EE 381 Homework 4 - Part 1

This homework is due with additional parts in dropbox (beginning of laboratory section) on 4-21-2021.

Poisson

Introduction

Suppose that we want to find the probability distribution of the number of automobile accidents at a particular intersection during a time period of one week. At first glance this random variable (RV), the number of accidents, may not seem even remotely related to a binomial RV, but we will see that an interesting relationship exists.

Think of the time period, one week in this example, as being split up into n subintervals, each of which is so small that at most one accident could occur in it with probability different from zero. Denoting the probability of an accident in any subinterval by p, we have, for all practical purposes,

P(no accidents occur in a subinterval) = $1 - p$

P(one accident occurs in a subinterval) = p

and

P(more than one accident occurs in a subinterval) = 0

Then the total number of accidents in the week is just the total number of subintervals that contain the accident. If the occurrence of accidents can be regarded as independent from interval to interval, the total number of accidents has a binomial distribution.

Although there is no unique way to choose the subintervals, and we therefor know neither n nor p, it seems reasonable that as we divide the week into greater number n of subintervals the probability p of one accident in one of these shorter subintervals will decrease. Letting $\lambda = np$ and taking the limit of the binomial probability ${}_nC_x\ p^x(1-p)^{(n-x)}$ as $n \to \infty$ we have,

$\lim_{n\to\infty} {}_nC_x\ p^x(1-p)^{(n-x)} = e^{-\lambda}\frac{\lambda^x}{x!}$. This derivation was covered in lecture.

A RV possessing this distribution is said to be Poisson RV. Hence, the number of accidents per week should possess the Poisson distribution.

The Poisson distribution often provides a good model for the probability distribution of the number of rare events that occur in space, time, volume or any other dimension, where λ is the average value of the RV. It provides a good model for the probability distribution of the number of automobile accidents, industrial accidents, or any other types of accidents in a given unit of time.

A significant portion of the preceding was adopted from: Mathematical Statistics with Applications; Wackerly, Mendenhall, and Sceafer 5th Ed. 1996 pages 112 & 113

Poisson Random Variable X (Distribution)

X is the number of occurrences of an event over some interval

The occurrences are independent of each other

The occurrences are random and are uniformly distributed over the interval of the event

The time between occurrences is exponentially distributed with the same parameter.

Both the mean and variance are equal to the parameter.

If X is a discrete random variable with pmf $f_X(x)$ and if $g(x)$ is any function of X, then

$$E[g(X)] = \sum_{\text{all } x} g(x) \cdot f_X(x)$$

If X is a continuous random variable with pdf $f_X(x)$ and if $g(x)$ is any function of X, then

$$E[g(X)] = \int g(x) \cdot f_X(x)$$

A food manufacturer uses an extruder (a machine that makes bite-size candies) that yields revenue for the firm at a rate of \$200 per hour when in operation. However, the extruder breaks down an average of two times every day it operates. If Y denotes the number of breakdowns per day, the daily revenue generated by the machine is $R = 1600 - 50Y^2$. Find the expected daily revenue for the extruder. {Hint: The expectation of the square of a RV was discussed when determining the formula for the variance of a binomial RV.} From Mathematical Statistics with Applications; Wackerly, Mendenhall, and Sceafer 5th Ed. 1996 page 118

A company makes reusable produce bags. Their annual profit, Q, in hundreds of thousands of dollars, can be expressed as a function of product demand, x:

$$Q = Q(x) = 2(1 - e^{-2x})$$

Suppose that the demand in thousands for their bags follows an exponential pdf, $f_X(x) = 6e^{-6x}, x > 0$.

Find the company's expected profit. From Marx page 169

Suppose that X is a RV whose pmf is nonzero only for three values -2, 1, and 2. Let $Y = g(X) = X^2$. Determine $E(Y)$. Marx page 167.

x	$f_X(x)$
-2	$^5/_8$
1	$^1/_8$
2	$^2/_8$

Of nine executives in a certain business firm, four are married, three have never married, and two are divorced. Three of the executives are to be selected for promotions. Let Y_1 denote the number of married executives, and Y_2 the number of never married executives among the three selected for promotion. Assuming that the three are randomly selected from the nine available, find the joint pmf of Y_1 and Y_2. (Wackerly, Mendenhall, and Scheaffer; 5th Ed. 1996)

$Cov(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$ and If X and Y are independent, then $Cov(X, Y) = 0$. The converse is false.