***Abstract*—In this paper, we propose a high-speed greedy sequential algorithm for the vertex coloring problem (VCP), based on the Wave Function Collapse algorithm, called Wave Function Collapse Coloring (WFC-C). An iteration of this algorithm goes through three distinct phases: vertex selection, color restriction through wave function collapsing, and domain propagation. In effect, WFC-C propagates color choices or "domain" restrictions beyond immediate neighbourhoods. This heuristic, combined with a series of other greedy optimizations, allows for a fast algorithm that prevents certain color conflicts. Through extensive experiments, we show that WFC-C remains competitive (and occasionally better) in terms of optimal coloring, and dramatically outperforms existing high-speed VCP, with on average speed differences ranging from 2000 times to 16000 times, on the most difficult instances of the DIMACS benchmark.**

1. Introduction

The Vertex Coloring Problem (VCP), a sub-problem of the Graph Coloring Problem, is an NP-hard combinatorics optimization problem with a wide range of applications, studied extensively in literature. VCP asks, in essence, to assign a color to every vertex such that no adjacent vertex shares a color. A common extension of the VCP is to find the minimum number of colors to create a valid coloring, called the chromatic number $\chi(G)$. Examples of this problem's applications include frequency assignment in networks [1], [2]; timetabling [3], [4]; register allocation in compilers [5], [6]. See [7] or [8] for a survey on the applications of VCP. While exact approaches to solving the VCP exist [9], [10], [11], [12], they are impractical for real-life applications as exact algorithms are unable to solve large graphs due to the amount of time required. Thus, researchers tend to concentrate on heuristic solutions. Traditionally, heuristic and metaheuristic algorithms for VCP can be split into three distinct categories: constructive approaches [13], [14], [15]; local searching (including simulated annealing [16], quantum annealing [17], tabu search [18], [19], [20], variable neighborhood searching [21]); and population-based approaches [22], [23], [24], [25], [26]. More recently, modern approaches have incorporated machine and statistical learning techniques. For example, [27] introduces a population-based approach with gradient descent optimization, and [28] uses probability learning on a local search algorithm to produce more optimal colorings. An exhaustive study of popular heuristic methods can be found in [29].

However, with the exception of sequential construction algorithms, modern literature places an emphasis on optimal coloring as opposed to time efficiency. Despite this focus on optimal coloring, fast graph coloring is essential in a large number of applications, such as computing the upper bounds in branch-and-bound algorithms for the maximum cliche problem [30], [31], or to use graph coloring-based compression techniques to speed up automatic differentiation [32], [33]. Many hybrid VCP algorithms use fast but inaccurate vertex coloring algorithms to generate a high estimate of the chromatic number and repeatedly lower this until a legal k-coloring cannot be reached while other algorithms optimize the initial, inaccurate coloring directly [17], [18], [19], [21], [22], [28]. In such applications, speed is more important than achieving an optimal coloring. Despite modern literature's focus on optimal VCP algorithms, high-speed vertex coloring is still vital to many crucial applications.

Approaches to high-speed VCP solutions generally consist of greedy and constructive algorithms. These algorithms iterate through a set of all vertices, assigning a color following some rules until a valid coloring is reached. Once a coloring is assigned, it is not reconsidered. Most effective high-speed VCP algorithms employ a dynamic ordering of vertices to produce more optimal coloring. Famous examples of these high-speed VCP algorithms are *maximum saturation degree* (DSatur) [13], *recursive largest first* (RLF) [14], and the iterated greedy algorithm (IG) [15]. These are the algorithms we compare our novel algorithm to. More recently, an algorithm proposed by [34] implements a greedy-style algorithm using bit-wise operations to increase time efficiency. However, the majority of these fast VCP solutions do not restrict colors of vertices beyond the immediate neighborhood, nor is there any metaheuristic processing to optimize coloring.

In this paper, we present a fast heuristic vertex coloring algorithm, hereafter called Wave Function Collapse Coloring (WFC-C). The key contribution provided in WFC-C is the propagation of color restrictions beyond the immediate neighborhood. Computational results show that WFC-C dramatically outperforms existing fast VCP algorithms in