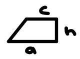
Aufgabe 1:

$$Tf = \frac{f(a)+f(b)}{2} \cdot (b-a)$$

a) $$Tf(h) = \sum_{i=0}^{n-1} \frac{y_i + y_{i+1}}{2} \cdot (x_{i+1} - x_i)$$

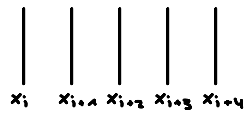
$x_0 = a$

$x_n = b$ Fläche Trapez: $\frac{a+c}{2} \cdot h$



$y_i = f(x_i)$

$$\frac{f(x_i)+f(x_{i+1})}{2} \cdot (x_{i+1} - x_i) = \frac{y_i + y_{i+1}}{2} (x_{i+1} - x_i)$$



aufsummieren: $$Tf(h) = \sum_{i=0}^{n-1} \frac{y_i + y_{i+1}}{2} (x_{i+1} - x_i)$$

b) $$Tf = h \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{n-1} f(x_i) \right)$$

$$Tf(...) = \sum_{i=0}^{n-1} \frac{y_i + y_{i+1}}{2} \underbrace{(x_{i+1} - x_i)}_{h}$$

$$= \sum_{i=0}^{n-1} \frac{y_i + y_{i+1}}{2} \cdot h$$

$$= h \cdot \sum_{i=0}^{n-1} \frac{f(x_i)+f(x_{i+1})}{2}$$

$$= h \cdot \left(\underbrace{\frac{f(x_0)}{2} + \frac{f(x_1)}{2}}_{i=0} + \underbrace{\frac{f(x_1)}{2} + \frac{f(x_2)}{2}}_{i=1} + \dots + \underbrace{\frac{f(x_{n-1})}{2} + \frac{f(x_n)}{2}}_{i=n-1} \right)$$

$$= h \cdot \left(\frac{f(x_0)}{2} + \sum_{i=1}^{n-1} \frac{\cancel{2} \cdot f(x_i)}{2} + \frac{f(x_n)}{2} \right)$$

$$= h \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{n-1} f(x_i) \right) \quad \square$$