1. a)

$$A = \begin{pmatrix} 20 & 10 & 0 \\ 50 & 30 & 20 \\ 200 & 150 & 100 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad b = \begin{pmatrix} 150 \\ 470 \\ 2150 \end{pmatrix}$$

LR-Zerlegung:

$a_{ij}$

$0 = i$

$1 = j$

$$A = \left(\begin{array}{ccc|c} 20 & 10 & 0 & 150 \\ 0 & 5 & 20 & 95 \\ 200 & 150 & 100 & 2150 \end{array}\right)$$

$\frac{50}{20} = 2.5$

$50 - 2.5 \cdot 20 = 0$

$30 - 2.5 \cdot 10 = 5$

$20 - 2.5 \cdot 0 = 20$

$470 - 2.5 \cdot 150 = 95$

$$A = \left(\begin{array}{ccc|c} 20 & 10 & 0 & 150 \\ 0 & 5 & 20 & 95 \\ 0 & 50 & 100 & 650 \end{array}\right)$$

$\frac{200}{20} = 10$

$200 - 10 \cdot 20 = 0$

$150 - 10 \cdot 10 = 50$

$100 - 10 \cdot 0 = 100$

$2150 - 10 \cdot 150 = 650$

$x_1 \quad x_2 \quad x_3$

$$A = \left(\begin{array}{ccc|c} 20 & 10 & 0 & 150 \\ 0 & 5 & 20 & 95 \\ 0 & 0 & -100 & -300 \end{array}\right)$$

$\frac{50}{5} = 10$

$50 - 10 \cdot 5 = 0$

$100 - 10 \cdot 20 = -100$

$650 - 10 \cdot 95 = -300$

R

$x_3 = 3$

$95 = 5x_2 + 60 \quad | -60$

$35 = 5x_2 \quad | : 5$

$7 = x_2$

$150 = 20x_1 + 10 \cdot 7 + 0 \cdot 3$

$150 = 20x_1 + 70 \quad | -70$

$80 = 20x_1 \quad | : 20$

$4 = x_1$

Es werden 4 Flüge des Typs A, 7 des Typs B und 3 des Typs C benötigt.

b)

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2.5 & 1 & 0 \\ 10 & 10 & 1 \end{pmatrix} \qquad b_2 = \begin{pmatrix} 120 \\ 350 \\ 1600 \end{pmatrix} \rightarrow b_2 = \begin{pmatrix} 120 \\ 50 \\ -100 \end{pmatrix}$$

$$Z_2 = Z_2 - \frac{50}{20} \cdot Z_1 \rightarrow 350 = 350 - \frac{50}{20} \cdot 120 = 50$$

$$Z_3 = Z_3 - \frac{200}{20} \cdot Z_1 \rightarrow 1600 = 1600 - \frac{200}{20} \cdot 120 = 400$$

$$Z_3 = Z_3 - \frac{50}{5} \cdot Z_2 \rightarrow 400 = 400 - \frac{50}{5} \cdot 50 = -100$$

$$A = \left(\begin{array}{ccc|c} 20 & 10 & 0 & 120 \\ 0 & 5 & 20 & 50 \\ 0 & 0 & -100 & -100 \end{array}\right)$$

$x_3 = 1$

$5x_2 = 50 - 20x_3$

$5x_2 = 30 \mid :5$

$x_2 = 6$

$20x_1 = 120 - 10x_2$

$20x_1 = 120 - 10 \cdot 6$

$20x_1 = 120 - 60$

$20x_1 = 60 \mid :20$

$x_1 = 3$

Es werden 3 Flüge des Typs A, 6 des Typs B und 1 des Typs C benötigt.