МИНИСТЕРСТВО ОБРАЗОВАНИЯ И НАУКИ
РОССИЙСКОЙ ФЕДЕРАЦИИ

НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ «МЭИ»

РАСЧЕТЫ НА ПРОЧНОСТЬ ЭЛЕМЕНТОВ ЭНЕРГЕТИЧЕСКОГО ОБОРУДОВАНИЯ

Курсовая работа
по курсу «Прикладная физика» 4 семестр

К защите
31.05.24

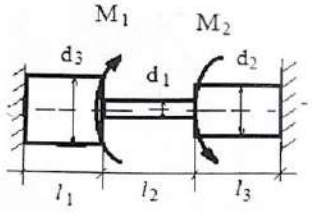
Студент: Маркаров М.Г.
Группа: ТФ-13-22
Преподаватель: Мамаева В.В.

Москва 2024

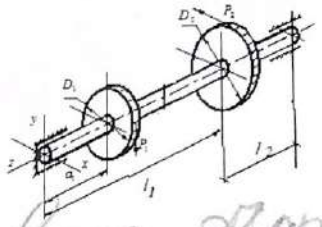
ИТАЭ Курсовая работа по курсу **Прикладная физика** Задание 6

Расчеты на прочность элементов энергетического оборудования

1



2,6



Вид Эпюр.

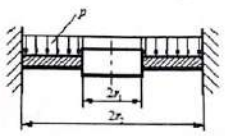
Группа ТФ- 13-22

Студент Марцаров М.Г.

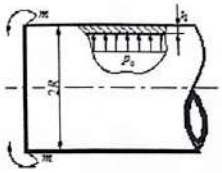
Вариант численных данных 15

Подпись преподавателя

4

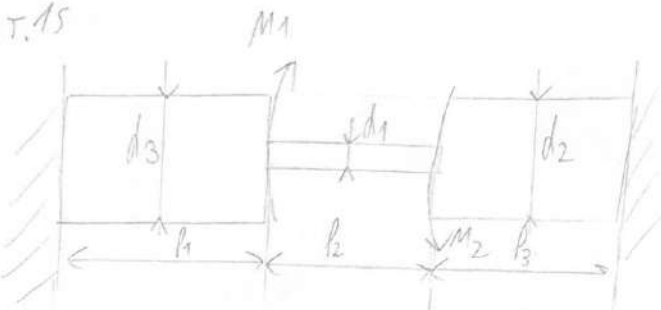


5



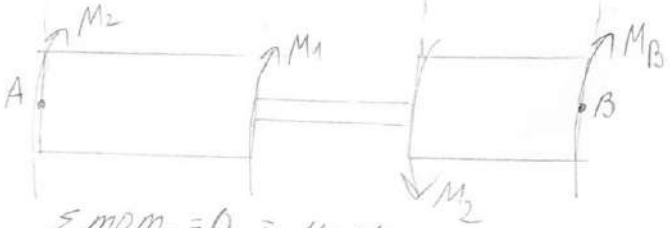
Вар. 15

№1



$l_1 = 2$ м
$l_2 = 0{,}6$ м
$l_3 = 2$ м
$d_1 = 10$ мм
$d_2 = 20$ мм
$d_3 = 28$ мм
$K = 3$
$M_2 = K M_1$

Система статически неопределима:



$\Sigma m_{om_z} = 0 = M_A + M_1 + M_B - M_2 = 0 \Rightarrow M_A + M_B = M_2 - M_1 \Rightarrow$

$\Rightarrow M_A + M_B = M_1(K-1)$. (2)

Точка A жестко не перемещается $\Rightarrow \varphi_A = 0$

$\varphi_A = \varphi_{M_1} + \varphi_{M_2} + \varphi_{M_B}$; $\varphi_i = \pm \frac{M_i l_i}{G I_i} \Rightarrow$

$\Rightarrow \varphi_A = -\frac{M_1 l_1}{G I_1} + \frac{M_2 l_1}{G I_1} + \frac{M_2 l_2}{G I_2} - \frac{M_B}{G}\left(\frac{l_1}{I_1} + \frac{l_2}{I_2} + \frac{l_3}{I_3}\right) = 0$ (1)

I_i - полярный момент инерции i-го интервала

$I_i = \frac{\pi d_i^4}{32}$; d_i - диаметр участка (не трубы, кольцо и пр.)

$I_1 = \frac{\pi d_3^4}{32}$; $I_2 = \frac{\pi d_1^4}{32}$; $I_3 = \frac{\pi d_2^4}{32}$

Из (1): $M_B = \frac{M_1}{\Sigma\left(\frac{l_i}{I_i}\right)}\left(-\frac{l_1}{I_1} + \frac{K l_1}{I_1} + \frac{K l_2}{I_2}\right) = M_1 \frac{\frac{32}{\pi}\left(\frac{2 \cdot 2}{(0{,}028)^4} + \frac{3 \cdot 0{,}6}{(0{,}01)^4}\right)}{\frac{32}{\pi}\left(\frac{2}{0{,}028^4} + \frac{0{,}6}{(0{,}01)^4} + \frac{2}{0{,}02^4}\right)}$ (2)

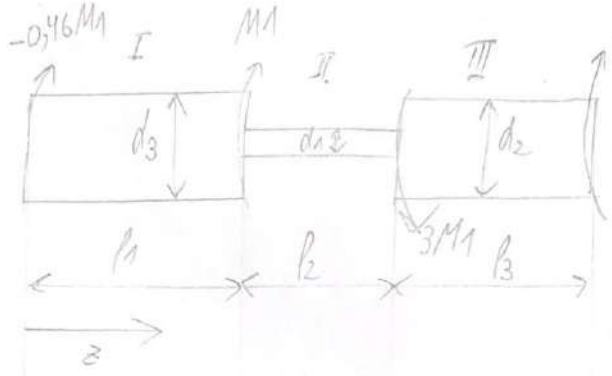
$= 2{,}462 M_1$

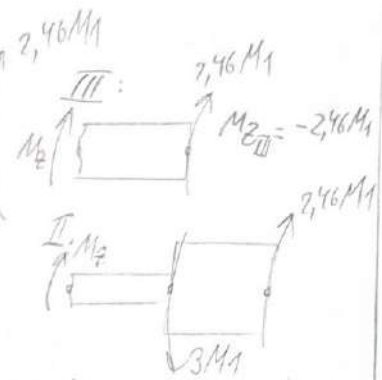
Из (2) $M_A = M_1(K-1) - M_B = 2M_1 - 2{,}462 M_1 = -0{,}46 M_1$

$M_A = -0{,}46 M_1$

$M_B = 2{,}462 M_1$

Построим эпюры крутящего момента M_z (см. след. лист)





$M_{z\,II} = 3M_1 - 2{,}46M_1 = 0{,}54M_1$

$I: M_z = 3M_1 - 2{,}46M_1 - M_1 = -0{,}46M_1$

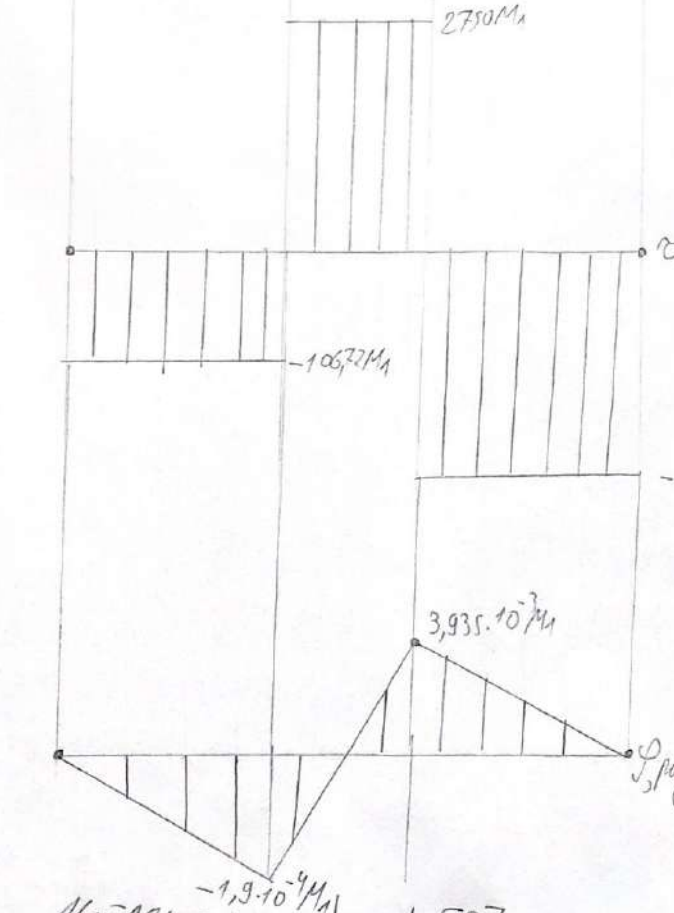
M_z, кН·м

$\tau_i = \frac{M_{zi}}{W_{pi}}$; $W_p = \frac{\pi d_i^3}{16}$

$\tau_I = \frac{-0{,}46M_1 \cdot 16}{\pi (0{,}028)^3} = -106{,}772M_1$ кПа

$\tau_{II} = \frac{0{,}54M_1 \cdot 16}{\pi (0{,}01)^3} = 2750M_1$ кПа

$\tau_{III} = \frac{-2{,}46M_1 \cdot 16}{\pi (0{,}02)^3} = -186M_1$ кПа



$\varphi_0 = 0$, заделка

$\varphi_{l_1} = \varphi_0 + \frac{M_{z\,I} \cdot l_1}{G I_1} = \frac{-0{,}46M_1 \cdot 2 \cdot 32}{80 \cdot 10^9 \pi (0{,}028)^4} = -1{,}9 \cdot 10^{-4} M_1$

$\varphi_{l_1+l_2} = -1{,}9 \cdot 10^{-4} M_1 + \frac{0{,}54 \cdot 0{,}6 \cdot 32}{80 \cdot 10^9 \pi \cdot (0{,}01)^4} = 3{,}935 \cdot 10^{-3} M_1$

$\varphi_{l_1+l_2+l_3} = 0$, заделка

Найдем $[M_1]$: $|\tau_{max}| \le [\tau]$

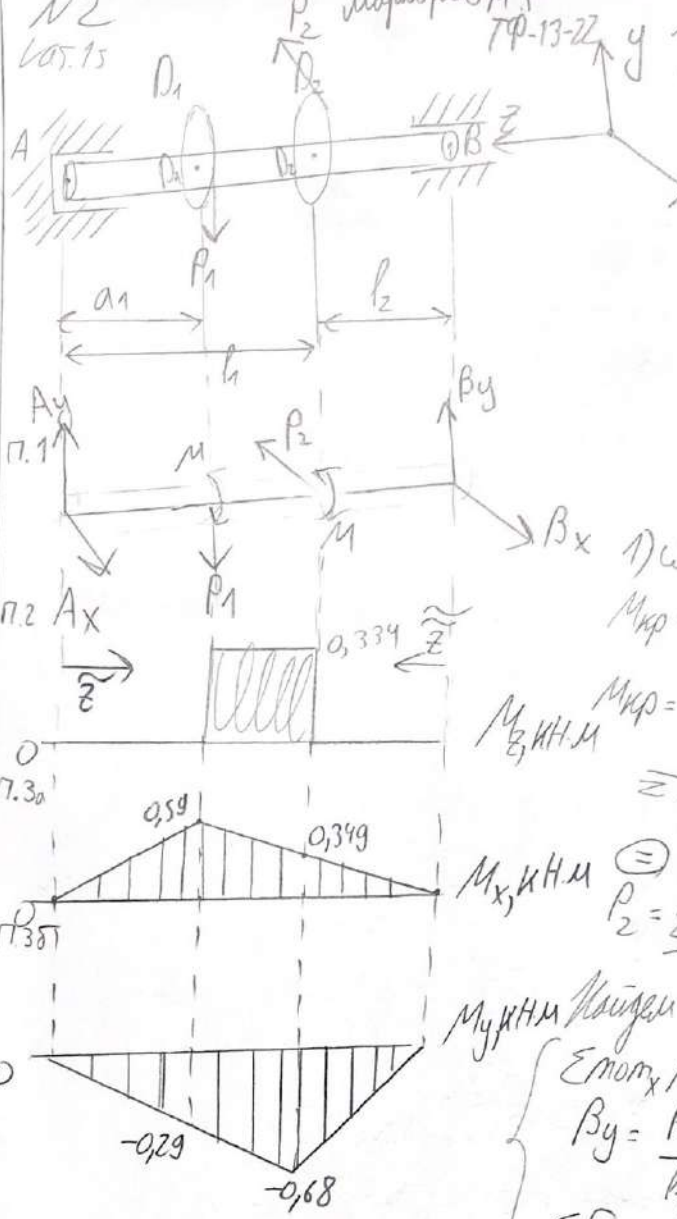
$[M_1] 2750 \cdot 10^3 \le 100 \cdot 10^6$

$[M_1] = 36{,}36$ Н·м

№2
Сб. 15

Марков М.Г.
ТФ-13-22

Расчет промежуточного вала редуктора



$N = 14 кВт$

$n_0 = 400 \frac{об}{мин}$

$D_1 = 0,3 м$

$D_2 = 0,35 м$

$l_1 = 0,8 м$

$l_2 = 0,65 м$

$a_1 = 0,35 м$

Ст. 35

1) $\omega = \frac{\pi n_0}{30} = \frac{400\pi}{30} \approx 41,888 \frac{рад}{с}$

$M_{кр} = \frac{N}{\omega} = \frac{14 \cdot 10^3}{41,888} = 0,334 кН \cdot м$

$M_{кр} = \frac{P_i D_i}{2} \quad \forall i = 1,2 \ldots N_{шлиц}$

$M_z, кН \cdot м$

$\Rightarrow P_1 = \frac{2 M_{кр}}{D_1} = \frac{2 \cdot 0,334}{0,3} (=)$

$M_x, кН \cdot м \; (=) \; 2,2267 кН \cdot м$

$P_2 = \frac{2 \cdot 0,334}{0,35} = 1,9086 кН$

$M_y, кН \cdot м$

Найдем опорные реакции:

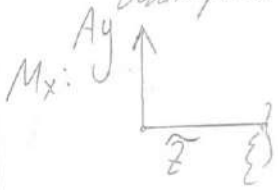
$$\begin{cases} \sum mom_x A = (l_1 + l_2) B_y - P_1 a_1 = 0 \\ B_y = \frac{P_1 a_1}{l_1 + l_2} = \frac{2,2267 \cdot 0,35}{0,8 + 0,65} = 0,5375 кН \\ \sum F_y = A_y + B_y - P_1 = 0 \Rightarrow A_y = P_1 - B_y \\ A_y = 2,2267 - 0,5375 = 1,6892 кН \end{cases}$$

$$\begin{cases} \sum mom_y A = l_1 P_2 - B_x (l_1 + l_2) = 0 \\ B_x = \frac{P_2 l_1}{l_1 + l_2} = \frac{1,9086 \cdot 0,8}{0,8 + 0,65} = 1,053 кН \\ \sum F_x = P_2 - A_x - B_x = 0 \Rightarrow A_x = P_2 - B_x \; (=) \\ (=) \; 1,9086 - 1,053 = 0,8556 кН \end{cases}$$

Построение M_x, M_y:

1) Введем ось $\tilde{z}$, нач. в т. A

интервал $0 \le \tilde{z} \le a_1$:



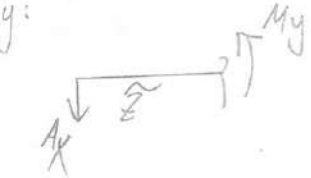
$\sum mom_{\xi} F_i = -A_y\tilde{z} + M_x = 0 \Rightarrow$

$M_x(\tilde{z}) = A_y\tilde{z}$

$M_x(0) = 0$

$M_x(a_1) = A_y a_1 = 1{,}6892 \cdot 0{,}35 = 0{,}5912$ кН·м

M_y:

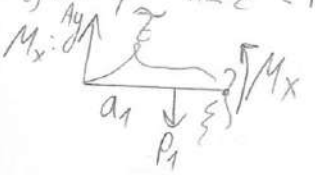


$M_y(\tilde{z}) = -A_x\tilde{z}$

$M_y(0) = 0$

$M_y(a_1) = -A_x a_1 = -0{,}35 \cdot 0{,}8556 = -0{,}29946$ кН·м

2) Интервал $a_1 \le \tilde{z} \le l_1$



$\sum mom_{\xi} F_i = -A_y\tilde{z} + M_x + P_1(\tilde{z} - a_1) = 0$

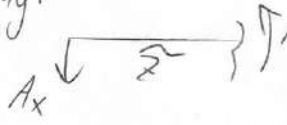
$\Rightarrow M_x(\tilde{z}) = A_y\tilde{z} - P_1(\tilde{z} - a_1)$

$M_x(a_1) = A_y a_1 = 0{,}5912$ кН·м

$M_x(l_1) = A_y l_1 - P_1(l_1 - a_1) = 1{,}6892 \cdot 0{,}8 - 2{,}2267(0{,}8 - 0{,}35) =$

$= 1{,}35136 - 1{,}002015 = 0{,}349345$ кН·м

M_y:



$M_y = -A_x\tilde{z}$

$M_y(a_1) = -A_x a_1 = -0{,}29946$ кН·м

$M_y(l_1) = -A_x l_1 = -0{,}8556 \cdot 0{,}8 = -0{,}68448$ кН·м

Введем ось $\tilde{\tilde{z}}$, нач. в т. B. Условие согласования осей:

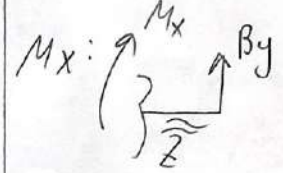
$f(\tilde{z}) = f(l_1 + l_2 - \tilde{\tilde{z}})$; $\tilde{z} \uparrow\downarrow \tilde{\tilde{z}}$

Интервал: $0 \le \tilde{\tilde{z}} \le l_2$

$M_y = -\tilde{\tilde{z}}B_x$ $M_y(0) = 0$

$M_y(l_2) = -B_x l_2 = -0{,}65 \cdot 1{,}053 \Rightarrow$

$\Rightarrow -0{,}68445$ кН·м



$M_x = B_y\tilde{\tilde{z}}$ $M_x(0) = 0$

$M_x(l_2) = B_y l_2 = 0{,}65 \cdot 0{,}5375 \Rightarrow$

$\Rightarrow 0{,}34934$ кН·м

Строим эпюры на пред. листе

П.4 Расчет на статическую прочность

Ищем опасную точку $\Rightarrow M_{экв}$ всей, $M_{экв} = \sqrt{\sum_{i=0,1,2} M_i^2}$ [i:0=x, 1=y, 2=z]

$\sigma_{экв} = \frac{M_{экв}}{W_{нэ}} \le [\sigma] \Leftrightarrow \sigma_{экв} \le \frac{\sigma_т}{[n]}$; $W_{нэ} = \frac{\pi \left(\frac{d}{2}\right)^3}{32}$

d - теор. диаметр.

Точки опасной точки: Точка должна лежать где-то между дисками, т.е. $\xi \in [a_1, b_1]$, иначе $M_z = 0$ и $M_{экв}$ не max.

Введем функцию: $f(\tilde{z}) = \sqrt{M_z^2(\tilde{z}) + M_x^2(\tilde{z}) + M_y^2(\tilde{z})}$; $\tilde{z} \in [a_1, b_1]$

M_z - const = 0,334

$M_x = A_y \tilde{z} - P_1(\tilde{z} - a_1) \le 1,6892\tilde{z} - 2,2267 \cdot x + 2,2267 \cdot 0,35 \le -0,5375\tilde{z} + 0,779$

$M_y = -A_x \tilde{z} = -0,8556\tilde{z}$

Найдем максимум $f(\tilde{z})$ на $[a_1, b_1]$:

$\max_{[35;08]} f(\tilde{z}) = \max \sqrt{0,334^2 + |-0,5375\tilde{z} + 0,779|^2 + |-0,8556\tilde{z}|^2} = f(b_1)$

В силу монотонного возрастания $f(\tilde{z})$ на $[a_1, b_1]$ (см. приложение)

$f(0,8) = \sqrt{0,334^2 + 0,349^2 + 0,68^2} = \sqrt{0,70187} = 0,8377 \Rightarrow$

$\Rightarrow \max M_{экв} = f(0,8) = 0,8377$ кН·м

$\frac{\max M_{экв} \cdot 32}{\pi \tilde{d}^3} \le \frac{\sigma_т}{[n]}$; $[n]$ берем 2,25. $\Rightarrow$

$\sigma_т$ (Ст.35) = 280 МПа

$\tilde{d} = \sqrt[3]{\frac{0,8377 \cdot 10^3 \cdot 32 \cdot 2,25}{\pi \cdot 280 \cdot 10^6}} \approx 0,0409$ м - теор. d

Из ГОСТа: $d = 45$ мм $= 0,045$ м

$W_{нэ} = \frac{\pi d^3}{32} = \frac{\pi \cdot 0,045^3}{32} = 8,946 \cdot 10^{-6}$ м^3

П.5 Расчет на сопротивление усталости.

$\sigma_в = 520$ Ст.35

$\sigma_т = 280$ МПа

$\tau_т = 150$

$\sigma_{-1} = 220$

$K_{F\sigma} \in [0,98; 0,96]$; $K_{F\sigma} = 0,96$

т. D_1 (диск)

$K_\sigma = 2,0468$ (из графика $K_\sigma(\sigma_в)$)

Воспользуемся пакетом Python, т.к. для $d = 45$ мм не выполнится условие $1{,}4 \le n \le 1{,}7$

Из результатов вып. эксперимента: (см. приложение)

Берем $d = 55$ мм $\Rightarrow K_\sigma = 0{,}734 \Rightarrow K = \frac{2{,}0468}{0{,}734 \cdot 0{,}9} = 3{,}098$

$n_{вып.} = 1{,}466$

$\tau_{max} = \frac{0{,}334 \cdot 10^3}{2 \cdot \frac{\pi (0{,}055)^3}{32}} \approx 10{,}22$ МПа

$n_\tau = \frac{150}{10{,}22} \approx 15$

$n_\sigma = \frac{\sigma_{-1}}{K\sigma_a} = \frac{220 \cdot \frac{\pi (0{,}055)^3}{32}}{3{,}098 \cdot \sqrt{0{,}59^2 + 0{,}29^2} \cdot 10^3} = 1{,}6174$

$\frac{1}{n} = \frac{1}{n_\tau} + \frac{1}{n_\sigma} \Rightarrow n = \frac{n_\tau n_\sigma}{\sqrt{n_\tau^2 + n_\sigma^2}} = 1{,}46 \in [1{,}4; 1{,}7]$

Условие выполнено.

Задача №2

FROM THE MAKERS OF WOLFRAM LANGUAGE AND MATHEMATICA

WolframAlpha

find maximum of function y=(0.334^2 + abs((-0.5375x+0.779))^2 + abs((-0.8556x))^2) at x from 0.35 to (

NATURAL LANGUAGE | MATH INPUT | EXTENDED KEYBOARD | EXAMPLES | UPLOAD | RANDOM

Input interpretation

maximize	function	$0.334^2 + \lvert -0.5375\,x + 0.779\rvert^2 + \lvert -0.8556\,x\rvert^2$
	domain	$0.35 \le x \le 0.8$

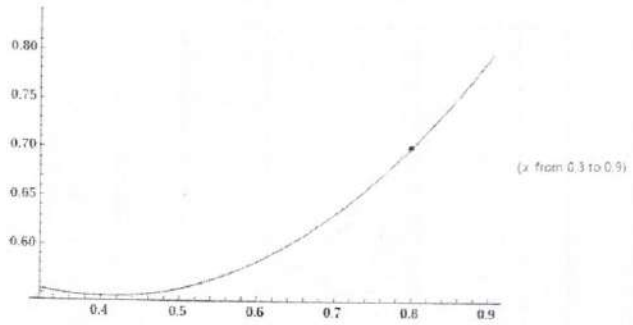
$|x|$ is the absolute value of x

Global maximum

Step-by-step solution

$\max\left\{0.334^2 + \lvert -0.5375\,x + 0.779\rvert^2 + \lvert -0.8556\,x\rvert^2 \;\middle|\; 0.35 \le x \le 0.8\right\} \approx 0.70187$ at $x = 0.8$

Plot



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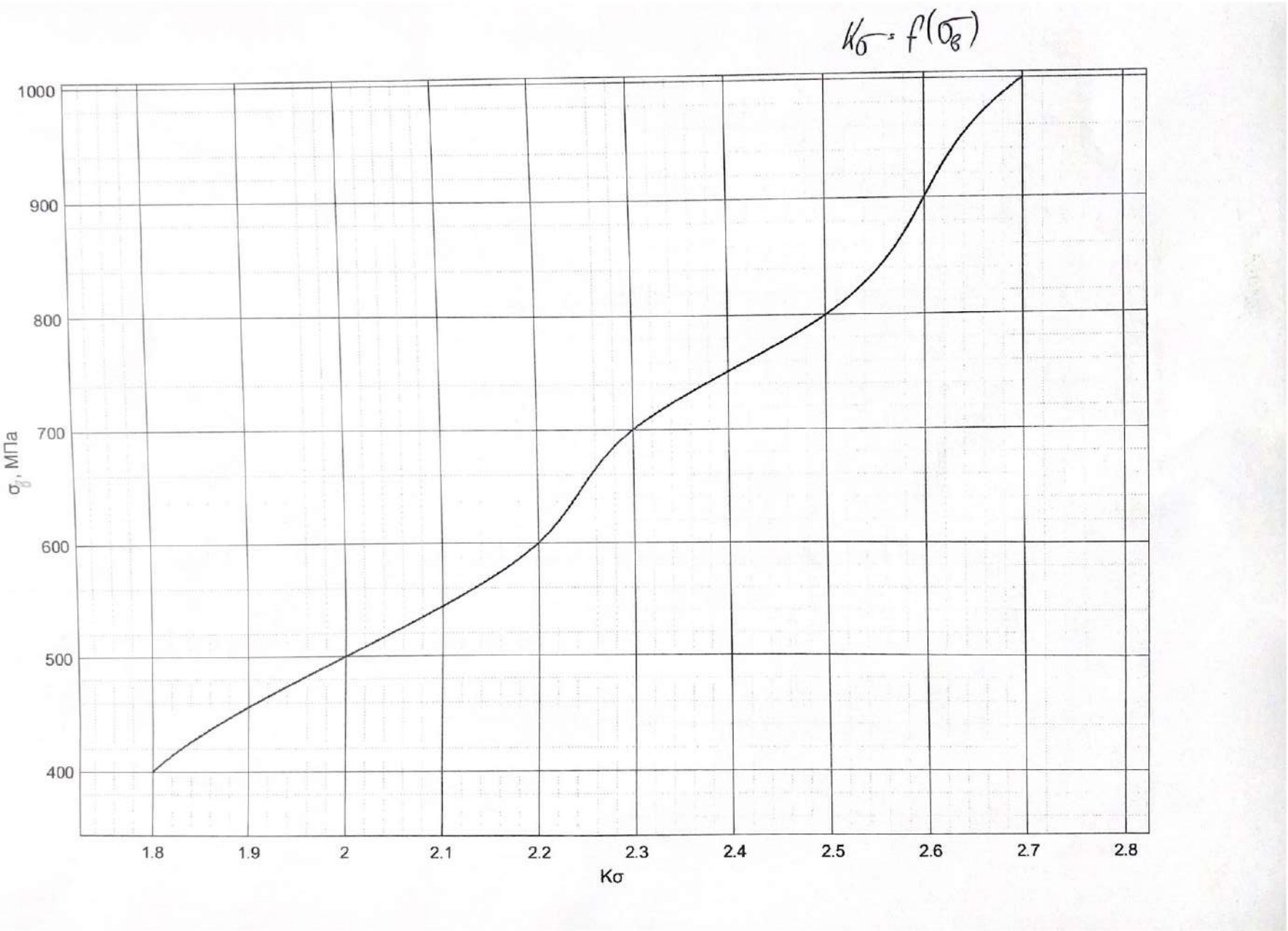
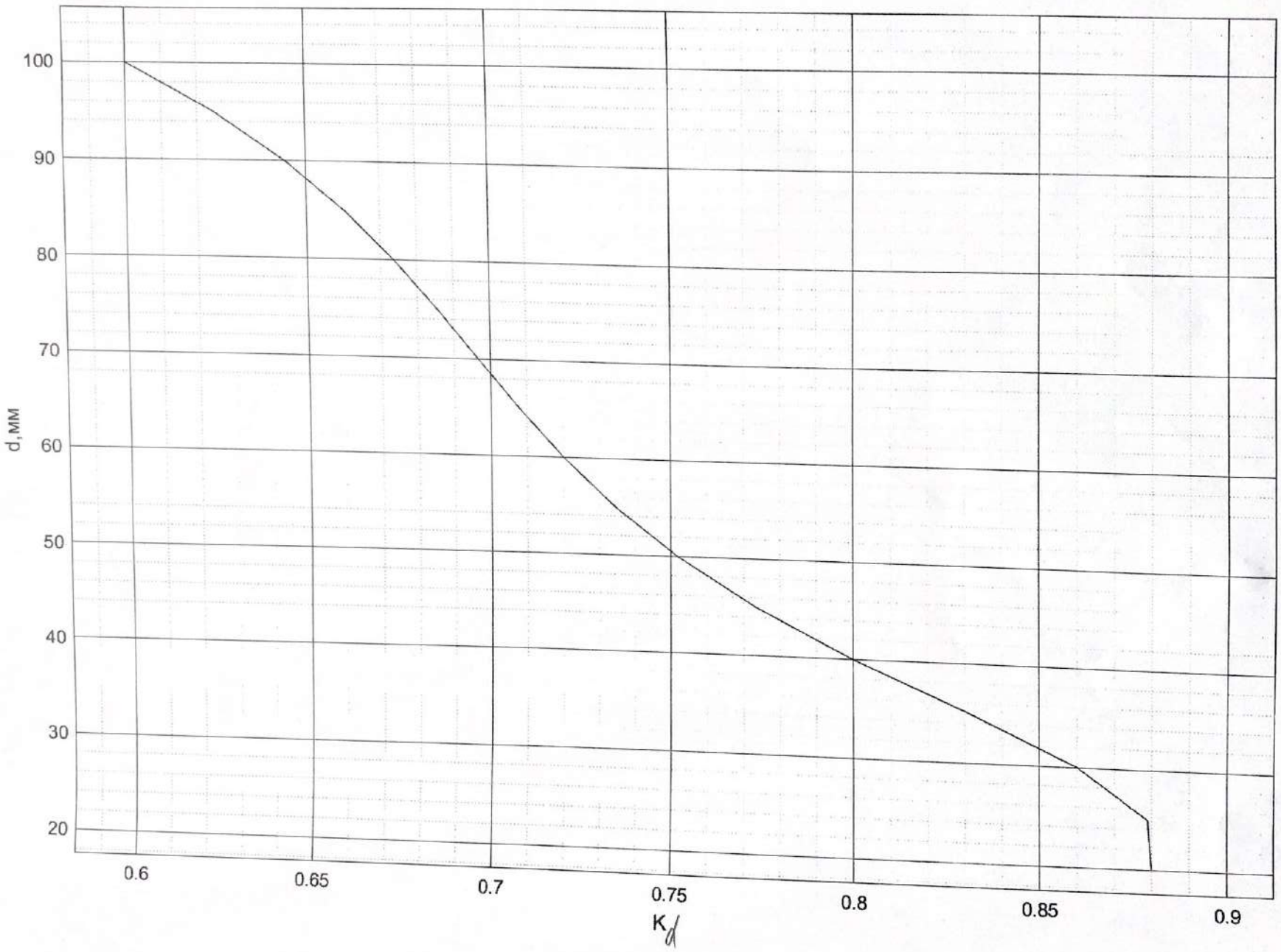
Программный написания интерполированного графика K:

```
Ksigma=[1.8,2.0,2.2,2.3,2.5,2.6,2.7];

sigma=[400,500,600,700,800,900,1000];
sigma_q = 400:5:1000;  % additional query points
F1 = griddedInterpolant(sigma,Ksigma,'spline'); % interpolant 1
Ksigma_Inter=F1(sigma_q);
plot(Ksigma_Inter,sigma_q)
% Анализ интерполированного графика
% 520--   2.04685714285714
% 820--   2.53028571428571
% 950--   2.63660714285714
%1000--   2.70000000000000
grid on
grid minor
set(0,'DefaultLineLineWidth',1)
ax=gca;
ax.GridColor='k';
ax.GridAlpha = 0.8;
ax.GridLineStyle = '-';
ax.GridLineStyle = '-';
```


Kσ = f(σв)
1000
900
800
700
600
500
400
σв, МПа
1.8
1.9
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
Kσ


100
90
80
70
60
50
40
30
20
d,мм
0.6
0.65
0.7
0.75
0.8
0.85
0.9
K_d

C: > Users > MSI > Desktop > Study > Сопромат > D finder.py > ...

Диск D_1 ; подбор d для $\Pi \in [1,4;1,7]$

```python
import numpy as np


ksigma=2.04586


kd=[0.8500000000000000,  0.8769230769230785,  0.8600000000000000  ,0.8310096153846154  ,0.8000000000000000  ,0.7734615384615390  ,0.751923076923077, 0.734423076923077,
kf=0.9
d=np.ndarray(17)
taumax=np.ndarray(17)
for i in range(17):
    d[i]=20+5*i
taunorm=150
ntau=np.ndarray(17)
nsigma=np.ndarray(17)
nFIN=np.ndarray(17)
W=np.ndarray(17)
for i in range(17):
    W[i]=1/32 * 3.14*(d[i])**3
    taumax[i]=0.334*10**3 /(2*W[i])
    ntau[i]=taunorm/taumax[i]
    nsigma[i] = 220* 10**3 *W[i]/(2.04586/(0.9*kd[i]) * 657.42)
    nFIN[i] = nsigma[i] * ntau[i]   / np.sqrt( nsigma[i]**2 + ntau[i]**2 )
    if(nFIN[i]/10000 >= 1.4 and nFIN[i]/10000 <= 1.7):
        print(nFIN[i]/10000,d[i],kd[i])
        break

```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS

```
PS C:\Users\MSI> & C:/Users/MSI/AppData/Local/Microsoft/WindowsApps/python3.11.exe "c:/Users/MSI/Desktop/Study/Сопромат/D finder.py"
1.46631588534199 55.0 0.734423076923077
PS C:\Users\MSI>
```

$\Pi =$

d K_d

Задача №3

$R_1 = 18см = 0{,}18м$
$R_2 = 0{,}21м$
$P_1 = 7 \cdot 10^6 Па$
$P_2 = 2 \cdot 10^6 Па$
$T_1^* = 170°C$
$T_2 = 130°C$
$\alpha = 1{,}25 \cdot 10^{-5} \frac{1}{C}$
$\nu = 0{,}3$
$E = 200 \cdot 10^9 Па$
$\sigma_m = 280 \cdot 10^6$

1) Эпюры напряжений $\sigma_r, \sigma_\theta, \sigma_z$ при (P). Строим на $[0{,}18; 0{,}21]$ по r

$$\sigma_{r,P} = \frac{P_1 R_1^2 - P_2 R_2^2}{R_2^2 - R_1^2} - \frac{(P_1 - P_2) R_1^2 R_2^2}{R_2^2 - R_1^2} \cdot \frac{1}{r^2}$$ из ф-лы Ламе:

$$\sigma_{\theta,P} = \frac{P_1 R_1^2 - P_2 R_2^2}{R_2^2 - R_1^2} + \frac{(P_1 - P_2) R_1^2 R_2^2}{R_2^2 - R_1^2} \cdot \frac{1}{r^2}$$

$$\sigma_{z,P} = \frac{P_1 R_1^2 - P_2 R_2^2}{R_2^2 - R_1^2} - const$$

2) Эпюры напряжений $\sigma_r, \sigma_\theta, \sigma_z$ при (T).

$$T(r) = T_2 + (T_1 - T_2) \frac{\ln(\frac{r}{R_2})}{\ln(\frac{R_1}{R_2})}, \quad \Delta T = 170 - 130 = 40°$$

$$K = \frac{\Delta T \cdot \alpha \cdot E}{2(1-\nu)\ln(\frac{R_2}{R_1})}$$

$$\sigma_{r,T} = -K\left[\ln(\frac{R_2}{r}) + \frac{R_1^2}{R_2^2 - R_1^2}\left(1 - \frac{R_2^2}{r^2}\right)\ln(\frac{R_2}{R_1})\right]$$

$$\sigma_{\theta,T} = K\left[1 - \ln(\frac{R_2}{r}) - \frac{R_1^2}{(R_2^2 - R_1^2)}\left(1 + \frac{R_2^2}{r^2}\right)\ln(\frac{R_2}{R_1})\right]$$

$$\sigma_{z,T} = K\left[1 - 2\ln(\frac{R_2}{r}) - \frac{2R_1^2}{R_2^2 - R_1^2}\ln(\frac{R_2}{R_1})\right]$$

3) Эпюры $\sigma_{r,P} + \sigma_{r,T}$; $\sigma_{\theta,P} + \sigma_{\theta,T}$; $\sigma_{z,P} + \sigma_{z,T}$

4) Находим опасную точку: R_2 + проверка на прочность по Сен-Венану

$\sigma_r = \sigma_3 \Rightarrow$ $\sigma_{экв} = \sigma_1 - \sigma_3 < [\sigma] = \frac{\sigma_m}{n}$

$\sigma_\theta = \sigma_1$

$|\sigma_3| = 2 \cdot 10^6$

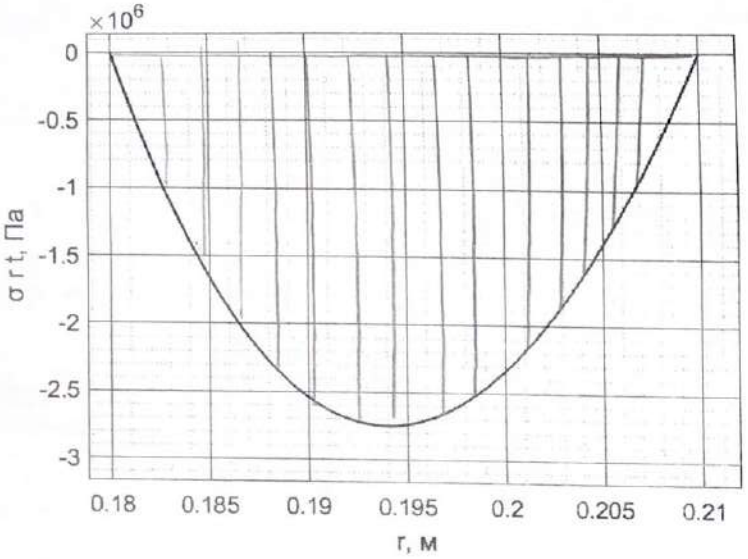
$\sigma_1 = 79{,}61\,03 \cdot 10^6 Па$

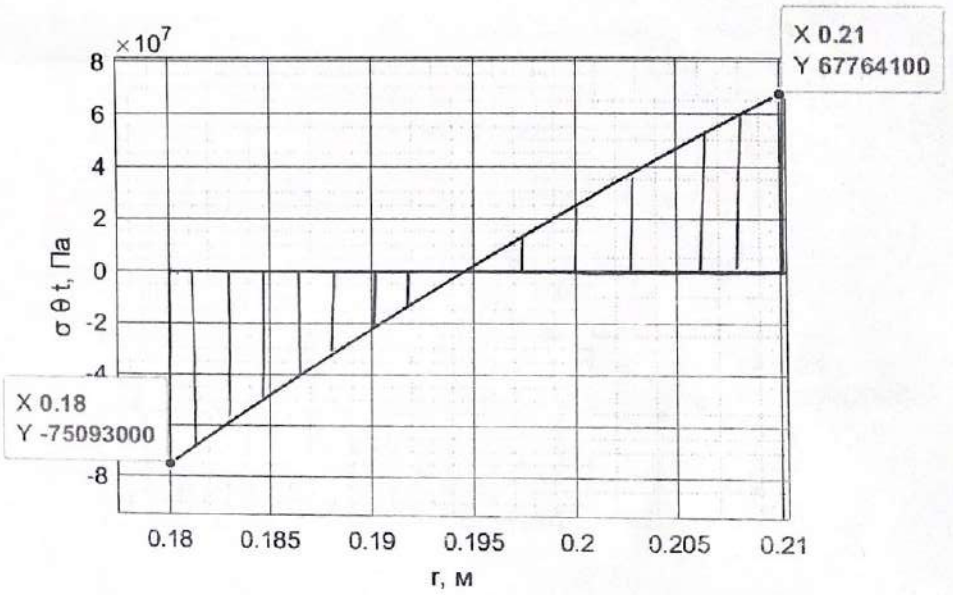
$n = \frac{\sigma_m}{\sigma_1 - \sigma_3} = \frac{280 \cdot 10^6}{(2 + 79{,}6103) \cdot 10^6} = 2{,}993 > 1.$

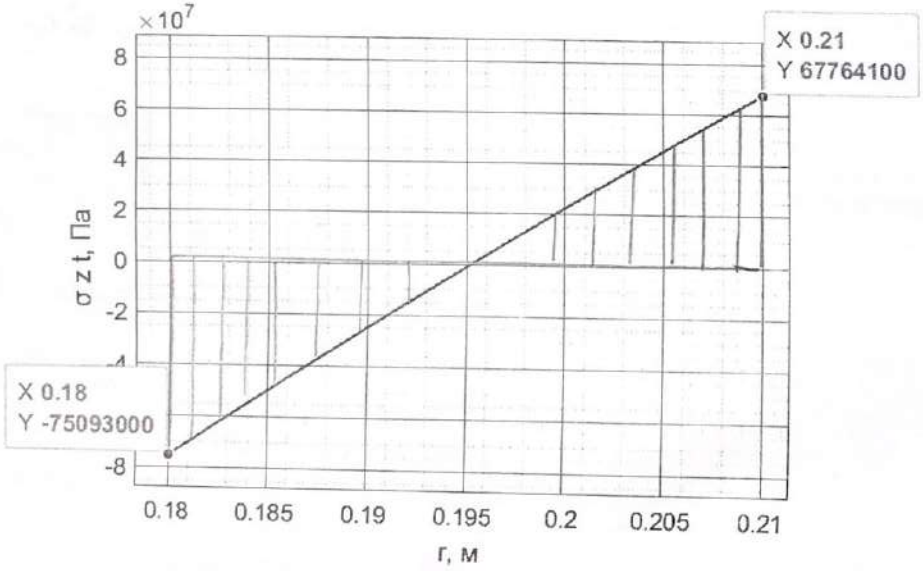
Сопромат, номер 3 в Matlab.

```
R1=0.18;
R2=0.21; % м
p1=7*10^6; % Па
p2=2*10^6;
T1=170; % celcius
T2=130;
deltaT=T1-T2;
E=200*10^9; % Па
alpha=1.25*10^(-5);  % inv celcius
PoissonCoeff=0.3;
sigmaflow=280 * 10^6; % предел текучести
K=E*alpha*deltaT./(2*(1-PoissonCoeff)*log(R2/R1));
r=linspace(R1,R2,250);

sigma_r_p=(p1.*R1.^2 - p2.*R2.^2)./(R2.^2 - R1.^2) - (p1-p2).*((R1.*R2).^2)./((R2.^2 - R1.^2).*r.^2);
sigma_theta_p=(p1.*R1.^2 - p2.*R2.^2)./(R2.^2 - R1.^2) + (p1-p2).*((R1.*R2).^2)./((R2.^2 -
R1.^2).*r.^2);
sigma_z_p=(p1.*R1.^2 - p2.*R2.^2)./(R2.^2 - R1.^2)+ 0*r;  % необходимый костыль матлаба
K=E*alpha*deltaT./(2*(1-PoissonCoeff)*log(R2/R1));

sigma_r_t=-K*(log(R2./r)+ (R1.^2)*(1-(R2.^2)./(r.^2))*log(R2./R1)/(R2.^2 - R1.^2));
sigma_theta_t=K*(1-log(R2./r) - (R1.^2)*(1+(R2.^2)./(r.^2))*log(R2./R1)/(R2.^2 - R1.^2));
sigma_z_t=K*(1-2*log(R2./r) - 2*(R1.^2)*log(R2./R1)/(R2.^2 - R1.^2));

tiledlayout(2,2)
nexttile

%plot(r,sigma_r_p)
%plot(r,sigma_r_t)
plot(r,sigma_r_p + sigma_r_t)

grid on
grid minor
```

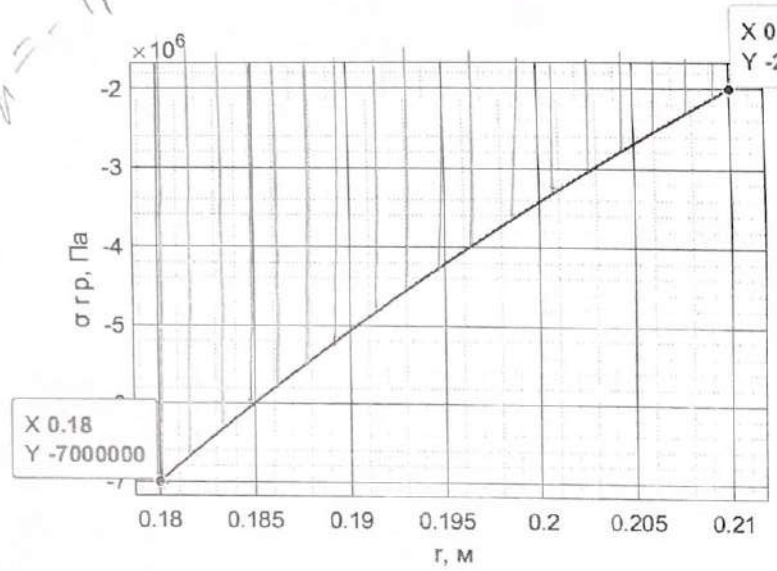
```
ax.GridLineStyle = '-';

% п.4

nexttile

%{ поиск опасной точки. Для этого вводим функцию равную сумме МОДУЛЕЙ всех

% суммарных напряжений и ищем ее максимум на интервале [R1,R2]

%}

sigma_sum=abs(sigma_z_p+sigma_z_t)+abs(sigma_theta_p + sigma_theta_t)+abs(sigma_r_p +
sigma_r_t);

plot(r,sigma_sum)

% Видно что максимум будет в т. R2

% расчет на прочность по Сен-Венану : sigma_eqv=sigma1 - sigma3; sigma1 =

% sigma theta, sigma 3 = sigma r

sigma_eqv=(sigma_theta_p(250)+sigma_theta_t(250)) - (sigma_r_p(250)-sigma_r_t(250)); % 250 т.к по
индексации [250]=R2

n=sigmaflow/sigma_eqv;

%disp(n);

n= 2.9333.
```

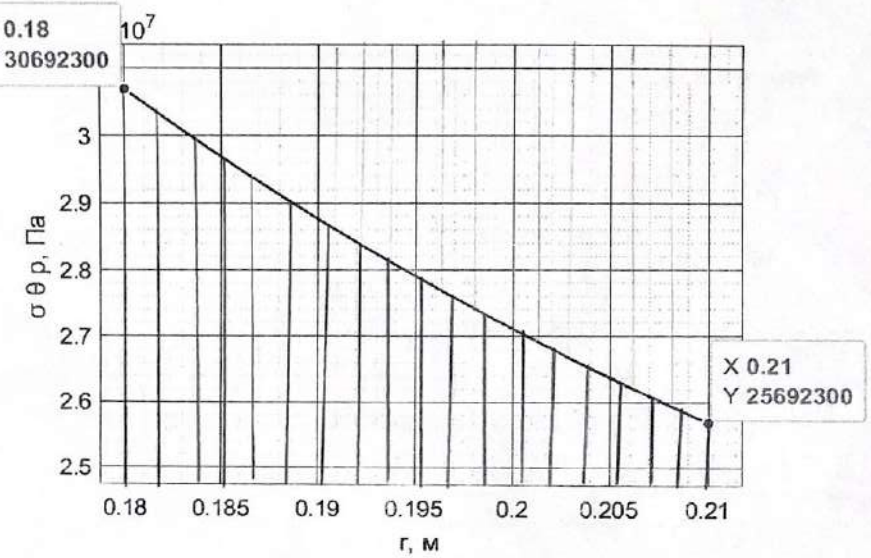

×10^6
0
-0.5
-1
-1.5
-2
-2.5
-3
σ r t, Па
0.18
0.185
0.19
0.195
0.2
0.205
0.21
r, м

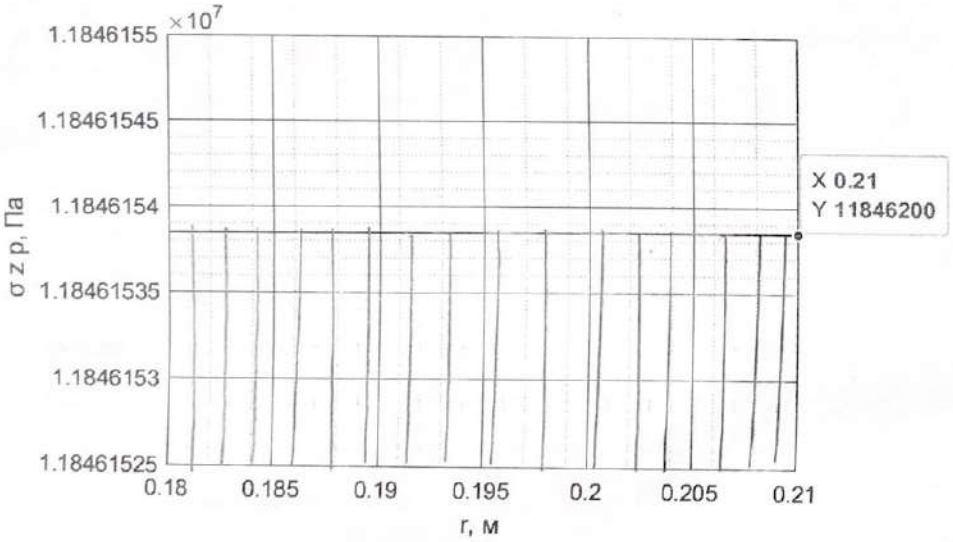

×10^7
8
6
4
2
0
-2
-8
σ θ t, Па
X 0.21
Y 67764100
X 0.18
Y -75093000
0.18
0.185
0.19
0.195
0.2
0.205
0.21
r, м

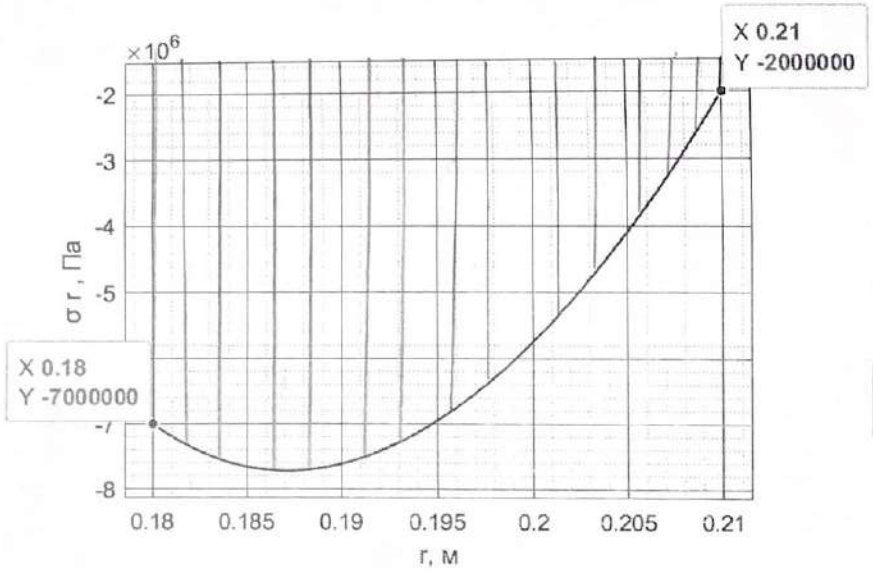

×10^7
8
6
4
2
0
-2
-8
σ z t, Па
X 0.21
Y 67764100
X 0.18
Y -75093000
0.18
0.185
0.19
0.195
0.2
0.205
0.21
r, м

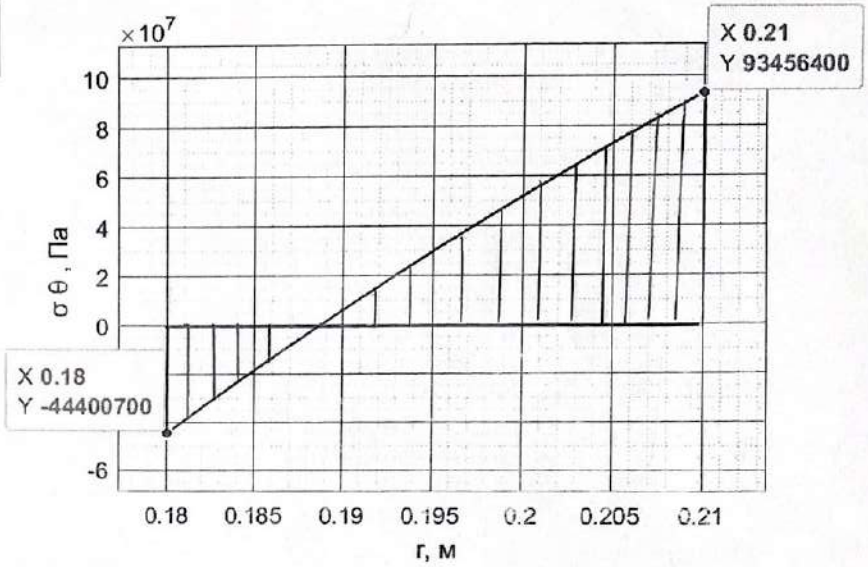
$\sigma_r = -P_1$

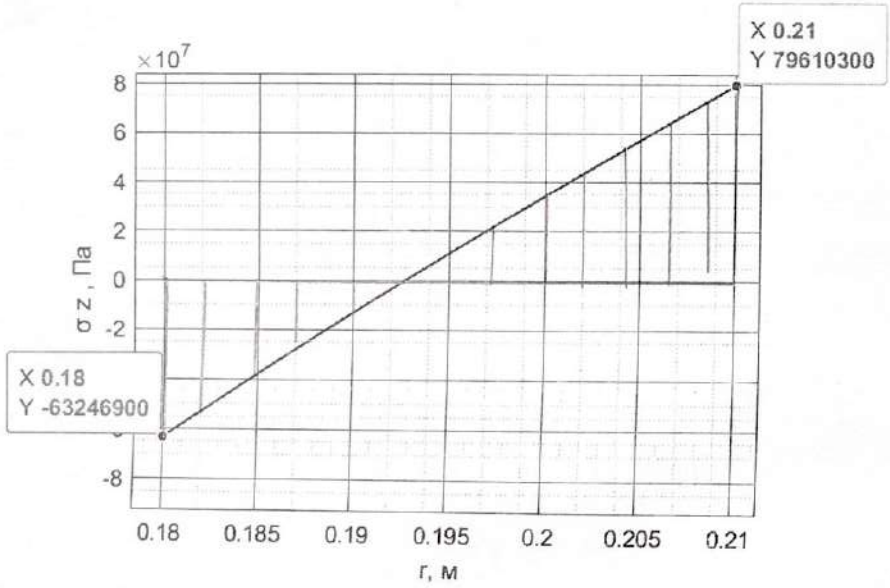

×10^6
-2
-3
-4
-5
σ r p, Па
X 0.21
Y -2000000
X 0.18
Y -7000000
0.18
0.185
0.19
0.195
0.2
0.205
0.21
r, м

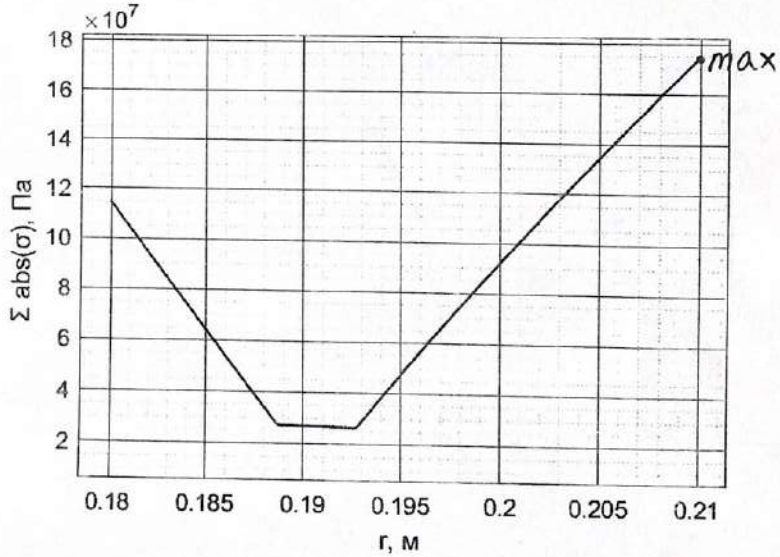

10^7
3
2.9
2.8
2.7
2.6
2.5
σ θ p, Па
X 0.18
Y 30692300
X 0.21
Y 25692300
0.18
0.185
0.19
0.195
0.2
0.205
0.21
r, м


×10^7
1.1846155
1.18461545
1.1846154
1.18461535
1.1846153
1.18461525
σ z p, Па
X 0.21
Y 11846200
0.18
0.185
0.19
0.195
0.2
0.205
0.21
r, м


X 0.21
Y -2000000
X 0.18
Y -7000000
×10^6
σ r , Па
-2
-3
-4
-5
-7
-8
0.18
0.185
0.19
0.195
0.2
0.205
0.21
r, м

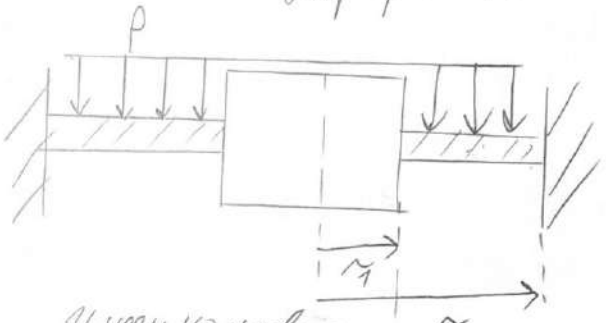

X 0.21
Y 93456400
X 0.18
Y -44400700
×10^7
σ θ , Па
10
8
6
4
2
0
-6
0.18
0.185
0.19
0.195
0.2
0.205
0.21
r, м


X 0.21
Y 79610300
X 0.18
Y -63246900
×10^7
σ z , Па
8
6
4
2
0
-2
-8
0.18
0.185
0.19
0.195
0.2
0.205
0.21
r, м


max
×10^7
Σ abs(σ), Па
18
16
14
12
10
8
6
4
2
0.18
0.185
0.19
0.195
0.2
0.205
0.21
r, м

Задача №4 Мартынов М.Г.

$\tilde{r}_1 = 0{,}3$ м
$\tilde{r}_2 = 1{,}4$ м
$h = 0{,}08$ м
$E = 200 \cdot 10^9$ Па
$\nu = 0{,}3$
$[\sigma] = 240 \cdot 10^6$ Па



Имеем кольцевую пластину.

$D = \frac{Eh^3}{12(1-\nu^2)}$ – ее цилиндрическая жесткость

Функция прогиба:

$$w(\tilde{r}) = \left[C_1 + C_2\tilde{r}^2 + C_3 \ln\tilde{r} + C_4 \ln\tilde{r}\cdot(\tilde{r}^2)\right] - \frac{p\tilde{r}^4}{64D}$$

Константы находим из граничных условий:

В $\tilde{r}_1$: скользящая заделка:

$Q(\tilde{r}_1) = 0$ как поперечная сила

$$Q(\tilde{r}_1) = D\left(\frac{d^3w}{d\tilde{r}^3} + \frac{1}{\tilde{r}}\frac{d^2w}{d\tilde{r}^2} - \frac{1}{\tilde{r}^2}\frac{dw}{d\tilde{r}}\right)\Big|_{\tilde{r}=\tilde{r}_1}$$

$\varphi|_{\tilde{r}_1} = \frac{dw}{d\tilde{r}}\Big|_{\tilde{r}_1}$ как угол поворота сечения $= 0$.

В $\tilde{r}_2$: заделка:

$w(\tilde{r}_2) = 0$ как ф-я прогиба

$\varphi|_{\tilde{r}_2} = \frac{dw}{d\tilde{r}}\Big|_{\tilde{r}_2} = 0$.

Из граничных условий находим $C_1 - C_4$.

Строим $M_{\tilde{r}}, M_\theta$ как $f(\tilde{r})$: $M_{\tilde{r}} = D\left(\frac{d^2w}{d\tilde{r}^2} + \frac{\nu}{\tilde{r}}\frac{dw}{d\tilde{r}}\right)$

$M_\theta = D\left(\frac{1}{\tilde{r}}\frac{dw}{d\tilde{r}} + \nu\frac{d^2w}{d\tilde{r}^2}\right)$

Находим $\sigma_{\tilde{r}}; \sigma_\theta$:

$\sigma_{\tilde{r}} = \frac{M_{\tilde{r}} \cdot 12}{h^3} z$, $\sigma_{\tilde{r}\,max}$: $z = \pm\frac{h}{2}$

И ту силу опасную точку

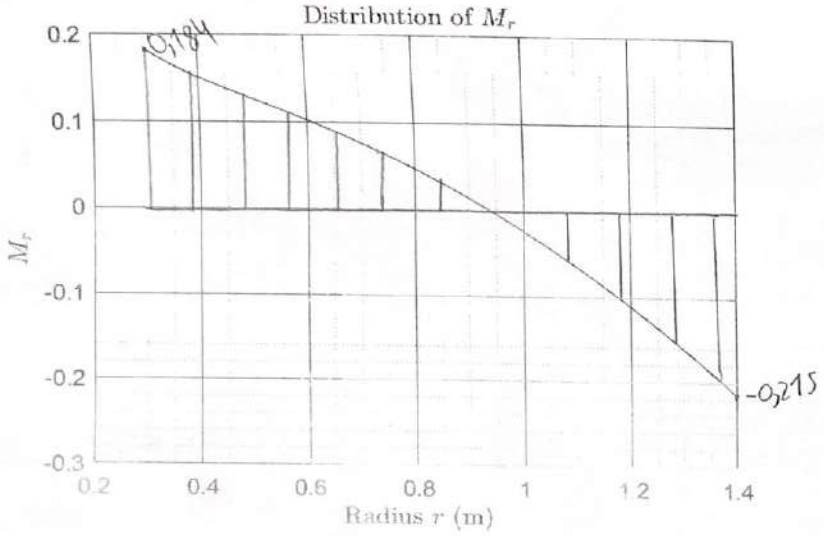
Опасная – т. $\tilde{r}_2$

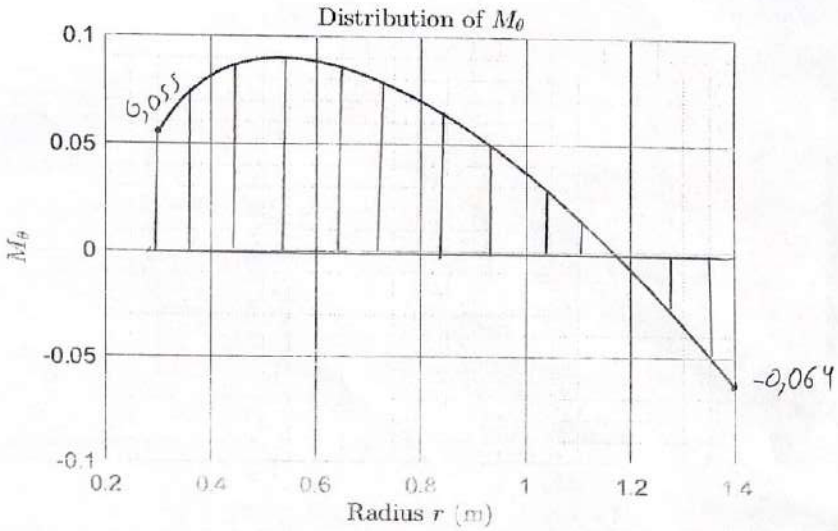
$\sigma_\theta = \frac{M_\theta \cdot 12}{h^3} z$, $\sigma_{\theta\,max}$: $z = \pm\frac{h}{2}$

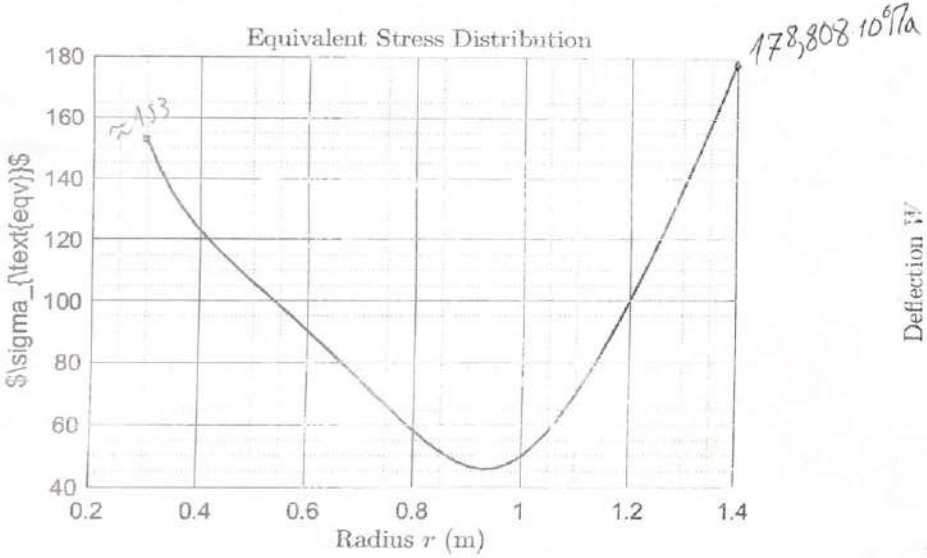
$\sigma_{экв}$ – по Мизесу. $\Rightarrow$

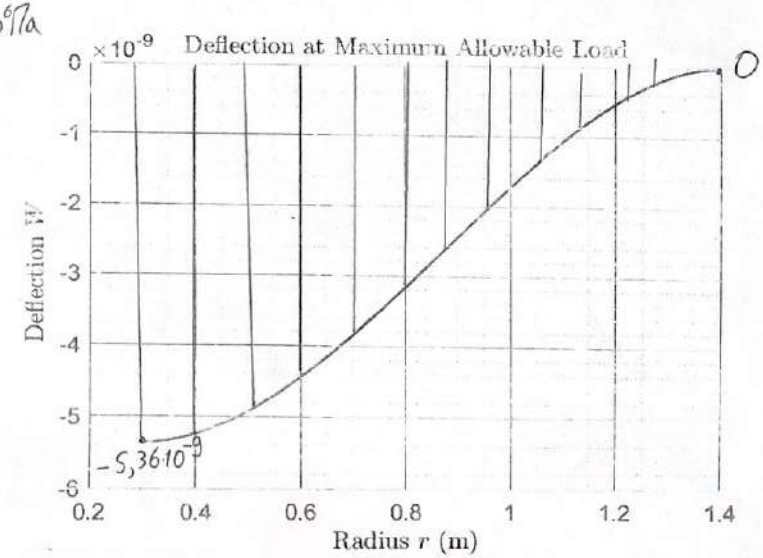
$$[p] \le \frac{[\sigma]}{\sigma_{экв}}$$

Далее строим $w(\tilde{r})$ при $[p]$.


Distribution of M_r
0,184
-0,215
M_r
Radius r (m)

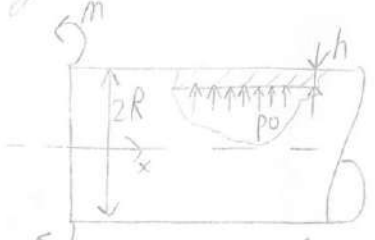

Distribution of M_θ
0,055
-0,064
M_θ
Radius r (m)


Equivalent Stress Distribution
178,808 10^6 Pa
≈ 153
σ_{eqv}
Radius r (m)


Deflection at Maximum Allowable Load
×10^-9
0
-5,36 10^-9
Deflection W
Radius r (m)

```
E = 200e9;
m = 0.3;
h = 0.08;
r1 = 0.3;
r2 = 1.4;
sigma = 240e6;

D = E * h^3 / (12 * (1 - m^2));

C = [1, r2^2, log(r2), log(r2) * r2^2;
     0, 2*r2, 1/r2, 2*r2*log(r2)+r2;
     0, 2*r1, 1/r1, 2*r1*log(r1)+r1;
     0, 0, 0, 4/r1];

b = [r2^4/(64*D); r2^3/(16*D); r1^3/(D*16); 1/(2*D)*r1];

consts = linsolve(C, b);
C1 = consts(1);
C2 = consts(2);
C3 = consts(3);
C4 = consts(4);


W = @(C1, C2, C3, C4, D, r, p) ((C1 + C2*r.^2 + C3*log(r) + C4*log(r).*r.^2).*p -
r.^4.*p./(64*D));
W1 = @(C2, C3, C4, D, r) (2*r*C2 + C3./r + C4.*(2*r.*log(r) + r) - r.^3./(16*D));
W2 = @(C2, C3, C4, D, r) (2*C2 - 1./r.^2.*C3 + C4.*(3 + 2*log(r)) - r.^2*3./(16*D));
M_r = @(W2, W1, r, D, m) (D*(W2(C2, C3, C4, D, r) + m./r.*W1(C2, C3, C4, D, r)));
M_theta = @(W2, W1, r, D, m) (D*(m*W2(C2, C3, C4, D, r) + 1./r.*W1(C2, C3, C4, D, r)));

sigma_r = @(M_r, h, r, D, m) (6*M_r(W2, W1, r, D, m)./h^2);
sigma_theta = @(M_theta, h, r, D, m) (6*M_theta(W2, W1, r, D, m)./h^2);
sigma_eqv = @(sigma_r, sigma_theta) (sqrt(sigma_r.^2 + sigma_theta.^2 - sigma_r .*
sigma_theta));

sigma_eqv_max = sigma_eqv(sigma_r(M_r, h, r2, D, m), sigma_theta(M_theta, h, r2, D, m));
p = sigma / (sigma_eqv_max * 10^6);

tiledlayout(2,2)

r_vals = linspace(r1, r2, 50);
titles = {'Distribution of $M_r$', 'Distribution of $M_{\theta}$', 'Equivalent Stress
Distribution', 'Deflection at Maximum Allowable Load'};
ylabels = {'$M_r$', '$M_{\theta}$', '$\sigma_{\text{eqv}}$', 'Deflection $W$'};
functions = {@(r) M_r(W2, W1, r, D, m), @(r) M_theta(W2, W1, r, D, m), @(r)
sigma_eqv(sigma_r(M_r, h, r, D, m), sigma_theta(M_theta, h, r, D, m)), @(r) W(C1, C2, C3,
C4, D, r, p)};

for i = 1:4
    nexttile
    plot(r_vals, functions{i}(r_vals))
    title(titles{i}, 'Interpreter', 'latex')
    xlabel('Radius $r$ (m)', 'Interpreter', 'latex')
    ylabel(ylabels{i}, 'Interpreter', 'latex')
    grid on
    grid minor
    set(0,'DefaultLineLineWidth',1)
    ax=gca;
    ax.GridColor='k';
    ax.GridAlpha = 0.8;
    ax.GridLineStyle = '-';
end
```

Задача № 5 Мартынов М.Г.



Имеем цилиндрическую оболочку толщины h, нагруженную осесимметричной нагрузкой.

$a = 0{,}05\,м$; $R = 0{,}9\,м$; $\Delta = 4\cdot10^{-5}\,м$; $p_0 = 0{,}6\cdot10^6\,Па$; $q = 9{,}2\cdot10^6\,\frac{Н}{м}$

$m = qa$, $[\sigma] = 240\cdot10^6\,Па$

Функция прогиба: $w(x) = C_1 e^{kx}\sin(kx) + C_2 e^{kx}\cos(kx) + \frac{p_0 R^2}{Eh}$

Имеем два краевых условия при $x = 0$

$$\begin{cases} M_x(0) = -m \\ Q(0) = 0 \end{cases} \Leftrightarrow \begin{cases} D\frac{d^2w}{dx^2}\Big|_0 = -m \\ D\frac{d^3w}{dx^3}\Big|_0 = 0 \end{cases} \Leftrightarrow \begin{cases} -D\cdot 2k^2 C_2 = -m \\ -D\cdot 2k^2(C_1 + C_2) = 0 \end{cases}$$

$C_2 = \frac{m}{D\cdot 2k^2}$; $C_1 = -C_2$

Считаем D: $D = \frac{Eh^3}{12(1-\mu^2)}$; $k = \sqrt[4]{\frac{Eh}{4DR^2}} \Rightarrow C_1, C_2$

Имеем $w(x)$, $\vartheta(x) = \frac{dw}{dx}$; $M_x = \left(\frac{Eh^3}{12(1-\mu^2)}\right)_D \frac{d^2w}{dx^2}$; $M_y = \mu D\frac{d^2w}{dx^2}$

$N_y = \mu\underbrace{(N_x)}_{0} + \frac{Eh}{R}w(x)$

$Q = D\frac{d^3w}{dx^3}$

Ищем σ_x; σ_y на $\pm\frac{h}{2}$ т.е. $\sigma_{x\,внутр}$ и $\sigma_{x\,внешн}$ и по ним строим $\sigma_{экв}$, где $\sigma_{экв}$ берем по Мизесу $\sigma_{экв} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y}$ на $\frac{h}{2}$ и $-\frac{h}{2}$ соотв.

Ищем максимумы $\sigma_{экв\,внутр}$; $\sigma_{экв\,внешн}$ и берем больший из них для проверки на прочность:

$$\max\left[\max\left(\sigma_{экв\,внешн};\ \sigma_{экв\,внутр}\right)\right] < \frac{[\sigma]}{n} \Rightarrow n = \ldots$$

$n = 1{,}58$ при $h = 9h^*$, где $h^* = p_0\frac{R}{\sigma}$ по безмоментной теории

при $h < 9h^*$: $n < 1{,}4 \div 1{,}8$

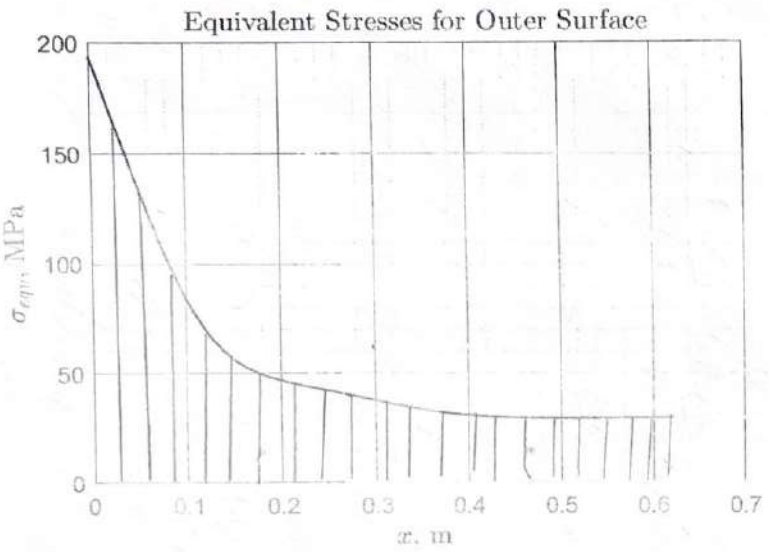
Результат работы программы: (+ эпюры)

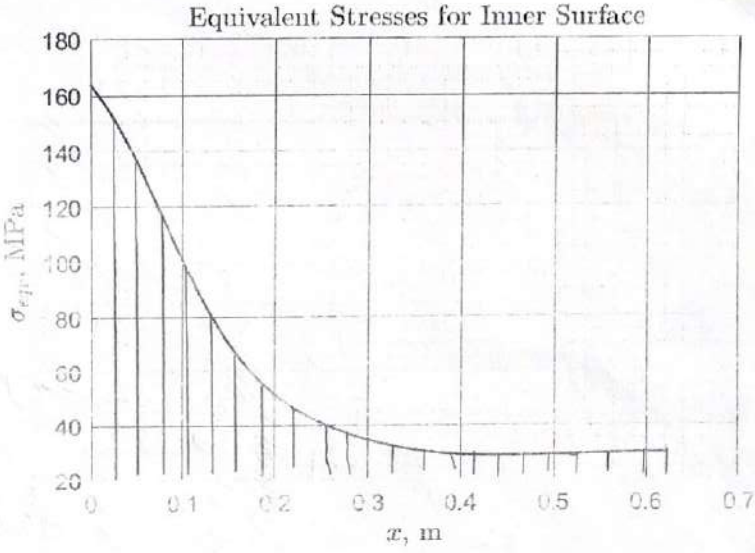
$h = 0{,}2025\,м$

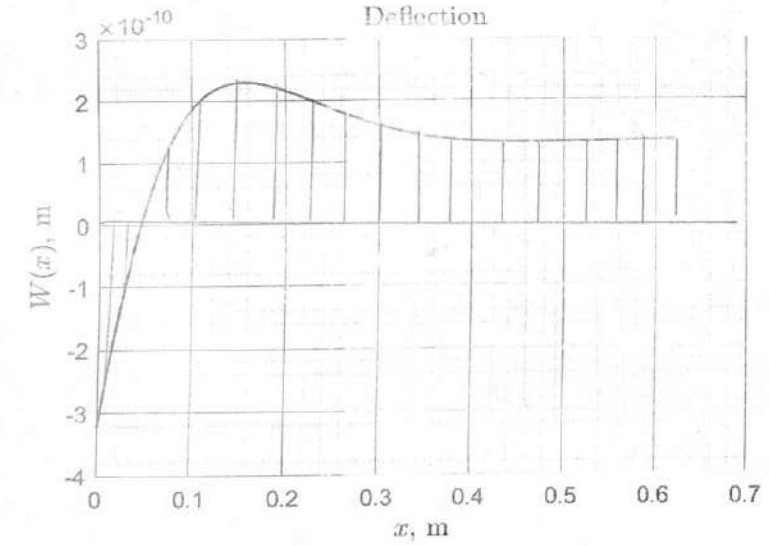
$k = 9{,}5215\,\frac{1}{м}$; $\lambda = 0{,}32995\,м$; $C_1 = -0{,}00036$; $C_2 = -C_1$

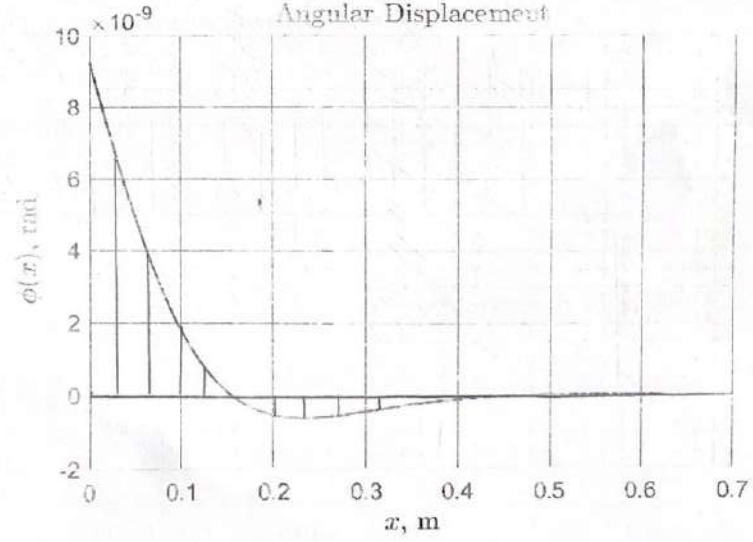
$\max\sigma_{экв\,внешн} = 151{,}58\,МПа$

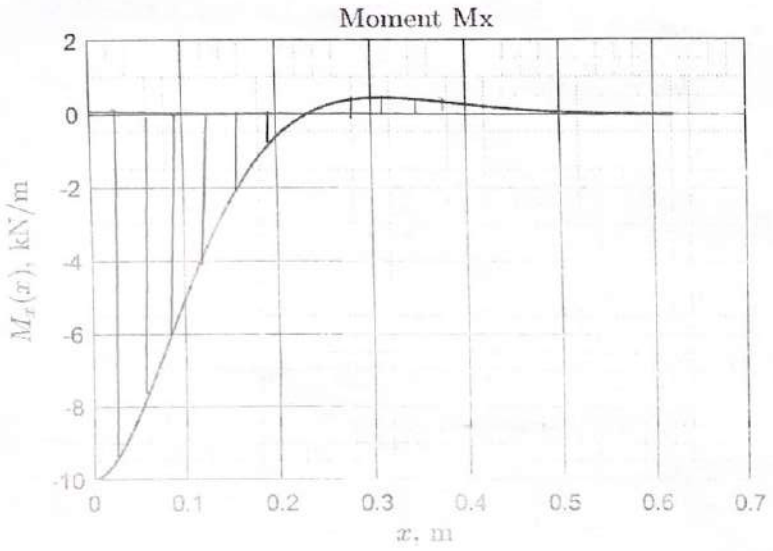
$\max\sigma_{экв\,внутр} = 129{,}09\,МПа$ $\Rightarrow$ $n = 1{,}5833$

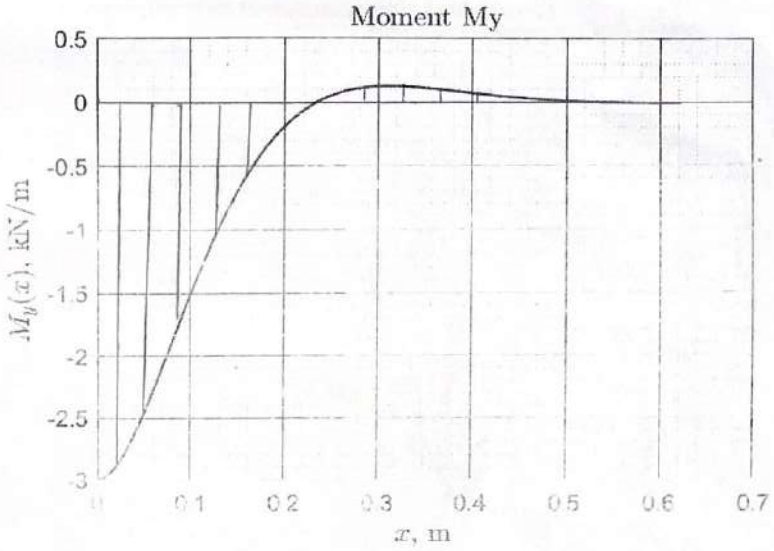

Equivalent Stresses for Outer Surface
200
150
100
50
0
σ_{equ}, MPa
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
x, m

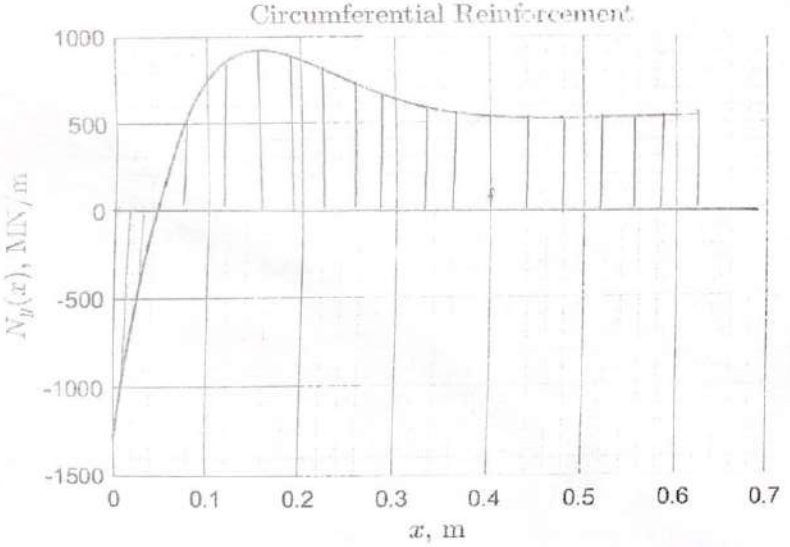

Equivalent Stresses for Inner Surface
180
160
140
120
100
80
60
40
20
σ_{equ}, MPa
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
x, m

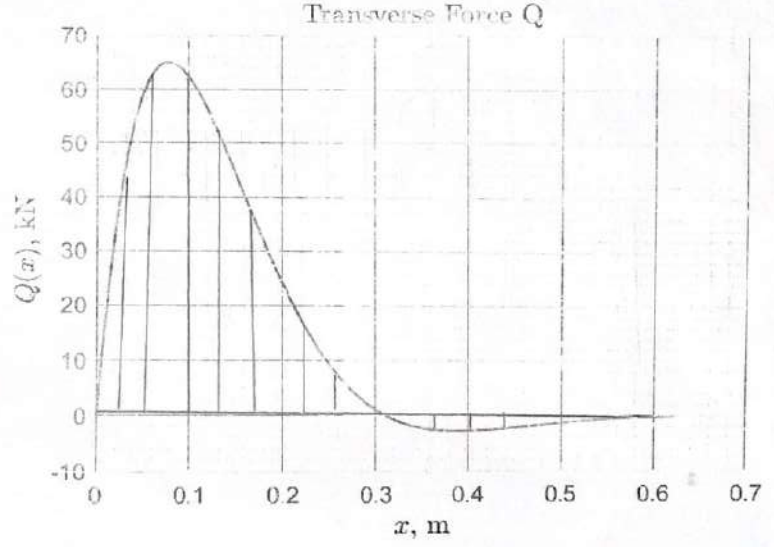

Deflection
$\times 10^{-10}$
3
2
1
0
-1
-2
-3
-4
$W(x)$, m
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
x, m


$\times 10^{-9}$
10
8
6
4
2
0
-2
$\phi(x)$, rad
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
x, m


Moment Mx
2
0
-2
-4
-6
-8
-10
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
x, m
$M_x(x)$, kN/m


Moment My
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
0.1
0.2
0.3
0.4
0.5
0.6
0.7
x, m
$M_y(x)$, kN/m


Circumferential Reinforcement
1000
500
0
-500
-1000
-1500
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
x, m
$N_y(x)$, MN/m


Transverse Force Q
70
60
50
40
30
20
10
0
-10
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
x, m
$Q(x)$, kN

Реализация номера 5 на языке Matlab, var.15

```
a = 0.05;
R = 0.99;
p0 = 0.6 * 10^6;
q = 0.2 * 10^6;
m = q * a;
E = 200 * 10^9;
sigma = 240 * 10^6;
Nx = 0;
mu = 0.3;

h = 8 * p0 * R / sigma; % оптимизированное значение толщины
disp(['Value of h=', num2str(h), ' m']);

D = E * h^3 / (12 * (1 - mu^2));
disp(['Cylindrical stiffness D=', num2str(D)]);

k = (E * h / (4 * D * R^2))^(1 / 4);
disp(['Wave number k=', num2str(k), ' m^-1']);

lambda = pi / k;
disp(['Edge effect length λ=', num2str(lambda), ' m']);

C = [0, 1; 1, 1]; % матрица краевых условий
b = [m / (2 * D * k^2); 0];  % вектор краевых условий

consts = linsolve(C, b);
C1 = consts(1);
C2 = consts(2);
disp('Coefficients C:');
disp(['C1=', num2str(C1)]);
disp(['C2=', num2str(C2)]);

x = linspace(0, 2 * lambda, 100);

% функции для вычисления величин
W = @(x) C1 * exp(-k * x) .* cos(k * x) + C2 * exp(-k * x) .* sin(k * x) + p0 * R^2 / (E * h);
phi = @(x) -k * exp(-k * x) .* ((C1 - C2) * cos(k * x) + (C1 + C2) * sin(k * x));
Mx = @(x) 2 * k^2 * exp(-k * x) .* D .* (C1 * sin(k * x) - C2 * cos(k * x));
My = @(x) mu * Mx(x);
Ny = @(x) mu * Nx + E * h / R * W(x);
Q = @(x) 2 * k^3 * D * exp(-k * x) .* ((C1 + C2) * cos(k * x) - (C1 - C2) * sin(k * x));
sigma_x_inner = @(x) Nx / h + 6 * Mx(x) / h^2;
sigma_x_outer = @(x) Nx / h - 6 * Mx(x) / h^2;
sigma_y_inner = @(x) Ny(x) / h + 6 * My(x) / h^2;
sigma_y_outer = @(x) Ny(x) / h - 6 * My(x) / h^2;
sigma_eqv_inner = @(x) sqrt(sigma_x_inner(x).^2 + sigma_y_inner(x).^2 - sigma_x_inner(x) .*
sigma_y_inner(x));
sigma_eqv_outer = @(x) sqrt(sigma_x_outer(x).^2 + sigma_y_outer(x).^2 - sigma_x_outer(x) .*
sigma_y_outer(x));

% Графики
figure('Position', [0, 0, 800, 600]);
subplot_titles = {'Moment Mx', 'Moment My', 'Circumferential Reinforcement', 'Transverse Force Q'};
plot_functions = {Mx, My, Ny, Q};
plot_ylabels = {'$M_x(x)$, kN/m', '$M_y(x)$, kN/m', '$N_y(x)$, MN/m', '$Q(x)$, kN'};
for i = 1:4
```

```
    subplot(2, 2, i);
    plot(x, plot_functions{i}(x) / (1e3), 'k');
    xlabel('$x$, m', 'Interpreter', 'latex', 'FontSize', 12);
    ylabel(plot_ylabels{i}, 'Interpreter', 'latex', 'FontSize', 12);
    title(subplot_titles{i}, 'Interpreter', 'latex', 'FontSize', 12);
    grid on;
    grid minor;
    set(gca, 'GridColor', 'k', 'GridAlpha', 0.8, 'GridLineStyle', '-');
end

% Графики эквивалентных напряжений и прочности
figure('Position', [0, 0, 800, 600]);
subplot_titles = {'Equivalent Stresses for Outer Surface', 'Equivalent Stresses for Inner Surface',
'Deflection', 'Angular Displacement'};
plot_functions = {sigma_eqv_outer, sigma_eqv_inner, W, phi};
plot_ylabels = {'$\sigma_{eqv}$, MPa', '$\sigma_{eqv}$, MPa', '$W(x)$, m', '$\phi(x)$, rad'};
for i = 1:4
    subplot(2, 2, i);
    plot(x, plot_functions{i}(x) / (1e6), 'k');
    xlabel('$x$, m', 'Interpreter', 'latex', 'FontSize', 12);
    ylabel(plot_ylabels{i}, 'Interpreter', 'latex', 'FontSize', 12);
    title(subplot_titles{i}, 'Interpreter', 'latex', 'FontSize', 12);
    grid on;
    grid minor;
    set(gca, 'GridColor', 'k', 'GridAlpha', 0.8, 'GridLineStyle', '-');
end

% Расчет коэффициента запаса прочности
sigma_eqv_inner_max = max(abs(sigma_eqv_inner(x)));
sigma_eqv_outer_max = max(abs(sigma_eqv_outer(x)));
disp(['Maximum equivalent stress for inner surface σ_eqv_inner_max=', num2str(sigma_eqv_inner_max /
(1e6)), ' MPa']);
disp(['Maximum equivalent stress for outer surface σ_eqv_outer_max=', num2str(sigma_eqv_outer_max /
(1e6)), ' MPa']);
if sigma_eqv_inner_max > sigma_eqv_outer_max
    n = sigma / sigma_eqv_inner_max;
else
    n = sigma / sigma_eqv_outer_max;
end
disp(['Safety factor n=', num2str(n)]);
```

Задача №6
Кат. 1s
$h_1 = 15$мм
$h_2 = 50$мм
$\varepsilon_1 = 3$мм
$\varepsilon_2 = 1$мм

Маршаров М.Г.
ТФ-13-22

Из №2: $N = 14$кВт
$n_0 = 400 \frac{об}{мин}$
$D_1 = 0,3$м
$D_2 = 0,35$м
$l_1 = 0,8$м $l_2 = 0,85$м

$a_1 = 0,35$м
Ст. 35 $\sigma_в = 520$
$\sigma_т = 280$ МПа
$\sigma_{-1} = 220$
$d = 55$мм (из кандидатов №2)

1) Массы дисков:

$$m_1 = \rho V_1 = \rho \pi \left(\frac{D_1}{2}\right)^2 h_1 = 7850 \cdot \pi \left(\frac{0,3}{2}\right)^2 \cdot 15 \cdot 10^{-3} = 8,3 \text{кг}$$

$$m_2 = \rho V_2 = 7850 \pi \left(\frac{0,35}{2}\right)^2 \cdot 50 \cdot 10^{-3} = 37,8 \text{кг}$$

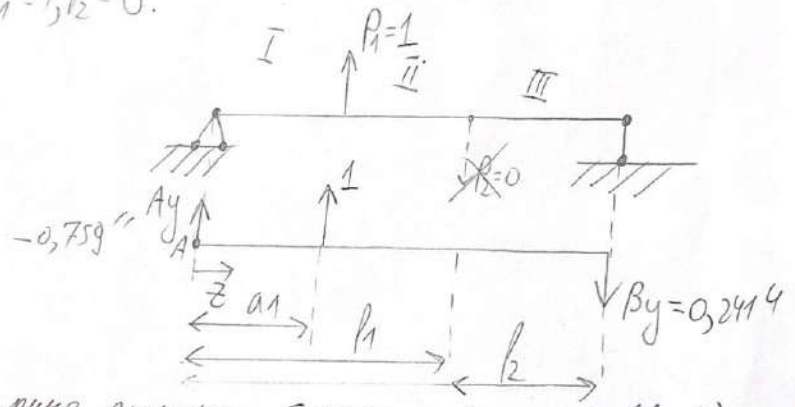
2) Найдем силы:

$$P_1 = m_1 \varepsilon_1 \Theta^2 = 8,3 \cdot 3 \cdot 10^{-3} \Theta^2 = 0,0249 \Theta^2$$

$$P_2 = m_2 \varepsilon_2 \Theta^2 = 37,8 \cdot 10^{-3} \Theta^2 = 0,0378 \Theta^2$$

3) Найдем моменты от ед. сил ($M_y|_{P_1=1, P_2=0}$ и $M_y|_{P_2=1, P_1=0}$)

$M_y|_{P_1=1, P_2=0}$.

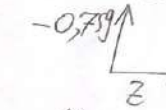


Опорные реакции: $\sum mom_A = 1 \cdot a_1 - B_y(l_1+l_2) = 0 \Rightarrow$

$$B_y = \frac{a_1}{l_1+l_2} = \frac{0,35}{0,8+0,85} = 0,2414$$

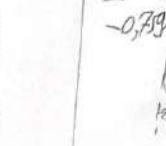
$$\sum F_y = 1 + A_y - B_y = 0 \Rightarrow A_y = B_y - 1 = -0,759$$

I: $0 < z < a_1$



$M_y = -0,759z$
$M_y(0) = 0$
$M_y(a_1 = 0,35) = -0,266$

II: $a_1 < z < l_1$



$\sum mom_k = -0,759z - M_y + 1(z - a_1) = 0 \Rightarrow M_y = -0,241z - 0,35$
$M_y(a_1 = 0,35) = -0,266$
$M_y(l_1 = 0,8) = -0,1572$

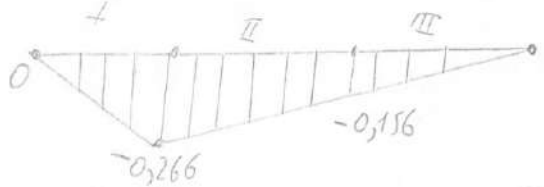
III $l_1 < z < l_1 + l_2$
Ф-я M_y продолжается без разрывов как на II-ом интервале и $M_y(l_1+l_2) = 0$

Построим эпюру $\overline{M_1} = M_y / P_1 = 1, P_2 = 0$. Запишем $\overline{M_1}$ как ф-ю:

на I: $-0{,}759z$, $0 \le z \le 0{,}35$

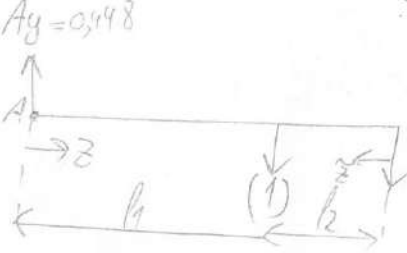
на II, III: $0{,}241z - 0{,}35$; $0{,}35 \le \tilde{z} \le 0{,}8$

III: $0{,}8 \le z \le 1{,}45$



$\overline{M_1}$

Теперь построим эпюру $\overline{M_2} = M_y / P_1 = 0, P_2 = 1$



Найдем опорные реакции

$\sum M_{om_A} = 1 \cdot l_1 + B_y(l_1 + l_2) = 0$

$B_y = -\frac{l_1}{l_1 + l_2} = -\frac{0{,}8}{0{,}8 + 0{,}65} = -0{,}552$

$\sum F_y = A_y - 1 + B_y = 0 \Rightarrow$

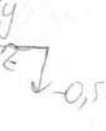
$A_y = B_y + 1 = 1 - 0{,}552 = 0{,}448$

I. $0 < z < l_1$:

$\Rightarrow M_y = 0{,}448z$ $M_y(0) = 0$

$M_y(l_1 = 0{,}8) = 0{,}3584$

II. $0 < \tilde{z} < l_2$



$\Rightarrow M_y = 0{,}552\tilde{z}$

$M_y(\tilde{z} = 0) = 0$

$M_y(\tilde{z} = l_2 = 0{,}65) = 0{,}3588$

Запишем $\overline{M_2}$ как ф-ю: (z)

на I: $0{,}448z$; $0 < z < 0{,}35$

II: $0{,}448z$; $0{,}35 < z < 0{,}8$

III. $-0{,}552z + 0{,}8$

$0{,}8 < z < 1{,}45$

Построим эпюру $\overline{M_2}$:

$\overline{M_2}$

Воспользуемся интегралом Максвелла Мора для поиска

$\delta_{ij} = \int_0^{l_1+l_2} \frac{M_j \cdot M_i}{EI} dz$; $I = \frac{\pi d^4}{64} \Rightarrow EI = 200 \cdot 10^9 \cdot \pi \cdot \frac{(55 \cdot 10^{-3})^4}{64} = 89836{,}1$

$\delta_{11} = \int_0^{l_1+l_2} \frac{\overline{M_1} \overline{M_1}}{EI} dz$. $\overline{M_1}$ - кусочно-заданная поэтому интеграл придется разбивать. $\int_0^{l_1+l_2} \ldots = \int_0^{a_1} \ldots + \int_{a_1}^{l_1+l_2} \ldots$

$$\delta_{11} = \frac{1}{EI}\left[\int_0^{0{,}35} (-0{,}759z)^2 dz + \int_{0{,}35}^{1{,}45} (0{,}241z - 0{,}35)^2 dz\right]$$

Все вычисления через пакет Wolfram Alpha (см. конец)

$\delta_{11} = 3{,}803 \cdot 10^{-7}$

$$\delta_{22} = \frac{1}{EI}\int_0^{l_1+l_2} \overline{M_2}\,\overline{M_2}\,dz = \frac{1}{EI}\left[\int_0^{0,8}(0,448z)^2 dz + \int_{0,8}^{1,45}(-0,552z+0,8)^2 dz\right]$$

$$= 6,907\cdot 10^{-7}$$

$$\delta_{21} = \frac{1}{EI}\int_0^{l_1+l_2} \overline{M_1}\,\overline{M_2}\,dz = \frac{1}{EI}\left[\int_0^{0,35}(-0,759z)(0,448z)dz + \int_{0,35}^{0,8}(0,241z-0,35)(0,448z)dz\right.$$

$$\left. + \int_{0,8}^{1,45}(0,241z-0,35)(-0,552z+0,8)dz\right] = -4,54\cdot 10^{-7}$$

Найдем $\Delta P_1, \Delta P_2$

$$\Delta P_1 = P_1\delta_{11} + P_2\delta_{12} = \Theta^2(0,0249\cdot 3,8\cdot 10^{-7} + 0,0378(-4,54\cdot 10^{-7})) =$$

$$-10^{-7}\Theta^2\cdot 0,07624$$

$$\Delta P_2 = P_1\delta_{21} + P_2\delta_{22} = \Theta^2(0,0249(-4,54)10^{-7} + 0,0378\cdot 6,907\cdot 10^{-7}) =$$

$$= 0,148\,\Theta^2\cdot 10^{-7}$$

Запишем ур-е движения:

$$\sum_{k=1}^{\hat{}} \ddot{u}_k m_k + \delta_{jk} + u_j = \Delta P_j\cos(\Theta t) \quad \forall j = \overline{1,n}$$

Решение ищем в виде: $u_1(t) = D_1\cos(\Theta t)$

$u_j'' = -\Theta^2 D_j\cos(\Theta t)$ $\quad u_2(t) = D_2\cos(\Theta t)$

$$\begin{cases} \ddot{u}_1 m_1\delta_{11} + m_2\ddot{u}_2\,\overline{\delta_{12}} + u_1 = \Delta P_1\cos(\Theta t) \\ \ddot{u}_1 m_1\delta_{21} + m_2\ddot{u}_2\,\overline{\delta_{22}} + u_2 = \Delta P_2\cos(\Theta t) \end{cases}$$

$$\begin{cases} -\Theta^2\delta_{11}m_1 D_1\cos(\Theta t) - \Theta^2\overline{\delta_{12}}m_2 D_2\cos(\Theta t) + D_1\cos(\Theta t) = \Delta P_1\cos(\Theta t) \\ -\Theta^2\delta_{21}m_1 D_1\cos(\Theta t) - \Theta^2\delta_{22}m_2 D_2\cos(\Theta t) + D_2\cos(\Theta t) = \Delta P_2\cos(\Theta t) \end{cases}$$

$$\begin{cases} D_1(1-\Theta^2\delta_{11}m_1) - D_2(m_2\delta_{12}\Theta^2) = \Delta P_1 \\ D_1(-\Theta^2\delta_{21}m_1) + D_2(1-m_2\delta_{22}\Theta^2) = \Delta P_2 \end{cases}$$

СЛАУ отн. $\overline{D} = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}$, $A\overline{D} = B$

Подставим $\delta_{ij}, m_j, \Delta P_j$

$$\begin{cases} D_1(1-\Theta^2\cdot 31,57\cdot 10^{-7}) + D_2(\Theta^2\cdot 171,6\cdot 10^{-7}) = -0,076\,\Theta^2 10^{-7} \\ D_1(\Theta^2\cdot 37,68\cdot 10^{-7}) + D_2(1-\Theta^2\cdot 261\cdot 10^{-7}) = 0,148\,\Theta^2 10^{-7} \end{cases}$$

Метод Крамера: $D_1 = \frac{\Delta_1}{\Delta}$; $D_2 = \frac{\Delta_2}{\Delta}$ $\quad \Delta_i$ - замена i-го столбца на B и его определитель

$\Delta = \det(C)$ - матрица СЛАУ

$$\Delta = 1{,}774 \cdot 10^{-11}\Theta^4 - 0{,}0000029\Theta^2 + 1$$
$$\Delta_1 = -5{,}56 \cdot 10^{-14}\Theta^4 - 7{,}6 \cdot 10^{-9}\Theta^2$$
$$\Delta_2 = 1{,}48 \cdot 10^{-8}\Theta^2 - 1{,}809 \cdot 10^{-14}\Theta^4$$

} скрины в конце

$$P_1 = \frac{\Delta_1}{\Delta} = \frac{-5{,}56 \cdot 10^{-14}\Theta^4 - 7{,}6 \cdot 10^{-9}\Theta^2}{1{,}774 \cdot 10^{-11}\Theta^4 - 0{,}0000029\Theta^2 + 1}$$

$$P_2 = \frac{\Delta_2}{\Delta} = \frac{1{,}48 \cdot 10^{-8}\Theta^2 - 1{,}809 \cdot 10^{-14}\Theta^4}{1{,}774 \cdot 10^{-11}\Theta^4 - 0{,}0000029\Theta^2 + 1}$$

$$\omega_1 = \frac{1}{\sqrt{\frac{m_1\delta_{11} + m_2\delta_{22}}{2} + \sqrt{\left(\frac{m_1\delta_{11} - m_2\delta_{22}}{2}\right)^2 + m_1 m_2 \delta_{12}\delta_{22}}}} \quad (=)$$

$(=)$ $\begin{vmatrix} m_1 = 8{,}3 \\ m_2 = 37{,}8 \\ \delta_{11} = 3{,}8 \cdot 10^{-7} \\ \delta_{22} = 6{,}9 \cdot 10^{-7} \\ \delta_{12} = -4{,}4 \cdot 10^{-7} \end{vmatrix} = 186{,}92 \frac{рад}{с}$

$$\omega_2 = \frac{1}{\sqrt{\frac{m_1\delta_{11} + m_2\delta_{22}}{2} - \sqrt{\left(\frac{m_1\delta_{11} - m_2\delta_{22}}{2}\right)^2 + m_1 m_2 \delta_{12}\delta_{21}}}} \quad (=)$$

$(=)$ $1275{,}38 \frac{рад}{с}$

$$n_1 = \frac{60\omega_1}{2\pi} = 1{,}785 \cdot 10^3 \frac{об}{мин}; \quad n_2 = \frac{60\omega_2}{2\pi} = 12{,}179 \cdot 10^3 \frac{об}{мин}$$

Первая форма колебаний:

$$\Theta = \omega_1 = 186{,}92 \frac{рад}{с}$$

$$P_I = \begin{pmatrix} P_1/\omega_1 \\ P_2/\omega_1 \end{pmatrix} = \begin{pmatrix} +0{,}0386 \\ 9{,}057 \end{pmatrix}$$

Вторая:

$$\Theta = \omega_2 = 1275{,}38 \frac{рад}{с}$$

$$P_{II} = \begin{pmatrix} P_1/\omega_2 \\ P_2/\omega_2 \end{pmatrix} = \begin{pmatrix} -0{,}21 \\ 0{,}59 \end{pmatrix}$$

FROM THE MAKERS OF WOLFRAM LANGUAGE AND MATHEMATICA

WolframAlpha

$$\frac{1}{89836.1}*\left(\int_0^{0.8}(0.448*z)^2\,dz+\int_{0.8}^{1.45}(-0.552*z+0.8)^2\,dz\right)$$

NATURAL LANGUAGE | MATH INPUT

Input interpretation

$$\frac{1}{89836.1}\left(\int_0^{0.8}(0.448\,z)^2\,dz+\int_{0.8}^{1.45}(-0.552\,z+0.8)^2\,dz\right)$$

Computation result

$$\frac{\int_0^{0.8}(0.448\,z)^2\,dz+\int_{0.8}^{1.45}(-0.552\,z+0.8)^2\,dz}{89836.1}=6.9074\times10^{-7}$$

POWERED BY THE WOLFRAM LANGUAGE

FROM THE MAKERS OF WOLFRAM LANGUAGE AND MATHEMATICA

WolframAlpha

$$\frac{1}{89836.1}*\left(\int_0^{0.35}(0.448*z)*(-0.759*z)\,dz+\int_{0.35}^{0.8}(0.241*z-0.35)*(0.448*z)\,dz+\int_{0.8}^{1.45}(0.241*z\right.$$

NATURAL LANGUAGE | MATH INPUT

Input interpretation

$$\frac{1}{89836.1}\left(\int_0^{0.35}(0.448\,z)(-0.759\,z)\,dz+\int_{0.35}^{0.8}(0.241\,z-0.35)(0.448\,z)\,dz+\int_{0.8}^{1.45}(0.241\,z-0.35)(-0.552\,z+0.8)\,dz\right)$$

Computation result

$$\frac{1}{89836.1}\left(\int_0^{0.35}(0.448\,z)(-0.759\,z)\,dz+\int_{0.35}^{0.8}(0.241\,z-0.35)(0.448\,z)\,dz+\int_{0.8}^{1.45}(0.241\,z-0.35)(-0.552\,z+0.8)\,dz\right)=-4.53823\times10^{-7}$$

POWERED BY THE WOLFRAM LANGUAGE

FROM THE MAKERS OF WOLFRAM LANGUAGE AND MATHEMATICA

WolframAlpha

$$\frac{1}{89836.1}*\left(\int_0^{0.35}(-0.759*z)^2\,dz+\int_{0.35}^{1.45}(0.241*z-0.35)^2\,dz\right)$$

NATURAL LANGUAGE | MATH INPUT

Input interpretation

$$\frac{1}{89836.1}\left(\int_0^{0.35}(-0.759\,z)^2\,dz+\int_{0.35}^{1.45}(0.241\,z-0.35)^2\,dz\right)$$

Computation result

$$\frac{\int_0^{0.35}(-0.759\,z)^2\,dz+\int_{0.35}^{1.45}(0.241\,z-0.35)^2\,dz}{89836.1}=3.80276\times10^{-7}$$

WolframAlpha

$$\frac{1}{\sqrt{\frac{10^{-7}(8.3*3.8+37.8*6.9)}{2}+\sqrt{\left(\frac{10^{-7}(8.3*3.8-37.8*6.9)}{2}\right)^2+8.3*37.8*10^{-14}*(4.54)^2}}}$$

NATURAL LANGUAGE | MATH INPUT

Input

$$\frac{1}{\sqrt{\frac{1}{2}\frac{8.3\cdot3.8+37.8\cdot6.9}{10^7}+\sqrt{\left(\frac{1}{2}\frac{8.3\cdot3.8-37.8\cdot6.9}{10^7}\right)^2+\frac{8.3\cdot37.8\cdot4.54^2}{10^{14}}}}}$$

Result

More digits

186.920...

Number line

Number name

one hundred eighty-six point nine two zero zero seven two one six one one nine

POWERED BY THE WOLFRAM LANGUAGE

FROM THE MAKERS OF WOLFRAM LANGUAGE AND MATHEMATICA

WolframAlpha

$$\frac{1}{\sqrt{\frac{10^{-7}(8.3*3.8+37.8*6.9)}{2}-\sqrt{\left(\frac{10^{-7}(8.3*3.8-37.8*6.9)}{2}\right)^2+8.3*37.8*10^{-14}*(4.54)^2}}}$$

NATURAL LANGUAGE | MATH INPUT

Input

$$\frac{1}{\sqrt{\frac{1}{2}\frac{8.3\cdot3.8+37.8\cdot6.9}{10^7}-\sqrt{\left(\frac{1}{2}\frac{8.3\cdot3.8-37.8\cdot6.9}{10^7}\right)^2+\frac{8.3\cdot37.8\cdot4.54^2}{10^{14}}}}}$$

Result

More digits

1275.38...

Number line

Number name