Master's thesis in
Applied Computer Science

CoolingGen

A parametric 3D-modeling software for turbine
blade cooling geometries using NURBS

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I hereby declare that this thesis has been written by myself and no other resources than those mentioned have been used.

Göttingen, June 29, 2022

Abstract

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1.1 Motivation

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2.1 Bézier Curves

Bézier curves are named after the french engineer Pierre Bézier, who famously utilized them in the 1960s to design car bodies for the automobile manufacturer Renault. Today, they are used in a wide variety of vector graphics applications (i.e. in font representation on computers). At first glance, the definition of the Bézier curve might seem cumbersome, but given the mathematical foundation and a few graphical representations, it becomes apparent why they are such a powerful tool in computer-aided design.

2.1.1 Definition

Definition 2.1.1. The *Bernstein basis polynomials* of degree n on the interval $[t_0, t_1]$ are defined as

$$b_{n,k,[t_0,t_1]}(t) := \frac{\binom{n}{k}(t_1 - t)^{n-k}(t - t_0)^k}{(t_1 - t_0)^n,} \tag{2.1}$$

for $k \in \{0 \ldots n\}$.

Theorem 2.1.2. The set of polynomials

$$\mathcal{P}_n := \{p : t \mapsto \sum_{k=0}^{n} c_k t^k, c_k \in \mathbb{R}\}$$

equipped with the usual operations is a vector space.

Theorem 2.1.3. The Bernstein basis polynomials of degree n form a basis of $\mathcal{P}_n$.

Theorem 2.1.4 (Theorem of Stone-Weierstrass)**.** Yet to come.

Definition 2.1.5. A *Bézier curve* of degree n is a parametric curve $C_{P,[t_0,t_1]} : [t_0, t_1] \to \mathbb{R}^3$ that has a representation

$$C_{P,[t_0,t_1]}(t) = \frac{\sum_{i=0}^{n} \binom{n}{k}(t_1 - t)^{n-k}(t - t_0)^k P_k}{(t_1 - t_0)^n}. \tag{2.2}$$

We call the elements of the set $P = \{P_1, P_2, \ldots, P_n\}$ the *control points* of C_P.

Remark. Let $t_0 = 0$ and $t_1 = 1$. Then 2.2 simplifies to

$$b_{n,k}(t) := b_{n,k,[0,1]}(t) = \binom{n}{k}(1-t)^{n-k}(t)^k. \tag{2.3}$$

Also, 2.1 simplifies to

$$C_P(t) := C_{P,[0,1]}(t) = \sum_{i=0}^{n}\binom{n}{k}(1-t)^{n-k}t^k P_k. \tag{2.4}$$

This case is the only case considered in this thesis.

2.1.2 Properties

2.1.3 De Casteljau's Algorithm

2.2 Non-Uniform Rational B-Splines (NURBS)

2.2.1 Definition

2.2.2 Properties

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