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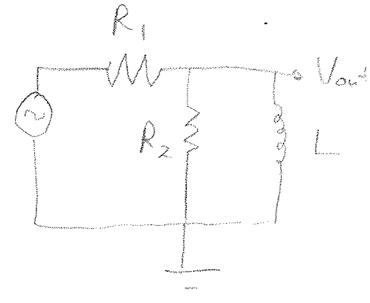
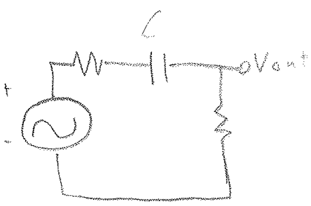
ENGR 40M Problem Set 5

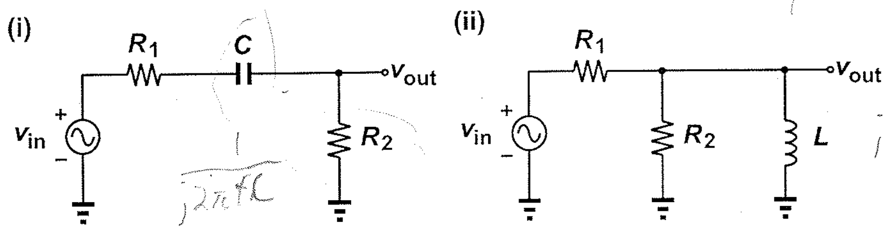
Due 1:30pm, August 14, 2017



Problem 1: Passive filters

(10 points) Please answer the follow questions based on the figure below.





$j2\pi fL$

(a) For the circuit in (i) please derive its transfer function, i.e., $\frac{V_{out}}{V_{in}}$ with respect to frequency, f.

$R_3 = (R_1 + C_c)$ $V_{out} = V_{in} \cdot \frac{R_2}{R_3 + R_2}$ $\frac{V_{out}}{V_{in}} = \frac{R_2}{\frac{1}{j2\pi fC} + R_1 + R_2}$

$R_2 = R_2$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{j2\pi f_c \cdot R_2}{1 + j2\pi fC \cdot R_2 + j2\pi fC \cdot R_1}}$$

(b) What kind of circuit is it? (*Hint: low-pass, high-pass, band-pass, or band-stop.*)

high pass. filter

(c) If the circuit in (ii) has the same transfer function, i.e., the same $\frac{V_{out}}{V_{in}}$ as that of the circuit in (i), what are the conditions that L, R_1 and R_2 need to satisfy?

Resistors in parallel

$\frac{R_2 \cdot R_1}{R_1 + R_2}$

$R_3 = R_2 + L$

$R_3 = \frac{R_2 \cdot j2\pi fl}{R_2 + j2\pi fl}$

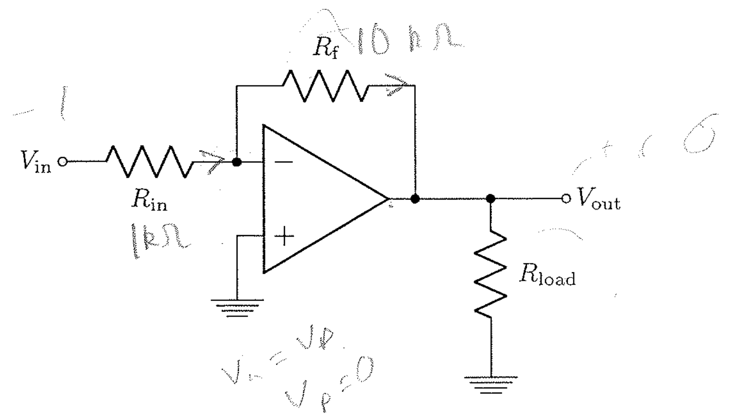
$$\boxed{\frac{V_{out}}{V_{in}} = \frac{(j2\pi fLR_2)}{\frac{R_1}{R_2 + j2\pi fl} + j2\pi fLR_2}}$$

Voltage divider $= V_{in} \cdot \frac{R_2}{R_1 + R_2}$

$\frac{\frac{R_2 \cdot j2\pi fl}{R_2 + j2\pi fl}}{R_1 + R_2 \cdot j2\pi fl / R_2 + j2\pi fl}$

Problem 2: Inverting amplifier

(15 points) Consider the inverting amplifier shown below, with $R_f = 10\,\mathrm{k\Omega}$ and $R_{in} = 1\,\mathrm{k\Omega}$.



(a) For $R_{load} = \infty$, find the gain of the circuit, V_{out}/V_{in}.

$$i_1 = \frac{V_{in} - 0}{1000\,\Omega} = \frac{0 - V_{out}}{10{,}000}$$

$$\frac{V_{in}}{1k} = \frac{-V_{out}}{10k}$$

$$10V_{in} = -V_{out} \rightarrow \boxed{\frac{V_{in}}{V_{out}} = -10}$$

(b) For $R_{load} = 200\,\Omega$, find the gain of the circuit, V_{out}/V_{in}.

$$\frac{V_{in} - 0}{1\,k\Omega} = \frac{-V_{out}}{10\,k\Omega}$$

$$\frac{V_{in}}{1} = \frac{-V_{out}}{10}$$

$$10V_{in} = -V_{out}$$

$$\boxed{\frac{V_{in}}{V_{out}} = -10}$$

$250mV = -10$

$250 \cdot -10$

$\rightarrow -2V$

(c) For $R_{\text{load}} = 200\,\Omega$, $V_{\text{in}} = 250\,\text{mV}$, find the power dissipated by R_{load}.

$250 \cdot -10$

$=$

$p = V \cdot i$ $\quad i = \frac{V}{R}$

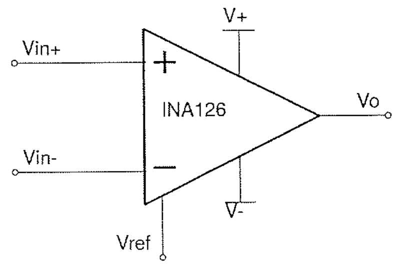
$p = \frac{V^2}{R}$

$p = \frac{(-2.5V)^2}{200\Omega} = \boxed{31.25\,mW}$

$p = \frac{V^2}{R}$

Problem 3: Instrumentation Amplifier (INA126P)

(15 points)

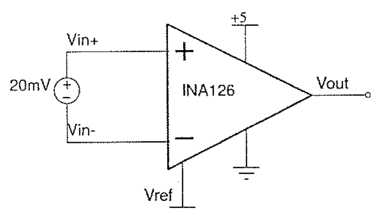


For this problem we consider a INA126P instrumentation amplifier (IA), which you will be using in lab 4. While similar to opamps, IAs don not follow the "golden rules". Instead, the behavior of the IA can be described by the following equation:

$$V_o = G \cdot (V_{\text{in}}^+ - V_{\text{in}}^-) + V_{\text{ref}},$$

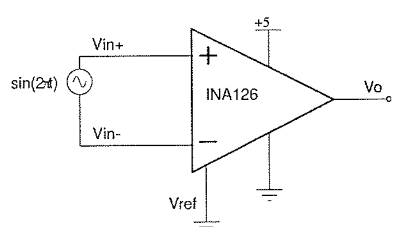
where G is the gain of the IA, V_{in}^+ and V_{in}^- are the input terminals, V_{ref} is called the "reference voltage", and V_+, V_- are used to supply power to the amplifier.

As a simple example, let us look at a DC input, assuming the IA has a gain G of 100.

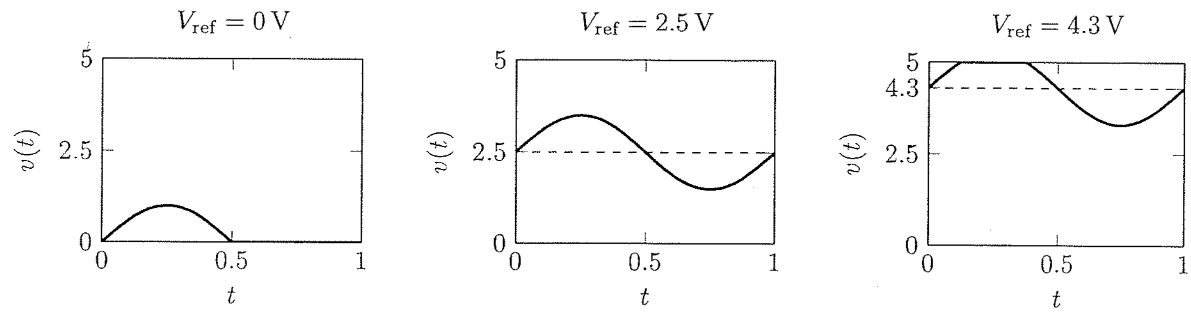


$$V_o = 100 \cdot 20\,\text{mV} + 0\,\text{V} = 2\,\text{V}.$$

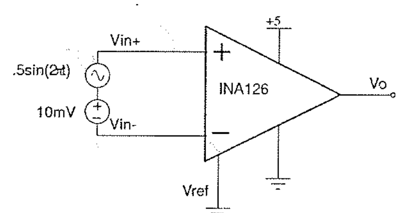
However, there is one additional consideration. The output voltage is limited by the supply voltages V_+ and V_-. Instead of going above V_+ or below V_-, the output signal will be cut off or 'clipped 'so that it stays within the supply voltages. Consider the following example.



Assume that our input is a 10 mV peak-to-peak (10 mVpp) sine wave between V_{in}^+ and V_{in}^- and $V_{\text{ref}} = 0\,\text{V}$. If $G = 100$, then we would ideally see a 1 Vpp sine wave at the output, centered at 0 V. However, since V_- is connected to GND, the output signal cannot be less than 0 V, and therefore, the output signal is clipped, as shown in the leftmost figure below. One solution to this problem is to change the reference voltage, which sets the "zero" reference of the output circuit. If our supply voltage range goes from $0 - 5\,\text{V}$, we can generate 2.5 V as the reference voltage, so we can support sine waves that go above and below the reference without going outside of our supply voltage range. With $V_{\text{ref}} = 2.5\,\text{V}$ the 1 Vpp sine wave would 2.5 V and go from $2 - 3\,\text{V}$ Other reference voltages are possible, but would cause the output signal to get clipped at lower amplitudes.



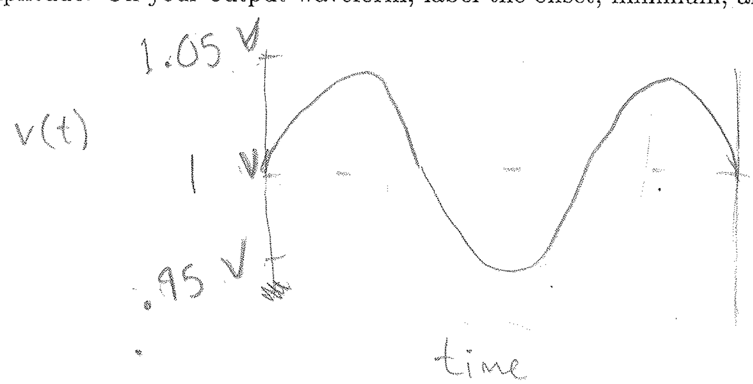
Let's start off with the below configuration. V_{ref} and V_{in}^{-} are both connected to gnd. V_{in}^{+} is connected to a a sine wave of amplitude 1 mVpp, centered at 10 mV (thus the max value is 10.5 mV, and the min value is 9.5 mV). Assume that $G = 100$.



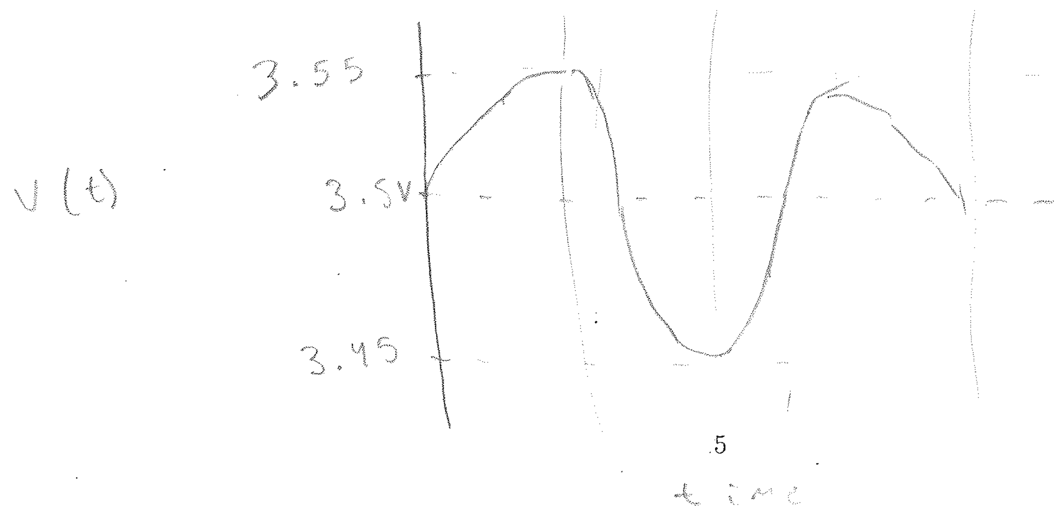
$V_{out} = 100(10\,mV) + 0$

$100 \cdot (10.5) + 2.5$

(a) Sketch the output waveform. Note that the gain applies to the offset voltage as well as the signal amplitude. On your output waveform, label the offset, minimum, and maximum voltages.



(b) Now we connect the reference pin to 2.5 V instead of ground. Sketch the new output waveform. Can you still see the full sine wave?



Yes, you can see the entire sine wave

$10\,mV + .5\,mV \quad +2.5$

(c) In truth, signals get cut off slightly before they hit the rails. To be safe, we will try to keep our entire output signal between 1 V and 4 V. With the same setup from (b), what is the largest possible gain that would not clip your output signal?

$4 = x\,(.0105) + 2.5$

$\boxed{142.857}$

(d) We can select the gain G of an IA using a resistor R_g. According to the datasheet for the INA126P,

$$V_o - V_{ref} = \left(5 + \frac{80\,\mathrm{k}\Omega}{R_g}\right) \cdot (V_{in+} - V_{in-})$$

Using this formula, what resistor value R_g will give you the gain computed in part (c)?

$142.857 = \left(5 + \frac{80{,}000}{R\,\Omega}\right)$

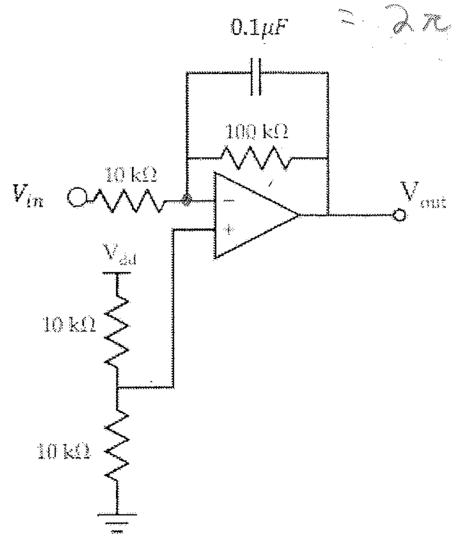
$137.857 = \frac{80{,}000}{R\,\Omega}$

$\boxed{R_g = 580\,\Omega}$

Problem 4: Operational Amplifier (LM4250CN)

(25 points) For this problem, we look at a simplified version of a subcircuit you will be using in lab 4. Assume $V_{dd} = 5\,\text{V}$.

$= \frac{1}{2\pi j f C} \quad \frac{1}{.2uF \cdot \pi f C}$



(a) Find Z_p, the overall impedance of the parallel combination of the $100\,\text{k}\Omega$ resistor and $0.1\,\mu\text{F}$ capacitor.

$\frac{R_1 \cdot R_2}{R_1 + R_2}$ 100,000

$$\frac{\frac{100k\pi}{.2uF \cdot \pi f}}{100k\Omega + \frac{1}{.2uF \cdot \pi f}} = \frac{100k\pi}{1 + (100k\Omega)(.2uF \cdot \pi f)}$$

$1\times10^5 \cdot .2\times10^{-6} = .2\times10^{-1}$

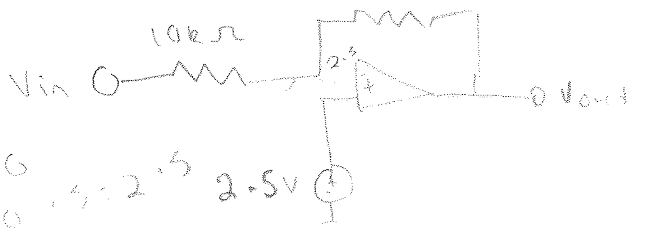
$.2\times10^{-1}$

$$\boxed{Z_p = \frac{100k\Omega}{1 + .01j2\pi f}}$$

(b) Using the ideal op-amp rules for current and voltage at the op-amp inputs and the impedances in this circuit, write the nodal equation for this circuit. Use your value of Z_p solved from part (a).
Hint: the nodal equation should be for the node at the opamp "-" input

$\frac{100k\pi}{1+.02j\pi f}$

$$\frac{V_{in} - 2.5V}{10k\Omega} = \frac{2.5 - V_{out}}{\frac{100k\Omega}{1+.02j\pi f}}$$



$\frac{10}{20} \cdot 5 = 2.5$

Nodal equation $\rightarrow$ $$\frac{V_{in} - 2.5V}{10k\Omega} = \frac{2.5 - V_{out}}{\left(\frac{100k\Omega}{1+.01 \cdot j2\pi f}\right)}$$

(c) Now using this equation, find an expression for the transfer function for this circuit as a function of frequency, which should be in the form $\frac{V_{out}-2.5}{V_{in}-2.5} = \frac{A}{1+B\cdot j2\pi f}$, where A and B depend only on the impedances in the circuit. The magnitude of the gain is the absolute value of this transfer function. Use $V_{dd} = 5\,V$.

$$\frac{(V_{in}-2.5)}{10k\Omega} = \frac{-(V_{out}-2.5)}{Z}$$

$$\frac{-Z\,(V_{in}-2.5)}{10k\Omega} = (V_{out}-2.5)$$

$$\frac{(V_{out}-2.5)}{(V_{in}-2.5)} = \frac{\left(\frac{-100k\Omega}{1+.02j\pi f}\right)}{10k\Omega} \cdot \frac{1+.02j\pi f}{1+.02j\pi f}$$

$$\boxed{\frac{(V_{out}-2.5)}{(V_{in}-2.5)} = \frac{-10\ \Omega}{(1+.01j2\pi f)}}$$

$1 = .02\cdot\pi\cdot f$

$1 = j2\pi f \cdot B$

$1 = 2\pi \cdot .01 \cdot x$

$B = .01$

(d) What is the magnitude of this transfer function, in dB, (i) at very low frequencies and (ii) at very high frequencies? (Remember, the magnitude in dB is $20\log_{10}$(magnitude).)

low frequencies (i.e. $f \to 0$)

$dB = 20\log(10)$

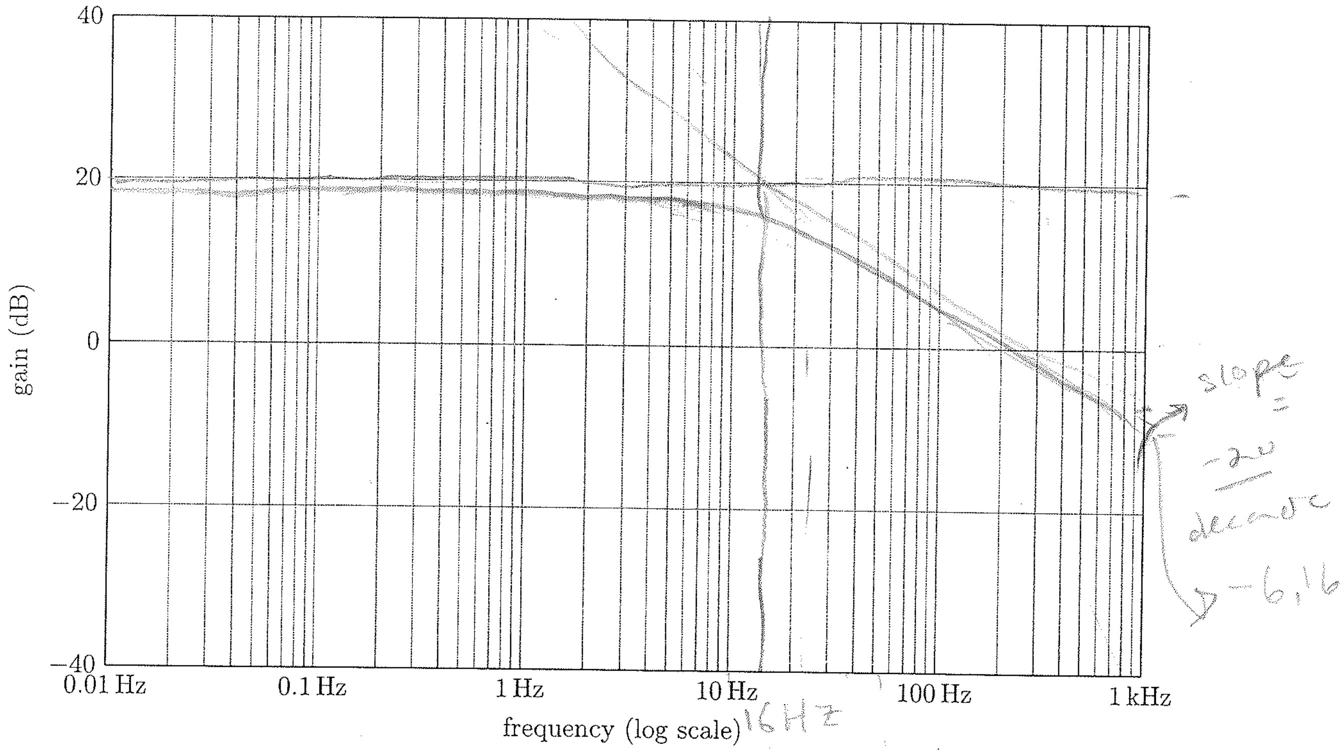
$\boxed{dB = 20 \text{ decibles}}$

high frequencies (i.e. $f \to \infty$)

$dB = 20\log(\sim 0)$

$\boxed{0\ dB}$

(e) Draw the magnitude Bode plot for this circuit, labeling the important cutoff frequencies. Use the figure below for your plot.



$1 = j \cdot 2 \cdot \pi \cdot .01 \cdot f$

$\frac{100}{50}{\pi} = f = 16$

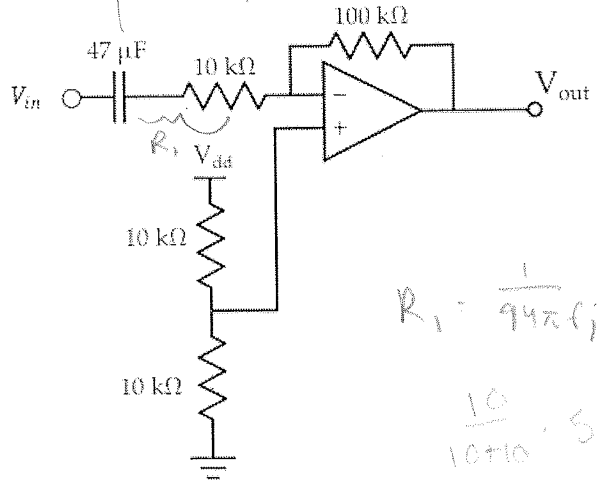
(f) By looking at the magnitude Bode plot, can you tell what this circuit does? (highpass, lowpass, bandpass, or bandstop?)

low pass filter

Problem 5: Another Op-amp Circuit

(15 points) The below circuit is a different simplification of the one you'll be making in lab 4. Assume $V_{dd} = 5\,\text{V}$.

$\frac{1}{2\pi j f C}$

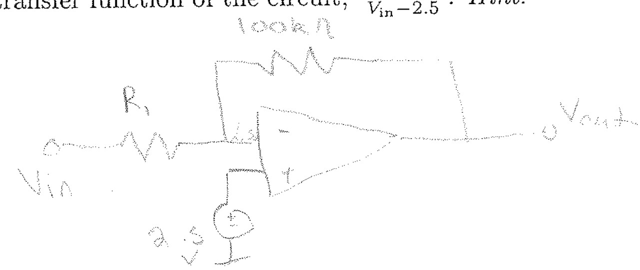


47
2
94

$R_1 = \frac{1}{94\pi f j C} + 10\,\text{k}\Omega$

$\frac{10}{10+10} \cdot 5 = 2.5$

(a) Using the same steps as above, find an equation for the transfer function of the circuit, $\frac{V_{out}-2.5}{V_{in}-2.5}$. *Hint: This transfer function will be in the form* $\frac{j2\pi f \cdot A}{1+j2\pi f \cdot B}$.



$$\frac{V_{in}-2.5}{R_1} = \frac{-(V_{out}-2.5)}{100\,\text{k}\Omega}$$

$$\frac{V_{in}-2.5}{V_{out}-2.5} = \frac{-R_1}{100\,\text{k}\Omega}$$

$$\frac{V_{out}-2.5}{V_{in}-2.5} = \frac{-100\,\text{k}\Omega}{\frac{1}{94\pi f j} + 10\,\text{k}\Omega}$$

$$\frac{V_{out}-2.5}{V_{in}-2.5} = \frac{(-100\,\text{k}\Omega)(94\pi f j)}{1 + 10\,\text{k}\Omega(94\pi f j)}$$

$$\frac{-4.72 \cdot 2\pi f j}{1 + .472 \cdot 2\pi f j} = \frac{V_{out}-2.5}{V_{in}-2.5}$$

(b) What is the magnitude of this transfer function, in dB, (i) at very low frequencies and (ii) at very high frequencies? (Remember, the magnitude in dB is $20\log_{10}(\text{magnitude})$.)

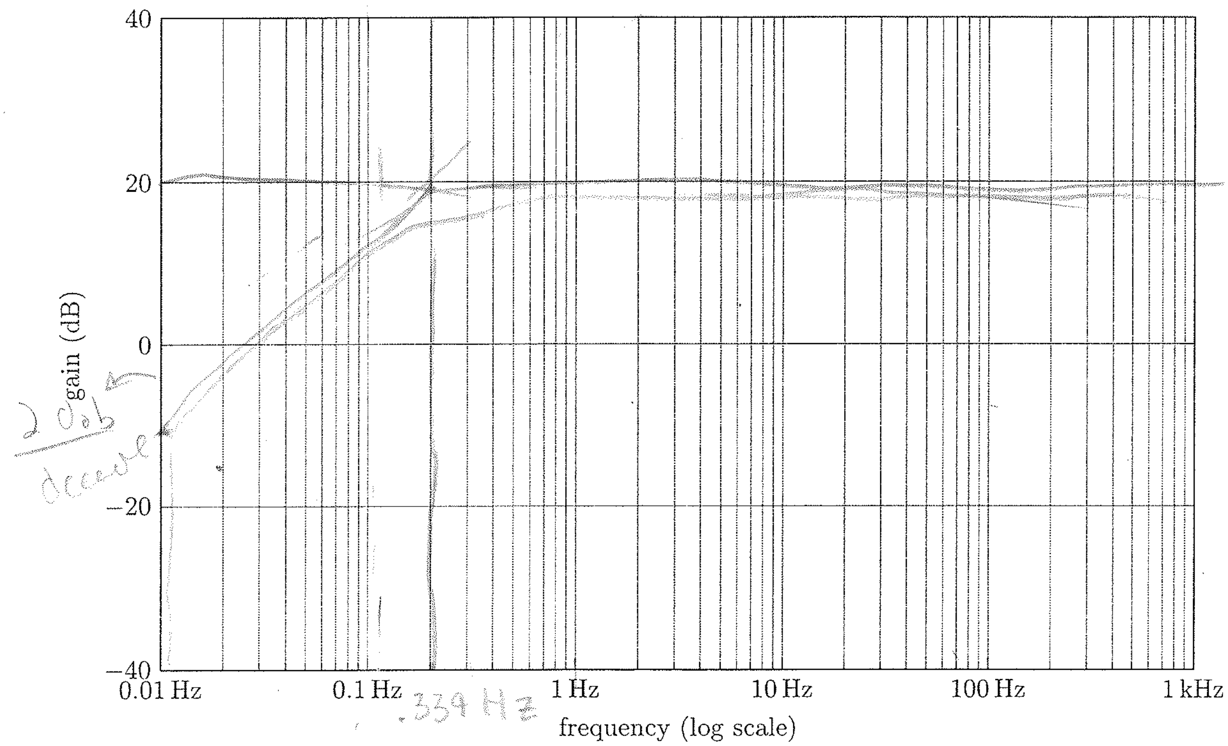
low freq $(f \to 0)$

$20 \log(0) = 0$

high freq $(f \to \infty)$

$20 \log(10) = 20$

(c) Draw the magnitude Bode plot for this circuit, labeling the important cutoff frequencies. Use the figure below for your plot.



y intercept = −10.84

$\frac{1}{B} = \frac{1}{47} = .02$

$1 = .94\pi f$

.34 Hz

(d) By looking at the magnitude Bode plot, can you tell what this circuit does? (highpass, lowpass, bandpass, or bandstop?)

high-pass filter

47μ $\frac{1}{94\pi j f 6} + 10k\Omega$

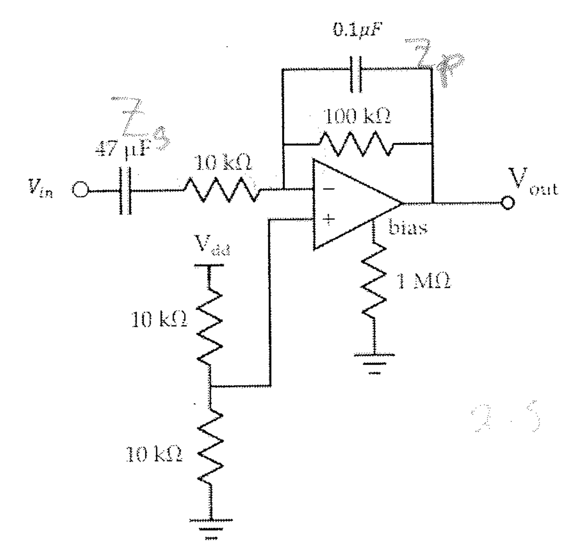
$= 94nf\pi j f + 1940$

Problem 6: Opamps: Final Version

(20 points) Now, below is the circuit we are doing in lab. Notice that all the resistors are the same and we have included both of the capacitors this time.

$100k\Omega$

$Z_p = \frac{}{14.02 j\pi f}$



$Z_s = \frac{1}{94nF \cdot \pi f j} + 10k\Omega$

2.5

(a) Write the nodal equation at the negative input as function of Z_s, the impedance of the 47 µF capacitor and the 10 kΩ resistor in series, and Z_p, the impedance of the 0.1 µF capacitor and the 100 kΩ resistor in parallel.

$\frac{V_{in} - 2.5}{Z_1} = \frac{2.5 - V_{out}}{Z_2} = \frac{V_{in} - 2.5}{Z_1} = \frac{-(V_{out} - 2.5)}{Z_2}$

$\frac{V_{in} - 2.5}{V_{out} - 2.5} = \frac{-Z_1}{Z_2} \quad \frac{V_{out} - 2.5}{V_{in} - 2.5} = \frac{-Z_p}{Z_s}$

(b) Find an equation for the transfer function in the last circuit by rearranging your nodal equation and substituting in Z_p and Z_s. You can leave the denominator in factored form (you don't have to multiply out to find the $(j2\pi f)^2$ term).

$$\frac{-4.7 \cdot 2\pi f j}{(1 + .01 \cdot 2\pi j f)(1 + .47 \cdot 2\pi j f)}$$

(c) What is the magnitude, in dB, of this transfer function at very low frequencies? At very high frequencies? What is the magnitude between the two cutoff frequencies? *Hint: In the factored transfer function, your denominator should contain two* $(1+j2\pi f\cdot A)$ *terms. Between the two cutoff frequencies,* $j2\pi f$ *will dominate the first of these terms, while the 1 will still dominate the second.*

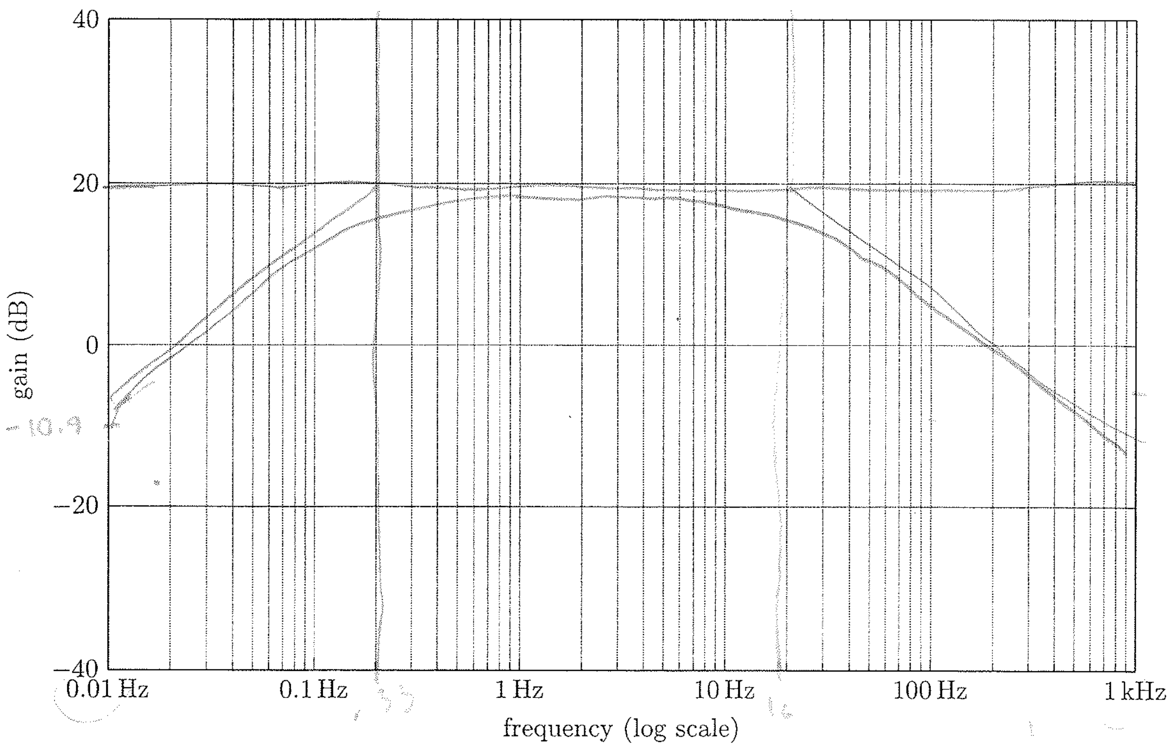
low $(f \to 0)$

as frequency approaches zero, magnitude approaches $-\infty$

high $(f \to \infty)$

as frequency approaches infinity, magnitude approaches $-\infty$

(d) Draw the magnitude bode plot for this circuit, labeling the important cutoff frequencies. Use the figure below for your plot.



$1 \cdot 47 \cdot 2\pi j f$

$.01 \cdot 2\pi j f \cdot 1$

$f = \frac{1}{.02\pi} = 15.9$

$f = \frac{1}{.47 \cdot 2 \cdot \pi} = .338$

(e) By looking at the magnitude bode plot, can you tell what this circuit does? (highpass, lowpass, bandpass, or bandstop?)

Bandpass

Problem 7: Reflection

(2 points)

(a) How long did it take you to complete this assignment?

Many hours (4+)

(b) Which problem was the most difficult, and why?

~~Pr~~ 4, had to understand

how to do it, after

that 5+6 were easier