**Problem 1:** Let P(n) be the statement that 12+22+ ... +n2 = for some positive integer n.

1. What is the statement P(1)?
2. Show that P(1)is true, completing the basis step of the proof.
3. What is the inductive hypothesis?
4. What do you need to prove in the inductive step?
5. Complete the inductive step, identifying where you use the inductive hypothesis.
6. State why these steps show that this formula is true whenever nis a positive integer.

**Answer:**

a. P(1) = 1(1+1)(1+2) / 6 = 1

b. P(1): 1 = 1

c. Assume for all positive integers n

P(n) = 12 + 22+ … + n2 = n(n+1)(2n+1) / 6 is true

d. Need to prove that P(n+1) is true

e. Given

P(n) = 12 + 22+ … + n2 = (n(n+1)(2n+1)) / 6 + (n+1)2

Focusing on RHS:

= (n(n+1) (2n+1) + 6(n+1)2) /6

= (n+1)(n(2n+1) + 6(n+1)) / 6

= (n+1)(2n2 + n + 6n + 6) / 6

= (n+ 1) (2n2 + 7n + 6) / 6

= (n+1)(n+2)(2n+3)) / 6

P(n+1) = (n+1)(n+2)(2n+3)) / 6

**Problem 2:**  Use mathematical induction to prove that 2 divides n2+n whenever nis a positive integer.

**Answer:**

Step 1: Let P(k) =  “2 divides k2+k”

Domain = { Positive integers}

k Z+

Step 2: P(1) is True because (12+1)/2 = 1

Step 3: Inductive Step

Assume P(K) is True for some k >0, k Z+

If k2 + k is a multiple of 2 for some k, then consider:

(k+1)2 + (k+1) = k2 + 2k + 1 + k + 1 = (k2 + k) + 2k + 2 = (k2 + k) + 2(k+1)

By our inductive hypothesis, k2 + k was an even number, because we are adding an even number, 2(k+1) to it, we still have an even number. Therefore, (k + 1)2 + (k+1) must be even as well.