Master's thesis in
Applied Computer Science

CoolingGen

A parametric 3D-modeling software for turbine
blade cooling geometries using NURBS

July 17, 2022

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I hereby declare that this thesis has been written by myself and no other resources than those mentioned have been used.

Göttingen, July 17, 2022

Abstract

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1 Introduction

1.1 Motivation

1.2 State of the Art

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2.1 Bézier Curves

Bézier curves are named after the french engineer Pierre Bézier, who famously utilized them in the 1960s to design car bodies for the automobile manufacturer Renault [Béz68]. Today, they are used in a wide variety of vector graphics applications (i.e. in font representation on computers). At first glance, the definition of the Bézier curve might seem cumbersome, but given the mathematical foundation and a few graphical representations, it becomes apparent why they are such a powerful tool in computer-aided design.

2.1.1 Definition

Definition 2.1.1. The *Bernstein basis polynomials* of degree n on the interval $[t_0, t_1]$ are defined as

$$b_{n,k,[t_0,t_1]}(t) := \frac{\binom{n}{k}(t_1 - t)^{n-k}(t - t_0)^k}{(t_1 - t_0)^n,} \tag{2.1}$$

for $k \in \{0 \dots n\}$.

Definition 2.1.2. A *Bézier curve* of degree n is a parametric curve $C_{P,[t_0,t_1]} : [t_0, t_1] \to \mathbb{R}^3$ that has a representation

$$C_{P,[t_0,t_1]}(t) = \sum_{k=0}^{n} b_{n,k,[t_0,t_1]}(t) P_k = \sum_{k=0}^{n} \frac{\binom{n}{k}(t_1 - t)^{n-k}(t - t_0)^k P_k}{(t_1 - t_0)^n}. \tag{2.2}$$

We call the elements of the set $P = \{P_1, P_2, \dots, P_n\}$ the *control points* of C_P.

Remark. Let $t_0 = 0$ and $t_1 = 1$. Then 2.2 simplifies to

$$b_{n,k}(t) := b_{n,k,[0,1]}(t) = \binom{n}{k}(1 - t)^{n-k} t^k. \tag{2.3}$$

Also, 2.1 simplifies to

$$C_P(t) := C_{P,[0,1]}(t) = \sum_{k=0}^{n} \binom{n}{k}(1 - t)^{n-k} t^k P_k. \tag{2.4}$$

This case is the only case considered in this thesis.

2.1.2 De Casteljau's Algorithm

The algorithm proposed by Paul de Faget de Casteljau calculates points on the Beziér curve in a recursive manner.

2.1.3 Properties

Algorithm 1 De Casteljau's algorithm

Input

$P = \{P_0, P_1, ..., P_n\}$ set of control points

t real number

Output

$P_0^{(n)} = C_P(t)$ the point on the Beziér curve

procedure DECASTELJAU(P, t)

  $P^{(0)} \leftarrow P$

  for $i = 1, 2, ..., n$ **do**

    for $j = 0, 1, ..., n - j$ **do**

      $P_j^{(i)} = (1 - t) \cdot P_j^{(i-1)} + t \cdot P_{j+1}^{(i-1)}$

  return $P_0^{(n)}$

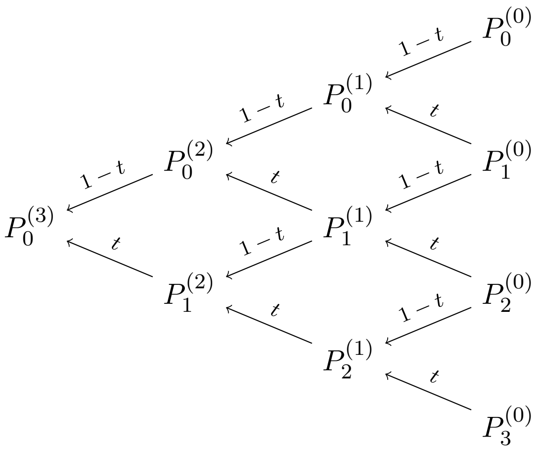

Figure 2.1: Beziér curves of different degrees and their control points

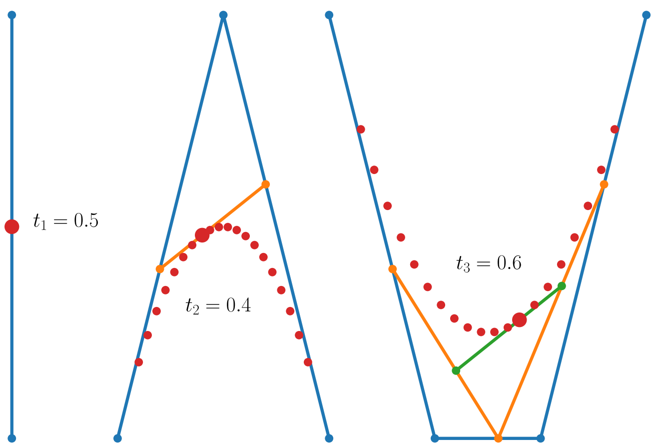

Figure 2.2: Beziér curves of different degrees and their control points

2.2 Non-Uniform Rational B-Splines (NURBS)

2.2.1 Definition

2.2.2 Properties

2.2.3 De Boor's Algorithm

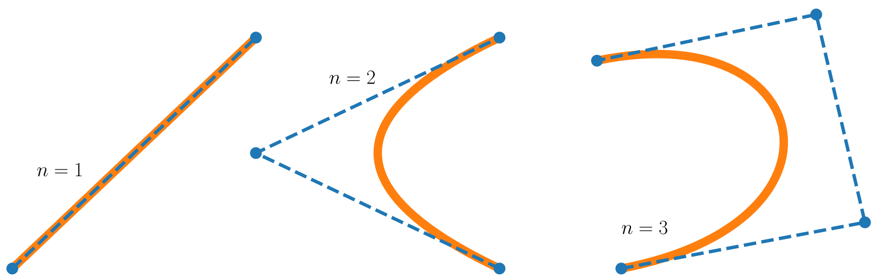

Figure 2.3: Beziér curves of different degrees and their control points

3 Results

3.1 Cooling Geometries And Their Parametrizations

3.1.1 Chambers

3.1.2 Turnarounds

3.1.3 Slots

3.1.4 Film Cooling Holes

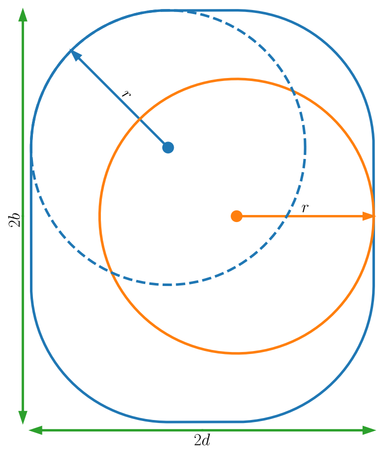

Figure 3.1: yeah

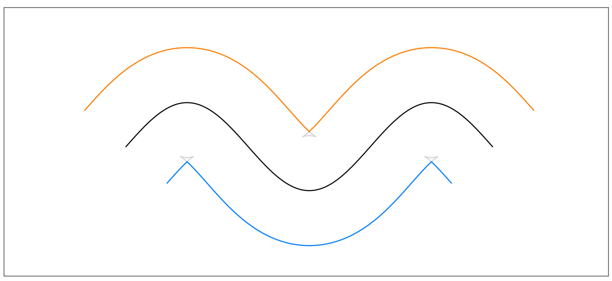

Figure 3.2: yeah

3.1.5 Impingement Inserts

3.2 Export for CENTAUR

3.3 Export for Open CASCADE

4 Discussion

4.1 Future Work

4.2 Conclusion

[Pie97]

5 References

[Béz68] Pierre E. Bézier. "How Renault Uses Numerical Control for Car Body Design and Tooling". In: *SAE Technical Paper Series*. SAE International, Feb. 1968. DOI: 10.4271/680010.

[Pie97] Les A. Piegl. *The NURBS book*. Springer, 1997, p. 646. ISBN: 3540615458.