CECS 329 Midterm, Part 1 of 3, Spring 2021, Dr. Ebert

A. Consider the alphabet

$$\Sigma = \left\{ \begin{matrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{matrix} \right\}$$

where each symbol is a triple of bits. Provide the state diagram for a DFA that accepts all words w over Σ for which the top layer of w plus the middle layer of w equals the bottom layer of w, where each layer is viewed as a binary number whose left most bit is the least significant bit, and whose rightmost bit is the most significant bit. For example, the DFA should accept

$$w_1 = \begin{matrix} 010110 \\ 001100 \\ 011001 \end{matrix}$$

since the top layer represents the number 26, the middle layer 12, the bottom layer 38, and $26 + 12 = 38$. However, it should not accept

$$w_2 = \begin{matrix} 11001 \\ 10100 \\ 10011 \end{matrix}$$

since the top layer represents the number 19, the middle layer 5, the bottom layer 25, and $19 + 5 \neq 25$. For each state of your DFA, write a few words that describe the purpose of the state and/or the situation that the state represents. (25 pts)

B. Provide a three-state NFA N that accepts $a^*b^*a^+$. Show the computation of the NFA on input aabba. In other words, show the sequence of subset states based on the input being read. (25 pts)

C. Consider the language

$$L = \{w\#1^k | w \in \{0,1\}^+ \text{ and } k = \text{ value of w as a binary number}\}.$$

For example, $110\#111111 \in L$ since the binary number to the left of # has the value of 6, and there are 6 1's to the right of #. Prove that L is not regular. (25 pts)