

1)

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -5 & 4 & 1 \\ 2 & -1 & 3 \end{pmatrix}$$

$$b = \begin{pmatrix} 1 \\ 9 \\ 5 \end{pmatrix}$$

$$v_1 = a_1 + \text{sign}(a_{11}) \cdot |a_{11}| \cdot e_1$$

$$u_1 = \frac{1}{|v_1|} \cdot v_1$$

$$H_1 = I_n - 2 \cdot u_1 \cdot u_1^T$$

$$a_1 = \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix}$$

$$v_1 = \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix} + 1 \sqrt{1^2 + 5^2 + 2^2} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 6.5 \\ -5 \\ 2 \end{pmatrix}$$

$$u_1 = \frac{1}{\sqrt{6.5^2 + (-5)^2 + 2^2}} \cdot \begin{pmatrix} 6.5 \\ -5 \\ 2 \end{pmatrix} = \begin{pmatrix} 0.77 \\ -0.59 \\ 0.24 \end{pmatrix}$$

$$H_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - 2 \begin{pmatrix} 0.77 \\ -0.59 \\ 0.24 \end{pmatrix} \begin{pmatrix} 0.77 & -0.59 & 0.24 \end{pmatrix} =$$

$$\begin{pmatrix} -0.18 & 0.91 & -0.37 \\ 0.91 & 0.30 & 0.28 \\ -0.37 & 0.28 & 0.89 \end{pmatrix} = Q_1 = H_1$$

$$Q_1 \cdot A = \begin{pmatrix} -5.49 & 4.38 & -0.73 \\ 0 & -0.93 & 3.88 \\ 0 & 0.97 & 1.85 \end{pmatrix} = A^*$$

$$\begin{pmatrix} -5.49 & 4.38 & -0.73 \\ 0 & -0.93 & 3.88 \\ 0 & 0.97 & 1.85 \end{pmatrix}$$

$$a_{1\text{neu}} = \begin{pmatrix} -0.93 \\ 0.97 \end{pmatrix}$$

$$v_2 = \begin{pmatrix} -0.93 \\ 0.97 \end{pmatrix} - 1 \cdot |a_{1\text{neu}}| \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -2.23 \\ 0.97 \end{pmatrix}$$

$$u_2 = \frac{1}{|v_2|} \cdot v_2 = \begin{pmatrix} -0.92 \\ 0.39 \end{pmatrix}$$

$$H_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 2 \cdot \begin{pmatrix} -0.92 \\ 0.39 \end{pmatrix} \begin{pmatrix} -0.92 & 0.39 \end{pmatrix}$$

$$= \begin{pmatrix} -0.69 & 0.72 \\ 0.72 & 0.69 \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.69 & 0.72 \\ 0 & 0.72 & 0.69 \end{pmatrix}$$

$$Q_2 \cdot A^* = \begin{pmatrix} -5.48 & 4.38 & -0.78 \\ 0 & 1.34 & -1.31 \\ 0 & 0 & 4.08 \end{pmatrix} = R$$

$$1) \quad Q_2 \cdot Q_1 = b = b_{\text{neu}}$$

$$2) \quad R \cdot x = b_{\text{neu}}$$

$$\begin{array}{c}
 Q_2 \quad 0 \\
 \begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.69 & 0.77 \\ 0 & 0.77 & 0.69 \end{pmatrix}
 \end{array}
 \begin{array}{c}
 Q_1 \\
 \begin{pmatrix} -0.18 & 0.91 & -0.37 \\ 0.91 & 0.30 & 0.28 \\ -0.37 & 0.28 & 0.89 \end{pmatrix}
 \end{array}
 \begin{array}{c}
 b \\
 \begin{pmatrix} 1 \\ 9 \\ 1 \end{pmatrix}
 \end{array}
 = b_{\text{neu}}
 \begin{array}{c}
 \\
 \begin{pmatrix} 6.21 \\ 1.34 \\ 8.17 \end{pmatrix}
 \end{array}$$

$$\begin{array}{c}
 R \\
 \begin{pmatrix} -5.48 & 4.38 & -0.78 \\ 0 & 1.34 & -1.31 \\ 0 & 0 & 4.08 \end{pmatrix}
 \end{array}
 \begin{array}{c}
 x \\
 \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}
 \end{array}
 = \begin{array}{c}
 b_{\text{neu}} \\
 \begin{pmatrix} 6.21 \\ 1.34 \\ 8.17 \end{pmatrix}
 \end{array}$$

Rückwärts einsetzen:

$$x_1 = 1$$

$$x_2 = 3$$

$$x_3 = 2$$

$$x = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$