CMPS 130
Midterm2 Review Problems

These are problems include some of the examples and exercises stated in class. Homework problems from hw4 through hw6 should also be considered good review problems.

1. Let $L \subseteq \{a, b\}^*$ correspond to the regular expression $(a+b)^* aa(a+b)^*$.
   a. Give a verbal description of *L*.
   b. Draw an NFA with no λ–transitions accepting *L*.
   c. Use the *subset construction* to obtain a DFA accepting *L*. Label the states in your DFA so as to make its relation to the NFA clear.
   d. Show that $\{\lambda, a, aa\}$ is a pairwise *L*-distinguishable set. Explain why no DFA with fewer than three states will accept *L*.
   e. Show that λ and *b* are *L*-indistinguishable.
   f. Draw a DFA with only three states accepting *L*. (Either run the state minimization algorithm on your DFA in (c), or just eyeball it.)

2. Let L_1 and L_2 be regular languages over Σ. Show that $L_1 \cap L_2$ is also regular. (Hint: This is hard to prove directly from the definition of regular language. Instead use some of the theorems we have proved in chapters 2 and 3.)

3. Let $M = (Q, \Sigma, q_0, A, \delta)$ be an NFA, and let $S_1 \subseteq S_2 \subseteq Q$.
   a. Write the recursive definitions of $\lambda(S_1)$ and $\lambda(S_2)$.
   b. Show that $\lambda(S_1) \subseteq \lambda(S_2)$ using structural induction.

4. Let $M_1 = (Q_1, \Sigma, q_1, A_1, \delta_1)$ and $M_2 = (Q_2, \Sigma, q_2, A_2, \delta_2)$ be NFA accepting languages L_1 and L_2 respectively. Recall the definition of the NFA $M_u = (Q_u, \Sigma, q_u, A_u, \delta_u)$ that accepts the union $L_1 \cup L_2$. (See webpage lecture notes 11-6-13 pages 4, 5 and 6.) Use structural induction to prove that for any $x \in \Sigma^*$:

$$\delta_u^*(q_u, x) = \delta_1^*(q_1, x) \cup \delta_2^*(q_2, x)$$

5. State the Pumping Lemma and use it to prove that $L = \{\, x \in \{a,b\}^* \mid n_a(x) = n_b(x) \,\}$ is not accepted by any DFA.

6. (This is problem 3.31 with the typo in the last sentence fixed.) Let $M = (Q, \Sigma, q_0, A, \delta)$ be a DFA and let $M_1 = (Q, \Sigma, q_0, A, \delta_1)$ be the NFA with no λ–transitions for which $\delta_1(q, \sigma) = \{\delta(q, \sigma)\}$ for every $q \in Q$ and $\sigma \in \Sigma$. Show for all $x \in \Sigma^*$ and $q \in Q$ that $\delta_1^*(q, x) = \{\delta^*(q, x)\}$. In addition also show that $L(M) = L(M_1)$.

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7. (This is problem 3.53 on p. 127). Let Σ_1 and Σ_2 be finite alphabets. A function $f:\Sigma_1^* \to \Sigma_2^*$ is called a *homomorphism* if $f(xy) = f(x)f(y)$ for all $x, y \in \Sigma_1^*$. Let f be such a function.
   a. Show that $f(\lambda) = \lambda$
   b. The *image* of $L_1 \subseteq \Sigma_1^*$ under f is the set $f(L_1) = \{ f(x) \mid x \in L_1 \} \subseteq L_2$. Show that if L_1 is regular, then $f(L_1)$ is regular. (Hint: use structural induction and the recursive definition of the set of regular languages over Σ_1.)
   c. The *preimage* of $L_2 \subseteq \Sigma_2^*$ under f is the set $f^{-1}(L_2) = \{ x \in L_1 \mid f(x) \in L_2 \} \subseteq L_1$. Show that if L_2 is regular, then $f^{-1}(L_2)$ is regular. (Hint: start with a DFA accepting L_2 then show how to construct from it a DFA accepting $f^{-1}(L_2)$.)

8. Let $L \subseteq \Sigma^*$ and $x, y \in \Sigma^*$. Write the definitions of $x \, \mathbf{I}_L \, y$ and L/x. Prove that $x \, \mathbf{I}_L \, y$ if and only if $L/x = L/y$.

9. Draw an NFA corresponding to the regular expression $(ba)^*(bab + aa)^*$. Draw the computation trees corresponding to the strings $babaa$, $babaaa$ and $babaaabab$. Which of these strings are accepted?

The following problems constitute hw7 which is due Friday after the midterm. Consider these as review problems.

Chapter 3: # 49ab, 51a, 52

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