Aufgabe 1:

i	0	1	2	3
x_i	4	6	8	10
y_i	6	3	9	0
a_i	6	3	9	0
h_i	2	2	2	-
c_i	0	?	?	0

$$A = \begin{pmatrix} 2(h_0+h_1) & h_1 \\ h_1 & 2(h_1+h_2) \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 3\frac{y_2-y_1}{h_1} - 3\frac{y_1-y_0}{h_0} \\ 3\frac{y_3-y_2}{h_2} - 3\cdot\frac{y_2-y_1}{h_1} \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 2 \\ 2 & 8 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 13.5 \\ -22.5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 8 & 2 \\ 0 & 7.5 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 13.5 \\ -25.875 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 2.55 \\ -3.45 \end{pmatrix}$$

$$b_i = \frac{y_{i+1}-y_i}{h_i} - \frac{h_i}{3}(c_{i+1}+2c_i)$$

$$b_0 = \frac{-3}{2} - \frac{2}{3}(2.55+0) = -3.2$$
$$b_1 = \frac{6}{2} - \frac{2}{3}(-3.45+2\cdot 2.55) = 1.9$$
$$b_2 = -\frac{9}{2} - \frac{2}{3}(0+2\cdot(-3.45)) = 0.1$$

$$d_i = \frac{1}{3h_i}(c_{i+1}-c_i)$$

$$d_0 = \frac{1}{6}(2.55-0) = 0.425$$
$$d_1 = \frac{1}{6}(-3.45-2.55) = -1$$
$$d_2 = \frac{1}{6}(0+3.45) = 0.575$$

$$S_i(x) = a_i + b_i(x-x_i) + c_i(x-x_i)^2 + d_i(x-x_i)^3$$

$$S_0(x) = 6 - 3.2(x-4) + 0 + 0.425(x-4)^3 = 0.425x^3 - 5.1x^2 + 17.2x - 8.4$$
$$S_1(x) = 3 + 1.9(x-6) + 2.55(x-6)^2 - 1(x-6)^3 = -x^3 + 20.55x^2 - 136.7x + 299.4$$
$$S_2(x) = 9 + 0.1(x-8) - 3.45(x-8)^2 + 0.575(x-8)^3 = -0.575x^3 + 10.35x^2 - 55.1x + 81.8$$