Quiz 3: January 22, 2015

Left Neighbor: ______________ Right Neighbor: ______________

This is a closed book quiz

TABLE 1 **Rules of Inference.**

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \land (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \land (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \land (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$[(p \lor q) \land \neg p] \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$[(p) \land (q)] \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$[(p \lor q) \land (\neg p \lor r)] \to (q \lor r)$	Resolution

Section TA: ______________________ Name: Solutions

1. (4 points) Let $Q(x)$ be the statement "$x = x^2$." If the domain consists of all integers, what are these truth values?

For this question, you were supposed to state TRUE or FALSE for each statement below. Many students circled statements without writing 0 or 1 or T or F. This is not a sufficient answer. Credit was generally given but a point or less was deducted.

   T (a) $Q(0)$

   F (b) $Q(-1)$

   T (c) $\exists x Q(x)$

   F (d) $\forall x \neg Q(x)$

2. (4 points) Express the negations of each of these statements so that all negation symbols immediately precede predicates:

   (a) $\exists z \forall y \forall x T(x, y, z)$

   $$\sim(\exists z\forall y\forall z T(x,y,z)) = \forall z\exists y\exists z\sim T(x,y,z)$$

   (b) $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$

   $$\sim(\exists x\exists y P(x,y) \wedge \forall x\forall y Q(x,y)) = \sim\exists x\exists y P(x,y) \vee \sim\forall x\forall y Q(x,y) = \forall x\forall y\sim P(x,y) \vee \exists x\exists y\sim Q(x,y)$$

3. (2 points) Determine whether each of these arguments is valid. If it is valid, what rule of inference is being used?

   (a) If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$.

   Invalid

   (b) if n is a real number with $n > 2$, then $n^2 > 4$. Suppose that $n^2 \leq 4$. Then $n \leq 2$

   Valid. We are given p -> q and ~q and it yields ~p.
   This is Modus Tollens aka Contraposition