Aufgabe 1:

a)

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -5 & 4 & 1 \\ 2 & -1 & 3 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 9 \\ 5 \end{pmatrix}$$

$v_1 = a_{11} + \operatorname{sign}(a_{11}) \cdot |a_1| \cdot e_1$

$u_1 = \frac{1}{|v_1|} \cdot v_1$

$H_1 = I_n - 2 u_1 \cdot u_1^T$

$$a_1 = \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix} \qquad v_1 = \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix} + 1 \cdot \sqrt{1+25+4} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 6.5 \\ -5 \\ 2 \end{pmatrix}$$

$$u_1 = \frac{1}{\sqrt{6.5^2+5^2+2^2}} \cdot \begin{pmatrix} 6.5 \\ -5 \\ 2 \end{pmatrix} = \begin{pmatrix} 0.77 \\ -0.59 \\ 0.24 \end{pmatrix}$$

$$H_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - 2 \cdot \begin{pmatrix} 0.77 \\ -0.59 \\ 0.24 \end{pmatrix} \cdot \begin{pmatrix} 0.77 & -0.59 & 0.24 \end{pmatrix} = \begin{pmatrix} -0.18 & 0.91 & -0.37 \\ 0.91 & 0.30 & 0.28 \\ -0.37 & 0.28 & 0.89 \end{pmatrix}$$

$Q_1 = H_1$

$$Q_1 \cdot A = \begin{pmatrix} -5.48 & 4.38 & -0.73 \\ 0 & -0.93 & 3.88 \\ 0 & 0.97 & 1.85 \end{pmatrix} = A^*$$

$$\begin{pmatrix} -5.48 & 4.38 & -0.73 \\ 0 & -0.93 & 3.88 \\ 0 & 0.97 & 1.85 \end{pmatrix} \Rightarrow A = \begin{pmatrix} -0.93 & 3.88 \\ 0.97 & 1.85 \end{pmatrix}$$

$$a_{1,neu} = \begin{pmatrix} -0.93 \\ 0.97 \end{pmatrix}$$

$$v_2 = \begin{pmatrix} -0.93 \\ 0.97 \end{pmatrix} \cdot (-1) \cdot \sqrt{(-0.93)^2 + (0.97)^2} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2.23 \\ 0.97 \end{pmatrix}$$

$$u_2 = \frac{1}{|v_2|} \cdot v_2 = \begin{pmatrix} -0.92 \\ 0.39 \end{pmatrix}$$

$$H_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 2 \cdot \begin{pmatrix} -0.92 \\ 0.39 \end{pmatrix} \cdot \begin{pmatrix} -0.92 & 0.39 \end{pmatrix} = \begin{pmatrix} -0.69 & 0.72 \\ 0.72 & 0.69 \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.69 & 0.72 \\ 0 & 0.72 & 0.69 \end{pmatrix}$$

$$Q_2 \cdot A^{*} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.69 & 0.72 \\ 0 & 0.72 & 0.69 \end{pmatrix} \cdot \begin{pmatrix} -5.48 & 4.38 & -0.73 \\ 0 & -0.93 & 3.88 \\ 0 & 0.97 & 1.85 \end{pmatrix} = \begin{pmatrix} -5.48 & 4.38 & -0.73 \\ 0 & 1.34 & -1.31 \\ 0 & 0 & 4.08 \end{pmatrix} = R$$

b) 1) $Q_2 \cdot Q_1 \cdot b = b_{neu}$

2) $R \cdot x = b_{neu}$

1) $$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.69 & 0.72 \\ 0 & 0.72 & 0.69 \end{pmatrix} \cdot \begin{pmatrix} -0.18 & 0.91 & -0.37 \\ 0.91 & 0.30 & 0.28 \\ -0.37 & 0.28 & 0.89 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 9 \\ 5 \end{pmatrix} = \begin{pmatrix} 6.21 \\ 1.34 \\ 8.17 \end{pmatrix}$$

2) $$\begin{pmatrix} -5.48 & 4.38 & -0.73 \\ 0 & 1.34 & -1.31 \\ 0 & 0 & 4.08 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6.21 \\ 1.34 \\ 8.17 \end{pmatrix}$$ $\Rightarrow$ Rückwärtseinsetzen

$x_3 = 2 \quad x_2 = 3 \quad x_1 = 1$

$$\Rightarrow x = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$