Aufgabe 2:

$z^4 + 4z^2 + 16 = 0$ biquadratisch $\boxed{\begin{matrix} u = z^2 \\ u^2 = z^4 \end{matrix}}$ $a_1 = 1 \quad a_2 = 4 \quad a_3 = 16 \quad \in \mathbb{R} \implies$ Lsg. paarweise

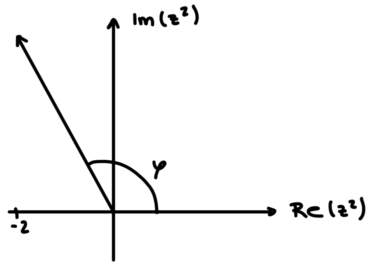
$u^2 + 4u + 16 = 0$

Mitternachtsformel

$\rightsquigarrow \; u_1 = -2 + \sqrt{-12} = -2 + \sqrt{12}\,i$

$u_2 = -2 - \sqrt{-12} = -2 - \sqrt{12}\,i$

$\underline{u_1:}$ $z_N^2 = -2 + \sqrt{12}\,i$



$\varphi = \pi - \tan^{-1}\left(\frac{\sqrt{12}}{2}\right) = \frac{2\pi}{3}$

$r = \sqrt{2^2 + \sqrt{12}^2} = \sqrt{16} = 4$

$z_E^2 = 4 \cdot e^{i \cdot \frac{2\pi}{3}}$

$z^n = r_0 \cdot e^{i\varphi}$

$z^2 = 4 \cdot e^{i\,2\frac{\pi}{3}}$

$\implies n = 2, \; \varphi = {}^{2\pi}/_3, \; r_0 = 4, \; k = 0,1 \qquad k \in [0, \dots, n-1]$

$z_k = \underbrace{r}_{\sqrt[n]{r_0}} \cdot e^{i \varphi_k}, \quad \varphi_k = \frac{\varphi + k \cdot 2\pi}{n}$

$k = 0: \quad z_0 = \sqrt[2]{4} \cdot e^{i \cdot \left(\frac{\frac{2\pi}{3} + 0 \cdot 2\pi}{2}\right)} = 2 \cdot e^{i \cdot 60°}$

$k = 1: \quad z_1 = \sqrt[2]{4} \cdot e^{i \cdot \left(\frac{\frac{2\pi}{3} + 1 \cdot 2\pi}{2}\right)} = 2 \cdot e^{i \cdot 240°}$

$\underline{u_2:}$ $z_N^2 = -2 - \sqrt{12}\,i$

$\varphi = \pi + \tan^{-1}\left(\frac{\sqrt{12}}{2}\right) = \frac{4\pi}{3}$

$r = \sqrt{2^2 + \sqrt{12}^2} = \sqrt{16} = 4$

$z_E^2 = 4 \cdot e^{i \cdot \frac{4\pi}{3}}$

$z^n = r_0 \cdot e^{i\varphi}$

$z^2 = 4 \cdot e^{i\,\frac{4\pi}{3}}$

$\implies n = 2, \; \varphi = {}^{4\pi}/_3, \; r_0 = 4, \; k = 0,1 \qquad k \in [0, \dots, n-1]$

$k = 0: \quad z_3 = \sqrt[2]{4} \cdot e^{i \cdot \left(\frac{\frac{4\pi}{3} + 0 \cdot 2\pi}{2}\right)} = 2 \cdot e^{i \cdot 120°}$ $\quad z_1^*$

$k = 1: \quad z_4 = \sqrt[2]{4} \cdot e^{i \cdot \left(\frac{\frac{4\pi}{3} + 1 \cdot 2\pi}{2}\right)} = 2 \cdot e^{i \cdot 300°}$ $\quad z_0^*$