

Aufgabe 2:

$$\frac{dy}{dx} = \frac{x^2}{y}, \quad 0 \leq x \leq 1.4, \quad y(0) = 2$$

a) $a = 0, b = 1.4, h = 0.7$

$$i=0: \quad x_0 = a = 0, \quad x_i = 0 + i \cdot 0.7, \quad n = \frac{b-a}{h} = \frac{1.4-0}{0.7} = 2$$

$$|y(x_0) - y_0| = |\sqrt{4} - 2| = 0$$

$$i=1: \quad x_1 = x_0 + h = 0 + 0.7 = 0.7 \quad y_1 = y_0 + h \cdot f(x_0, y_0) = 2 + 0.7 \cdot \frac{0^2}{2} = 2$$

$$|y(x_1) - y_1| = \left| \sqrt{\frac{2 \cdot 0.7^3}{3} + 4} - 2 \right| = 0.0564$$

$$i=2: \quad x_2 = x_1 + h = 0.7 + 0.7 = 1.4 \quad y_2 = y_1 + h \cdot f(x_1, y_1) = 2 + 0.7 \cdot \frac{0.7^2}{2} = 2.1715$$

$$|y(x_2) - y_2| = \left| \sqrt{\frac{2 \cdot 1.4^3}{3} + 4} - 2.1715 \right| = 0.2429$$

b)

$$i=0: \quad x_0 = 0 \quad y_0 = 2 \quad x_{n/2} = x_0 + \frac{h}{2} = 0 + \frac{0.7}{2} = 0.35 \quad y_{n/2} = y_0 + \frac{h}{2} \cdot f(x_0, y_0) = 2 + 0.35 \cdot \left(\frac{0^2}{2} \right) = 2$$

$$x_1 = x_0 + h = 0.7 \quad y_1 = y_0 + h \cdot f(x_{n/2}, y_{n/2}) = 2 + 0.7 \cdot \frac{0.35^2}{2} = 2.042875$$

$$|y(x_0) - y_0| = |\sqrt{4} - 2| = 0$$

$$i=1: \quad x_1 = 0.7 \quad y_1 = 2.0429 \quad x_{n/2} = x_1 + \frac{h}{2} = 0.7 + 0.35 = 1.05$$

$$y_{n/2} = y_1 + \frac{h}{2} \cdot f(x_1, y_1) = 2.0429 + 0.35 \cdot \frac{0.7^2}{2} = 2.1268$$

$$x_2 = x_1 + h = 1.4 \quad y_2 = y_1 + h \cdot f(x_{n/2}, y_{n/2}) = 2.0429 + 0.7 \cdot \frac{1.05^2}{2} = 2.4057$$

$$|y(x_1) - y_1| = \left| \sqrt{\frac{2 \cdot 0.7^3}{3} + 4} - 2.0429 \right| = 0.0135$$

$$i=2: \quad x_2 = 1.4 \quad y_2 = 2.4057 \quad |y(x_2) - y_2| = \left| \sqrt{\frac{2 \cdot 1.4^3}{3} + 4} - 2.4057 \right| = 0.0087$$

c)

$$i=0: \quad x_0 = 0 \quad y_0 = 2 \quad x_1 = x_0 + h = 0.7 \quad y_1^{\text{Euler}} = y_0 + h \cdot f(x_0, y_0) = 2 + 0.7 \cdot \frac{0^2}{2} = 2$$

$$k_1 = f(x_0, y_0) = \frac{0^2}{2} = 0 \quad k_2 = f(x_1, y_1^{\text{Euler}}) = \frac{0.7^2}{2} = 0.245$$

$$y_1 = y_0 + h \cdot \frac{k_1 + k_2}{2} = 2 + 0.7 \cdot \frac{0 + 0.245}{2} = 2.0858 \quad |y(x_0) - y_0| = |\sqrt{4} - 2| = 0$$

$$i=1: \quad x_1 = 0.7 \quad y_1 = 2.0858 \quad x_2 = 1.4 \quad y_2^{\text{Euler}} = y_1 + h \cdot f(x_1, y_1) = 2.0858 + 0.7 \cdot \frac{0.7^2}{2.0858} = 2.2502$$

$$k_1 = f(x_1, y_1) = \frac{0.7^2}{2.0858} = 0.2349 \quad k_2 = f(x_2, y_2^{\text{Euler}}) = \frac{1.4^2}{2.2502} = 0.8710$$

$$y_2 = y_1 + h \cdot \frac{k_1 + k_2}{2} = 2.0858 + 0.7 \cdot \frac{0.2349 + 0.8710}{2} = 2.4729$$

$$|y(x_1) - y_1| = \left| \sqrt{\frac{2 \cdot 0.7^3}{3} + 4} - 2.0858 \right| = 0.0294$$

$$i=2: \quad x_2 = 1.4 \quad y_2 = 2.4729 \quad |y(x_2) - y_2| = \left| \sqrt{\frac{2 \cdot 1.4^3}{3} + 4} - 2.4729 \right| = 0.0585$$