C++ now

A New Dragon in the Den

Fast Conversion from Floating Point Numbers

Cassio Neri

2024

Hercules slaying the Hydra

Hercules slaying the Hydra
Louis Chéron
Oil painting, ca. 1690-1725
Victoria & Albert Museum, London, UK

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Bicho de sete cabeças.

Seven-headed beast.

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Medieval French Apocalypse Tapestry, Produced between 1377 and 1382.
Château d'Angers, France
Credits: Jean-Pierre Dalbéra, CC BY 2.0, via Wikimedia Commons.

Album The Number of the Beast
Iron Maiden, 1982.

Scream for me Aspen !!!

Bruce Dickinson in Brno, Czech Republic, June 8, 2014
Credits: Vaclav Salek/CTK Photo/Alamy Live News

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Bicho de sete cabeças.

Seven-headed beast.

Não é um bicho de sete cabeças.

It's not a seven-headed beast.

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Bicho de sete cabeças.

Seven-headed beast.

Não é um bicho de sete cabeças.

It's not a seven-headed beast.

It's not rocket science.

A New Dragon in the Den

Fast conversion from floating-point numbers

Cassio Neri

(Independent Researcher)

C++Now 2024 - Aspen

π

https://godbolt.org/z/a8qETqzbj

```
🚫 sprintf(s1.data(), "%g", pi);                         // 3.14159
🚫 snprintf(s2.data(), s2.size(), "%g", pi);             // 3.14159
   string s3 = (stringstream{} << pi).str();            // 3.14159
   string s4 = to_string(pi);                           // 3.141593
   to_chars(s5.data(), s5.data() + s5.size(), pi); // 3.141592653589793
   string s6 = format("{}", pi);                        // 3.141592653589793
```

π

https://godbolt.org/z/a8qETqzbj

```cpp
sprintf(s1.data(), "%g", pi);                         // 3.14159
snprintf(s2.data(), s2.size(), "%g", pi);             // 3.14159
string s3 = (stringstream{} << pi).str();             // 3.14159
string s4 = to_string(pi);                            // 3.141593
to_chars(s5.data(), s5.data() + s5.size(), pi);       // 3.141592653589793
string s6 = format("{}", pi);                         // 3.141592653589793
```

https://godbolt.org/z/5aseWr7fz

```python
s1 = f'{pi}'           // 3.141592653589793
s2 = str(pi)           // 3.141592653589793
s3 = pi.__str__()      // 3.141592653589793
s4 = "{}".format(pi)   // 3.141592653589793
s5 = "%s" % pi         // 3.141592653589793
```

JS

https://onecompiler.com/javascript/428wnkarf

```javascript
const s1 = "" + pi;          // 3.141592653589793
const s2 = pi.toString();    // 3.141592653589793
const s3 = String(pi);       // 3.141592653589793
const s4 = `${pi}`;          // 3.141592653589793
```

https://godbolt.org/z/8M483Kvba

```java
String s1 = "" + pi;                    // 3.141592653589793
String s2 = Double.toString(pi);        // 3.141592653589793
String s3 = String.valueOf(pi);         // 3.141592653589793
String s4 = String.format("%g", pi);    // 3.14159
```

https://godbolt.org/z/Y46WY4ooY

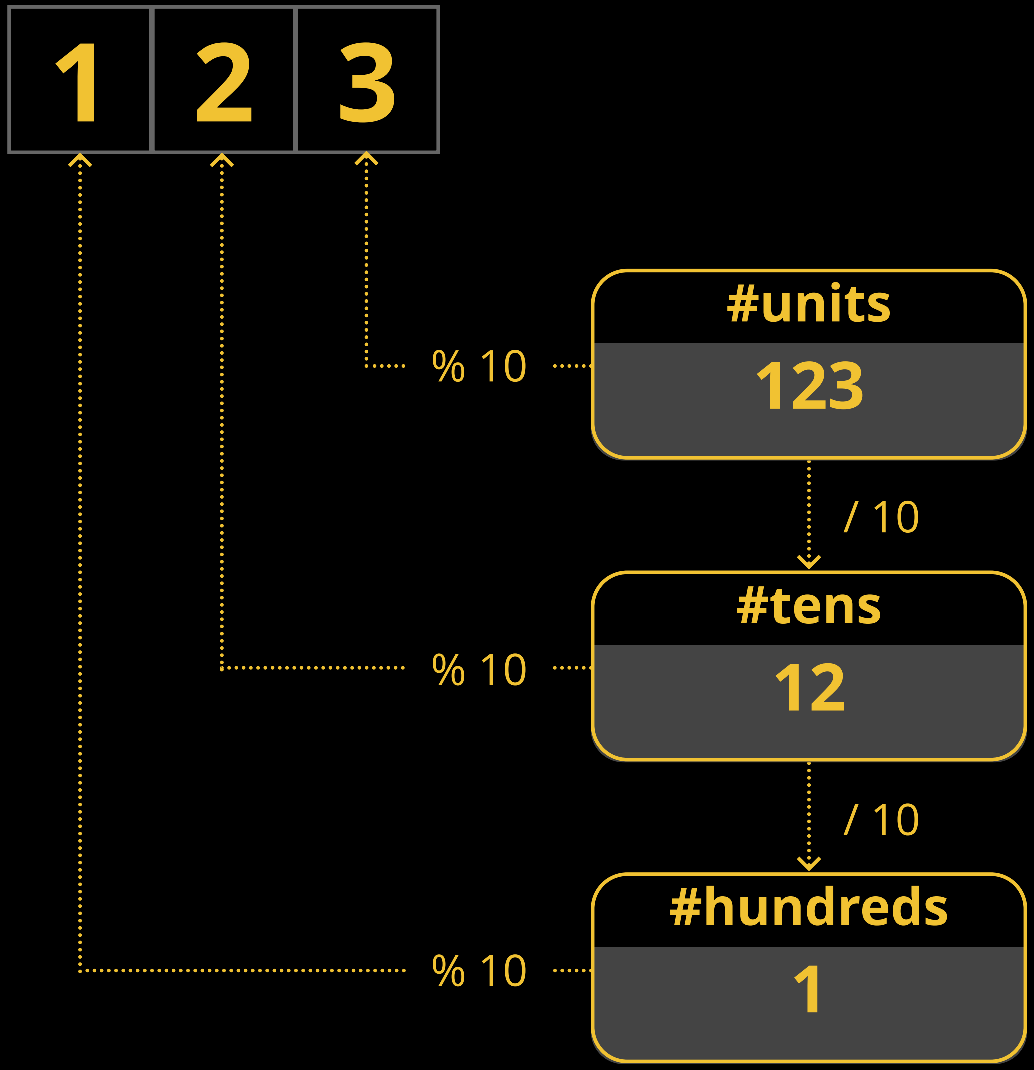
```rust
let s1 = pi.to_string();    // 3.141592653589793
let s2 = format!("{}", pi); // 3.141592653589793
```

.NET

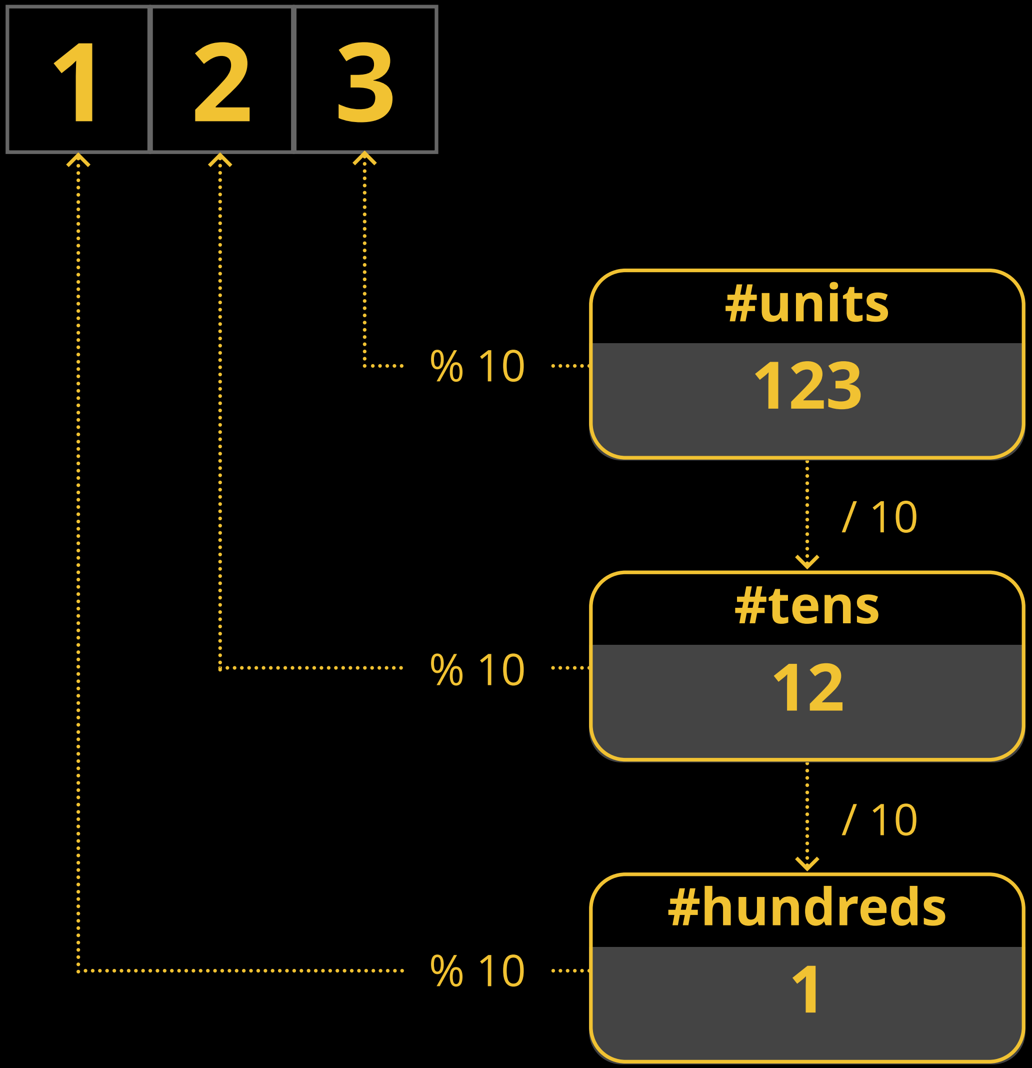
https://godbolt.org/z/czhrT1hhh

```csharp
string s1 = pi.ToString();              // 3.141592653589793
string s2 = string.Format("{0}", pi);   // 3.141592653589793
string s3 = $"{pi}";                    // 3.141592653589793
```

Integer to string



Integer to string



https://godbolt.org/z/1G9hbKMnP

```
string convert(unsigned n) {

  size_t size = number_of_digits(n);
  string str(size, '\0');
  char* p = &str.back();

  do {

    *p = n % 10 + '0';
    n /= 10;

    --p;

  } while(n);

  return str;
}
```

Minimum strictly positive `float`

$$2^{-149} = \begin{aligned}&0.000,000,000,000,000,000,000,000,000,000,\\&000,000,000,000,001,401,298,464,324,817,\\&070,923,729,583,289,916,131,280,261,941,\\&876,515,771,757,068,283,889,791,082,685,\\&860,601,486,638,188,362,121,582,031,25\end{aligned}$$

(149 digits after the dot) 🙁

Minimum strictly positive `float`

$$
\begin{aligned}
2^{-149} = \; & 0.000,000,000,000,000,000,000,000,000,000, \\
& 000,000,000,000,001,401,298,464,324,817, \\
& 070,923,729,583,289,916,131,280,261,941, \\
& 876,515,771,757,068,283,889,791,082,685, \\
& 860,601,486,638,188,362,121,582,031,25
\end{aligned}
$$

(149 digits after the dot) ☹️

$\cong 10^{-45}$ 🙂

Mechanical counter

Mechanical counter

Mechanical counter

Decimal fixed-point representation

1234

Decimal fixed-point representation

12.34

Decimal floating-point representation

exponent

mantissa

$$10^{1} \times 2.34$$

Decimal floating-point representation

exponent

mantissa

$$10^{1} \times 2.34$$

0,0 0 0,0 0 0,0 2 3.4 0

Decimal floating-point representation

exponent

mantissa

$10^{1} \times 2.34$

0,0 0 0,0 0 0,0 2 3.4 0 🚫

Decimal floating-point representation

exponent $\neq 0$

$\Longrightarrow$

mantissa's first digit $\neq 0$

(scientific notation)

exponent

mantissa

$$10^{1} \times 2.34$$

0,0 0 0,0 0 0,0 2 3.4 0

Decimal floating-point representation

exponent

mantissa

$$10^{9} \times 9.99 \times 10^{-3}$$

9,9 9 0,0 0 0.0 0 0,0 0

Decimal floating-point representation

biased exponent $\neq 0$

$\Longrightarrow$

mantissa's first digit $\neq 0$

(scientific notation)

biased exponent

mantissa

$$10^{9-3} \times 9.99$$

9,9 9 0,0 0 0.0 0 0,0 0

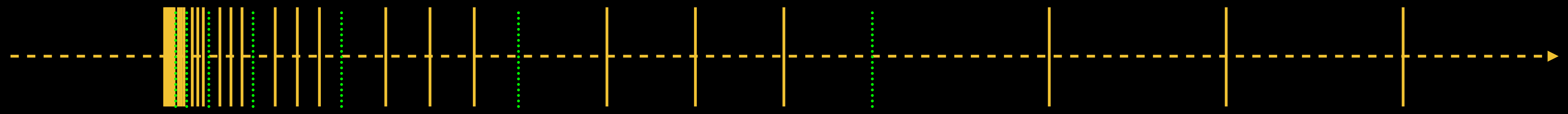
Binary floating-point representation

Exponent and mantissa are in binary.

biased exponent

mantissa

$$2^{000-11} \times 0.00$$



Binary floating-point representation

Exponent and mantissa are in binary.

biased exponent $\neq 0$

$\implies$

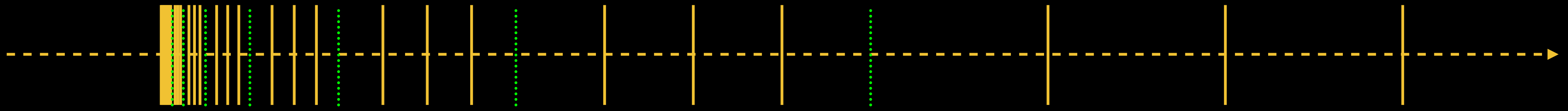
mantissa's first bit $= 1$

(normal)

biased exponent

mantissa

$$2^{000-11} \times 0.00$$



Binary floating-point representation

Exponent and mantissa are in binary.

biased exponent $\neq 0$

$\Longrightarrow$

mantissa's first bit $= 1$

(normal)

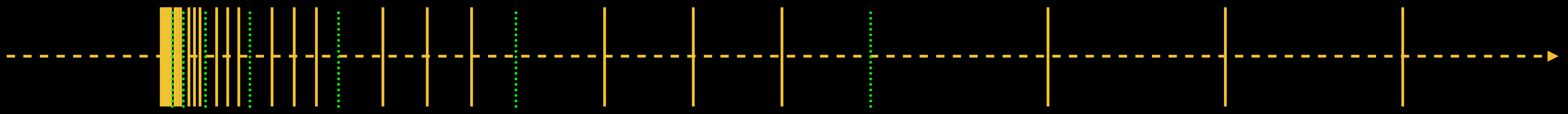
biased exponent

mantissa

$$2^{000-11} \times 0.00$$

Centred value

Uncentred value



Binary floating-point representation

Exponent and mantissa are in binary.

biased exponent $\neq 0$

$\Longrightarrow$

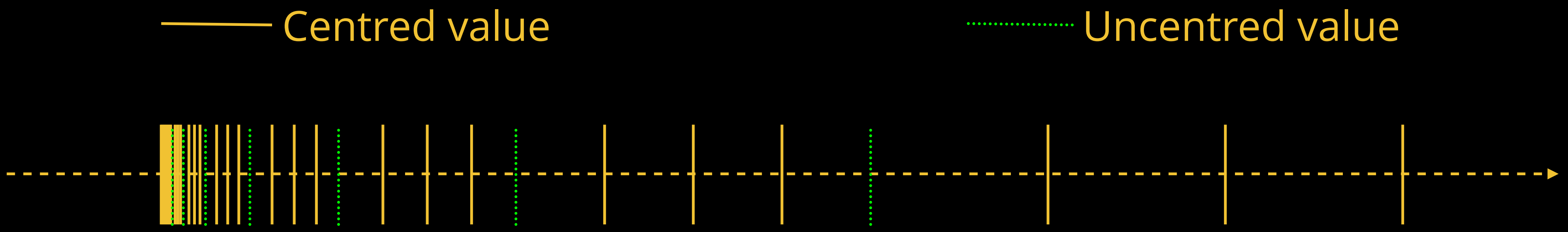
mantissa's first bit $= 1$

(normal)

biased exponent

mantissa

$$2^{000-11} \times 1.00$$



IEEE-754 representation

- *Binary32* (`float` and `std::float32_t`)
- *Binary64* (`double` and `std::float64_t`)
- *Binary128* (`__float128` and `std::float128_t`)
- Others

Binary32

0	1	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Binary32

									mantissa																						
									23																						
0	1	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Binary32

	biased exponent								mantissa																						
	8								23																						
0	1	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Binary32

±	biased exponent								mantissa																						
1	8								23																						
0	1	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Binary32

<table>
<tr><td>±</td><td colspan="8">biased exponent</td><td colspan="23">mantissa</td></tr>
<tr><td>1</td><td colspan="8">8</td><td colspan="23">23</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table>

0 sign: **+** (**0**) or **–** (**1**)

Binary32

<table>
<tr><td>±</td><td colspan="8">biased exponent</td><td colspan="23">mantissa</td></tr>
<tr><td>1</td><td colspan="8">8</td><td colspan="23">23</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table>

Binary32

±	biased exponent								mantissa																						
1	8								23																						
0	1	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$$\begin{array}{r} 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0 \\ -\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1 \\ \hline 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1 \end{array}$$

biased exponent $\neq 0 \implies$ normal

bias: **01111111** (127, normal) or **011111110** (126, subnormal)

+

Binary32

±	biased exponent								mantissa																						
1	8								23																						
0	1	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

biased exponent $\neq 0 \implies$ normal

$$+2^{00000011} \times 1.$$

Binary32

https://godbolt.org/z/KvMT57Pjs

<table>
<tr><td>±</td><td colspan="8">biased exponent</td><td colspan="23">mantissa</td></tr>
<tr><td>1</td><td colspan="8">8</td><td colspan="23">23</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table>

$$+2^{00000011} \times 1.01011000000000000000000$$

https://godbolt.org/z/KvMT57Pjs

Binary32

±	biased exponent								mantissa																						
1	8								23																						
0	1	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$$+2^{00000011} \text{x} 1.01011000000000000000000$$

$$+2^{3} \text{x} \quad 1.3\ 4\ 3,7\ 5$$

Binary32

https://godbolt.org/z/KvMT57Pjs

±	biased exponent	mantissa
1	8	23
0	1 0 0 0 0 0 1 0	0 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

$$+2^{-00010100} \times 1010110000000000000000000$$

$$+2^{-20} \times 11{,}272{,}192$$

Quiz

What's the output?

https://godbolt.org/z/EzWTGEoP8

```cpp
double x = 1.0 / 3.0;
std::cout << std::format("{}", x);
```

Options:

(a) 0.3

(b) 0.333333333333333**3**

(c) 0.333333333333333**4**

(d) One third

(e) *None of the above*

Quiz

What's the output?

https://godbolt.org/z/EzWTGEoP8

```
double x = 1.0 / 3.0;
std::cout << std::format("{}", x);
```

Options:

(a) 0.3

(b) 0.3333333333333333

(c) 0.3333333333333334

(d) One third

(e) *None of the above*

Quiz

What's the output?

https://godbolt.org/z/EzWTGEoP8

```cpp
double x = 1.0 / 3.0;
std::cout << std::format("{}", x);
```

https://godbolt.org/z/4qP56b6Ks

```cpp
float x = 1.f / 3.f;
std::cout << std::format("{}", x);
```

Options:

double	float
(a) 0.3	(a) 0.3
✔ (b) 0.33333333333333333	(b) 0.33333333
(c) 0.33333333333333334	(c) 0.33333334
(d) One third	(d) One third
(e) *None of the above*	(e) *None of the above*

Quiz

What's the output?

https://godbolt.org/z/EzWTGEoP8

```
double x = 1.0 / 3.0;
std::cout << std::format("{}", x);
```

https://godbolt.org/z/4qP56b6Ks

```
float x = 1.f / 3.f;
std::cout << std::format("{}", x);
```

Options:

(a) 0.3
(b) 0.3333333333333333**3**
(c) 0.3333333333333333**4**
(d) One third
(e) *None of the above*

(a) 0.3
(b) 0.3333333**3**
(c) 0.3333333**4**
(d) One third
(e) *None of the above*

Quiz

What's the output?

https://godbolt.org/z/EzWTGEoP8

```cpp
double x = 1.0 / 3.0;
std::cout << std::format("{}", x);
```

https://godbolt.org/z/4qP56b6Ks

```cpp
float x = 1.f / 3.f;
std::cout << std::format("{}", x);
```

Options:

(a) 0.3
(b) 0.33333333333333333 ✔
(c) 0.33333333333333334
(d) One third
(e) *None of the above*

(a) 0.3
(b) 0.33333333
(c) 0.33333334 ✔
(d) One third
(e) *None of the above*

https://godbolt.org/z/17jjd6Kv1

https://godbolt.org/z/43EnYG6Pj

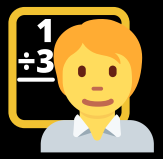
.NET

https://godbolt.org/z/Yax6xdYf3

https://godbolt.org/z/9P6s4zK49

⅓

$$0.333{,}333{,}333{,}333{,}333{,}333{,}333{,}333{,}\overline{3}$$



⅓

https://godbolt.org/z/9P6s4zK49

$$0.333{,}333{,}333{,}333{,}333{,}333{,}333{,}333{,}\overline{3} = 2^{-25} \times \left(11{,}184{,}810 + \frac{2}{3}\right)$$

⅓

0.333,333,343,267,440,795,898,437,5 = $2^{-25} \times 11{,}184{,}811$

$0.333{,}333{,}333{,}333{,}333{,}333{,}333{,}333{,}\overline{3} = 2^{-25} \times \left(11{,}184{,}810 + \frac{2}{3}\right)$

0.333,333,313,465,118,408,203,125,0 = $2^{-25} \times 11{,}184{,}810$

https://godbolt.org/z/9P6s4zK49

⅓

$0.333,333,343,267,440,795,898,437,5 = 2^{-25} \times 11,184,811$

💻

$0.333,333,333,333,333,333,333,333,\bar{3} = 2^{-25} \times (11,184,810 + \frac{2}{3})$

🧑‍🏫

$0.333,333,313,465,118,408,203,125,0 = 2^{-25} \times 11,184,810$

https://godbolt.org/z/9P6s4zK49

⅓

$0.333,333,343,267,440,795,898,437,5 = 2^{-25} \times 11,184,811$

$3,333,333,333,333,333,\overline{3} = 2^{-25} \times (11,184,810 + \frac{2}{3})$

$0.333,333,313,465,118,408,203,125,0 = 2^{-25} \times 11,184,810$

https://godbolt.org/z/9P6s4zK49

⅓

$$0.333{,}333{,}343{,}267{,}440{,}795{,}898{,}437{,}5 = 2^{-25} \times 11{,}184{,}811$$

$$3{,}333{,}333{,}333{,}333{,}333{,}\overline{3} = 2^{-25} \times (11{,}184{,}810 + \frac{2}{3})$$

$$0.333{,}333{,}313{,}465{,}118{,}408{,}203{,}125{,}0 = 2^{-25} \times 11{,}184{,}810$$

https://godbolt.org/z/9P6s4zK49

⅓

$0.333{,}333{,}343{,}267{,}440{,}795{,}898{,}437{,}5 = 2^{-25} \times 11{,}184{,}811$

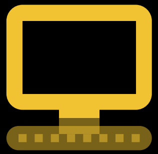
$3{,}333{,}333{,}333{,}333{,}333{,}\overline{3} = 2^{-25} \times \left(11{,}184{,}810 + \frac{2}{3}\right)$

$0.333{,}333{,}313{,}465{,}118{,}408{,}203{,}125{,}0 = 2^{-25} \times 11{,}184{,}810$

⅓

https://godbolt.org/z/9P6s4zK49

0.333,333,343,267,440,795,898,437,5 = $2^{-25} \times 11,184,811$



3,333,333,333,333,333,$\bar{3}$ = $2^{-25} \times (11,184,810 + \frac{2}{3})$

0.333,333,313,465,118,408,203,125,0 = $2^{-25} \times 11,184,810$

https://godbolt.org/z/9P6s4zK49

⅓

$0.333{,}333{,}343{,}267{,}440{,}795{,}898{,}437{,}5 = 2^{-25} \times 11{,}184{,}811$

$3{,}333{,}333{,}333{,}333{,}333{,}\bar{3} = 2^{-25} \times (11{,}184{,}810 + \frac{2}{3})$

$0.333{,}333{,}313{,}465{,}118{,}408{,}203{,}125{,}0 = 2^{-25} \times 11{,}184{,}810$

⅓

https://godbolt.org/z/9P6s4zK49

0.333,333,343,267,440,795,898,437,5 = $2^{-25} \times 11{,}184{,}811$

$\ldots 3{,}333{,}333{,}333{,}333{,}333{,}\overline{3}$ = $2^{-25} \times \left(11{,}184{,}810 + \frac{2}{3}\right)$

0.333,333,313,465,118,408,203,125,0 = $2^{-25} \times 11{,}184{,}810$

⅓

https://godbolt.org/z/9P6s4zK49

$$0.333,333,343,267,440,795,898,437,5 = 2^{-25} \times 11,184,811$$

$$3,333,333,333,333,333,\overline{3} = 2^{-25} \times \left(11,184,810 + \frac{2}{3}\right)$$

$$0.333,333,313,465,118,408,203,125,0 = 2^{-25} \times 11,184,810$$

⅓

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$$0.333,333,343,267,440,795,898,437,5 = 2^{-25} \times 11,184,811$$

$$3,333,333,333,333,333,\overline{3} = 2^{-25} \times \left(11,184,810 + \frac{2}{3}\right)$$

$$0.333,333,313,465,118,408,203,125,0 = 2^{-25} \times 11,184,810$$

⅓

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$$0.333,333,343,267,440,795,898,437,5 = 2^{-25} \times 11,184,811$$

$$0.333,333,333,333,333,333,333,\overline{3} = 2^{-25} \times (11,184,810 + \frac{2}{3})$$

$$0.333,333,313,465,118,408,203,125,0 = 2^{-25} \times 11,184,810$$

https://godbolt.org/z/9P6s4zK49

⅓

$0.333,333,343,267,440,795,898,437,5 = 2^{-25} \times 11,184,811$

$3,333,333,333,333,333,\bar{3} = 2^{-25} \times (11,184,810 + \frac{2}{3})$

$0.333,333,313,465,118,408,203,125,0 = 2^{-25} \times 11,184,810$

⅓

https://godbolt.org/z/9P6s4zK49

0.333,333,343,267,440,795,898,437,5 = $2^{-25} \times 11{,}184{,}811$

0.333,333,34

3,333,333,333,333,333,$\bar{3}$ = $2^{-25} \times \left(11{,}184{,}810 + \frac{2}{3}\right)$

0.333,333,313,465,118,408,203,125,0 = $2^{-25} \times 11{,}184{,}810$

Jorge Luis Borges, The Book of Imaginary Beings

“There is something in the dragon’s image that fits man’s imagination.”

Dragon's Den

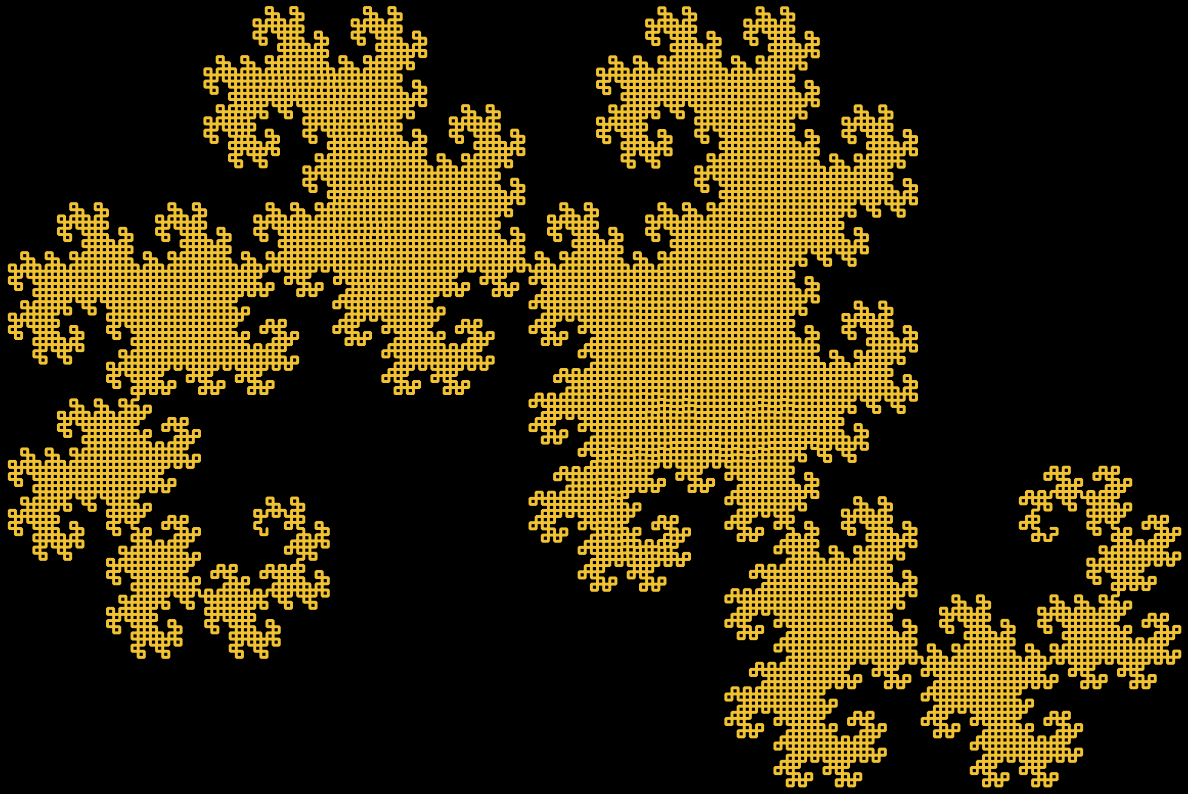
Dragon's Den

Copyright: ABC

Copyright: BBC

Another dragon's den

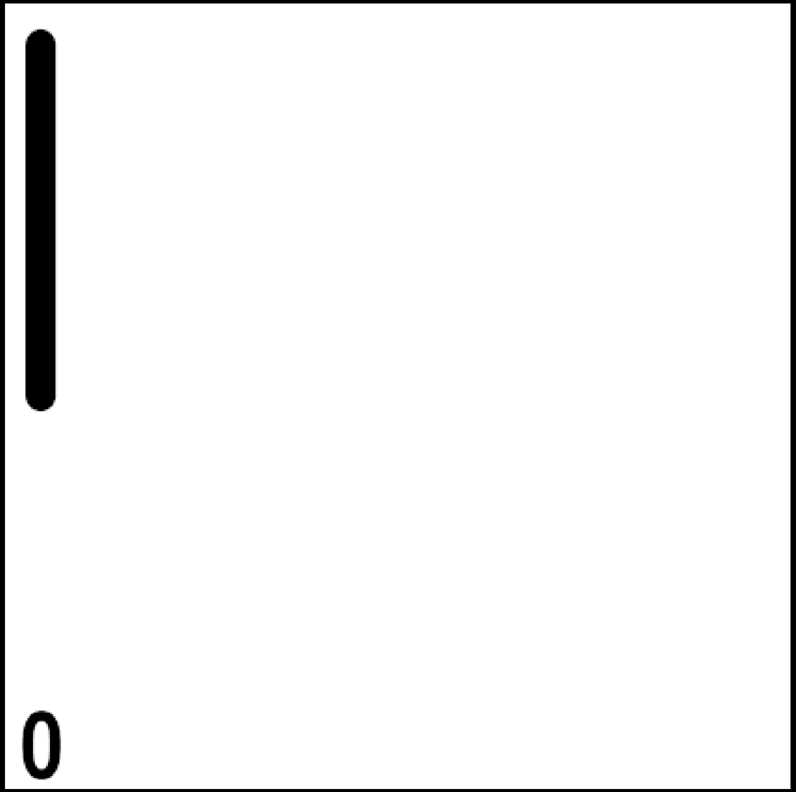
1990 - *Dragon* Guy L. Steele, Jon L. White



Heighway Dragon Curve

Another dragon's den

1990 - *Dragon* Guy L. Steele, Jon L. White



Heighway Dragon Curve (orders 0 to 15)
Public domain

Another dragon's den

1990 - *Dragon*	Guy L. Steele, Jon L. White
1996	Robert G. Burger and R. Kent Dybvig

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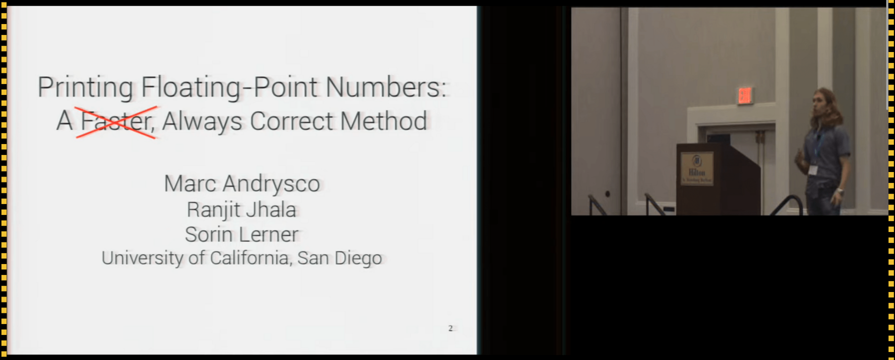

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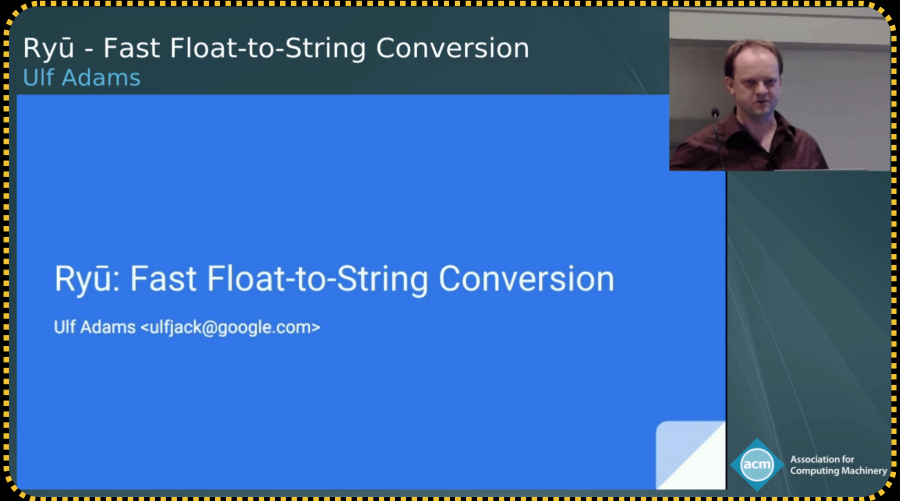

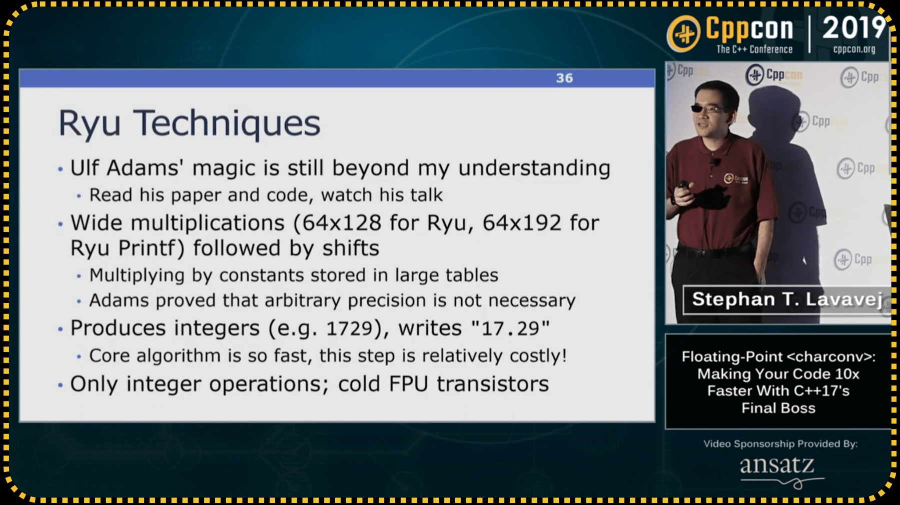

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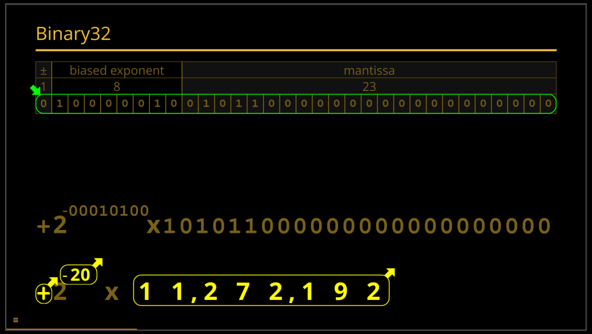
2018 - *Ryū*	Ulf Adams
2020 - *Schubfach*	Raffaello Giulietti
2020 - *Grisù-Exact*	Junekey Jeon
2022 - *Dragonbox*	Junekey Jeon

{fmt}

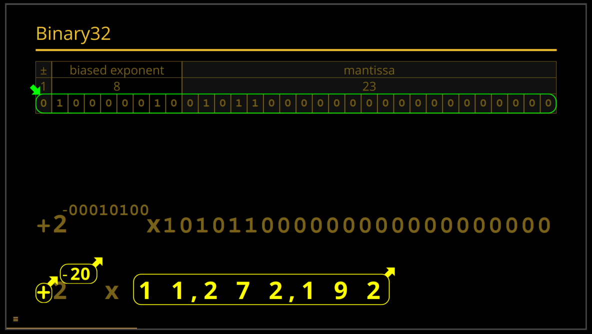
Three steps

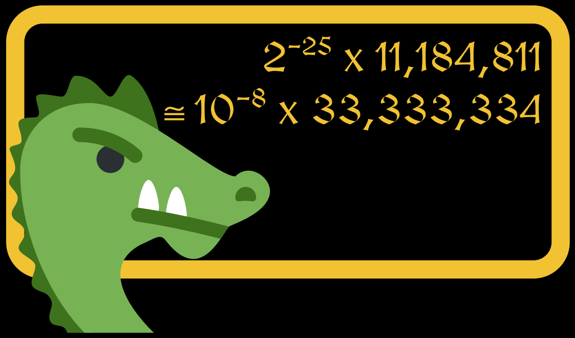
Three steps



decode representation

Three steps



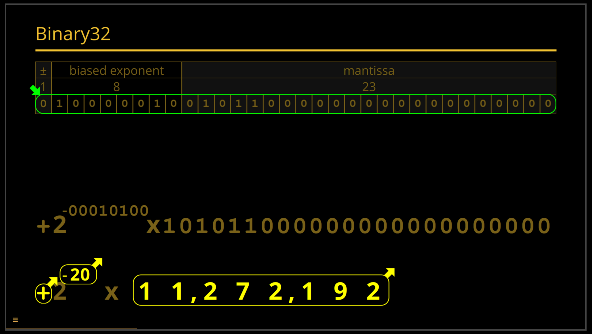


decode representation

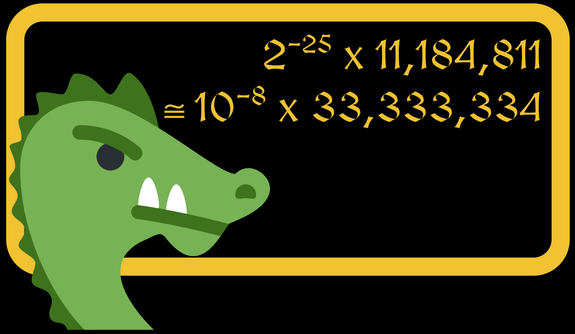
↓

convert binary to decimal

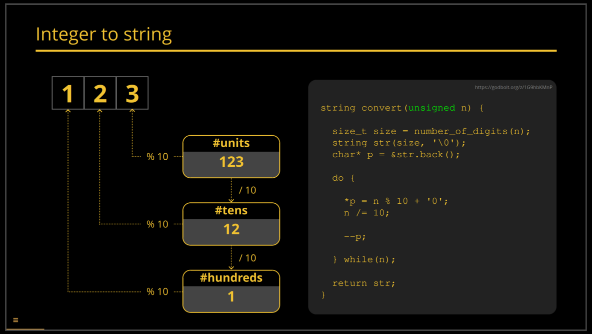
Three steps



decode representation



convert binary to decimal



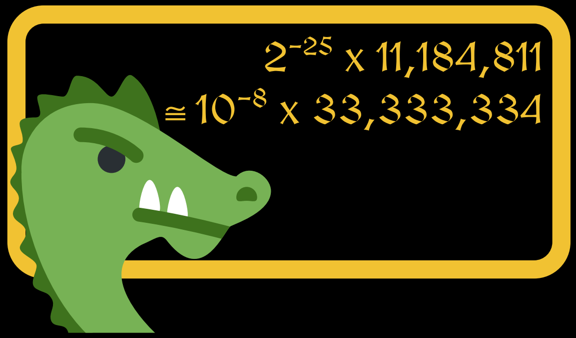
convert exponent and mantissa to string

Dragon's problem

Given $m \in \mathbb{N}$ and $E \in \mathbb{Z}$,

find $n \in \mathbb{N}$ and $F \in \mathbb{Z}$ such that

$$n \times 10^F \cong m \times 2^E.$$



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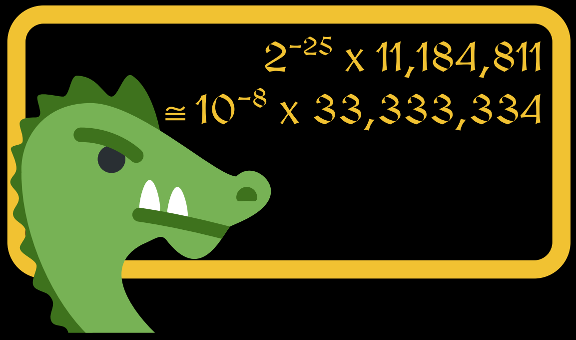
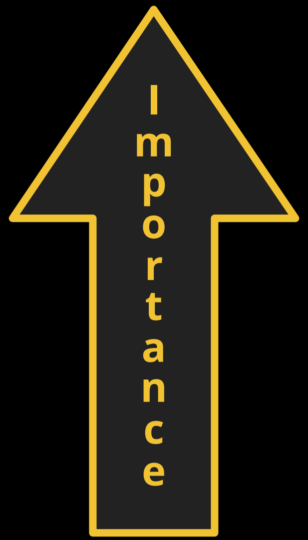
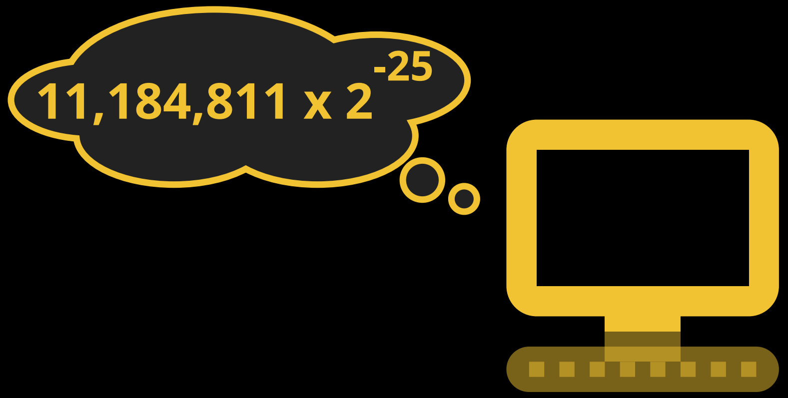


- No information loss
- As short as possible
- As close as possible
- Tiebreak rules



No information loss

https://godbolt.org/z/9P6s4zK49



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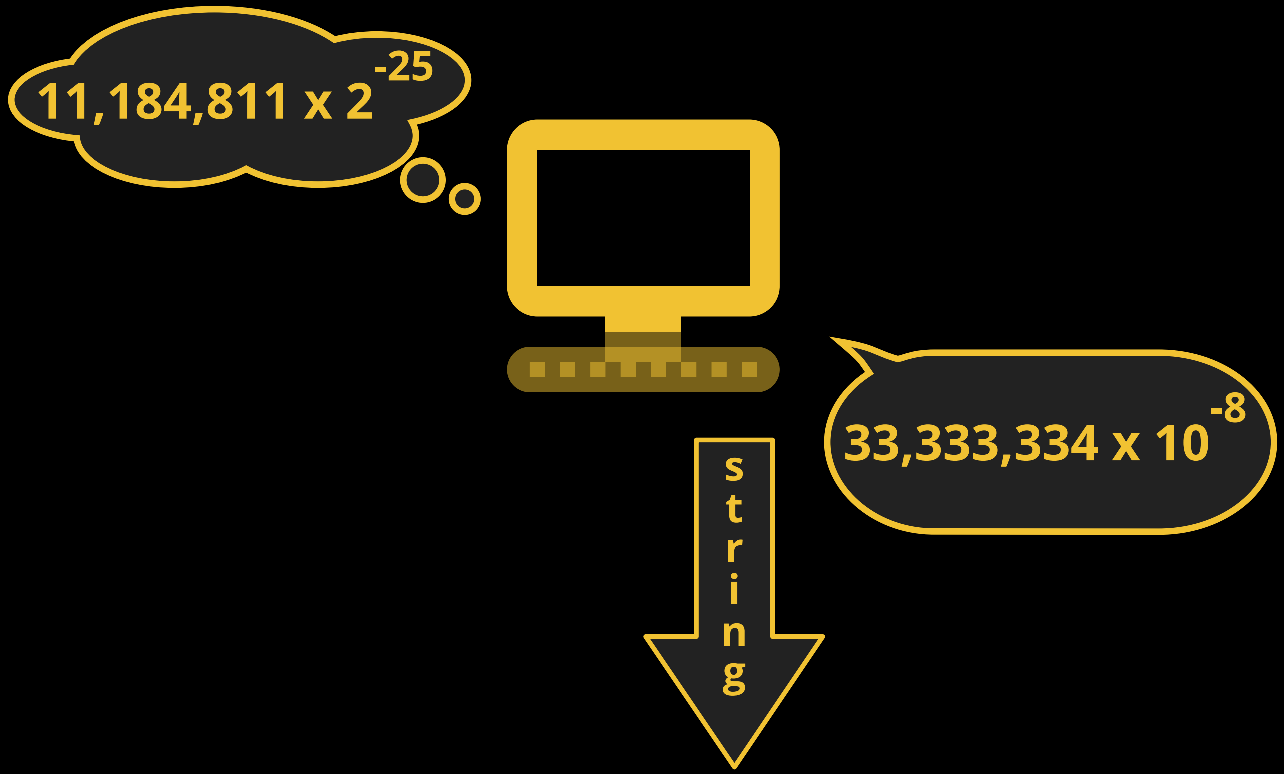

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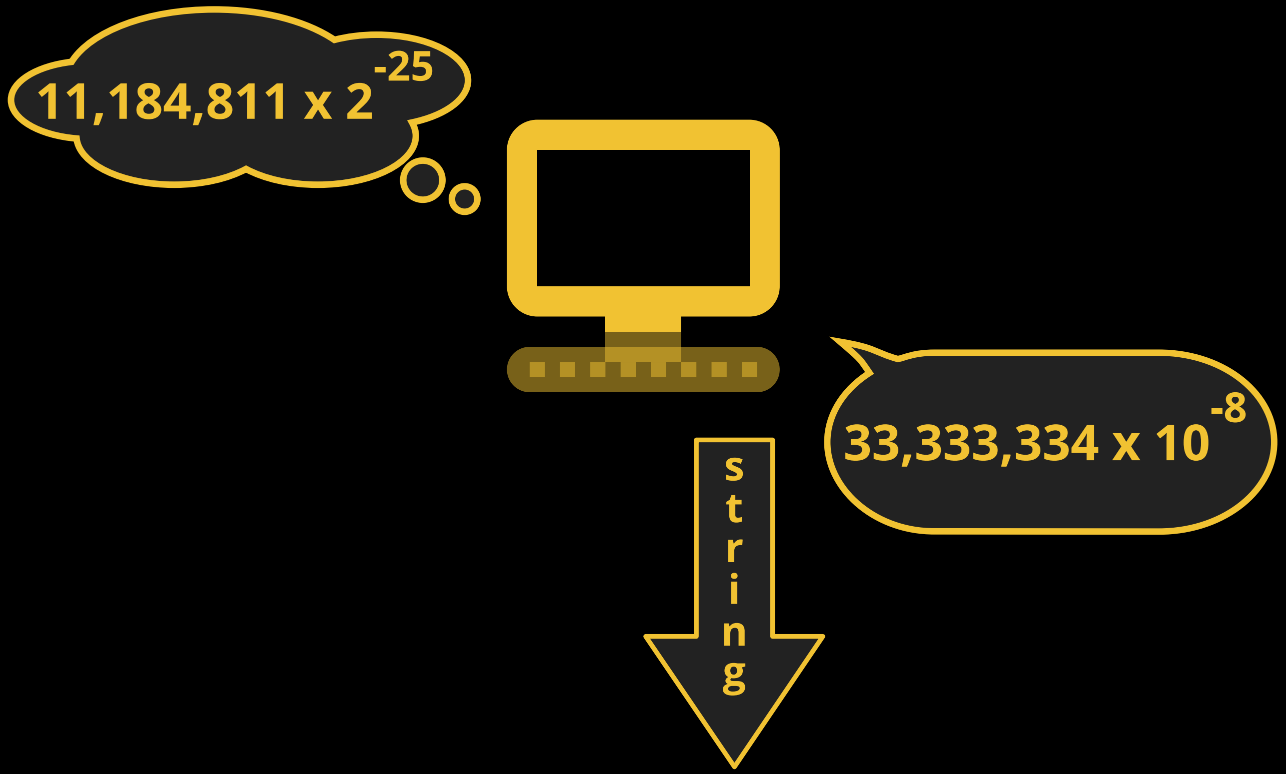
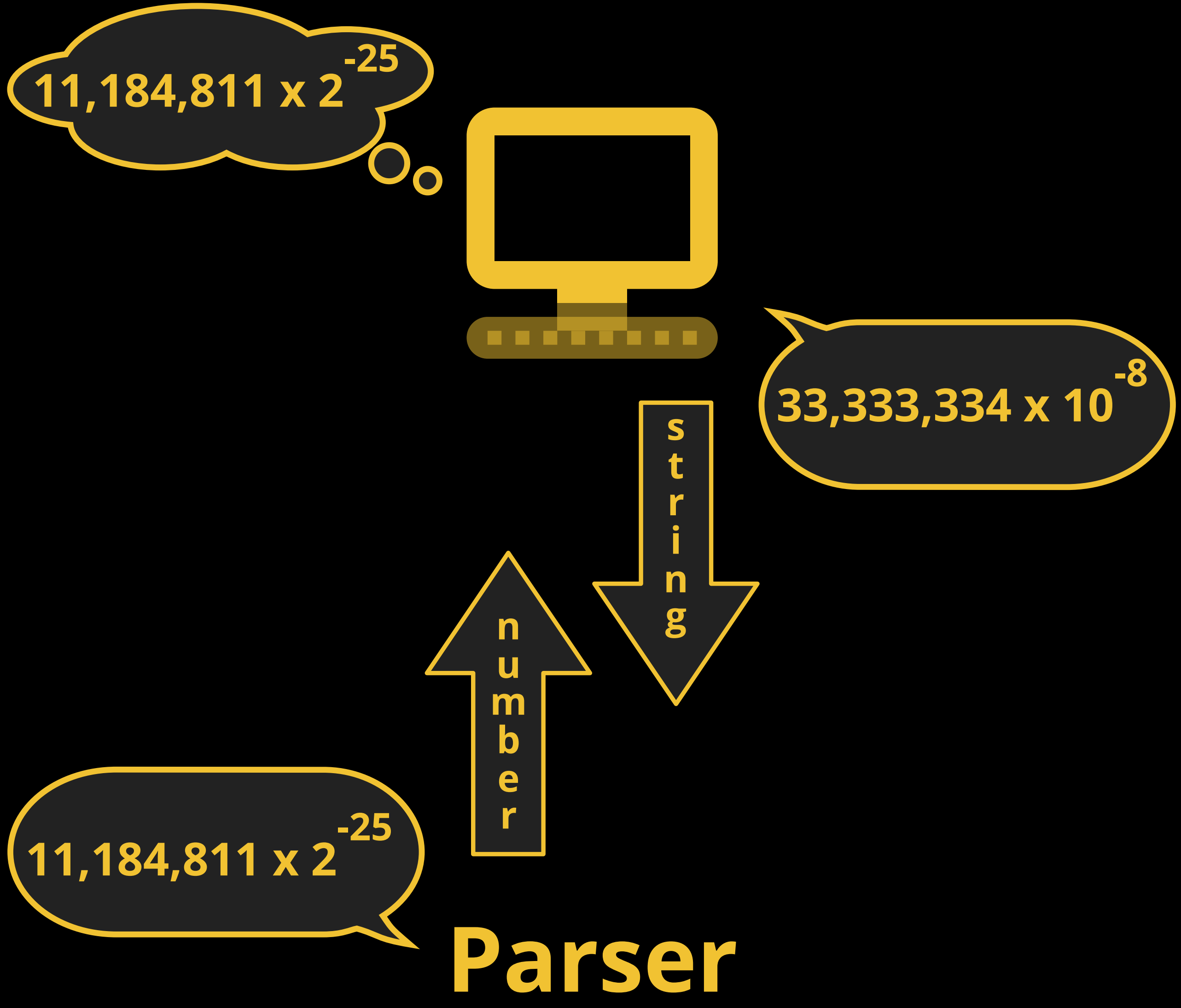


Parser

https://godbolt.org/z/9P6s4zK49

No information loss



No information loss

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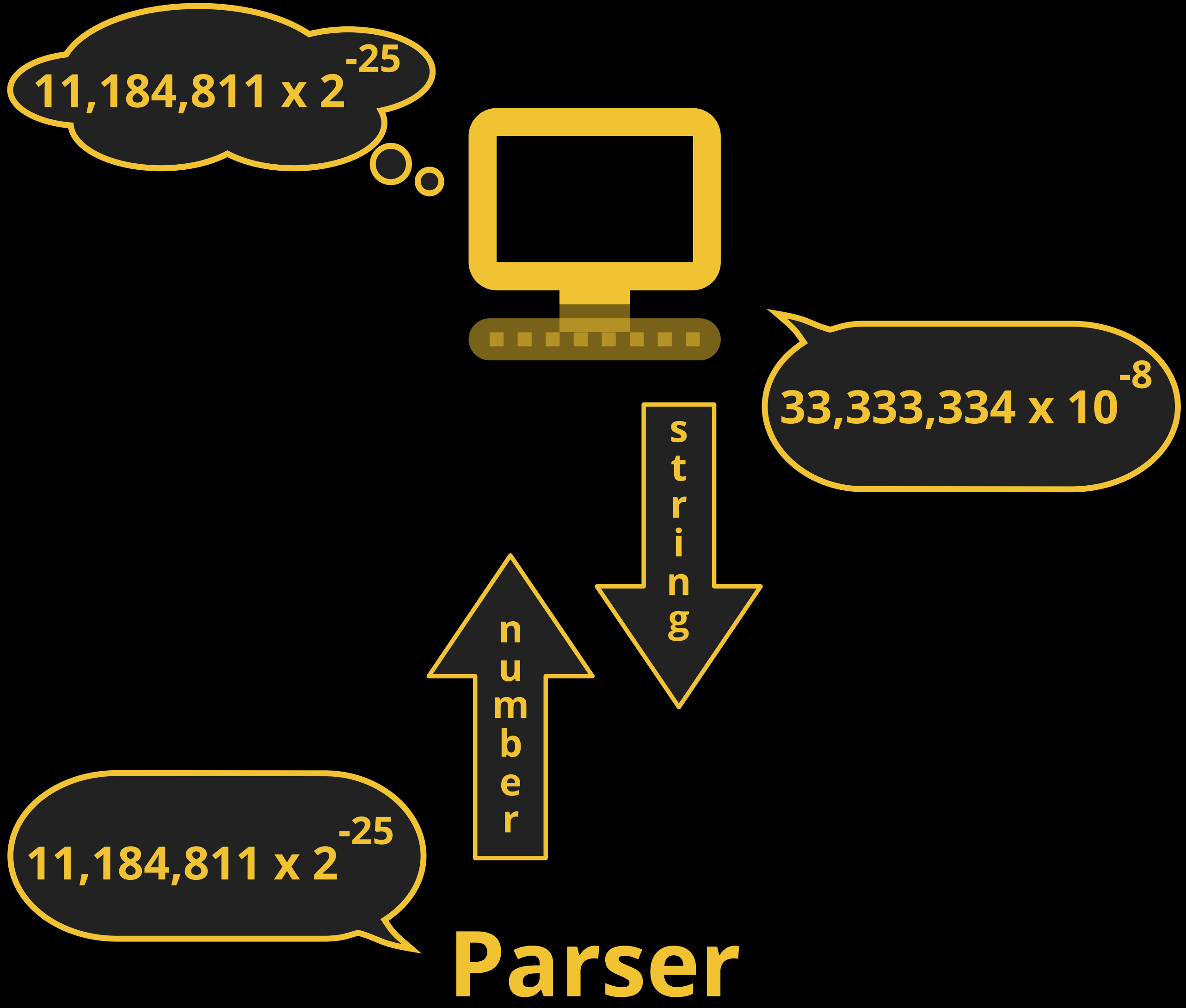

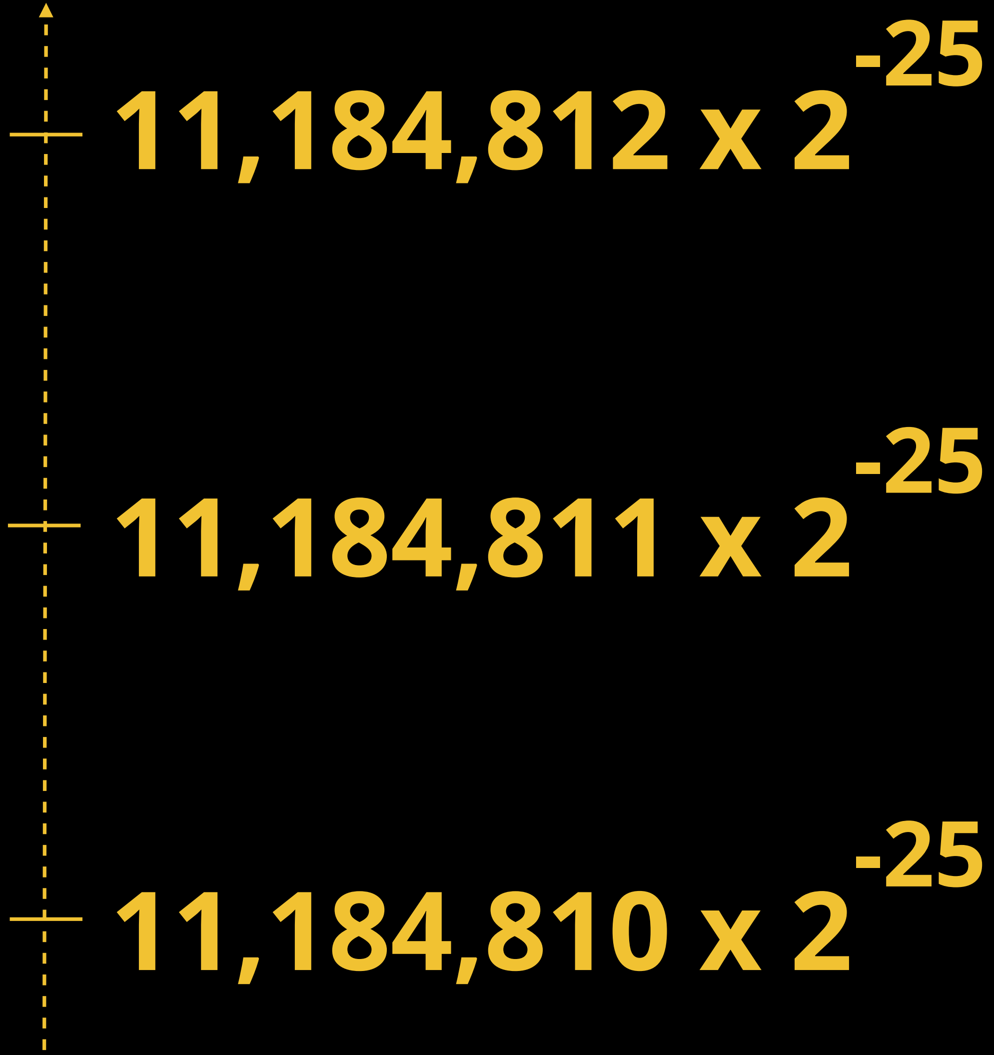

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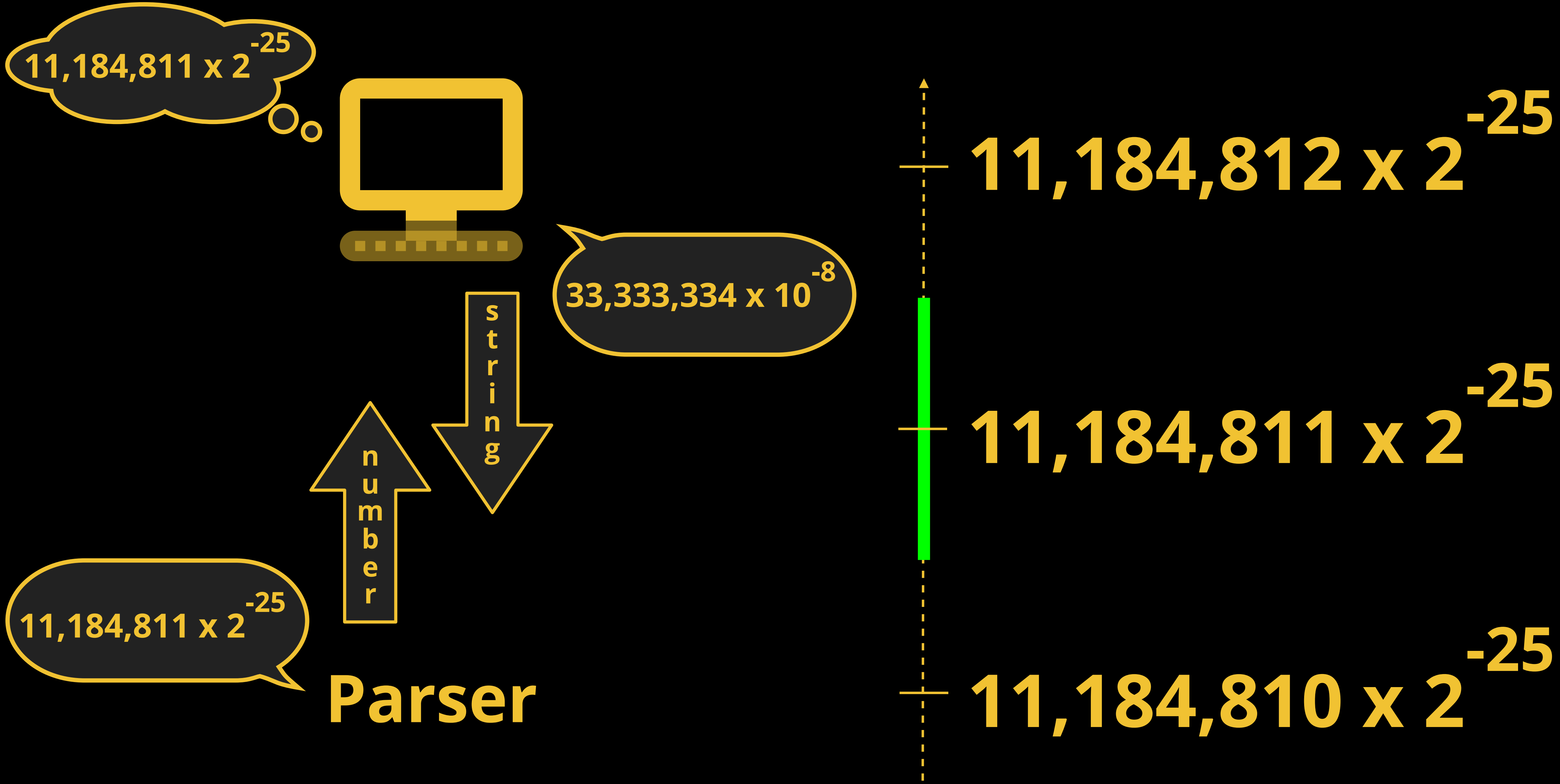

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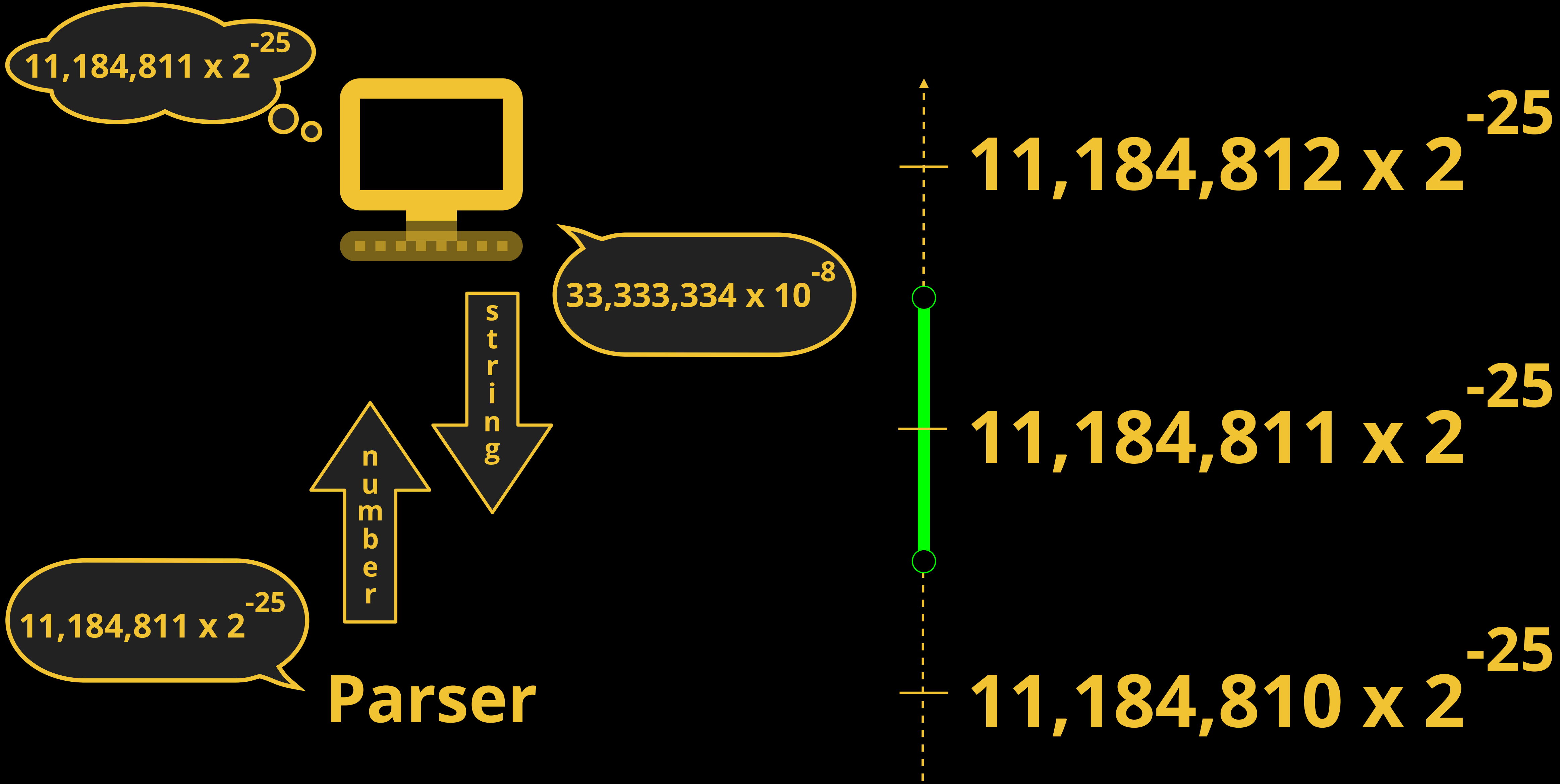

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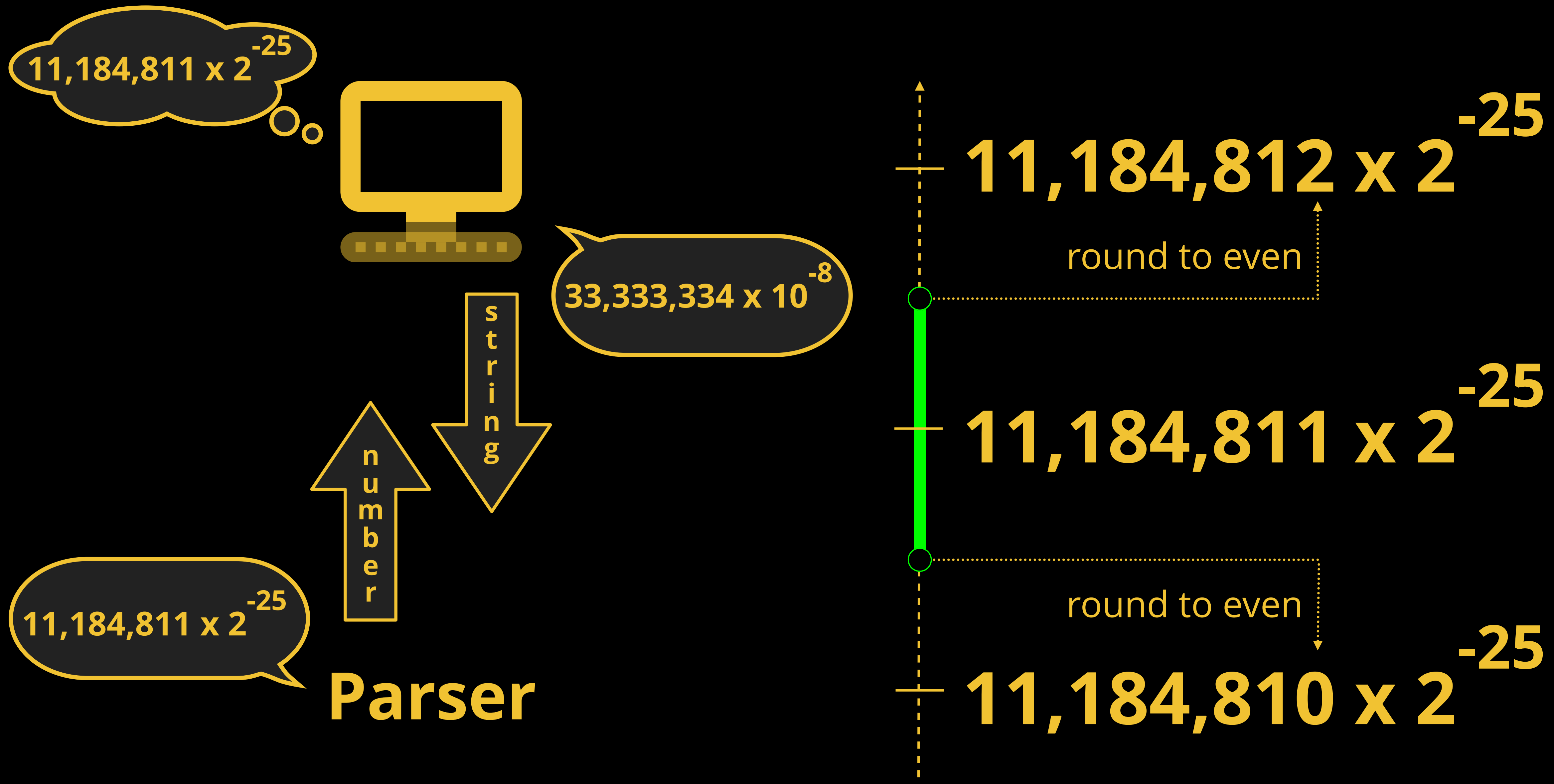

As short as possible

https://godbolt.org/z/3czzTzsTs

$$9{,}765{,}625 \times 2^{10} = 10 \times 10^{9}$$

$$= 1 \times 10^{10}$$

As short as possible

https://godbolt.org/z/3czzTzsTs

$$\boxed{9{,}765{,}625 \times 2^{10}} = 10 \times 10^{9}$$

$$= \boxed{1 \times 10^{10}}$$

Only the mantissa matters.

As short as possible

https://godbolt.org/z/5xKzvfzfx

$$13{,}421{,}773 \times 2^{-27} \cong 100 \times 10^{-3}$$

$$\cong 10 \times 10^{-2}$$

$$\cong 1 \times 10^{-1}$$

As short as possible

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$$13{,}421{,}773 \times 2^{-27} \cong 100 \times 10^{-3}$$

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only representations of the form $n \times 10^{F}$ matter

As short as possible

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$\cong$ 

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$$\cong$$

only representations of the form $n \times 10^{F}$ matter

the larger F,

the shorter n

https://godbolt.org/z/64ro6669j

As short as possible

$16{,}570{,}089 \times 2^{-27}$

$123{,}456{,}784 \times 10^{-9}$

$123{,}456{,}782 \times 10^{-9}$

$123{,}456{,}780 \times 10^{-9}$

As short as possible

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$$12{,}345{,}678 \times 10^{-8}$$

mantissa is multiple of 10
$\Longrightarrow$ we can do better

As short as possible

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mantissa is multiple of 10

$\Longrightarrow$ we can do better

https://godbolt.org/z/1hPP6hK6j

As short as possible

$30{,}000{,}002 \times 10^{-8}$

$10{,}066{,}330 \times 2^{-25}$

$30{,}000{,}001 \times 10^{-8}$

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As short as possible

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$$10{,}066{,}330 \times 2^{-25}$$

$$30{,}000{,}002 \times 10^{-8}$$

$$30{,}000{,}001 \times 10^{-8}$$

$$30{,}000{,}000 \times 10^{-8}$$

$$3 \times 10^{-1}$$

mantissa is multiple of 10

$\Longrightarrow$ we can do better

As short as possible

https://godbolt.org/z/1hPP6hK6j

$$30{,}000{,}002 \times 10^{-8}$$

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$$30{,}000{,}000 \times 10^{-8}$$

mantissa is multiple of 10

$\Longrightarrow$ we can do better

$$\boxed{3 \times 10^{-1}}$$

Pythagoras

"Leave the roads;

take the trails."

Guaraní people

Guaraní mythology

https://tinyurl.com/75xhk7ef

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1️⃣ God Tupã and goddess Arasy created the first humans, Rupave and Sypave.

Guaraní mythology

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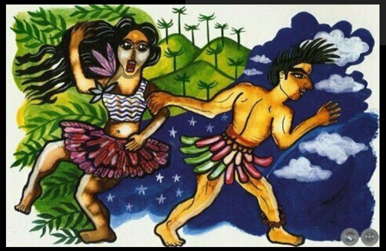

6️⃣ Arasy, placed a curse upon Taú's seven sons, who were born as terrifying beasts.

Guaraní mythology

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6️⃣ Arasy, placed a curse upon Taú's seven sons, who were born as terrifying beasts.

7️⃣ Tejú Jaguá was the first legendary monster.

Tejú Jaguá - I

$$n \times 10^F \cong m \times 2^E$$

Tejú Jaguá - I

$$n \times 10^{F} \cong m \times 2^{E}$$

Tejú Jaguá - I

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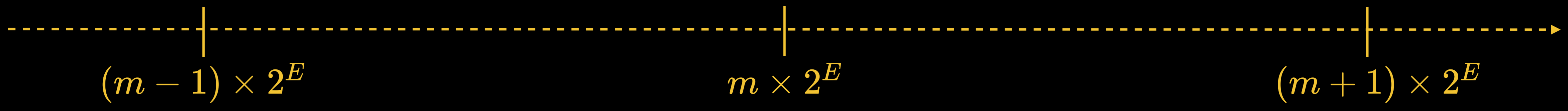

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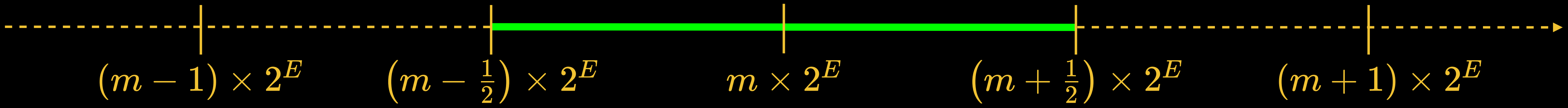

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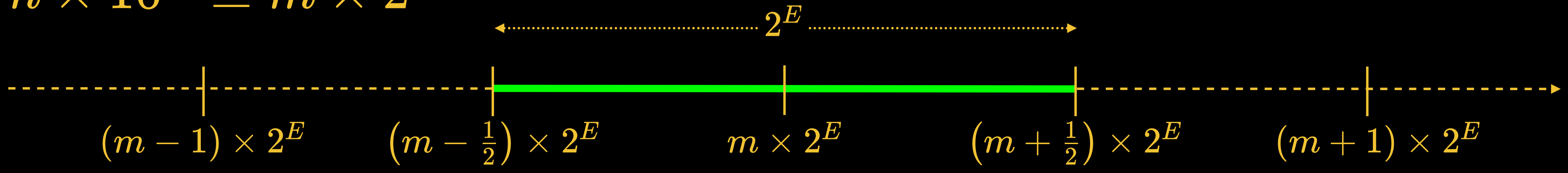

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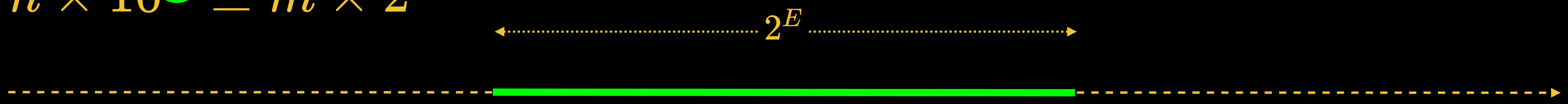

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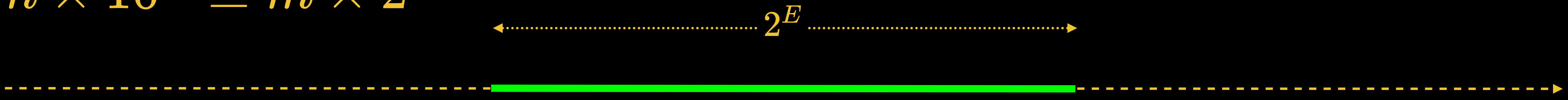

- The larger F, the shorter n.

Tejú Jaguá - I

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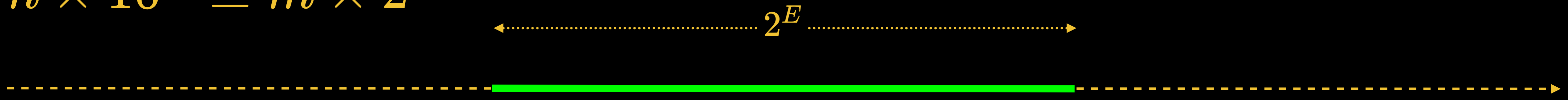
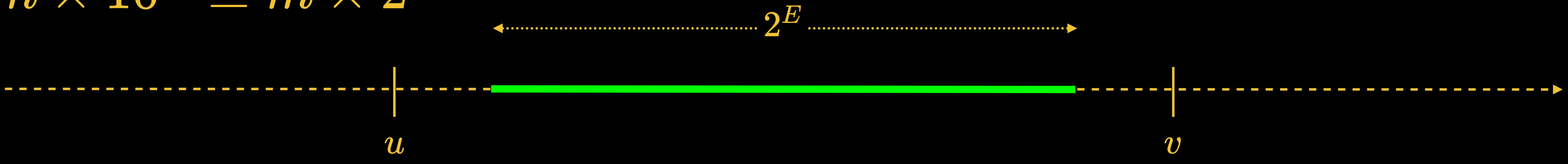


- The larger F, the shorter n.
- We wish to ensure that $n \times 10^F$ falls inside the permissible interval.

Tejú Jaguá - I

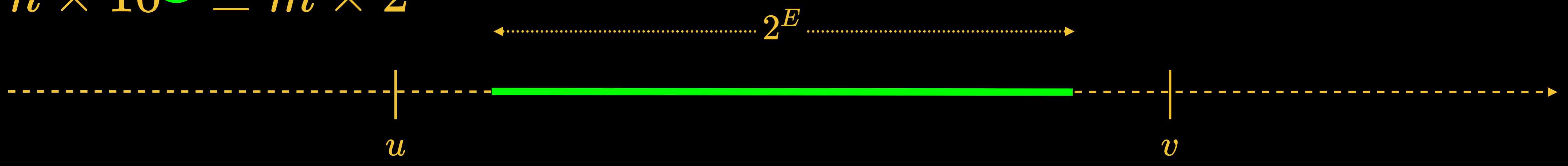
$$n \times 10^{F} \cong m \times 2^{E}$$



- The larger F, the shorter n.
- We wish to ensure that $n \times 10^F$ falls inside the permissible interval.
- If we fail, then two **consecutive** numbers u and v of this form fall outside the permissible interval and thus, $10^F = v - u > 2^E$.

Tejú Jaguá - I

$$n \times 10^{F} \cong m \times 2^{E}$$



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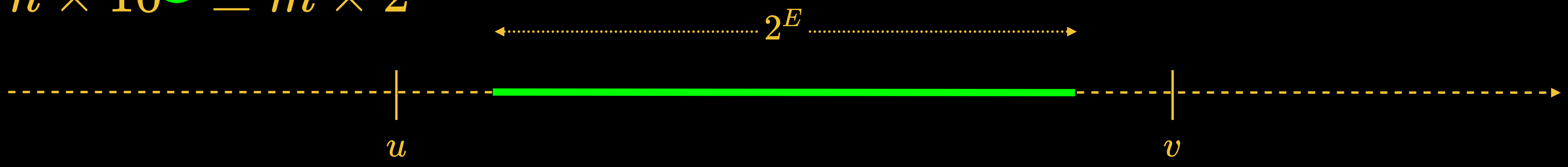

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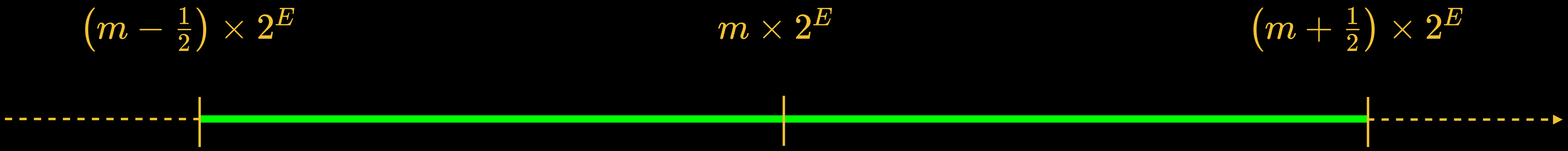
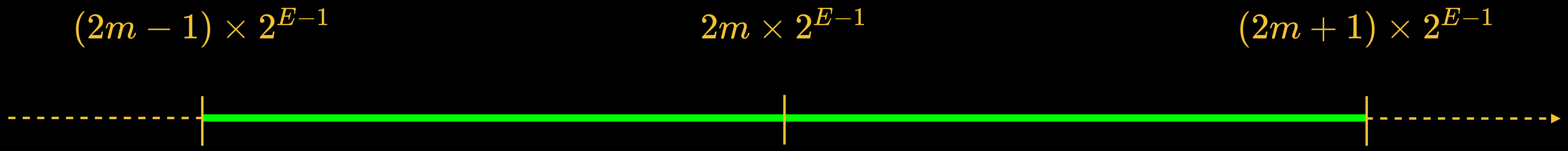
$$10^F \leq 2^E < 10^{F+1} \iff F \leq E \times \log(2) < F + 1 \iff \boxed{F = \lfloor E \times \log(2) \rfloor}$$

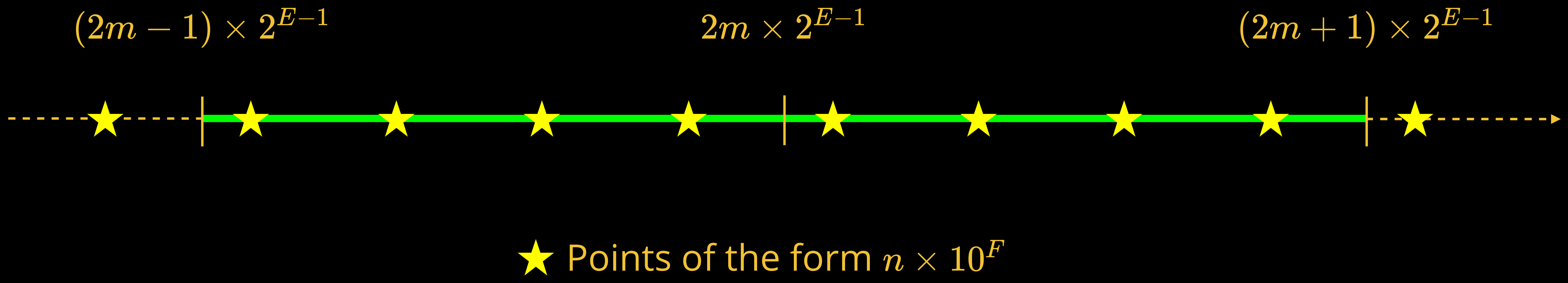
Tejú Jaguá - II



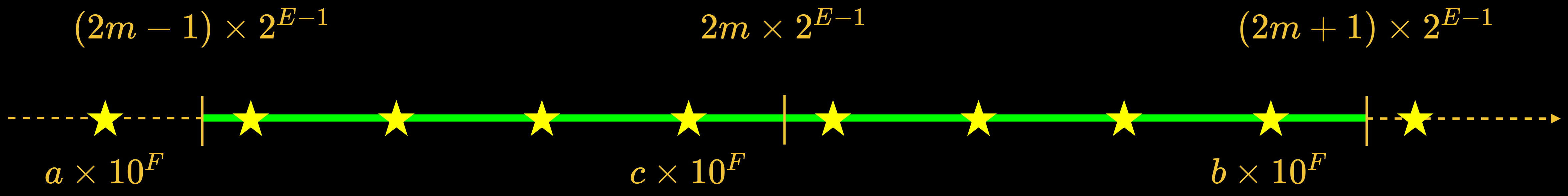
Tejú Jaguá - II



Tejú Jaguá - II

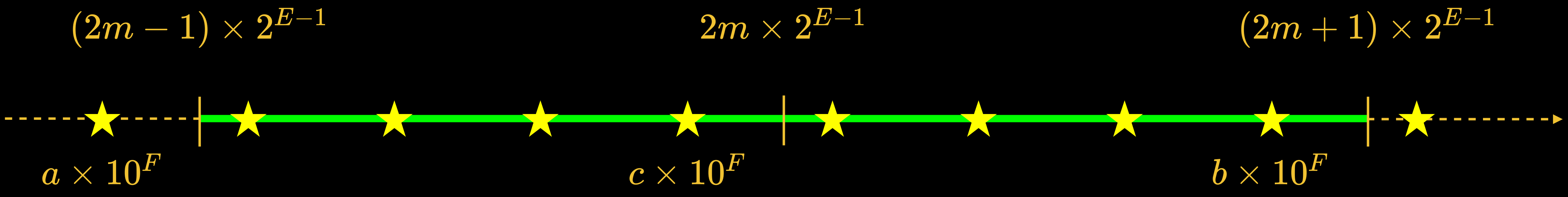


Tejú Jaguá - II



★ Points of the form $n \times 10^F$

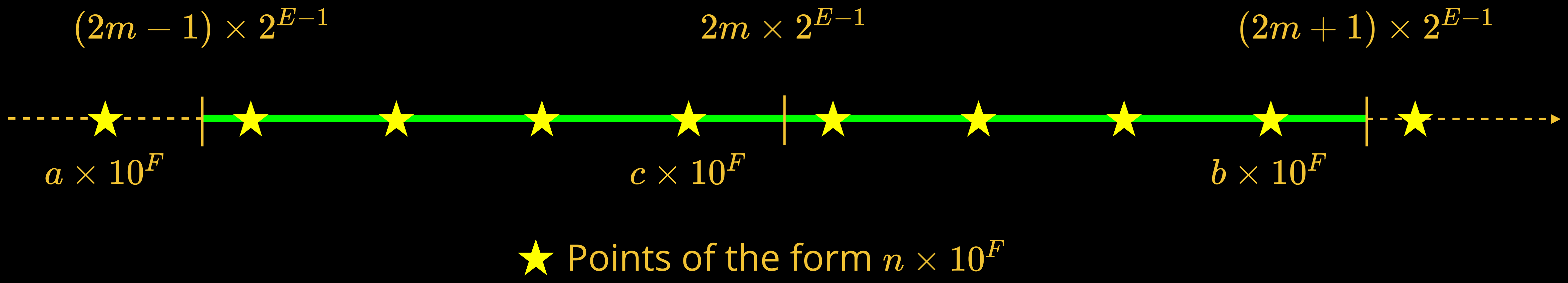
Tejú Jaguá - II



★ Points of the form $n \times 10^F$

$$a \times 10^F \leq (2m-1) \times 2^{E-1} < (a+1) \times 10^F$$

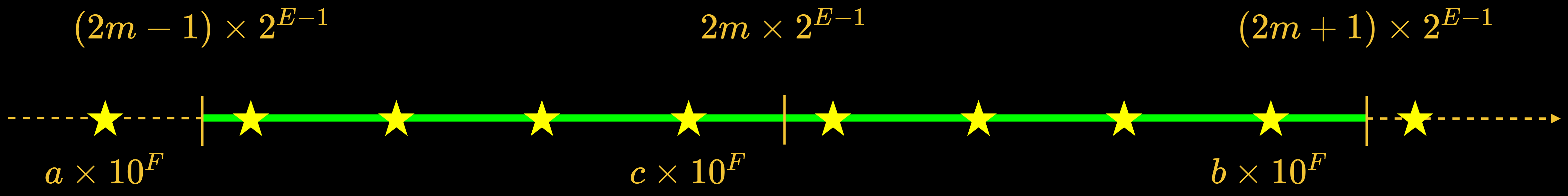
Tejú Jaguá - II



$$a\times 10^F \le (2m-1)\times 2^{E-1} < (a+1)\times 10^F$$

$$a \le (2m-1)\times \frac{2^{E-1}}{10^F} < a+1$$

Tejú Jaguá - II



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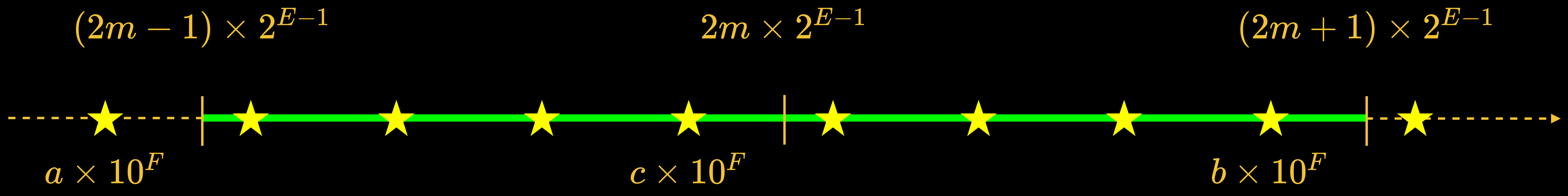
$$a = \left\lfloor (2m-1) \times \frac{2^{E-1}}{10^F} \right\rfloor$$

Tejú Jaguá - II



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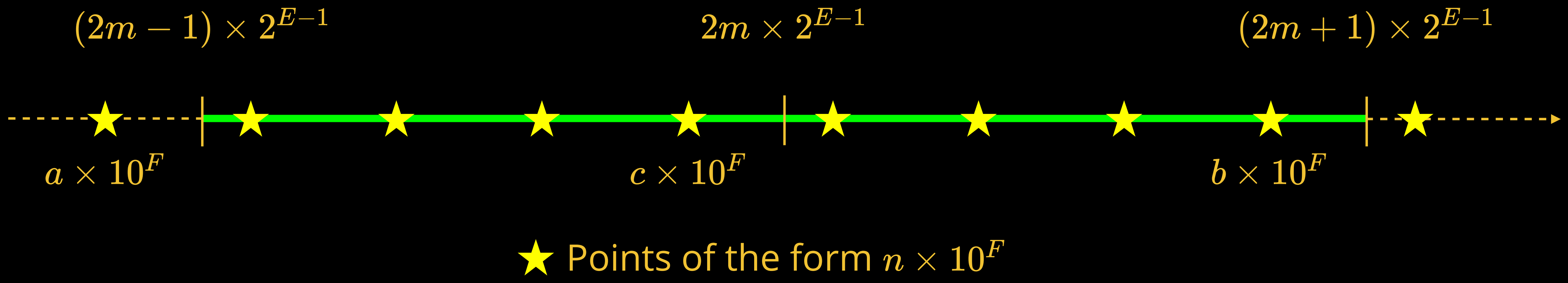
$$a = \left\lfloor (2m-1)\times \frac{2^{E-1}}{10^F} \right\rfloor$$

$$b = \left\lfloor (2m+1)\times \frac{2^{E-1}}{10^F} \right\rfloor$$

Tejú Jaguá - II



$$a\times 10^F \le (2m-1)\times 2^{E-1} < (a+1)\times 10^F$$

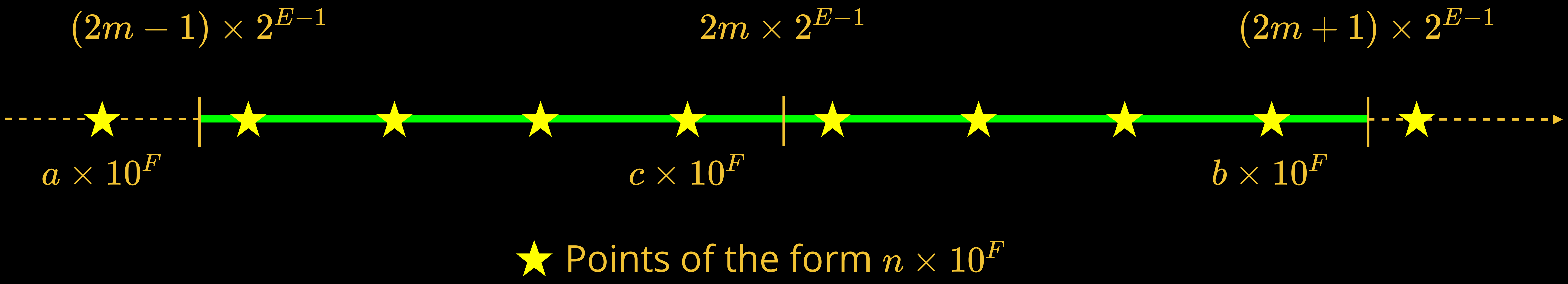
$$b = \left\lfloor (2m+1)\times \frac{2^{E-1}}{10^F} \right\rfloor$$

$$a \le (2m-1)\times \frac{2^{E-1}}{10^F} < a+1$$

$$a = \left\lfloor (2m-1)\times \frac{2^{E-1}}{10^F} \right\rfloor$$

$$c = \left\lfloor 2m\times \frac{2^{E-1}}{10^F} \right\rfloor$$

Tejú Jaguá - II



$$a\times 10^F \le (2m-1)\times 2^{E-1} < (a+1)\times 10^F$$

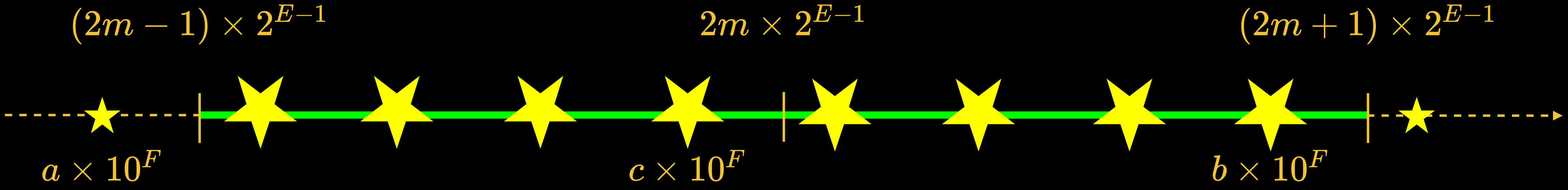
$$a \le (2m-1)\times \frac{2^{E-1}}{10^F} < a+1$$

$$\boxed{a = \left\lfloor (2m-1)\times \frac{2^{E-1}}{10^F} \right\rfloor}$$

$$\boxed{b = \left\lfloor (2m+1)\times \frac{2^{E-1}}{10^F} \right\rfloor}$$

$$\boxed{c = \left\lfloor 2m\times \frac{2^{E-1}}{10^F} \right\rfloor}$$

Tejú Jaguá - III

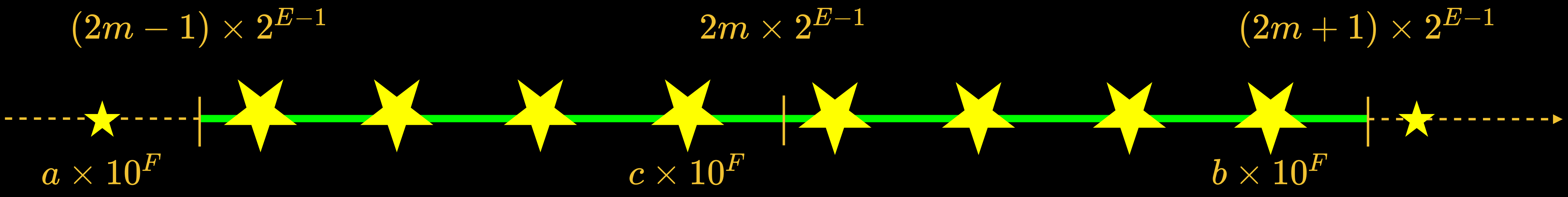


★ Points of the form $n \times 10^F$

Is any of those ★ of the form $s \times 10^F$ where s is a multiple of 10?

If so, then we can do better.

Tejú Jaguá - III



★ Points of the form $n\times 10^F$

Is any of those ★ of the form $s\times 10^F$ where s is a multiple of 10?

🤔 If so, then we can do better.

A good **candidate** is $s = 10\left\lfloor \frac{b}{10} \right\rfloor$.

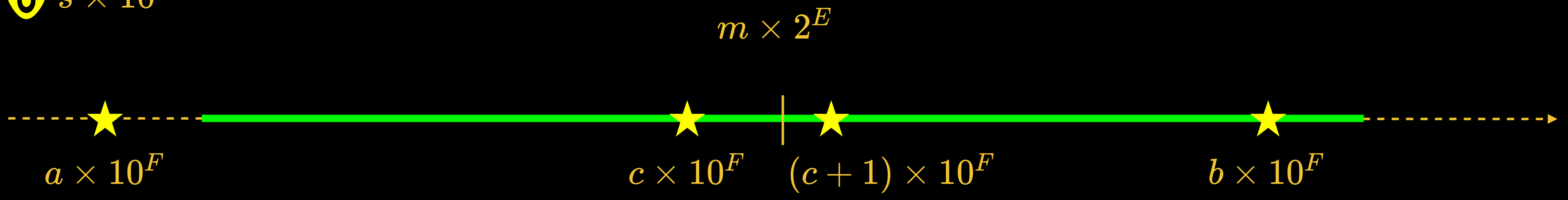
Return

- the shortest[*], $s \times 10^F$, if you **can**; or
- the closest, $c \times 10^F$ or $(c+1) \times 10^F$, if you **must**.

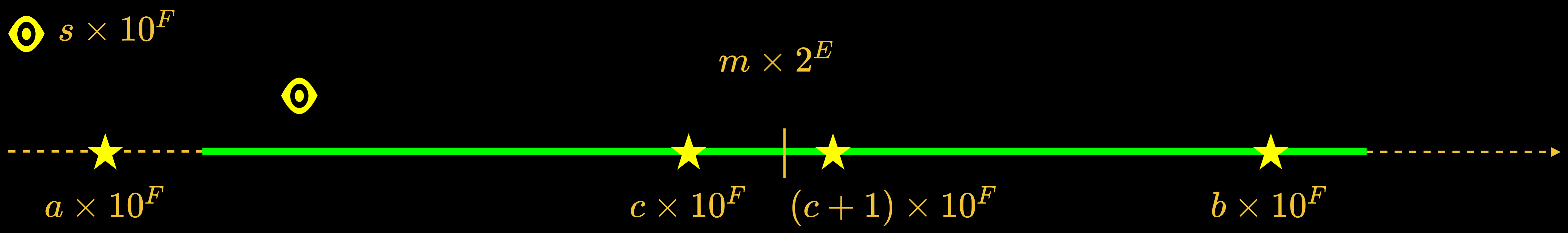
(*) After removing trailing zeros and adjusting exponent.

Tejú Jaguá - V

$s \times 10^F$



Tejú Jaguá - V



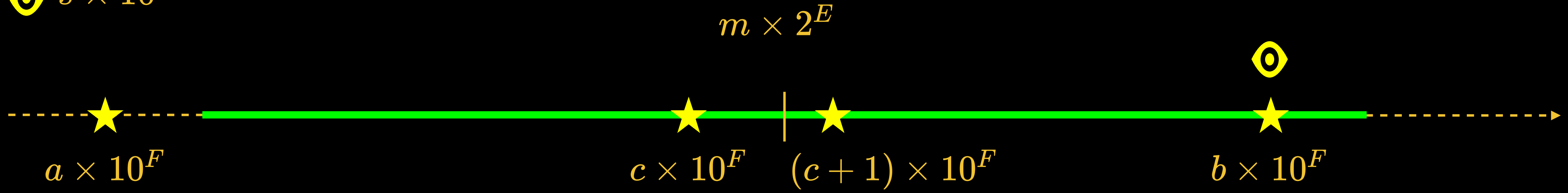
if $a < s$

  if $s < b$

    return the shortest, $s \times 10^F$

Tejú Jaguá - V

$s \times 10^F$



if $a < s$

if $s < b$

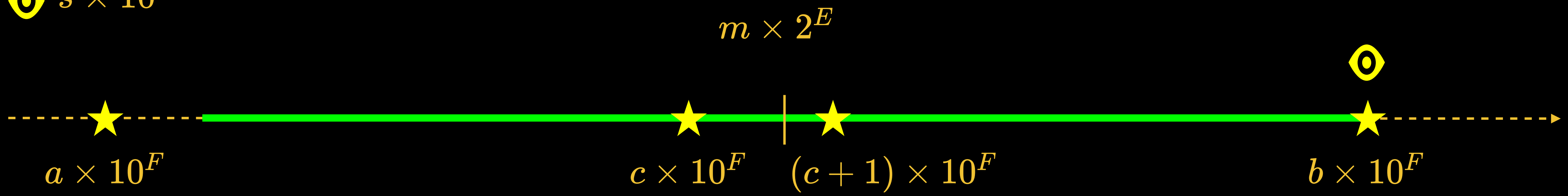
return the shortest, $s \times 10^F$

if b is not a tie

return the shortest, $s \times 10^F$

Tejú Jaguá - V

$s \times 10^F$



if $a < s$

if $s < b$

return the shortest, $s \times 10^F$

if b is not a tie

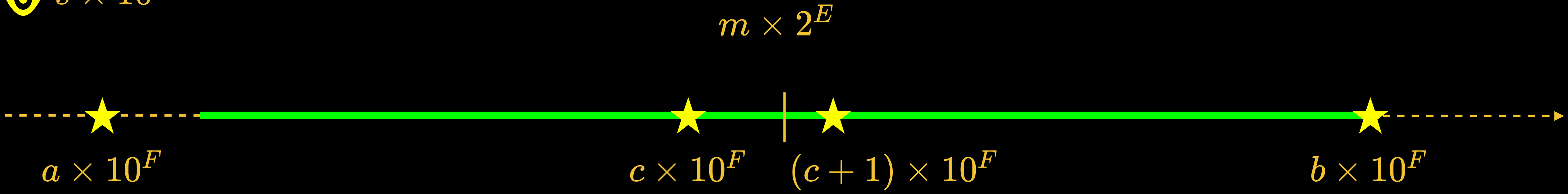
return the shortest, $s \times 10^F$

if m wins against $m + 1$

return the shortest, $s \times 10^F$

Tejú Jaguá - V

$s \times 10^F$



if $a < s$

    if $s < b$

        return the shortest, $s \times 10^F$

    if b is not a tie

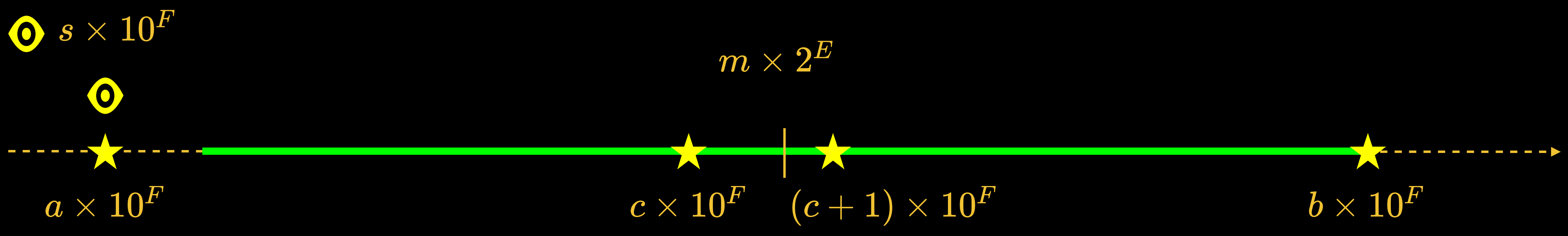
        return the shortest, $s \times 10^F$

    if m wins against $m + 1$

        return the shortest, $s \times 10^F$

    return the closest, $c \times 10^F$ or $(c+1) \times 10^F$

Tejú Jaguá - V



if $a < s$

if $s < b$

return the shortest, $s \times 10^F$

if b is not a tie

return the shortest, $s \times 10^F$

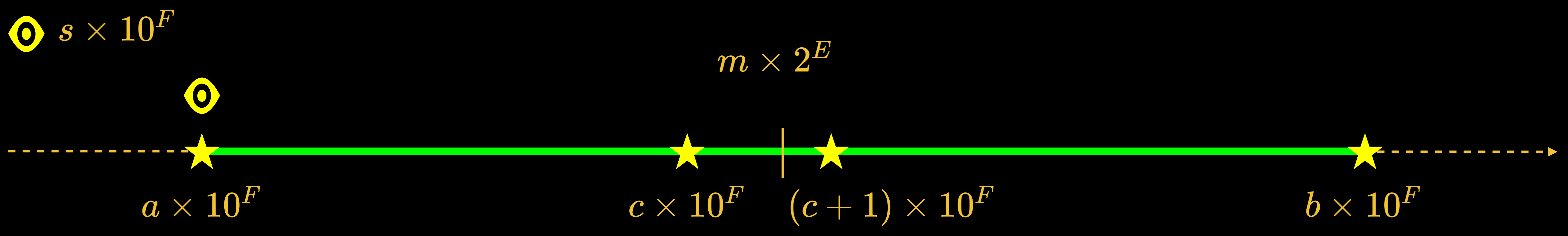
if m wins against $m + 1$

return the shortest, $s \times 10^F$

return the closest, $c \times 10^F$ or $(c + 1) \times 10^F$

if $s = a$

Tejú Jaguá - V



if $a < s$

  if $s < b$

    return the shortest, $s \times 10^F$

  if b is not a tie

    return the shortest, $s \times 10^F$

  if m wins against $m + 1$

    return the shortest, $s \times 10^F$

  return the closest, $c \times 10^F$ or $(c+1) \times 10^F$

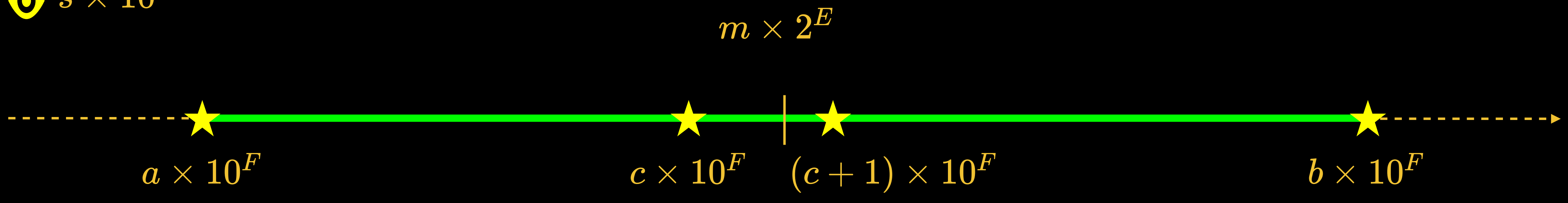
if $s = a$

  if a is a tie and m wins against $m - 1$

    return the shortest, $s \times 10^F$

Tejú Jaguá - V

$s \times 10^F$



if $a < s$

- if $s < b$
  - return the shortest, $s \times 10^F$
- if b is not a tie
  - return the shortest, $s \times 10^F$
- if m wins against $m+1$
  - return the shortest, $s \times 10^F$
- return the closest, $c \times 10^F$ or $(c+1) \times 10^F$

if $s = a$

- if a is a tie and m wins against $m-1$
  - return the shortest, $s \times 10^F$

return the closest, $c \times 10^F$ or $(c+1) \times 10^F$

Practicalities

$$F = \lfloor E \times \log(2) \rfloor = \lfloor E \times 0.301... \rfloor$$

$$\boxed{F = \left\lfloor \frac{1,292,913,987 \times E}{2^{32}} \right\rfloor}$$

Valid $\forall E \in [-112,815\ ,\ 112,815]$

Practicalities

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Valid $\forall E \in [-112,815\ ,\ 112,815]$

$$a = \left\lfloor M \times \frac{2^{E-1}}{10^F} \right\rfloor$$

$$\boxed{a = \left\lfloor \frac{M \times M(E)}{2^K} \right\rfloor}$$

K = twice the size of floating-point type

$M(E) \cong 2^K \times \dfrac{2^{E-1}}{10^F}$ can be precomputed and stored in a look-up table

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Detecting ties becomes a divisibility check by 5^F using the *minverse* algorithm

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Optimisation for c

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Detecting ties becomes a divisibility check by 5^F using the *minverse* algorithm

Optimisation for integers

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Optimisation for c

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Valid $\forall E \in [-112,815\ ,\ 112,815]$

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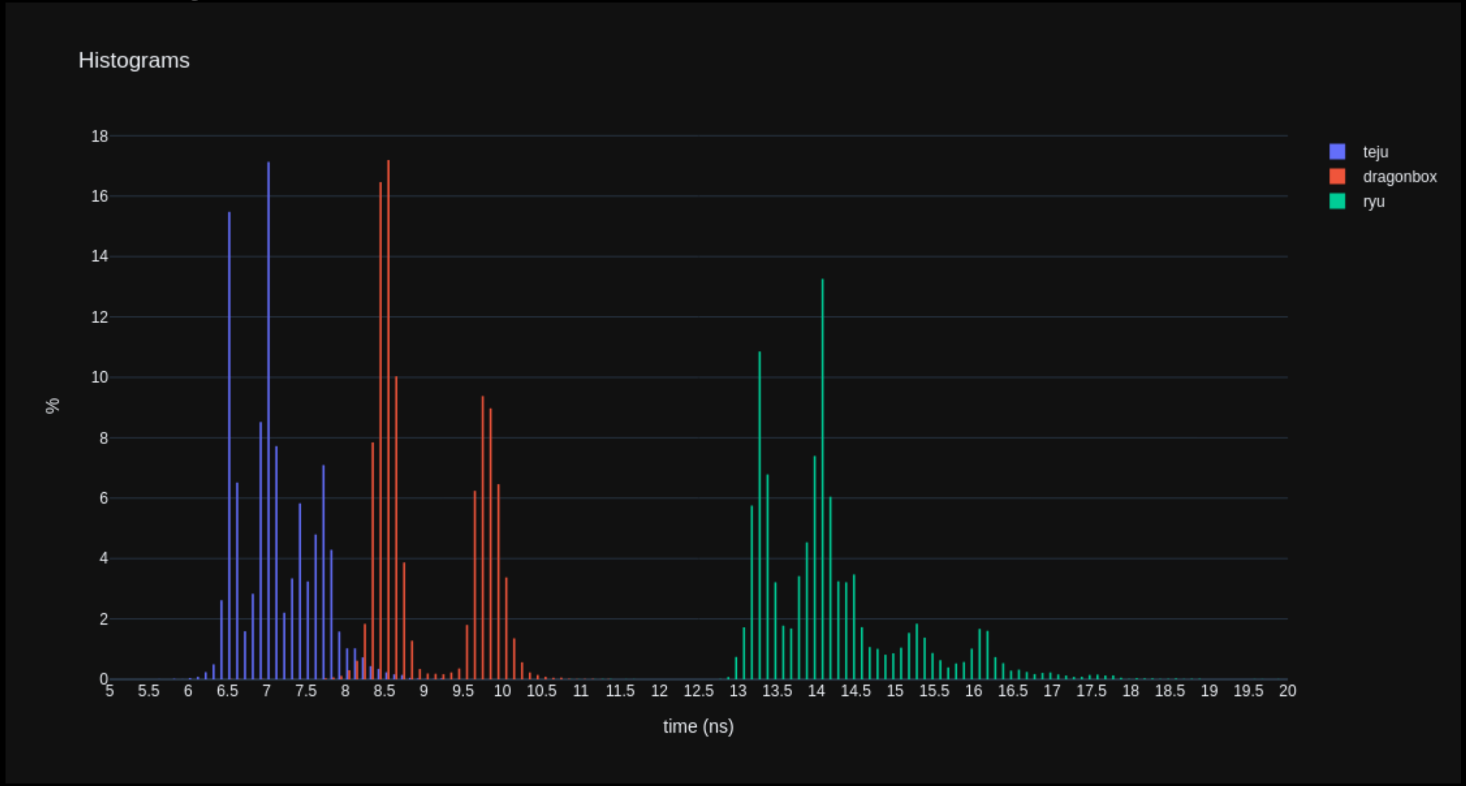
Optimisation for c

Other tricks

Benchmarks (centred)

Intel i7 (10510U) - gcc 13.2.1



teju x dragonbox	
wins	99.5%
ties	0.0%
losses	0.5%

teju x ryu	
wins	100.0%
ties	0.0%
losses	0.0%

	mean	σ	min	median	max	relative
teju	7.14	0.50	2.84	7.06	9.63	1.00
dragonbox	9.04	0.66	6.52	8.64	11.39	1.27
ryu	14.16	0.98	3.40	14.01	20.50	1.98

Benchmarks (uncentred)

Intel i7 (10510U) - gcc 13.2.1

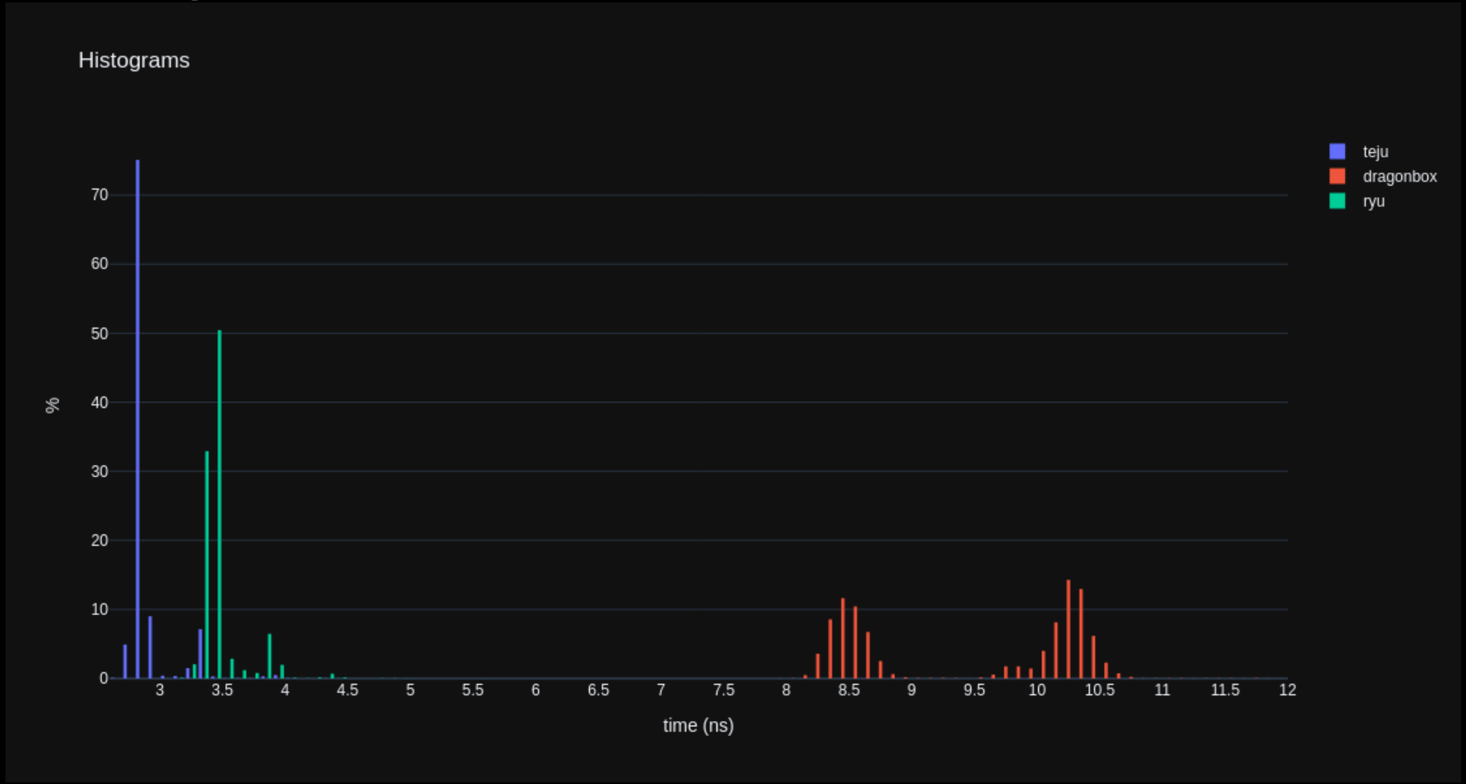
teju x dragonbox	
wins	38.2%
ties	0.0%
losses	61.8%

teju x ryu	
wins	100.0%
ties	0.0%
losses	0.0%

	mean	σ	min	median	max	relative
teju	6.78	0.95	2.79	6.79	17.69	1.00
dragonbox	6.72	0.47	5.95	6.53	8.69	0.99
ryu	13.76	2.79	3.34	13.58	45.12	2.03

Benchmarks (integers)

Intel i7 (10510U) - gcc 13.2.1



teju x dragonbox	
wins	100.0%
ties	0.0%
losses	0.0%

teju x ryu	
wins	100.0%
ties	0.0%
losses	0.0%

	mean	σ	min	median	max	relative
teju	2.92	0.18	2.61	2.86	5.05	1.00
dragonbox	9.46	0.90	7.92	10.00	12.58	3.25
ryu	3.47	0.18	3.13	3.42	5.51	1.19

Joseph Campbell

“The very cave you are afraid to enter turns out to be the source of what you are looking for.”

Tejú Jaguá

Tejú Jaguá

Thank you

Cristina Acosta
Lorenz Schneider
Manuel Caicoya
Victor Bogado