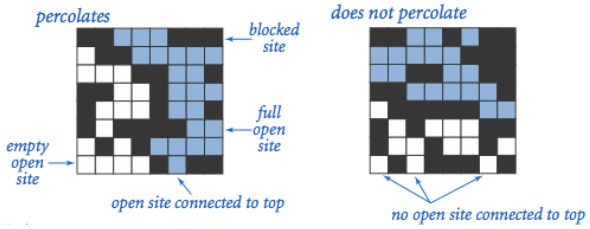
Programming Assignment 1: Percolation

Write a program to estimate the value of the *percolation threshold* via Monte Carlo simulation.

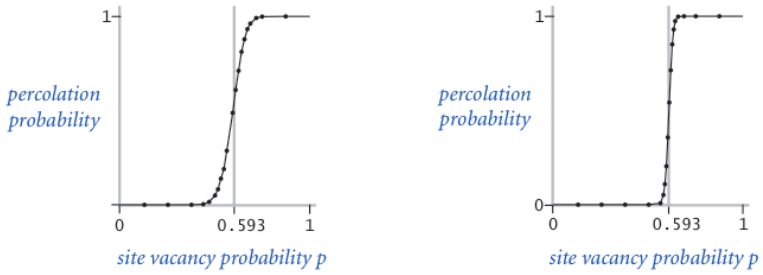
Install a Java programming environment. Install a Java programming environment on your computer by following these step-by-step instructions for your operating system [Mac OS X · Windows · Linux]. After following these instructions, the commands `javac-algs4` and `java-algs4` will classpath in both stdlib.jar and algs4.jar: the former contains libraries for reading data from *standard input*, writing data to *standard output*, drawing results to *standard draw*, generating random numbers, computing statistics, and timing programs; the latter contains all of the algorithms in the textbook.

Percolation. Given a composite systems comprised of randomly distributed insulating and metallic materials: what fraction of the materials need to be metallic so that the composite system is an electrical conductor? Given a porous landscape with water on the surface (or oil below), under what conditions will the water be able to drain through to the bottom (or the oil to gush through to the surface)? Scientists have defined an abstract process known as *percolation* to model such situations.

The model. We model a percolation system using an *N*-by-*N* grid of *sites*. Each site is either *open* or *blocked*. A *full* site is an open site that can be connected to an open site in the top row via a chain of neighboring (left, right, up, down) open sites. We say the system *percolates* if there is a full site in the bottom row. In other words, a system percolates if we fill all open sites connected to the top row and that process fills some open site on the bottom row. (For the insulating/metallic materials example, the open sites correspond to metallic materials, so that a system that percolates has a metallic path from top to bottom, with full sites conducting. For the porous substance example, the open sites correspond to empty space through which water might flow, so that a system that percolates lets water fill open sites, flowing from top to bottom.)



The problem. In a famous scientific problem, researchers are interested in the following question: if sites are independently set to be open with probability *p* (and therefore blocked with probability 1 − *p*), what is the probability that the system percolates? When *p* equals 0, the system does not percolate; when *p* equals 1, the system percolates. The plots below show the site vacancy probability *p* versus the percolation probability for 20-by-20 random grid (left) and 100-by-100 random grid (right).



When *N* is sufficiently large, there is a *threshold* value *p** such that when *p* < *p** a random *N*-by-*N* grid almost never percolates, and when *p* > *p**, a random *N*-by-*N* grid almost always percolates. No mathematical solution for determining the percolation threshold *p** has yet been derived. Your task is to write a computer program to estimate *p**.

Percolation data type. To model a percolation system, create a data type `Percolation` with the following API:

```
public class Percolation {
   public Percolation(int N)              // create N-by-N grid, with all sites blocked
   public void open(int i, int j)         // open site (row i, column j) if it is not already
   public boolean isOpen(int i, int j)    // is site (row i, column j) open?
   public boolean isFull(int i, int j)    // is site (row i, column j) full?
   public boolean percolates()            // does the system percolate?
}
```

By convention, the indices *i* and *j* are integers between 1 and *N*, where (1, 1) is the upper-left site: Throw a `java.lang.IndexOutOfBoundsException` if either *i* or *j* is outside this range. The constructor should take time proportional to N^2; all methods should take constant time plus a constant number of calls to the union-find methods `union()`, `find()`, `connected()`, and `count()`.

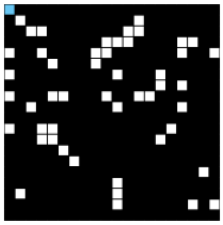
Monte Carlo simulation. To estimate the percolation threshold, consider the following computational experiment:

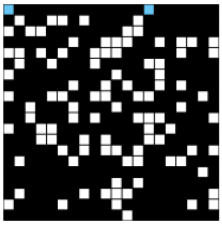
- Initialize all sites to be blocked.
- Repeat the following until the system percolates:

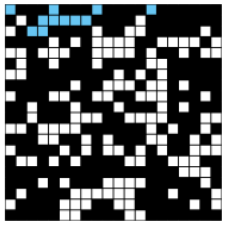
- Choose a site (row *i*, column *j*) uniformly at random among all blocked sites.
- Open the site (row *i*, column *j*).

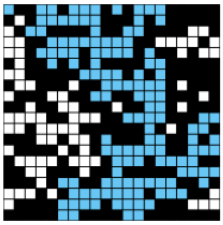
- The fraction of sites that are opened when the system percolates provides an estimate of the percolation threshold.

For example, if sites are opened in a 20-by-20 lattice according to the snapshots below, then our estimate of the percolation threshold is 204/400 = 0.51 because the system percolates when the 204th site is opened.


50 open sites


100 open sites


150 open sites


204 open sites

By repeating this computation experiment *T* times and averaging the results, we obtain a more accurate estimate of the percolation threshold. Let x_t be the fraction of open sites in computational experiment *t*. The sample mean μ provides an estimate of the percolation threshold; the sample standard deviation σ measures the sharpness of the threshold.

$$\mu = \frac{x_1 + x_2 + \cdots + x_T}{T}, \quad \sigma^2 = \frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \cdots + (x_T - \mu)^2}{T - 1}$$

Assuming *T* is sufficiently large (say, at least 30), the following provides a 95% confidence interval for the percolation threshold:

$$\left[\mu - \frac{1.96\sigma}{\sqrt{T}},\ \mu + \frac{1.96\sigma}{\sqrt{T}}\right]$$

To perform a series of computational experiments, create a data type `PercolationStats` with the following API.

```
public class PercolationStats {
   public PercolationStats(int N, int T)    // perform T independent computational experiments on an N-by-N grid
   public double mean()                     // sample mean of percolation threshold
   public double stddev()                   // sample standard deviation of percolation threshold
   public double confidenceLo()             // returns lower bound of the 95% confidence interval
   public double confidenceHi()             // returns upper bound of the 95% confidence interval
   public static void main(String[] args)   // test client, described below
}
```

The constructor should throw a `java.lang.IllegalArgumentException` if either $N \le 0$ or $T \le 0$.

Also, include a `main()` method that takes two *command-line arguments* *N* and *T*, performs *T* independent computational experiments (discussed above) on an *N*-by-*N* grid, and prints out the mean, standard deviation, and the *95% confidence interval* for the percolation threshold. Use *standard random* from our standard libraries to generate random numbers; use *standard statistics* to compute the sample mean and standard deviation.

```
% java PercolationStats 200 100
mean                    = 0.5929934999999997
stddev                  = 0.00876990421552567
95% confidence interval = 0.5912745987737567, 0.5947124012262428

% java PercolationStats 200 100
mean                    = 0.592877
stddev                  = 0.009990523717073799
95% confidence interval = 0.5909188573514536, 0.5948351426485464


% java PercolationStats 2 10000
mean                    = 0.666925
stddev                  = 0.11776536521033558
95% confidence interval = 0.6646167988418774, 0.6692332011581226

% java PercolationStats 2 100000
mean                    = 0.6669475
stddev                  = 0.11775205263262094
95% confidence interval = 0.666217665216461, 0.6676773347835391
```

Analysis of running time and memory usage (optional and not graded). Implement the `Percolation` data type using the quick-find algorithm QuickFindUF.java from `algs4.jar`.

- Use the *stopwatch data type* from our standard library to measure the total running time of `PercolationStats`. How does doubling *N* affect the total running time? How does doubling *T* affect the total running time? Give a formula (using tilde notation) of the total

running time on your computer (in seconds) as a single function of both *N* and *T*.

- Using the 64-bit memory-cost model from lecture, give the total memory usage in bytes (using tilde notation) that an *N*-by-*N* percolation system uses. Count all memory that is used, including memory for the union-find data structure.

Now, implement the `Percolation` data type using the weighted quick-union algorithm WeightedQuickUnionUF.java from `algs4.jar`. Answer the questions in the previous paragraph.

Deliverables. Submit only `Percolation.java` (using the weighted quick-union algorithm as implemented in the `WeightedQuickUnionUF` class) and `PercolationStats.java`. We will supply `stdlib.jar` and `WeightedQuickUnionUF`. Your submission may not call any library functions other than those in `java.lang`, `stdlib.jar`, and `WeightedQuickUnionUF`.