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DLR

Master's thesis in
Applied Computer Science

# CoolingGen

A parametric 3D-modeling software for turbine blade cooling geometries using NURBS

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I hereby declare that this thesis has been written by myself and no other resources than those mentioned have been used.

Göttingen, July 5, 2022

## Abstract

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# Introduction

## 1.1 Motivation

## 1.2 State of the Art

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# Methods

## 2.1 Bézier Curves

Bézier curves are named after the french engineer Pierre Bézier, who famously utilized them in the 1960s to design car bodies for the automobile manufacturer Renault [Béz68]. Today, they are used in a wide variety of vector graphics applications (i.e. in font representation on computers). At first glance, the definition of the Bézier curve might seem cumbersome, but given the mathematical foundation and a few graphical representations, it becomes apparent why they are such a powerful tool in computer-aided design.

### 2.1.1 Definition

**Definition 2.1.1.** The *Bernstein basis polynomials* of degree $n$ on the interval $[t_0, t_1]$ are defined as

$$b_{n,k,[t_0,t_1]}(t) := \frac{\binom{n}{k}(t_1 - t)^{n-k}(t - t_0)^k}{(t_1 - t_0)^n,} \tag{2.1}$$

for $k \in \{0 \dots n\}$.

**Theorem 2.1.2.** The set of polynomials

$$\mathcal{P}_n := \{p : t \mapsto \sum_{k=0}^{n} c_k t^k, c_k \in \mathbb{R}\}$$

equipped with the usual operations is a vector space.

**Theorem 2.1.3.** The Bernstein basis polynomials of degree $n$ form a basis of $\mathcal{P}_n$.

**Theorem 2.1.4** (Theorem of Stone-Weierstrass)**.** Yet to come.

**Definition 2.1.5.** A *Bézier curve* of degree $n$ is a parametric curve $C_{P,[t_0,t_1]} : [t_0, t_1] \to \mathbb{R}^3$ that has a representation

$$C_{P,[t_0,t_1]}(t) = \frac{\sum_{i=0}^{n} \binom{n}{k}(t_1 - t)^{n-k}(t - t_0)^k P_k}{(t_1 - t_0)^n}. \tag{2.2}$$

We call the elements of the set $P = \{P_1, P_2, \dots, P_n\}$ the *control points* of $C_P$.

**Remark.** Let $t_0 = 0$ and $t_1 = 1$. Then 2.2 simplifies to

$$b_{n,k}(t) := b_{n,k,[0,1]}(t) = \binom{n}{k}(1-t)^{n-k}(t)^k. \tag{2.3}$$

Also, 2.1 simplifies to

$$C_P(t) := C_{P,[0,1]}(t) = \sum_{i=0}^{n} \binom{n}{k}(1-t)^{n-k}t^k P_k. \tag{2.4}$$

This case is the only case considered in this thesis.

### 2.1.2 Properties

### 2.1.3 De Casteljau's Algorithm

## 2.2 Non-Uniform Rational B-Splines (NURBS)

### 2.2.1 Definition

### 2.2.2 Properties

### 2.2.3 De Boor's Algorithm

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[Pie97]

# Chapter 5

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