Module 2. Univariate statistics

Data Science & AI

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2023–2024

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Contents

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Learning Goals

- Descriptive statistics
- Central tendency and dispersion for each measurement level
- Know formulas, being able to calculate
- Suitable visualization techniques for each measurement level

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Central Tendency and Dispersion

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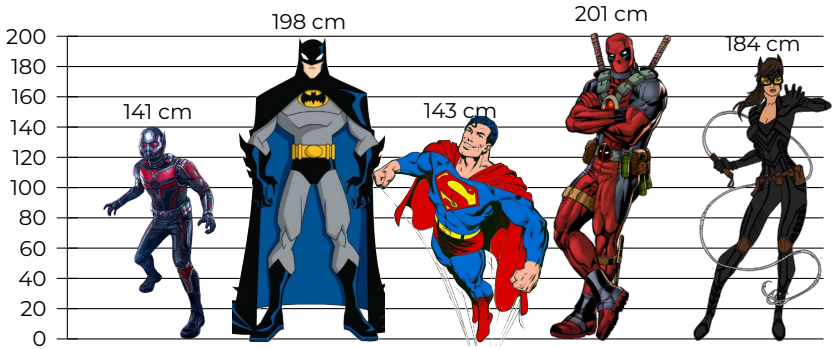
How tall are my friends?

Remember our superheroes:



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Measure of Central Tendency

What value is representative of the entire group?

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Mean or Average

Arithmetic mean

The arithmetic mean (notation: $\overline{x}$) is the sum of all values divided by the number of values

$$\overline{x} = \frac{1}{n} \sum_{x=1}^{n} x_i$$

x_1	x_2	x_3	x_4	x_5
141	198	143	201	184

Mean or Average

Q1 What happens if Ant-Man shrinks to a size of 10 cm?

Q2 The arithmetic mean of 15 numbers is 12. What number should be added to get a mean of 13?

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Median

Median

To find the median, sort all values and pick the middle number

- Odd number of values: no problem
- Even number of values: average of the middle two

x_1	x_2	x_3	x_4	x_5
141	198	143	201	184

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Median

Q1 What happens if Ant-Man shrinks to a size of 10 cm?

Q2 What is the median of the number of people saved by Batman during the last eight years?

4	7	11	16	20	22	25	26

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Mode

Mode

The mode is the value that appears most often in a dataset.

Number of people saved by Superman during the last 15 years:

3	7	5	13	20	23	39	23	40	23	14	12	56	23	29

Number of people saved by Batman during the last 8 years:

4	7	11	16	20	22	25	26

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Measures of Dispersion

How large are the differences within the group?

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Range

Range

The range of a dataset is the absolute value of the difference between the highest and the lowest value.

x_1	x_2	x_3	x_4	x_5
141	198	143	201	184

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Quartiles

Quartiles

The quartiles of a sorted set of numbers are the three values that divide the set into 4 equally large subsets. Notation: Q_1, Q_2, Q_3

Number of people saved by Superman during the last 15 years:

3	7	5	13	20	23	39	23	40	23	14	12	56	23	29

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Calculating Quartiles

- Different software programs have slightly different ways of calculating quartiles.
- The following method is easy to perform by hand. Start by sorting the values.
  - When n is odd.
    - The median (Q_2) is the middle value (as before).
    - Leave out the median. Q_1 is the median of the first half, Q_3 is the median of the second half.
  - When n is even.
    - The median (Q_2) is the average of the two middle values.
    - Q_1 is the median of the first half, Q_3 is the median of the second half.

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Interquartile Range (IQR)

Interquartile Range

The interquartile range is the difference between the third and first quartile $|Q_3 - Q_1|$.

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Variance and Standard Deviation

Variance

The variance (s^2 or σ^2) is the mean squared difference between the values of a data set and the arithmetic mean.

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

Standard deviation

The standard deviation (s or σ) is the square root of the variance

x_1	x_2	x_3	x_4	x_5
141	198	143	201	184

Properties of the Standard Deviation

- Can the standard deviation be negative?

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Properties of the Standard Deviation

- Can the standard deviation be negative?
- What is the smallest possible value? What does this imply?
- What effect do outliers have on the standard deviation?
- What is the unit of the standard deviation (in relation to the unit of the variable)?
- How do you interpret the standard deviation combined with the average?

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Properties of the Standard Deviation

Why $n - 1$ in the denominator and not n?
You can prove the reason for the change mathematically, but we will investigate it empirically

See Python example code in `demo-analysis-1-var.ipynb`

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This news item reports on high prices for houses and flats. Do the numbers give a good idea of the situation?

Remember!

Providing only a center value is never sufficient!

- What is the dispersion?
- How is the data distributed? Normal distribution?
- Is the group sufficiently homogeneous?

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Central Tendency and Dispersion: Summary

Measurement Level	Center	Spread Distribution
Qualitative	Mode	—
Quantitative	Average/Mean Median	Variance, Standard Deviation Range, Interquartile Range

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Summary of Symbols

	Population	Sample
number of elements	N	n
average or mean	μ	$\overline{x}$
variance	$\sigma^2 = \frac{\sum(x_i-\mu)^2}{N}$	$s^2 = \frac{\sum(x_i-\overline{x})^2}{n-1}$
standard deviation	σ	s

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Data visualisation

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Chart type overview

Measurement level	Chart type
Qualitative	Bar chart
Quantitative	Boxplot Histogram Density plot

See Python-example code in `demo-analysis-1-var.ipynb`

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Pie Chart

Attention!

Avoid using a pie chart!

Disadvantages:

- Comparing angles is harder than comparing length
- Unusable for data with many categories

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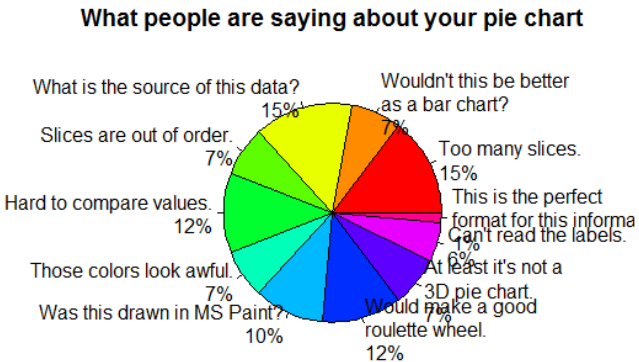
Pie Chart



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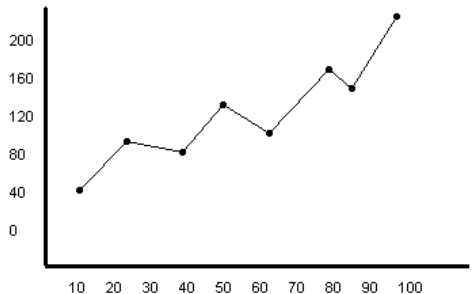
Interpretation of Charts



Tips:

- Label the axes
- Add a clear title
- Name the unit (and, if necessary, order of magnitude)
- Add a label that clarifies the chart

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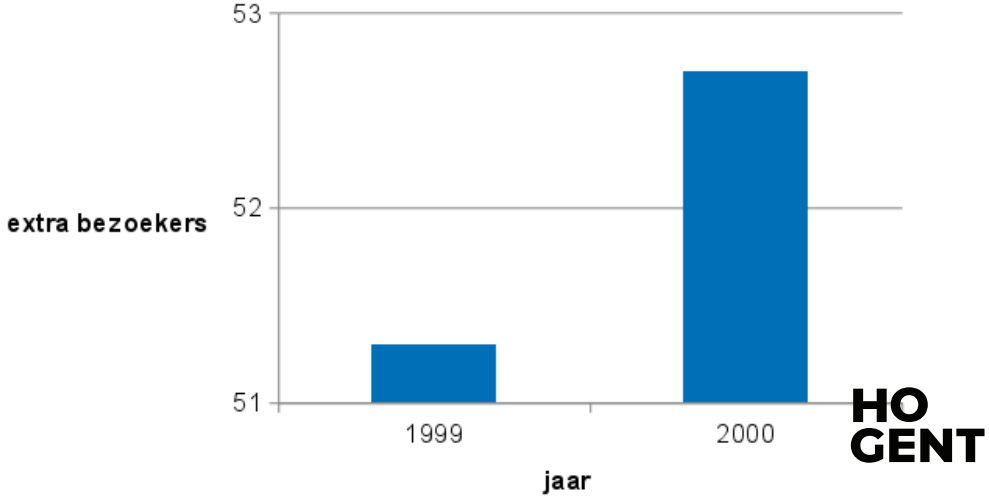
Data distortion

= misrepresenting data so that invalid conclusions are drawn



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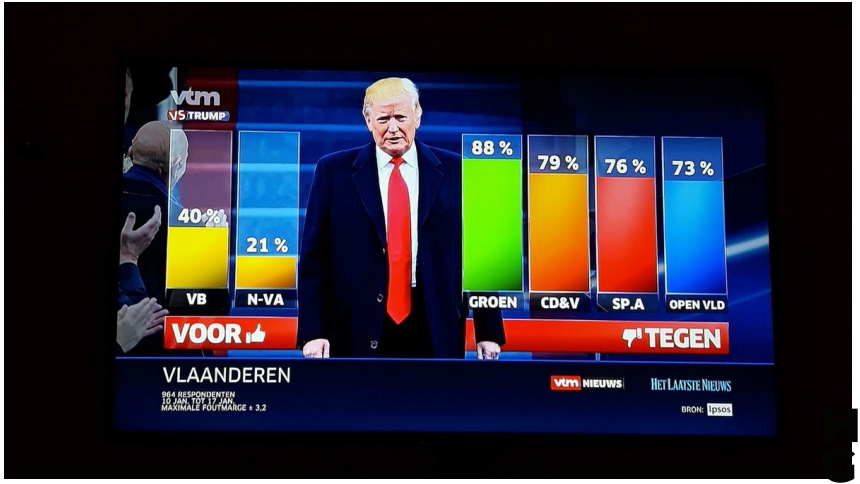
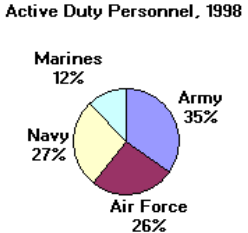
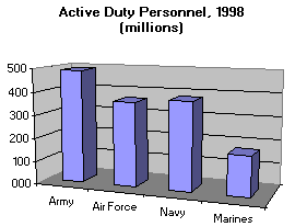
Data distortion

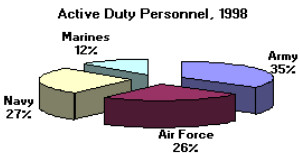


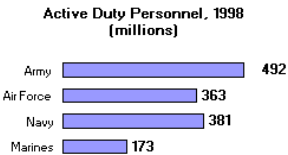
Data distraction

- Avoid bells and whistles
- Minimize “ink to data” ratio









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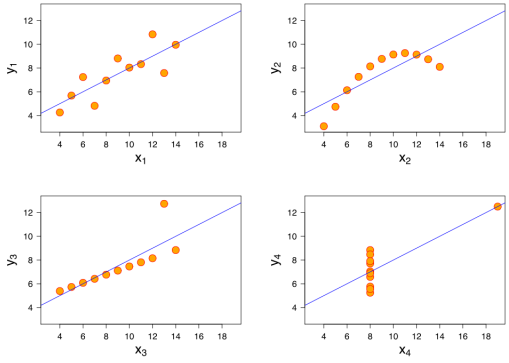
The importance of data visualization

Anscombe's Quartet are four completely different datasets with the same measurements of central tendency and dispersion.



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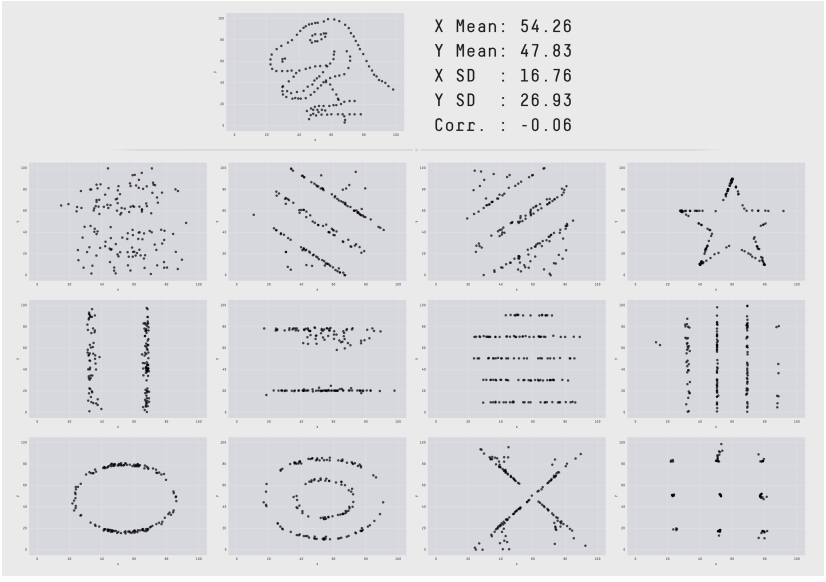

“The Datasaurus Dozen” (Source: https://www.autodeskresearch.com/publications/samestats)