CE16 Larrabee Winter '15 Name: Solution

1. **(2 points)** Do a change of index from the following sum so that your new sum starts with a lower index of 3. Make sure the sum adds up the exact same numbers. $\sum_{k=1}^{n} i$

$$\sum_{k=3}^{n+2} (k-2)$$

2. **(3 points)** Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$ with $f(a) = 4$, $f(b) = 2$, $f(c) = 4$, and $f(d) = 3$.

   (a) Is f one-to-one? If it is, say why, and if it is not, give a very similar function that is one-to-one.

   no, $f(a)=1$, $f(b)=2$, $f(c)=3$, $f(d)=4$

   (b) Is f onto? If it is, say why, and if it is not, give a very similar function that is onto.

   no, $f(a)=1$, $f(b)=2$, $f(c)=3$, $f(d)=4$

   (c) Is f a one-to-one correspondence? If it is, say why, and if it is not, give a very similar function that is a one-to-one correspondence.

   no, $f(a)=1$, $f(b)=2$, $f(c)=3$, $f(d)=4$

3. **(2 points)** Prove or disprove that $\lceil xy \rceil = \lceil x \rceil \lceil y \rceil$ for all real numbers x and y.

Disprove by counter example.

Let $x = 0.5$ $y = 0.5$; so

$\lceil x \rceil = 1$ $\lceil y \rceil = 1$.

$\lceil xy \rceil = \lceil 0.25 \rceil = 1$ and $\lceil x \rceil \lceil y \rceil = 1 \times 1 = 1$. $\lceil x y \rceil \neq \lceil x \rceil \lceil y \rceil$

4. **(3 points)** Prove that $\lceil x \rceil = -\lfloor -x \rfloor$ for all real numbers x.
Hint: one method of doing this would involve analyzing two separate cases (though you can do it without cases). Feel free to use the back.

Assume $x > 0$, let $x = a+b$ where $a \in \mathbb{Z}$ and $0 \leq b < 1$.

$\lceil x \rceil = a+1$. $\lfloor -x \rfloor = -a-1$.

$\lceil x \rceil = -\lfloor -x \rfloor$

Assume $x < 0$. let $x = -a-b$ where $a \in \mathbb{Z}$, and $0 \leq b < 1$.

$\lceil x \rceil = -a+1$ $\lfloor -x \rfloor =$ $a-1$

$\lceil x \rceil = -\lfloor -x \rfloor$

Assume $x = 0$. $\lceil x \rceil = -\lfloor -x \rfloor = 0$