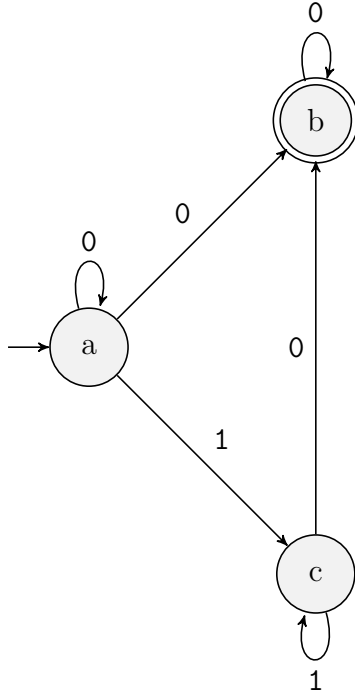


## CECS 329 Midterm, Part 3 of 3, Fall 2021, Dr. Ebert

Please show all work and/or explain. Points will NOT be awarded for answers that have no justification. Please upload a single file to the drop box before the deadline.

- A. Convert the NFA below to a DFA by using the method of subset states. Provide the state diagram for the DFA. (15 points)



- B. Use mathematical induction to prove that, if two bit strings  $x$  and  $y$  have the same length and numerical value, then it must be true that  $x = y$ , meaning that the bit sequence of  $x$  is the same bit sequence as  $y$ . For example, if  $x = 0111$ ,  $x$  has numerical value  $0 + 4 + 2 + 1 = 7$ , and it is the only bit string of length four having this numerical value, meaning that, if  $y$  has numerical value 7, then necessarily  $y = 0111$ . Perform the induction on  $n = |x| = |y| \geq 1$ . Hint: make use of the fact that, for any integer  $k \geq 0$ ,

$$1 + 2 + \cdots + 2^k = 2^{k+1} - 1,$$

and use cases with respect to the most significant bits of  $x$  and  $y$ .

1. Prove the basis step. (5 pts)
  2. State the inductive assumption and what needs to be shown in the inductive step. (10 pts)
  3. Based on your answer to 2, complete the inductive step. (10 pts)
- C. Let  $L_1$  denote all binary words with an even number of zeros, and  $L_2$  denote all binary words that have the subword 111. Provide the state diagram of an NFA that accepts the language  $(L_1 \circ L_2)^* \cup L_2$ . (15 pts)

- D. For the Turing machine  $M$  with state diagram shown below, draw the computation tree  $T(M, 10)$ ; i.e. the computation tree of  $M$  on input 10. Is this an accepting computation? Explain. (20 pts)

