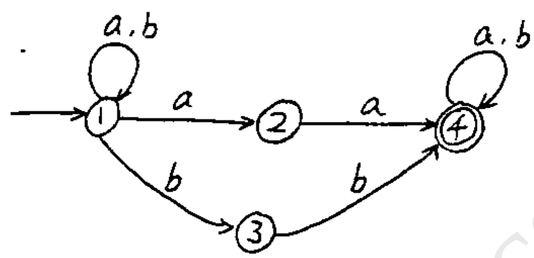
Handout 14

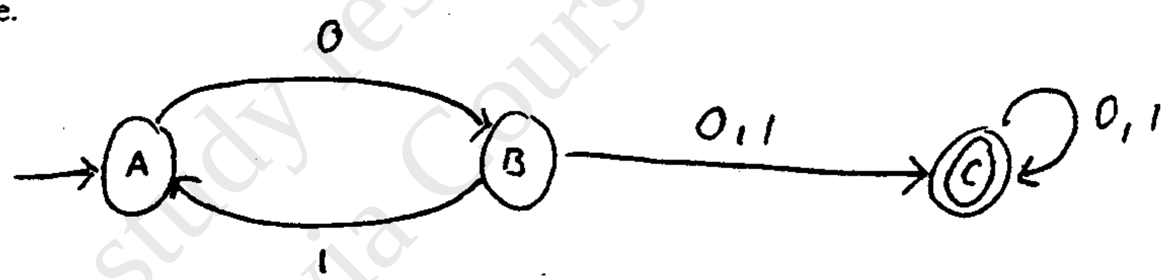
CIS 130 - FALL 96 SOLUTIONS!

1. Give an NFA that accepts all strings over $\{a, b\}$ containing two consecutive a's or two consecutive b's (non-exclusive 'or').

$$(a+b)^* (aa+bb)(a+b)^*$$



2. For each state of the following ~~D~~NFA give a regular expression for all the strings that lead to that state.

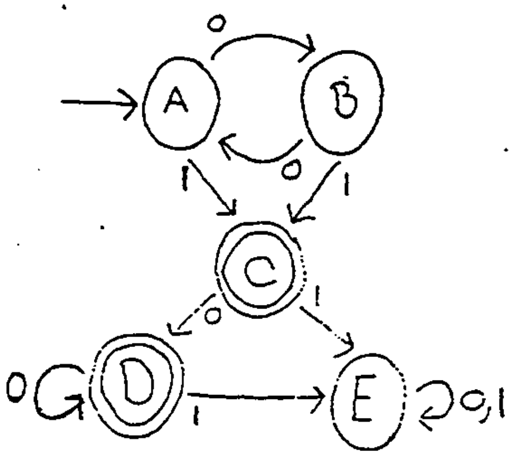


Ⓐ : $(01)^*$

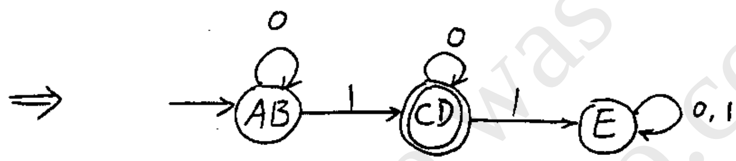
Ⓑ : $0(10)^*$

Ⓒ : $0(10)^*(0+1)^+$

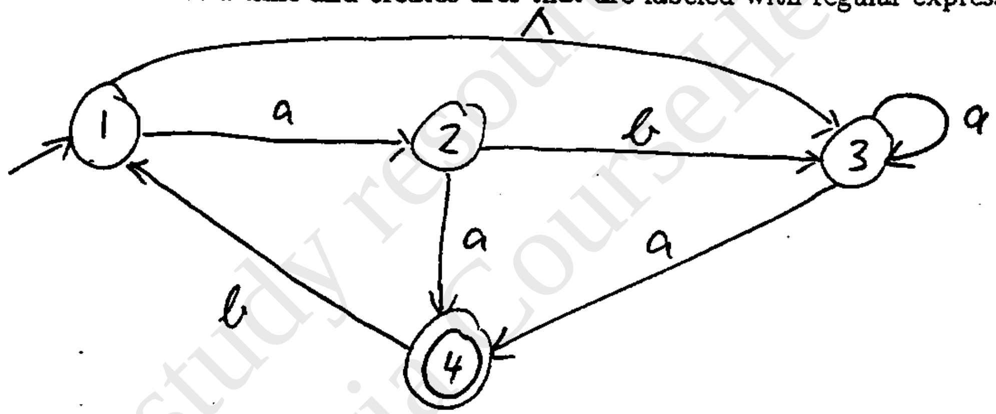
4 Minimize the following FA. Show your work. Draw the new FA in case the number of states was reduced.

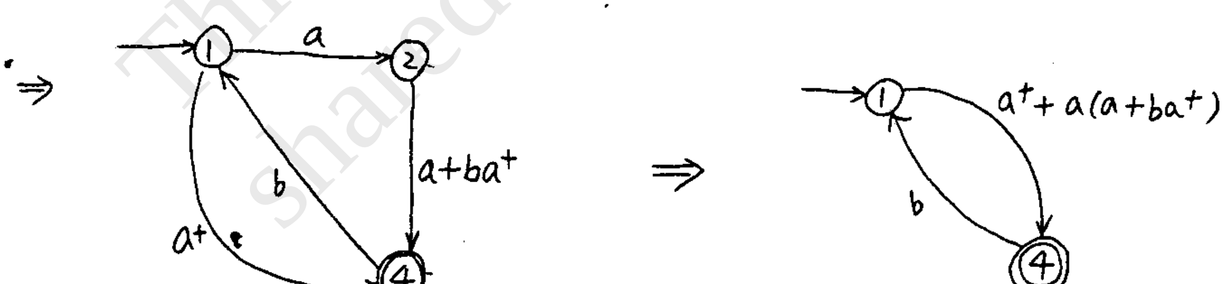


	A	B	C	D
B	0			
C	1	1		
D	1	1	0	
E	2	2	1	1



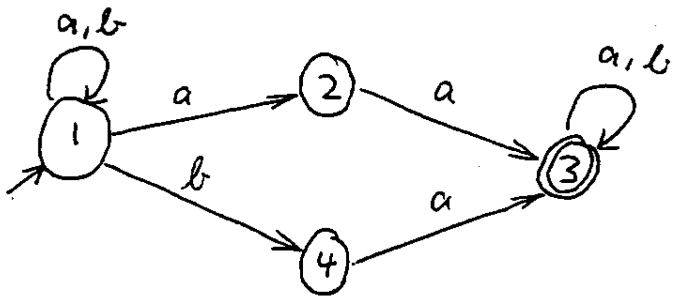
3. Convert the following λ-NFA into a regular expression. Use the algorithm given in class that eliminates one node at a time and creates arcs that are labeled with regular expressions.



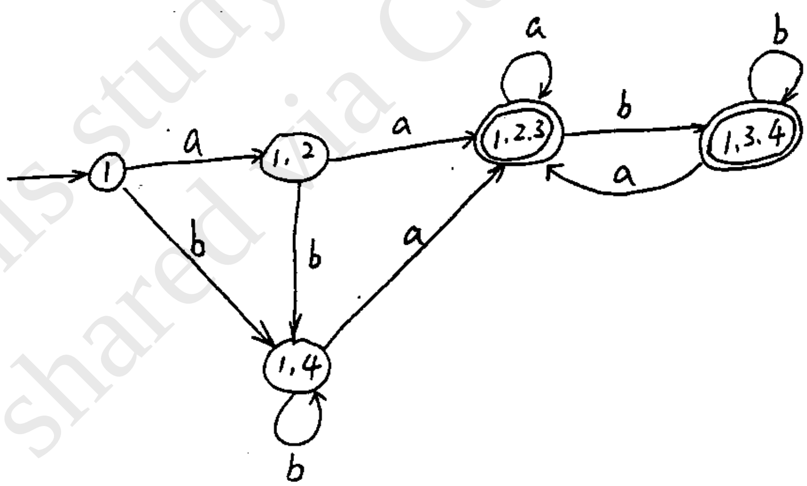


$$(a^+ + a(a+ba^+))\,(b\,(a^+ + a(a+ba^+)))^*$$

5. Use the "subset construction method" to convert the following NFA to an FA. Show the steps of your construction as well as the final FA.



q	$\delta(q, a)$	$\delta(q, b)$
{1}	{1, 2}	{1, 4}
{1, 2}	{1, 2, 3}	{1, 4}
{1, 4}	{1, 2, 3}	{1, 4}
{1, 2, 3}	{1, 2, 3}	{1, 3, 4}
{1, 3, 4}	{1, 2, 3}	{1, 3, 4}



6 Show that $L := \{ww : w \in \{a,b\}^*\}$ is not regular by exhibiting an infinite set that is pairwise distinguishable.

Hint: Use $S = \{ba^i : i \geq 1\}$.

Prove for each pair of words of S that it is pairwise distinguishable.

Proof: Let x and y be two arbitrary distinct elements of S. That is,

$$x = ba^j, \quad y = ba^k, \text{ and } j \neq k, \ j \geq 1, \ k \geq 1$$

Select $z = ba^j$,

$$xz = (ba^j)(ba^j) \in L$$
$$yz = (ba^k)(ba^j) \notin L, \quad \text{since } k \neq j.$$

It follows that the infinite set S is pairwise distinguishable w.r.t. L.
Thus L can't be regular since distinguishable words have to end up in different states in a DFA that accepts L.

7 Show that that if a L over some finite alphabet Σ is regular then the language $\tilde{L}$ of all suffixes of words in L is also regular.

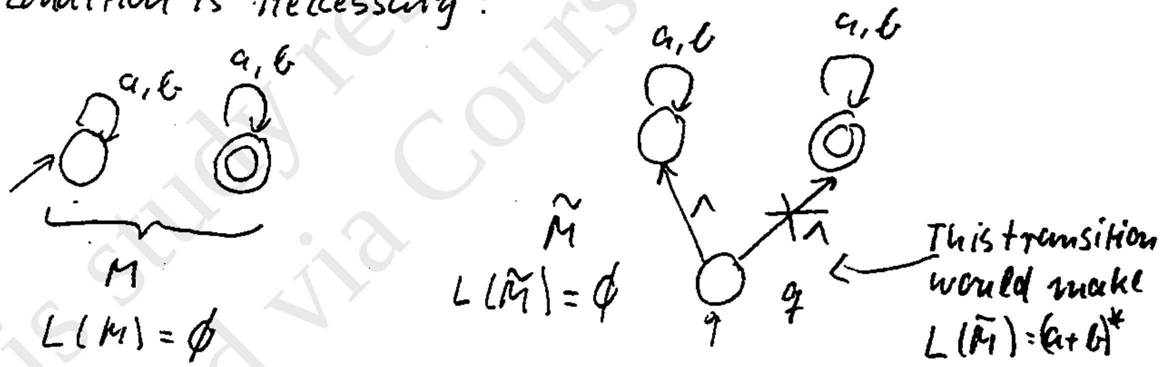
Formally $\tilde{L}$ is defined as $\{w \in \Sigma^* | \exists v \in \Sigma^* \text{ such that } vw \in L\}$. $= SUFF(L)$

Hint: There are many solutions to this problems. One solution uses other similar closure properties of regular sets that were discussed in class.

I) Since L is regular, there is an FA M that accepts it. We will construct a λNFA $\tilde{M}$ that accepts $\tilde{L}$. Since λNFA's accept regular languages we are done.

To construct $\tilde{M}$ from M, add a new state q to M and make it the startstate of the new machine. Add λ transition to all states of M reachable from the old startstate of M

↑ This condition is necessary!



II) $SUFF(L) = REV(PREV(REV(L)))$

$REV(L) = \{w \in \Sigma^* : w^R \in L\}$

$PREV(L) : \{w \in \Sigma^* : \exists v \in \Sigma^* \text{ such that } wv \in L\}$

We showed in class that regular languages are closed under the operations REV and PREV. Thus they are closed under PREV as well.

8 Use the Pumping Lemma for regular languages to show that $\{a^i b^j | 0 \leq 2i \leq j\}$ is not regular.

You can use the following Pumping Lemma:

For every regular language L there is a constant N such that each word $x \in L$ of length at least N can be written as uvw such that the following holds:

i) $|uv| \leq N$,

ii) v is not the empty word and

iii) for all $i \geq 0$, $uv^iw \in L$.

Assume L is regular. Then the PL holds for L.
Let N be the constant of the PL.
Let $x = a^N b^{2N}$. Since $x \in L$ and $|x| \geq N$,
x can be written as uvw such that i), ii) and iii) of PL hold.

i) implies that $uv \in a^*$

ii) implies that $v \in a^+$.

iii) implies that

$$uv^2w = a^{N+|v|} b^{2N} \in L$$

By the definition of L this means that

$$2(N+|v|) \leq 2N$$

This can't be true since $|v| \geq 1$.
We have a contradiction to the assumption that L is regular!