2 - Univariate statistics

Central Tendency and Dispersion

Measure of Central Tendency

What value is representative of entire group?

Mean vs average

- **Arithmic mean** $\overline{x}$ = sum of all values divided by number of values

$$\overline{x} = \sum_{i=1}^{n} x_i$$

- **Median** = middle number of all values sorted
- **Mode** = value that appears most often in dataset

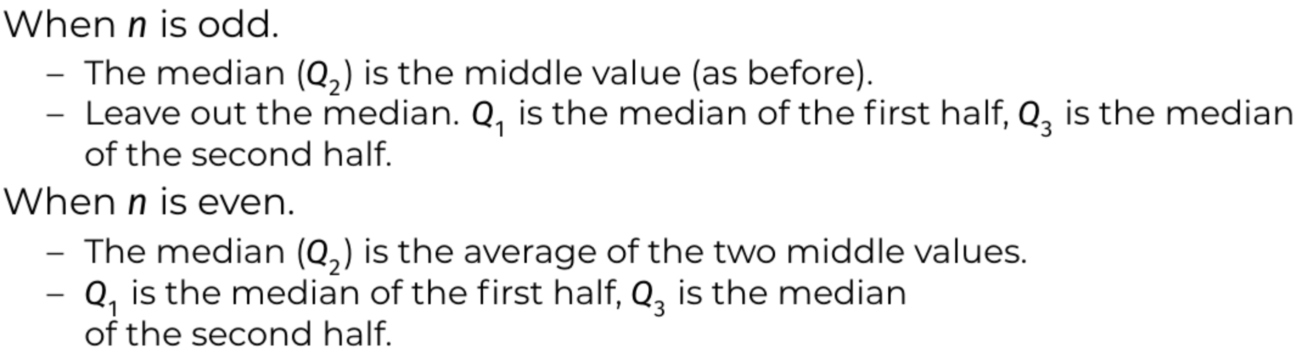
Measures of Dispersion

How large are differences within group?

- **Range** = absolute value of difference between highest & lowest value

$$abs(x_{\min} - x_{\max})$$

- **Quartiles** of sorted set of numbers = 3 values Q_1, Q_2 and Q_3 that divide set into 4 equally large subsets
  - Calculation:



When n is odd.

- The median (Q_2) is the middle value (as before).
- Leave out the median. Q_1 is the median of the first half, Q_3 is the median of the second half.

When n is even.

- The median (Q_2) is the average of the two middle values.
- Q_1 is the median of the first half, Q_3 is the median of the second half.

- **Interquartile Range (IQR)** = difference between third & first quartile

$$|Q_3 - Q_1|$$

- **Variance** (s^2 or σ^2) = mean squared diffference between values of a data set and arithmic mean

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2$$

- **Standard deviation** (s or σ) is square root of variance

Properties of standard deviation σ

Property	Reason
$\sigma \geq 0$	$\sigma = \sqrt{\sigma^2}$ and number under $\sqrt{}$ is never < 0

Property	Reason
Smallest possible value is 0	If all values are equal to eachother. It means that there is no spread.
Outliers affect σ more	their difference with the mean is squared
The unit of standard deviation σ is the same as the unit of the original variable x	because x is squared and then the root is taken

When you're working with a sample (not the whole population), you use $n-1$ in the denominator to get an unbiased estimate of the population variance. This is called *Bessel's correction*.

The variance formula involves squaring the differences from the mean. When you use the sample mean, you're essentially using a statistic that's already tailored to the data. That costs you 1 degree of freedom; you've used one piece of information (the mean) to calculate the rest.

That's why we divide by $n-1$:

- n data points
- 1 constraint (the mean is fixed once you pick the rest)
- $\implies n-1$ degrees of freedom

Summary

Central tendency & dispersion

Measurement level	Center	Spred Distribution
Qualitative	Mode	/
Quantitative	Average/Mean	Variance, Standard Deviation
	Median	Range, IQR

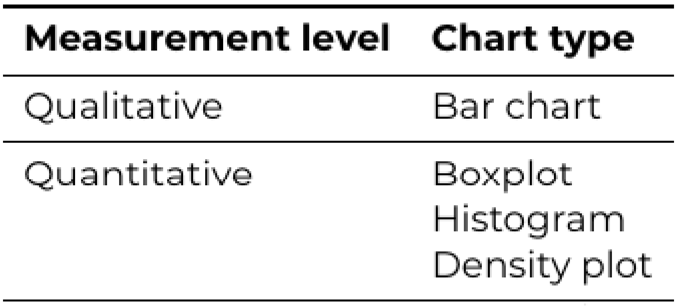
Symbols

	Population	Sample
# of elements	N	n
average / mean	μ	$\overline{x}$
variance	$\sigma^2 = \frac{1}{N}\sum\left(x_i-\mu\right)^2$	$s^2 = \frac{1}{n-1}\sum\left(x_i-\overline{x}\right)^2$
standard deviation	σ	s

Data visualisation

Simple Graphs

Chart type overview



Measurement level	Chart type
Qualitative	Bar chart
Quantitative	Boxplot Histogram Density plot

Pie chart

Disadvantages:

- Comparing angles << comparing length
- unusable fir data with many categories

Interpretation of Charts

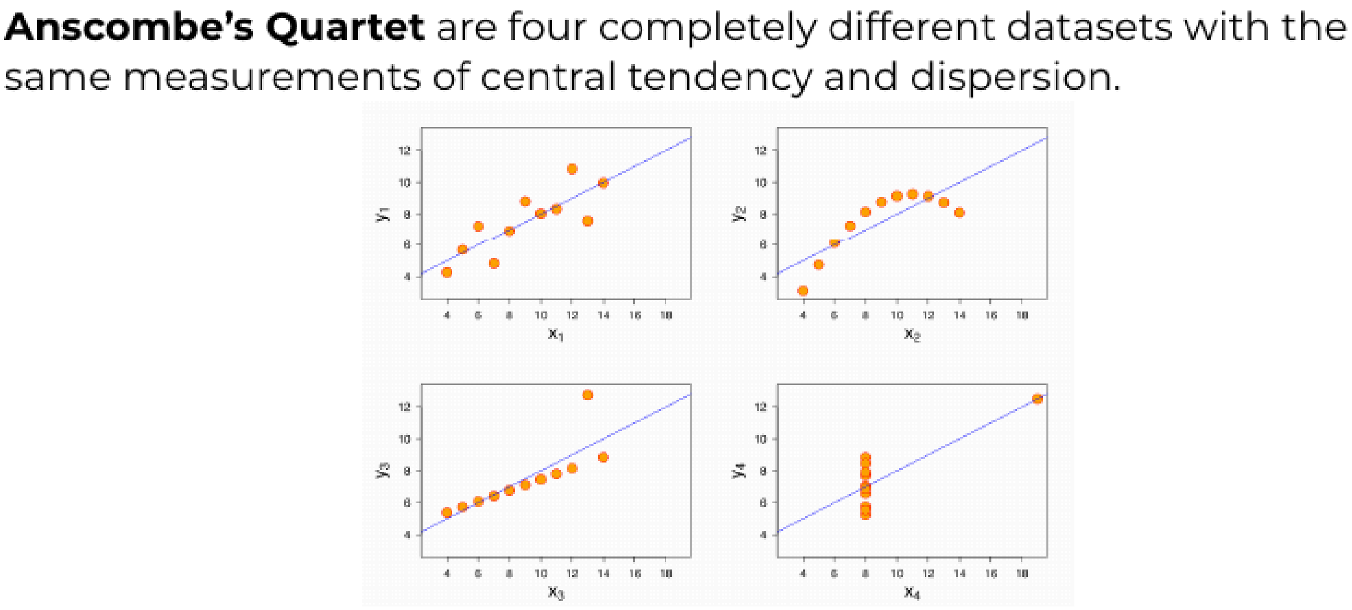
Tips:

- Label the axes
- Clear title
- Name the unit (and, if necessary, order of magnitude)
- Add a label that clarifies the chart
- Avoid bells & whistles
- Minimize "ink to data" ratio

Data distortion

= misrepresenting data so that invalid conclusions are drawn

Importance of visualization



Anscombe's Quartet are four completely different datasets with the same measurements of central tendency and dispersion.