2. a)

$$A = \begin{pmatrix} 0.8 & 2.2 & 3.6 \\ 2.0 & 3.0 & 4.0 \\ 1.2 & 2.0 & 5.8 \end{pmatrix} \qquad b = \begin{pmatrix} 2.4 \\ 1.0 \\ 4.0 \end{pmatrix}$$

$$A^* = \begin{pmatrix} 2.0 & 3.0 & 4.0 \\ 0.8 & 2.2 & 3.6 \\ 1.2 & 2.0 & 5.8 \end{pmatrix} \qquad P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$z_2 = z_2 - \frac{0.8}{2.0} z_1$$

$$z_3 = z_3 - \frac{1.2}{2.0} z_1 \longrightarrow A_2^* = \begin{pmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 0.2 & 3.4 \end{pmatrix}$$

$$z_3 = z_3 - \frac{0.2}{1} z_2 \longrightarrow A_3^* = \begin{pmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{0.8}{2} & 1 & 0 \\ \frac{1.2}{2.0} & \frac{0.2}{1} & 1 \end{pmatrix} \qquad P = P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{0.8}{2} & 1 & 0 \\ \frac{1.2}{2.0} & \frac{0.2}{1} & 1 \end{pmatrix} \quad R = \begin{pmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix} \quad P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

b)

$$Ax = b$$

$$LRx = Pb$$

① $Ly = Pb$ geg: L, Pb

ges: y

② $Rx = y$ geg: R, y

ges: x

$$Ly = \begin{pmatrix} 1 & 0 & 0 \\ \frac{0.8}{2} & 1 & 0 \\ \frac{1.2}{2.0} & \frac{0.2}{1} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2.4 \\ 1.0 \\ 4.0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2.4 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ \frac{0.8}{2} & 1 & 0 \\ \frac{1.2}{2.0} & \frac{0.2}{1} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2.4 \\ 4 \end{pmatrix}$$

↳ Vorwärtseinsetzen (zuerst y_1, dann y_2, y_3)

$$y_1 = 1$$
$$\frac{0.8}{2} y_1 + y_2 = 2.4$$
$$\frac{1.2}{2.0} y_1 + 0.2 y_2 + y_3 = 4$$

$y_1 = 1$

$y_2 = 2$

$y_3 = 3$

(2)

$$\begin{pmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

↳ Rückwärtseinsetzen (zuerst $x_3, \dots$)

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 1 \\ x_2 + 2x_3 &= 2 \\ 3x_3 &= 3 \end{aligned}$$

$x_3 = 1 \quad x_2 = 0 \quad x_1 = -1.5$

$$x = \begin{pmatrix} -1.5 \\ 0 \\ 1 \end{pmatrix}$$

c) Unsere Lösung ist identisch.

```python
import scipy as sp
import numpy as np

A = np.array([[0.8, 2.2, 3.6], [2.0, 3.0, 4.0], [1.2, 2.0, 5.8]])

result = sp.linalg.lu(A)

print("P =\n", result[0])
print("L =\n", result[1])
print("R =\n", result[2])
```

```
P =
 [[0. 1. 0.]
 [1. 0. 0.]
 [0. 0. 1.]]
L =
 [[1.  0.  0. ]
 [0.4 1.  0. ]
 [0.6 0.2 1. ]]
R =
 [[2. 3. 4.]
 [0. 1. 2.]
 [0. 0. 3.]]
```