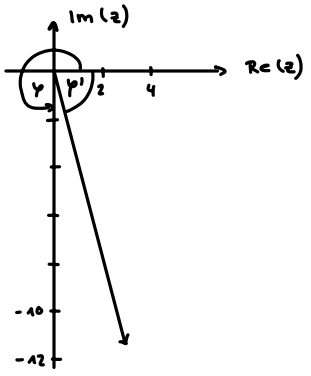
Aufgabe 1:

$z_N = x + iy$

$z_E = r \cdot e^{i\varphi}$

$z_T = r \cdot (\cos(\varphi) + i \cdot \sin(\varphi))$

a) $z = 3 - 11i$



$r = \sqrt{11^2 + 3^2} = \sqrt{130} \sim 11.402$

$\varphi = 2\pi - \tan^{-1}\left(\frac{11}{3}\right) = 4.9786 \sim 316.95°$

$z_E = 11.402 \cdot e^{i \cdot 4.9786}$

$z_T = 11.402 \cdot (\cos(4.9786) + i \cdot \sin(4.9786))$

$z_N^* = 3 + 11i \qquad \varphi = \tan^{-1}\left(\frac{11}{3}\right)$

$z_E^* = 11.402 \cdot e^{i \cdot (-4.9786)} = 11.402 \cdot e^{i \cdot (-4.9786 + 2\pi)} = 11.402 \cdot e^{i \cdot 1.3045} = 11.402 \cdot e^{i(\tan^{-1}(\frac{11}{3}))} = 11.402 \cdot e^{i \cdot 74.7°}$

$z_T^* = 11.402 \cdot (\cos(1.3045) + i \cdot \sin(1.3045))$

b) $z = 4(\cos(-40°) + i \cdot \sin(-40°)) + 2e^{i \cdot 30°} - 3 + 1.5i$

$= 3.064 - 2.751 \cdot i + 2 \cdot \cos(30°) + 2 \cdot i \cdot \sin(30°) - 3 + 1.5i$

$= 3.064 - 2.751 \cdot i + 1.732 + i - 3 + 1.5 \cdot i$

$= \underline{\underline{1.796 - 0.071 \cdot i}}$

$z^* = \underline{\underline{1.796 + 0.071 \cdot i}}$

c) $z_1 = \frac{2+i}{1-2i} = \frac{(2+i)\cdot(1+2i)}{(1-2i)\cdot(1+2i)} = \frac{2+5i+2i^2}{1-4i^2} = \frac{2+5i-2}{1+4} = \frac{5i}{5} = i$

$z_1^* = -i \qquad r = \sqrt{0^2 + 1^2} = 1, \quad \varphi = \frac{3\pi}{2} \qquad z_{1,E}^* = 1 \cdot e^{i \cdot \frac{3\pi}{2}}$

$z_3 = 4 \cdot (\cos(30°) + i \cdot \sin(30°)) \Rightarrow r = 4, \quad \varphi = 30° = \frac{\pi}{6} \qquad z_{3,E} = 4 \cdot e^{i \cdot \frac{\pi}{6}}$

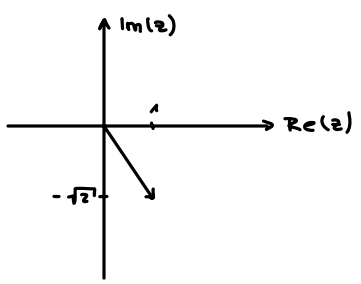
$z_{2,E} = 2 \cdot e^{-i \cdot \frac{\pi}{3}}, \qquad 0.5 \cdot z_2 = e^{-i\frac{\pi}{3}}$

$\frac{z_1^* \cdot z_3}{0.5 \cdot z_2} = \frac{e^{i \cdot \frac{3\pi}{2}} \cdot 4e^{i\pi/6}}{e^{-i\pi/3}} = e^{i \cdot \frac{3\pi}{2}} \cdot 4e^{i\pi/6} \cdot e^{i\pi/3}$

$= e^{i\,5\pi/6} \cdot 4e^{i\pi/6} \cdot e^{i\,2\pi/6}$

$= 4e^{i \cdot 2\pi} = 4 \cdot e^{i \cdot 0} = 4 \cdot 1 = \underline{\underline{4}}$

d) $(1 - \sqrt{2}i)^3$



$\varphi = 2\pi - \tan^{-1}\left(\frac{\sqrt{2}}{1}\right) \sim 5.3279$

$r = \sqrt{2 + 1^2} = \sqrt{3}$

$z_E = \sqrt{3} \cdot e^{i \cdot 5.3279}$

$z_E^3 = \sqrt{3}^3 \cdot e^{i \cdot 3 \cdot 5.3279} = \sqrt{3}^3 \cdot e^{i \cdot 15.9837} = \sqrt{3}^3 \cdot e^{i \cdot (15.9837 - 4\pi)}$

$= \sqrt{3}^3 \cdot e^{i \cdot 3.4173} = 3 \cdot \sqrt{3}\,(\cos(3.4173) + i \cdot \sin(3.4173))$

$= \underline{\underline{-5 - 1.415 \cdot i}}$