Biomedical Signal Analysis

Hsiao-Lung Chan, Ph.D.

Department of Electrical Engineering,

Chang-Gung University, Taiwan

chanhl@mail.cgu.edu.tw

Outline

- Spectral analysis
- Digital filters

Joseph Fourier

lived from 1768 to 1830

Fourier studied the mathematical theory of heat conduction. He established the partial differential equation governing heat diffusion and solved it by using infinite series of trigonometric functions.

Find out more at:
http://www-history.mcs.st-andrews.ac.uk/history/Mathematicians/Fourier.html

Fourier transform

$$\underbrace{X(f)}_{\text{Spectrum at frequency } f} = \int_{-\infty}^{\infty} x(t)\underbrace{e^{-j2\pi ft}}_{\text{Basis function for frequency } f}dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

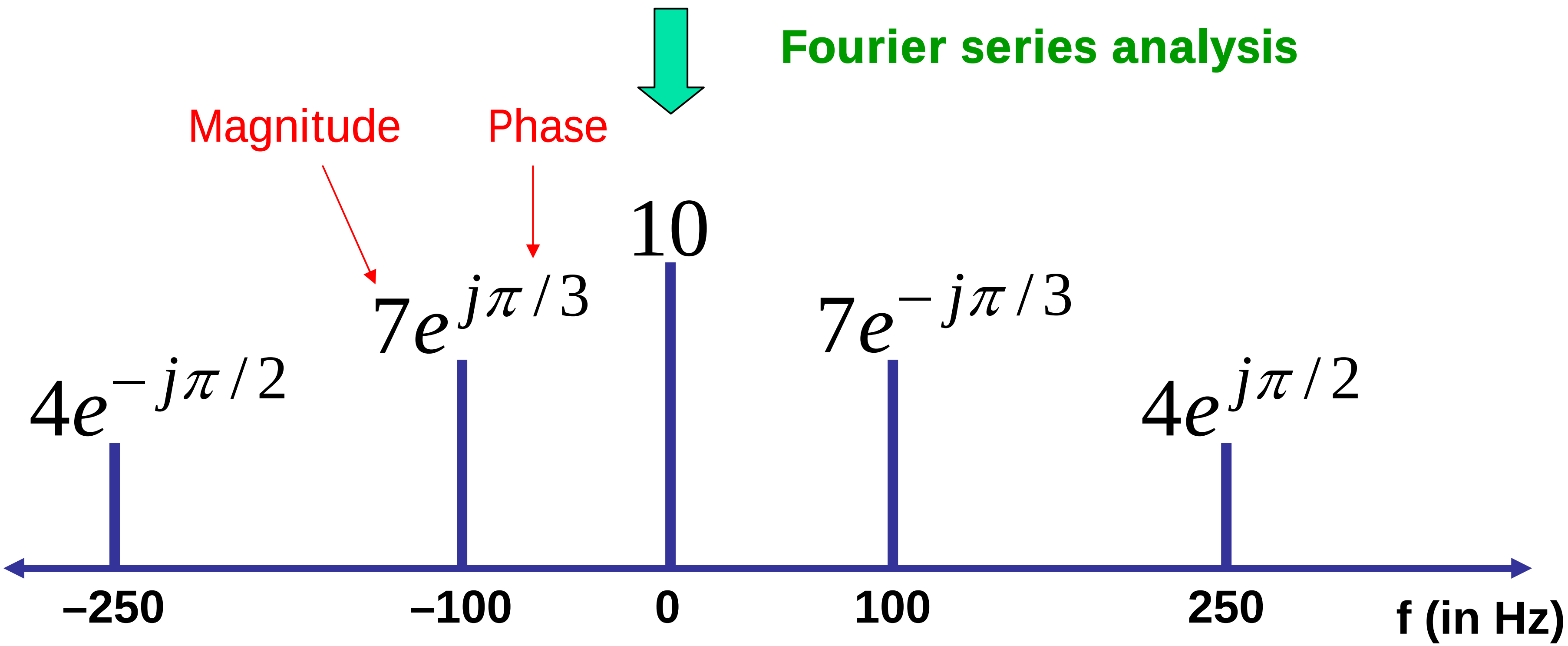
Inverse Fourier transform

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{-j2\pi ft}df$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

Example

$$x(t) = 10 + 14\cos(2\pi(100)t - \pi/3) + 8\cos(2\pi(250)t + \pi/2)$$



Magnitude and phase

$$X(f) = |X(f)|\, e^{j\phi(f)}$$

where magnitude and phase spectra

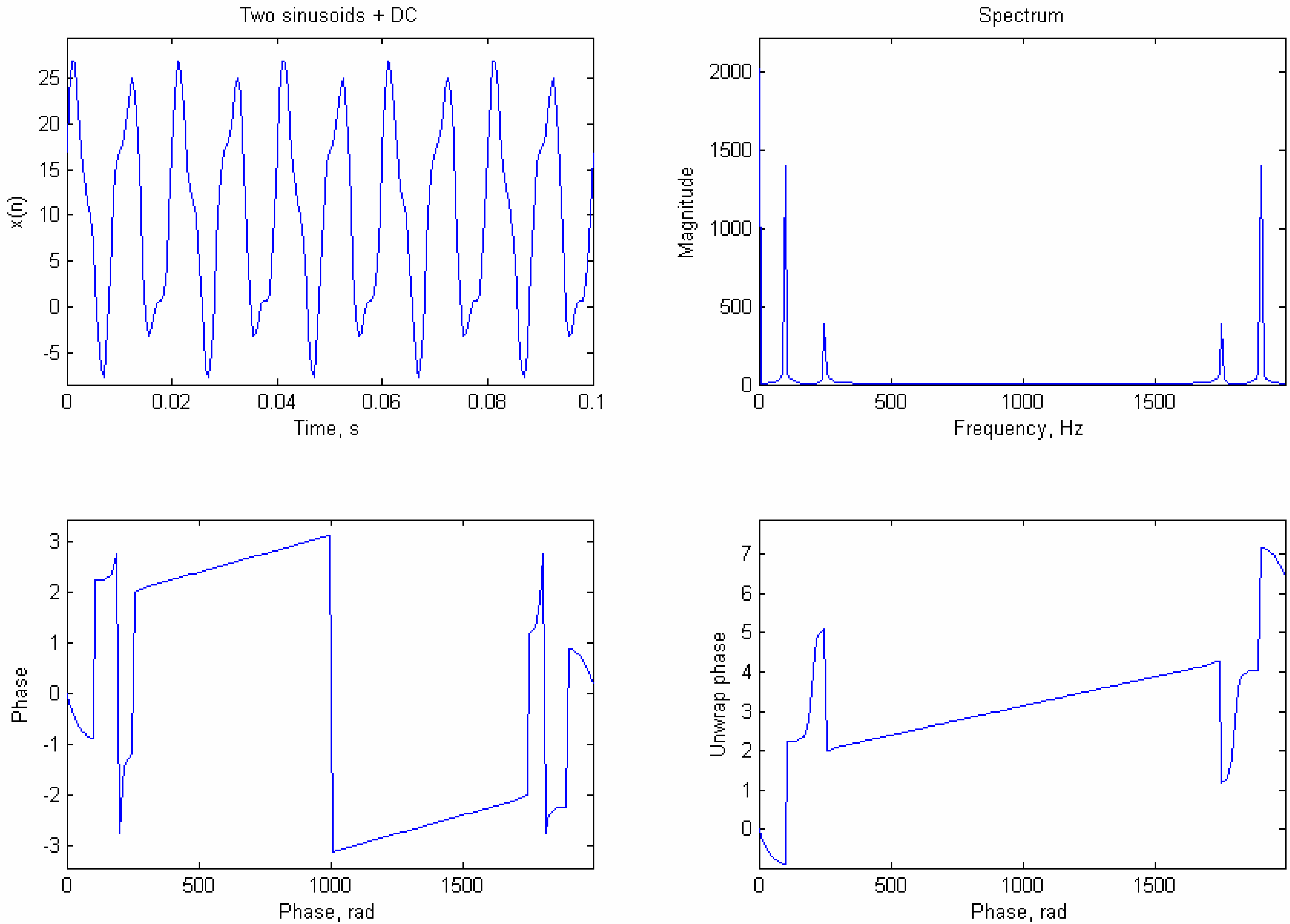
$$|X(f)| = \sqrt{\{\mathrm{Re}[X(f)]\}^2 + \{\mathrm{Im}[X(f)]\}^2}$$

$$\phi(f) = \tan^{-1}\left\{\frac{\mathrm{Im}[X(f)]}{\mathrm{Re}[X(f)]}\right\}$$

```
% Generating two sinusoids and one DC component
fs=2000;   % sampling at 2 kHz
t=0:1/fs:0.1;
x=10 + 14*cos(2*pi*100*t-pi/3) + 4*cos(2*pi*250*t-pi/2);
subplot(2,2,1)
plot(t,x)
ylabel('x(n)')
xlabel('Time, s')
title('Two sinusoids + DC')
axis([min(t) max(t) min(x)*1.1 max(x)*1.1])
```

```
% Spectral analysis
Xf=fft(x);
resolution=fs/length(Xf);
f=(0:length(Xf)-1)*resolution;
Xf_mag = abs(Xf);  % magnitude of spectrum
subplot(2,2,2)
plot(f,Xf_mag)
xlabel('Frequency, Hz')
ylabel('Magnitude')
title('Spectrum')
axis([min(f) max(f) 0 max(Xf_mag)*1.1])
```

```
Xf_phase = angle(Xf);                    % phase of spectrum
subplot(2,2,3)
plot(f,Xf_phase)
xlabel('Phase, rad');   ylabel('Phase');
axis([min(f) max(f) min(Xf_phase)*1.1 max(Xf_phase)*1.1])

Xf_phase = unwrap(Xf_phase);     % Unwrap phase angle
subplot(2,2,4)
plot(f,Xf_phase)
xlabel('Phase, rad');    ylabel('Unwrap phase');
axis([min(f) max(f) min(Xf_phase)*1.1 max(Xf_phase)*1.1])
```

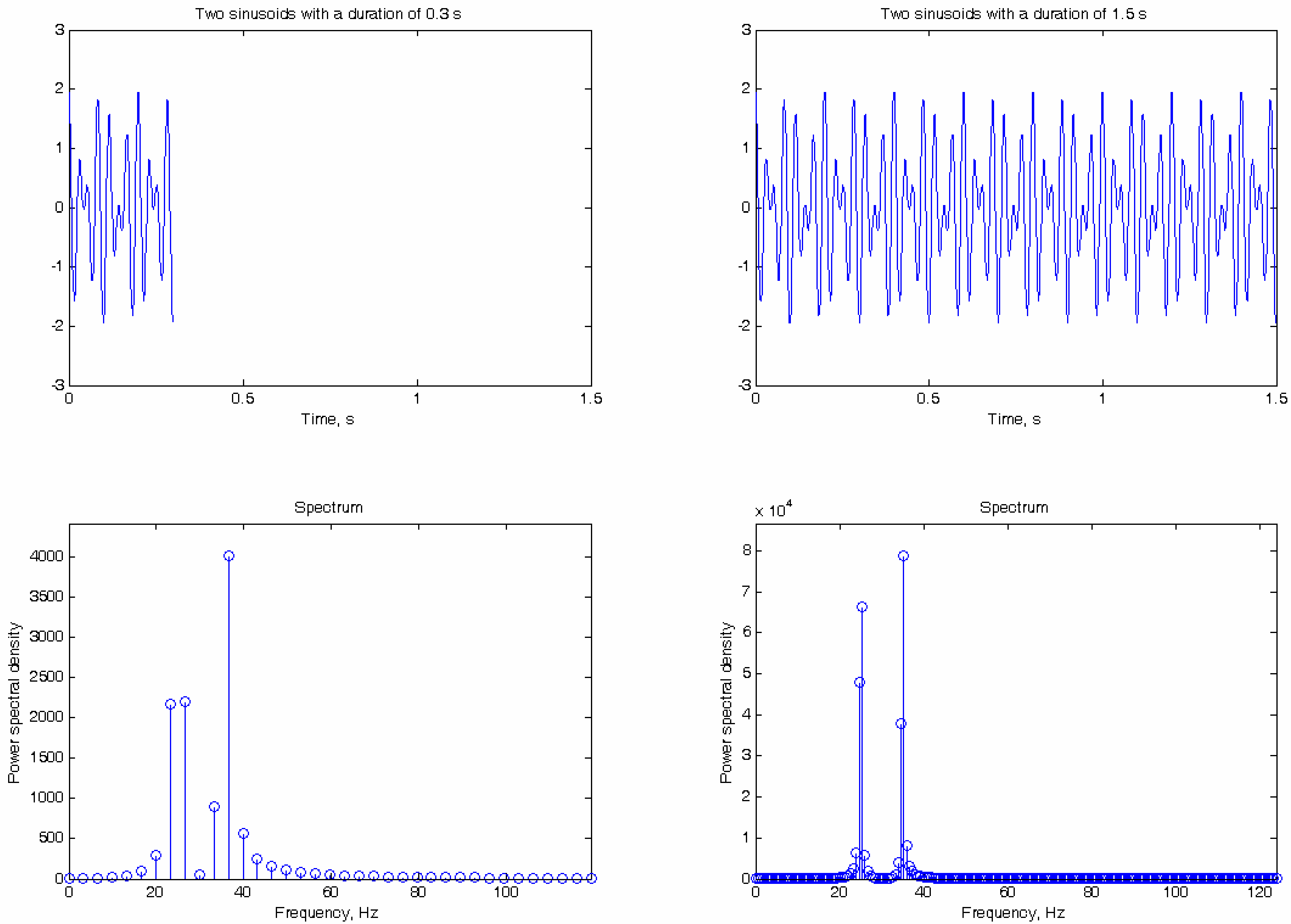

Two sinusoids + DC
x(n)
Time, s
Spectrum
Magnitude
Frequency, Hz
Phase
Phase, rad
Unwrap phase
Phase, rad

Effect of data points on spectral analysis

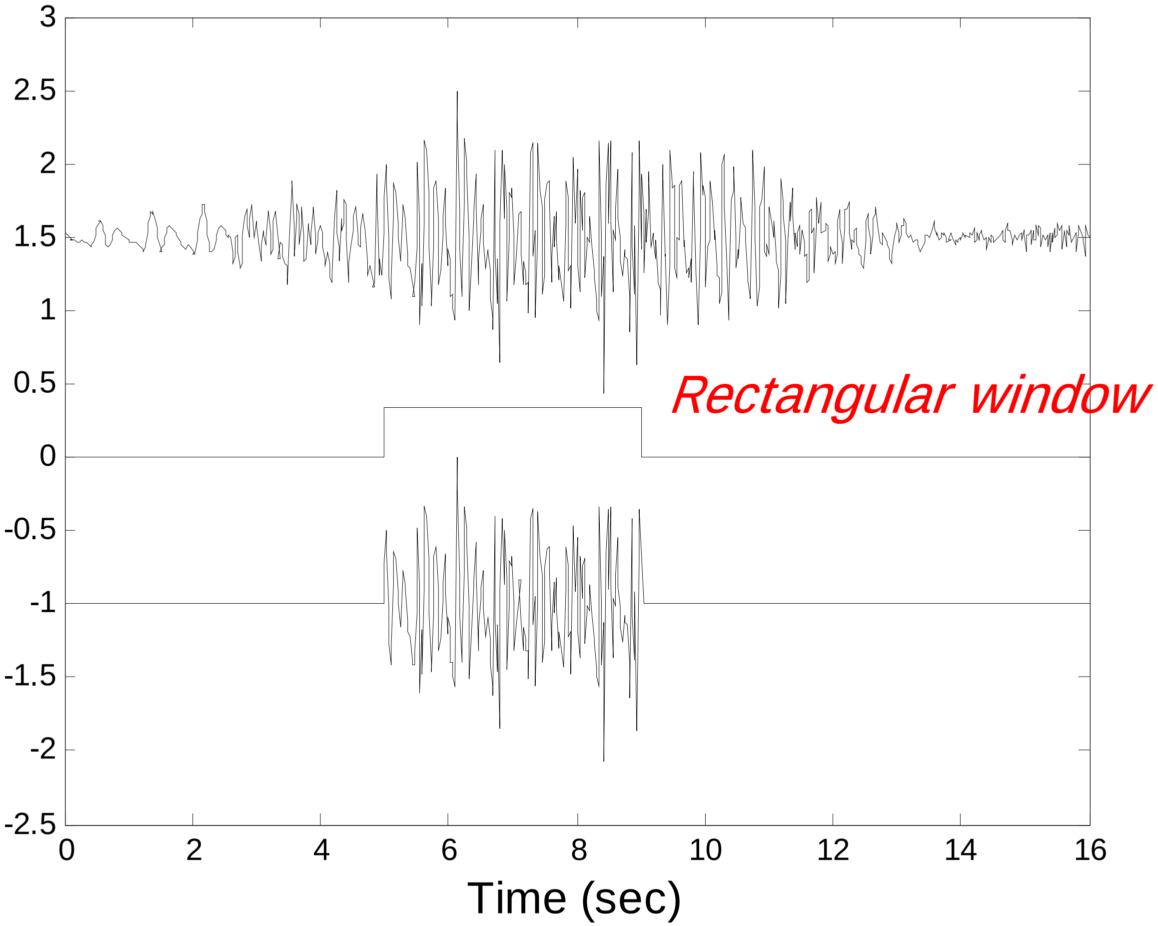
```
fs=500;
t=0:1/fs:0.3;
x = cos(2*pi*25*t) + cos(2*pi*35*t+pi/10);

Xf=fft(x);
resolution=fs/length(Xf);
f=(0:length(Xf)-1)*resolution;
Xf_power = Xf.*conj(Xf);  % power spectral density
index=1:length(Xf)/4;
stem(f(index),Xf_power(index))
```

Effect of data points on spectral analysis

```
% Improve resolution by increase data length
t=0:1/fs:0.3*5;
x = cos(2*pi*25*t) + cos(2*pi*35*t+pi/10);

Xf=fft(x);
resolution=fs/length(Xf);
f=(0:length(Xf)-1)*resolution;
Xf_power = Xf.*conj(Xf);  % power spectral density
index=1:length(Xf)/4;
stem(f(index),Xf_power(index))
```


Two sinusoids with a duration of 0.3 s
Time, s
Two sinusoids with a duration of 1.5 s
Time, s
Spectrum
Power spectral density
Frequency, Hz
Spectrum
x 10^4
Power spectral density
Frequency, Hz

Data length: truncation



Window functions

Rectangular:

$$w(n) = 1$$

Blackman:

$$w(n) = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right)$$

Hamming:

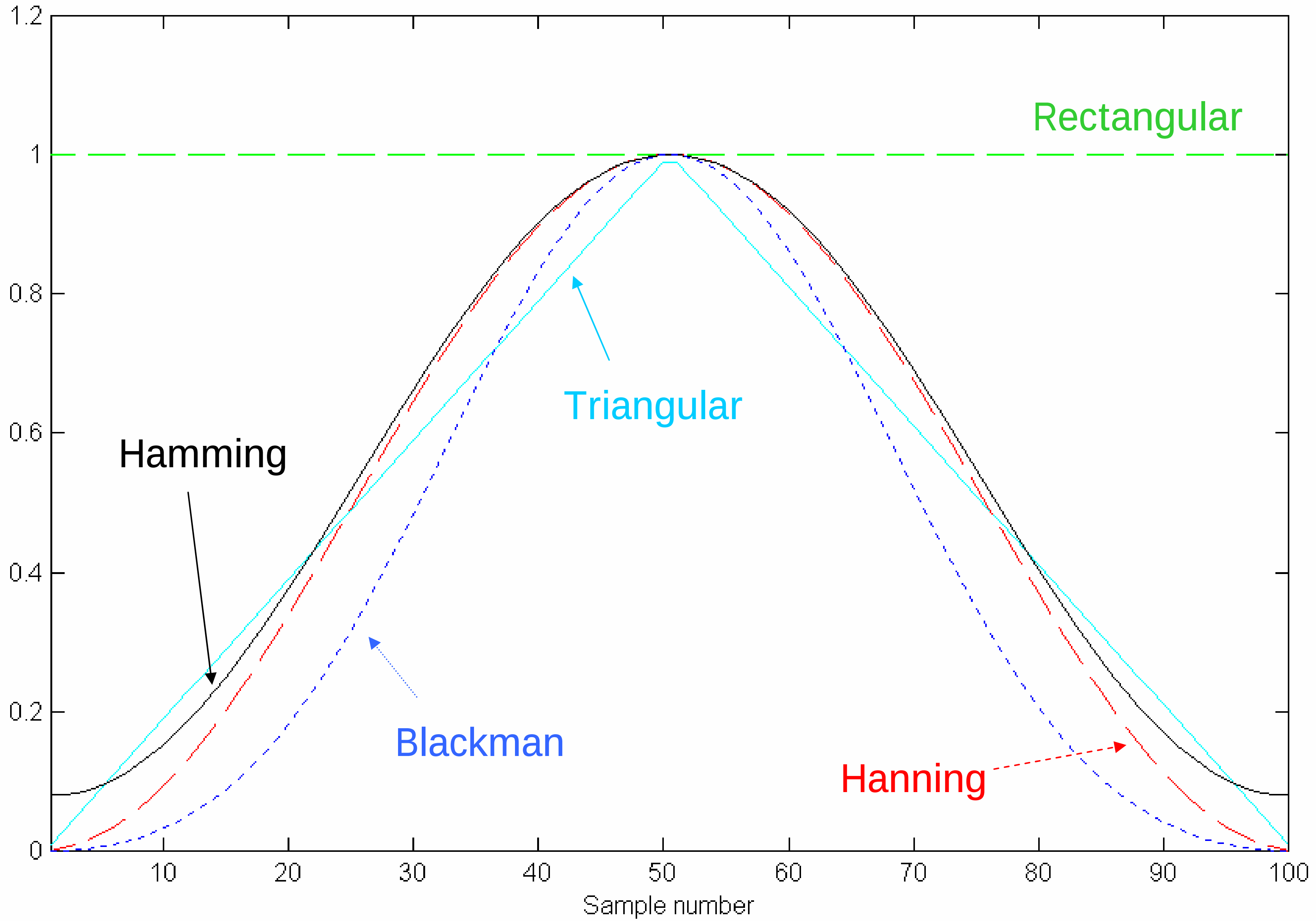
$$w(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)$$

Bartlett (Triangular):

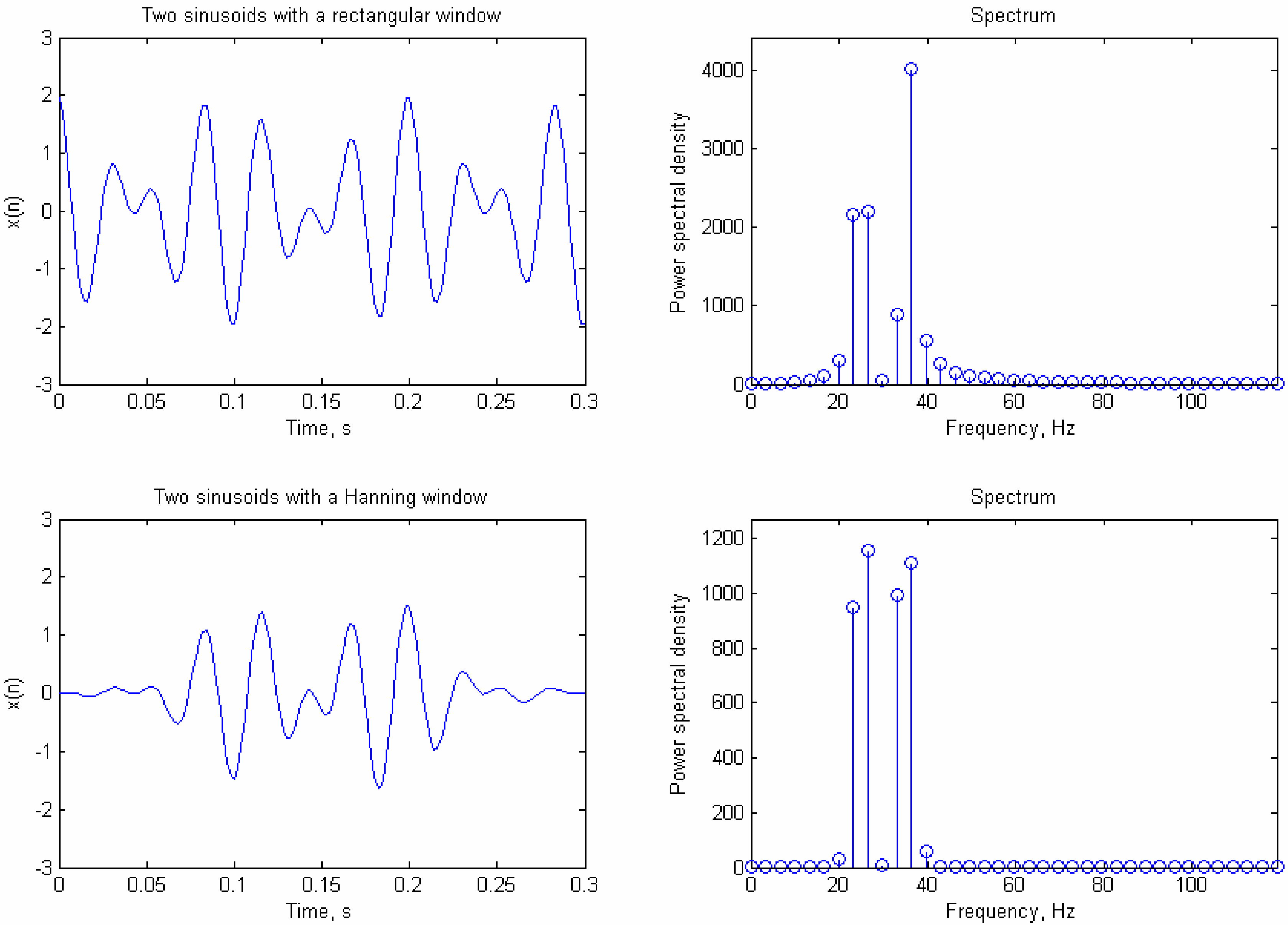
$$w(n) = \begin{cases} \dfrac{2n}{N-1}, & 0 \le n \le \dfrac{N-1}{2} \\ 2 - \dfrac{2n}{N-1}, & \dfrac{N-1}{2} \le n \le N-1 \end{cases}$$

Hanning:

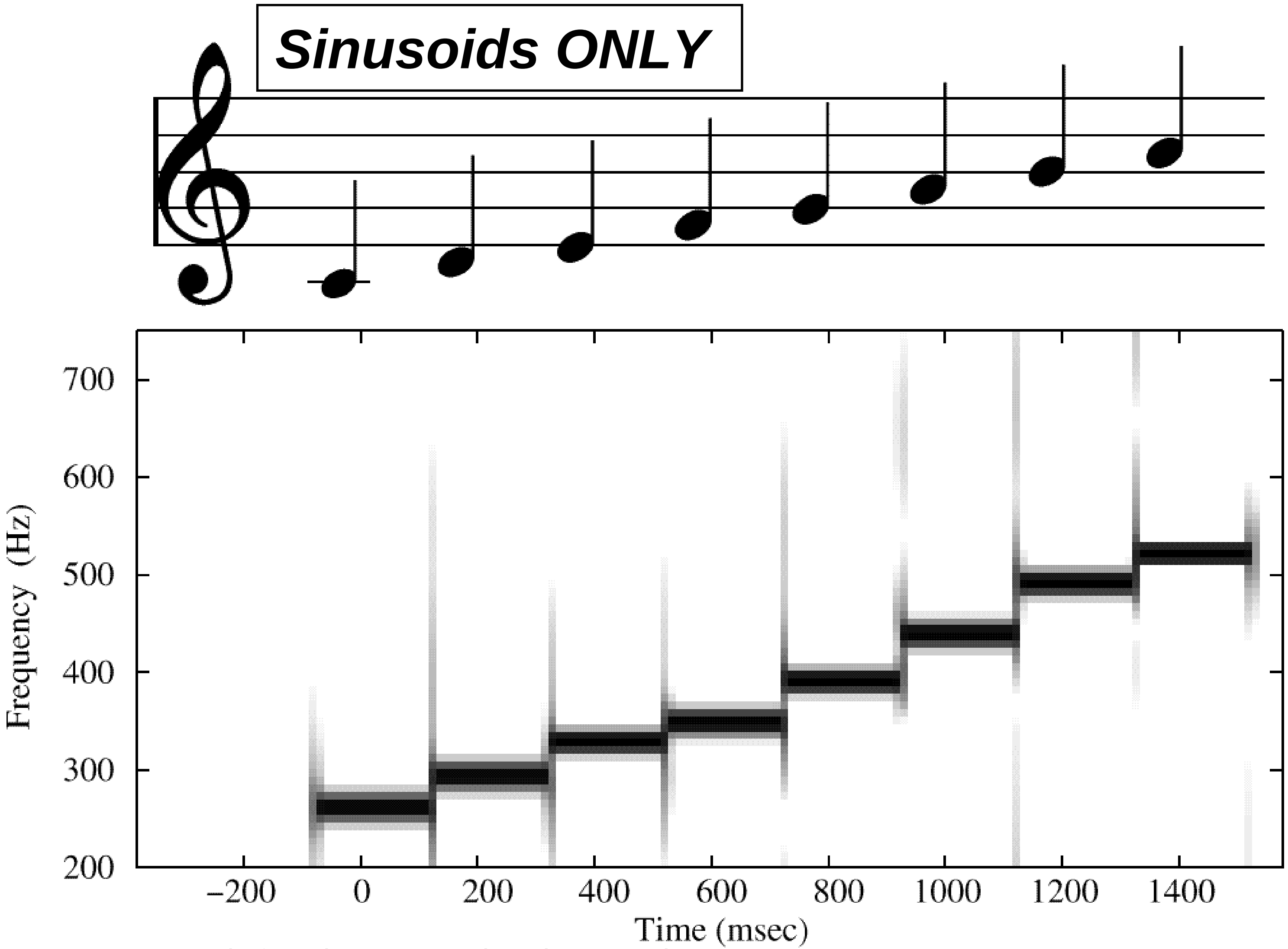
$$w(n) = 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right)$$


1.2
1
0.8
0.6
0.4
0.2
0
10
20
30
40
50
60
70
80
90
100
Sample number
Rectangular
Triangular
Hamming
Blackman
Hanning

```
fs=500;
t=0:1/fs:0.3;
x = cos(2*pi*25*t) + cos(2*pi*35*t+pi/10);

Xf=fft(x);
resolution=fs/length(Xf);
f=(0:length(Xf)-1)*resolution;
Xf_power = Xf.*conj(Xf);    % power spectral density
index=1:length(Xf)/4;
stem(f(index),Xf_power(index))

% Using Hanning window
x=x.*hanning(length(x))';
Xf=fft(x);
resolution=fs/length(Xf);
f=(0:length(Xf)-1)*resolution;
Xf_power = Xf.*conj(Xf);  % power spectral density
index=1:length(Xf)/4;
stem(f(index),Xf_power(index))
```

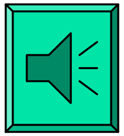

Two sinusoids with a rectangular window
x(n)
Time, s
Spectrum
Power spectral density
Frequency, Hz
Two sinusoids with a Hanning window
x(n)
Time, s
Spectrum
Power spectral density
Frequency, Hz

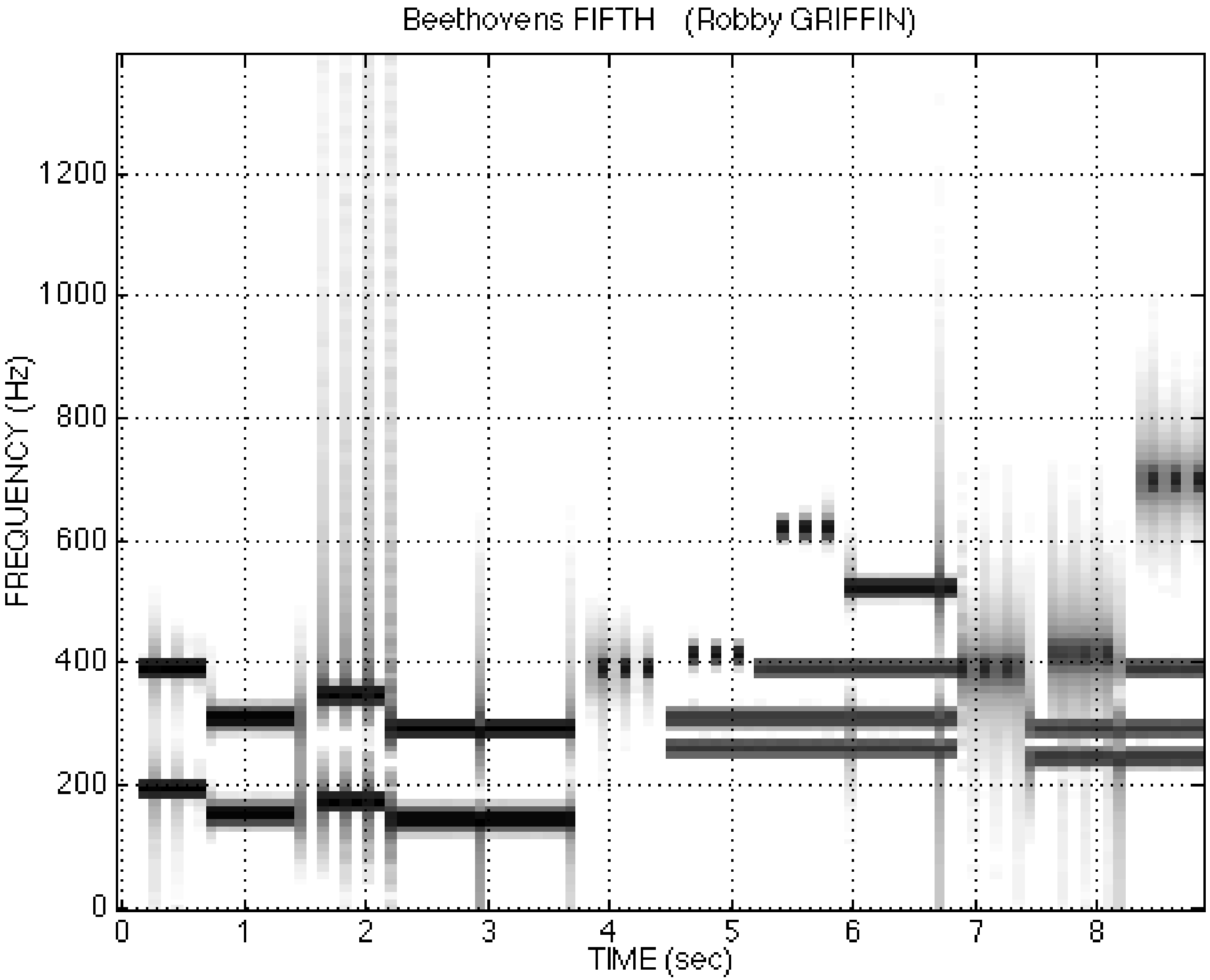
Spectrogram of C-Scale



From J.H. McClellan, R.W. Schafer, Signal Processing First, Prentice-Hall, 2003.

Spectrogram of LAB SONG

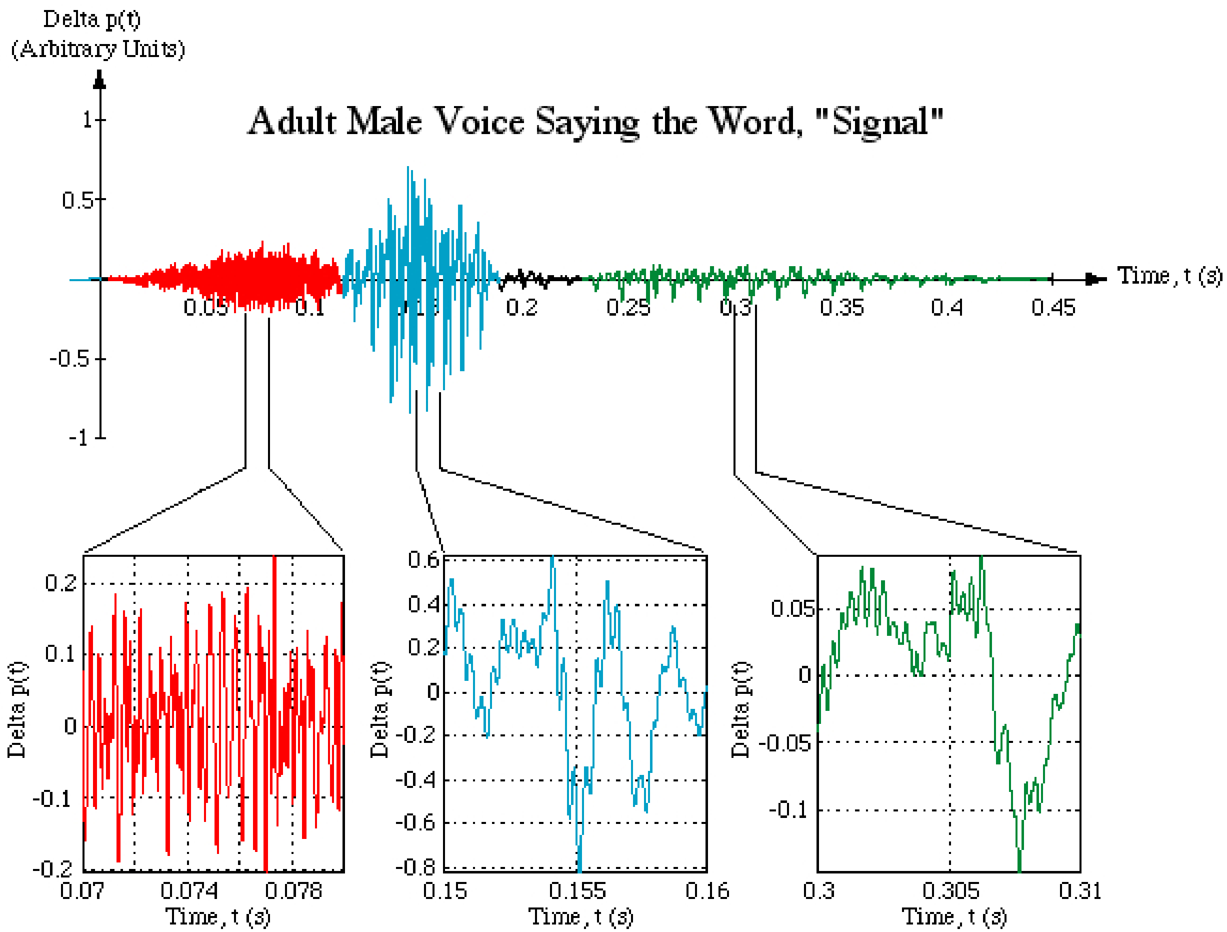




From J.H. McClellan, R.W. Schafer, Signal Processing First, Prentice-Hall, 2003.

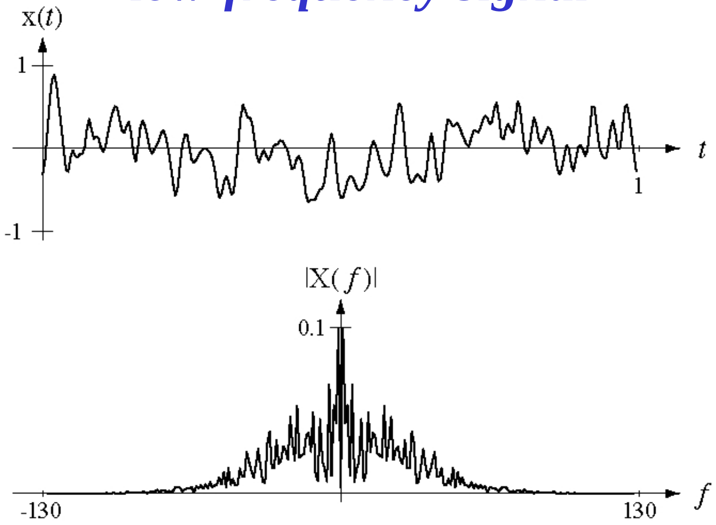
Recorded sound

"s" "i" "gn" "al"

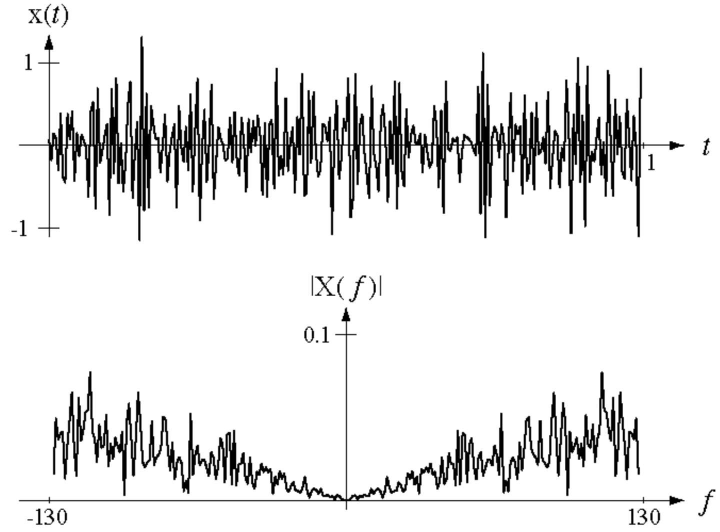


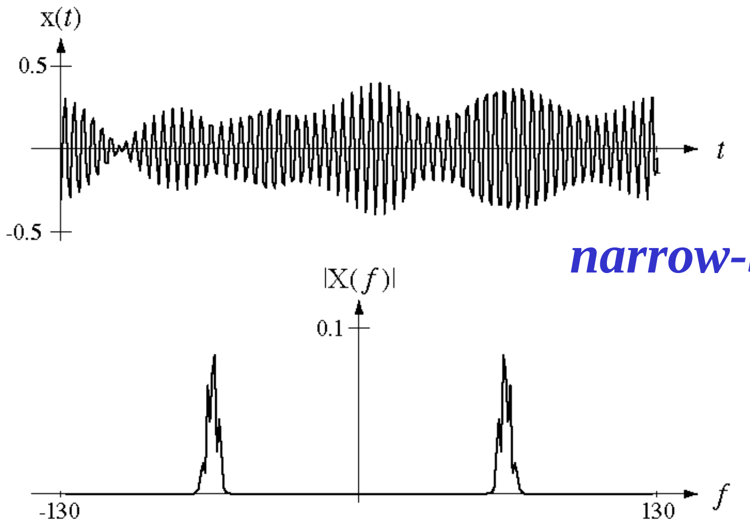
Fourier transform of signals

low-frequency signal



high-frequency signal

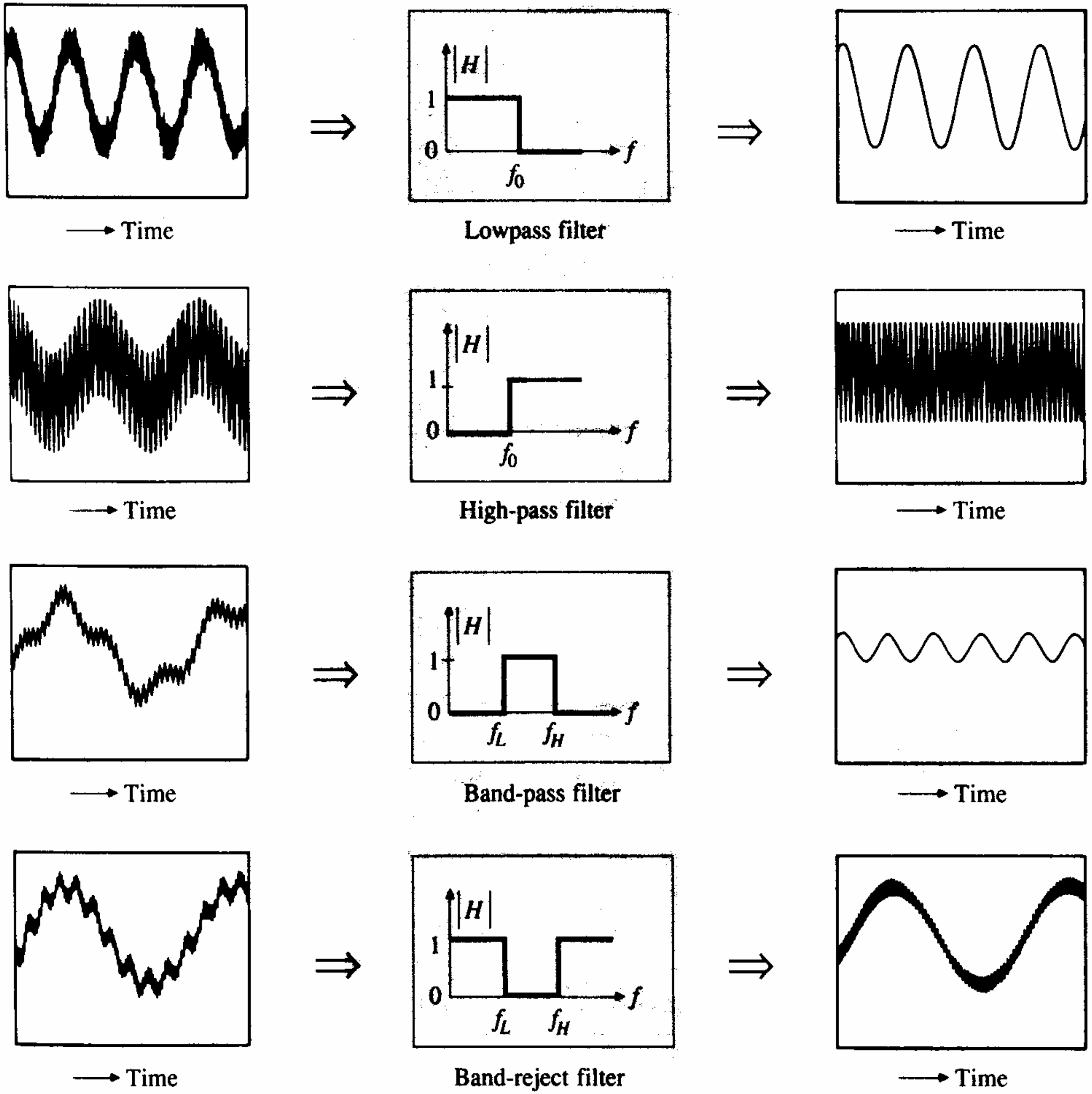




narrow-band signal

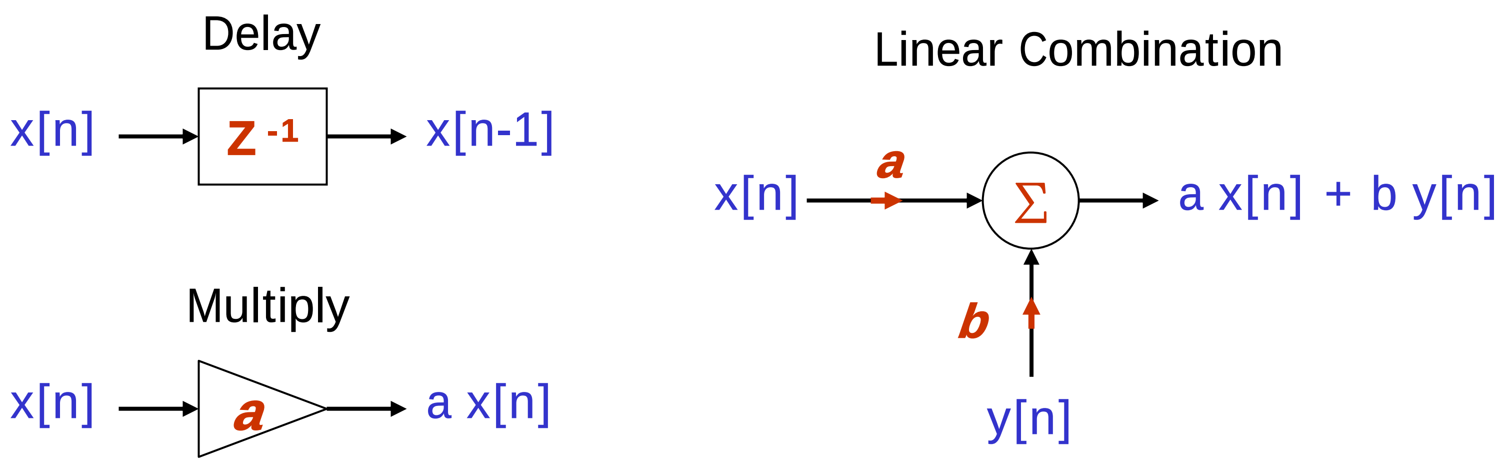
From .J.Roberts, Signals and Systems, McGraw-Hill, 2003.

Ideal filters



S. Franco, "Design with Operational Amplifiers and Analog Integrated Circuits", Second Edition, 1998.

Z transform



	Digital signal	z transform	Analog signal
Input signal	x[n]	X(z)	x(t)
Delay one sample	x[n-1]	$Z^{-1}X(z)$	x(t-T)
Multiply	a x[n]	a X(z)	a x(t)
Linear combination	a x[n] + b y[n]	a X(z) + b Y(z)	a x(t) + b y(t)

Example 1: Perform the running average of last six digital sample

$$y[n] = \frac{x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] + x[n-5]}{6}$$

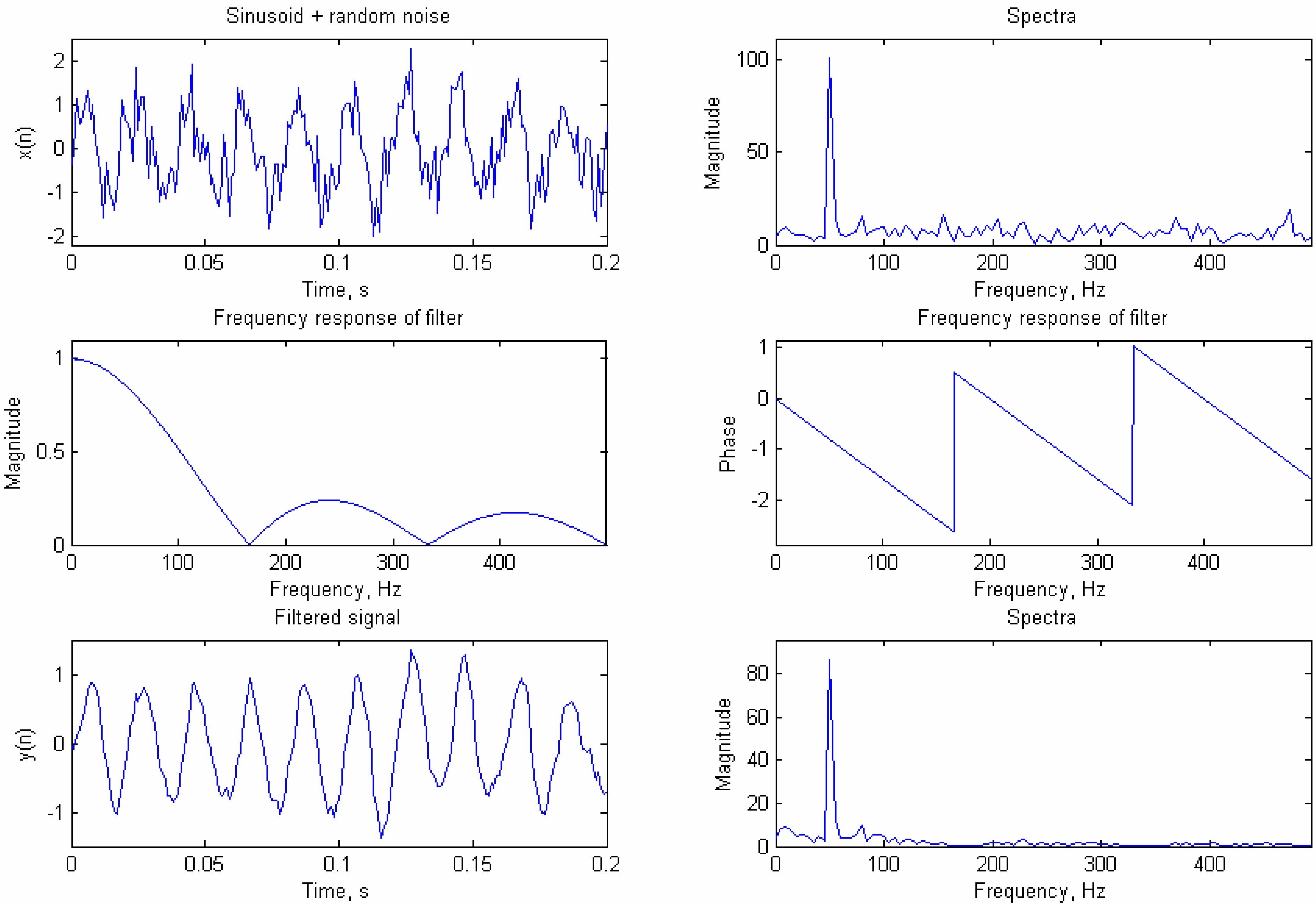
$$Y(z) = \frac{X(z) + z^{-1}X(z) + z^{-2}X(z) + z^{-3}X(z) + z^{-4}X(z) + z^{-5}X(z)}{6}$$

Transfer function

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5}}{6}$$

```
fs=1000;
t=0:1/fs:0.2;
x = sin(2*pi*50*t) + 0.5*randn(size(t));

Xf=fft(x);
resolution=fs/length(Xf);
f=(0:length(Xf)-1)*resolution;
Xf_magnitude = abs(Xf);
subplot(3,2,2)
index=1:length(Xf_magnitude)/2;
plot(f(index),Xf_magnitude(index))

b=[1/6 1/6 1/6 1/6 1/6 1/6];
a=[1];
NFFT=1024;
[h,f] = freqz(b,a,NFFT);
```

```
f=f/pi*fs/2;
h_magnitude=abs(h);
h_phase=phase(h);
subplot(3,2,3)
plot(f,h_magnitude);
subplot(3,2,4)
plot(f,h_phase);

y=filter(b,a,x);
t=(0:length(y)-1)/fs;

Yf=fft(y);
resolution=fs/length(Yf);
f=(0:length(Yf)-1)*resolution;
Yf_magnitude = abs(Yf);

subplot(3,2,6)
index=1:length(Yf_magnitude)/2;
plot(f(index),Yf_magnitude(index
```


Sinusoid + random noise
x(n)
Time, s
Spectra
Magnitude
Frequency, Hz
Frequency response of filter
Magnitude
Frequency, Hz
Frequency response of filter
Phase
Frequency, Hz
Filtered signal
y(n)
Time, s
Spectra
Magnitude
Frequency, Hz

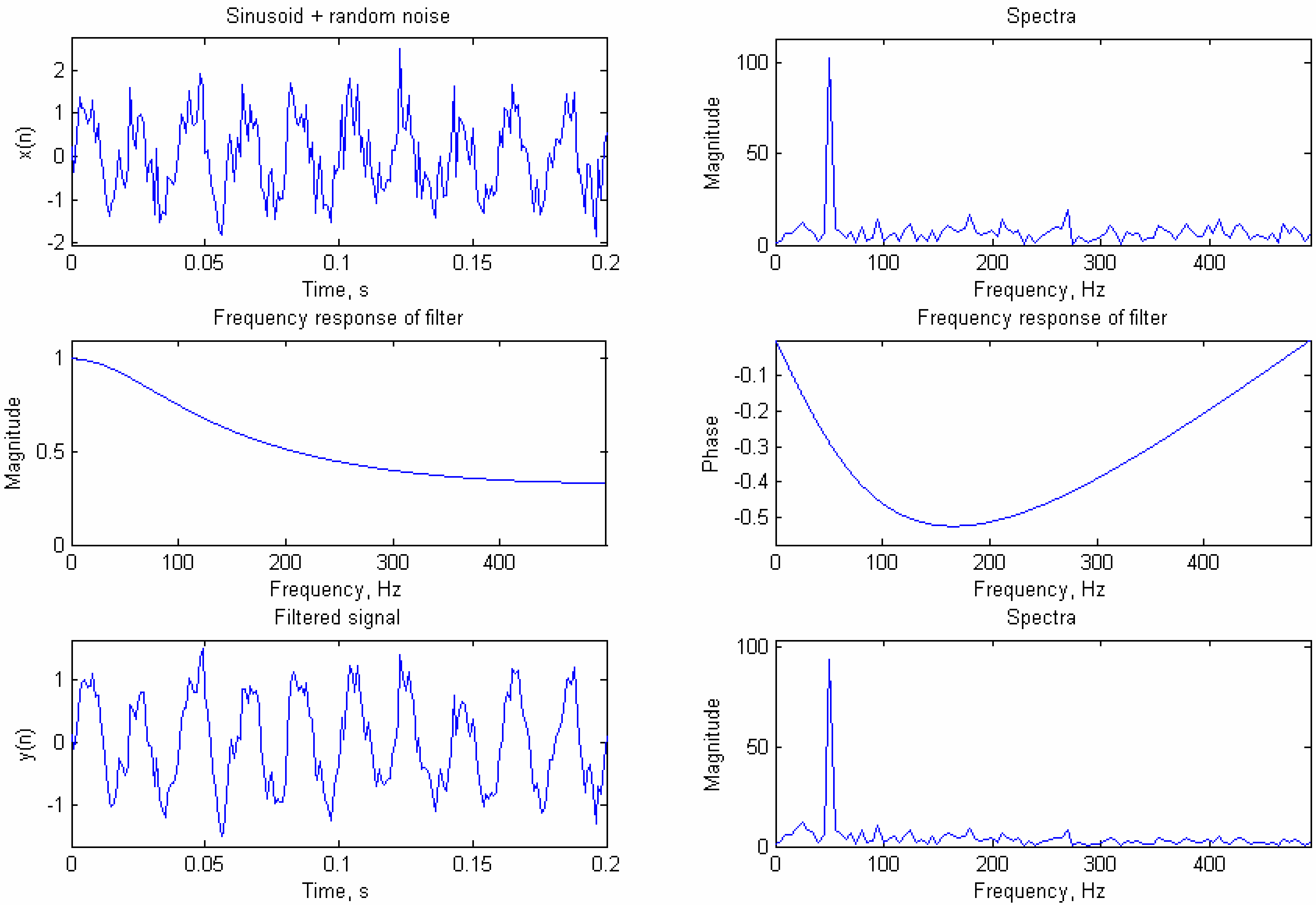
Example 2: Perform the average of current data and last filter output

$$y(n) = \frac{y(n-1) + x(n)}{2}$$

$$Y(z) = \frac{z^{-1}Y(z) + X(z)}{2}$$

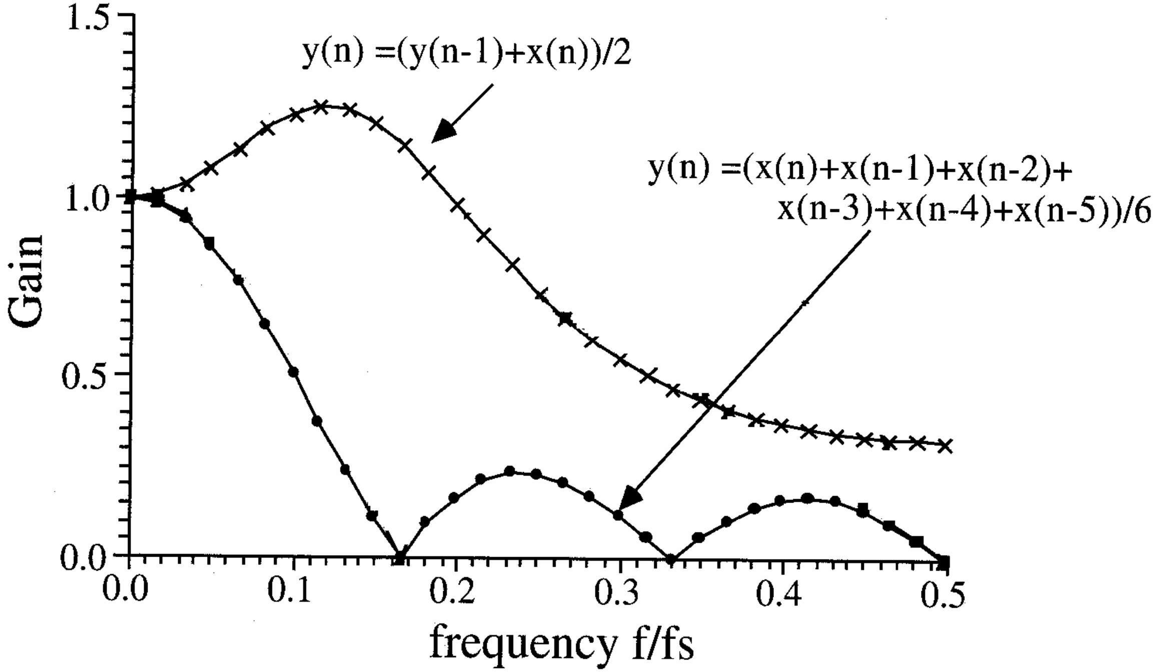
Transfer function

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2 - z^{-1}}$$


Sinusoid + random noise
x(n)
Time, s
Spectra
Magnitude
Frequency, Hz
Frequency response of filter
Magnitude
Frequency, Hz
Frequency response of filter
Phase
Frequency, Hz
Filtered signal
y(n)
Time, s
Spectra
Magnitude
Frequency, Hz

Frequency response of example 1 and 2

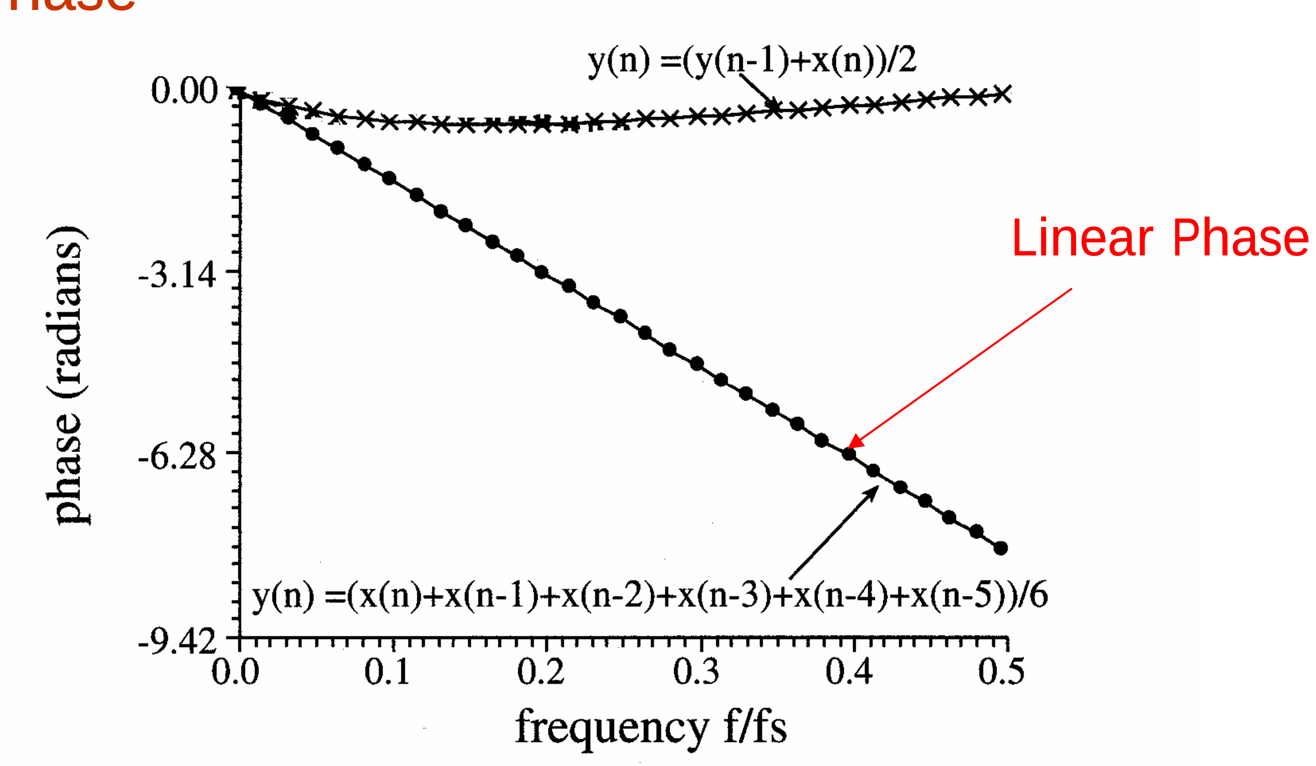
Magnitude



From Jonathan W. Valvano, Embedded Microcomputer Systems, real time interfacing, Brooks/Cole, 2000.

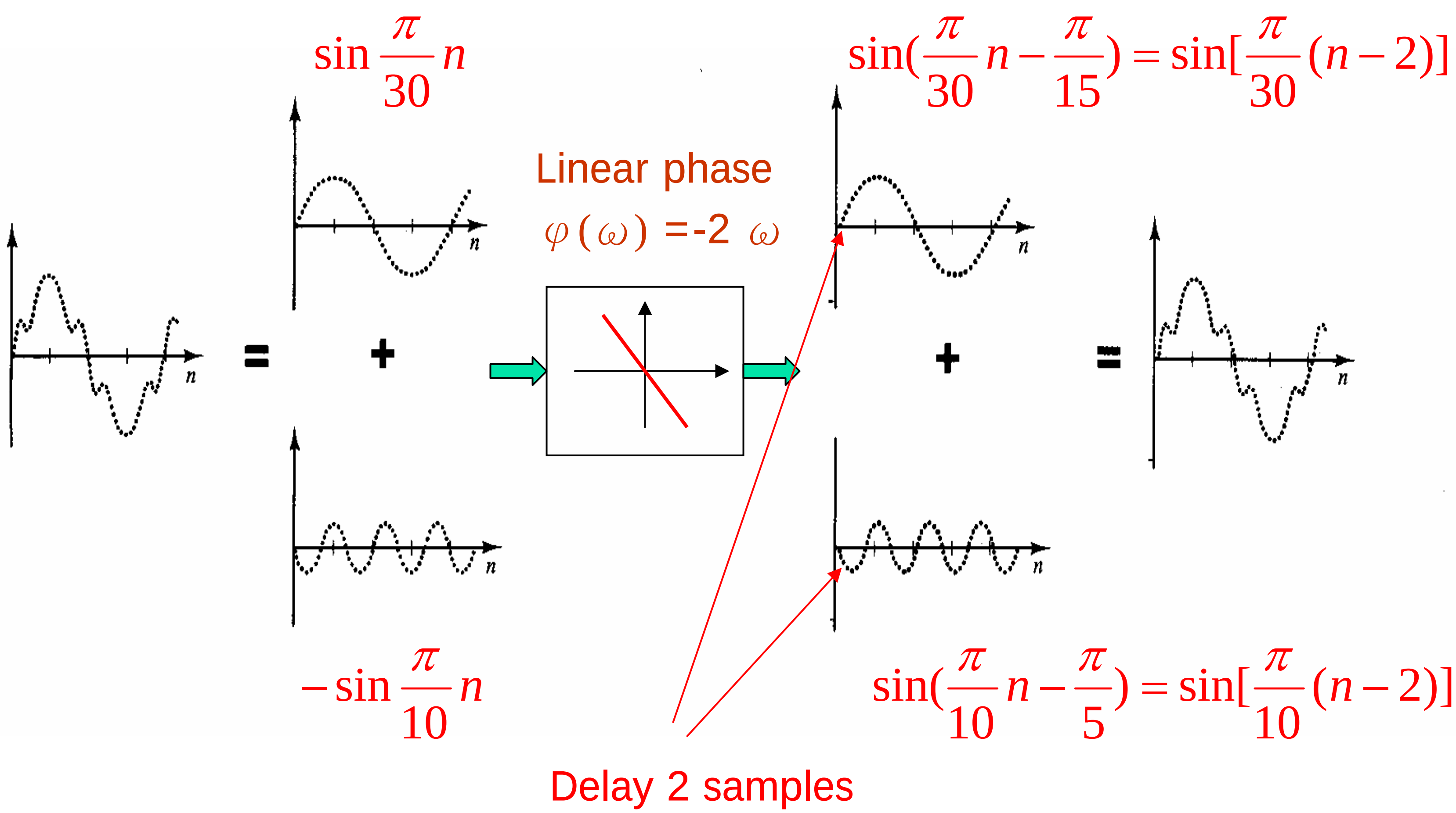
Frequency Response of Example 1 and 2

Phase



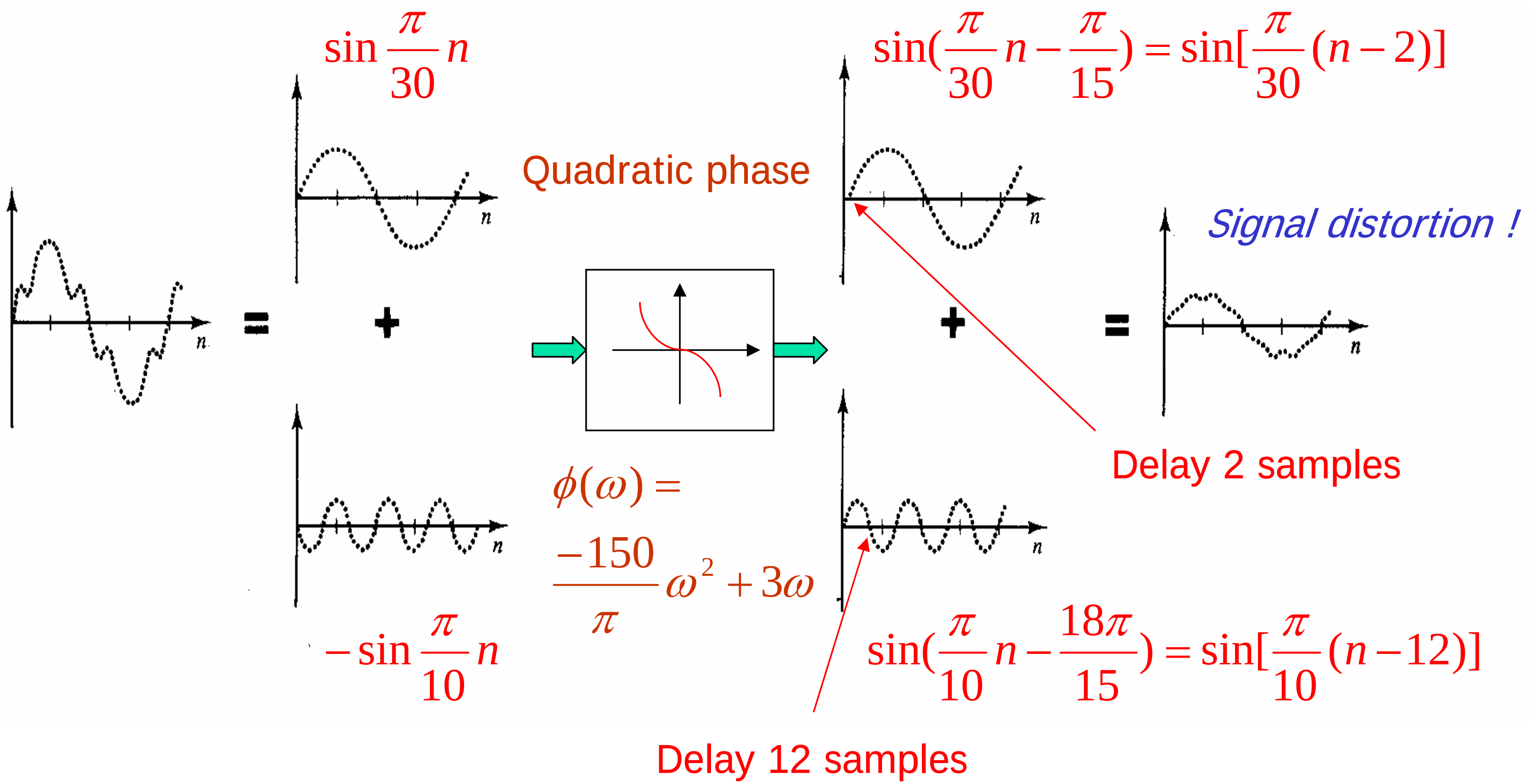
From Jonathan W. Valvano, Embedded Microcomputer Systems, real time interfacing, Brooks/Cole, 2000.

Linear Phase



Modified from L.Ludeman, Fundamentals of digital signal processing,Harper & Row, 1986.

Nonlinear Phase



Modified from L.Ludeman, Fundamentals of digital signal processing,Harper & Row, 1986.

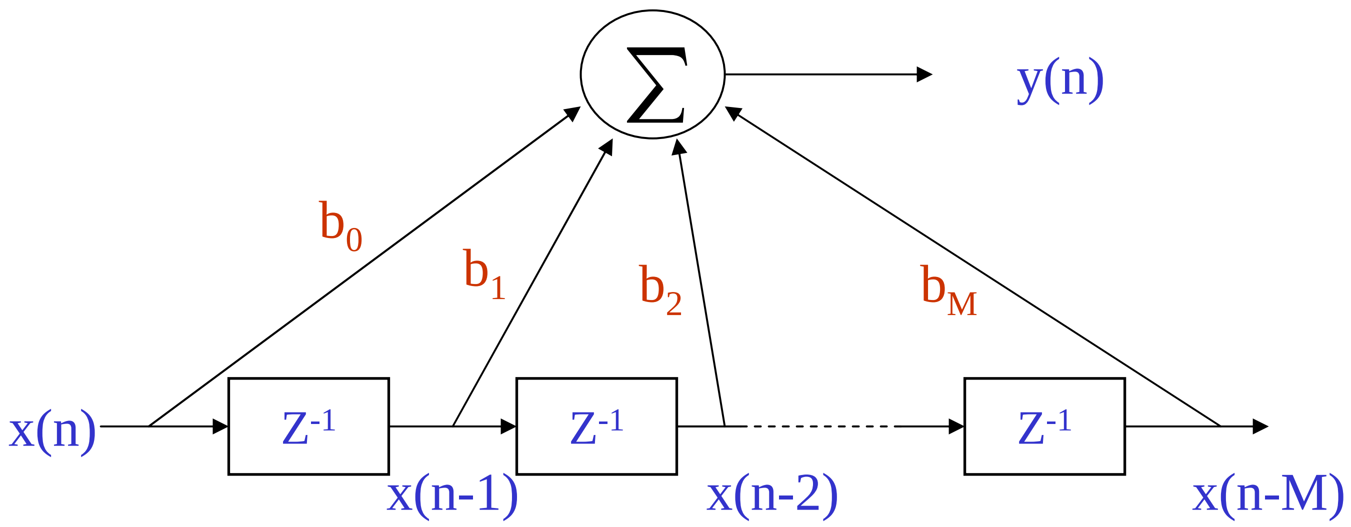
Group delay

$$\tau = -\frac{d\phi(\omega)}{d\omega}$$

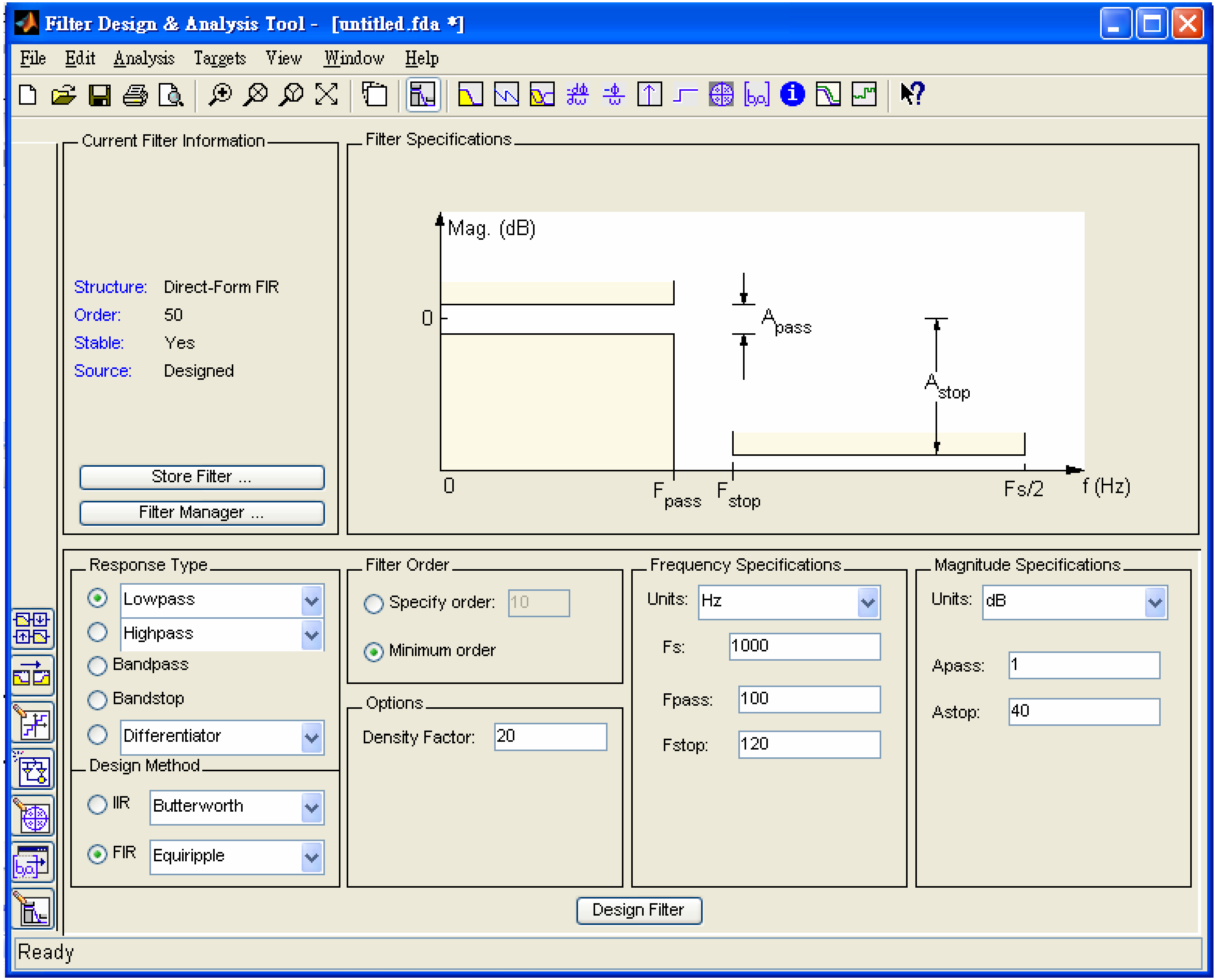
- Linear phase yield constant group delay

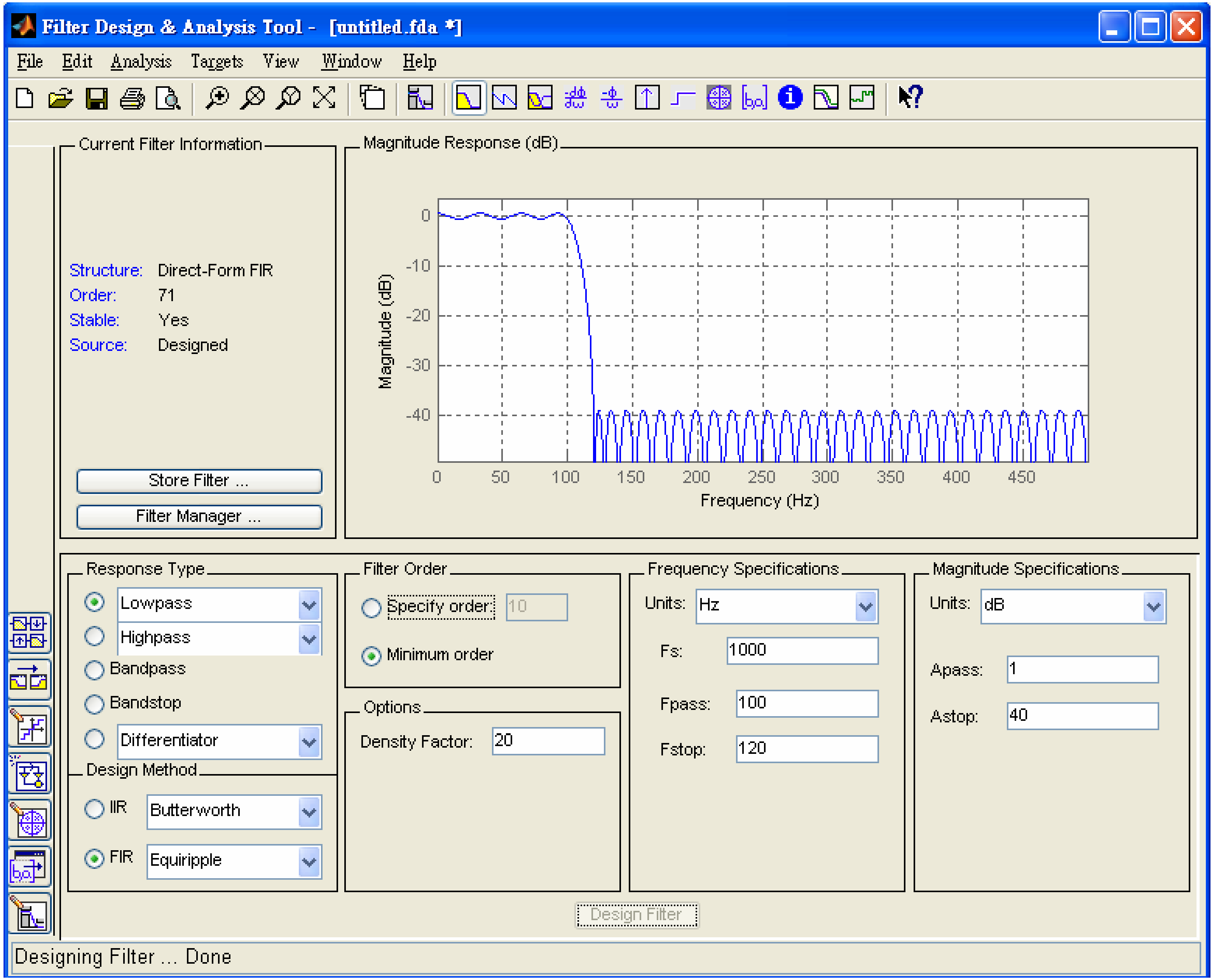
Finite impulse response (FIR) filter

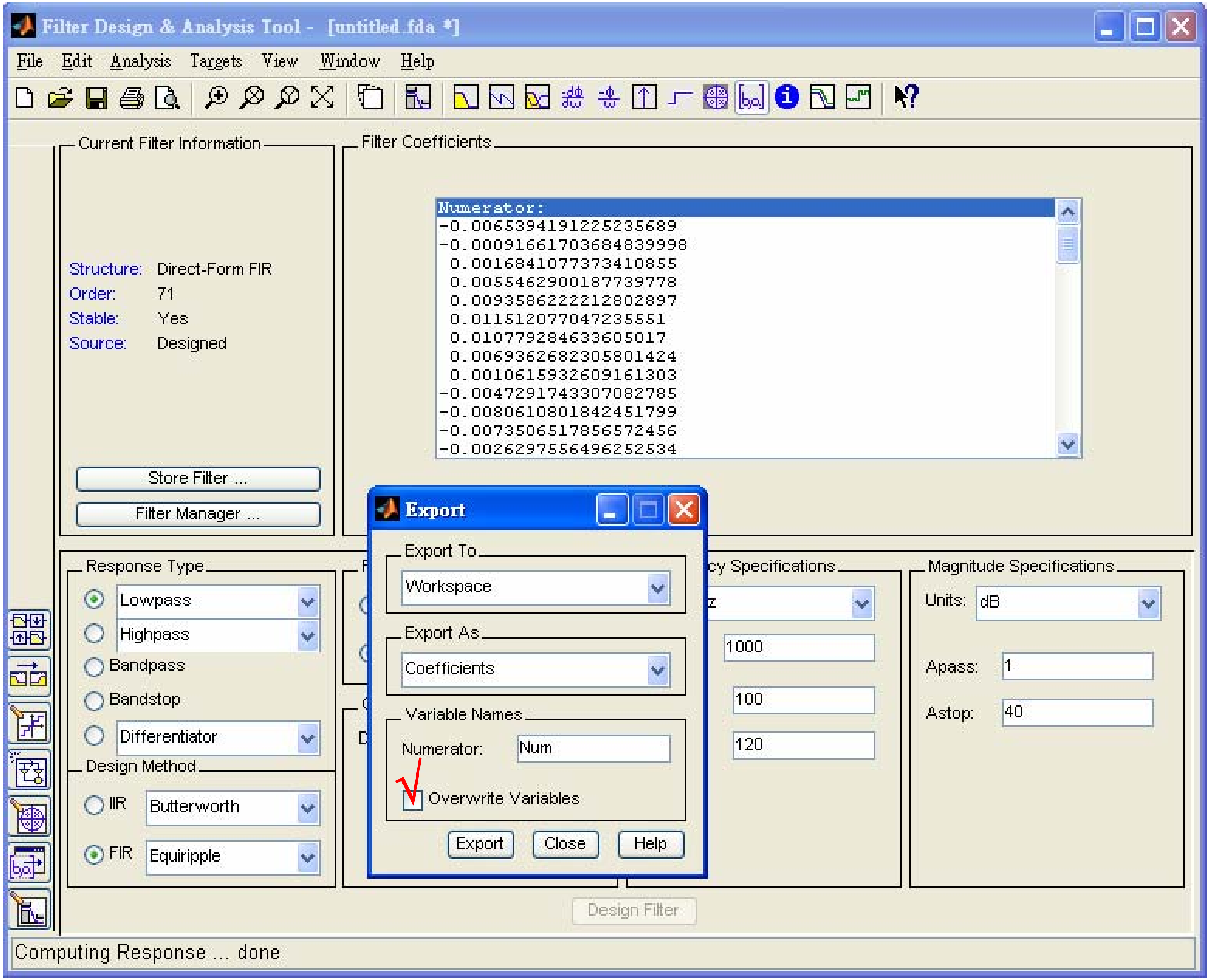
$$y(n) = \sum_{k=0}^{M} b_k x(n-k) \qquad H(z) = \frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} \cdots + b_N z^{-M}$$



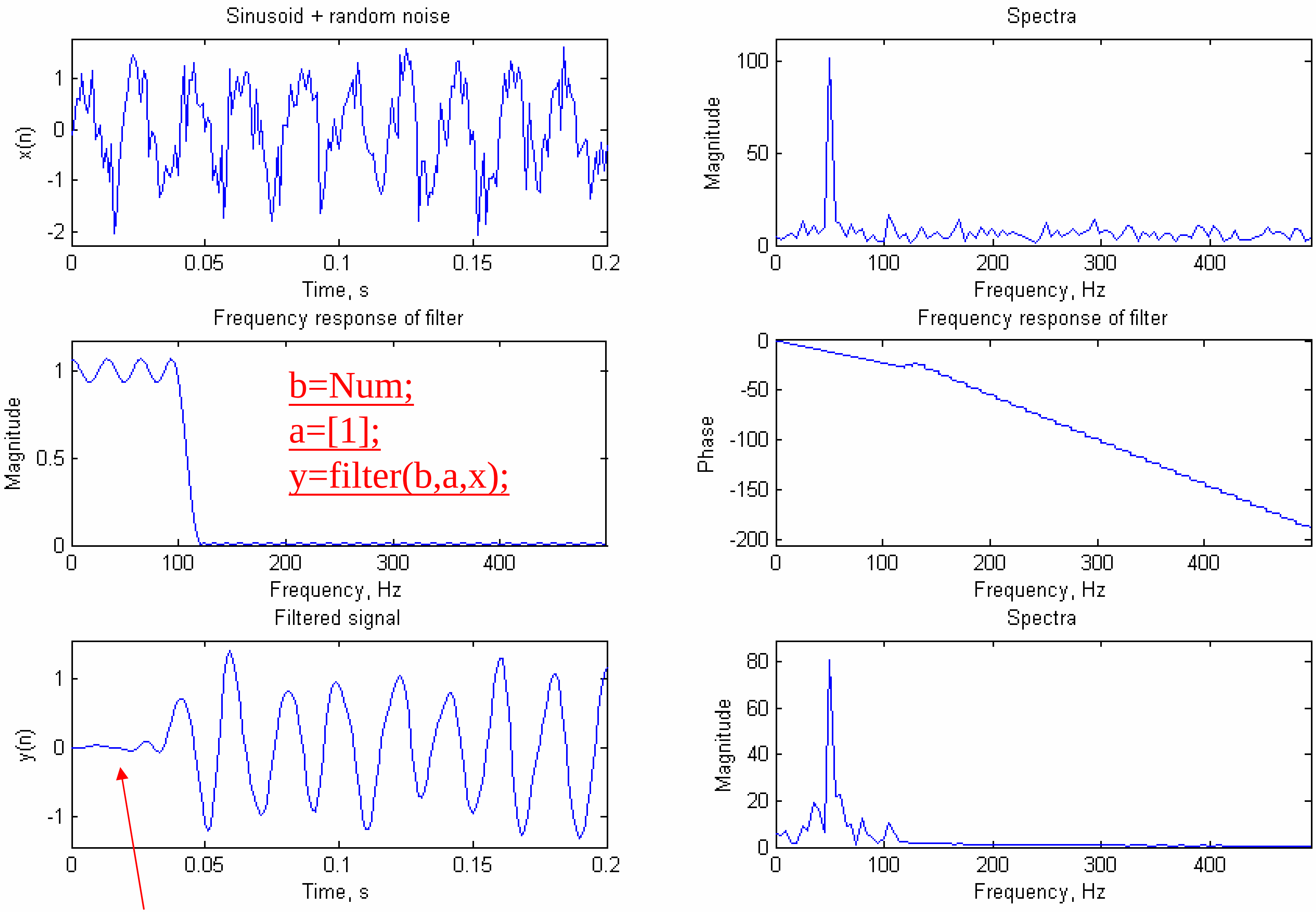
- FIR posses linear-phase property if filter coefficients are symmetry or anti-symmetry around the center


Filter Design & Analysis Tool - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 50
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Filter Specifications
Mag. (dB)
0
Apass
Astop
0
Fpass
Fstop
Fs/2
f (Hz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR
Butterworth
FIR
Equiripple
Filter Order
Specify order:
10
Minimum order
Options
Density Factor:
20
Frequency Specifications
Units:
Hz
Fs:
1000
Fpass:
100
Fstop:
120
Magnitude Specifications
Units:
dB
Apass:
1
Astop:
40
Design Filter
Ready


Filter Design & Analysis Tool - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 71
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
Magnitude (dB)
0
-10
-20
-30
-40
0 50 100 150 200 250 300 350 400 450
Frequency (Hz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR Butterworth
FIR Equiripple
Filter Order
Specify order: 10
Minimum order
Options
Density Factor: 20
Frequency Specifications
Units: Hz
Fs: 1000
Fpass: 100
Fstop: 120
Magnitude Specifications
Units: dB
Apass: 1
Astop: 40
Design Filter
Designing Filter ... Done



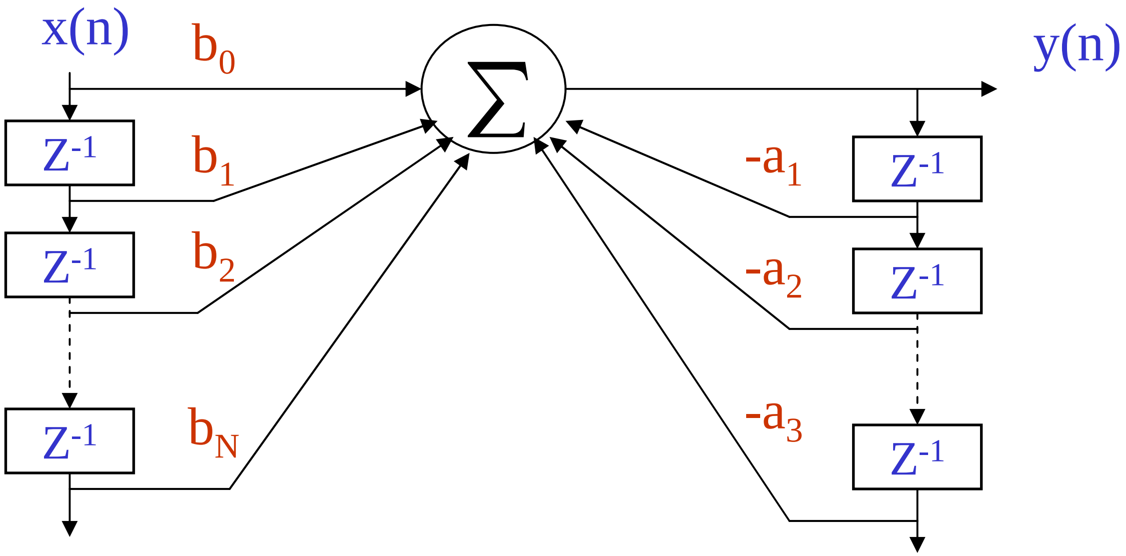
Filter Design & Analysis Tool - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 71
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Filter Coefficients
Numerator:
-0.0065394191225235689
-0.00091661703684839998
0.0016841077373410855
0.0055462900187739778
0.0093586222212802897
0.011512077047235551
0.010779284633605017
0.0069362682305801424
0.0010615932609161303
-0.0047291743307082785
-0.0080610801842451799
-0.0073506517856572456
-0.0026297556496252534
Export
Export To
Workspace
Export As
Coefficients
Variable Names
Numerator: Num
Overwrite Variables
Export Close Help
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR Butterworth
FIR Equiripple
cy Specifications
1000
100
120
Magnitude Specifications
Units: dB
Apass: 1
Astop: 40
Design Filter
Computing Response ... done

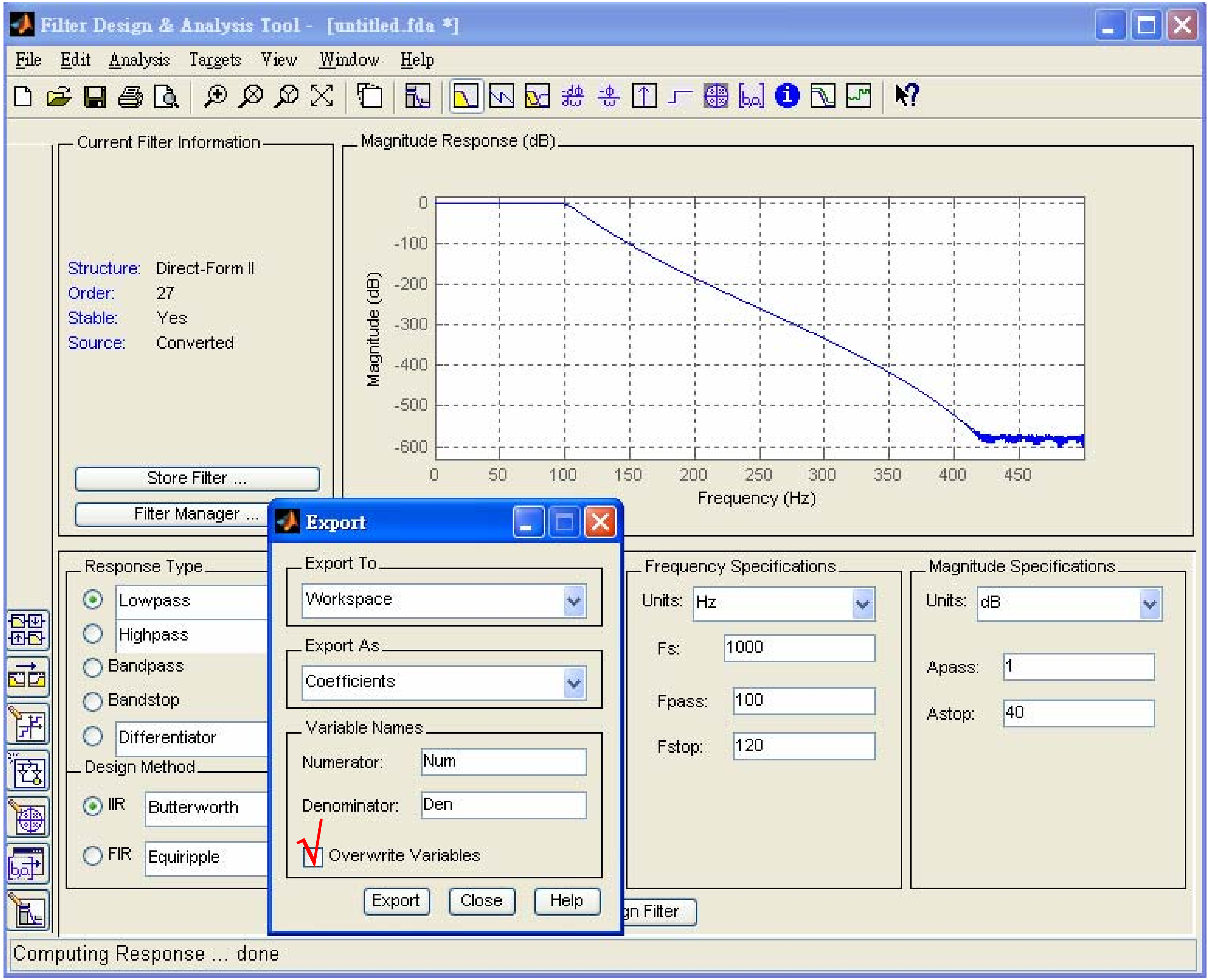


Transient. Group delay = filter length / 2

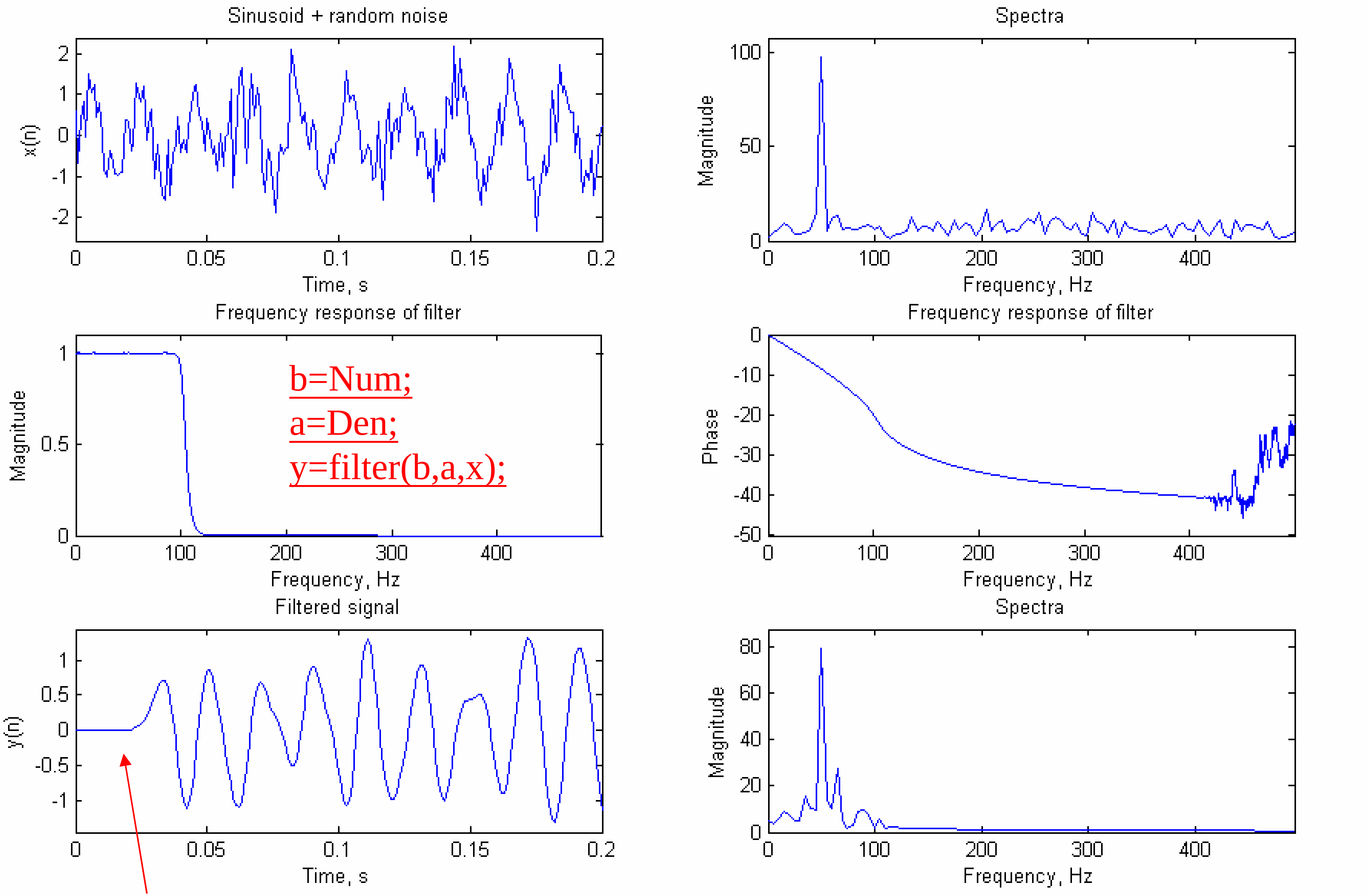
Infinite impulse response (IIR) filter

$$\sum_{p=0}^{N} a_p y(n-p) = \sum_{q=0}^{M} b_q x(n-q) \qquad H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{q=0}^{M} b_q z^{-q}}{\sum_{p=0}^{N} a_p z^{-p}}$$



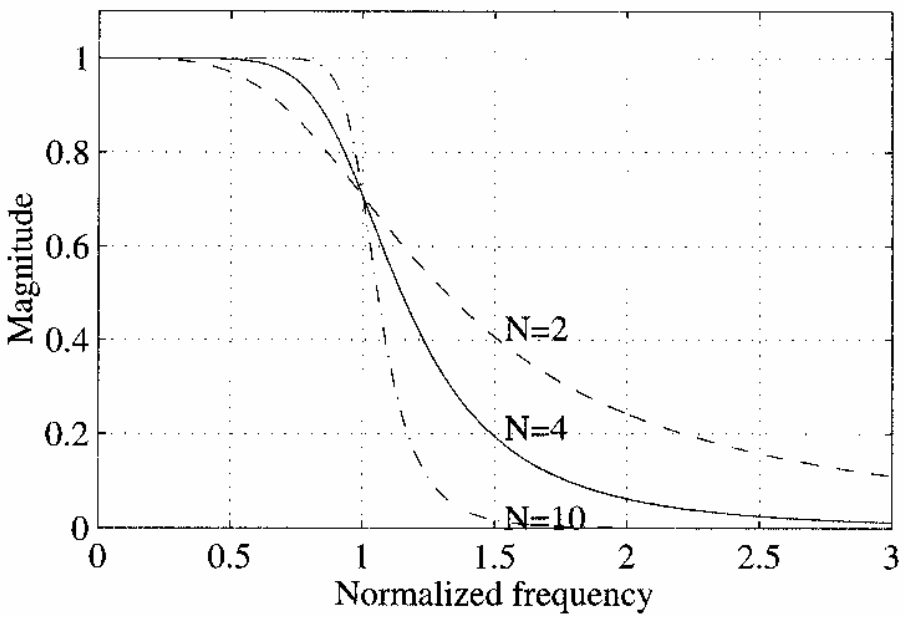


Filter Design & Analysis Tool - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form II
Order: 27
Stable: Yes
Source: Converted
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
Magnitude (dB)
Frequency (Hz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR
Butterworth
FIR
Equiripple
Export
Export To
Workspace
Export As
Coefficients
Variable Names
Numerator: Num
Denominator: Den
√
Overwrite Variables
Export Close Help
Frequency Specifications
Units: Hz
Fs: 1000
Fpass: 100
Fstop: 120
Magnitude Specifications
Units: dB
Apass: 1
Astop: 40
Computing Response ... done



Transient. Group delay is not constant, changed as frequency

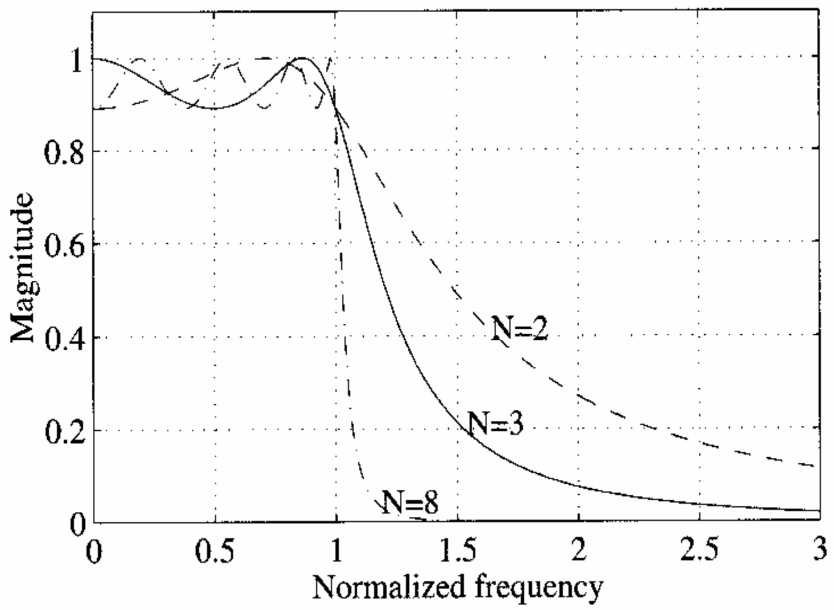
Butterworth approximation



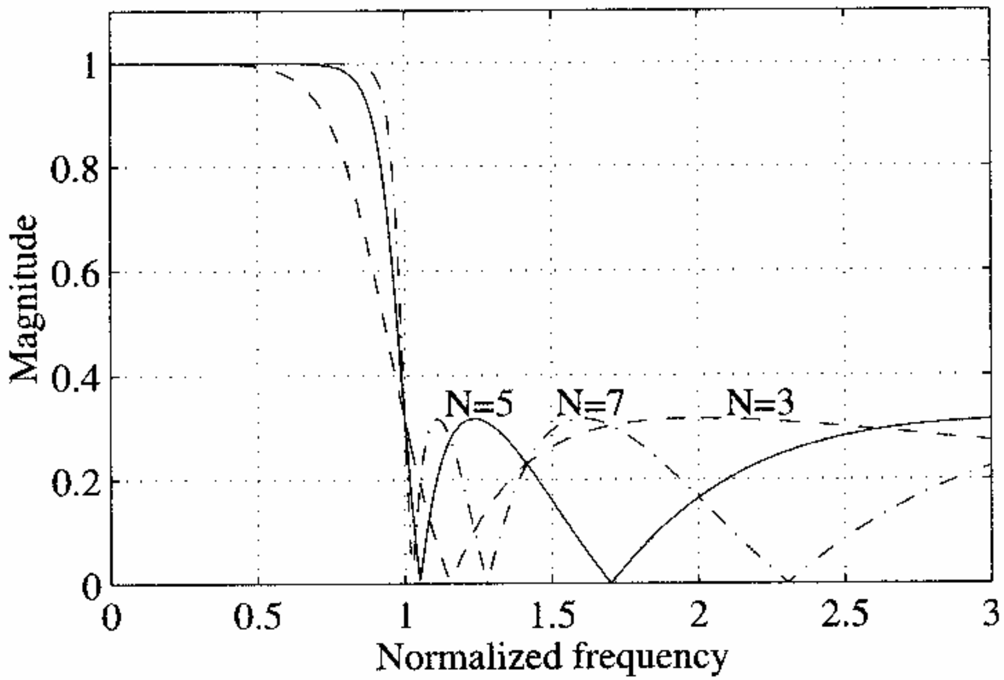
ω / ω_c

$$|H(j\omega)|^2 = \frac{1}{1+(\omega/\omega_c)^{2N}}$$

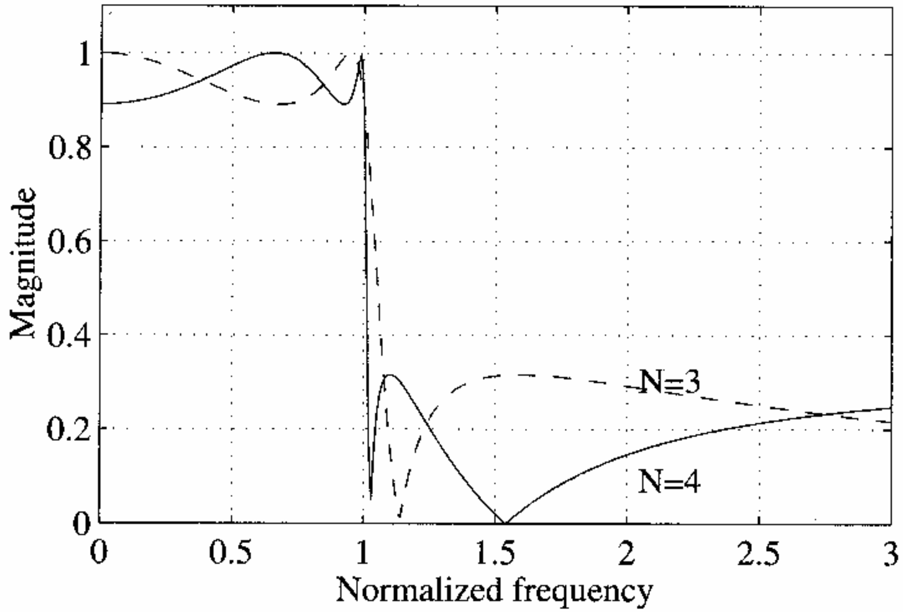
Type I Chebyshev approximation



Type II Chebyshev approximation



Elliptic approximation



- Equiripple in both passband and stopband
- Meet filter requirements with the lowest order

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