ETH Zürich
D-ITET
Biomedical Engineering

Master Studies

Biomedical Imaging

Homework #7 - X-ray Imaging 3

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1 TASK 1

Given the parameters in Table 1.1,

Tube Voltage(kV)	50
Anode Current(mA)	100
Anode Material	Tungsten
Relative X-ray Yield(kV^{-1})	$10^{-15} \times Z$
X-ray Burst Duration(ms)	10

Table 1.1: Parameters

The number of photons N_0 incident to the object during an X-ray burst is given by,

$$N_0 = \frac{Z \times 10^{-15} V_{tube} I_{anode} \Delta t}{e} = \frac{74 \times 10^{-15} \times 50 \times 100 \times 10^{-3} \times 10 \times 10^{-3}}{1.6 \times 10^{-19}} = 2.3125 \times 10^4 \tag{1.1}$$

According to Beer Lambert's law,

$$N = N_0 e^{-\int \mu(x) dx} \tag{1.2}$$

Where $\mu(x)$ is the mass attenuation coefficient.
The results of back-projection with Poisson noises is shown in Figure 1.1.

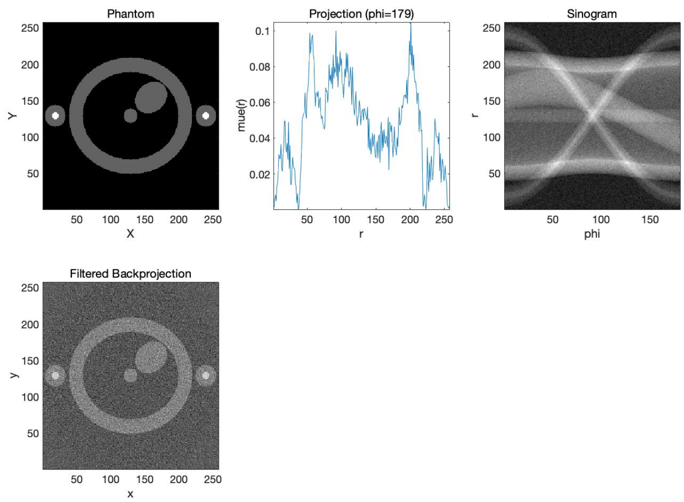

Figure 1.1: Back-projection with Poisson Noises

2 TASK 2

The 2-D Gaussian filter is given by,

$$G(u, v) = e^{-\frac{u^2+v^2}{2\sigma_g^2}} \tag{2.1}$$

Where σ_g is the the standard deviation of the corresponding Gaussian distribution. The relationship between σ_g and *FWHM*(i.e. Full Width at Half Maximum) is given by,

$$\sigma_g = \frac{FWHM}{2\sqrt{2\ln 2} \times resolution\ factor} \tag{2.2}$$

The results of back-projection applied Gaussian filter is shown in Figure 1.2.

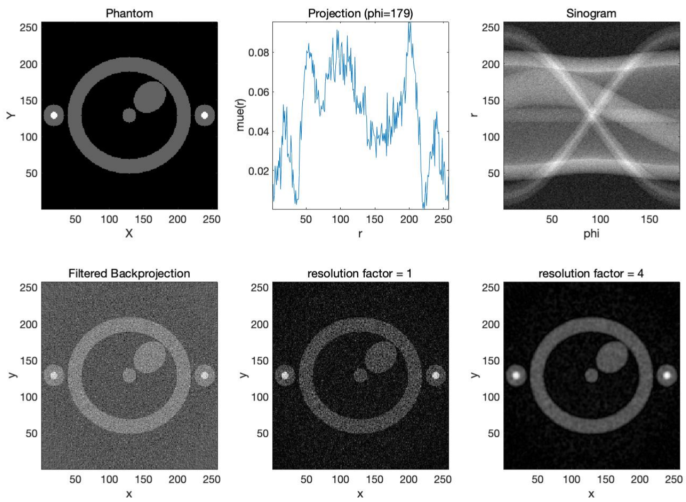

Figure 2.1: Back-projection with Gaussian Filter

According to Equation 2.2, the standard deviation σ_g is inversely proportional to the resolution factor given a fixed full width at half maximum. That is to say, the larger the resolution factor is, the smaller σ_g is. In another word, the larger the resolution factor is, the narrower the bandwidth of the passing frequency is.

As is shown in the bottom middle and bottom right of Figure 2.1, when the resolution factor is increased from 1 to 10, the filtered back-projection gets smoother yet more blurred. This is because with a larger resolution factor of 10, the corresponding Gaussian filter allows only less higher frequencies to pass. That is to say, the filter gets rid of more high frequency noises as well as details and sharp edges which are also contained in the high frequency component of the spectrum of the original image.

Therefore, we can conclude that the reduction of the noises is at the cost of the resolution of the filtered image. The price of filtering out more noises is having lower resolution.

3 TASK 3

The SNR of the heart is given by,

$$SNR = \frac{\bar{I}_{heart}}{SD(I_{heart})} \tag{3.1}$$

Where $\bar{I}_{heart}$ and $SD(I_{heart})$ is the mean value and standard deviation of the intensity of the heart region each respectively.
The SNR of back-projection with double and triple tube current are given in Figure 3.1 and 3.2.

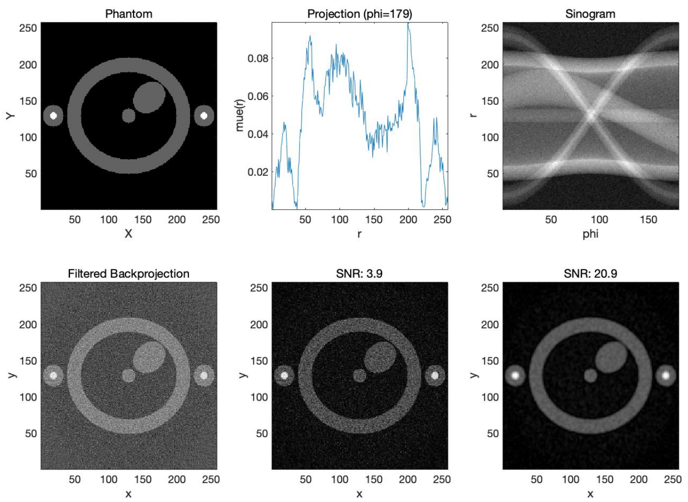

Figure 3.1: SNR of Back-projection with Double Tube Current

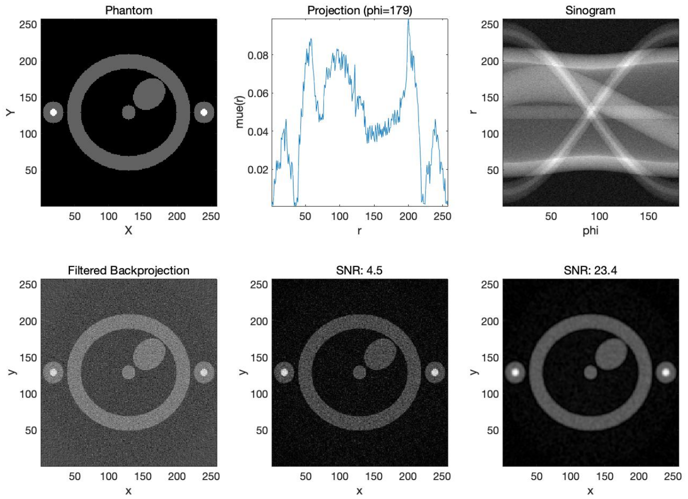

Figure 3.2: SNR of Back-projection with Triple Tube Current

The SNR of back-projection with burst duration of 20 and 40 milliseconds are given in Figure 3.3 and 3.4

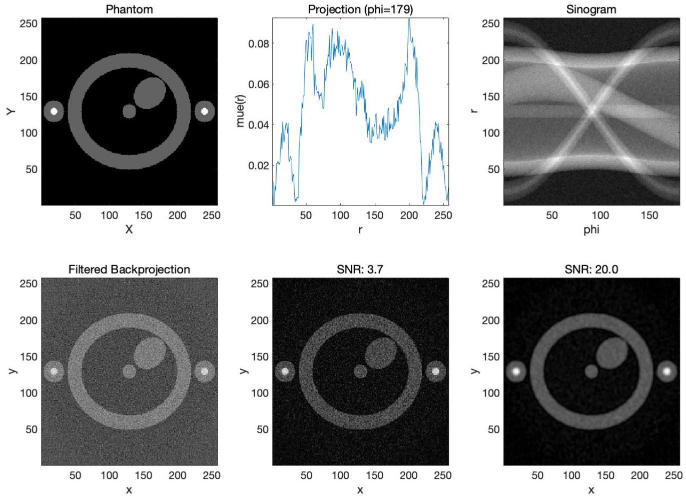

Figure 3.3: SNR of Back-projection with Burst Duration of 20 Milliseconds

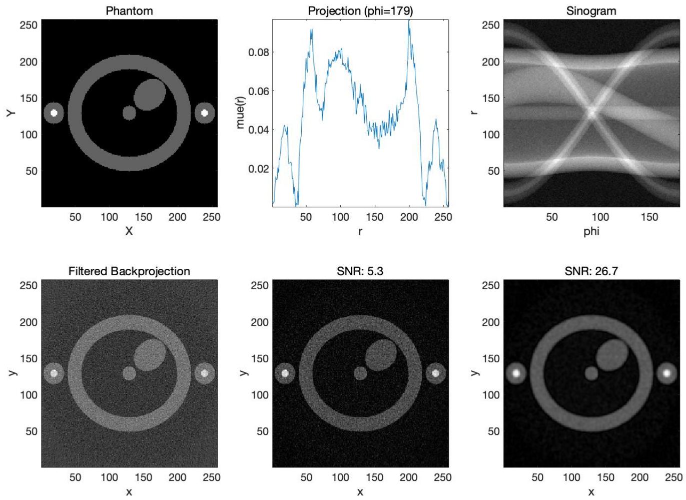

Figure 3.4: SNR of Back-projection with Burst Duration of 40 Milliseconds

From the previous lecture, we know that the SNR is in proportional to the square root of tube current multiplied by burst time and detector thickness, given by,

$$SNR \propto \sqrt{I_Q \Delta t \Delta Z} \tag{3.2}$$

The relationships between SNR and tube current and burst duration are given in the Figure 3.5 and 3.6 each respectively.

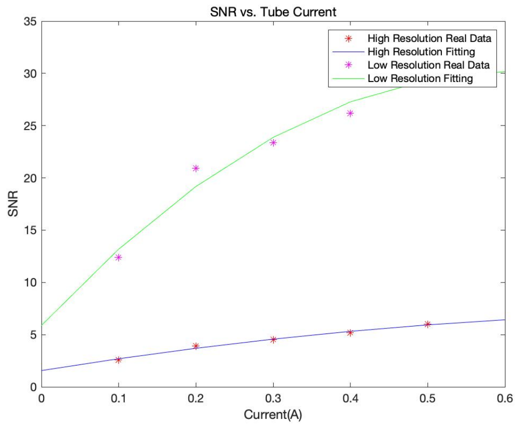

Figure 3.5: SNR vs. Tube Current

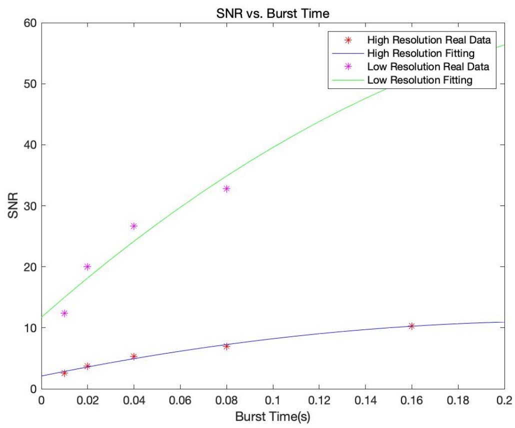

Figure 3.6: SNR vs. Burst Duration

4 Task 4

The heart beat is simulated by changing the length of the minor axis(i.e. b) sinusoidally in different projection angles. Since the period of one heart beat cycle is 1 second and the full cycle of the projection is from 0 to π, we can easily derive the angular frequency given by,

$$\omega = \frac{\pi}{T} = \pi \tag{4.1}$$

The result of back-projection of a beating heart is shown in Figure 4.1.

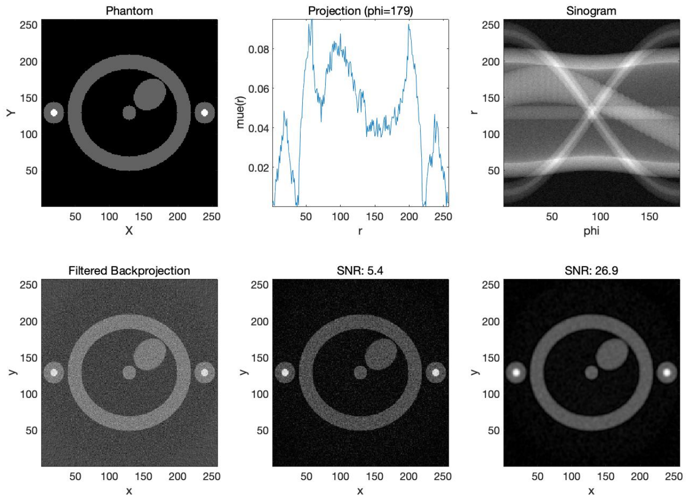

Figure 4.1: Back-projection of Beating Heart, Undersampling Factor = 1

As is shown in the Figure 4.1, the signal to noise ratio(SNR) drops compared to a stationary heart.
The results of of back-projection of a beating heart with increasing undersampling factors are shown in Figure 4.2, 4.3 and 4.4.

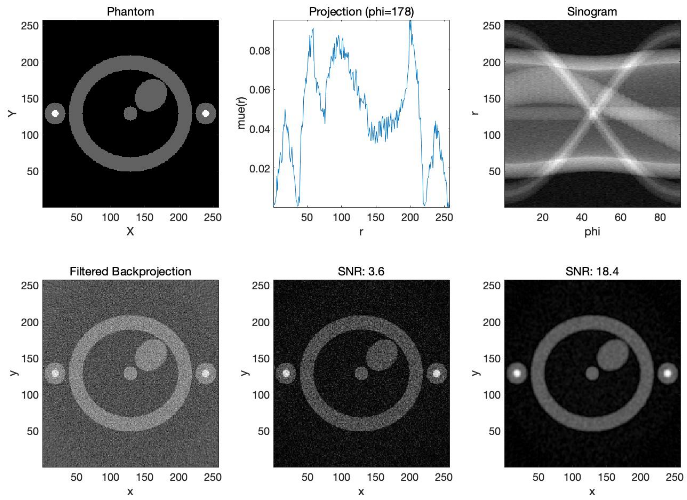

Figure 4.2: Back-projection of Beating Heart, Undersampling Factor = 2

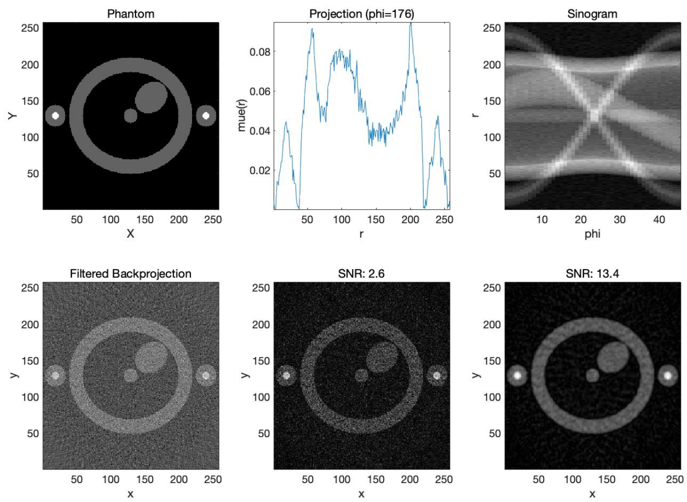

Figure 4.3: Back-projection of Beating Heart, Undersampling Factor = 4

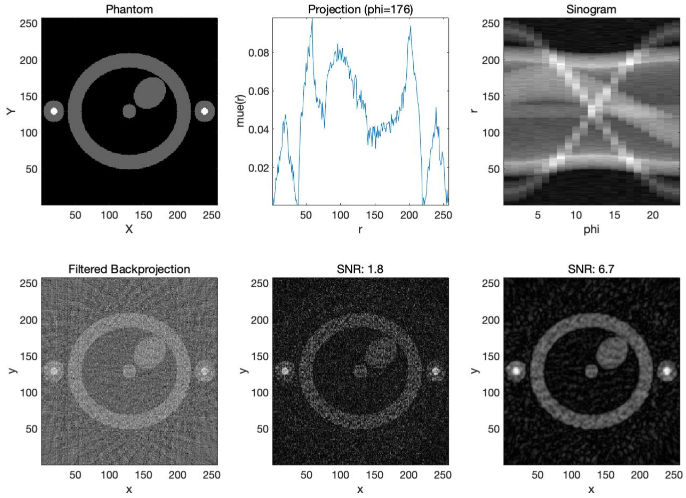

Figure 4.4: Back-projection of Beating Heart, Undersampling Factor = 8

We can see that the signal to noise ratio drops as undersampling factor increases. The relationship between signal to noise ratio and undersampling factor is shown in Figure 4.5.

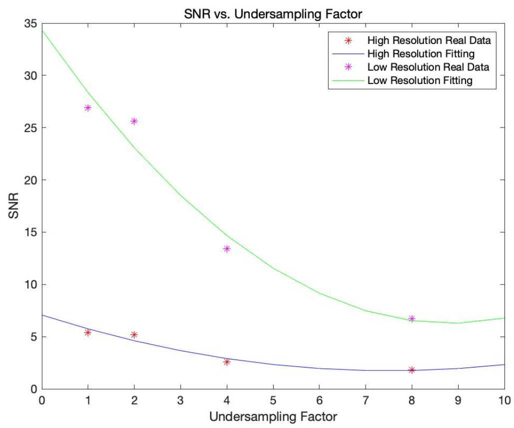

Figure 4.5: SNR vs. Undersampling Factor

From the results above, we can conclude that there is a trade-off between signal to noise ratio and undersampling factor. Thus, if we want to decrease the motion artifacts by speeding up the projection(i.e. increasing the undersampling factor), we will have a reconstructed image with lower quality(i.e. smaller signal to noise ratio).
From the aforementioned above, our goal is to decrease the motion artifacts while keep a relatively high signal to noise ratio. Thus, we can increase the burst time and tube current in the first place to make up for the signal to noise ratio reduced by increasing the undersampling factor. However, this method is not always possible in practical because the CT equipment is already set up.
The alternative way is to adjust the sampling to heart beat cycle so that each projection is taken at the beginning or the end of the heart beat cycle. Thus, the heart can be

viewed as a stationary object. However this method is at the cost of scanning time. Also, the heart beat cycle can by fully and accurately predicted.