COMP0124 Multi-Agent Artificial Intelligence Group Project

Real-Time Bidding Strategy - Team 10

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ABSTRACT

Real-time bidding (RTB) is the auctioning of online advertising impressions that occurs whenever a user loads a webpage containing advertising slots serviced by ad-exchanges. For every auction, advertisers will be provided a myriad of information such as page information and user details, this will then be used by the advertisers to determine how much they are willing to bid for the impression.

In this project, we explore different bidding strategies in both single and multi-agent environments, with the objective of maximising clicks from a set of advertising impressions. We explored bidding strategies in a single agent environment, evaluating basic strategies such as constant and random bidding, before implementing more efficient strategies that use click-through-rate classifiers such as linear and non-linear bidding. Finally, we put everything together and implement a bidding strategy that will be useful in a multi-agent environment.

The source code is hosted here: boonwj/real-time-bidding

1. INTRODUCTION

Real-Time Bidding: The advancement of the internet has enabled marketing to be targeted and specifically driven to consumers via online advertising [2, 4]. Advertisers use Real-Time Bidding (RTB) [12] to bid for impression in real-time as they are being generated. The second price sealed-bid auction is used in RTB by ad exchanges [15] as the auction encourages all users to bid their true value.

The RTB process unfolds in three main steps [16], first, the publisher provides its inventory to an ad exchange; who is responsible for holding an auction. During which the DSPs, on behalf of the advertisers will place a bid on the impressions. This bid value is usually dependant on how much advertisers value the impression. After the bidding is completed, the winner is chosen and the winner's ad is served on the publisher's website.

Even though a second priced auction encourages users to bid their true value, in the real-world there are other factors that prevent one from doing so, such as the (ir)rationality of rival bidders and an advertiser's budget constraints. Therefore, there have been multiple research into the best ways of optimising bidding in RTB [15, 19].

Objective: The aim of this paper is to develop bidding strategies to maximise the number of clicked impressions within a dataset of impressions. We implement and evaluate different strategies in this paper, from basic strategies such as constant and random bidding to more efficient strategies that uses click-through-rate classifiers such as linear and non-linear bidding.

Single-Agent Environment: Evaluating both basic and advanced strategies in a single-agent environment, we found that more advanced strategies using Click-Through-Rate (CTR) estimators consistently outperforms basic strategies. With a budget of 6,250 fen, our best basic strategy achieved around 68 clicks in 146,864 impression whereas our best advanced strategies achieved 160 clicks in 141,861 impressions.

Multi-Agent Environment: In a multi-agent environment with homogeneous agents, it was found that as the number of agents increase, each agent's average impressions and clicks drops. In our tests with 50 to 100 agents, using our single-agent optimal random bidding range, none of the agents were able to finish their budget. Hence, in a homogeneous multi-agent environment with limited impressions to cover, it is mutually beneficial to bid in a such that more impressions could be shared among all agents.

Finally when we compared all the different bidding strategies together, we combined a non-linear and linear single-agent strategy such that it outperforms all our existing strategies in a multi-agent environment. Achieving 86 clicks in 6,148 impressions among 30 agents.

2. RELATED WORK

Optimal Real-time Bidding Strategies Many studies on optimal bidding strategies focus on big picture analytical solutions that presumes market environments to be static and does not change with the actions of any current bidders. [1, 10, 13, 16, 18, 20].

However, there have been other novel studies that expanded upon the domain, for example Zhang et al. [18] presented that it is possible to realise budget-constrained optimal bidding in a static environment with other ad bidders and Zhu et al., [20] proposed a two-stage bandit modelling approach where each bidding decision becomes autonomous with time. There was also a reinforcement learning approach to model optimal bidding that was presented by Wang et al., [13] and Cai et al., [1].

Furthermore, the focus of existing computation bidding techniques [10, 18] is on the benefit of a particular merchant or advertiser's micro-level optimisation. Notwithstanding, it is possible for the benefit of a party to disregard and harm the benefit of other parties when they are optimised; this is as a result of competition in the RTB auction [8]. The perception of the ad system is that it is possible for the micro-level optimisation not to use the full dynamics of the ad environment so that social optimality can be realised better [13].

Multi-Agent Strategy Tan [11] provides a complete discussion of Q learning to implement the multi-agent bidding strategy, where when a bidder changes strategies, it affects other bidders such that they change their strategies too. There have also been other studies that discussed how to coordinate agents using shared parameters [6] and/or effective learning protocol communication [3]. Lowe et al. [9] introduce the Multi-agent Deep deterministic policy gradient (DDPG) that provides a centralised agent augmented with the policies defined for each agent.

3. DATASET

3.1 Data Format

In this study, we were provided with historical ad impressions data from the iPinYou dataset. In total we are provided with 3 sets of data (training, validation and testing). The training and validation data set is meant for our analysis and training, whereas the testing is where we will be applying our bidding strategies on.

Table 1 details the key fields within them. The data contains many features to describe every impression, ranging from users (e.g. usertag), advertisers (e.g. advertiser Id), publisher (e.g. domain, url, urlid, slotid, slotwidth, slotheigh, slotvisibility, slotformat) and context (e.g. weekday, hour, city, region, browser and OS). All these information are provided to advertisers so they can determine how valuable an impression is to them before they bid.

For a more detailed description of some columns can be found in the iPinYou benchmarking paper [19].

3.2 Basic Statistics

A brief analysis of the data is first done on our dataset to get basic information as shown in table 2. For the exploration of the data, we turn to the training dataset which contains the most data with 2,430,981 impressions. Within it, all money related fields such as bid price, paying price and floor price are in the currency CNY fen which follows the cost-per-mille (CPM) pricing model.

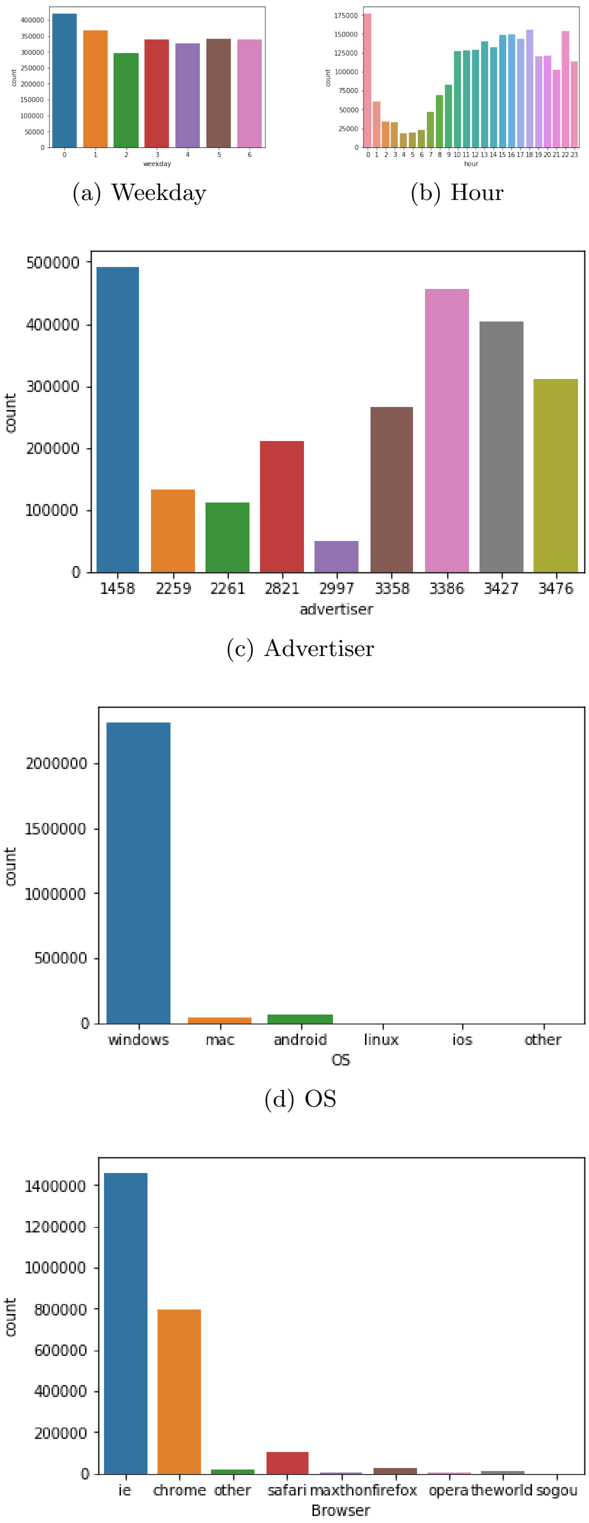
We also looked into user feedback information as seen in Figure 1 and in Figure 2 we see how much are click impressions generally cost. We have also identified the following key observations from the data.

- Number of clicks and therefore CTR is extremely low (1,793 clicks, 0.074% CTR)
- Payprices for all impressions range between 0 to 300
- Most of the clicked impressions are generally below 100
- Certain fields are actually categorical but requires post processing (e.g. usertags)

4. APPROACH AND RESULTS

In this section, we discuss the approaches taken to implement different bidding strategies. For all the approaches, we work with a budget of 6,250 CNY fen to maximise the number of click impressions from the validation dataset, containing 303,375 impressions.

In addition, depending on the scenario, the winning criterion used will differ. This is shown in Table 3.



(a) Weekday

(b) Hour

(c) Advertiser

(d) OS

(e) Browser

Figure 1: User Feedback

Table 1: Key columns in dataset

Col	Field	Description	Example
		Key columns not used in training	
1	click	Show if it is a clicked impression	1
4	bidid	Identifier for each impression up for bid	b7bea80521fdecd95d2d761a38c91c3f09618066
22	payprice	Pay price of the impression	100
		Columns used in training	
2	weekday	Day of impression	3
3	hour	Hour of impression	20
6	useragent	OS and browser information	windows_ie
8	region	Origin of user	79
9	city	City of user	135
10	adexchange	Adexchange hosting the bid	2.0
15	slotwidth	Width of impression	160
16	slotheight	Height of impression	90
17	slotvisibility	If the ad slot is above the fold or not	2
18	slotformat	Type of advertising slot	0
19	slotprice	Floor price of impression	5
23	keypage	Hash of keypage of impression	bebefa5efe83beee17a3d245e7c5085b
24	advertiser	ID of advertiser	3358
25	usertag	User tags in iPinYou's audience database	13866,10111

Table 2: Dataset Overview

Dataset	Impressions	Click	avgBidprice	avgPayprice	avgCTR	avgCPM	avgeCPC
Training	2,430,981	1,793	272.96	78.15	0.074%	78.15	105.95
Validation	303,925	202	273.053	78.23	0.066%	78.23	105.72
Testing	303,925	-	-	-	-	-	-

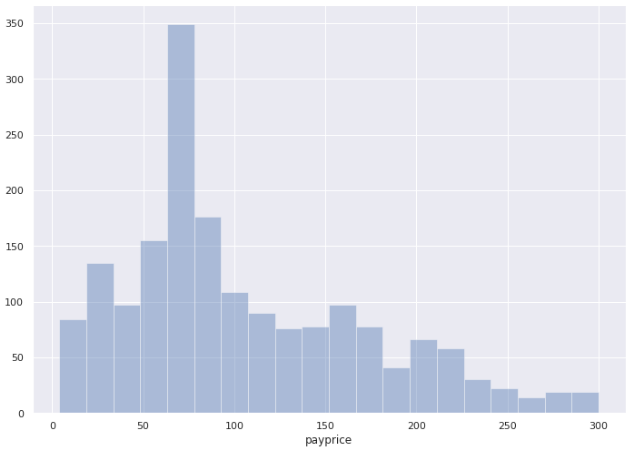


Figure 2: Distribution of payprices for clicked impressions

Table 3: Overview of Winning Criterion

Criterion	Scenario	Condition
1	Single Agent	bid $\geq$ payprice
2	Multi Agent	bid $\geq$ payprice & otherSubmittedBids

4.1 Basic Bidding Strategies

For basic bidding strategies, they do not depend on the many features provided in each impression. They are simply determined by basic information such as the agent's budget, impression prices and total impressions up for bid.

4.1.1 Constant Bidding Strategy

In a constant bidding strategy, a constant bid is chosen and applied regardless of the information provided in every impression. For this strategy, we will be working in a single-agent environment using winning criterion 1.

To obtain the optimal constant value, we set bid prices from 1 to 200 and validate the performance. The results are illustrated in Figure 3 where we observe that the performance improves linearly with bid prices till it reaches a peak at 77 before falling off. Table 4 shows the top 3 results obtained from our constant bidding finding.

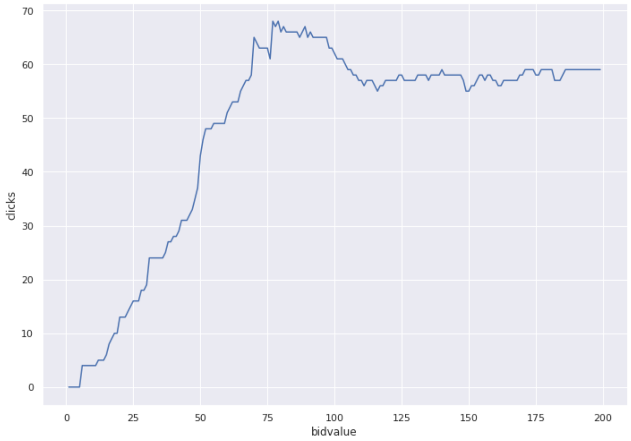


Figure 3: Clicks at different constant bid prices

Table 4: Constant Bidding Results

Click	Range	Imps	CTR	CPC	Spent
68	77	146,864	0.00046	91.91	6,249.99
68	79	145,916	0.00047	91.91	6,249.99
67	78	146,311	0.00046	93.28	6,249.99

4.1.2 Random Bidding Strategy

In a random bidding strategy, bids are randomly selected from within a certain range. We empirically determine this optimal range under winning criterion 1 by testing it against the validation dataset. To account for the random nature of the bids, 10 sets of bids are generated for every range and we take the average performance from them.

Table 5: Random Bidding Results

Click	Range	Imps	CTR	CPC	Spent
55.7	[1, 300]	100,911.1	0.00055	112.20	6,249.96
61.9	[1, 150]	137,880.7	0.00045	100.96	6,249.97
55.1	[150, 300]	87,857.7	0.00063	113.42	6,249.93
67.6	[50, 100]	151,761.3	0.00045	92.45	6,249.97
64.8	[75, 80]	149,350.8	0.00043	96.45	6,249.96
57.7	[75, 150]	120,793.7	0.00048	108.31	6,249.96
47.9	[1, 100]	116,589.2	0.00041	87.52	4,192.38
29.9	[1, 75]	84,554.8	0.00035	76.81	2,296.67

Table 5 shows the results of the tested ranges. The ranges $[1, 300]$, $[1, 150]$ and $[150, 300]$ were used to covers all possible bids in the validation set dataset and generate a baseline performance of around 55.7 clicks. It also showed that the optimal range would be located in lower half of the bid range.

Focusing on the lower half of the bids, it was found that the best performing range was between 50 to 100 with an average of 67.6 clicks. This result was further supported by the payprice distribution we saw in Figure 2, where most clicked impressions had a payprice of under 100.

4.1.3 Homogeneous Random Bidding Agents

We next look at how the environment changes with the introduction of multiple bidding agents using winning criterion 2.

With the introduction of multiple agents, the auction environment becomes more complex where every agent's bids affects the final payprice and individual agent's budgets have to be accounted for. Since we are using homogeneous agents, we expect every agent to achieve similar performances.

Table 6 shows our test results. We first used the single agent optimal random bidding range (50-100) and tested it against 50, 75 and 100 homogenous agents. As expected, the average agent's performance was much lower compared to the single agent environment where clicks dropped from 67.6 to around 1-3 clicks. Also, we found that the average number of clicks, impressions and budget spending reduces with increasing number of agents, in fact, none of the agents went out of budget (OOB) in our tests.

We then tested a high bid range (300-350) that guarantees winning of all available impressions in the dataset. We observe that the average performance of all agents has improved, but even then, none of the agents went out of the budget.

Finally, we tested with a narrow bidding range (300-310) and found that even though the average performance was similar, some agents performed much better and went out of the budget. We found that because of the narrower range, it increased the odds of similar bids where the simulation environment simply awards it to the first agent. Since the bidding order is always the same, some agents therefore get an inherent advantage to win over others, thereby creating this disparity.

From our results, in a multi-agent environment, it is possible for all agents to win every impression without exhausting their budget. But this is only when the win probability of all agents are even. So in an optimal homogeneous multi-agent environment, every agent would bid at price ranges that ensures all impressions in the dataset can be won (>300) and set a range large enough such that every agent has an equal probability of winning.

Table 6: Performance of different agent counts and bids

Agents	Range	Click	Imps	Spent	OOB
50	[50, 100]	2.52	4553.60	668.47	0
75	[50, 100]	1.68	3037.13	447.83	0
100	[50, 100]	1.26	2278.51	336.74	0
50	[300, 350]	4.04	6087.50	2588.04	0
75	[300, 350]	2.69	4052.33	1727.95	0
100	[300, 350]	2.02	3039.25	1296.92	0
50	[300, 310]	4.04	6087.50	2353.53	9
75	[300, 310]	2.69	4052.33	1569.18	9
100	[300, 310]	2.02	3039.25	1176.90	9

4.2 Advanced Bidding Strategies

Next we look into advanced bidding strategies that use CTR estimation models to determine click probability of impressions. These strategies vary the bids such that higher click probabilities will be more aggressively bid for.

4.2.1 CTR Estimation

Models: We considered two different CTR estimation models; Logistic Regression (LR) and Gradient Boosting Regression Tree (GBRT). The first one is a type of probabilistic statistical classification model which uses a binary predictor and the latter is a non-linear model which has the advantage of learning from the continuous features instead of binary features as required in the LR model. For LR, we use the implementation provided by scikit-learn and for GBRT, we use the open source library, XGBoost.

Features: As mentioned in Table 1 we selected 14 categorical features from the dataset to be used for training. Additionally, we performed feature engineering on the following 3 fields before applying one hot encoding on them. In total, we have used 606 binary features for the training of our classifier.

- useragent: Split into OS and browser details, the original useragent field is removed
- slotprices: Labelled into the ranges $[0, 10]$, $(10, 50]$, $(50, 100]$, $(100, 150]$, $(150, +\infty]$
- usertag: Split into binary features for each user

However, during the training stage we faced two problems. Firstly the imbalance of negative results (99.926% of training set) caused our resultant models to become improperly trained. Secondly, as we feature engineered and increased the size of our training data, there was a point where our devices were no longer able to load it in memory. Thus, we turned to negative downsampling [7] to solve both problems.

Negative Downsampling: In negative downsampling, we reduce the negative data from the training set to remove the bias and also reduce the data size of the training data. To determine the best rate to use, we tried different downsampling rates on our models and evaluated them with the widely used, area under ROC curve (AUC) measurement. [17, 5]. Our results is presented in Figure 4 where we found that except on the lowest sample rate, GBRT consistently showed better results as compared to LR. So our best performing model is the GBRT model at a downsampling rate of 0.0005, providing an AUC score of 0.7785. The chosen model is then used for subsequent bidding strategies.

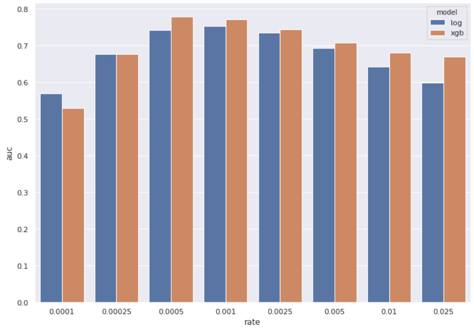


Figure 4: AUC scores at different downsampling rates

Re-calibration: Negative downsampling helped improve the model and training times, however, the CTR estimates by the model were also affected by the negative downsampling and cannot be used as it is. A re-calibration formula 1 needs to be applied to the model's CTR to get the real predicted CTR [7] by the model.

$$\mathrm{CTR_{pred}} = \frac{\mathrm{CTR_{model}}}{\mathrm{CTR_{model}} + \frac{1-\mathrm{CTR_{model}}}{downsampleRate}} \tag{1}$$

4.2.2 *Linear Bidding Strategy*

In a linear bidding strategy, the bid values are determined in a linear proportion to the predicted CTR of the impression. Formula 2 shows how bids are computed in a linear bidding strategy, $\mathrm{CTR_{pred}}$ is obtained from the CTR estimation model, whereas $\mathrm{CTR_{avg}}$ is obtained from the training dataset.

$$\mathrm{bid_{value}} = \mathrm{bid_{base}} * \frac{\mathrm{CTR_{pred}}}{\mathrm{CTR_{avg}}} \tag{2}$$

We then tune $\mathrm{bid_{base}}$ by trying different values to empirically determine the best value. Figure 5 shows our observation where our best performing $\mathrm{bid_{base}}$ value lies between 124 and 129. Table 7 shows the top 3 performing base bids from this range, where 129 was our optimal $\mathrm{bid_{base}}$.

4.2.3 *Non-Linear Bidding Strategy*

In a non-linear bidding strategy, the bid values are determined in a non-linear relationship with the predicted CTR of an impression. We implement 2 different Optimal Real-Time Bidding (ORTB) strategies [18] and evaluate the best performing results they get in our dataset.

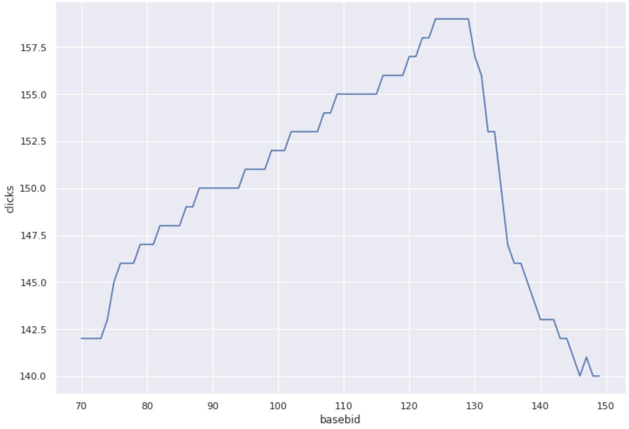


Figure 5: Click results at different base bids

Table 7: Optimal base bid results

Click	$\mathrm{bid_{base}}$	Imps	CTR	CPC	Spent
159	129	138,040	0.00115	39.31	6,250.00
159	128	137,414	0.00116	39.03	6,205.56
159	127	136,647	0.00116	38.72	6,156.00

Formulas 3 and 4 below shows the 2 bidding functions in ORTB, the equations uses the predicted CTR from our CTR estimation model and tuning parameters c and λ which are determined empirically from the dataset.

$$\mathrm{bid_{ortb1}} = \sqrt{\frac{c}{\lambda} * \mathrm{CTR_{pred}} + c^2} - c \tag{3}$$

$$\mathrm{bid_{ortb2}} = c * [(\frac{\mathrm{CTR_{pred}} + \sqrt{c^2\lambda^2 + \mathrm{CTR_{pred}^2}}}{c\lambda})^{\frac{1}{3}} - (\frac{c\lambda}{\mathrm{CTR_{pred}} + \sqrt{c^2\lambda^2 + \mathrm{CTR_{pred}^2}}})^{\frac{1}{3}}] \tag{4}$$

We used different combinations of the 2 tuning parameters λ $(1e^{-7} - 1e^{-5})$ and c $(10 - 150)$ to find our optimal results. Tables 8 and 9 shows our best performing configurations for the respective ORTB bidding strategies.

Table 8: ORTB 1 Performance

Click	c	λ	Imps	CTR	CPC	Spent
158	100	$2.0e^{-6}$	144,609	0.00109	39.56	6,249.97
158	95	$2.0e^{-6}$	144,342	0.00109	39.20	6,193.43
156	90	$2.0e^{-6}$	143,160	0.00109	39.12	6,103.36

Table 9: ORTB 2 performance

Click	c	λ	Imps	CTR	CPC	Spent
160	150	$3.4e^{-6}$	141,861	0.00113	38.59	6,174.66
155	90	$3.0e^{-6}$	143,269	0.04173	38.57	5,978.37
152	70	$2.5e^{-6}$	149,213	0.00102	41.12	6,249.97

In terms of clicks, the ORTB strategies were comparable to the linear strategy (± 1 click), however ORTB was more efficient with the budget and were able to win more impressions (ortb1: +6,569, ortb2: +3,821) using the same budget.

4.3 Multi-agent Bidding Strategy

Finally, we look at how the different strategies work in a multi-agent environment. We put all 5 different optimal strategies discussed above together in a winning criterion 2 auction to see how they perform. The results as seen in Table 10 shows that surprisingly, our best performing single-agent strategy, ORTB2, performed badly with only 30 clicks. ORTB1 and linear bidding performed best with 63 clicks and 62 clicks respectively. ORTB1 also had the best CTR and CPC to achieve the best performance while spending only 2744.33. It looks to be the best strategy to implement for our multi-agent strategy.

However as mentioned in section 4.1.3, given the limited number of impressions, the environment changes with the addition of multiple agents. So to more closely recreate the final environment of 29 groups, we look to optimise the strategies using 30 agents, using 6 agents for each strategy mentioned above. In this case, the linear bidding strategy dominated and won most of the click impressions. From this, we determine that ORTB1's performance earlier was likely due to it performing well after the linear strategy ran out of the budget, but since there are more agents to carry on the linear strategy, ORTB1 did not have a chance.

Table 10: Strategies compared

Strat	Click	Imp	CTR	CPC	Spent	OOB
5 Agents - 1 agent per strat						
ortb1	63	16,112	0.003910	43.56	2,744.33	0
linear	62	26,938	0.002302	100.80	6,250.10	134,106
ortb2	29	38,766	0.000748	215.52	6,250.24	293,019
const	7	72,953	0.000096	892.87	6,250.09	262,310
rand	2	53,419	0.000037	3,125.05	6,250.11	211,173
30 agents - 6 agents per strat - Top 5						
linear$_4$	48	18,591	0.002582	130.22	6,250.71	290,984
linear$_0$	45	18,427	0.002442	138.89	6,250.12	96,126
linear$_3$	42	18,295	0.002296	148.81	6,250.15	192,926
linear$_5$	11	2,459	0.004473	72.68	799.51	0
rand$_5$	8	29,223	0.000274	481.70	3,853.58	0

Though ORTB1 was more efficient, it was too conservative in a multi-agent setting and it gets outbid in the presence of multiple aggressive bidders. So we tried making it more aggressive by combining both linear and ORTB1 as shown in formula 5. The strategy makes it bid more aggressively when confident and less aggressively otherwise. We also included a sensitivity constant c to tune the effect of the ratio. Table 11 shows that by reducing the sensitivity to 0.45, we spent our budget more effectively and got our best performance at 86 clicks.

$$\mathrm{bid}_{\mathrm{linORTB1}} = c * \frac{\mathrm{CTR}_{\mathrm{pred}}}{\mathrm{CTR}_{\mathrm{avg}}} * \mathrm{bid}_{\mathrm{ortb1}} \qquad (5)$$

A supplementary strategy we then considered was to conserve our budget early by bidding less for the first half of the auction, then increasing our bid proportionally in the latter half. The idea is to let the competition tire out before coming in. From our finding in Table 12, the original bidding strategy still provided the best results. So we stuck with that for our final multi-agent bidding strategy.

Table 11: Linear ORTB Performance

Strat	Click	Imp	CTR	CPC	Spent	OOB
31 agents - 6 agents per strat + linORTB1 (c = 1.0)						
linORTB1	55	12,215	0.0045	113.64	6,250.26	128,967
linear$_3$	48	18,588	0.0025	130.21	6,250.10	290,335
linear$_0$	33	24,608	0.0013	189.59	6,256.71	192,339
31 agents - 6 agents per strat + linORTB1 (c = 1.2)						
linORTB1	47	15,226	0.0030	132.99	6,250.58	109,607
31 agents - 6 agents per strat + linORTB1 (c = 0.8)						
linORTB1	63	9,797	0.0064	99.21	6,250.26	154,987
31 agents - 6 agents per strat + linORTB1 (c = 0.45)						
linORTB1	86	6,148	0.0139	72.69	6,251.56	269,478

Table 12: Budget Conservation Performance

Bid Ratio	Click	Imp	CTR	CPC	Spent	OOB
100%	86	6,148	0.0139	72.69	6,251.56	269,478
90% / 110%	84	6,128	0.0137	74.40	6,250.06	279,027
75% / 125%	82	6,541	0.0125	76.22	6,250.09	300,585
50% / 150%	79	7,723	0.0102	76.81	6,068.48	0

5. LIMITATION AND FUTURE WORK

Improve CTR classifiers: In our study, we only explored 2 CTR classifiers, LR and GBRT and we focused on feature engineering instead of optimising the classifiers' parameters. Choosing better classifiers or fine tuning existing ones should create a more accurate CTR estimator such that the bidding strategies will be more effective.

Explore other bidding strategies: We only managed to explore the non-linear ORTB strategy in our study, however there are other strategies, such as lift-based bidding[14] that was not evaluated. Those strategies could have possibly been more efficient than the one that was used in our study. With time we would explore further to make a more informed decision on the non-linear strategies to use.

Multi-agent strategy: In our multi-agent strategy, we created bidding strategies that were more effective against what we have discovered so far. It did not consider changes in strategies that were implemented by other agents. A better way would have been to create a Stochastic game for multi-agent reinforced learning [8] and use the algorithms to create the ideal bidding strategy.

6. CONCLUSION

In this study, we explored multiple bidding strategies in both single and multi-agent environments. In the single agent environment, we found that basic bidding strategies produced only limited results as compared to more advanced strategies that use a CTR estimator. This is seen where our best performing basic bidding strategy only produced 77 clicks as compared to the 160 from our advanced bidding strategy. In the multi-agent environment the strategies that previously worked in a single environment will need to be adapted to handle other agents. We modified our original non-linear ORTB 1 bidding strategy to be more sensitive to handle bids from other agents and achieved the best performing result when compared with 30 other agents in our experimentation.

7. REFERENCES

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