## Aufgabe 1

$$I = \int_1^2 \ln(x^2)\,dx$$

$$\left|\int_a^b f(x)dx - Rf(h)\right| \le \frac{h^2}{24}(b-a)\cdot \max_{x\in[a,b]} |f''(x)| \le 10^{-5}$$

$$\left|\int_a^b f(x)dx - Tf(h)\right| \le \frac{h^2}{12}(b-a) \max_{x\in[a,b]} |f''(x)| \le 10^{-5}$$

$$\left|\int_a^b f(x)dx - Sf(h)\right| \le \frac{h^4}{2880}(b-a) \max_{x\in[a,b]} |f^{(4)}(x)| \le 10^{-5}$$

$$f(x) = \ln(x^2)$$
$$f'(x) = \frac{1}{x^2}\cdot 2x = \frac{2}{x} = 2x^{-1}$$
$$f''(x) = -2x^{-2} = \frac{-2}{x^2}$$
$$f'''(x) = \frac{4}{x^3}$$
$$f^{IV}(x) = \frac{-12}{x^4}$$

$$\max_{x\in[1,2]} |f''(x)| = \max \left|-\frac{2}{x^2}\right| = \frac{2}{1} = \underline{\underline{2}}$$

$$\max |f^{IV}(x)| = \max\left|\frac{-12}{x^4}\right| = \frac{12}{1} = 12$$

$$\frac{h^2}{24}(b-a)\cdot \max_{x\in[a,b]} |f''(x)| \le 10^{-5} \qquad \frac{h^2}{24}(2-1)\cdot 2 \le 10^{-5} \Rightarrow h \le \sqrt{10^{-5}\cdot 12} = 0.01095 \Rightarrow n = \frac{1}{h} = \underline{\underline{91.28}}$$

$$\frac{h^2}{12}(b-a) \max_{x\in[a,b]} |f''(x)| \le 10^{-5} \qquad \frac{h^2}{12}(2-1)\cdot 2 \le 10^{-5} \Rightarrow h \le \sqrt{10^{-5}\cdot 6} = 0.00774 \Rightarrow n = \frac{1}{h} = \underline{\underline{129.1}}$$

$$\frac{h^4}{2880}(b-a) \max_{x\in[a,b]} |f^{(4)}(x)| \le 10^{-5} \qquad \frac{h^4}{2880}(2-1)\cdot 12 \le 10^{-5} \Rightarrow h \le \sqrt[4]{10^{-5}\cdot 240} = 0.22133 \Rightarrow n = \frac{1}{h} = \underline{\underline{4.51}}$$

Rf = 92 Subintervalle
Tf = 130 Subintervalle
Sf = 5 Subintervalle