

Master's thesis in  
Applied Computer Science

## CoolingGen

A parametric 3D-modeling software for turbine  
blade cooling geometries using NURBS

July 9, 2022

Institute for Numerical and Applied Mathematics  
at the Georg-August-University Göttingen

Institute for Propulsion Technology at the  
German Aerospace Center in Göttingen

Bachelor's and master's theses at the Center for  
Computational Sciences at the  
Georg-August-University Göttingen

Julian Lüken  
`julian.lueken@dlr.de`

Georg-August-University Göttingen  
Institute of Computer Science

☎ +49 (551) 39-172000

☎ +49 (551) 39-14403

✉ [office@cs.uni-goettingen.de](mailto:office@cs.uni-goettingen.de)

[www.informatik.uni-goettingen.de](http://www.informatik.uni-goettingen.de)

I hereby declare that this thesis has been written by myself and no other resources than those mentioned have been used.

A handwritten signature in blue ink, appearing to read 'Lüken', with a stylized, flowing script.

Göttingen, July 9, 2022

## Abstract

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## **Zusammenfassung**

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# 1 Introduction

## 1.1 Motivation

## 1.2 State of the Art

## 1.3 Problem Statement

## 2 Methods

### 2.1 Bézier Curves

Bézier curves are named after the french engineer Pierre Bézier, who famously utilized them in the 1960s to design car bodies for the automobile manufacturer Renault [Béz68]. Today, they are used in a wide variety of vector graphics applications (i.e. in font representation on computers). At first glance, the definition of the Bézier curve might seem cumbersome, but given the mathematical foundation and a few graphical representations, it becomes apparent why they are such a powerful tool in computer-aided design.

#### 2.1.1 Definition

**Definition 2.1.1.** The *Bernstein basis polynomials* of degree  $n$  on the interval  $[t_0, t_1]$  are defined as

$$b_{n,k,[t_0,t_1]}(t) := \frac{\binom{n}{k}(t_1 - t)^{n-k}(t - t_0)^k}{(t_1 - t_0)^n}, \quad (2.1)$$

for  $k \in \{0 \dots n\}$ .

**Definition 2.1.2.** A *Bézier curve* of degree  $n$  is a parametric curve  $C_{P,[t_0,t_1]} : [t_0, t_1] \rightarrow \mathbb{R}^3$  that has a representation

$$C_{P,[t_0,t_1]}(t) = \sum_{k=0}^n b_{n,k,[t_0,t_1]}(t)P_k = \sum_{k=0}^n \frac{\binom{n}{k}(t_1 - t)^{n-k}(t - t_0)^k}{(t_1 - t_0)^n} P_k. \quad (2.2)$$

We call the elements of the set  $P = \{P_1, P_2, \dots, P_n\}$  the *control points* of  $C_P$ .

**Remark.** Let  $t_0 = 0$  and  $t_1 = 1$ . Then 2.2 simplifies to

$$b_{n,k}(t) := b_{n,k,[0,1]}(t) = \binom{n}{k}(1 - t)^{n-k}t^k. \quad (2.3)$$

Also, 2.1 simplifies to

$$C_P(t) := C_{P,[0,1]}(t) = \sum_{k=0}^n \binom{n}{k}(1 - t)^{n-k}t^k P_k. \quad (2.4)$$

This case is the only case considered in this thesis.



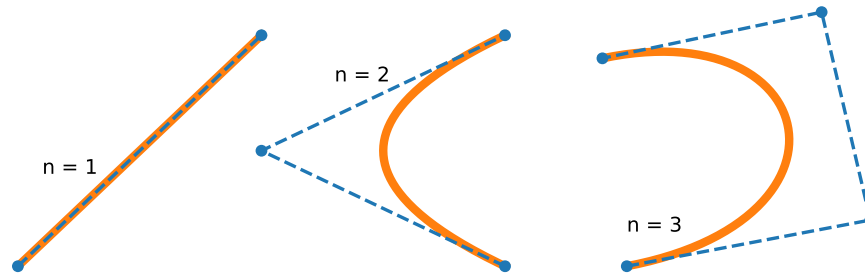


Figure 2.1: Beziér curves of different degrees and their control points

### 2.1.2 Properties

### 2.1.3 De Casteljau's Algorithm

## 2.2 Non-Uniform Rational B-Splines (NURBS)

### 2.2.1 Definition

### 2.2.2 Properties

### 2.2.3 De Boor's Algorithm

## 2.3 Methods on NURBS Objects

### 2.3.1 Affine Transformations

### 2.3.2 The Frenet-Serret Apparatus

### 2.3.3 Finding Intersections

### 2.3.4 Interpolation

## 2.4 Jet Engine Design Specifics

### 2.4.1 Fundamental Terms

### 2.4.2 The S2M Net

### 2.4.3 Fillet Creation

## 3 Results

### 3.1 Cooling Geometries And Their Parametrizations

#### 3.1.1 Chambers

#### 3.1.2 Turnarounds

#### 3.1.3 Slots

#### 3.1.4 Film Cooling Holes

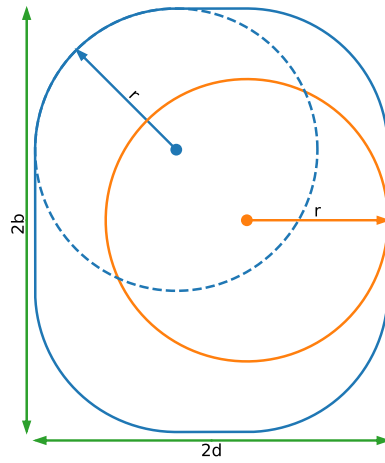


Figure 3.1: yeah

#### 3.1.5 Impingement Inserts

### 3.2 Export for CEN TAUR

### 3.3 Export for Open CASCAD E

## 4 Discussion

### 4.1 Future Work

### 4.2 Conclusion

[Pie97]

## 5 References

- [Béz68] Pierre E. Bézier. “How Renault Uses Numerical Control for Car Body Design and Tooling”. In: *SAE Technical Paper Series*. SAE International, Feb. 1968. DOI: 10 . 4271/680010.
- [Pie97] Les A. Piegl. *The NURBS book*. Springer, 1997, p. 646. ISBN: 3540615458.