ETH Zürich
D-ITET
Biomedical Engineering

Master Studies

Biomedical Imaging

Homework #5 - X-ray Imaging 1

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1 TASK 1

The linear attenuation coefficient μ decreases non-linearly when anode voltage increases. There exists a certain anode voltage where for anode voltages smaller than it the decreasing is faster and for anode voltage larger than it the decreasing is slower. This is due to the fact that at lower energies(correspondingly lower anode voltages), the Photo Effect τ predominantly contributes to the attenuation coefficient, while at higher energies(correspondingly higher anode voltages) the contribution from Compton Effect σ becomes predominant.

$$\mu = \tau + \sigma + \kappa \tag{1.1}$$

Note that κ is the contribution from Pair Production which requires higher energy and thus it is not considered in medical imaging.
As is shown in the third figure in Figure 1, we can observe a staircase artifacts on the curve from the results of summation along the vertical direction. This is due to the fact that the projection is based on discrete objects. Thus it will lead to discontinuity in a certain range.

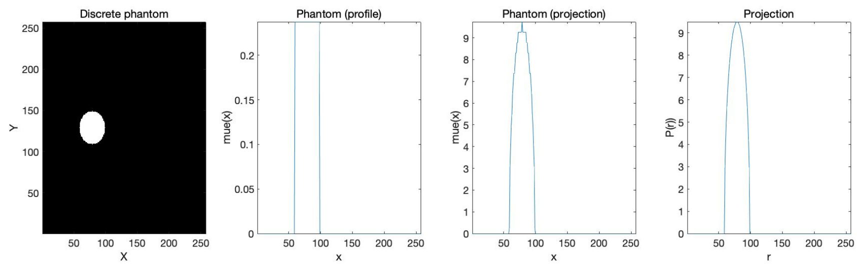

Figure 1.1: Projection

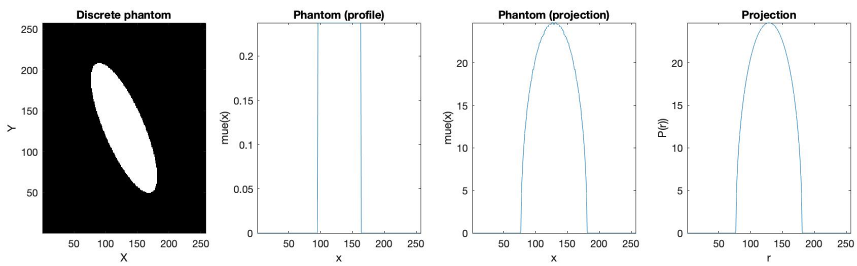

Figure 1.2: Projection of another Eclipse with oblique angle = $\pi/6$

2 TASK 2

The details of the projection is given as follows.
Consider the coordinates transformation as follows:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} cos\varphi & sin\varphi \\ -sin\varphi & cos\varphi \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix} \tag{2.1}$$

The eclipse is now given as:

$$DQ \left\| \vec{x} - \vec{x_0} \right\| \leq 1 \tag{2.2}$$

Where D,Q are the characteristic matrices of the eclipse. For a eclipse with half axes of a,b and tilt angle of θ relative to the x-axis, D,Q are given as:

$$D = \begin{pmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{b} \end{pmatrix} \tag{2.3}$$

$$Q = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \tag{2.4}$$

After the projection, the eclipse is given as:

$$As^2 + Bs + C \leq 1 \tag{2.5}$$

Solving for the boundary points, we obtain:

$$s_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{2.6}$$

Where:

$$A = \left\| DQ \begin{pmatrix} sin\varphi \\ -cos\varphi \end{pmatrix} \right\|^2 \tag{2.7}$$

$$B = 2DQ \begin{pmatrix} sin\varphi \\ -cos\varphi \end{pmatrix} DQ(r \begin{pmatrix} cos\varphi \\ sin\varphi \end{pmatrix} - \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}) \tag{2.8}$$

$$C = \left\| DQ(r \begin{pmatrix} cos\varphi \\ sin\varphi \end{pmatrix} - \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}) \right\|^2 \tag{2.9}$$

For each pair of boundary points(i.e. boundary points with the same r), μ is given as the integral between the two boundaries:

$$\mu = \int_{s_1}^{s_2} \mu d\mu = \mu_0 \left\| s_1 - s_2 \right\| \tag{2.10}$$

As is shown in the last figure in Figure 1, the staircase artifacts in the third figure disappears. This is because the projection now(i.e. integral) is continuous.

2.1 RESULT

From this image we can clearly see the result of line integral:
The oblique angle of the eclipse is $\frac{\pi}{4}$, and the projection angle is $\frac{\pi}{2}$. We can easily see the difference between the two pictures on the right side (angle = $\frac{\pi}{2}$ and 0 seperately).

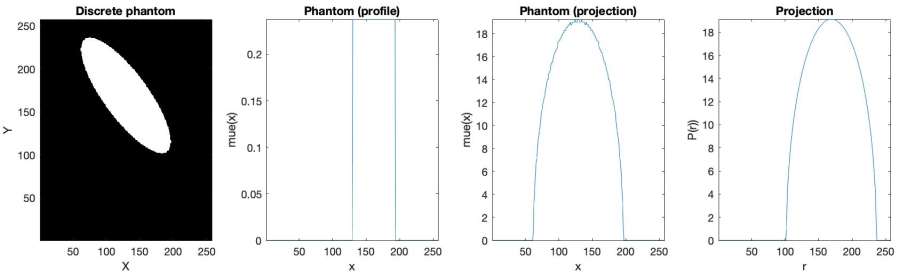

Figure 2.1: Projection angle = $\pi/2$

3 Task 3

3.1 Anode Voltage

With higher anode voltage, the energy of photons increases. Therefore more photons will interact with medium visa Compton scattering rather than photoelectric effect. In Compton scattering, secondary photons will be created which lead to blurring. As we can see from the two pictures below (on the left side):

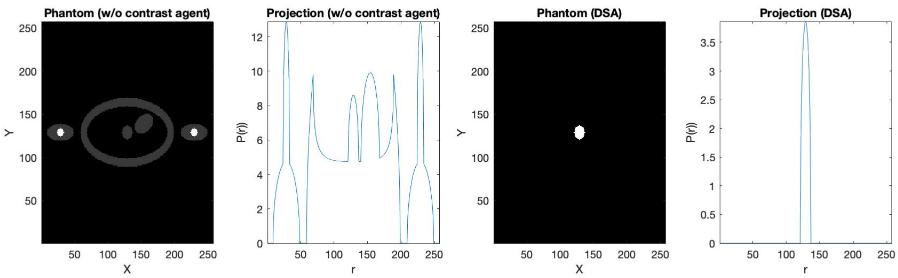

Figure 3.1: Anode Voltage = 50 keV

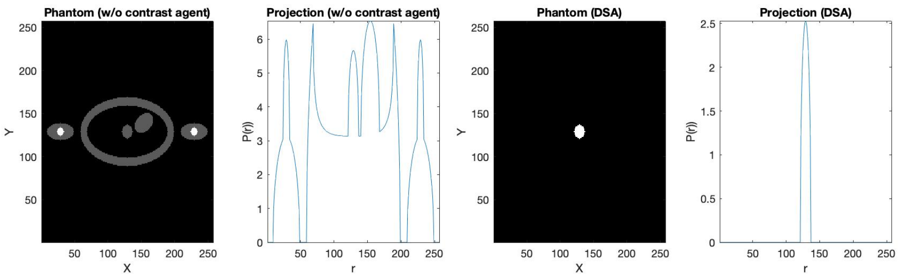

Figure 3.2: Anode Voltage = 150 keV

Although the image of 50 keV has better definition (the range of y-axis is higher), in both cases the blood are hard to recognize (very close to the heart). This is what we can get from Beer-Lambert's exponential decay law:

$$I(z) = I_0 exp(-\mu z) \tag{3.1}$$

When two materials have similar attenuation coefficient μ, the intensity after projection will be very close and therefore hard to differentiate. We can write the contrast using

Beer-Lambert law:

$$C = \frac{I_1 - I_2}{I_1} = \frac{e^{-\mu_1 z} - e^{-\mu_2 z}}{e^{-\mu_1 z}} = 1 - e^{(\mu_1 - \mu_2)z} \tag{3.2}$$

If the attenuation coefficients are very close, the contrast can be simplified to:

$$C = |\mu_1 - \mu_2| z \tag{3.3}$$

3.2 Digital Subtraction Angiography (DSA)

An Iodine contrast agent is now added to the blood, which will double the mass attenuation coefficient. When the image with no contrast agent was substracted from the image with contrast agent, we can get a image solely about the blood.
The resulted is shown on the picture above (right side). Now the subtracted projection is only the projection of blood vessels.