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HW 4

1) a) Let $Y \sim$, and let $P(Y=y) = f_Y(y) = (1-p)^y p$, $y \in \{0, 1, \dots\}$

$$f_Y(y) = (1-p)^y p$$
$$= \exp\{y\ln(1-p) + \ln(p)\}$$
$$= \exp\{y\theta - (-\ln(1-e^\theta))\}$$
$$= \exp\{\frac{y\theta - b(\theta)}{a(\phi)} + c(y,\phi)\}$$

let $\theta = \ln(1-p)$
$e^\theta = 1-p$
$p = 1-e^\theta$

where
$\theta = \ln(1-p)$
$b(\theta) = -\ln(1-e^\theta)$
$a(\phi) = 1$
$\phi = 1$
$c(y,\phi) = 0$

b) $E[Y] = b'(\theta)$
$$= \frac{\partial}{\partial\theta}(-\ln(1-e^\theta))$$
$$= -\frac{1}{1-e^\theta}(-e^\theta)$$
$$= \frac{e^\theta}{1-e^\theta} = \mu$$

$\mu - \mu e^\theta = e^\theta$
$\mu = e^\theta(\mu+1)$
$e^\theta = \frac{\mu}{\mu+1}$
$\theta = \ln(\frac{\mu}{\mu+1})$

$$g(\mu) = \ln(1-p)$$
$$= \ln(1-(1-e^\theta))$$
$$= \ln(e^\theta)$$
$$= \theta$$
$$= \ln(\frac{\mu}{\mu+1})$$

2) a) Y_i = the number of times an individual woke up in an 8 hour overnight observation period, for $i \in \{1, \dots, 32\}$. Let l_i = light level of the i^{th} observation, let n_i = noise level of i^{th} observation, and let m_i = if the i^{th} individual is a morning owl or not. $m_i = \begin{cases} 1, & \text{if } i^{th} \text{ ind. is morning owl} \\ 0, & \text{otherwise} \end{cases}$, let $O_i = \begin{cases} 1, & \text{if } i^{th} \text{ individual is a night owl} \\ 0, & \text{otherwise} \end{cases}$. Let $\mu_i = E[Y_i] = \lambda$

Random Component: $Y_i \overset{indep}{\sim} Poisson(\mu_i)$ for $i \in \{1, \dots, 32\}$

Systematic Component: $\eta_i = \beta_1 l_i + \beta_2 n_i + \beta_3 m_i + \beta_4 O_i$

Link Function: $g(\mu_i) = \ln(\mu_i) = \eta_i$

b) i) Yes

ii) 4

c) i) Yes, fit2 is nested within fit1.

H_0: fit2 is true, H_a: fit1 is true,

H_0: $2\sum_{i=1}^{n}\{y_i \ln(\frac{y_i}{\hat{\mu}_i}) - (y_i - \hat{\mu}_i)\} - 2\sum_{i=1}^{n}\{y_i \ln(\frac{y_i}{\hat{\mu}_i}) - (y_i - \hat{\mu}_i)\} \sim \chi_1^2$

ii) $g(\mu_n) = \ln(\mu_n) = \eta_n$

$\mu_n = e^{\eta_n}$

$\mu_n = e^{(25)(-.317)+4.144}$

$= 28.5455$

$P(Y=2) = \frac{28.54^2 e^{-28.54}}{2!}$

$= 1.132 \times 10^{-10}$

3) No, $g(\mu) = \ln(\mu)$ results in the following: $\mu = e^{\eta}$, which allows μ to take on any value in the range $(0, \infty)$, when in actuality, this value can only ~~take on~~ exist in the range $(0, 1)$.

4) H_0: Model 1 is correct, H_a: Model 2 is correct.

Under the null hypothesis, $D_1 - D_0 \sim \chi^2_3$, we reject the H_0 in favor of H_a if our observed value is greater than the critical value $\chi^2_{3,.01} = 11.345$

Since $D_1 - D_0 = 13.4$, we reject the null hypothesis, and favor Model 2.