Section TA: Luke Name: Solution

1. **(4 points)** Prove if $a, b, c, d, m \in \mathbb{Z}$ and $m \geq 2$ and $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then $(a+c) \equiv (b+d) \pmod{m}$.

Suppose $a, b, c, d, m \in \mathbb{Z}$ and $m \geq 2$ and
$a \equiv b \pmod{m}$ and
$c \equiv d \pmod{m}$.

$\exists s, t \in \mathbb{Z}$ s.t. $a = b + sm$ (by def) (1)
$c = d + tm$ (by def.) (2)
$a + c = b + d + sm + tm$ (adding (1) and (2))
$a + c = (b+d) + (s+t)m$ (arithmetic)
$\therefore (a+c) \equiv (b+d) \pmod{m}$ (by def.)

2. **(6 points)** Prove for all positive integers n, $5 | (7^n - 2^n)$ by induction on n.

Induction Hypothesis (1 point):

Suppose $5 | (7^k - 2^k)$ for $k \in \mathbb{Z}^+$

Basis step (2 points):

Let $k = 1$, $5 | (7^1 - 2^1)$
$5 | 5$ $\longrightarrow$ Every # divides itself!

Inductive Step (3 points):

$$7^{k+1} - 2^{k+1} = 7 \cdot 7^k - 2 \cdot 2^k$$
$$= 7 \cdot 7^k - 2 \cdot 2^k + \overbrace{2^k \cdot 7 - 2^k \cdot 7}^{0}$$
$$= 7(7^k - 2^k) + 2^k(7-2)$$
$$= 7(7^k - 2^k) + 2^k(5)$$

Since $5 | (7^k - 2^k)$ (our I.H.), then $5 | 7(7^k - 2^k)$, also $5 | 2^k(5)$, hence $5 | 7^{k+1} - 2^{k+1}$.
From induction, the result is true.