1.

$$A = \begin{pmatrix} 20\,000 & 30000 & 10000 \\ 10\,000 & 17\,000 & 6000 \\ 2000 & 3000 & 2000 \end{pmatrix} \qquad b = \begin{pmatrix} 5.2 \cdot 10^6 \\ 3.0 \cdot 10^6 \\ 7.6 \cdot 10^5 \end{pmatrix}$$

$$A = \begin{pmatrix} 20 & 30 & 16 \\ 10 & 17 & 6 \\ 2 & 3 & 2 \end{pmatrix} \qquad b = \begin{pmatrix} 5200 \\ 3000 \\ 760 \end{pmatrix}$$

a)
$$\left(\begin{array}{ccc|c} 20 & 30 & 16 & 5200 \\ 10 & 17 & 6 & 3000 \\ 2 & 3 & 2 & 760 \end{array}\right)$$

$$Z_2 = Z_2 - \frac{10}{20} Z_1$$

$$\left(\begin{array}{ccc|c} 20 & 30 & 16 & 5200 \\ 0 & 2 & 1 & 400 \\ 2 & 3 & 2 & 760 \end{array}\right)$$

$$Z_3 = Z_3 - \frac{2}{20} Z_1$$

$$\left(\begin{array}{ccc|c} 20 & 30 & 16 & 5200 \\ 0 & 2 & 1 & 400 \\ 0 & 0 & 1 & 240 \end{array}\right)$$

$$20x_1 + 30x_2 + 10x_3 = 5200$$

$$2x_2 + x_3 = 400$$

$$x_3 = 240$$

$\rightarrow x_3 = 240 \quad x_2 = 80 \quad x_1 = 20$

b)

$$L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{10} & 0 & 1 \end{pmatrix} \qquad R = \begin{pmatrix} 20 & 30 & 10 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

c)

$$b_{neu} = \begin{pmatrix} 5720 \\ 3306 \\ 836 \end{pmatrix}$$

1) $Ly = b_{neu}$ geg: $L, b_{neu}$ ges: $y$

2) $Rx = y$ geg: $R, y$ ges: $x$

$$b_{neu}^{*} = \begin{pmatrix} 5720 \\ 440 \\ 264 \end{pmatrix}$$

$$z_2 = z_2 - \frac{10}{20} z_1 \longrightarrow z_2 = 3300 - \frac{10}{20} \cdot 5720 = 440$$

$$z_3 = z_3 - \frac{2}{20} z_1 \longrightarrow z_3 = 836 - \frac{2}{20} \cdot 5720 = 264$$

$$\left(\begin{array}{ccc|c} 20 & 30 & 10 & 5720 \\ 0 & 2 & 1 & 440 \\ 0 & 0 & 1 & 264 \end{array}\right)$$

$x_3 = 264$

$x_2 = \frac{440 - 264}{2} = 88$

$x_1 = \frac{5720 - 2640 - (30 \cdot 88)}{20} = 22$

$x_1 = 22$

$x_2 = 88$

$x_3 = 264$