Minor-Embedding For Quantum Computing

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CanaDAM

D-Wave
The Quantum Computing Company™

Brief Overview of Adiabatic Quantum Computing

Adiabatic quantum computing is a technology to find low-energy samples of *Ising problems* given by a *Hamiltonian* matrix,

$$\mathcal{H}_P(\sigma) = \sum_{i=1}^{n} h_i \sigma_i + \sum_{i \neq j} J_{i,j} \sigma_i \sigma_j$$

where σ is a spin vector $\sigma \in \{\pm 1\}^n$. These samples are produced in a process called *adiabatic annealing*, where a system of qubits is prepared in the *ground-state* of a trivial problem $\mathcal{H}_T$, and the problem is gradually transformed into the problem Hamiltonian:

$$\mathcal{H}(s) = \Delta(s)\mathcal{H}_T + \epsilon(s)\mathcal{H}_P, \qquad \mathcal{H}(0) = \mathcal{H}_T, \qquad \mathcal{H}(1) = \mathcal{H}_P.$$

The Adiabatic Theorem (google it) states that careful applications of this process can find minimum-energy solutions σ to $\mathcal{H}_P$. For the remainder of this talk, we will simply use $\mathcal{H}$ in place of $\mathcal{H}_P$.

Application 1: Max Cut

Proposition

Let G be a graph, and $\mathcal{H}(\sigma) = \sum_{ij \in E(G)} \sigma_i \sigma_j$. Minimizing $\mathcal{H}$ is equivalent to finding a max-cut of G.

Let σ be a spin vector, and let A be the set of nodes with $\sigma_a = -1$ and B be the set of nodes b with $\sigma_b = 1$. Then,

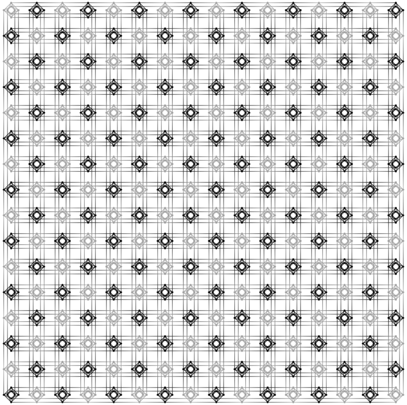
$$\begin{aligned} \mathcal{H}(\sigma) &= \sum_{ij \in E(A)} 1 + \sum_{ij \in E(B)} 1 + \sum_{ij \in E(A,B)} -1 \\ &= |E(A)| + |E(B)| - |E(A,B)|. \end{aligned}$$

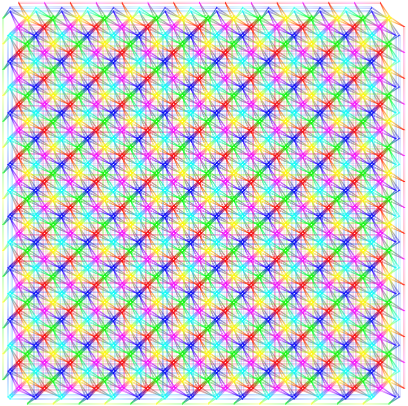
Subtracting $C = |E(G)|$ from both sides, we have

$$\mathcal{H}(\sigma) = -2|E(A,B)| + C. \quad \square$$

Motivation for Minor-Embedding

Existing adiabatic quantum computers do not implement completely-connected problems directly. The current generation of D-Wave systems implements a Chimera™ topology (left), and the forthcoming generation implements a Pegasus™ topology (right)





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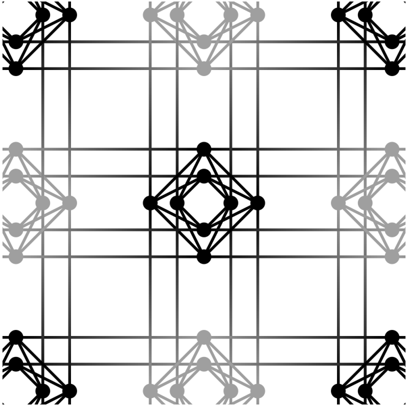
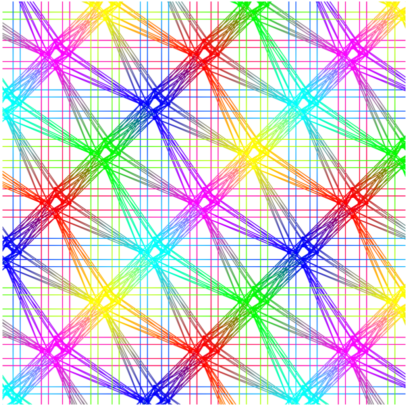




Motivation for Minor-Embedding

Problems that are *native* to the QPU have $-5 \leq h_i < 5$, and if ij is an edge in the qubit topology $\mathcal{T}$, $-2 \leq J_{ij} < 1$ (otherwise $J_{ij} = 0$). When we set $J_{ij} = -2$, the qubits i and j are encouraged to take the same spin value,

σ	$\mathcal{H}(\sigma)$
$\uparrow\uparrow$	-2
$\uparrow\downarrow$	2
$\downarrow\uparrow$	2
$\downarrow\downarrow$	-2

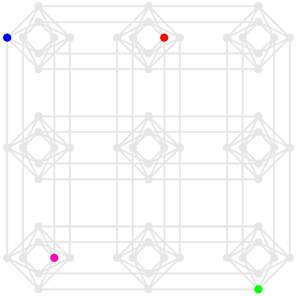
Ideally, these states occur with frequency according to a Boltzmann distribution, $\mathbf{P}[\sigma] \propto e^{\mathcal{H}(\sigma)/kT}$, and we heuristically assume that this system of two qubits will behave as a single qubit. Practically speaking, this provides a mechanism to *contract* edges of the qubit topology.

To solve a problem whose underlying graph is not native to the QPU, we must find a *minor embedding* $\varepsilon : V(G) \hookrightarrow V(\mathcal{T})$.

See Choi, **Minor-Embedding in Adiabatic Quantum Computation: I. The Parameter Setting Problem**, Quantum Information Processing, 7, 193–209, 2008. (arXiv:0804.4884)

Heuristic for Minor-Embedding (K_4 into C_3 example)

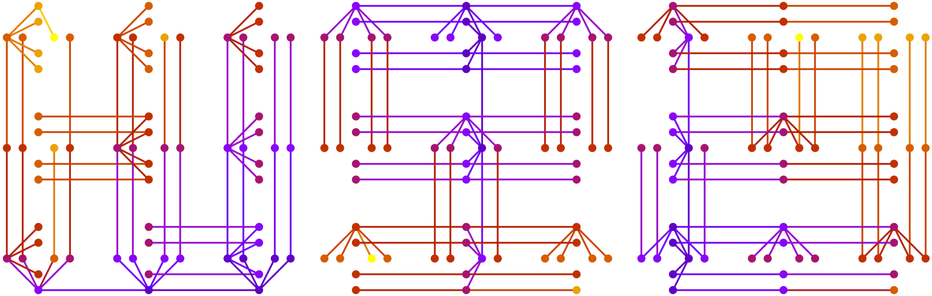
We begin with a randomized "embedding" which is not a proper minor – we've selected qubits spread far apart for the sake of illustration



See Cai, Macready and Roy, **A practical heuristic for finding graph minors**, (arXiv:1406.2741) and https://github.com/dwavesystems/minorminer

Heuristic for Minor-Embedding (K_4 into C_3 example)

We remove one *chain*, and compute distances from its neighboring chains to all qubits with node-weighted Dijkstra's algorithm (heatmap: )



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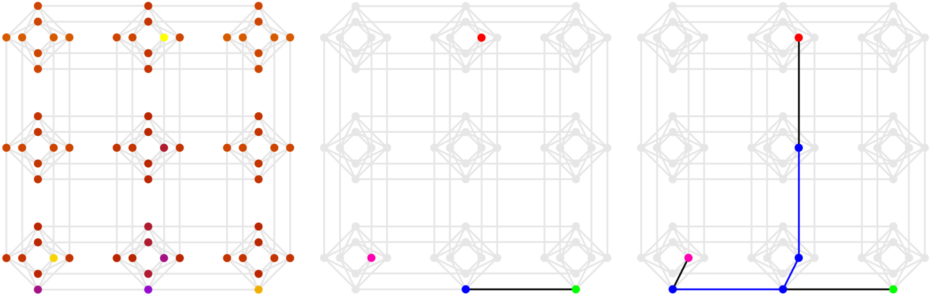
Heuristic for Minor-Embedding (K_4 into C_3 example)

We sum those distances, and find a node which minimizes that sum to find a new root, and follow shortest paths to find a new chain.



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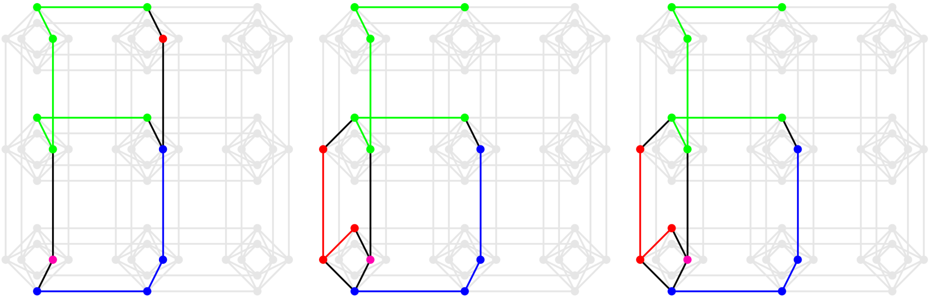
Heuristic for Minor-Embedding (K_4 into C_3 example)

We repeat that process for each node in K_4, and find an embedding.



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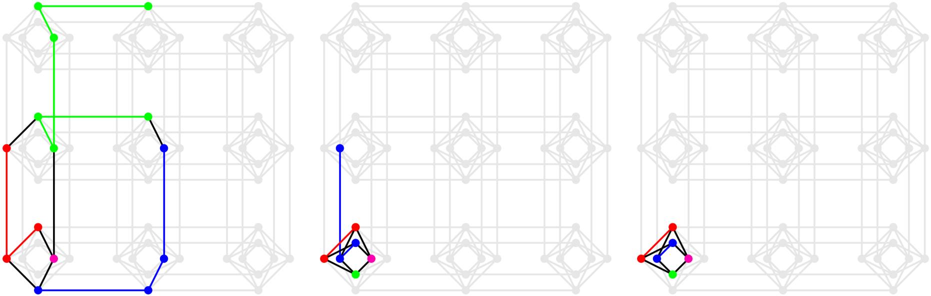
Heuristic for Minor-Embedding (K_4 into C_3 example)

This embedding is crufty because these slides were generated using a very simplified form of the algorithm. Taking a couple more passes, we get something tidier.



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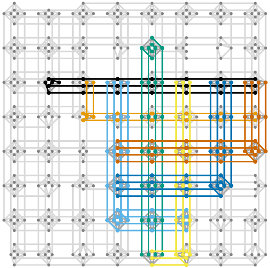
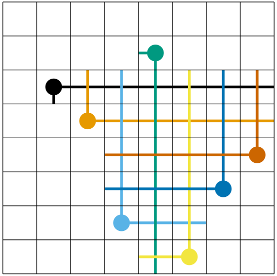
Polynomial Algorithm for Max-Clique-Embedding

Theorem

Given an induced subgraph G of a Chimera graph C_n, we can find a maximum-size native clique in time $O(n^5)$.

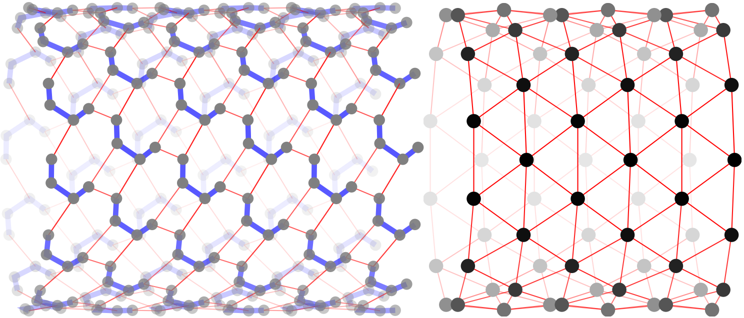




See **B.**, King and Roy, **Fast clique minor generation in Chimera qubit connectivity graphs**, Quantum Information Processing, 15(1), 495–508 (2016) (arXiv:1507.04774)

Application 2: Materials Simulation

Another application is the simulation of materials. Here, we represent lattices of spins, and compare their behaviors with those seen in nature.



King et al, **Observation of topological phenomena in a programmable lattice of 1,800 qubits**, Nature, 560 (2018)

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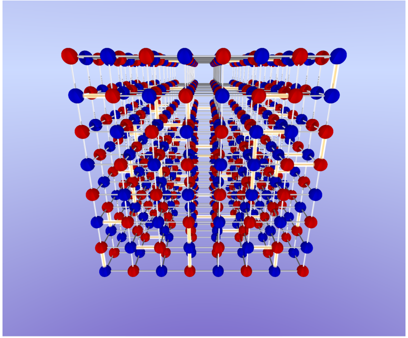
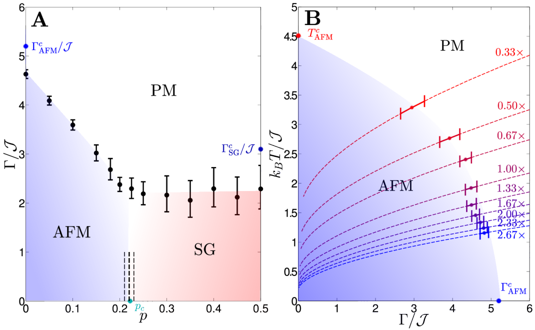
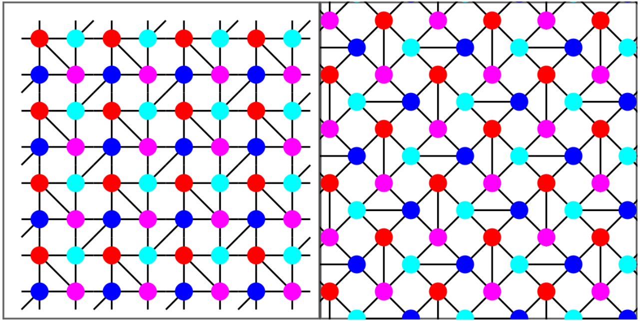




See Harris et al, **Phase transitions in a programmable quantum spin glass simulator**, Science, 361, 162–165 (2018)

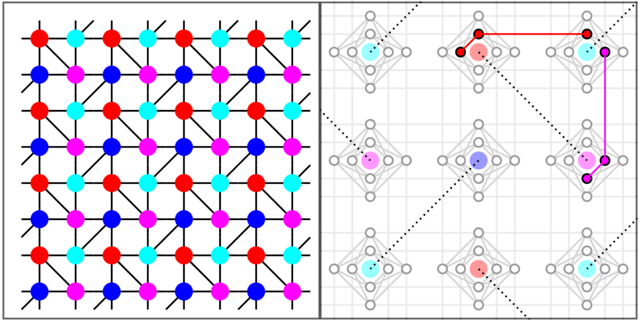
Future Work: Shastry-Sutherland Lattice

Another lattice currently under study, the *Shastry-Sutherland lattice*, presents an interesting challenge. There are two "natural" drawings, differing by a 45° rotation.



Future Work: Shastry-Sutherland Lattice

It's beneficial to embed lattices such that the embedding respects the automorphism groups of both Chimera and the embedded lattice. This is not possible, using the “grid" drawing:



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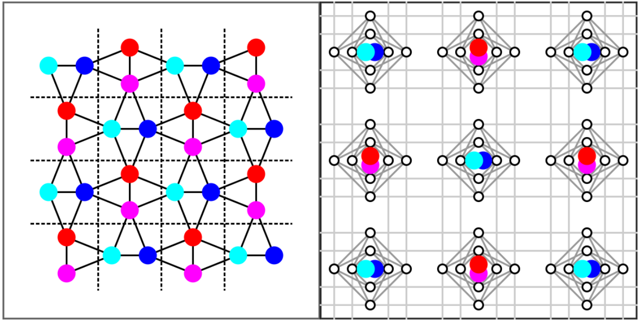
Taking the 45° rotation, we find a fully symmetric embedding: every symmetry of the lattice is a symmetry of the embedding.



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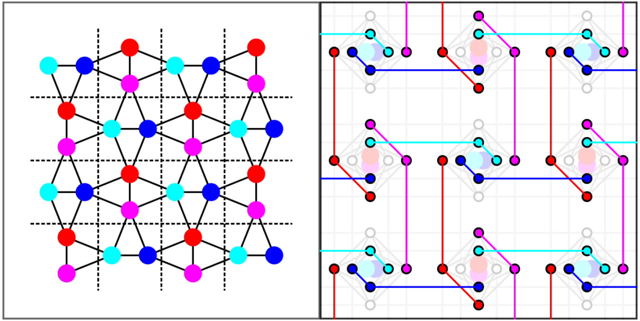

D:wave