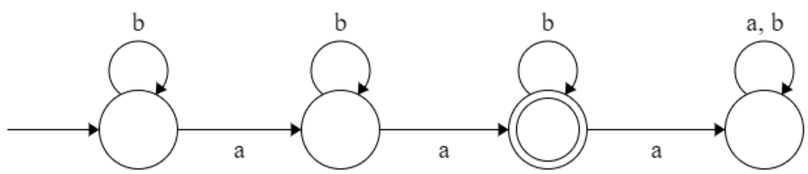
CMPS 130
Spring 2016
Homework Assignment 3 Solutions
Problems are from Martin 4th edition.

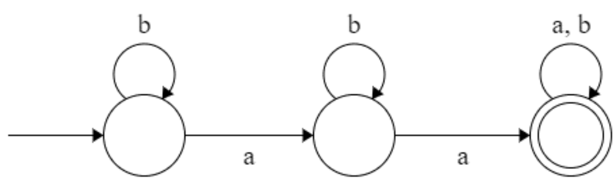
Chapter 2 (p.77): 1abcdefgh, 2abcde, 3abc, 4, 6, 7, 10abc

1. Problem 2.1a-h
   In each part below, draw an FA accepting the indicated language over $\{a, b\}$.
   a. The language of all strings containing exactly two *a*'s.
   b. The language of all strings containing at least two *a*'s.
   c. The language of all strings that do not end with *ab*.
   d. The language of all strings that begin or end with *aa* or *bb*.
   e. The language of all strings not containing the substring *aa*.
   f. The language of all strings in which the number of *a*'s is even.
   g. The language of all strings in which both the number of *a*'s and the number of *b*'s are even.
   h. The language of all strings containing no more than one occurrence of the string *aa*. (Note the string *aaa* contains two occurrences of *aa*.)

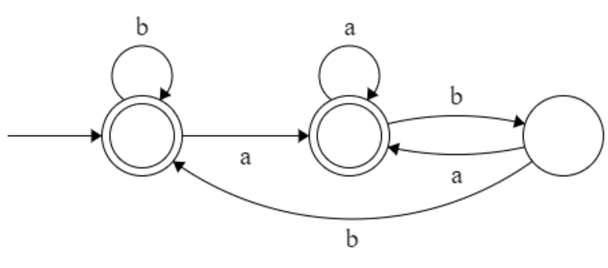
   Solution:
   a.



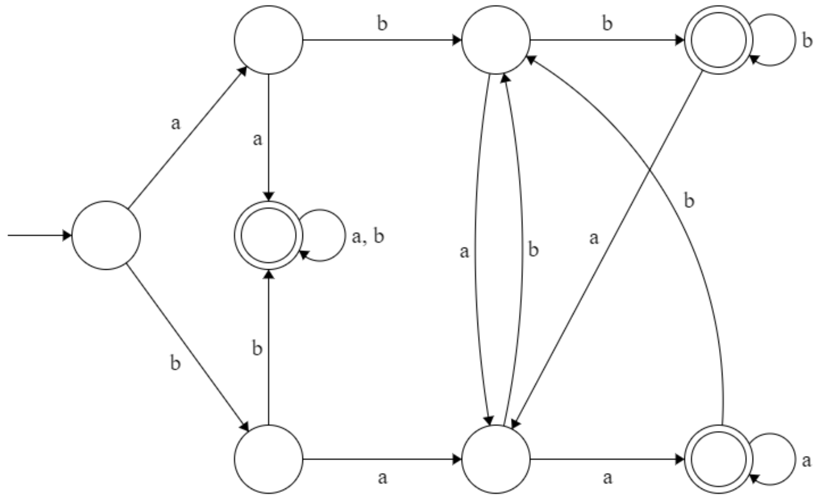
   b.



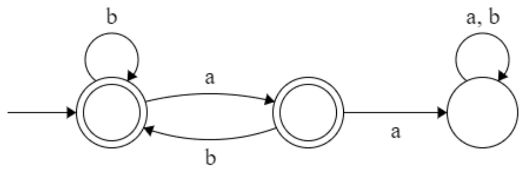
   c.



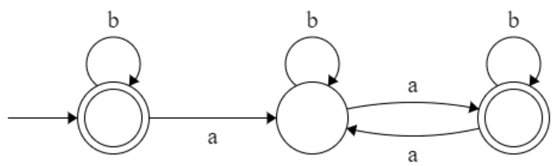
d.



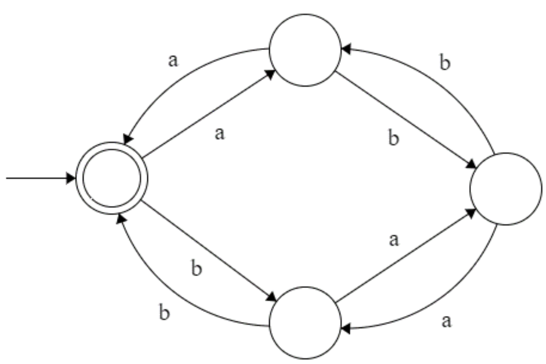
e.



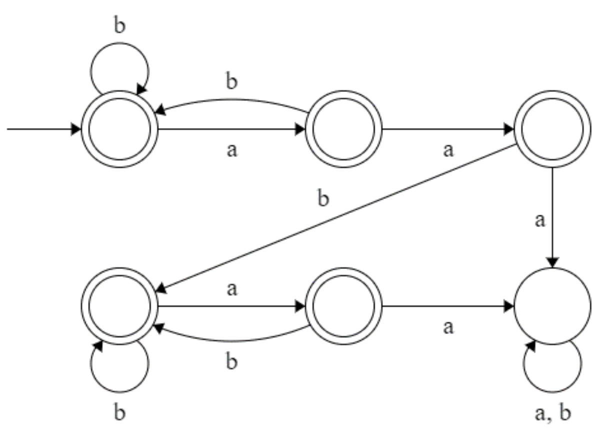
f.



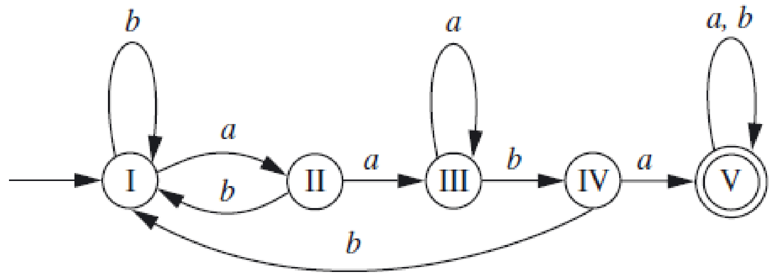
g.



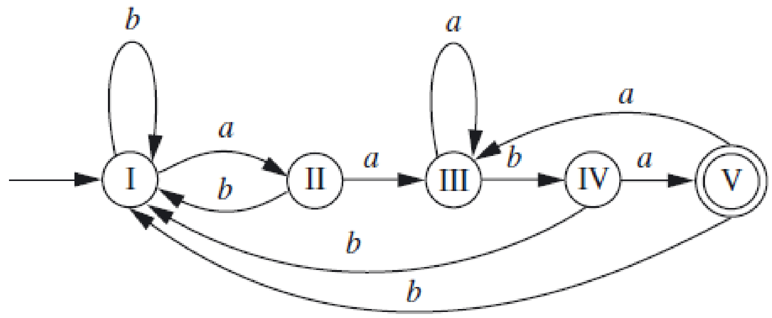
h.



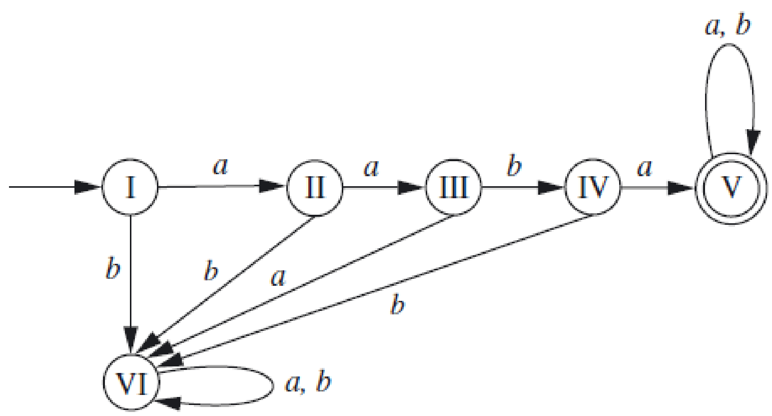
2. Problem 2.2a-e
   For each of the RAs pictured in Fig. 2.43, give a simple verbal description of the language it accepts.



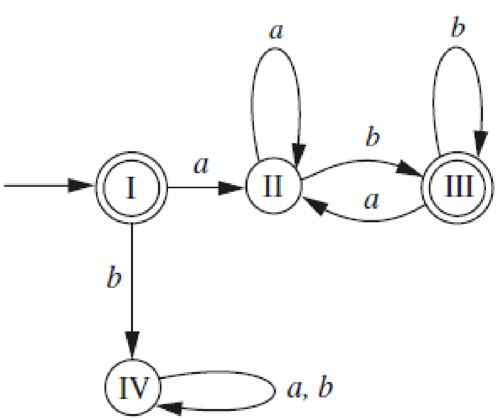
(a)



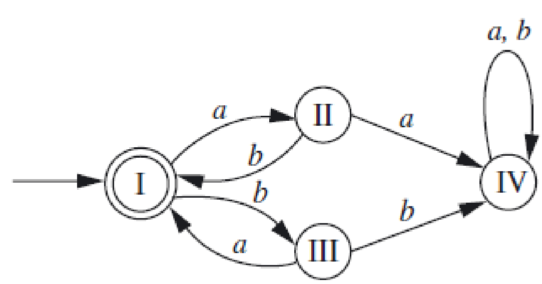
(b)



(c)



(d)



(e)

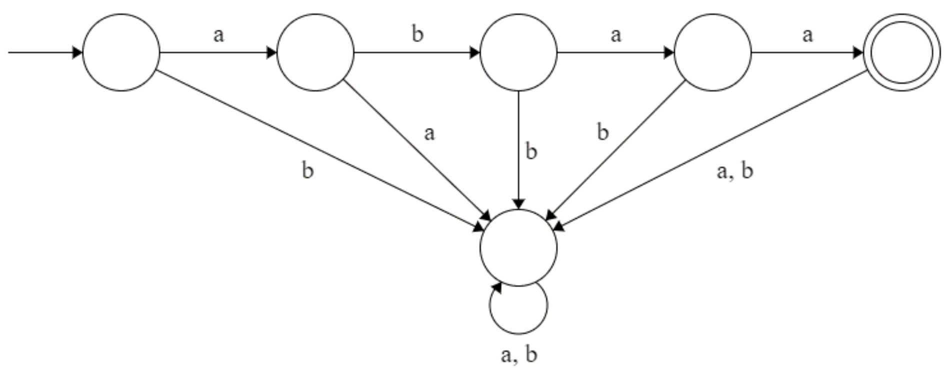
Solution:

a. L={ all strings that contain *aaba* }
b. L={ all strings that end with *aaba* }
c. L={ all strings that begin with *aaba* }
d. $L=\{\lambda\} \cup$\{ all strings that begin with *a* and end with *b* \}
   = { all strings that do not begin with *b* and do not end with *a* }
e. L={ concatenations of zero or more instances of the strings *ab* and *ba* }
   $= \{ab, ba\}^*$

3. Problem 2.3abc
   a. Draw a transition diagram for an FA that accepts the string *abaa* and no other strings.
   b. For a string $x \in \{a, b\}^*$ with $|x| = n$, how many states are required for an FA accepting *x* and no other strings? For each of these states, describe the strings that cause the FA to be in that state.
   c. For a string $x \in \{a, b\}^*$ with $|x| = n$, how many states are required for an FA accepting the language of all strings in $\{a, b\}^*$ that begin with *x*? For each of these states, describe the strings that cause the FA to be in that state.

Solution:

a.

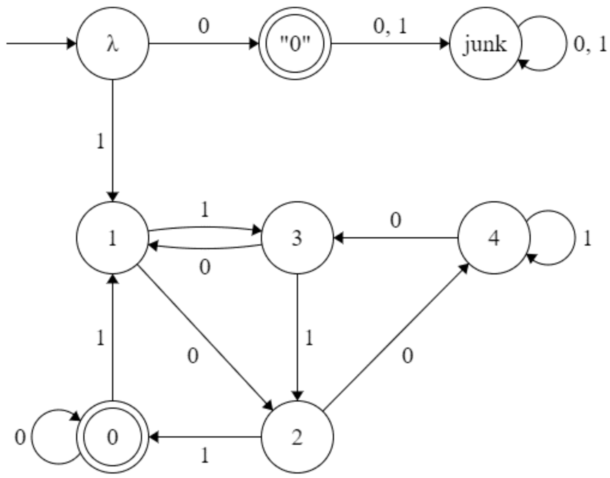


b. Given $x \in \{a, b\}^*$ with $|x| = n$, an FA accepting the language $\{x\}$ must contain $n + 2$ states. One state is a trap for any string that does not begin with a prefix of *x*. The remaining $n + 1$ states correspond to the $n + 1$ prefixes of *x* (including *x* itself.) The one accepting state corresponds to the string *x*.

c. Given $x \in \{a, b\}^*$ with $|x| = n$, an FA accepting the language $\{ xy \mid y \in \{a, b\}^* \}$ must contain $n + 2$ states. One state is a trap for any string that does not begin with a prefix of *x* (and therefor does not begin with *x*). One state (the accepting state) corresponds to all strings beginning with *x*. The remaining *n* states correspond to the *n* prefixes of *x*, other than *x* itself.

4. Problem 2.4
Example 2.7 describes an FA accepting L_3, the set of strings in $\{0, 1\}^*$ that are binary representations of integers divisible by 3. Draw a transition diagram for an FA accepting L_5.

Solution:



5. Problem 2.6
Suppose $M = (Q, \Sigma, q_0, A, \delta)$ is an FA, q is an element of Q, and $\delta(q, \sigma) = q$ for every $\sigma \in \Sigma$. Show using structural induction that for every $x \in \Sigma^*$, $\delta^*(q, x) = q$.

Proof: Suppose that for some particular $q \in Q$, we have $\delta(q, \sigma) = q$ for all $\sigma \in \Sigma$.

I. $\delta^*(q, \lambda) = q$ by the recursive definition of δ^*.

II. Let $x \in \Sigma^*$ and assume $\delta^*(q, x) = q$. We must show that $\delta^*(q, x\sigma) = q$ for any $\sigma \in \Sigma$. Choose an arbitrary $\sigma \in \Sigma$, then

$$\begin{aligned}\delta^*(q, x\sigma) &= \delta(\delta^*(q, x), \sigma) && \text{by the recursive definition of } \delta^* \\ &= \delta(q, \sigma) && \text{by the induction hypothesis} \\ &= q && \text{by our assumption on } q\end{aligned}$$

The result now follows for all $x \in \Sigma^*$ by structural induction.

6. Problem 2.7
Let $M = (Q, \Sigma, q_0, A, \delta)$ be an FA. Let $M_1 = (Q, \Sigma, q_0, R, \delta)$, where R is the set of states p in Q for which $\delta^*(p, z) \in A$ for some string z. What is the relationship between the language accepted by M_1 and the language accepted by M ? Prove your answer.

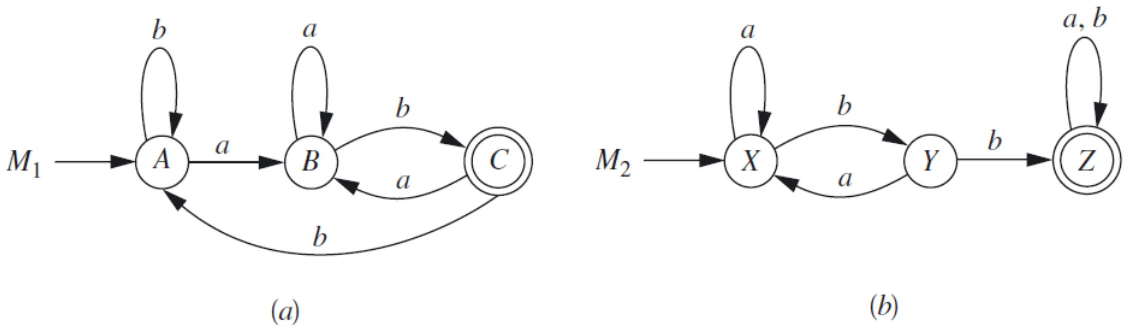
Solution:
Let L be the language accepted by M, and let L_1 be that accepted by M_1. Then L_1 consists of all prefixes of strings in L. In other words $L_1 = \{\, x \in \Sigma^* \mid \exists z\colon\ xz \in L \,\}$.

Proof: We must show that x is accepted by M_1 if and only if $xz \in L$ for some $z \in \Sigma^*$. We have

$$\begin{aligned} x \text{ is accepted by } M_1 &\Leftrightarrow \delta^*(q_0, x) \in R && \text{by the definition of acceptance} \\ &\Leftrightarrow \exists z\colon\ \delta^*(\delta^*(q_0, x), z) \in A && \text{by the definition of } R \\ &\Leftrightarrow \exists z\colon\ \delta^*(q_0, xz) \in A && \text{by a Theorem proved in class} \\ &\Leftrightarrow \exists z\colon\ xz \in L \end{aligned}$$

7. Problem 2.10abc

Let M_1 and M_2 be the RAs pictured in Figure 2.44, accepting languages L_1 and L_2 respectively.

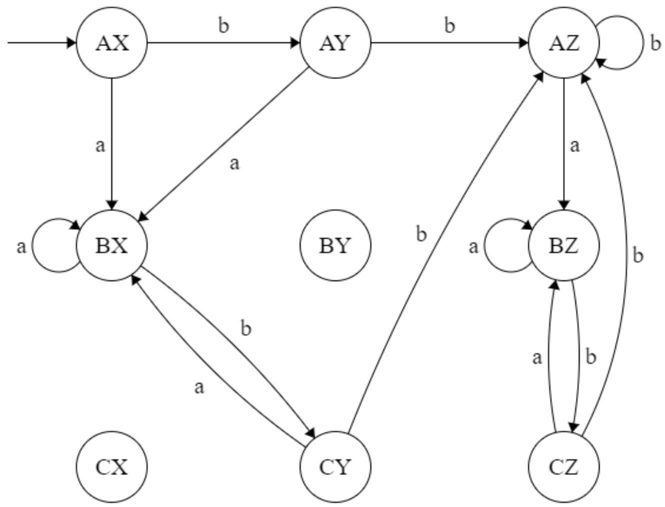


Draw RAs accepting the following languages.

a. $L_1 \cup L_2$
b. $L_1 \cap L_2$
c. $L_1 - L_2$

Solution:

We draw one transition diagram and give the accepting states separately. Outgoing edges from unreachable edges are not drawn, and are left out of the accepting sets.



a. Accepting states for $L_1 \cup L_2$: { AZ, BZ, CZ, CY }
b. Accepting states for $L_1 \cap L_2$: { CZ }
c. Accepting states for $L_1 - L_2$: { CY } (leaving out CX since it is unreachable)