1. a) $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

$F(x): \quad 230x^4 + 18x^3 + 9x^2 - 221x - 9 = 0 \quad | + 221x$

$230x^4 + 18x^3 + 9x^2 - 9 = 221x \quad | : 221$

$$\frac{230x^4 + 18x^3 + 9x^2 - 9}{221} = x$$

Startwert: $x_0 = 0 \rightarrow \frac{230 \cdot 0^4 + 18 \cdot 0^3 + 9 \cdot 0^2 - 9}{221} = \frac{-9}{221} =$

-0.040723

$x_1 = -0.040723 \rightarrow \frac{230 \cdot x_1^4 + 18 \cdot x_1^3 + 9 \cdot x_1^2 - 9}{221}$

$= -0.040659$

$x_2 = -0.040659 \rightarrow \frac{230 \cdot x_2^4 + 18 \cdot x_2^3 + 9 \cdot x_2^2 - 9}{221}$

$= -0.040659 \rightarrow$ Konvergiert

Startwerd: $x_0 = 1 \rightarrow \frac{230 \cdot 1^4 + 18 \cdot 1^3 + 9 \cdot 1^2 - 9}{221} = 1.122172$

$x_1 = 1.122172 \rightarrow \frac{230 \cdot x_1^4 + 18 \cdot x_1^3 + 9 \cdot x_1^2 - 9}{221}$

$= 1.77599$

$x_2 = 1.77599 \rightarrow \frac{230 \cdot x_2^4 + 18 \cdot x_2^3 + 9 \cdot x_2^2 - 9}{221}$

$= 10.81000 \rightarrow$ Divergiert

Zeigen das F(x) divergiert bei 1:

$$F(x) = \frac{230x^4 + 18x^3 + 9x^2 - 9}{221}$$

$$F'(x) = 4 \cdot \left(\frac{230}{221}\right)x^3 + 3\left(\frac{18}{221}\right)x^2 + 2\left(\frac{9}{221}\right)x$$

$$= \frac{920x^3 + 54x^2 + 18x}{221}$$

$$F'(0.9) = \frac{920 \cdot 0.9^3 + 54 \cdot 0.9^2 + 18 \cdot 0.9}{221} = 3.3059 > 1 \checkmark$$

-> Fixpunktgleichung divergiert

b) Banach!

Wenn

i) $F: [a,b] \to [a,b]$

ii) $\alpha \leq |F'(x_0)|$

Dann $x_0 \in [a,b]$

$0 < \alpha < 1$

$F: [-0.5, 0.5] \to [-0.5, 0.5]$

$$F(x) = \frac{230x^4 + 18x^3 + 9x^2 - 9}{221} = x$$

$F(-0.5) = 0.024$ ✓

$F(0) = -0.040724$ ✓

$F(0.5) = 0.0447$ ✓

Im Intervall

$$F'(x) = \frac{920x^3 + 54x^2 + 18x}{221} = x$$

$F'(0.5) = 0.6222 = \alpha$

$0 < \alpha < 1$ ✓

c) $|x_n - \bar{x}| \leq \frac{\alpha^n}{1-\alpha} \cdot (x_1 - x_0) \leq 10^{-9}$

$$\frac{0.6222^n}{1 - 0.6222} \cdot (-0.040723 - 0) \leq 10^{-9}$$

$$\frac{0.6222^n}{0.378} \cdot 0.040723 \leq 10^{-9} \quad | \cdot 0.378$$

$$0.6222^n \cdot 0.040723 \leq 3.78 \cdot 10^{-10} \quad | : 0.040723$$

$$0.6222^n \leq 0.92826 \cdot 10^{-9} \quad \log$$

$$38.97 \leq n$$

$\to$ 39 Iterationen