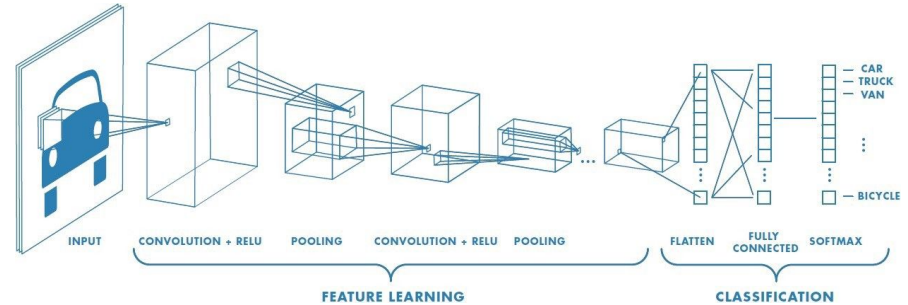




# **Graph Convolutional Network**

## **Neural Graph Collaborative Filtering**

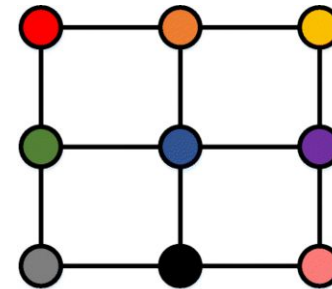
# Graph Neural Network



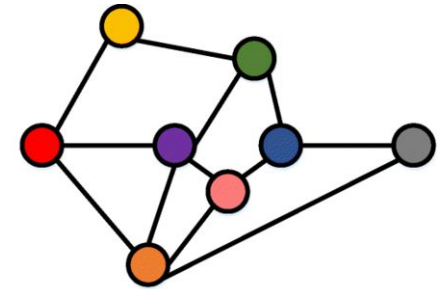
- Applicable to non-euclidean space

✂ Convolutional Neural Network

$$x_{ij}^{\ell} = \sum_{a=0}^{m-1} \sum_{b=0}^{m-1} \omega_{ab} y_{(i+a)(j+b)}^{\ell-1}$$



**CNN**  
In Euclidean Space



**GNN**  
In Non-Euclidean Space

Lin et al., (2020) A Survey on Deep Learning-Based Vehicular Communication Applications

<https://towardsdatascience.com/a-comprehensive-guide-to-convolutional-neural-networks-the-eli5-way-3bd2b1164a53>

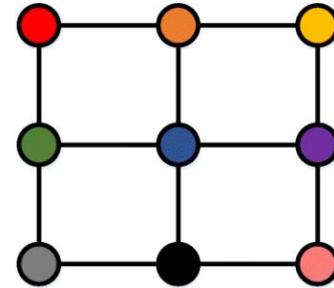
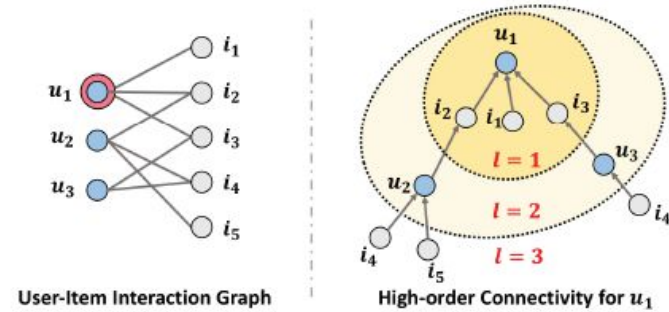
# Graph Neural Network

- Applicable to non-euclidean space

✂ Graph Convolutional Neural Network

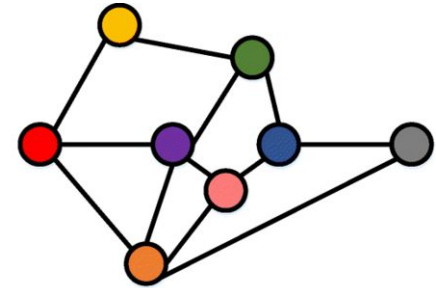
$$H^{(l+1)} = \sigma\left(\tilde{D}^{-\frac{1}{2}}\tilde{A}\tilde{D}^{-\frac{1}{2}}H^{(l)}W^{(l)}\right)$$

Work of art:  
showing that the network following this equation works well!



**CNN**

In Euclidean Space



**GNN**

In Non-Euclidean Space



# Prior Works

대충 설명할 예정 ㅋㅋ

Regularization Term

Complex Eigenvalue Computation

Not Transductive

⇒ GCN!

⇒ GAT! (consider feature nodes as well)



# FAST APPROXIMATE CONVOLUTIONS ON GRAPHS


1. Graph Laplacian
  - Normalized:  $L = I_N - D^{-\frac{1}{2}}AD^{-\frac{1}{2}}$
  - Graph Fourier Transform (convolution!!)
2. Chebyshev Polynomials

$$T_1(x)=x, T_2(x)=2x^2-1, \dots \Rightarrow$$

$$g_{\theta'} \star x \approx \sum_{k=0}^K \theta'_k T_k(\tilde{L})x$$

3. Renormalization Trick

$$g_{\theta} \star x \approx \theta \left( I_N + D^{-\frac{1}{2}}AD^{-\frac{1}{2}} \right) x \quad I_N + D^{-\frac{1}{2}}AD^{-\frac{1}{2}} \rightarrow \tilde{D}^{-\frac{1}{2}}\tilde{A}\tilde{D}^{-\frac{1}{2}}, \quad \tilde{A} = A + I_N \quad \tilde{D}_{ii} = \sum_j \tilde{A}_{ij}$$



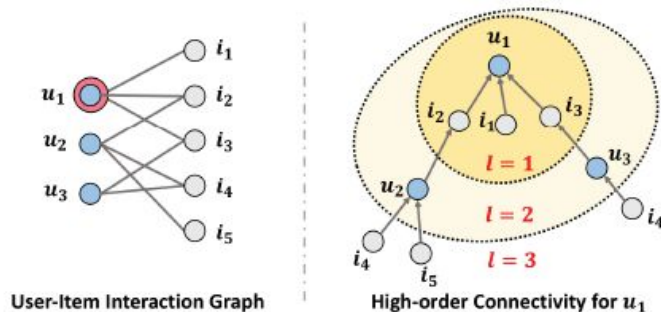
## 코드 구현

깃허브, 노션 보면서 대충 할 예정 ㅋㅋ

- Pytorch로 구현 언젠가는 할 예정
- Torch-geo에 GCN layer로 predefined 되어있긴 함
- CSR 형태에 대해 이해해야함!
  - sparse matrix의 경우,  $N \times N$  형태가 아니라 값이 있는 행, 열 index와 value만  $P \times 3$  형태로 저장한다.

# Neural Graph Collaborative Filtering

- GNN에게는 CF signal 정보를 담는 structural benefit 존재
  - 그림은 얼핏 보면 트리 구조 같지만 (실제로 트리 형태로 propagation 되기도 하고)
  - GCN 형태로 학습시키면 효율적이다.
- Architecture
  - Embedding Layer
  - Embedding Propagation Layer
    - Message Construction
    - Message Aggregation
- Training details

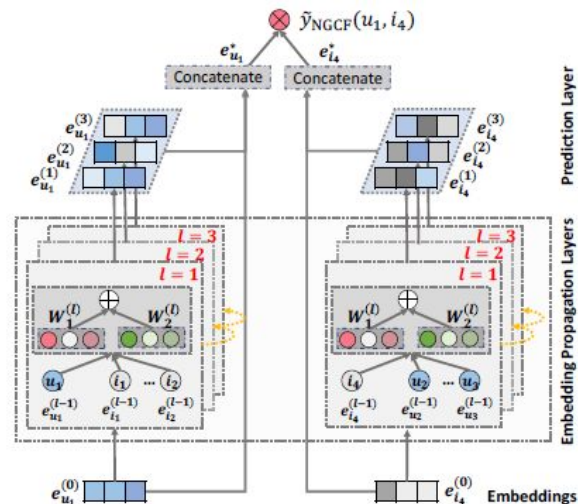


# Architecture

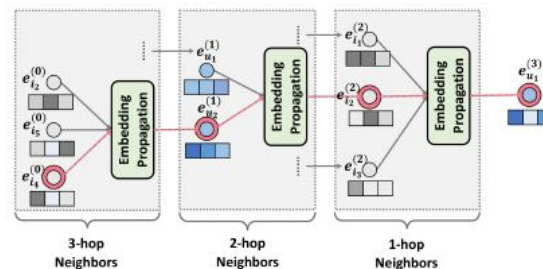
## 1. Message Propagation / Aggregation (+ Layer Orders)

$$\mathbf{e}_u^{(l)} = \text{LeakyReLU}\left(\mathbf{m}_{u \leftarrow u}^{(l)} + \sum_{i \in \mathcal{N}_u} \mathbf{m}_{u \leftarrow i}^{(l)}\right)$$

$$\begin{cases} \mathbf{m}_{u \leftarrow i}^{(l)} = p_{ui} \left( \mathbf{W}_1^{(l)} \mathbf{e}_i^{(l-1)} + \mathbf{W}_2^{(l)} (\mathbf{e}_i^{(l-1)} \odot \mathbf{e}_u^{(l-1)}) \right) \\ \mathbf{m}_{u \leftarrow u}^{(l)} = \mathbf{W}_1^{(l)} \mathbf{e}_u^{(l-1)}, \end{cases}$$



$$\mathbf{E}^{(l)} = \text{LeakyReLU}\left((\mathcal{L} + \mathbf{I})\mathbf{E}^{(l-1)}\mathbf{W}_1^{(l)} + \mathcal{L}\mathbf{E}^{(l-1)} \odot \mathbf{E}^{(l-1)}\mathbf{W}_2^{(l)}\right)$$



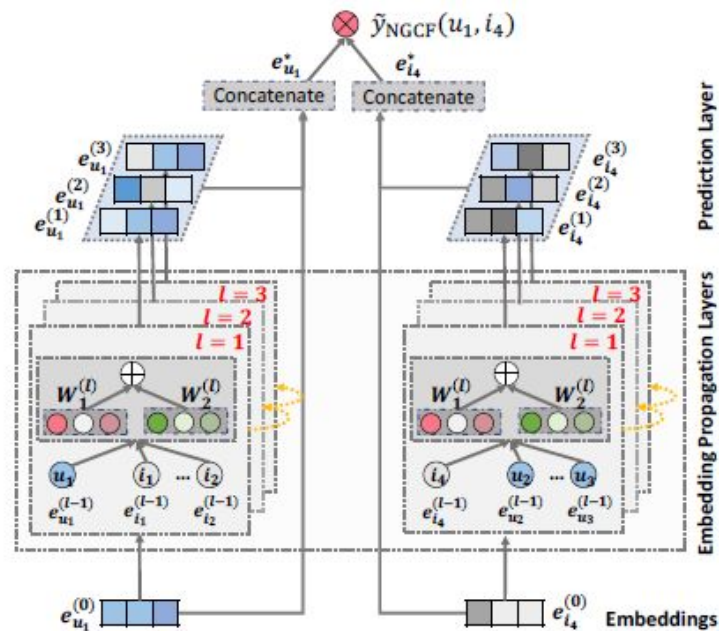
## 2. Graph Formulation (items/users are linked only to vice versa)


$$\mathcal{L} = \mathbf{D}^{-\frac{1}{2}} \mathbf{A} \mathbf{D}^{-\frac{1}{2}} \text{ and } \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{R} \\ \mathbf{R}^\top & \mathbf{0} \end{bmatrix}$$



# Training Detail

1. Concatenate every order of representation.
  2. Inner product
- 
3. Pairwise BPR loss
  4. Randomly sampled triples





## 코드 구현

아마 이쯤 되면 조원들 모두 지쳐있을 것이라 예상.

설명 블로그와 원저자 깃허브 슬랙에 올려두었으며, 심화 과제가 따로 있으니 턴을 넘기기로 함.

## 추신

앞서 언급한 **Graph Attention Network (GAT)** 계열의 **MultiSAGE**가 성능이 조금 더 잘나오는 듯 하다.

다음엔 **GAT**랑 **MultiSAGE** 공부해서 오도록 하겠다.