EE 381 Homework 3 - Part 1

This homework is due with additional parts in dropbox (beginning of laboratory section) on 3-8-2021.

Normal Distribution

The normal random variable (rv) X has the probability density function (pdf)

$$f_X(x) = \frac{e^{-\frac{(x-\mu)^2}{(2\sigma^2)}}}{\sigma\sqrt{2\pi}}$$

Where μ is the mean and σ is the standard deviation. The pdf is bell shaped. The mean, median, and mode are equal. The pdf is symmetric about the mean. The area under the part of the pdf within one standard deviation of the mean is approximately 68%, within two standard deviations about 95%, and within three standard deviations about 99.7%. `The pdf is integrated numerically. Because of this and other reasons the standard normal random variable, Z, with mean zero, $\mu = 0$, and standard deviation one, $\sigma = 1$, is used to evaluate problems involving normal random variables. These are reasons for there being a standard normal table.

$$f_Z(z) = \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}$$

The reader may recall the past discussion of a function of a random variable. The formula used was

$$f_U(u) = f_Y(y)\left|\frac{dy}{du}\right|$$

for the function $U = h(Y)$. We can apply this here to justify the z-score, $z = \frac{X-\mu}{\sigma}$.

Exercise.

Show the pdf for the rv X is as above by using the linear function $X = Z\sigma + \mu$ and the pdf for Z given above. The rv X is a function of Z and by doing the transformation you get the pdf of X.

Central Limit Theorem, C.L.T.

When the sampling from a population that is not normally distributed if the conditions are suitable the distribution of the sample means, $\bar{x}$, will approach a normal distribution. The usual requirement is that the sample size, n, be thirty or greater. If the population distribution is already normally distributed then the sample size, n, can be less than 30.

The z-score used for the C.L.T. is a generalization of the one familiar in applications of the normal distribution.

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$z = \frac{S_n - n\mu}{\sqrt{n}\sigma}$$

where $S_n = \sum_i X_i$ is the sum of n independently and identically distributed random variable.

The C.L.T. states that

$\mu_{\bar{x}} = \mu$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

The distribution of the sample means, $\bar{x}$, approaches a normal distribution as $n \to \infty$.

Exercise.

Suppose that 100 dice are tossed. Let Y_i be the number showing on the i^{th} die. Estimate the probability that the sum of the Y_i's exceeds 370.

Exercise.

The average cholesterol content of a certain brand of eggs is 215 milligrams, and the standard deviation is 15 milligrams. The variable is normally distributed.

If a single egg is selected, find the probability that the cholesterol content will be greater than 220 milligrams.

If a sample of 25 eggs is selected, find the probability that the mean of the cholesterol content will be greater than 220 milligrams.