(B) $\frac{7!}{5!}$

(C) $\frac{12!}{7!5!}$

(D) 2^{12}

(E) $7!5!$

---

62. Let R be the set of real numbers with the topology generated by the basis $\{[a, b)$: $a < b$, where $a, b \in R\}$. If X is the subset $[0, 1]$ of R, which of the following must be true?

I. X is compact.
II. X is Hausdorff.
III. X is connected.

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

GO ON TO THE NEXT PAGE.

SCRATCHWORK

63. Let R be the circular region of the xy-plane with center at the origin and radius 2. Then $\int_R\int e^{-(x^2 + y^2)}dx\,dy =$

(A) 4π

(B) πe^{-4}

(C) $4\pi e^{-4}$

(D) $\pi(1 - e^{-4})$

(E) $4\pi(e - e^{-4})$

---

64. Let V be the real vector space of real-valued functions defined on the real numbers and having derivatives of all orders. If D is the mapping from V into V that maps every function in V to its derivative, what are all the eigenvectors of D?

(A) All nonzero functions in V

(B) All nonzero constant functions in V

(C) All nonzero functions of the form $ke^{\lambda x}$, where k and λ are real numbers

(D) All nonzero functions of the form $\sum_{i=0}^{k} c_i x^i$, where $k > 0$ and the c_i's are real numbers

(E) There are no eigenvectors of D.

GO ON TO THE NEXT PAGE.

SCRATCHWORK

65. If f is a function defined by a complex power series expansion in $z - a$ which converges for $|z - a| < 1$ and diverges for $|z - a| > 1$, which of the following must be true?

(A) $f(z)$ is analytic in the open unit disk with center at a.

(B) The power series for $f(z + a)$ converges for $|z + a| < 1$.

(C) $f'(a) = 0$

(D) $\int_C f(z)dz = 0$ for any circle C in the plane.

(E) $f(z)$ has a pole of order 1 at $z = a$.

---

66. Let n be any positive integer and $1 \leq x_1 < x_2 < \ldots < x_{n+1} \leq 2n$, where each x_i is an integer. Which of the following must be true?

I. There is an x_i that is the square of an integer.
II. There is an i such that $x_{i+1} = x_i + 1$.
III. There is an x_i that is prime.

(A) I only
(B) II only
(C) I and II
(D) I and III
(E) II and III

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST.

HOW TO SCORE YOUR TEST (GR9767)

Total Subject Test scores are reported as three-digit scaled scores with the third digit always zero. The maximum possible range for all Subject Test total scores is from 200 to 990. The actual range of scores for a particular subject test, however, may be smaller. The possible range for GRE Mathematics Test scores is from 400 to 990. The range for different editions of a given test may vary because different editions are not of precisely the same difficulty. The differences in ranges among different editions of a given test, however, usually are small. This should be taken into account, especially when comparing two very high scores. In general, differences between scores at the 99th percentile should be ignored. **The score conversion table provided shows the score range for this edition of the test only.**

The work sheet on page 82 lists the correct answers to the questions. Columns are provided for you to mark whether you chose the correct (C) answer or an incorrect (I) answer to each question. Draw a line across any question you omitted, because it is not counted in the scoring. At the bottom of the page, enter the total number correct and the total number incorrect. Divide the total incorrect by 4 and subtract the resulting number from the total correct. This is the adjustment made for guessing. Then round the result to the nearest whole number. This will give you your raw total score. Use the total score conversion table to find the scaled total score that corresponds to your raw total score.

Example: Suppose you chose the correct answers to 48 questions and incorrect answers to 15. Dividing 15 by 4 yields 3.75. Subtracting 3.75 from 48 equals 44.25, which is rounded to 44. The raw score of 44 corresponds to a scaled score of 930.

WORK SHEET for the GRE Mathematics Test, Form GR9767
Answer Key and Percentages* of Examinees Answering Each Question Correctly

QUESTION Number	Answer	P+	TOTAL C	I
1	C	76		
2	D	86		
3	C	87		
4	B	85		
5	E	81		
6	C	88		
7	B	80		
8	E	72		
9	A	79		
10	B	55		
11	A	82		
12	C	73		
13	A	70		
14	C	65		
15	B	61		
16	D	83		
17	E	80		
18	E	63		
19	B	59		
20	D	48		
21	D	70		
22	B	73		
23	C	71		
24	C	48		
25	D	73		
26	B	69		
27	E	61		
28	C	63		
29	B	58		
30	D	56		
31	C	62		
32	D	56		
33	A	70		
34	A	54		
35	B	48		

QUESTION Number	Answer	P+	TOTAL C	I
36	D	37		
37	B	32		
38	B	51		
39	B	50		
40	E	49		
41	E	43		
42	D	59		
43	B	60		
44	D	37		
45	A	51		
46	C	45		
47	E	32		
48	A	35		
49	E	42		
50	D	37		
51	D	32		
52	D	51		
53	D	30		
54	A	38		
55	D	34		
56	E	17		
57	C	21		
58	C	39		
59	A	29		
60	E	23		
61	E	25		
62	A	32		
63	D	21		
64	C	35		
65	A	39		
66	E	53		

Correct (C) _____

Incorrect (I) _____

Total Score:

C – I/4 = _____

Scaled Score (SS) = _____

*The P+ column indicates the percent of GRE Mathematics Test examinees who answered each question correctly; it is based on a sample of December 1997 examinees selected to represent all GRE Mathematics Test examinees tested between October 1, 1996 and September 30, 1999.

Score Conversions and Percents Below* for GRE Mathematics Test, Form GR9767

TOTAL SCORE					
Raw Score	Scaled Score	%	Raw Score	Scaled Score	%
50-66	990	82	23	690	33
49	980	81	22	680	31
48	970	80	21	670	30
47	960	79	20	660	28
46	950	77	19	650	26
45	940	75	18	640	24
44	930	74	17	630	23
43	920	72	16	610	20
42	910	71	15	600	18
41	890	67	14	590	17
40	880	66	13	580	15
39	870	64	12	570	13
38	860	62	11	560	12
37	850	61	10	550	11
36	840	59	9	540	10
35	830	57	8	530	8
34	820	55	7	510	6
33	800	52	6	500	5
32	790	51	5	490	4
31	780	49	4	480	2
30	770	47	3	470	2
29	760	45	2	460	1
28	750	43	1	450	1
27	740	41	0	440	0
26	730	39			
25	720	38			
24	700	35			

*Percent scoring below the scaled score is based on the performance of 7,092 examinees who took the GRE Mathematics Test between October 1, 1996 and September 30, 1999. This percent below information was used for score reports during the 2000-01 testing year.

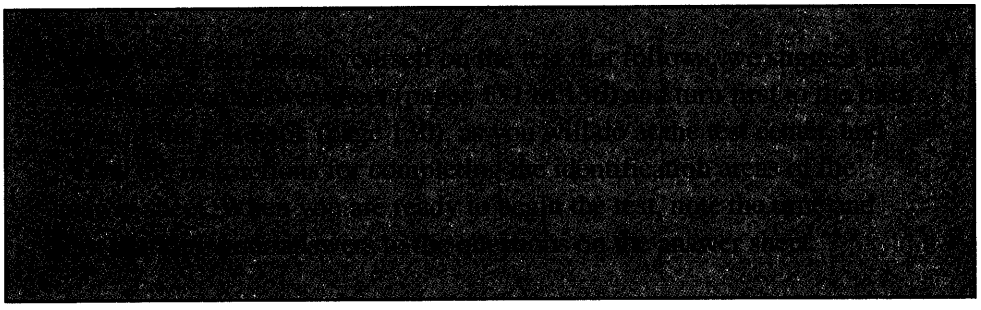
EVALUATING YOUR PERFORMANCE (GR9767)

Now that you have scored your test, you may wish to compare your performance with the performance of others who took this test. A worksheet and table are provided, both using performance data from GRE Mathematics Test examinees.

The worksheet (on page 82) is based on the performance of a sample of the examinees who took this particular test in December 1997. This sample was selected to represent the total population of GRE Mathematics Test examinees tested between October 1996 and September 1999. On the worksheet you used to determine your score is a column labeled "P+." The numbers in this column indicate the percent of the examinees in this sample who answered each question correctly. You may use these numbers as a guide for evaluating your performance on each test question.

The table (on page 83) contains, for each scaled score, the percentage of examinees tested between October 1996 and September 1999 who received lower scores. Interpretive data based on the scores earned by examinees tested in this three-year period were used by admissions officers in 2000-2001. These percentages appear in the score conversion table in a column to the right of the scaled scores. For example, in the percent column opposite the scaled score of 870 is the number 64. This means that 64 percent of the GRE Mathematics Test examinees tested between October 1996 and September 1999 scored lower than 870. To compare yourself with this population, look at the percent next to the scaled score you earned on the practice test. This number is a reasonable indication of your rank among GRE Mathematics Test examinees if you followed the test-taking suggestions in this practice book.

It is important to realize that the conditions under which you tested yourself were not exactly the same as those you will encounter at a test center. It is impossible to predict how different test-taking conditions will affect test performance, and this is only one factor that may account for differences between your practice test scores and your actual test scores. By comparing your performance on this practice test with the performance of other GRE Mathematics Test examinees, however, you will be able to determine your strengths and weaknesses and can then plan a program of study to prepare yourself for taking the GRE Mathematics Test under standard conditions.



FORM GR9767

67

GRE®

THE GRADUATE RECORD EXAMINATIONS®

MATHEMATICS TEST

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Princeton, N.J. 08541

MATHEMATICS TEST

Time—170 minutes

66 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is the best of the choices offered and then mark the corresponding space on the answer sheet.

Computation and scratchwork may be done in this examination book.

Note: In this examination:

(1) All logarithms are to the base e unless otherwise specified.
(2) The set of all x such that $a \leq x \leq b$ is denoted by $[a, b]$.

---

1. If $F(x) = \int_e^x \log t \, dt$ for all positive x, then $F'(x) =$

(A) x

(B) $\frac{1}{x}$

(C) $\log x$

(D) $x \log x$

(E) $x \log x - 1$

---

2. If $F(1) = 2$ and $F(n) = F(n-1) + \frac{1}{2}$ for all integers $n > 1$, then $F(101) =$

(A) 49
(B) 50
(C) 51
(D) 52
(E) 53

GO ON TO THE NEXT PAGE.

SCRATCHWORK

3. If $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is invertible under matrix multiplication, then its inverse is

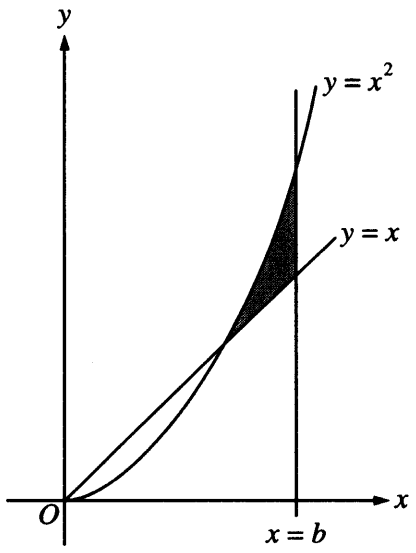
(A) $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

(B) $\frac{1}{a^2 + b^2}\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

(C) $\frac{1}{a^2 + b^2}\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

(D) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

(E) $\frac{1}{a^2 - b^2}\begin{pmatrix} -b & a \\ a & b \end{pmatrix}$



4. If $b > 0$ and if $\int_0^b x\,dx = \int_0^b x^2\,dx$, then the area of the shaded region in the figure above is

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

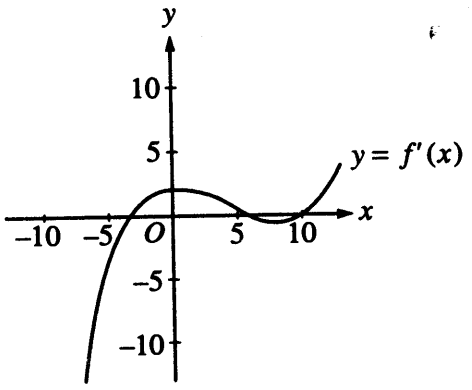
(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$

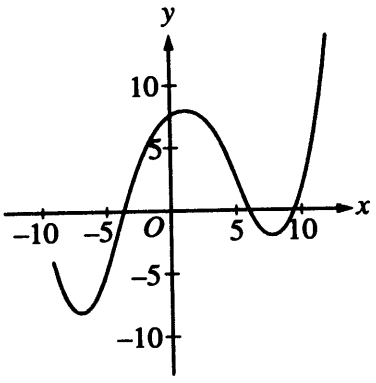
GO ON TO THE NEXT PAGE.

SCRATCHWORK

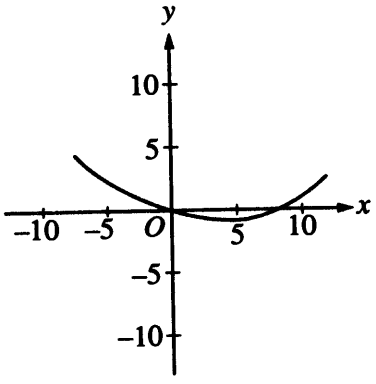


5. If the figure above is the graph of $y = f'(x)$, which of the following could be the graph of $y = f(x)$?

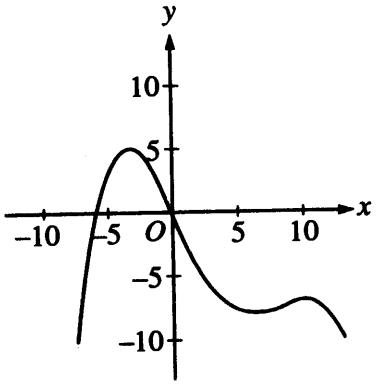
(A)



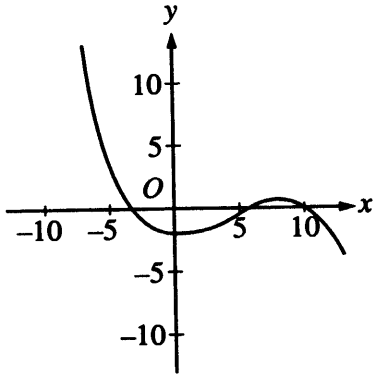
(B)



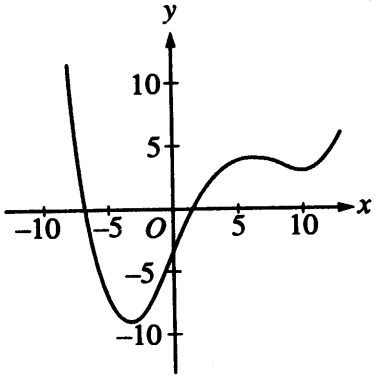
(C)



(D)



(E)



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SCRATCHWORK

6. Consider the following sequence of instructions.

1. Set $k = 999$, $i = 1$, and $p = 0$.
2. If $k > i$, then go to step 3; otherwise go to step 5.
3. Replace i with $2i$ and replace p with $p + 1$.
4. Go to step 2.
5. Print p.

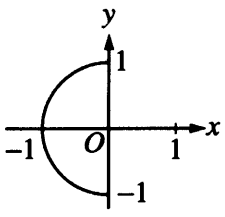
If these instructions are followed, what number will be printed at step 5 ?

(A) 1
(B) 2
(C) 10
(D) 512
(E) 999

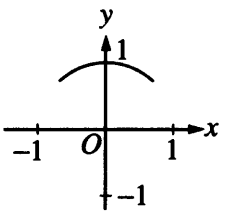
---

7. Which of the following indicates the graph of $\left\{(\sin t, \cos t): -\frac{\pi}{2} \leq t \leq 0\right\}$ in the xy-plane?

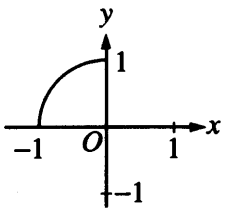
(A)



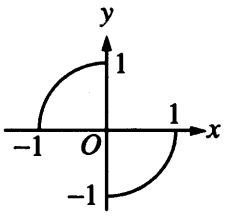
(D)



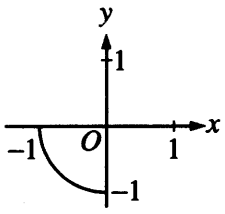
(B)



(E)



(C)



GO ON TO THE NEXT PAGE.

SCRATCHWORK

8. $\int_0^1 \frac{x}{1+x^2}\,dx =$

(A) 1

(B) $\frac{\pi}{4}$

(C) $\tan^{-1}\frac{\sqrt{2}}{2}$

(D) $\log 2$

(E) $\log \sqrt{2}$

---

9. If S is a nonempty finite set with k elements, then the number of one-to-one functions from S onto S is

(A) $k!$

(B) k^2

(C) k^k

(D) 2^k

(E) 2^{k+1}

---

10. Let g be the function defined on the set of all real numbers by

$$g(x) = \begin{cases} 1 & \text{if } x \text{ is rational,} \\ e^x & \text{if } x \text{ is irrational.} \end{cases}$$

Then the set of numbers at which g is continuous is

(A) the empty set
(B) $\{0\}$
(C) $\{1\}$
(D) the set of rational numbers
(E) the set of irrational numbers

---

11. For all real numbers x and y, the expression $\frac{x+y+|x-y|}{2}$ is equal to

(A) the maximum of x and y

(B) the minimum of x and y

(C) $|x+y|$

(D) the average of $|x|$ and $|y|$

(E) the average of $|x+y|$ and $x-y$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

12. Let B be a nonempty bounded set of real numbers and let b be the least upper bound of B. If b is not a member of B, which of the following is necessarily true?

(A) B is closed.
(B) B is not open.
(C) b is a limit point of B.
(D) No sequence in B converges to b.
(E) There is an open interval containing b but containing no point of B.

---

13. A drawer contains 2 blue, 4 red, and 2 yellow socks. If 2 socks are to be randomly selected from the drawer, what is the probability that they will be the same color?

(A) $\frac{2}{7}$

(B) $\frac{2}{5}$

(C) $\frac{3}{7}$

(D) $\frac{1}{2}$

(E) $\frac{3}{5}$

---

14. Let R be the set of real numbers and let f and g be functions from R into R. The negation of the statement

"For each s in R, there exists an r in R such that if $f(r) > 0$, then $g(s) > 0$."

is which of the following?

(A) For each s in R, there does not exist an r in R such that if $f(r) > 0$, then $g(s) > 0$.
(B) For each s in R, there exists an r in R such that $f(r) > 0$ and $g(s) \leq 0$.
(C) There exists an s in R such that for each r in R, $f(r) > 0$ and $g(s) \leq 0$.
(D) There exists an s in R and there exists an r in R such that $f(r) \leq 0$ and $g(s) \leq 0$.
(E) For each r in R, there exists an s in R such that $f(r) \leq 0$ and $g(s) \leq 0$.

---

15. If g is a function defined on the open interval (a, b) such that $a < g(x) < x$ for all $x \in (a, b)$, then g is

(A) an unbounded function
(B) a nonconstant function
(C) a nonnegative function
(D) a strictly increasing function
(E) a polynomial function of degree 1

GO ON TO THE NEXT PAGE.

SCRATCHWORK

16. For what value (or values) of m is the vector $(1, 2, m, 5)$ a linear combination of the vectors $(0, 1, 1, 1)$, $(0, 0, 0, 1)$, and $(1, 1, 2, 0)$?

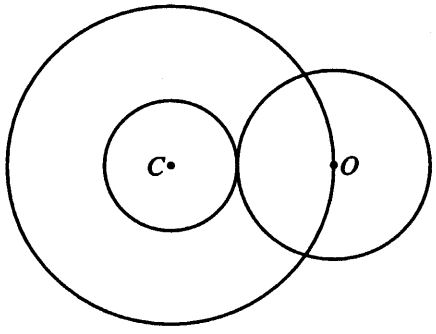
(A) For no value of m
(B) -1 only
(C) 1 only
(D) 3 only
(E) For infinitely many values of m

---

17. For a function f, the finite differences $\Delta f(x)$ and $\Delta^2 f(x)$ are defined by $\Delta f(x) = f(x + 1) - f(x)$ and $\Delta^2 f(x) = \Delta f(x + 1) - \Delta f(x)$. What is the value of $f(4)$, given the following partially completed finite difference table?

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
1	-1	4	
2		-2	6
3			
4			

(A) -5
(B) -1
(C) 1
(D) 3
(E) 5



18. In the figure above, the annulus with center C has inner radius r and outer radius 1. As r increases, the circle with center O contracts and remains tangent to the inner circle. If $A(r)$ is the area of the annulus and $a(r)$ is the area of the circular region with center O, then $\lim_{r \to 1^-} \frac{A(r)}{a(r)} =$

(A) 0
(B) $\frac{2}{\pi}$
(C) 1
(D) $\frac{\pi}{2}$
(E) ∞

GO ON TO THE NEXT PAGE.

SCRATCHWORK

19. Which of the following are multiplication tables for groups with four elements?

I.

	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

II.

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	a
d	d	c	a	b

III.

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	c	d
d	d	c	d	c

(A) None
(B) I only
(C) I and II only
(D) II and III only
(E) I, II, and III

---

20. Which of the following statements are true for every function f, defined on the set of all real numbers, such that $\lim_{x \to 0} \frac{f(x)}{x}$ is a real number L and $f(0) = 0$?

I. f is differentiable at 0.

II. $L = 0$

III. $\lim_{x \to 0} f(x) = 0$

(A) None
(B) I only
(C) III only
(D) I and III only
(E) I, II, and III

---

21. What is the area of the region bounded by the coordinate axes and the line tangent to the graph of $y = \frac{1}{8}x^2 + \frac{1}{2}x + 1$ at the point $(0, 1)$?

(A) $\frac{1}{16}$

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) 1

(E) 2

GO ON TO THE NEXT PAGE.

SCRATCHWORK

22. Let Z be the group of all integers under the operation of addition. Which of the following subsets of Z is NOT a subgroup of Z ?

(A) $\{0\}$
(B) $\{n \in Z: n \geq 0\}$
(C) $\{n \in Z: n$ is an even integer$\}$
(D) $\{n \in Z: n$ is divisible by both 6 and 9$\}$
(E) Z

---

23. In the Euclidean plane, point A is on a circle centered at point O, and O is on a circle centered at A. The circles intersect at points B and C. What is the measure of angle BAC ?

(A) $60°$
(B) $90°$
(C) $120°$
(D) $135°$
(E) $150°$

---

24. Which of the following sets of vectors is a basis for the subspace of Euclidean 4-space consisting of all vectors that are orthogonal to both $(0, 1, 1, 1)$ and $(1, 1, 1, 0)$?

(A) $\{(0, -1, 1, 0)\}$
(B) $\{(1, 0, 0, 0), (0, 0, 0, 1)\}$
(C) $\{(-2, 1, 1, -2), (0, 1, -1, 0)\}$
(D) $\{(1, -1, 0, 1), (-1, 1, 0, -1), (0, 1, -1, 0)\}$
(E) $\{(0, 0, 0, 0), (-1, 1, 0, -1), (0, 1, -1, 0)\}$

---

25. Let f be the function defined by $f(x, y) = 5x - 4y$ on the region in the xy-plane satisfying the inequalities $x \leq 2$, $y \geq 0$, $x + y \geq 1$, and $y - x \leq 0$. The maximum value of f on this region is

(A) 1
(B) 2
(C) 5
(D) 10
(E) 15

GO ON TO THE NEXT PAGE.

SCRATCHWORK

26. Let f be the function defined by

$$f(x) = \begin{cases} -x^2 + 4x - 2 & \text{if } x < 1, \\ -x^2 + 2 & \text{if } x \geq 1. \end{cases}$$

Which of the following statements about f is true?

(A) f has an absolute maximum at $x = 0$.
(B) f has an absolute maximum at $x = 1$.
(C) f has an absolute maximum at $x = 2$.
(D) f has no absolute maximum.
(E) f has local maxima at both $x = 0$ and $x = 2$.

---

27. Let f be a function such that $f(x) = f(1 - x)$ for all real numbers x. If f is differentiable everywhere, then $f'(0) =$

(A) $f(0)$
(B) $f(1)$
(C) $-f(0)$
(D) $f'(1)$
(E) $-f'(1)$

---

28. If V_1 and V_2 are 6-dimensional subspaces of a 10-dimensional vector space V, what is the smallest possible dimension that $V_1 \cap V_2$ can have?

(A) 0
(B) 1
(C) 2
(D) 4
(E) 6

---

29. Assume that p is a polynomial function on the set of real numbers. If $p(0) = p(2) = 3$ and $p'(0) = p'(2) = -1$, then $\int_0^2 xp''(x)\,dx =$

(A) -3
(B) -2
(C) -1
(D) 1
(E) 2

GO ON TO THE NEXT PAGE.

SCRATCHWORK

30. Suppose B is a basis for a real vector space V of dimension greater than 1. Which of the following statements could be true?

(A) The zero vector of V is an element of B.
(B) B has a proper subset that spans V.
(C) B is a proper subset of a linearly independent subset of V.
(D) There is a basis for V that is disjoint from B.
(E) One of the vectors in B is a linear combination of the other vectors in B.

---

31. Which of the following CANNOT be a root of a polynomial in x of the form $9x^5 + ax^3 + b$, where a and b are integers?

(A) -9

(B) -5

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) 9

---

32. When 20 children in a classroom line up for lunch, Pat insists on being somewhere ahead of Lynn. If Pat's demand is to be satisfied, in how many ways can the children line up?

(A) 20!

(B) 19!

(C) 18!

(D) $\frac{20!}{2}$

(E) $20 \cdot 19$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

33. How many integers from 1 to 1,000 are divisible by 30 but not by 16 ?

(A) 29
(B) 31
(C) 32
(D) 33
(E) 38

---

34. Suppose f is a differentiable function for which $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} f'(x)$ both exist and are finite. Which of the following must be true?

(A) $\lim_{x\to\infty} f'(x) = 0$

(B) $\lim_{x\to\infty} f''(x) = 0$

(C) $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} f'(x)$

(D) f is a constant function.

(E) f' is a constant function.

---

35. In xyz-space, an equation of the plane tangent to the surface $z = e^{-x}\sin y$ at the point where $x = 0$ and $y = \frac{\pi}{2}$ is

(A) $x + y = 1$
(B) $x + z = 1$
(C) $x - z = 1$
(D) $y + z = 1$
(E) $y - z = 1$

---

36. For each real number x, let $\mu(x)$ be the mean of the numbers 4, 9, 7, 5, and x; and let $\eta(x)$ be the median of these five numbers. For how many values of x is $\mu(x) = \eta(x)$?

(A) None
(B) One
(C) Two
(D) Three
(E) Infinitely many

GO ON TO THE NEXT PAGE.

SCRATCHWORK

37. $\sum_{k=1}^{\infty} \frac{k^2}{k!} =$

(A) e

(B) $2e$

(C) $(e + 1)(e - 1)$

(D) e^2

(E) ∞

---

38. Which of the following integrals on the interval $\left[0, \frac{\pi}{4}\right]$ has the greatest value?

(A) $\int_0^{\frac{\pi}{4}} \sin t \, dt$

(B) $\int_0^{\frac{\pi}{4}} \cos t \, dt$

(C) $\int_0^{\frac{\pi}{4}} \cos^2 t \, dt$

(D) $\int_0^{\frac{\pi}{4}} \cos 2t \, dt$

(E) $\int_0^{\frac{\pi}{4}} \sin t \cos t \, dt$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

39. Consider the function f defined by $f(x) = e^{-x}$ on the interval $[0, 10]$. Let $n > 1$ and let $x_0, x_1, \ldots, x_n$ be numbers such that $0 = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = 10$. Which of the following is greatest?

(A) $\sum_{j=1}^{n} f(x_j)(x_j - x_{j-1})$

(B) $\sum_{j=1}^{n} f(x_{j-1})(x_j - x_{j-1})$

(C) $\sum_{j=1}^{n} f\left(\frac{x_j + x_{j-1}}{2}\right)(x_j - x_{j-1})$

(D) $\int_0^{10} f(x)\,dx$

(E) 0

---

40. A fair coin is to be tossed 8 times. What is the probability that more of the tosses will result in heads than will result in tails?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{87}{256}$

(D) $\frac{23}{64}$

(E) $\frac{93}{256}$

---

41. The function $f(x, y) = xy - x^3 - y^3$ has a relative maximum at the point

(A) (0, 0)

(B) (1, 1)

(C) (–1, –1)

(D) (1, 3)

(E) $\left(\frac{1}{3}, \frac{1}{3}\right)$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

42. Consider the points $A = (-1, 2)$, $B = (6, 4)$, and $C = (1, -20)$ in the plane. For how many different points D in the plane are A, B, C, and D the vertices of a parallelogram?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

---

43. If A is a 3×3 matrix such that $A\begin{pmatrix}0\\1\\2\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$ and $A\begin{pmatrix}3\\4\\5\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix}$, then the product $A\begin{pmatrix}6\\7\\8\end{pmatrix}$ is

(A) $\begin{pmatrix}0\\0\\1\end{pmatrix}$

(B) $\begin{pmatrix}-1\\2\\0\end{pmatrix}$

(C) $\begin{pmatrix}1\\-1\\0\end{pmatrix}$

(D) $\begin{pmatrix}9\\10\\11\end{pmatrix}$

(E) not uniquely determined by the information given

---

44. Let f denote the function defined for all $x > 0$ by $f(x) = (\sqrt{x})^x$. Which of the following statements is FALSE?

(A) $\lim_{x \to 0^+} f(x) = 1$

(B) $\lim_{x \to \infty} f(x) = \infty$

(C) $f(x) = x^{x/2}$ for all $x > 0$.

(D) The derivative $f'(x)$ is positive for all $x > 0$.

(E) The derivative $f'(x)$ is increasing for all $x > 0$.

GO ON TO THE NEXT PAGE.

SCRATCHWORK

45. An experimental car is found to have a fuel efficiency $E(v)$, in miles per gallon of fuel, where v is the speed of the car, in miles per hour. For a certain 4-hour trip, if $v = v(t)$ is the speed of the car t hours after the trip started, which of the following integrals represents the number of gallons of fuel that the car used on the trip?

(A) $\displaystyle\int_0^4 \frac{v(t)}{E(v(t))}\,dt$

(B) $\displaystyle\int_0^4 \frac{E(v(t))}{v(t)}\,dt$

(C) $\displaystyle\int_0^4 \frac{tv(t)}{E(v(t))}\,dt$

(D) $\displaystyle\int_0^4 \frac{tE(v(t))}{v(t)}\,dt$

(E) $\displaystyle\int_0^4 v(t)E(v(t))\,dt$

---

46. For $0 < t < \pi$, the matrix $\begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$ has distinct complex eigenvalues λ_1 and λ_2. For what value of t, $0 < t < \pi$, is $\lambda_1 + \lambda_2 = 1$?

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

(E) $\frac{2\pi}{3}$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

47. Let x and y be uniformly distributed, independent random variables on $[0, 1]$. The probability that the distance between x and y is less than $\frac{1}{2}$ is

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

(E) $\frac{3}{4}$

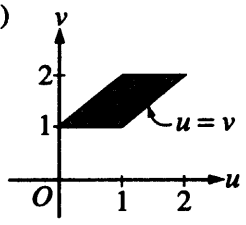
---

48. Consider the change of variables from the xy-plane to the uv-plane given by the equations

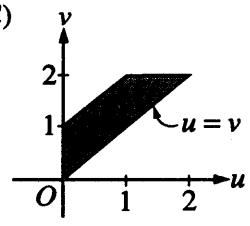
$$u = x^{1/3} + y$$
$$v = 1 + y.$$

Under this transformation, the image of the region $\{(x, y)\colon 0 \le x \le 1 \text{ and } 0 \le y \le 1\}$ is which of the following shaded regions?

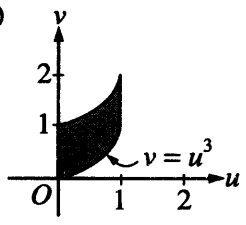
(A)



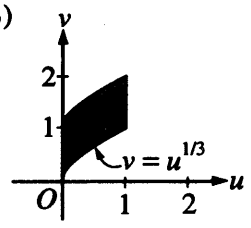
(C)



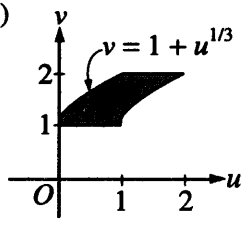
(E)



(B)



(D)



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SCRATCHWORK

49. If f is a continuous function on the set of real numbers and if a and b are real numbers, which of the following must be true?

I. $\int_a^b f(x)\,dx = \int_{a+3}^{b+3} f(x-3)\,dx$

II. $\int_a^b f(x)\,dx = \int_a^3 f(x)\,dx - \int_b^3 f(x)\,dx$

III. $\int_{3a}^{3b} f(x)\,dx = 3\int_a^b f(3x)\,dx$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

---

50. How many continuous real-valued functions f are there with domain $[-1, 1]$ such that $(f(x))^2 = x^2$ for each x in $[-1, 1]$?

(A) One
(B) Two
(C) Three
(D) Four
(E) Infinitely many

---

51. Let D be the region in the xy-plane in which the series $\sum_{k=1}^{\infty} \frac{(x+2y)^k}{k}$ converges. Then the interior of D is

(A) an open disk
(B) the open region bounded by an ellipse
(C) the open region bounded by a quadrilateral
(D) the open region between two parallel lines
(E) an open half plane

GO ON TO THE NEXT PAGE.

SCRATCHWORK

52. Consider the following system of linear equations over the real numbers, where x, y, and z are variables and b is a real constant.

$$\begin{aligned} x + y + z &= 0 \\ x + 2y + 3z &= 0 \\ x + 3y + bz &= 0 \end{aligned}$$

Which of the following statements are true?

I. There exists a value of b for which the system has no solution.
II. There exists a value of b for which the system has exactly one solution.
III. There exists a value of b for which the system has more than one solution.

(A) II only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

---

53. In the complex plane, let C be the circle $|z| = 2$ with positive (counterclockwise) orientation. Then

$$\int_C \frac{dz}{(z-1)(z+3)^2} =$$

(A) 0

(B) $2\pi i$

(C) $\frac{\pi i}{2}$

(D) $\frac{\pi i}{8}$

(E) $\frac{\pi i}{16}$

---

54. The inside of a certain water tank is a cube measuring 10 feet on each edge and having vertical sides and no top. Let $h(t)$ denote the water level, in feet, above the floor of the tank at time t seconds. Starting at time $t = 0$, water pours into the tank at a constant rate of 1 cubic foot per second, and simultaneously, water is removed from the tank at a rate of $0.25h(t)$ cubic feet per second. As $t \to \infty$, what is the limit of the volume of the water in the tank?

(A) 400 cubic feet
(B) 600 cubic feet
(C) 1,000 cubic feet
(D) The limit does not exist.
(E) The limit exists, but it cannot be determined without knowing $h(0)$.

GO ON TO THE NEXT PAGE.

SCRATCHWORK

55. Suppose that f is a twice-differentiable function on the set of real numbers and that $f(0)$, $f'(0)$, and $f''(0)$ are all negative. Suppose f'' has all three of the following properties.

I. It is increasing on the interval $[0, \infty)$.
II. It has a unique zero in the interval $[0, \infty)$.
III. It is unbounded on the interval $[0, \infty)$.

Which of the same three properties does f necessarily have?

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

---

56. For every set S and every metric d on S, which of the following is a metric on S ?

(A) $4 + d$
(B) $e^d - 1$
(C) $d - |d|$
(D) d^2
(E) $\sqrt{d}$

---

57. Let R be the field of real numbers and $R[x]$ the ring of polynomials in x with coefficients in R. Which of the following subsets of $R[x]$ is a subring of $R[x]$?

I. All polynomials whose coefficient of x is zero
II. All polynomials whose degree is an even integer, together with the zero polynomial
III. All polynomials whose coefficients are rational numbers

(A) I only
(B) II only
(C) I and III only
(D) II and III only
(E) I, II, and III

---

58. Let f be a real-valued function defined and continuous on the set of real numbers R. Which of the following must be true of the set $S = \{f(c)\colon\ 0 < c < 1\}$?

I. S is a connected subset of R.
II. S is an open subset of R.
III. S is a bounded subset of R.

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

GO ON TO THE NEXT PAGE.

SCRATCHWORK

59. A cyclic group of order 15 has an element x such that the set $\{x^3, x^5, x^9\}$ has exactly two elements. The number of elements in the set $\{x^{13n}: n \text{ is a positive integer}\}$ is

(A) 3
(B) 5
(C) 8
(D) 15
(E) infinite

---

60. If S is a ring with the property that $s = s^2$ for each $s \in S$, which of the following must be true?

I. $s + s = 0$ for each $s \in S$.
II. $(s + t)^2 = s^2 + t^2$ for each $s, t \in S$.
III. S is commutative.

(A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

---

61. What is the greatest integer that divides $p^4 - 1$ for every prime number p greater than 5 ?

(A) 12
(B) 30
(C) 48
(D) 120
(E) 240

---

62. The coefficient of x^3 in the expansion of $(1 + x)^3(2 + x^2)^{10}$ is

(A) 2^{14}

(B) 31

(C) $\binom{3}{3} + \binom{10}{1}$

(D) $\binom{3}{3} + 2\binom{10}{1}$

(E) $\binom{3}{3}\binom{10}{1}2^9$

GO ON TO THE NEXT PAGE.

SCRATCHWORK

63. At how many points in the xy-plane do the graphs of $y = x^{12}$ and $y = 2^x$ intersect?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

---

64. Suppose that f is a continuous real-valued function defined on the closed interval $[0, 1]$. Which of the following must be true?

I. There is a constant $C > 0$ such that $|f(x) - f(y)| \leq C$ for all x and y in $[0, 1]$.

II. There is a constant $D > 0$ such that $|f(x) - f(y)| \leq 1$ for all x and y in $[0, 1]$ that satisfy $|x - y| \leq D$.

III. There is a constant $E > 0$ such that $|f(x) - f(y)| \leq E|x - y|$ for all x and y in $[0, 1]$.

(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

---

65. Let $p(x)$ be the polynomial $x^3 + ax^2 + bx + c$, where a, b, and c are real constants. If $p(-3) = p(2) = 0$ and $p'(-3) < 0$, which of the following is a possible value of c ?

(A) -27

(B) -18

(C) -6

(D) -3

(E) $-\frac{1}{2}$

---

66. In the xy-plane, if C is the circle $x^2 + y^2 = 9$, oriented counterclockwise, then $\oint_C -2y\,dx + x^2\,dy =$

(A) 0
(B) 6π
(C) 9π
(D) 12π
(E) 18π

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST.

SCRATCHWORK

I

NOTE: To ensure prompt processing of test results, it is important that you fill in the blanks exactly as directed.

SUBJECT TEST

A. Print and sign your full name in this box:

PRINT: _____
(LAST) (FIRST) (MIDDLE)

SIGN: _____

Copy this code in box 6 on your answer sheet. Then fill in the corresponding ovals exactly as shown.

6. TITLE CODE				
6	7	6	1	0
0	0	0	0	●
1	1	1	●	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
●	6	●	6	6
7	●	7	7	7
8	8	8	8	8
9	9	9	9	9

Copy the Test Name and Form Code in box 7 on your answer sheet.

TEST NAME Mathematics

FORM CODE GR9367

GRADUATE RECORD EXAMINATIONS SUBJECT TEST

B. The Subject Tests are intended to measure your achievement in a specialized field of study. Most of the questions are concerned with subject matter that is probably familiar to you, but some of the questions may refer to areas that you have not studied.

Your score will be determined by subtracting one-fourth the number of incorrect answers from the number of correct answers. Questions for which you mark no answer or more than one answer are not counted in scoring. If you have some knowledge of a question and are able to rule out one or more of the answer choices as incorrect, your chances of selecting the correct answer are improved, and answering such questions will likely improve your score. It is unlikely that pure guessing will raise your score; it may lower your score.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not spend too much time on questions that are too difficult for you. Go on to the other questions and come back to the difficult ones later if you can.

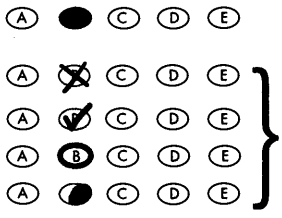
YOU MUST INDICATE ALL YOUR ANSWERS ON THE SEPARATE ANSWER SHEET. No credit will be given for anything written in this examination book, but you may write in the book as much as you wish to work out your answers. After you have decided on your response to a question, fill in the corresponding oval on the answer sheet. BE SURE THAT EACH MARK IS DARK AND COMPLETELY FILLS THE OVAL. Mark only one answer to each question. No credit will be given for multiple answers. Erase all stray marks. If you change an answer, be sure that all previous marks are erased completely. Incomplete erasures may be read as intended answers. Do not be concerned that the answer sheet provides spaces for more answers than there are questions in the test.

Example:

What city is the capital of France?

(A) Rome
(B) Paris
(C) London
(D) Cairo
(E) Oslo

Sample Answer



DO NOT OPEN YOUR TEST BOOK UNTIL YOU ARE TOLD TO DO SO.

DO NOT USE INK

Use only a pencil with soft, black lead (No. 2 or HB) to complete this answer sheet.
Be sure to fill in completely the space that corresponds to your answer choice.
Completely erase any errors or stray marks.

GRADUATE RECORD EXAMINATIONS® - GRE® - SUBJECT TEST

SIDE 1

1. NAME Enter your last name, first name initial (given name), and middle initial if you have one. Omit spaces, apostrophes, Jr., II., etc.

Last Name only (Family or Surname) - first 15 letters | First Name Initial | Middle Initial

2.

YOUR NAME: _____
(Print) Last Name (Family or Surname) First Name (Given) M.I.

MAILING ADDRESS: _____
(Print) P.O. Box or Street Address

City State or Province

Country Zip or Postal Code

CENTER: _____
City State or Province

Country Center Number Room Number

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE:
YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

3. DATE OF BIRTH

Month	Day	Year
Jan.		
Feb.		
Mar.	0 0	0 0
April	1 1	1 1
May	2 2	2 2
June	3 3	3 3
July	4 4	4 4
Aug.	5 5	5 5
Sept.	6 6	6 6
Oct.	7 7	7 7
Nov.	8 8	8 8
Dec.	9 9	9 9

4. SOCIAL SECURITY NUMBER (U.S.A. only)

5. REGISTRATION NUMBER (from your admission ticket)

6. TITLE CODE (on back cover of your test book)

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SHADED AREA FOR ETS USE ONLY

Item responses continued on reverse side.

SIDE 2

SUBJECT TEST

COMPLETE THE CERTIFICATION STATEMENT, THEN TURN ANSWER SHEET OVER TO SIDE 1.

CERTIFICATION STATEMENT

Please write the following statement below, DO NOT PRINT.
"I certify that I am the person whose name appears on this answer sheet. I agree not to disclose the contents of the test I am taking today to anyone."
Sign and date where indicated.

SIGNATURE: _____ DATE: ___/___/___
Month Day Y

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE: ●
YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

115 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	147 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	179 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	211 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
116 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	148 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	180 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	212 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
117 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	149 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	181 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	213 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
118 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	150 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	182 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	214 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
119 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	151 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	183 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	215 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
120 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	152 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	184 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	216 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
121 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	153 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	185 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	217 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
122 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	154 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	186 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	218 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
123 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	155 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	187 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	219 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
124 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	156 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	188 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	220 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
125 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	157 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	189 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	221 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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127 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	159 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	191 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	223 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
128 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	160 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	192 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	224 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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131 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	163 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	195 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	227 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
132 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	164 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	196 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	228 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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136 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	168 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	200 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	232 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
137 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	169 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	201 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	233 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
138 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	170 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	202 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	234 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
139 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	171 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	203 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	235 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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143 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	175 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	207 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	239 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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146 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	178 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	210 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	242 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

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TR	TW	TFS	TCS	1R	1W	1FS	1CS	2R	2W	2FS	2CS
FOR ETS USE ONLY				3R	3W	3FS	3CS	4R	4W	4FS	4CS
				5R	5W	5FS	5CS	6R	6W	6FS	6CS

DO NOT USE INK

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Be sure to fill in completely the space that corresponds to your answer choice.
Completely erase any errors or stray marks.

GRADUATE RECORD EXAMINATIONS® - GRE® - SUBJECT TEST

SIDE 1

1. NAME

Enter your last name, first name initial (given name), and middle initial if you have one.
Omit spaces, apostrophes, Jr., II., etc.

Last Name only (Family or Surname) - first 15 letters

First Name Initial

Middle Initial

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE:

YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

Item responses continued on reverse side.

2.

YOUR NAME: _____
(Print) Last Name (Family or Surname) First Name (Given) M.I.

MAILING ADDRESS: _____
(Print) P.O. Box or Street Address

City State or Province

Country Zip or Postal Code

CENTER: _____
City State or Province

Country Center Number Room Number

SIGNATURE: _____

3. DATE OF BIRTH

Month	Day	Year
Jan.		
Feb.		
Mar.	0 0	0 0
April	1 1	1 1
May	2 2	2 2
June	3 3	3 3
July	4 4	4 4
Aug.	5 5	5 5
Sept.	6 6	6 6
Oct.	7 7	7 7
Nov.	8 8	8 8
Dec.	9 9	9 9

4. SOCIAL SECURITY NUMBER

(U.S.A. only)

5. REGISTRATION NUMBER

(from your admission ticket)

6. TITLE CODE

(on back cover of your test book)

7. TEST NAME

(on back cover of your test book)

FORM CODE (on back cover of your test book)

8. TEST BOOK SERIAL NUMBER

(red number in upper right corner of front cover of your test book)

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SIDE 2

SUBJECT TEST

COMPLETE THE CERTIFICATION STATEMENT, THEN TURN ANSWER SHEET OVER TO SIDE 1.

CERTIFICATION STATEMENT

Please write the following statement below, DO NOT PRINT.
"I certify that I am the person whose name appears on this answer sheet. I also agree not to disclose the contents of the test I am taking today to anyone."
Sign and date where indicated.

SIGNATURE: _____ DATE: ____/____/____
Month Day Year

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE: ●
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115 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	147 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	179 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	211 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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TR	TW	TFS	TCS	1R	1W	1FS	1CS	2R	2W	2FS	2CS
FOR ETS USE ONLY				3R	3W	3FS	3CS	4R	4W	4FS	4CS
				5R	5W	5FS	5CS	6R	6W	6FS	6CS

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Completely erase any errors or stray marks.

GRADUATE RECORD EXAMINATIONS® - GRE® - SUBJECT TEST

SIDE 1

1. NAME Enter your last name, first name initial (given name), and middle initial if you have one. Omit spaces, apostrophes, Jr., II., etc.

Last Name only (Family or Surname) - first 15 letters | First Name Initial | Middle Initial

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE: ●

YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

Item responses continued on reverse side.

2.

YOUR NAME: _____
(Print) Last Name (Family or Surname) First Name (Given) M.I.

MAILING ADDRESS: _____
(Print) P.O. Box or Street Address

City State or Province

Country Zip or Postal Code

CENTER: _____
City State or Province

Country Center Number Room Number

SIGNATURE: _____

3. DATE OF BIRTH

Month: Jan., Feb., Mar., April, May, June, July, Aug., Sept., Oct., Nov., Dec.
Day
Year

4. SOCIAL SECURITY NUMBER (U.S.A. only)

5. REGISTRATION NUMBER (from your admission ticket)

6. TITLE CODE (on back cover of your test book)

7. TEST NAME (on back cover of your test book)

FORM CODE (on back cover of your test book)

8. TEST BOOK SERIAL NUMBER (red number in upper right corner of front cover of your test book)

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SIDE 2

SUBJECT TEST

COMPLETE THE CERTIFICATION STATEMENT, THEN TURN ANSWER SHEET OVER TO SIDE 1.

CERTIFICATION STATEMENT

Please write the following statement below, DO NOT PRINT.
"I certify that I am the person whose name appears on this answer sheet. I also agree not to disclose the contents of the test I am taking today to anyone."
Sign and date where indicated.

SIGNATURE: _____ DATE: ___/___/___ Month Day Year

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115	A	B	C	D	E	147	A	B	C	D	E	179	A	B	C	D	E	211	A	B	C	D	E			
116	A	B	C	D	E	148	A	B	C	D	E	180	A	B	C	D	E	212	A	B	C	D	E			
117	A	B	C	D	E	149	A	B	C	D	E	181	A	B	C	D	E	213	A	B	C	D	E			
118	A	B	C	D	E	150	A	B	C	D	E	182	A	B	C	D	E	214	A	B	C	D	E			
119	A	B	C	D	E	151	A	B	C	D	E	183	A	B	C	D	E	215	A	B	C	D	E			
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128	A	B	C	D	E	160	A	B	C	D	E	192	A	B	C	D	E	224	A	B	C	D	E			
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132	A	B	C	D	E	164	A	B	C	D	E	196	A	B	C	D	E	228	A	B	C	D	E			
133	A	B	C	D	E	165	A	B	C	D	E	197	A	B	C	D	E	229	A	B	C	D	E			
134	A	B	C	D	E	166	A	B	C	D	E	198	A	B	C	D	E	230	A	B	C	D	E			
135	A	B	C	D	E	167	A	B	C	D	E	199	A	B	C	D	E	231	A	B	C	D	E			
136	A	B	C	D	E	168	A	B	C	D	E	200	A	B	C	D	E	232	A	B	C	D	E			
137	A	B	C	D	E	169	A	B	C	D	E	201	A	B	C	D	E	233	A	B	C	D	E			
138	A	B	C	D	E	170	A	B	C	D	E	202	A	B	C	D	E	234	A	B	C	D	E			
139	A	B	C	D	E	171	A	B	C	D	E	203	A	B	C	D	E	235	A	B	C	D	E			
140	A	B	C	D	E	172	A	B	C	D	E	204	A	B	C	D	E	236	A	B	C	D	E			
141	A	B	C	D	E	173	A	B	C	D	E	205	A	B	C	D	E	237	A	B	C	D	E			
142	A	B	C	D	E	174	A	B	C	D	E	206	A	B	C	D	E	238	A	B	C	D	E			
143	A	B	C	D	E	175	A	B	C	D	E	207	A	B	C	D	E	239	A	B	C	D	E			
144	A	B	C	D	E	176	A	B	C	D	E	208	A	B	C	D	E	240	A	B	C	D	E			
145	A	B	C	D	E	177	A	B	C	D	E	209	A	B	C	D	E	241	A	B	C	D	E			
146	A	B	C	D	E	178	A	B	C	D	E	210	A	B	C	D	E	242	A	B	C	D	E			

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A. fill in both ovals here: ○ – ○ B. sign your full name here:

TR	TW	TFS	TCS	1R	1W	1FS	1CS	2R	2W	2FS	2CS
FOR ETS USE ONLY				3R	3W	3FS	3CS	4R	4W	4FS	4CS
				5R	5W	5FS	5CS	6R	6W	6FS	6CS

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