CMPS 130
Midterm1 Review Problems

These are problems include some of the examples and exercises stated in class. All homework problems (through hw3) should also be considered as good review problems.

1. Write down all relations on the set $A = \{a, b\}$ and determine which of them are reflexive, symmetric or transitive.

2. Let $f : A \to B$ and define a relation *R* on *A* by: xRy if and only if $f(x) = f(y)$.
   a. Prove that *R* is an equivalence relation.
   b. What is $[x]$ under this relation?
   c. Let $A = B = \mathbf{R}$ and define $f : \mathbf{R} \to \mathbf{R}$ by $f(x) = x(x-1)(x+1)$. Determine $[0]$.

3. Let $L_1 = \{ab, ba\}$ and $L_2 = \{\lambda, aab, bab\}$.
   a. Determine $L_1 L_2$.
   b. Determine L_1^3

4. Show that if $L_1 L = L$ for every $L \subseteq \{a,b\}^*$, then $L_1 = \{\lambda\}$.

5. Let *S* be a set. Prove that no function $f : S \to 2^S$ is onto. (Note 2^S denotes the power set of *S*, i.e. the set of all subsets of *S*.)

6. Recall the language $\text{Expr} \subseteq \{a, (,), +, \cdot\}^*$ was defined recursively as

   (1) $a \in \text{Expr}$
   (2) If $x \in \text{Expr}$, then $(x) \in \text{Expr}$.
   (3) If $x, y \in \text{Expr}$, then $x + y \in \text{Expr}$.
   (4) If $x, y \in \text{Expr}$, then $x \cdot y \in \text{Expr}$.

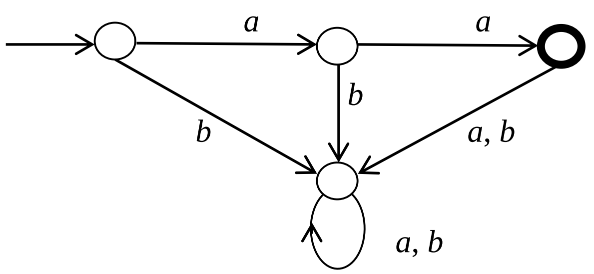
   Prove that $\forall x \in \text{Expr} : |x|$ is odd. (Hint: use structural induction on *x*.)

7. Recall the language $\text{Bal} \subseteq \{\,(\,,\ )\,\}^*$.
   a. Write down the recursive definition of Bal.
   b. Prove that $\forall x \in \text{Bal} : |x|$ is even.
   c. Prove that $\forall x \in \text{Bal} : n_((x) = n_)(x)$.
   d. Prove that $\forall x \in \text{Bal}$: if *z* is any prefix of *x*, then $n_)(z) \le n_((z)$.

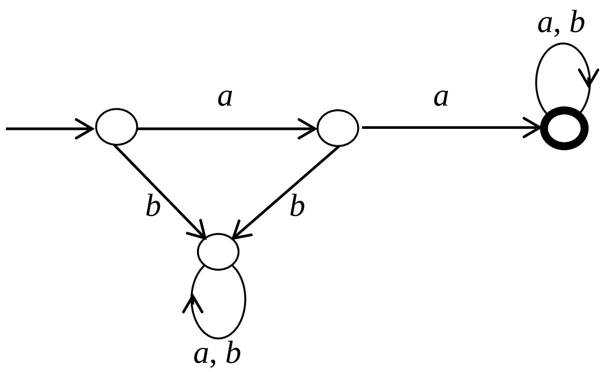
8. Let Σ be a finite alphabet.
   a. Write down the recursive definition of Σ^*.
   b. Write down the recursive definition of the reversal function $r : \Sigma^* \to \Sigma^*$.
   c. Prove that $\forall\, x, y \in \Sigma^* :\ r(xy) = r(y)r(x)$.

9. Write a simple description of the language $L \subseteq \{a,b\}^*$ accepted by the following finite automata. No justification is required.

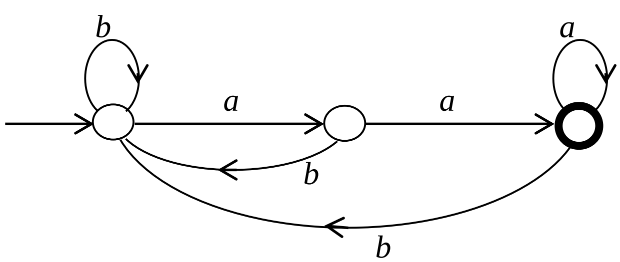
a.



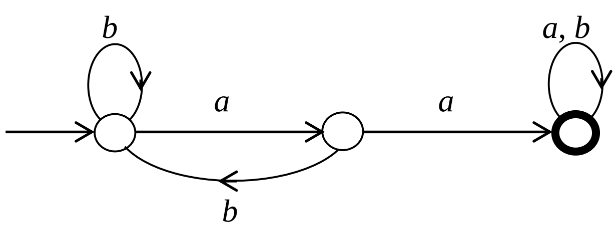
b.



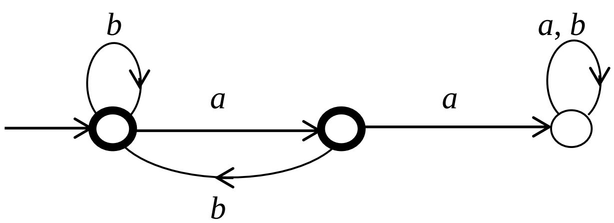
c.



d.



e.



10. Draw a state transition diagram for a DFA accepting the language

$$L = \{x \in \{a,b\}^* \mid x \text{ contains the substring } bba\}$$

11. Draw a state transition diagram for a DFA accepting the language

$$L = \{x \in \{a,b\}^* \mid x \text{ does not contain the substring } ababba\}$$

12. Given a DFA $M = (Q, \Sigma, q_0, A, \delta)$

    a. Write the recursive definition of the extended transition function $\delta^* : Q \times \Sigma^* \to Q$.

    b. Prove that $\delta^*(q, \sigma) = \delta(q, \sigma)$ for any $q \in Q$ and $\sigma \in \Sigma$.

    c. Prove that for all $x, y \in \Sigma^*$ and $q \in Q$: $\delta^*(q, xy) = \delta^*(\delta^*(q, x), y)$.