ETH Zürich
D-ITET
Biomedical Engineering

Master Studies

Biomedical Imaging

Homework #3 - Ultrasound 2

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1 Task 1

If the single-element array is viewed as a limiting case of a single-transducer beam as discussed in the previous lecture, the beam characteristics(incl. Near Field Boundary, Lateral Resolution and Broadening Angle θ) are given as the following:

$$NFB \approx \frac{r^2}{\lambda} \tag{1.1}$$

$$\theta = \arcsin\frac{0.61\lambda}{r} \tag{1.2}$$

$$lateral\ resolution = 2r + 2(Z - NFB)tan\theta \tag{1.3}$$

where r is the radius of the beam and λ is the wavelength.
Since the array elements are assumed to be long in the z direction and of negligible width in the y direction(i.e. r $\approx$ 0), we get:

$$NFB \approx 0 \tag{1.4}$$

$$\theta = \lim_{r \to 0} \arcsin\frac{0.61\lambda}{r} \tag{1.5}$$

Apparently, there exists no $\theta \in \mathbb{R}$ that satisfies equation (1.5). However, by Taylor Series Expansion of arcsin(x) at $x = \infty$, we can obtain an approximation of $\theta \in \mathbb{C}$ that satisfies equation (1.5) where the real part of θ is $\frac{\pi}{2}$.
Thus, the lateral resolution within the depth of NFB is approximately 0 while outside of NFB it rapidly deteriorates up to infinity.

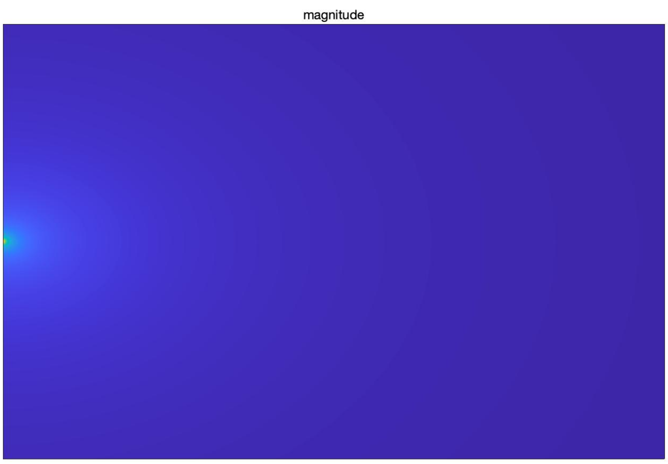

Figure 1.1: Magnitude, Single Transducer

2 TASK 2

The results of varying the number of transducer elements are as follows:

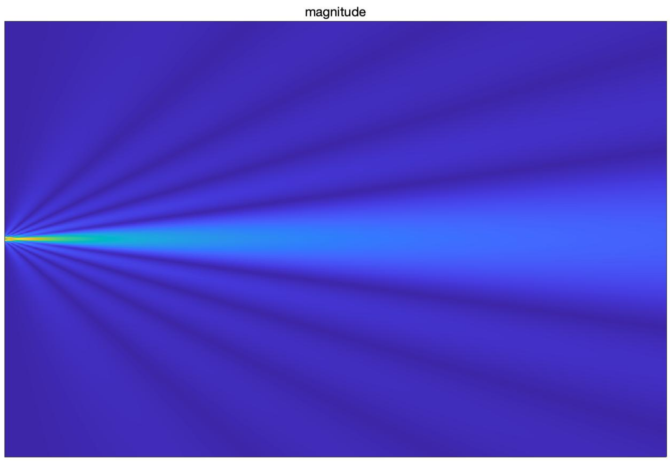

Figure 2.1: Magnitude, 10 Transducers

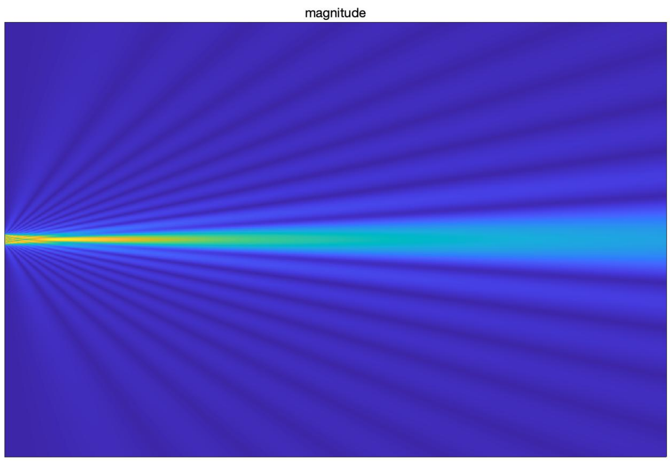

Figure 2.2: Magnitude, 20 Transducers

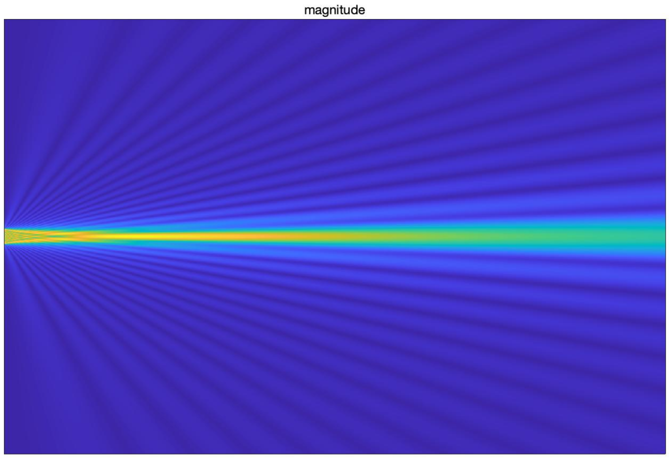

Figure 2.3: Magnitude, 30 Transducers

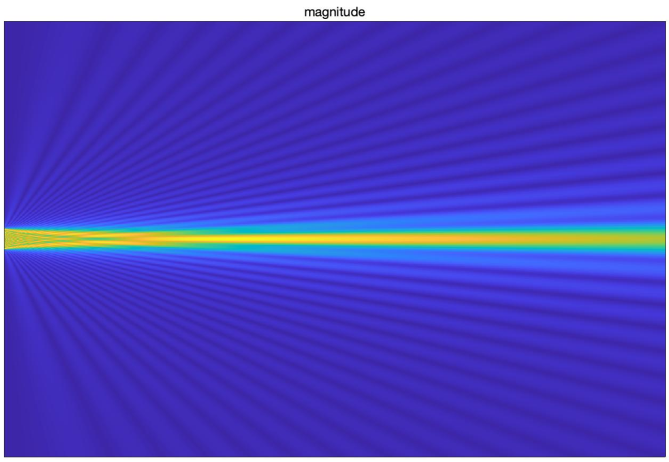

Figure 2.4: Magnitude, 40 Transducers

As is shown in the figures above, the array with more transducer elements forms comparatively narrower beam with successively-decreasing sidelobes.
This can be accounted by constructive interference. The beams from different transducer elements with the same phase constructively interfere with each other and the intensity of interference decreases as the opening angle increases. The strongest interference happens at which the opening angle equals to zero and thus forms the mainlobe. The interference happening at increasingly-larger opening angles forms sidelobes that decreases in succession.

3 TASK 3

To achieve constructive interference, the phase difference between each two point wave sources that form the new wave front must be equal to the phase difference between the sinusoidal voltage waves that generate them. This can be given by the following equation:

$$\Delta\Phi = kysin(\alpha) \tag{3.1}$$

Where k is a constant translating the distance into phase difference:

$$k = \frac{2\pi}{\lambda} \tag{3.2}$$

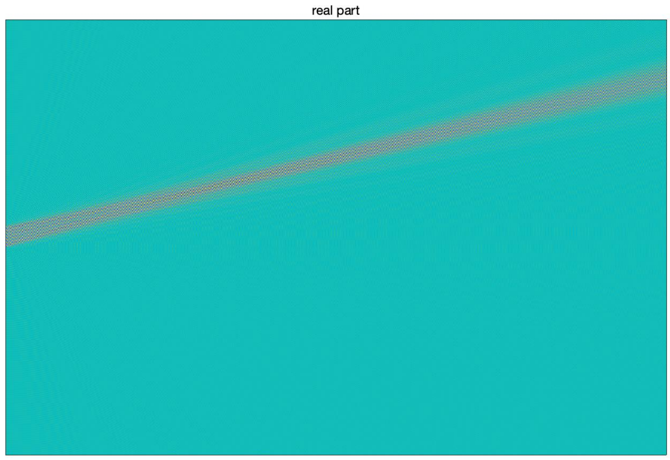

Figure 3.1: Real Part, Deflection

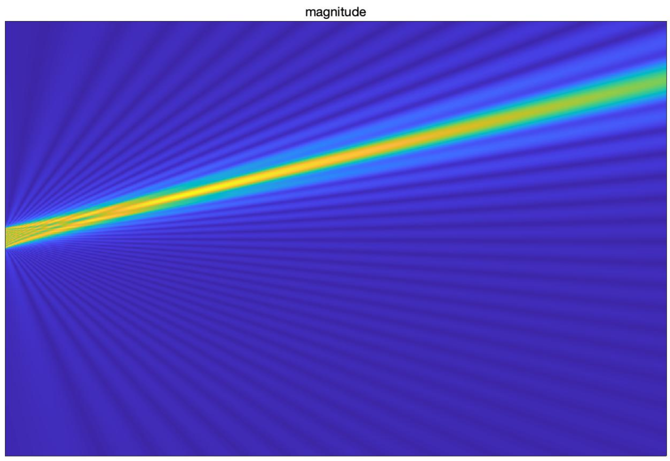

Figure 3.2: Magnitude, Deflection

4 TASK 4

According to the previous lecture, grating lobes occur when the array's pitch is greater than $\frac{\lambda}{2}$. In this case, we set the pitch to λ, the result is given as follows:

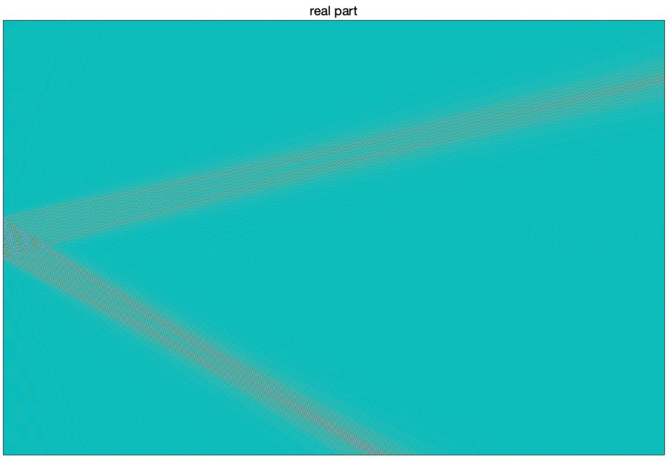

Figure 4.1: Real Part, Grating Lobes

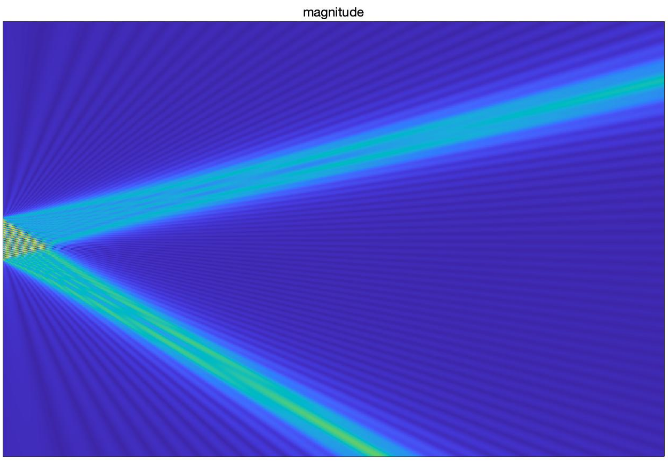

Figure 4.2: Magnitude, Grating Lobes

5 TASK 5

The difference of the travelling distance from the phased array to focusing point between the central transducer element and the one next to it is given by:

$$\Delta x = \sqrt{f^2 + y^2} - f \tag{5.1}$$

Thus the phase difference is given by:

$$\Delta \Phi = k(\sqrt{f^2 + y^2} - f) \tag{5.2}$$

By Taylor Expansion we obtain:

$$\Delta \Phi \approx kf(1 + \frac{y^2}{2f^2} - 1) = \frac{y^2}{2f} \tag{5.3}$$

As is shown in the equation (5.3), the phase difference is a parabolic function of the interval between two adjacent elements where the distance from the phased array to the focusing point is the focus.

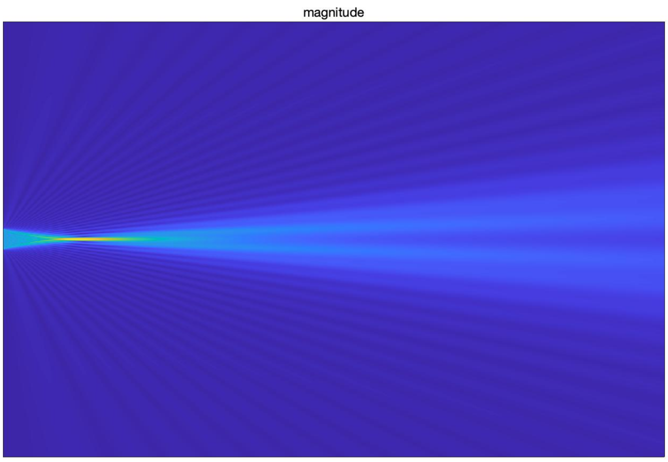

Figure 5.1: Magnitude, Focus at 5cm

6 TASK 6

The Gaussian Bell function with the mean of 0 and the standard deviation of σ is given by:

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp \frac{-y^2}{2\sigma^2} \tag{6.1}$$

Thus when the amplitude of the outer elements(i.e. $y = \frac{pitch \times N}{2}$) drops to 10 percent of that of the central element, we can easily derive that $\sigma = 4.6 \times 10^{-3}$.

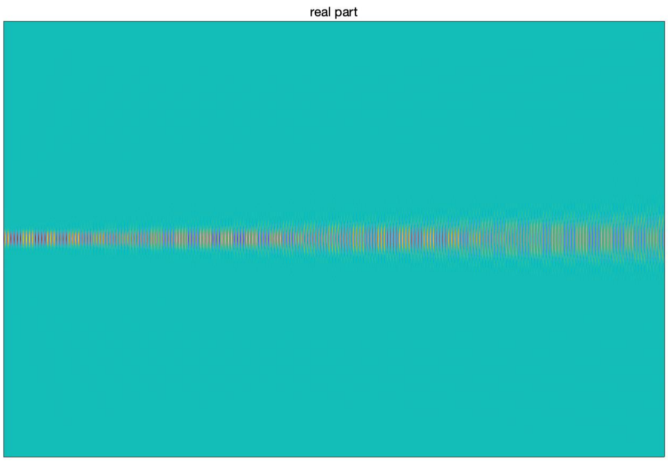

Figure 6.1: Real Part, Apodization

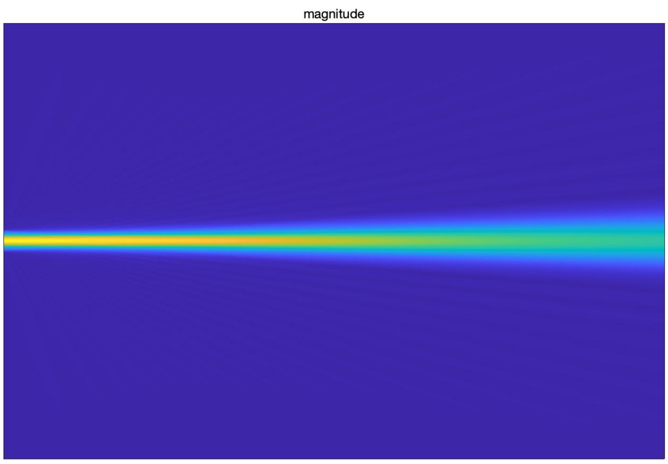

Figure 6.2: Magnitude, Apodization