EE 381 Homework 2 - Part 2

This homework is due with additional parts in dropbox (beginning of laboratory section) on 2-22-2021.

Material below (more formally stated) was discussed in lecture.

Definition

Let Y denote any random variable (rv). The *cumulative distribution function* (CDF) of Y denoted by $F_Y(y)$, is given by $F_Y(y) = P(\{Y \leq y\})$ for $-\infty < y < \infty$.

Properties of the cumulative distribution function

If $F_Y(y)$ is a cumulative distribution function, then

1.) $\lim_{y \to -\infty} F_Y(y) = F_Y(-\infty) = 0.$
2.) $\lim_{y \to \infty} F_Y(y) = F_Y(\infty) = 1.$
3.) $F_Y(y)$ is a nondecreasing function of y. If y_1 and y_2 are any values such that $y_1 < y_2$, then $F_Y(y_1) \leq F_Y(y_2)$.

Definition

Let $F_Y(y)$ be the cumulative distribution function for a <u>continuous</u> rv Y. Then $f_Y(y)$, is given by

$$f_Y(y) = \frac{dF_Y(y)}{dy} = F_Y'(y)$$

wherever the derivative exists, is called the *probability density function* (pdf) for the rv Y.

Properties of the probability density function

If $f_y(y)$ is a probability density function, then

1.) $f_Y(y) \geq 0$ for any value of y.
2.) $\int_{-\infty}^{\infty} f_Y(y)dy = 1.$

Properties of the probability mass function

If $f_y(y)$ is a probability mass function, then

1.) $f_Y(y) \geq 0$ for any value of y.
2.) $\sum_{-\infty}^{\infty} f_Y(y)dy = 1.$

If the rv Y has the probability density function $f_Y(y)$ and $a \leq b$, then the probability that Y falls in the interval (a, b) is

$$P(a < Y < b) = \int_a^b f_Y(y)dy$$

The discrete analog of the pdf is the probability mass function (pmf). For the discrete rv Y with CDF $F_Y(y)$ the pmf $f_Y(y) = P(\{Y = y\})$. We have using the CDF

$$f_Y(x) = \begin{cases} F_Y(1), & y = 1 \\ F_Y(y) - F_Y(y-1), & y = 2, 3, 4 \dots \end{cases}$$

Problems

Suppose that X possesses the pdf

$$f_X(x) = \begin{cases} cy, & 0 < x < 2 \\ 0, & \text{o.w.} \end{cases}$$

a.) Find the value of c that makes $f_X(x)$ a pdf.

b.) Find $F_X(x)$.

c.) Graph $f_X(x)$ and $F_X(x)$.

d.) Use the CDF to find $P(1 < X < 2)$.

e.) Use the pdf and plane geometry to find $P(1 < X < 2)$.

Urn I contains 3 red chips and 4 white chips; urn II has 6 red and 4 white. One chip is drawn from urn I, two are drawn from urn II. Let a denote the number of red chips in the sample coming from urn I; let b denote the number of red chips in the sample coming from urn II. (From Larsen & Marx page 111)

a.) List all possible outcomes (a, b) and compute the probability associated with each.

b.) Define the rv $Y(\{(a, b)\}) = ab$. Find $f_Y(y)$ for all y.