CMPS 130
Final Review Problems

These problems are meant as review of the material covered since the last midterm. Consult the previous review sheets and midterm solutions for review of prior material.

1. Write down productions for a CFG G that generates the language $\text{NonPal} = \{x \in \{a, b\}^* \mid x^r \neq x\}$. Give leftmost derivations in this grammar for the strings *aab*, *babbb*, *ababbab*. Draw the corresponding derivation trees in each case.

2. Draw the nondeterministic top-down PDA *NT*(*G*) corresponding to the CFG you wrote in problem (1). Give a sequence of moves in this PDA showing that the string *aaba* is accepted, displaying the machine configuration at each step. Alongside this trace give the leftmost derivation of this string. (See class notes from 12-2-13 p.5.)

3. Repeat problems (1) and (2) for the following languages. Make up your own example strings in each case.
   a. $L = \{x \in \{a, b\}^* \mid n_a(x) = n_b(x)\}$
   b. $L = \{a^i b^j \mid i \leq j\}$
   c. $L = \{a^i b^j \mid j = 2i\}$

4. Prove that the union of two CFLs is a CFL. (This is part (1) of Theorem 4.9 whose proof is on p.136 of the text. The proof is also in the lecture notes from 11-18-13 pages 1-6.)

5. Let $\mathcal{F}$ denote the set of finite languages and $\mathcal{R}$ the set of regular languages over an alphabet Σ.
   a. Prove that every $L \in \mathcal{F}$ is a CFL. (Use the recursive definition of $\mathcal{F}$, structural induction and Theorem 4.9.)
   b. Prove that every $L \in \mathcal{R}$ is a CFL. (Use the recursive definition of $\mathcal{R}$, structural induction and Theorem 4.9. This is also problem 4.21 in the text.)

6. For each of the DFAs pictured in figure 2.44 (p.79 of the text), write down a regular grammar that generates the language accepted by the DFA.

7. Draw a PDA that accepts the language of even length palindromes over $\{a, b\}$.

8. Draw a DPDA that accepts the language $L = \{x \in \{a, b\}^* \mid n_a(x) = n_b(x)\}$.

9. State the Pumping Lemma for CFLs

10. Prove that the language $L = \{t \in \{a, b, c\}^* \mid n_a(t) = n_b(t) = n_c(t)\}$ is not context free.

11. Problem 6.2abc on p. 220 of the text. (This is the pumping for CFLs lemma again.)

12. Draw a transition diagram for a Turing Machine accepting the language represented by the following regular expression: $(a+b)^* aba (a+b)^*$.

13. Trace the TM in Figure 7.5 of the text (p. 232), accepting the language $\{ xx \mid x \in \{a, b\}^* \}$ on the input strings *aa* and *ab*. Show the machine configuration at each step.