

Master's thesis in  
Applied Computer Science

## CoolingGen

A parametric 3D-modeling software for turbine  
blade cooling geometries using NURBS

June 27, 2022

Institute for Numerical and Applied Mathematics  
at the Georg-August-University Göttingen

Institute for Propulsion Technology at the  
German Aerospace Center in Göttingen

Bachelor's and master's theses at the Center for  
Computational Sciences at the  
Georg-August-University Göttingen

Julian Lüken  
`julian.lueken@dlr.de`

Georg-August-University Göttingen  
Institute of Computer Science

☎ +49 (551) 39-172000

☎ +49 (551) 39-14403

✉ [office@cs.uni-goettingen.de](mailto:office@cs.uni-goettingen.de)

[www.informatik.uni-goettingen.de](http://www.informatik.uni-goettingen.de)

I hereby declare that this thesis has been written by myself and no other resources than those mentioned have been used.

A handwritten signature in blue ink, appearing to read 'Lüken', written in a cursive style.

Göttingen, June 27, 2022

## **Abstract**

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Motivation . . . . .	1
1.2	State of the Art . . . . .	1
1.3	Problem Statement . . . . .	1
<b>2</b>	<b>Methods</b>	<b>2</b>
2.1	Bézier Curves . . . . .	2
2.1.1	Definition . . . . .	2
2.1.2	Properties . . . . .	3
2.1.3	De Casteljau's Algorithm . . . . .	3
2.2	Non-Uniform Rational B-Splines (NURBS) . . . . .	3
2.2.1	Definition . . . . .	3
2.2.2	Properties . . . . .	3
2.2.3	De Boor's Algorithm . . . . .	3
2.3	Methods on NURBS Objects . . . . .	3
2.3.1	Projection, Translation and Rotation . . . . .	3
2.3.2	The Frenet-Serret Apparatus . . . . .	3
2.3.3	Finding Intersections . . . . .	3
2.3.4	Interpolation . . . . .	3
2.4	Jet Engine Design Specifics . . . . .	3
2.4.1	Fundamental Terms . . . . .	3
2.4.2	The S2M Net . . . . .	3
2.4.3	Fillet Creation . . . . .	3
<b>3</b>	<b>Results</b>	<b>4</b>
3.1	Cooling Geometries And Their Parametrizations . . . . .	4
3.1.1	Chambers . . . . .	4
3.1.2	Turnarounds . . . . .	4
3.1.3	Slots . . . . .	4
3.1.4	Film Cooling Holes . . . . .	4
3.1.5	Impingement Inserts . . . . .	4
3.2	Export for CENTAUR . . . . .	4
3.3	Export for Open CASCADE . . . . .	4
<b>4</b>	<b>Discussion</b>	<b>5</b>
4.1	Future Work . . . . .	5
4.2	Conclusion . . . . .	5

# Chapter 1

## Introduction

### 1.1 Motivation

### 1.2 State of the Art

### 1.3 Problem Statement

# Chapter 2

## Methods

### 2.1 Bézier Curves

Bézier curves are named after the french engineer Pierre Bézier, who famously utilized them in the 1960s to design car bodies for the automobile manufacturer Renault. Today, they are used in a wide variety of vector graphics applications (i.e. in font representation on computers). At first glance, the definition of the Bézier curve might seem cumbersome, but given the mathematical foundation and a few graphical representations, it becomes apparent why they are such a powerful tool in computer-aided design.

#### 2.1.1 Definition

**Definition 2.1.1.** The *Bernstein basis polynomials* of degree  $n$  on the interval  $[t_0, t_1]$  are defined as

$$b_{n,k,[t_0,t_1]}(t) := \frac{\binom{n}{k}(t_1 - t)^{n-k}(t - t_0)^k}{(t_1 - t_0)^n}, \quad (2.1)$$

for  $k \in \{0 \dots n\}$ .

**Theorem 2.1.2.** The set of polynomials

$$\mathcal{P}_n := \{p : t \mapsto \sum_{k=0}^n c_k t^k, c_k \in \mathbb{R}\}$$

equipped with the usual operations is a vector space.

**Theorem 2.1.3.** The Bernstein basis polynomials of degree  $n$  form a basis of  $\mathcal{P}_n$ .

**Theorem 2.1.4** (Theorem of Stone-Weierstrass). Yet to come.

**Definition 2.1.5.** A *Bézier curve* of degree  $n$  is a parametric curve  $C_{P,[t_0,t_1]} : [t_0, t_1] \rightarrow \mathbb{R}^3$  that has a representation

$$C_{P,[t_0,t_1]}(t) = \frac{\sum_{i=0}^n \binom{n}{i} (t_1 - t)^{n-i} (t - t_0)^i P_i}{(t_1 - t_0)^n}. \quad (2.2)$$

We call the elements of the set  $P = \{P_1, P_2, \dots, P_n\}$  the *control points* of  $C_P$ .

**Remark.** Let  $t_0 = 0$  and  $t_1 = 1$ . Then 2.2 simplifies to

$$b_{n,k}(t) := b_{n,k,[0,1]}(t) = \binom{n}{k} (1-t)^{n-k} t^k. \quad (2.3)$$

Also, 2.1 simplifies to

$$C_P(t) := C_{P,[0,1]}(t) = \sum_{i=0}^n \binom{n}{k} (1-t)^{n-k} t^k P_k. \quad (2.4)$$

This case is the only case considered in this thesis.

### 2.1.2 Properties

### 2.1.3 De Casteljau's Algorithm

## 2.2 Non-Uniform Rational B-Splines (NURBS)

### 2.2.1 Definition

### 2.2.2 Properties

### 2.2.3 De Boor's Algorithm

## 2.3 Methods on NURBS Objects

### 2.3.1 Projection, Translation and Rotation

### 2.3.2 The Frenet-Serret Apparatus

### 2.3.3 Finding Intersections

### 2.3.4 Interpolation

## 2.4 Jet Engine Design Specifics

### 2.4.1 Fundamental Terms

### 2.4.2 The S2M Net

### 2.4.3 Fillet Creation



# Chapter 3

## Results

### 3.1 Cooling Geometries And Their Parametrizations

#### 3.1.1 Chambers

#### 3.1.2 Turnarounds

#### 3.1.3 Slots

#### 3.1.4 Film Cooling Holes

#### 3.1.5 Impingement Inserts

### 3.2 Export for CENTAUR

### 3.3 Export for Open CASCADE

## Chapter 4

# Discussion

### 4.1 Future Work

### 4.2 Conclusion