Aufgabe 1:

$$f(x_1, x_2) = \begin{pmatrix} 20 - 18x_1 - 2x_2^2 \\ -4x_2 \cdot (x_1 - x_2^2) \end{pmatrix} \qquad x^{(0)} = \begin{pmatrix} 1.1 \\ 0.9 \end{pmatrix}$$

$$Df(x_1, x_2) = \begin{pmatrix} -18 & -4x_2 \\ -4x_2 & -4x_1 + 12x_2^2 \end{pmatrix}$$

1. Iteration:

$$Df(x^{(0)})\, s^{(0)} = -f(x^{(0)})$$

$$\Rightarrow \begin{pmatrix} -18 & -3.6 \\ -3.6 & 5.32 \end{pmatrix} s^{(0)} = \begin{pmatrix} 1.42 \\ 1.044 \end{pmatrix} \Rightarrow s^{(0)} = \begin{pmatrix} -0.104 \\ 0.126 \end{pmatrix}$$

$$x^{(1)} = x^{(0)} + s^{(0)} = \begin{pmatrix} 1.1 \\ 0.9 \end{pmatrix} + \begin{pmatrix} -0.104 \\ 0.126 \end{pmatrix} = \begin{pmatrix} 0.996 \\ 1.026 \end{pmatrix}$$

$$\|f(x^{(1)})\|_2 = \left\| \begin{pmatrix} -0.032 \\ 0.231 \end{pmatrix} \right\|_2 = 0.233$$

$$\|x^{(1)} - x^{(0)}\|_2 = \left\| \begin{pmatrix} 0.996 \\ 1.026 \end{pmatrix} - \begin{pmatrix} 1.1 \\ 0.9 \end{pmatrix} \right\|_2 = 0.163$$

2. Iteration:

$$Df(x^{(1)})\, s^{(1)} = -f(x^{(1)})$$

$$\Rightarrow \begin{pmatrix} -18 & -4.103 \\ -4.103 & 8.644 \end{pmatrix} s^{(1)} = \begin{pmatrix} 0.032 \\ -0.231 \end{pmatrix} \Rightarrow s^{(1)} = \begin{pmatrix} 0.004 \\ -0.025 \end{pmatrix}$$

$$x^{(2)} = x^{(1)} + s^{(1)} = \begin{pmatrix} 0.996 \\ 1.026 \end{pmatrix} + \begin{pmatrix} 0.004 \\ -0.025 \end{pmatrix} = \begin{pmatrix} 1 \\ 1.001 \end{pmatrix}$$

$$\|f(x^{(2)})\|_2 = \left\| \begin{pmatrix} -0.001 \\ 0.008 \end{pmatrix} \right\|_2 = 0.008$$

$$\|x^{(1)} - x^{(0)}\|_2 = \left\| \begin{pmatrix} 1 \\ 1.001 \end{pmatrix} - \begin{pmatrix} 0.996 \\ 1.026 \end{pmatrix} \right\|_2 = 0.025$$