$\in \mathbb{R}$ = Log paarweise

2. $z^4 + 4z^2 + 16 = 0$

$u = z^2$ $\quad u^2 + 4u + 16 = 0$ $\quad \sqrt{12 \cdot -1} = \sqrt{12} \cdot \sqrt{-1}$

$u^2 = z^4$

Mitternachtsformel

$u_1 = -2 + \sqrt{-12} = -2 + \sqrt{12} \cdot i$

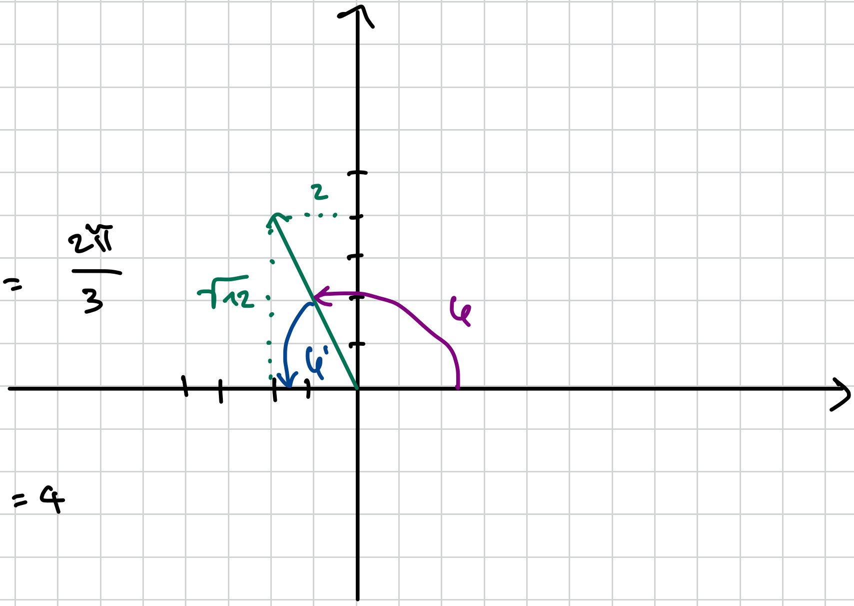
$u_2 = -2 - \sqrt{-12} = -2 + \sqrt{12} \cdot i$

$\boxed{u_1}$

$z^2 = -2 + \sqrt{12}\, i$

Was ist φ?

$\varphi = \pi - \tan^{-1}\left(\frac{\sqrt{12}}{2}\right) = \frac{2\pi}{3}$



Was ist r?

$r = \sqrt{2^2 + \sqrt{12}^2} = 4$

$z_E^2 = 4 \cdot e^{i \frac{2\pi}{3}}$

$z^n = r_0 e^{i u}$

$z^2 = 4 \cdot e^{i \frac{2\pi}{3}}$

$n = 2$

$\varphi = \frac{2\pi}{3}$

$r_0 = 4$

$k = 0, 1$

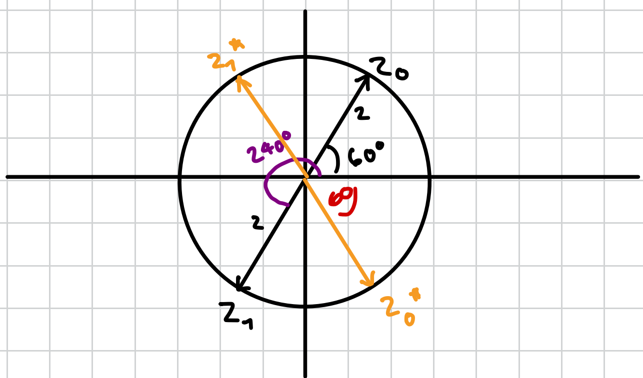
Lösung: $z_k = r \cdot e^{i \varphi_k}$

$r \to \sqrt[n]{r_0}$

$\varphi_k \to \frac{\varphi + k \cdot 2\pi}{n}$

$k_0 = 0 \quad z_0 = \sqrt[2]{4} \cdot e^{i\left(\frac{2\pi}{3} + 0 \cdot 2\pi}{2}\right)} = 2 \cdot e^{i \cdot 60°}$

$k_1 = 1 \quad z_1 = \sqrt[2]{4} \cdot e^{i\left(\frac{\frac{2\pi}{3} + 1 \cdot 2\pi}{2}\right)} = 2 \cdot e^{i \cdot 240°}$



$z_0 = 2e^{i 60°}$

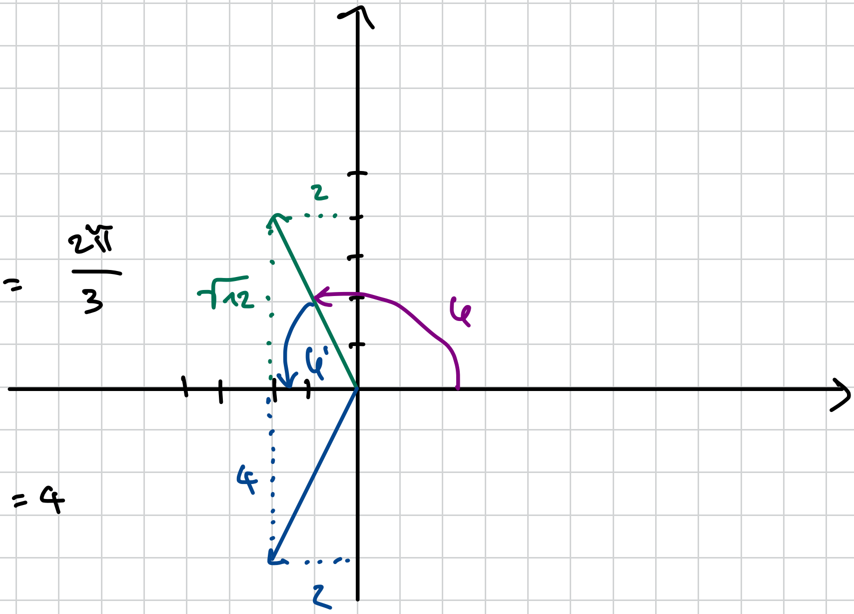
$z_0^* = 2e^{i 300°}$

$\boxed{n_2}$

$$z^2 = \boxed{-2 - \sqrt{12}\, i}$$

Was ist φ?

$$\varphi = \pi + \tan^{-1}\left(\frac{\sqrt{12}}{2}\right) = \frac{2\pi}{3}$$



Was ist r?

$$r = \sqrt{2^2 + \sqrt{12}^2} = 4$$

$$z_E^2 = 4 \cdot e^{i\frac{4\pi}{3}}$$

$$\left.\begin{array}{l} z^n = r_0 e^{i\varphi} \\ z^2 = 4 \cdot e^{i\frac{4\pi}{3}} \end{array}\right\} \quad \begin{array}{l} n = 2 \\ \varphi = \frac{4\pi}{3} \\ r_0 = 4 \\ k = 0, 1 \end{array}$$

(z_1^*)

$$k_0 = 0 \qquad z_0 = \sqrt[2]{4} \cdot e^{i\left(\frac{\frac{4\pi}{3} + 0 \cdot 2\pi}{2}\right)} = 2 \cdot e^{i \cdot 120°}$$

$$k_1 = 1 \qquad z_1 = \sqrt[2]{4} \cdot e^{i\left(\frac{\frac{4\pi}{3} + 1 \cdot 2\pi}{2}\right)} = 2 \cdot e^{i \cdot 300°}$$

(z_0^*)