Aufgabe 1:

a)

$$A = \begin{pmatrix} 20'000 & 30'000 & 10'000 \\ 10'000 & 17'000 & 6'000 \\ 2'000 & 3'000 & 2'000 \end{pmatrix} \qquad b = \begin{pmatrix} 5.2 \cdot 10^6 \\ 3 \cdot 10^6 \\ 0.76 \cdot 10^6 \end{pmatrix}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 20 & 30 & 10 & 5200 \\ 10 & 17 & 6 & 3000 \\ 2 & 3 & 2 & 760 \end{array}\right) \quad II - \tfrac{1}{2} I \Rightarrow z_2 = z_2 - \tfrac{1}{2} z_1$$

$$\Rightarrow \left(\begin{array}{ccc|c} 20 & 30 & 10 & 5200 \\ 0 & 2 & 1 & 400 \\ 2 & 3 & 2 & 760 \end{array}\right) \quad III - \tfrac{1}{10} I \Rightarrow z_3 - \tfrac{1}{10} z_1$$

$$\Rightarrow \left(\begin{array}{ccc|c} 20 & 30 & 10 & 5200 \\ 0 & 2 & 1 & 400 \\ 0 & 0 & 1 & 240 \end{array}\right)$$

$$\Rightarrow \underline{\underline{x_3 = 240}},$$

$$2x_2 + x_3 = 400 \Rightarrow x_2 = \frac{400 - 240}{2} = \underline{\underline{80}}$$

$$20x_1 + 30x_2 + 10x_3 = 5200 \Rightarrow x_3 = \frac{5200 - 2400 - 2400}{20} = \frac{400}{20} = \underline{\underline{20}}$$

$$\Rightarrow x = \begin{pmatrix} 240 \\ 80 \\ 20 \end{pmatrix}$$

b) $$L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{10} & 0 & 1 \end{pmatrix} \qquad R = \begin{pmatrix} 20 & 30 & 10 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

c) $$\tilde{b} = \begin{pmatrix} 5720 \\ 3300 \\ 836 \end{pmatrix}$$

$$Ly = \tilde{b} \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5720 \\ \frac{1}{2} & 1 & 0 & 3300 \\ \frac{1}{10} & 0 & 1 & 836 \end{array}\right)$$

$$y_1 = 5720$$

$$y_2 = 3300 - \tfrac{1}{2} y_1 = 440$$

$$y_3 = 836 - \tfrac{1}{10} y_1 = 264$$

$$\Rightarrow y = \begin{pmatrix} 5720 \\ 440 \\ 264 \end{pmatrix}$$

$$Rx = y \Rightarrow \left(\begin{array}{ccc|c} 20 & 30 & 10 & 5720 \\ 0 & 2 & 1 & 440 \\ 0 & 0 & 1 & 264 \end{array}\right)$$

$$\Rightarrow \underline{\underline{x_3 = 264}} \qquad 2x_2 + x_3 = 440 \Rightarrow x_2 = \frac{440 - 264}{2} = \underline{\underline{88}}$$

$$20x_1 + 30x_2 + 10x_3 = 5720 \Rightarrow x_3 = \frac{5720 - 2640 - 2640}{20} = \frac{440}{20} = \underline{\underline{22}}$$

$$\Rightarrow x = \begin{pmatrix} 22 \\ 88 \\ 264 \end{pmatrix}$$