FOUNTAINHEAD

"Math for Big Data"

~ Big Data, Small Data, No Data ~

Baruch College

Wednesday 27th January 2016

FOUNTAINHEAD

A Series of 3 Talks

This talk is the first in a series of three talks:

1. "Math for Big Data" [Wed 27th January 2016]
2. "Big Data Algorithms" [Thu 28th January 2016]
3. "Parallel C++11: Advanced C++ Skills for Quantitative Finance" [POSTPONED]

Along the way I'll talk about … Slime mold, Bitcoin, Climate change, Quantum computing, Fallibility, Apophenia, Probably Approximately Correct, Complexity … and more!

FOUNTAINHEAD

Three Talks in One!

This talk is really three talks in one:

1. Complexity, Fallibility and Finance.
2. Big Data in Finance (BDiF). The MFE course.
3. Mathematics for Big Data, with examples.

FOUNTAINHEAD

#1 Complexity, Fallibility and Finance

FOUNTAINHEAD

Complexity

Pretty much any problem can be classified as one of four problem types:

1. Simple. Easily knowable.
2. Complicated. Not simple, but still knowable and predictable.
3. Complex. Not fully knowable, limited predictability.
4. Chaotic. Neither knowable nor predictable.

FOUNTAINHEAD

Fallibility

There are only two reasons for making mistakes:

1. Ignorance.
2. Incompetence.
3. (And, maybe, “Act of God”. Though, surprisingly, that argument doesn’t work very often!)
4. (Test, test, test! Oh, libc let me down! Won’t fly.

FOUNTAINHEAD

Finance

How is all this relevant to what we do outside the classroom?

- In finance, we have simple, complicated and complex problems.
- The first thing you must do is identify what type of problem you are trying to solve.
- If and when you fail, you must identify why: ignorance, or incompetence?
- Much of what professors do—as teachers—is to help you with these key issues.

FOUNTAINHEAD

#2 Big Data in Finance
Course Outline

FOUNTAINHEAD

BDiF Course Logistics

This is the fourth year the course is running and it's been substantially revamped:

- Seven classes.
- Focus is on practical hands-on skills.
- Three hands-on assignments. (Previously six, plus project!)
- One final exam.
- Revamped based on feedback, technology change, jobs market.

FOUNTAINHEAD

BDiF Course Topics

BDiF follows a methodology for tackling Big Data projects:

- D1: Data Exploration ~ Know thy data.
- D2: Data Programming ~ Get the plumbing right.
- D3: Data Analysis ~ Use the right analysis tools.
- D4: Data Insights ~ Apply domain expertise to profit from data.

FOUNTAINHEAD

BDiF Course Data

BDiF will expose you to all the key characteristics of Big Data:

- V1: Volume.
- V2: Velocity.
- V3: Value.
- V4: Variety.
- V5: Veracity.

FOUNTAINHEAD

BDiF Course Models

With BDiF you will tackle various types of Big Data models:

- M1: Big Data in → Small Data out. E.g. CVA, VaR.
- M2: Small Data in → Big Data out. E.g. Bitcoin.
- M3: Big Data in → Big Data out. E.g. Sorting.
- M4: No Data in → Big Data out. E.g. Theoretical models.
- M5: Small Data in → Big Data out over time. E.g. Bitcoin.
- M6: Streaming Data in → Streaming Data out. E.g. Real-time.

FOUNTAINHEAD

BDiF Course Skills

Students who complete BDiF will walk away with a number of useful (and bankable!) hands-on skills, such as:

- Spark and no SQL.
- Parallel programming and running at scale.
- Graphs over Big Data.
- Data visualization.
- CVA, Bitcoin, Pattern Matching (with & without machine learning).
- Programming patterns. Big Data patterns.

FOUNTAINHEAD

BDiF Big Data

BDiF: Will expose you to real-world financial data:

- Absolute need to get your hands dirty and work with real financial Big Data.
- Know the tools.
- Need to know what tool to apply to what problem. Match capabilities to requirements. (Capability matrix.)
- All real data sets are living, breathing things. None are static.

FOUNTAINHEAD

BDiF Programming

BDiF: What does it mean to be a good programmer?

- Understand the problem and domain.
- Know what users want.
- Iterate to a good solution.
- Know the language.
- Know the tools (Github, etc.).
- Build a development toolchain.
- Have a development methodology (story about John O'Leary).

FOUNTAINHEAD

BDiF Portfolio of Projects

BDiF: Why have a personal portfolio of Big Data projects?

- There is a world of difference between talking about something and showing something.
- Show your boss - get a raise!
- Switching jobs (either internally or somewhere else).
- Starting points for more ambitious projects.
- Tip: extend your portfolio → contribute to open source projects.

FOUNTAINHEAD

#3 Mathematics for Big Data

FOUNTAINHEAD

What is Big Data?

Informal definition:

- End user: anything that doesn't fit in Excel.
- Technologist: anything for which the first thought is "Hadoop".

More formally:

- Anything that is "Big" in one or more of the "V"-axes.

FOUNTAINHEAD

Data on Big Data

Some Big Data data points (just a snapshot as of 2014):

- 1 TB: Structured trading data collected by the NYSE each day.
- 90% of stored data generated in the last 2 years [IBM].
- MapReduce: 1 PB of 100-byte records sorted in 33 min versus 6 hours in 2008.
- $600 hard drive will store all the world's music [McKinsey].

For comparison:

- Entire works of William Shakespeare is about 5 MB of text data.
- Human brain holds around 1.25 TB.
- Human brain performs at roughly 100 teraflops.
- 340,000 people working on Big Data in 2018 [McKinsey].

FOUNTAINHEAD

Why Math for Big Data?

Why is thinking mathematically about Big Data important?

- Gives us answers before we begin.
- Allows formalization of the problem and the data. Data models.
- Allows validation of the models.
- Points to underlying processes (power outages→power laws).
- Provides a big chest of useful tools.
- Helps with data selection.
- Helps with data representation.
- Counterbalances human tendency to apophenia.

FOUNTAINHEAD

Math for Big Data ~ A Survey

Where is mathematics currently being best applied to Big Data?

- Benford's law. Fraud.
- Power laws (80-20).
- Bayesian statistics.
- Optimization.
- Machine learning and AI. Why did Google give away TensorFlow?
- Transforms (Fourier, Laplace, etc.).
- Queuing and network theorems.
- Graphs.

FOUNTAINHEAD

V1: Volume

Sparse Sampling

FOUNTAINHEAD

V1: Sparse Sampling

Definition:

"Via optimized sampling, a signal can be recovered from fewer samples than required by the Shannon-Nyquist theorem."

Also known as "Compressed sensing", "Compressive sensing", or "Compressive sampling".

FOUNTAINHEAD

V1: Sparse Sampling

Key characteristics of sparse sampling:

- Shannon-Nyquist sampling theorem: must sample at least twice as fast as the highest frequency present in the signal.
- Sparcity: Information rate may be much smaller than suggested by its bandwidth. Sparse in the sense its information is compressible.
- Incoherence: Data sparse in their own domain may be spread out in the domain in which they are acquired.
- Non-intuitive!

FOUNTAINHEAD

V1: Sparse Sampling

Key benefits of sparse sampling:

- Selective and efficient data sampling can capture most of the information in a signal and condense it into a small amount of data.
- Non-adaptive. Signal independent (one-time set up).
- No need to comprehend the whole signal.
- Can reconstruct the full-length signal from the small amount of collected data.

FOUNTAINHEAD

V1: Sparse Sampling

The sensing problem:

$$y_k = \langle f, \varphi_k \rangle$$

$$k = 1, \ldots, m$$

The signal is obtained by linear functionals to record values. Correlate the object we wish to acquire with the waveforms (phi).

FOUNTAINHEAD

V1: Sparse Sampling

Sparcity:

$$f(t) = \sum_{i=1}^{n} x_i \psi_i(t)$$

When a signal has a sparse expansion, you can discard terms with small coefficients without much information loss.

FOUNTAINHEAD

V1: Sparse Sampling

Incoherence:

$$\mu(\Phi,\Psi) = \sqrt{n} \cdot \max_{1 \le k,j \le n} \left| \left\langle \varphi_k, \psi_j \right\rangle \right|$$

Coherence measures the largest correlation between any two elements of the sensing basis (phi) and representation basis (psi). If they contain correlated elements, the coherence is large.

FOUNTAINHEAD

V1: Sparse Sampling

Undersampling and sparse signal recovery:

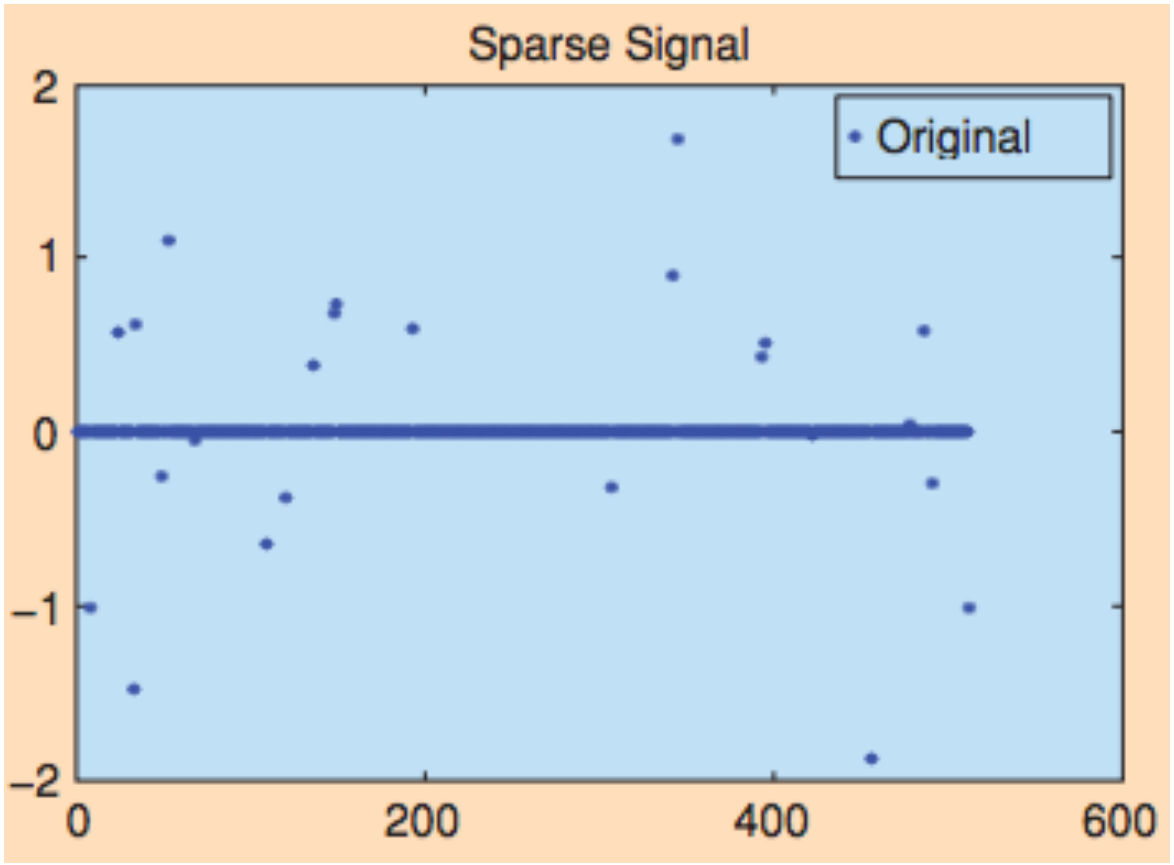
$$y_k = \langle f, \varphi_k \rangle$$

$$k \in M, M \subset \{1, \ldots, n\}, m < n$$

Ideally, would like to measure all the n coefficients of f, but we only get to observe a subset of these when collecting data. Can recover signal by optimization.

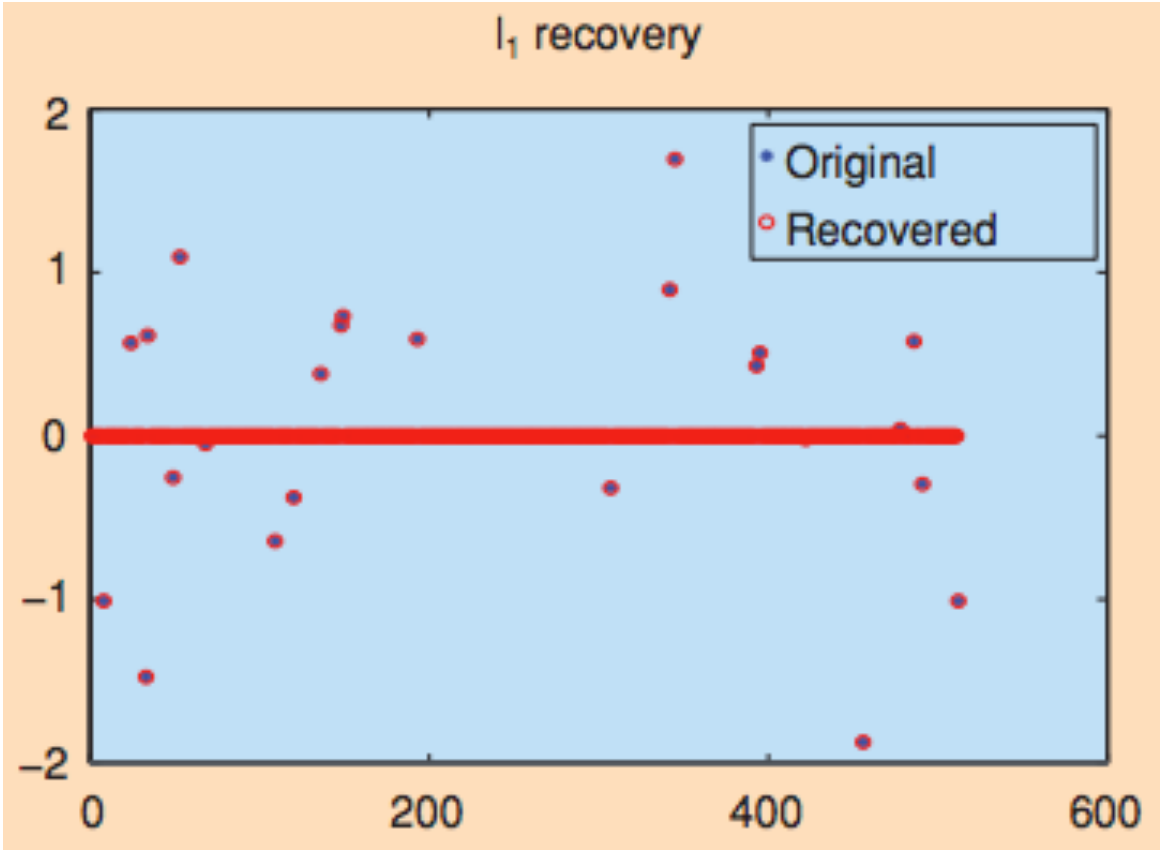
FOUNTAINHEAD

V1: Sparse Sampling



FOUNTAINHEAD

V1: Sparse Sampling



FOUNTAINHEAD

V2: Velocity

Bitcoin Transactions

FOUNTAINHEAD

V2: Bitcoin

Definition:

"Bitcoin is a P2P distributed value storage and transactional system centered on a public ledger called the blockchain."

Bitcoin is part of a wider group of emerging cryptocurrencies.

FOUNTAINHEAD

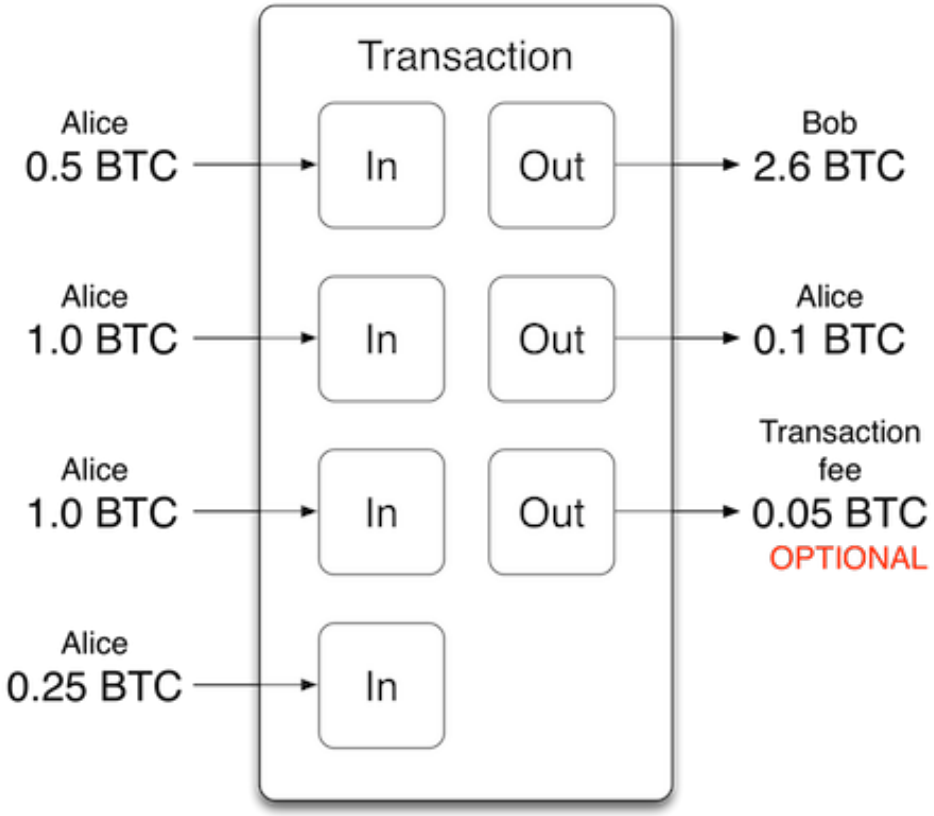
V2: Bitcoin Stats

Some bitcoin stats (as of Jan 27 2016):

- Market cap: $5.9 billion.
- BTC/USD: $390.
- Network hashing rate: 967,800,000 Thash/sec. (>> Top500)
- Daily transactions: 225,000.
- Daily transaction volume: $150 million.
- Largest recent transaction: 135,800 BTC (~$53 million)
- Average transaction confirmation time: ~8.9 minutes.

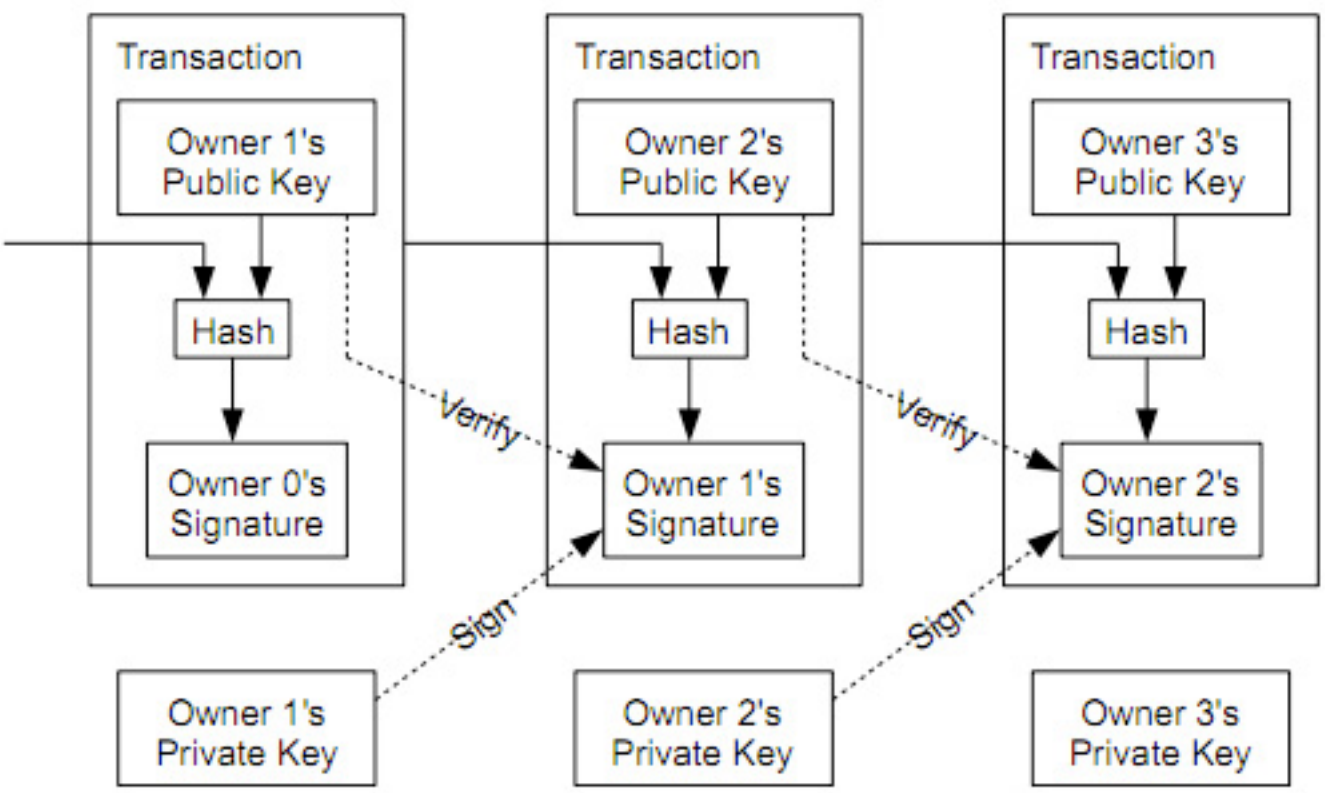
FOUNTAINHEAD

V2: Bitcoin Transaction



FOUNTAINHEAD

V2: Bitcoin Blockchain



FOUNTAINHEAD

V2: Bitcoin

Bitcoin as a currency:

- Non-fiat. Like gold. 1oz gold = 1 toga + 1 belt + 1 pair of shoes.
- Fixed issuance. Prices float. Deflationary.
- 21 million bitcoins. Each bitcoin divisible into 100 million satoshis.

Bitcoin as a technology:

- Distributed.
- Consensus through proof-of-work. No trust.
- 1000 bytes per transaction. 1MB per block. 1 block every 10 mins.

FOUNTAINHEAD

V2: Bitcoin as Currency

Enough units of bitcoin to go around? (Global currency)

- Total spendable units: 13,500,000 x 100,000,000 = 13.5 x 10^14.
- Evenly divide among world's population of 7.25 billion.
- Each person would have around 186,000 satoshis.

Compare with the U.S. dollar. (National currency)

- Total spendable units: $1.29 trillion in circulation. 1.29 x 10^14.
- Evenly divide among world's population of 7.25 billion.
- Each person would have around 17,800 cents. (100,000 all ccys)

FOUNTAINHEAD

V2: Bitcoin as Technology

Theoretical transactional bandwidth:

- Every 10 mins, 1MB block added to blockchain.
- 1MB block / 200 bytes per transaction every 10 mins.
- Theoretical transaction rate of ~10 tx per second.
- More detailed analysis (network/queuing) gives ~7 tx per second.

Compare with other payment networks:

- VISA 2,000 tx per second. 47,000 tx per second peak rate.
- PayPal average tx rate is about 115 tx per second.
- Velocity across all transaction processors?

FOUNTAINHEAD

V3: Value

Half-Life of Data

FOUNTAINHEAD

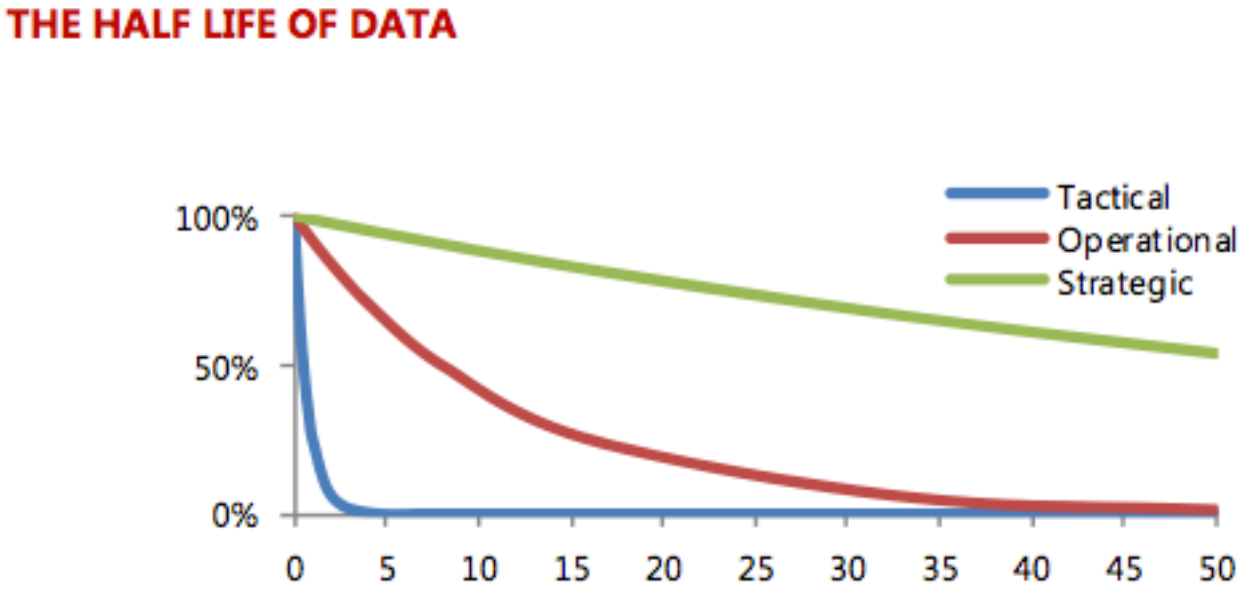
V3: Half-Life of Data

Definition:

"The half-life of any piece of data is the time it takes for its economic value to half."

FOUNTAINHEAD

V3: Half-Life of Data



FOUNTAINHEAD

V3: Half-Life of Data

From a report by Nucleus Research using data from 47 companies:

- Tactical: 30 minutes or less.
- Operational: 1 day. Natural cadence of business operations.
- Strategic: 90 days.

FOUNTAINHEAD

V3: Half-Life of Data

Half-life formula:

$$V(t) = V_0 \left(\frac{1}{2} \right)^{t/t_{1/2}}$$

FOUNTAINHEAD

V3: Half-Life of Data

So why is half-life important? Why is it useful?

- Makes you focus on the most valuable data.
- Provides criteria for when to throw away, or ignore, data.
- Helps when making tradeoff between accuracy and speed.

Incidentally, a weighting function for time series data is just another alternative expression of the half-life concept.

FOUNTAINHEAD

V4: Variety

Curse of Dimensionality

FOUNTAINHEAD

V4: Curse of Dimensionality

Definition:

"The *curse of dimensionality* refers to various phenomena that arise when analyzing and organizing data in high-dimensional spaces (often with hundreds or thousands of dimensions) that do not occur in low-dimensional settings such as the three-dimensional physical space of everyday experience."

FOUNTAINHEAD

V4: Curse of Dimensionality

Curse of dimensionality in a nutshell:

- When dimensionality increases, volume of space increases dramatically to make data sparse.
- Sparsity is a problem for any method that requires statistical significance.
- To make results statistically significant, data size grows exponentially with the dimensionality.
- A problem with the data? Or a problem with the algorithms?

FOUNTAINHEAD

V4: Curse of Dimensionality

Areas in which the curse of dimensionality is problematic:

- Combinatorics.
- Sampling.
- Optimization.
- Machine learning.
- Bayesian statistics.
- Distance functions.
- K-nearest neighbor classification.
- … and many more!

FOUNTAINHEAD

V4: Curse of Dimensionality

Strategies to deal with the curse of dimensionality:

- Pre-processing to a lower dimensional form. c.f. WWII Spitfire.
- Dimension reduction. E.g. principal component analysis.
- Better data organization. Make data less sparse along certain axes.
- Better algorithms.

FOUNTAINHEAD

V5: Veracity

What is Randomness?

FOUNTAINHEAD

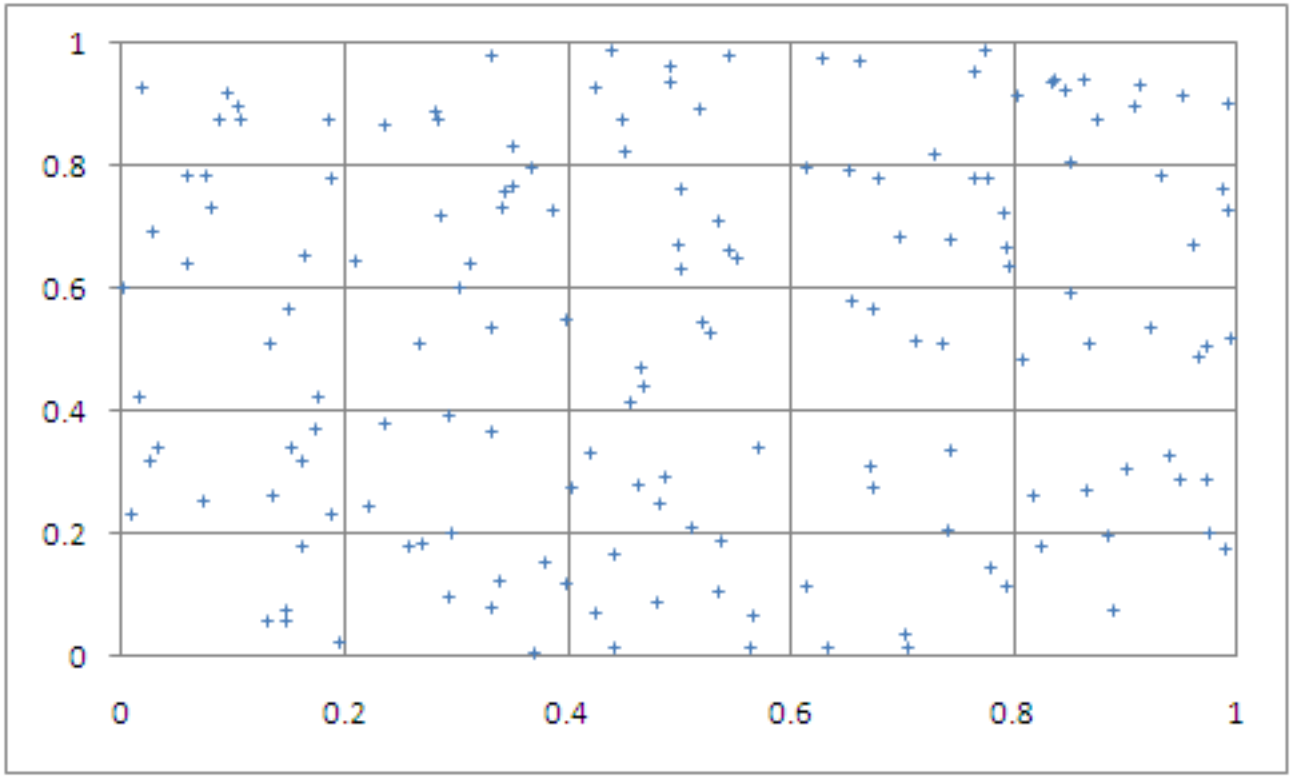
V5: Randomness

Definition:

"Randomness means lack of pattern or predictability in events. Randomness suggests a non-order or non-coherence in a sequence of symbols or steps, such that there is no intelligible pattern or combination."

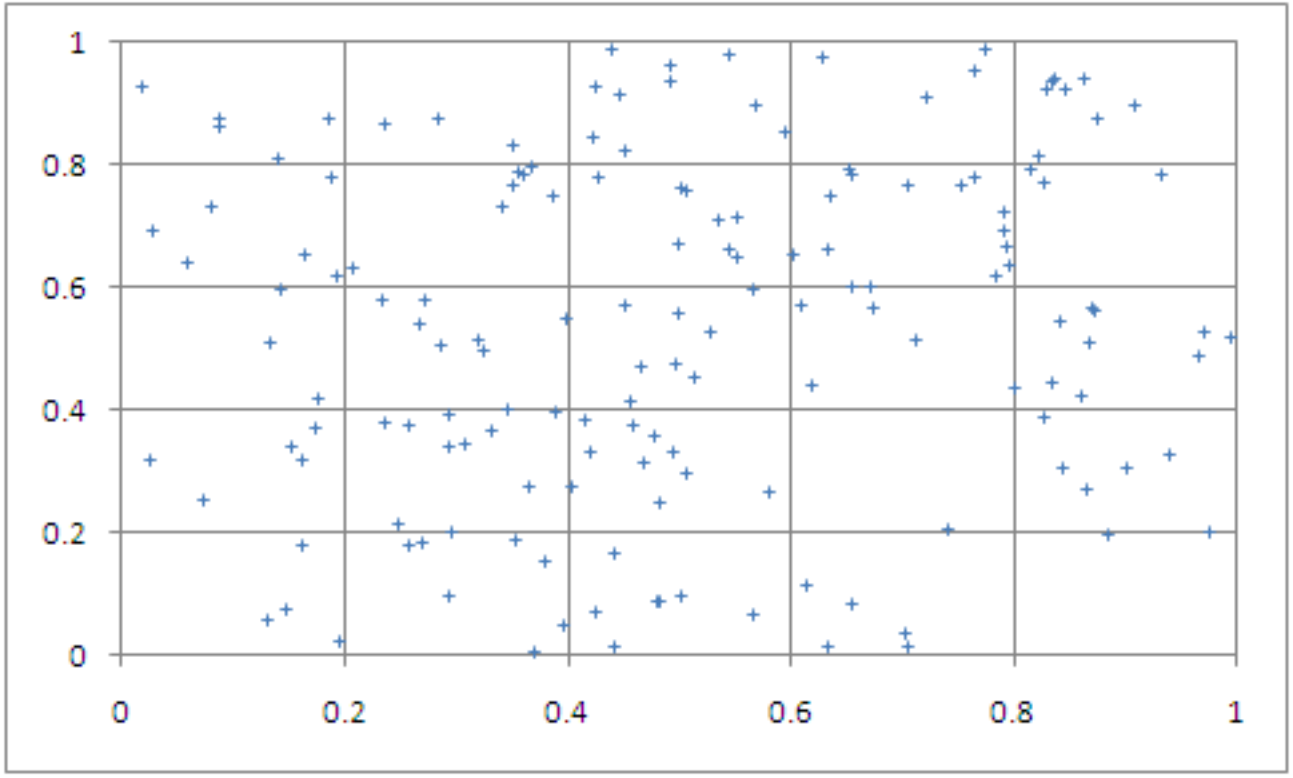
FOUNTAINHEAD

V5: Randomness – Chart 1



FOUNTAINHEAD

V5: Randomness – Chart 2



FOUNTAINHEAD

V5: Randomness

This is how charts 1 and 2 were generated:

- Produce 158 random points [a(n), b(n)], n=1,...,158
- Produce 158 random deviates u(n), v(n), n=1,...,158
- Define x(n) as follows for n>1: if u(n) < r, then x(n) = a(n), else x(n) = s*v(n)*a(n) + [1-s*v(n)]*x(n-1), with x(1)=a(1)
- Define y(n) as follows for n>1: if u(n) < r, then y(n) = b(n), else y(n) = s*v(n)*b(n) + [1-s*v(n)]*y(n-1), with y(1)=b(1)
- Chart 1: x(n)=a(n), y(n)=b(n)
- Chart 2: r=0.5, s=0.5

FOUNTAINHEAD

V5: Randomness

Towards a test of randomness:

- Nearest neighbors. But what about points at the boundaries?
- Correlation. Both charts show zero correlation!
- Number of points in sub-squares. Chi-square test. Sparseness?
- Compare with random simulation, then apply tests (e.g. k-th neighbors). “If it walks like a duck and quacks like a duck …”
- Large number of points in high dimensions. Curse of dimensionality. Apophenia.

FOUNTAINHEAD

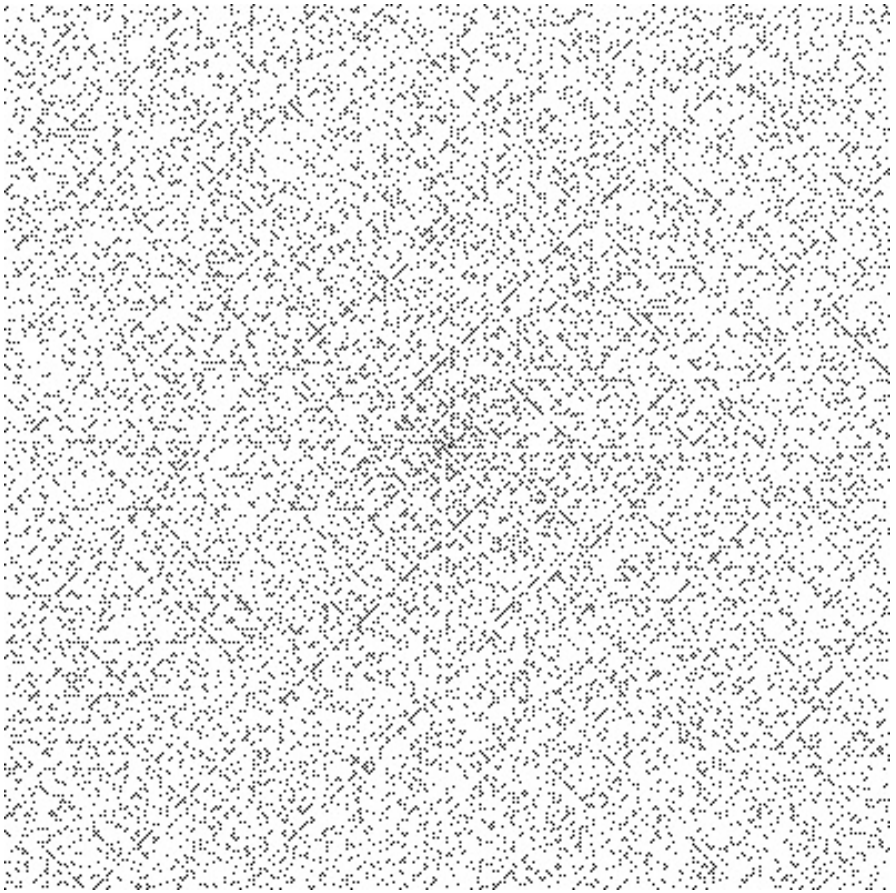
V5: Randomness

Some extra wrinkles:

- Is the number 6 random?
- Scale. HHHHH unlikely in a short sequence. Certain in a long one.
- Local randomness.
- No definitive tests. NIST tests made up from 15 separate tests.
- Plot and then view. Ulam Spiral. Apophenia.

FOUNTAINHEAD

V5: Randomness



FOUNTAINHEAD

V5: Apophenia