CECS 329 Writing Assignment 8

Due December 7th, 2021

Instructions

Submitting your work

Submit a single file with your handwritten solutions to the drop box by Tuesday, December 7th, 8:00 am. Make sure you provide your name and SID in the upper-right corner of your solution. Show all necessary steps in your solutions. Points will be lost otherwise.

Late submissions

Should you submit after the dropbox deadline, solutions received no later than 30 minutes after the deadline will lose 20% of the earned points. Solutions received after 30 minutes but before 60 minutes shall lose lose 50% of the earned points. All other late submissions will not be graded.

Problems

A. An instance of the `Set Splitting` decision problem is an integer $n \geq 1$ and a collection of subsets $C = \{C_1, \ldots, C_m\}$, where each $C_i \subseteq \{1, 2, \ldots, n\}$. The problem is to decide whether or not there exist two sets A and B such that

1. $A \cup B = \{1, 2, \ldots, n\}$
2. $A \cap B = \emptyset$
3. $C_i \cap A \neq \emptyset$, for all $i = 1, 2, \ldots, m$
4. $C_i \cap B \neq \emptyset$, for all $i = 1, 2, \ldots, m$.

1. Show that $n = 10$ and

   $$C = \{\{3, 7, 10\}, \{1, 2, 9\}, \{4, 6, 8\}, \{2, 3, 4\}, \{6, 8, 9\}, \{4, 5, 7\}, \{1, 5, 7\}, \{1, 6, 9\}, \{2, 5, 8\}, \{4, 7, 10\}\}$$

   represents a positive instance of Set Splitting. Do this by providing A and B and verifying that they satisfy all four of the above conditions. (7 pts)

2. Provide a nondeterministic polynomial-time algorithm that decides `Set Splitting`. Give a precise running time for your algorithm and defend your answer. (10 pts)

B. Recall that

$$E_{\text{TM}} = \{\langle M \rangle \mid M \text{ is a Turing machine and } L(M) = \emptyset\}.$$

Show that A_{halt} is functional reducible to E_{TM}. (10 pts)