CMPS 101
Midterm 2 review
Solutions to selected problems

Problem 6

Let *G* be a directed graph. Determine whether, at any point during a Depth First Search of *G*, there can exist an edge of the following kind.

a. A tree edge which joins a white vertex to a gray vertex.
b. A back edge which joins a black vertex to a white vertex.
c. A forward edge which joins a gray vertex to a black vertex.
d. A cross edge which joins a black vertex to a gray vertex.

Answers:

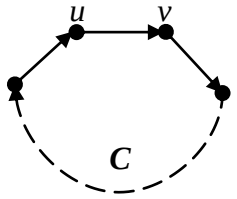
a. no
b. no
c. yes
d. no

Problem 8

Let *G* be a directed graph. Prove that if *G* contains a directed cycle, then *G* contains a back edge. (Hint: use the white path theorem.)

Proof:

Suppose *G* contains a directed cycle, call it *C*. Let *v* be the first vertex on *C* to be discovered by DFS, and let *u* be the vertex on *C* which precedes *v*.



Since no vertex on *C* is discovered before *v*, at the time of discovery of *v* the vertices of *C* form a path from *v* to *u* consisting of white vertices. By the white-path theorem, *u* becomes a descendent of *v* in some DFS tree. Therefore (*u*, *v*) is a back edge. / / /