**Problem 1:** Let P(n) be the statement that a postage of n cents can be formed using just 4-cent and 7-cent stamps.  The parts of this problem outline a strong induction proof that P(n) is true for all integers n >= 18.

1. Show that the statements P (18), P (19), P (20) and P (21) are true, completing the basis step of the proof.
2. What is the inductive hypothesis of a proof by strong mathematical induction that P(n)is true for all integers n >= 18?
3. Complete the inductive step for some k >= 21.
4. Explain why these steps show that P(n) is true for all integers n >= 18.

**Answer:**

1. Basis step
   1. n = 18 and n = 19 and n = 21
   2. P (18) is true, because 18 cents can be formed using 2 7-cent stamps and 1 4-cent stamp (since 2\*7 + 4\*1 = 18)
   3. P (19) is true, because 19 cents can be formed using 1 7-cent stamp and 3 4-cent stamps (since 1\*7 + 3\*4 = 7 + 12 = 19)
   4. P (20) is true, because 20 cents can be formed using 5 4-cent stamps (since 5\*4 = 20)
   5. P (21) is true, because 21 cents can be formed using 3 7-cent stamps (since 3\*7 = 21)
2. Inductive Hypothesis
   1. We assume that P (18), P (19), …. P(k) are all true, so any postage between 18 and k cents can be formed using just 4-cent stamps and 7-cent stamps
3. Inductive Step
   1. By b. we know P(k-3) is true and so a postage of k – 3 cents can be formed using just 4-cent stamps and 7-cent stamps
   2. Since k + 1 = (k – 3) + 4, P (k + 1) is then true, because the number of 7-cent stamps are the same for k + 1 as for k -3, while the number of 4-cent stamps is 1 for k + 1 than for k + 3
4. Explanation
   1. By the principle of strong induction, P (n) is true for all positive integers n with n >= 18.

**Problem 2:**  Which amounts of money can be formed using just $2-bills and $5-bills?  Prove your answer using Strong Mathematical Induction.

**Answer:**

Let P(n) be “n dollars can be formed using $2 and $5 dollar bills.”

Basis step: n = 4 and n = 5

P (4) is true, because 4 dollars can be formed using 2 2-dollar bills

2 \* 2 = 4

P (5) is true, because 5 dollars can be formed using one 5-dollar bill

1 \* 5 = 5

Inductive step: We assume that P (4), P (5), …. P(k) are all true, thus all amounts from 4 to k dollars can be formed using just 2-dollar and 5-dollar bills.

We then need to prove that P (k + 1) is true. Since P (k – 1) is true, P (k + 1) is also true

Conclusion: By the principle of strong induction, P (n) is true for all positive integers n.