Aufgabe 1:

$x_0 = 0$ $\quad y_0 = 1013$

$x_1 = 2500$ $\quad y_1 = 747$

$x_2 = 5000$ $\quad y_2 = 540$

$x_3 = 10'000$ $\quad y_3 = 226$

$x = 3750$

$$P_3(x) = \sum_{i=0}^{3} l_i(x) \cdot y_i$$

$$l_0(x) = \frac{x-x_1}{x_0-x_1} \cdot \frac{x-x_2}{x_0-x_2} \cdot \frac{x-x_3}{x_0-x_3} = \frac{(x-2500)\cdot(x-5000)\cdot(x-10'000)}{(0-2500)\cdot(0-5000)\cdot(0-10'000)} = -\frac{(x-2500)\cdot(x-5000)\cdot(x-10'000)}{125'000'000}$$

$$\Rightarrow l_0(3750) = -\frac{5}{64}$$

$$l_1(x) = \frac{(x-x_0)\cdot(x-x_2)\cdot(x-x_3)}{(x_1-x_0)\cdot(x_1-x_2)\cdot(x_1-x_3)} = \frac{(x-0)\cdot(x-5000)\cdot(x-10'000)}{(2500)\cdot(2500-5000)\cdot(2500-10'000)} = \frac{x\cdot(x-5000)\cdot(x-10'000)}{46'875'000'000}$$

$$\Rightarrow l_1(3750) = \frac{5}{8}$$

$$l_2(x) = \frac{(x-x_0)\cdot(x-x_1)\cdot(x-x_3)}{(x_2-x_0)\cdot(x_2-x_1)\cdot(x_2-x_3)} = \frac{x\cdot(x-2500)\cdot(x-10'000)}{5000\cdot(5000-2500)\cdot(5000-10'000)} = \frac{x\cdot(x-2500)\cdot(x-10'000)}{-62'500'000'000}$$

$$\Rightarrow l_2(3750) = \frac{15}{32}$$

$$l_3(x) = \frac{(x-x_0)\cdot(x-x_1)\cdot(x-x_2)}{(x_3-x_0)\cdot(x_3-x_1)\cdot(x_3-x_2)} = \frac{x\cdot(x-2500)\cdot(x-5'000)}{10'000\cdot(10'000-2500)\cdot(10'000-5'000)} = \frac{x\cdot(x-2500)\cdot(x-5'000)}{375'000'000'000}$$

$$\Rightarrow l_3(3750) = -\frac{1}{64}$$

$$P_3(3750) = -\frac{5}{64}\cdot 1013 + \frac{5}{8}\cdot 747 + \frac{15}{32}\cdot 540 - \frac{1}{64}\cdot 226 = \frac{40'789}{64} \approx \underline{\underline{637.328}}$$