ETH Zürich
D-ITET
Biomedical Engineering

Master Studies

Biomedical Imaging

Homework #10 - MRI 1

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1 TASK 1

According to Boltzmann statistics, the two population number is,

$$\frac{n_{down}}{n_{up}} = e^{-\frac{\Delta E}{k_B T}} \tag{1.1}$$

The energy gap is,

$$\Delta E = \hbar \gamma B_0 \tag{1.2}$$

with $\frac{\gamma}{2\pi} = 42.576 MHz/T$ for protons, $B_0 = 3T$, and $T = 310K$, we obtain:

$$\frac{n_{down}}{n_{up}} = 0.999980227 \tag{1.3}$$

Thus,

$$\frac{\Delta n}{n} = 9.88659 \times 10^{-6} \tag{1.4}$$

2 Task 2

In fully relaxed transverse magnetization, excitation with flip angle θ results in,

$$M_z' = M_z^s \cos\theta \tag{2.1}$$

Where M_z^s is the magnetization in steady-state before the next excitation pulse. Subsequent relaxation during the pulse interval T_R results in:

$$M_z^(\prime\prime) = M_0 + (M_z' - M_0)e^{-\frac{T_R}{T_1}} \tag{2.2}$$

According to the steady-state assumption,

$$M_z'' = M_0 + (M_z' - M_0)e^{-\frac{T_R}{T_1}} = M_z^s \tag{2.3}$$

Thus,

$$M_z^s = M_0 \frac{1 - e^{-\frac{T_R}{T_1}}}{1 - \cos^{-\frac{T_R}{T_1}}} \tag{2.4}$$

Hence,

$$M_{xy} = M_z^s \sin\theta = M_0 \sin\theta \frac{1 - e^{-\frac{T_R}{T_1}}}{1 - \cos^{-\frac{T_R}{T_1}}} \tag{2.5}$$

We can obtain the optimal flip angle which maximizes M_{xy} by setting the derivative with respect to θ to 0, i.e.

$$\frac{dM_{xy}}{d\theta} = M_0(1 - e^{-\frac{T_R}{T_1}}) \frac{\cos\theta(1 - \cos^{-\frac{T_R}{T_1}}) - \sin^2\theta e^{-\frac{T_R}{T_1}}}{(1 - \cos^{-\frac{T_R}{T_1}})^2} = 0 \tag{2.6}$$

And we obtain,

$$\theta = \cos^{-1} e^{-\frac{T_R}{T_1}} \tag{2.7}$$

The Matlab simulation results are shown as follows.

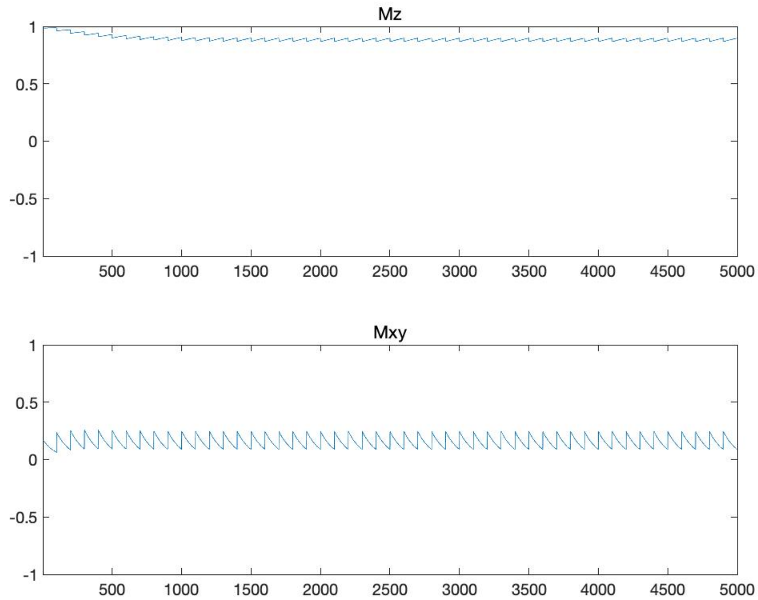

Figure 2.1: θ = 10°

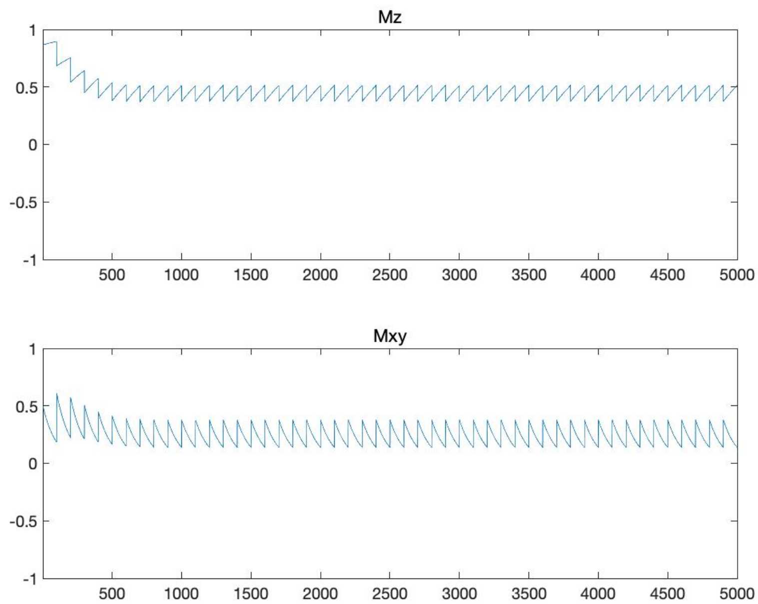

Figure 2.2: θ = 30°

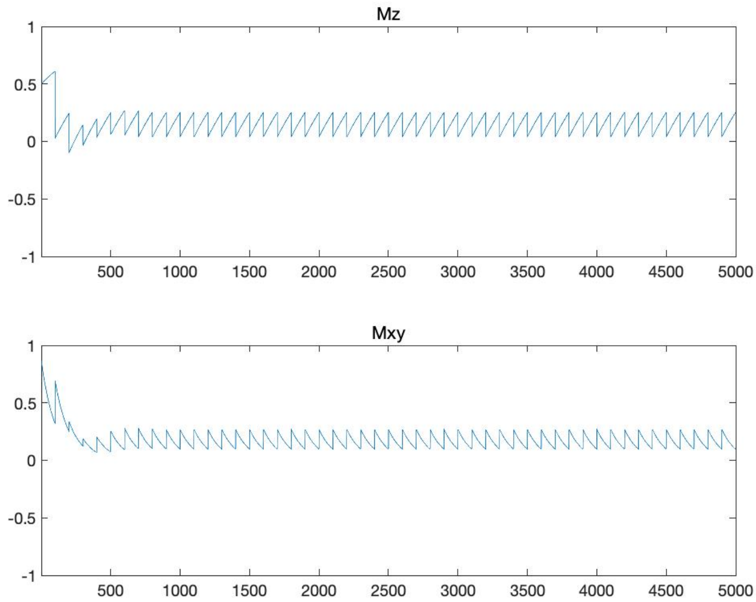

Figure 2.3: θ = 60°

Consider the case of our example where T_1 = 200 and T_R = 50, the optimal $\theta \approx 39°$.

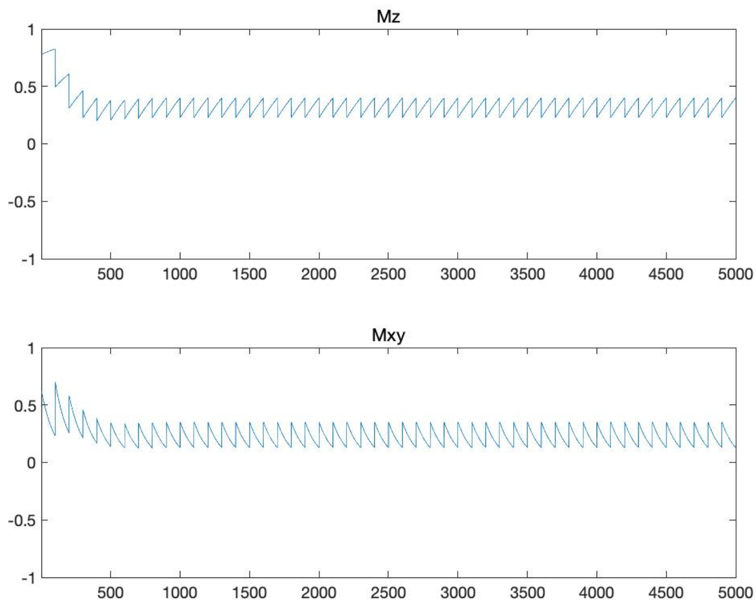

Figure 2.4: θ = 39°