CECS 474 - Homework 1

Solution

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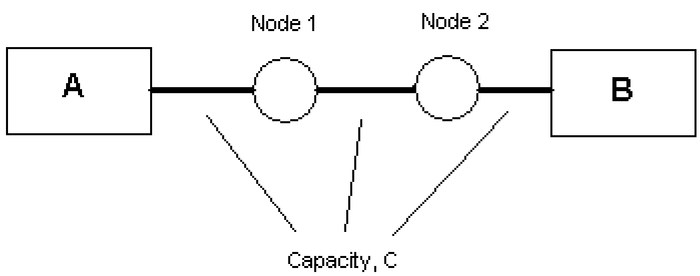
Problem 1:

Assume:

- Propagation delay is τ for each link respectively.
- No errors occur during transmission
- All nodes have the same capacity, C bits/second.
- The size of the file is M (in bits).



a) If message switching, then node 1 will only send the whole file to the next node after it has completely received it. Hence the total time needed to send the file from A to B is:

$$t_{Total} = 3 \times (\frac{M}{C} + \tau) \text{ seconds}$$

If we assume that τ is negligible, then t_{Total} = 30 s.

b) If the message is divided into L=1500 bits per packet, we will have M/L=15*(10^6)/1500 = 10000 packets.

When the first packet leaves A, B needs to wait for $t_{First} = 3 \times (\frac{L}{C} + \tau)$ seconds to get it.

After the first packet was received, B will get one packet every $\frac{L}{C}$ second (parallelism effect) and there are still M/L-1 = 9999 packets to send, therefore the total time taken to receive the whole file completely is:

$$t_{Total} = 3 \times \left(\frac{L}{C} + \tau\right) + \left(\frac{M}{L} - 1\right) \times \frac{L}{C} = \left(\frac{M}{L} + 2\right) \times \frac{L}{C} + 3\tau \text{ seconds}$$

Numerically, if we assume that τ is negligible, then t_{Total} = 10.002 s.

Hence we can conclude that packet switching is more efficient than message switching. Another issue not covered here is that the larger a packet is, the higher the probability it will have error(s). Moreover, it is worth noting that this study does not take into account the overhead issue. In fact, each packet will need some extra overhead. See next question.

Problem 2:

a) Since the propagation delay is zero, the time for a packet to traverse one link is the transmission time L/C. Because a router first receives a packet entirely before sending it back out again, the time for the packet to traverse n links from A to B is simply n times as much:

$$n \cdot L / C.$$

b) Using too many very small packets causes too much data to be sent, because of the per-packet overhead H, leading to a large delay. On the other hand, using only one large packet prevents any parallelism-with multiple packets. Indeed in packet switching, one packet can be using the j^{th} link at the same time the next packet is using the $j-1^{st}$ link, and so on.

Suppose we use k packets. Since the question asks the optimal way to break up the **file** into packets, therefore each of them has size $H+\frac{S}{k}$. After B receives the first packet, the rest will immediately follow. From part (a), the time for the first packet to get from A to B is $n \cdot (H+\frac{S}{k})/C$. The time for the remaining k-1 packets to arrive is simply $(k-1)(H+\frac{S}{k})/C,$ since we neglect the propagation delays, for a total of $(n+k-1)(H+\frac{S}{k})/C$. To find the k that minimizes the delay, we set the derivative with respect to k to zero:

$$\frac{d}{dk}\left[(n+k-1)\frac{(H+\frac{S}{k})}{C}\right]=0$$

$$\frac{d}{dk}\left[(n+k-1)\left(H+\frac{S}{k}\right)\right]=0$$

$$(n+k-1)\left(\frac{-S}{k^2}\right)+\left(H+\frac{S}{k}\right)(1)=0$$

$$(n+k-1)(-S)+(Hk^2+Sk)=0$$

$$Hk^2=(n-1)S$$

$$k=\sqrt{(n-1)\frac{S}{H}}$$

Of course, in general k will not be an integer, so given actual numbers for S, H, and n, we would need to evaluate the total delay for $\lfloor k \rfloor$ packets and $\lceil k \rceil$ packets, and use whichever was better.

Note: It is trivial to show that what we have indeed is a minimum by differentiating the above expression again.

What that means is that there is a trade-off between having very small packets which is good for parallelism but bad for overhead and large packets which is good for minimizing the overheads but bad for parallelism.

Problem 3:

1) b $t_{tansmission}$ = $\frac{30 \cdot 10^6 \, bits}{10 \cdot 10^6 \, bps} = 3 \sec$

2) b $t_{end\text{-}to\text{-}end}$ = $t_{transmission} + \tau$ = $3 + \frac{10000 \cdot 10^3}{2 \cdot 10^8} = 3.05 \sec$

3) d Total Bits Transmitted in 0.05 sec = 10Mbps * 0.05 = 500,000 bits

4) a Time taken from source to the router = $3 + \frac{5000 \cdot 10^3}{2 \cdot 10^8} = 3.025 \sec$

Time taken from router to destination is the same, hence total time =6.05sec

5) b When source starts to send the first packet, the destination needs to wait for: $2 \cdot (\frac{10 \cdot 10^6}{10 \cdot 10^6} + \frac{5000000}{200000000}) = 2 \cdot (1.025) = 2.05 \sec$ to get it, after that it will need to wait additional 2*(1.0)=2 sec to get the remaining 2 packets. The total time is: 2.05+2 = 4.05sec

6) a 10Mbps/10 channels = 1 Mbps/channel.

Problem 4:

a. d_{prop} = $\frac{m}{s} \sec$

b. d_{trans} = $\frac{L}{R} \sec$

c. End-to-end delay = d_{prop} + d_{trans}

d. At t= d_{trans}, the last bit just leaves the source.

e. At t= d_{trans}, if d_{prop} is greater than d_{trans}, the first bit is still on the link.

f. At t= d_{trans}, since d_{prop} is less than d_{trans}, the first bit has already been received by the destination.

g. Given:

L =1000 bits

s = 2.5*10^8 meters

R=284 kbps

m=?

To get d_{trans} = d_{prop}, we set:

$$\frac{L}{R}=\frac{m}{s} \Rightarrow m = s\cdot\frac{L}{R} = 2.5\cdot10^8\cdot(\frac{1000}{284\cdot10^3}) \approx 880.28Km$$

Problem 5:

a. When circuit switching is used, only $\left\lfloor \frac{2Mbps}{300kbps} \right\rfloor = \lfloor 6.67 \rfloor = 6$ users can be supported.

b. Since a user is only active 12% of the time, hence the probability p that a user is transmitting, is 0.12

c. Given that the total number of users is M, assuming that the users are all independent and given p (q=1-p). The probability of having n of them transmitting at the same time is: $C_n^M \cdot p^n \cdot q^{M-n}$

d. To find the probability that 7 or more users are transmitting, first find the probability that 6 or less users are transmitting:

Let: P_A=$P_{(6 \text{ or less users are transmitting})}$ = $\sum_{k=0}^{6} C_k^M p^k q^{M-k}$

$P_{(7 \text{ or more users are transmitting})}$ = 1- P_A

Alternative Solution: $P_{(7 \text{ or more users are transmitting})}$ = $\sum_{k=7}^{M} C_k^M p^k q^{M-k}$

We can see that the probability that 7 or more users are transmitting is very small and hence we can share the link between more than 6 users by using statistical multiplexing without having to worry too much about excessive delays.

Problem 6:

We assume no errors on the links and negligible processing times.

a. If we consider the fluid case, then the delivery of the file is like water going through pipes of different sizes, then clearly the throughput γ in bps is min(R_1,R_2,R_3) = 500 kbps. Is that a good assumption? Let's see.

If we assume message switching (including store and forward at each link), then the file will be completely received at B, T seconds after A started transmitted, with T = (M/R_1 +τ_1) + (M/R_2 +τ_2) + (M/R_3 +τ_3). Then the throughput γ' is M/T bps. Assuming that we can neglect the propagation delays, then the throughput is 1/(1/R_1+1/R_2+1/R_3) = 285 kbps << γ = 500 kbps!

If we assume packet switching such as M is fragmented in K packets of length L (to simplify we assume M=KL and neglect overhead), then we have to be careful about computing the time T' at which the complete file will be received at B since the links are no more identical. Since R_1 is the bottleneck (it would be different if R_2 was the bottleneck), we have T' = $KL/R_1 + L/R_2 + L/R_3 + \tau_1 + \tau_2 + \tau_3$. Assuming that we can neglect the propagation delays, and if K/R_1 is much larger

than $1/R_2+1/R_3$, then the throughput γ'' is M/T' bps = $R_1 = \gamma$! (under very strong assumptions)

b. We have compute γ, γ', γ'' for the three cases, then T, T' and T'' are just M/γ, M/γ' and M/γ'' resp. yielding 65.536s, 113.536s, and the last one will depend on the values of K and L.

Problem 7

a) 2 users can be supported because each user requires half of the link bandwidth.

b) Since each user requires 1Mbps when transmitting, if two or fewer users transmit simultaneously, a maximum of 2Mbps will be required. Since the available bandwidth of the shared link is 2Mbps, there will be no queuing delay before the link. Whereas, if three users transmit simultaneously, the bandwidth required will be 3Mbps which is more than the available bandwidth of the shared link. In this case, there will be queuing delay before the link.

c) Probability that a given user is transmitting = 0.2

d) Probability that all three users are transmitting simultaneously = $\binom{3}{3} p^3 (1-p)^{3-3}$ = $(0.2)^3$ = 0.008. Since the queue grows when all the users are transmitting, the fraction of time during which the queue grows (which is equal to the probability that all three users are transmitting simultaneously) is 0.008.

Problem 8

The queuing delay is 0 for the first transmitted packet, *L/R* for the second transmitted packet, and generally, *(n-1)L/R* for the n^{th} transmitted packet. Thus, the average delay for the *N* packets is

$$(L/R + 2L/R + ... + (N-1)L/R)/N = L/RN(1 + 2 + ... + (N-1)) = LN(N-1)/(2RN)$$

$$= (N-1)L/(2R)$$

Note that here we used the well-known fact that

$$1 + 2 + ... + N = N(N+1)/2$$