ETH Zürich
D-ITET
Biomedical Engineering

Master Studies

Biomedical Imaging

Homework #8 - Nuclear Imaging 2

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1 TASK 1

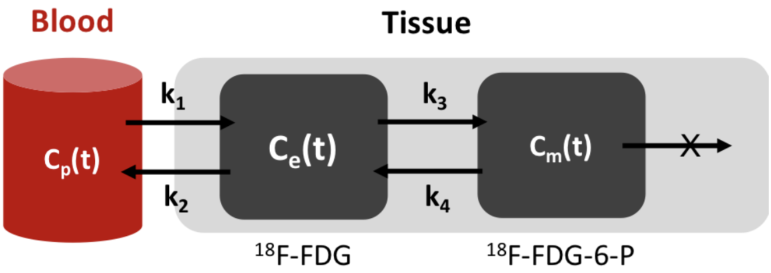

Figure 1.1: Flowchart

As is shown in Figure 1.1, assuming that the input from the blood plasma is a delta function and $k_4 = 0$, we can easily derive,

$$\begin{aligned} \frac{dc_e(t)}{dt} &= -k_2 c_e - k_3 c_e \\ \frac{dc_m(t)}{dt} &= k_3 c_m \end{aligned} \tag{1.1}$$

2 TASK 2

Solving the equation 1.1, we obtain the he impulse response functions of the extracellular compartment $c_e(t)$ and the metabolized compartment $c_m(t)$ given by,

$$
\begin{aligned}
c_e(t) &= k_1 e^{-t(k_2+k_3)} \\
c_m(t) &= \frac{-k_1 k_3 e^{-t(k_2+k_3)}}{k_2+k_3} + \frac{k_1 k_3}{k_2+k_3}
\end{aligned}
\tag{2.1}
$$

Now implement convolution of the impulse response functions with the blood plasma input $c_p(t)$ and inspect the tissue concentration-time curves for $c_e(t)$ and $c_m(t)$ using the following values: $k_1 = 0.1 min^{-1}$, $k_2 = 0.3 min^{-1}$, $k_3 = 0.5 min^{-1}$. The results are shown in Figure 2.1.

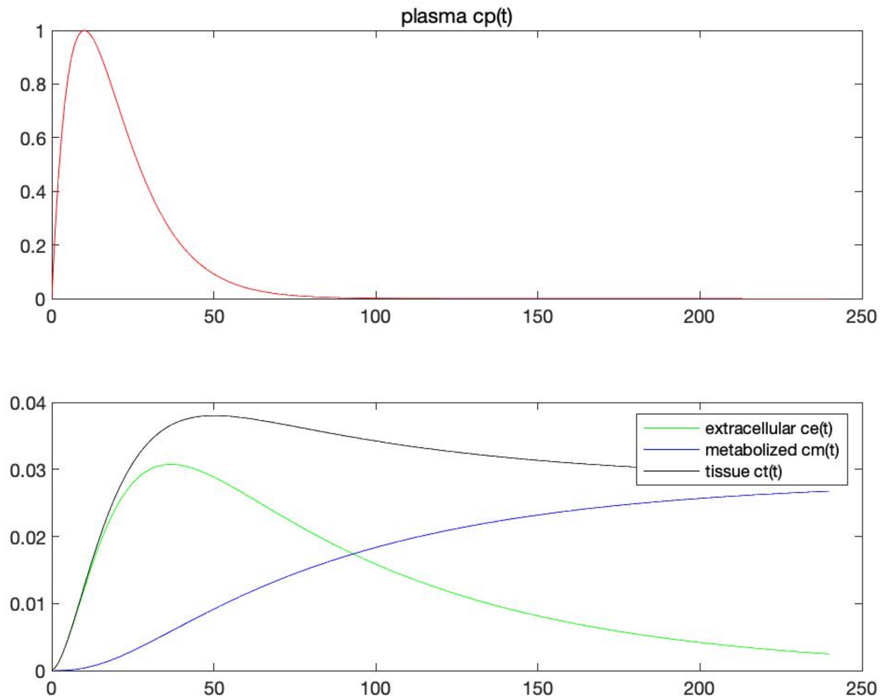

Figure 2.1: Results of Task 2

For most tumor cells, they take up all the energy(glucose) they can have. So there is hardly backward reaction. Thus, we can assume $k_4 = 0$.

3 TASK 3

Adding Poisson noise to $c_p(t)$ and $c_m(t)$ by converting concentrations into photon counts such as to obtain a peak SNR of 100 of the blood plasma signal, the resulting concentration-time curves are shown in Figure 3.1.

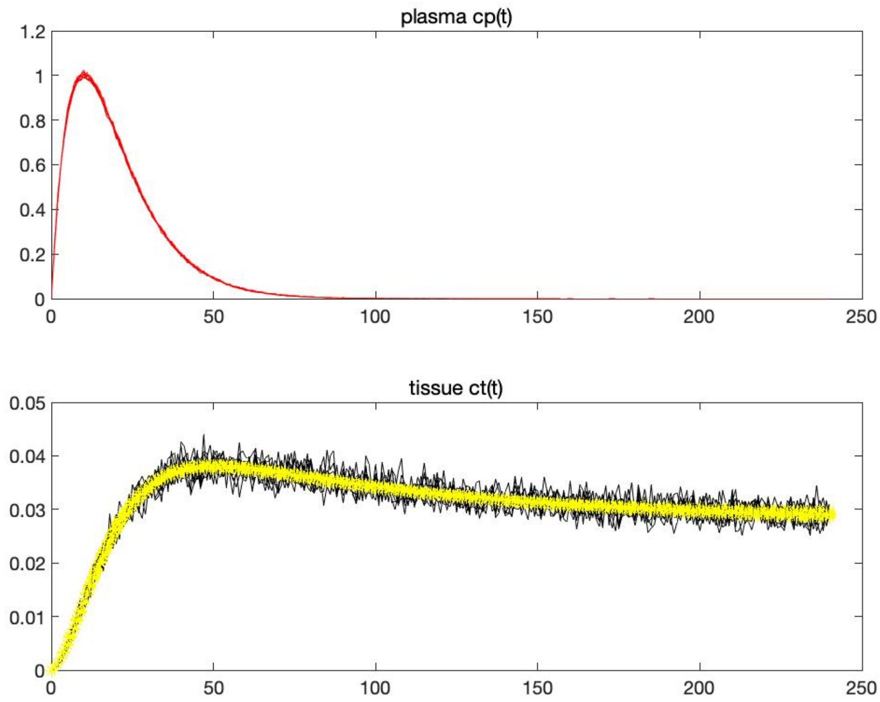

Figure 3.1: Concentration-Time Curves

Implementing the fit function to determine the rate constants (k_1, k_2, k_3) from noisy $c_p(t)$ and $c_m(t)$ input, the mean and standard deviation of the fitted rate constants (k_1, k_2, k_3) for multiple repetitions of adding noise and fitting the noisy data when $SNR = 100$ and $SNR = 10$ are shown in Table 3.1 and 3.2 each respectively.

	k_1	k_2	k_3
Mean	0.10	0.32	0.52
Standard Deviation	0.00	0.06	0.08

Table 3.1: Mean and Standard Deviation of k_1, k_2, k_3, SNR=100

	k_1	k_2	k_3
Mean	0.09	0.63	2.20
Standard Deviation	0.01	0.67	2.21

Table 3.2: Mean and Standard Deviation of k_1, k_2, k_3, SNR=10

We can easily observe that when SNR is reduced from 100 to 10, both the mean and standard deviation of k_1, k_2, k_3 increase. The conclusion is that in order to estimate the true constants we need input PET data with sufficiently large SNR.