Module 5. Bivariate analysis: qualitative — quantitative

Data Science & AI

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Effect size

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Learning goals

- Apply the *t*-test for two samples
- Calculate effect size using Cohen's *d*
- Visualization

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Bivariate analysis: overview

Independent	**Dependent**	**Test/Metric**
Qualitative	Qualitative	χ^2-test Cramér's V
Qualitative	Quantitative	two-sample t-test Cohen's d
Quantitative	Quantitative	— Regression, correlation

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Example research questions

- Are male penguins larger than females?
- Do men get a higher salary than women?
- Does a new vaccine protect against a disease?
- Does “retrieval practice” improve learning outcomes (i.e. student grades)?
- …

In these examples, what is the independent/dependent variable?

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Data visualization

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Data visualization

- Chart types for quantitative data
- Grouped by qualitative variable

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Data visualization

- Chart types for quantitative data
- Grouped by qualitative variable

Suitable chart types:

- Grouped boxplot
- Grouped density plot
- Bar chart with error bars
- ...

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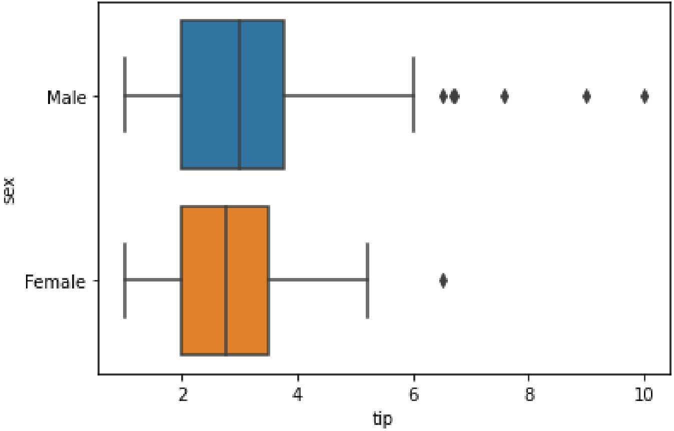
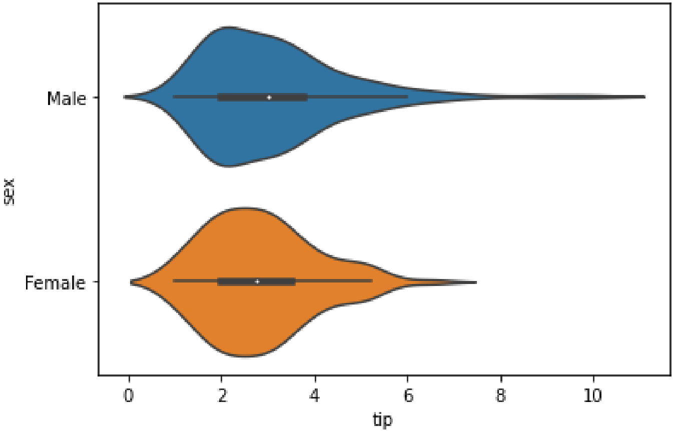
Grouped boxplot

(source code: see `demo-5.01-2sample-t.ipynb`)



Grouped violin plot



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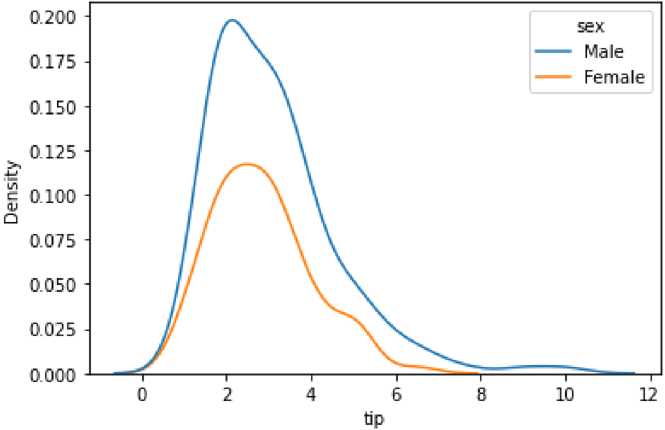
Grouped density plot



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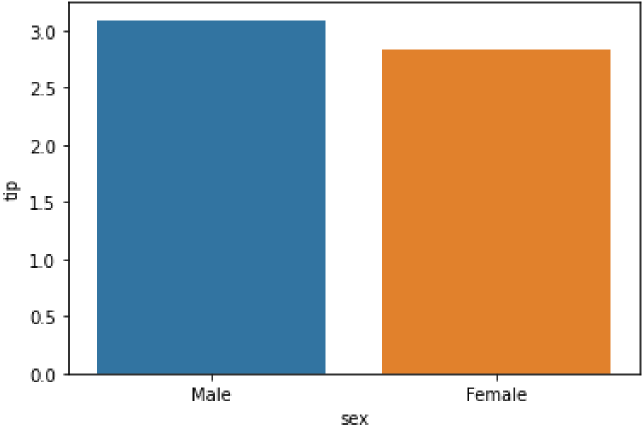
Beware! Bar chart of group means



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Beware! Bar chart of group means

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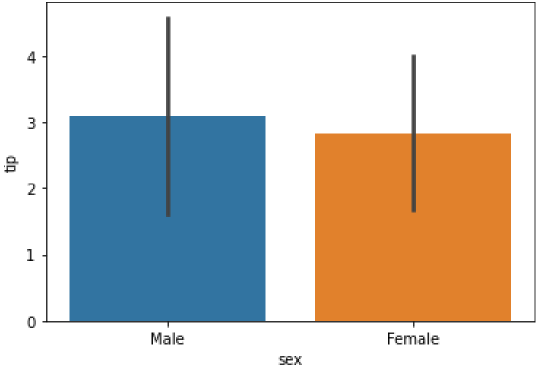
Bar chart with error bars

- Always add error bars!
- Only makes sense for normally distributed data
- Example: 1 standard deviation:



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Two-sample t-test

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Comparing two population means

Are the population means of two populations different?

We use two *samples* to perform an appropriate statistical test.

Correct test depends on:

- Independent samples
- Paired samples

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Independent samples

In a clinical study, the aim is to determine whether a new drug has a delayed (i.e. higher) reaction time as a side effect.

- Control group: 6 participants receive placebo
- Intervention group: 6 participants receive medicine

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Independent samples

In a clinical study, the aim is to determine whether a new drug has a delayed (i.e. higher) reaction time as a side effect.

- Control group: 6 participants receive placebo
- Intervention group: 6 participants receive medicine

Next, the reaction time (in ms) is measured:

- Control group: 91, 87, 99, 77, 88, 91 ($\overline{x}$ = 88.83)
- Intervention group: 101, 110, 103, 93, 99, 104 ($\overline{y}$ = 101.67)

Are there significant differences between the intervention and control group?

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Independent samples

Testing procedure

1. Hypotheses:
   - $H_0: \mu_1 - \mu_2 = 0$
   - $H_1: \mu_1 - \mu_2 < 0$
2. Significance level: $\alpha = 0.05$
3. Test statistic is based on:
   - $\overline{x} - \overline{y} = -12.833$
   - $\overline{x}$ = estimation for μ_1 (control group)
   - $\overline{y}$ = estimation for μ_2 (intervention group)
   - Takes the variances of the samples into account. For completeness the test statistic is

$$t = \frac{\overline{x} - \overline{y}}{\sqrt{s_x^2/n_x + s_y^2/n_y}}.$$

4. Calculate p

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Independent samples

Calculation in Python

The calculation of the test statistic and the associated *p*-value is done by `stats.ttest_ind()`

```
control = np.array([91, 87, 99, 77, 88, 91])
treatment = np.array([101, 110, 103, 93, 99, 104])
stats.ttest_ind(a=control, b=treatment,
                alternative='less', equal_var=False)
```

Result:

```
Ttest_indResult(statistic=-3.445612673536487,
  pvalue=0.003391230079206901)
```

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Independent samples

Testing procedure (continued)

5. Draw conclusion based on p-value.
   $p \approx 0.00339 < \alpha = 0.05$. We reject the null hypothesis. In this sample, there is reason to assume that the drug does indeed increase reaction time.

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Paired samples

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Paired samples

A study examined whether cars that run on petrol with additives have a lower consumption, i.e. a higher miles per gallon.
For 10 cars, the consumption was measured (expressed in miles per gallon) for both fuel types:

Car	1	2	3	4	5	6	7	8	9	10
Regular petrol	16	20	21	22	23	22	27	25	27	28
With additives	19	22	24	24	25	25	25	26	28	32

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Paired samples

Testing procedure

1. Hypotheses:
   - H_0: $\mu_{X-Y} = 0$
   - H_1: $\mu_{X-Y} < 0$
2. Significance level: $\alpha = 0.05$
3. Test statistic:
   - Based on the mean of the difference $\overline{d} = \overline{x - y}$

Regular petrol	16	20	21	22	23	22	27	25	27	28
With additives	19	22	24	24	25	25	25	26	28	32
Difference	-3	-2	-3	-2	-2	-3	2	-1	-1	-4

   - For completeness, the test statistic is

$$t = \frac{\overline{d}}{s_d / \sqrt{n}}$$

4. Calculate p

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Paired samples

Calculation in Python

```
regular = np.array([16, 20, 21, 22, 23, 22, 27, 25, 27, 28])
additives = np.array([19, 22, 24, 24, 25, 25, 26, 26, 28, 32])
stats.ttest_rel(regular, additives, alternative='less')
```

Result:

```
Ttest_relResult(statistic=-4.47213595499958,
    pvalue=0.0007749429585091)
```

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Paired samples

Testing procedure

5. Draw conclusion based on p-value.
   $p \approx 0.0007749 < \alpha = 0.05$. We reject the null hypothesis. In this sample, there is reason to assume that the fuel with additives leads to lower fuel consumption, i.e. higher miles per gallon.

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Effect size

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Effect size

Effect size

The effect size is a metric which expresses how great the difference between two groups is

- Control group vs. intervention group
- Can be used in addition to hypothesis test
- Often used in educational sciences
- There are several definitions, here: *Cohen's d*

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Cohen's *d*

$$d = \frac{\overline{x_1} - \overline{x_2}}{s}$$

where $\overline{x_1}$ and $\overline{x_2}$ represent the sample means
and s the pooled standard deviation, i.e. standard deviation of both groups combined:

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

with n_1, n_2 the sample sizes, and s_1, s_2 the standard deviations of both groups

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Interpretation Cohen's *d*

$\lvert d \rvert$	**Effect Size**
0.01	Very small
0.2	Small
0.5	Average
0.8	Large
1.2	Very large
2.0	Huge

In educational sciences (John Hattie):

- 0,4 = tipping point for desired effects
- effect size ***d* = 1**: process material that would normally take 1 year in 6 months!

E.g. `https://visible-learning.org/backup-hattie-ranking-256-effects-2017/`

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Typical approach research in education

- Research question: Is X a good learning strategy, in other words, does this have a positive effect on final results?
- Control group uses “traditional” approach
- Intervention group uses X
- Followed by an evaluation moment
- Determine scores, calculate *d*

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