Q) 


→ Entangled states : 4 are there

$\frac{1}{\sqrt{2}}[|00\rangle + |11\rangle]$ , $\frac{1}{\sqrt{2}}[|00\rangle - |11\rangle]$

$\frac{1}{\sqrt{2}}[|01\rangle + |10\rangle]$ , $\frac{1}{\sqrt{2}}[|01\rangle - |10\rangle]$

These are called bell states.

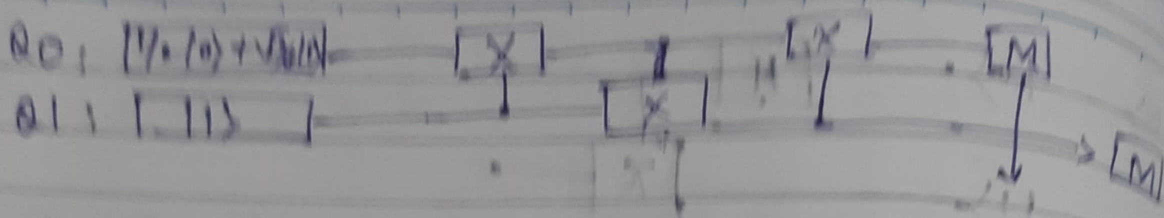
swap gate

Q) $\begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} \times \begin{bmatrix} I & 0 \\ 0 & X \end{bmatrix} \times \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix}$

$\begin{bmatrix} XI & 0 \\ 0 & XI \end{bmatrix} \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} = \begin{bmatrix} X^2 & 0 \\ 0 & X \end{bmatrix}$

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\quad X^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\begin{bmatrix} X^2 & 0 \\ 0 & X \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ = CNOT gate




NOTE: H, X, Phase, CNOT, Y, Z & swap gates to be remembered.

Q) Are $|00\rangle$, $|10\rangle$, $|01\rangle$, $|11\rangle$ normalised?

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$|01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\langle 00|01\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = 0$$

$$\langle 00|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = 0$$

$$\langle 00|11\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

They are normalised.

Q) $|\alpha\rangle = \frac{\sqrt{3}}{2}|00\rangle + \frac{1}{2}|01\rangle$

$P(|\alpha\rangle = |10\rangle) = \frac{\sqrt{3}}{2}\langle 10|00\rangle + \frac{1}{2}\langle 00|01\rangle$

$= 0$

$P(|\alpha\rangle = |00\rangle) = \frac{3}{4}$ $\quad P(|\alpha\rangle = |11\rangle) = 0.$

$P(|\alpha\rangle = |01\rangle) = \frac{1}{4}.$

→ * CNOT $= \begin{bmatrix} I & 0 \\ 0 & X \end{bmatrix}$, 4x4

→ SWAP $= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$|\psi_1\rangle = a_1|0\rangle + b_1|1\rangle$

$|\psi_2\rangle = a_2|0\rangle + b_2|1\rangle$

$SWAP\,|\psi_1\psi_2\rangle = |\psi_2\psi_1\rangle$

→ Toffoli gate

$\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & X \end{bmatrix}$ 8 X 8

## ★ UNIVERSAL QUBIT GATE

→ Not possible for 1 gate to behave like every other gate becos there is infinite no of g matrices.

→ In quantum computing, 1 gate can approximate other gates.

→ Rotation gates : $R_z(\theta)$, $R_y(\theta)$ & $R_x(\theta)$

where $\theta \neq \pi$

$$R_z(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$

$$\theta = \frac{\pi}{4}, \quad R_z(\theta) = T = \begin{bmatrix} 1 & 0 \\ 0 & 1/\sqrt{2} + \frac{i}{\sqrt{2}} \end{bmatrix}$$

$$T^2 = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = R_z(\pi/2) = S \text{ gate}$$

$$T^4 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = R_z(\pi) = Z \text{ gate}$$

$$T^{7} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1/\sqrt{2} + \frac{i}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & +1/\sqrt{2} - \frac{i}{\sqrt{2}} \end{bmatrix} T = \begin{bmatrix} 1 & 0 \\ 0 & 1/\sqrt{2}(1-i) \end{bmatrix} = T^{\dagger}$$

$$H\,T\,H = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1/\sqrt{2}(1+i) \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1/\sqrt{2}(1+i) & -1/\sqrt{2}(1+i) \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1+\frac{1}{\sqrt{2}}(1+i) & 1-1/\sqrt{2}(1+i) \\ 1-1/\sqrt{2}(1+i) & 1+1/\sqrt{2}(1+i) \end{bmatrix}$$

$$= H R_z(\pi/4) H = R_x(\pi/4)$$

$$H\,S\,H = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$$

$$= H R_z(\pi/2) H = R_x(\pi/2)$$

$$H\,Z\,H = H R_z(\pi) H$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & +1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = X$$

$$= R_x(\pi)$$

$$HR_z(\theta)H = R_x(\theta)$$

The set {H, T} will be our universal state for single qubit

→ Property of multi-qubits which does seen in single — — Entanglement

→ The set {H, T, CNOT} will be universal gate for multi-qubit system

→ Qubits can only be on the bloch sphere & not inside it becos it is always normalised to unit length.

→ $\frac{\delta}{\pi}$ is irrational & small

If I keep adding $P(\frac{\delta}{\pi})$ to $|\psi\rangle$ then, it will never reach a the starting point but it reaches every other point possible due to which universal gates are possible but with a small approxiation error which is neglected!

→ If a state vector can be represented on the block sphere by normalizing it, then they are in pure states.

→ If they are inside a block sphere then they are called mixed states.

→ $|0\rangle\langle 0| = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$|1\rangle\langle 1| = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$|0\rangle\langle 1| = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$|1\rangle\langle 0| = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

For pure states, trace of outer product is equal to 1 & can be represented as a linear combination. Outer products are called density matrix represented by $\rho$.

For mixed states, the trace will be less than 1, & can't be determining probability.