Let A, B, and C be sets. Use the definition of set equality to prove that

$(A-C) \cap (C-B)=0$

PROOF

1. Let $x \in (A-C) \cap (C-B)$
   a. Using the definition of intersection, x is in the intersection when it is in both sets (duh)
      i. $x \in (A-C) \wedge x \in (C-B)$
   b. Using the definition of the difference A-C we know that x is in A and x is not in C
      i. $x \in A \wedge \neg(x \in C) \wedge x \in C \wedge \neg(x \in B)$
   c. Negation law for propositions
      i. $x \in A \wedge G \wedge \neg(x \in B)$
   d. Domination law
      i. G
   e. Emptyset doesn't contain elements, so statement $x \in 0$ is false always.
   f. By definition of subset, we show that $(A-C) \cap (C-B) \subseteq 0$
2. Empty set is a subset of every set
   a. $0 \subseteq (A-C)$
3. Conclusion:
   a. Since $(A-C) \cap (C-B) \subseteq 0$ and $0 \subseteq (A-C)$, the two sets have to be the same

$(A-C) \cap (C-B)=0$