

a) Let  $a$  and  $b$  both be the least element of a poset  $(S, R)$

By definition of the least element,  $a$  is smaller than or equal to all elements in  $S$

$$a \leq s \text{ for all } s \in S$$

But  $b$  is also an element in  $S$ :

$$a \leq b$$

$b$  is also the least element of  $S$ . By the definition of the least element,  $b$  is smaller than or equal to all elements in  $S$ .

$$b \leq s \text{ for all } s \in S$$

But  $a$  is also an element of  $S$ :

$$b \leq a$$

Since  $a \leq b$  and  $b \leq a$ :

$$a = b$$

This implies that if a poset contains multiple least elements, then these least elements are identical and are one unique least element.

b) Let  $a$  and  $b$  both be the greatest element of a poset  $(S, R)$

By the definition of the greatest element,  $a$  is larger than or equal to all elements in  $S$

$$s \leq a \text{ for all } s \in S$$

But,  $b$  is also an element in  $S$ :

$$b \leq a$$

$b$  is also the greatest element of  $S$

By the definition of the greatest element,  $b$  is larger than or equal to all elements in  $S$

$$s \leq b \text{ for all } s \in S$$

but  $a$  is also an element of  $S$ :

$$a \leq b$$

Since  $a \leq b$  and  $b \leq a$

$$a = b$$

Which implies that if a poset contains multiple greatest elements, then these greatest elements are identical and thus there is exactly one unique greatest element in a poset.