Neural Networks for Exotic Option Pricing

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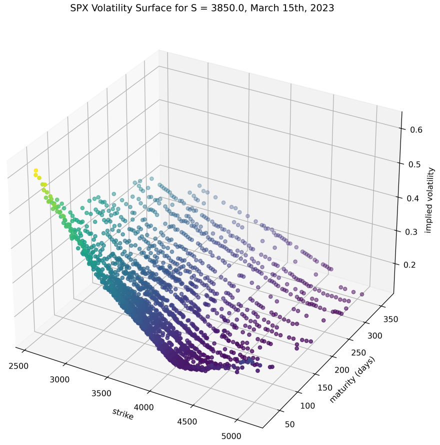
Overview

In this session we explore a proposed method for estimating nonlinear stochastic functions in the context of path dependent financial derivatives

1. Introduction
2. Pricing Model
3. Exotic Payoff Specifications
4. Implementation
5. Challenges
6. Appendix

What are financial derivatives?



Financial Derivatives

We can begin thinking about the problem by advancing on the illustrious concept of the Myron Black and Fischer Scholes pricing model for European call options.

$$C_t = S_t N(d_1) - K e^{-r(T-t)} N(d_2) \tag{1}$$

where

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \quad \text{and} \tag{2}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{3}$$

Financial Derivatives

Problems with the classical Black-Scholes model (as presented):

1. Constant volatility
2. Arbitrage free
3. Instantaneous returns
4. No dividends
5. Cannot price complex financial derivatives

The Heston (1993) Model

1. Geometric Brownian Motion
2. Arbitrary correlation between asset volatility and return

$$dS_t = \left(r - \frac{v_t}{2}\right) dt + \sqrt{v_t} \left(\rho dW_t + \sqrt{1-\rho^2} dB_t\right) \tag{4}$$

$$dv_t = \kappa(\theta - v_t)dt + \eta\sqrt{v_t}dW_t \tag{5}$$

1. v_0 represents the initial variance,
2. θ is the long-run variance,
3. ρ is the correlation between the log-price process and its volatility,
4. κ is the mean reversion of the variance to θ,
5. η is the volatility of the variance process, and
6. B_t, W_t are continuous random walks.

Applications

Calibration of the Heston (1993) model permits usage of more sophisticated pricing algorithms in determining a correct asset price. We are able to leverage path-dependent dynamics of the underlying price to create more sophisticated financial derivative products. One such example is the *Asian* option:

$$C_t^{\text{Asian}} = e^{-r(T-t)} \times \frac{1}{m} \sum_{i=1}^{m} (S_T^{\text{avg}} - K)^+ \tag{6}$$

where S_T^{avg} is the average price of the underlying spot price

Applications

$$C^{\text{Asian}} = F_t(S_0, \kappa, \theta, \rho, \eta, v_0, r, g, K, n, P, D^{\text{call/put}}, D^{\text{Arithmetic/Geometric}}) \tag{7}$$

where the price of an Asian option is a function of:

1. the underlying security price S (4),
2. the mean reversion speed κ of the variance process to θ (5),
3. the long-run mean of the variance process θ (5),
4. the correlation ρ between the variance process v_t and the underlying price process X_t (4),
5. the volatility of the variance process η (5),
6. the initial volatility v_0 (5),
7. the risk-free rate r,
8. the dividend rate g,
9. the strike price K,
10. the number of fixing dates n,
11. the number of past fixings$P$ which for our current purposes is always equal to zero,
12. a logical operator $D^{\text{call/put}}$ to denote the underlying European option payoff function, and
13. a logical operator $D^{\text{Arithmetic/Geometric}}$ denoting the averaging type (6),

Applications

Barrier Options

An even less trivial application could be pricing of barrier options.

$$\begin{aligned} C^{\text{payoff}}_{\text{UpIn}} &= \mathbb{1}_{\{S_t > B \, \forall \, t\}} \times (S_T - K)^+, \\ C^{\text{payoff}}_{\text{UpOut}} &= \mathbb{1}_{\{S_t < B \, \forall \, t\}} \times (S_T - K)^+, \\ C^{\text{payoff}}_{\text{DownIn}} &= \mathbb{1}_{\{S_t < B \, \forall \, t\}} \times (S_T - K)^+, \text{ and} \\ C^{\text{payoff}}_{\text{DownOut}} &= \mathbb{1}_{\{S_t > B \, \forall \, t\}} \times (S_T - K)^+ \end{aligned} \tag{8}$$

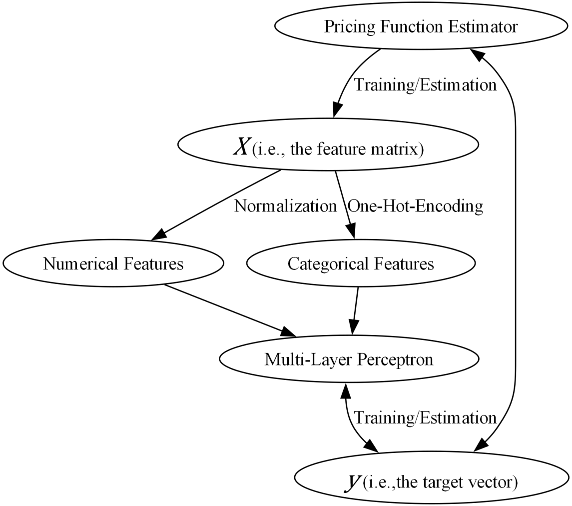
Applications

$$C^{\text{Barrier}} = F_t(S_0, \kappa, \theta, \rho, \eta, v_0, r, g, K, B, R, D^{\text{call/put}}, D^{\text{DownIn/DownOut/UpIn/UpOut}}) \tag{9}$$

Where the price of one Barrier option contract C^{Barrier} is a function of

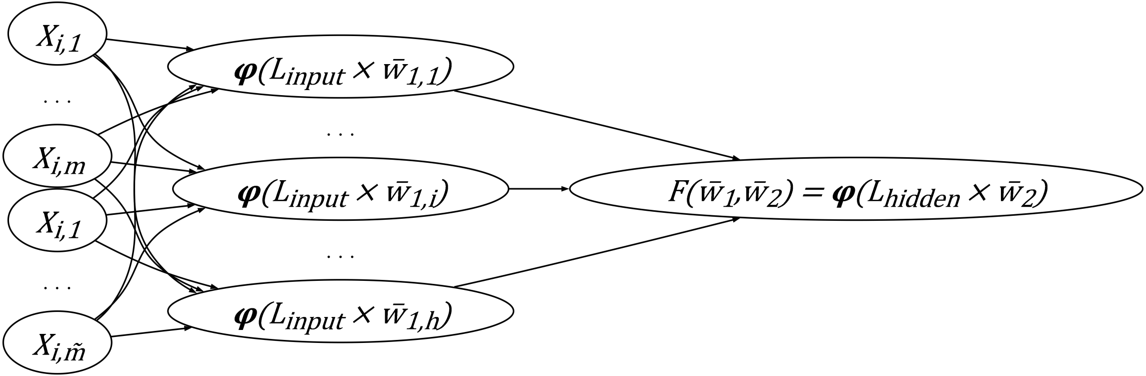
1. the underlying security price S (4),
2. the mean reversion speed κ of the variance process to θ (5),
3. the long-run mean of the variance process θ (5),
4. the correlation ρ between the variance process v_t and the underlying price process X_t (4),
5. the volatility of the variance process η (5),
6. the initial volatility v_0 (5),
7. the risk-free rate r,
8. the dividend rate g,
9. the strike price K,
10. the barrier level B,
11. the rebate R,
12. a logical operator $D^{\text{call/put}}$ to denote the underlying European option payoff function, and
13. a logical operator $D^{\text{DownIn/DownOut/UpIn/UpOut}}$ denoting the barrier contract type (8),

Model Specification



Model Specification

Multilayer Perceptron Specification



Implementation Hurdles

1. High initialisation costs
2. Continuous maintenance
3. Abnormal distributions
4. Idiosyncratic risk

References

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The End

