3b - Hypothesis testing

Hypothesis

- **Hypothesis** - statement yet to be (dis)proven
- **Hypothesis Test** - test to (dis)prove hypothesis
- **Null Hypothesis (H_0)** - Base hypothesis, assumed to be true
- **Alternative Hypothesis (H_1, H_a)** - Conclusion of H_0 is rejected

Elements of testing procedure

- **Test statistic** - value that is calculated from sample
- **Region of Acceptance** - region of values supporting H_0
- **Critical Region / Region of rejection** - region of values rejecting H_0
- **Significance level** - probability of falsely rejecting H_0

Testing procedure

1. Formulate H_0 and H_1
2. Determine significance level (α)
3. Calculate test statistic
4. Determine critical region / probability value
5. Draw conclusions

Probability Value

p**-value** = probability under H_0 to obtain value for the test statistic that is at least as extreme as observed value

- $p < \alpha \implies$ reject H_0
- $p > \alpha \implies$ accept H_0

Critical Region

Critical region = collection of all values of test statistic for which we can reject H_0

Right-tailed testing

look for value g for which $\mathcal{P}(M > g) = \alpha$

determine z_α for which $\mathcal{P}(Z > z_\alpha) = \alpha$, so:

$$g = \mu + z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$$

left of g: accept, right of g: reject

In python

$\alpha = 0.05 \implies z_\alpha = 1.645$
($z_\alpha =$ `stats.norm.isf(1-0.95)`)

Left-tailed testing

$$g = \mu - z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$$

Two-tailed testing

$$g = \mu \pm z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$$

Summary of testing procedures

Goal	Test regarding the value of the population mean μ using a sample of n independent values		
Prerequisite	De population has a random distribution, n is sufficiently large		
Test Type	Two-tailed	Left-tailed	Right-tailed
H_0	$\mu = \mu_0$	$\mu = \mu_0$	$\mu = \mu_0$
H_1	$\mu \neq \mu_0$	$\mu < \mu_0$	$\mu > \mu_0$
Critical Region	$\lvert\overline{x}\rvert > g$	$\overline{x} < -g$	$\overline{x} > g$
Test statistic	$z = \frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$		

Requirements for z-test

- Sample is random
- Sample size ≥ 30
- test statistic $\sim \mathrm{Nor}$
- stadard deviation of population, σ, is known

Student's t-test

Requirements

- sample is random (?)
- variable is normally distributed

Test

Critical value g is:

$$g = \mu \pm t \cdot \frac{s}{\sqrt{n}}$$

- t = derived from student's t-distr. based on number of degrees of freedom, $n-1$
  - t = `t.isf()` in python

apart from t, procedure is identical to z-test

Errors in Hypothesis Tests

Conclusion	Reality	
	H_0 True	**H_1 True**
H_0 not rejected	Correct inference	Type II error (false negative)
H_0 rejected	Type I error (false positive)	Correct inference

P(type I error) = α (= significance level)
P(type II error) = β
Calculating β is ***not*** trivial, but if $\alpha \searrow$ then $\beta \nearrow$