CECS 329 Final A, Problem 1, Fall 2021, Dr. Ebert

Instructions

Rules for Completing the Problem

When solving this problem you may only reference course artifacts, including the lecture notes and recordings, textbook, and any notes/solutions you have handwritten during the course and prior to the quiz. Communicating with others (whether inside or outside of class) or finding solutions online is considered cheating and is grounds for receiving an F grade for the class. You show sufficient work to receive full credit.

Submitting your work

Submit a single file with your handwritten solution to the appropriate drop box by 10:00 am. Make sure you provide your name and SID in the upper-right corner of your solution.

Late submissions

Should you submit after the dropbox deadline, solutions received no later than 10 minutes after the deadline will lose 20% of the earned points. Solutions received between 11 and 20 minutes after the deadline will 50% of the points. All other late submissions will not be graded.

Problem 1

An instance of `Set Cover` is a triple $(\mathcal{S}, m, k)$, where $\mathcal{S} = \{S_1, \ldots, S_n\}$ is a collection of n subsets, where $S_i \subseteq \{1, \ldots, m\}$, for each $i = 1, \ldots, n$, and a nonnegative integer k. The problem is to decide if there are k subsets $S_{i_1}, \ldots, S_{i_k}$ for which

$$S_{i_1} \cup \cdots \cup S_{i_k} = \{1, \ldots, m\}.$$

a. Verify that $(\mathcal{S}, m, k)$ is a positive instance of `Set Cover`, where $m = 9$, $k = 4$, and

$$\mathcal{S} = \{\{1,3,5\}, \{3,7,9\}, \{2,4,5\}, \{2,6,7\}, \{6,7,9\}, \{2,7,9\}, \{1,3,7\}, \{4,5,8\}\}.$$

Provide a desired solution. (5 pts)

b. Define the size parameters for a problem instance of `Set Cover`. Hint: there are two of them. (5 pts)

c. Provide a polynomial-time nondeterministic program for deciding `Set Cover` in polynomial time. (10 pts)

d. Prove that your algorithm does in fact run in polynomial time and provide its big-O time complexity as a function of the size parameters. (5 pts)