ETH Zürich
D-ITET
Biomedical Engineering

Master Studies

Biomedical Imaging

Homework #2 - Ultrasound 1

Xin Wu
Boqi Chen

DATE: *4th October 2020*
DUE : *5th October 2020*

Boqi Chen, bochen@student.ethz.ch
Xin Wu, xinwuxin@student.ethz.ch

1 Exercise 1

Pressure reflection coefficient:

$$r = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{1.1}$$

Pressure transmission coefficient:

$$t = \frac{2Z_2}{Z_2 + Z_1} \tag{1.2}$$

where Z_1, Z_2 is the wave impedance of the first and second material each separately. Let p_i , p_r and p_t be the pressure amplitude of the incident beam, reflected beam and the transmitted wave each separately, we obtain:

$$p_r = rp_i = \frac{Z_2 - Z_1}{Z_2 + Z_1} p_i \tag{1.3}$$

$$p_t = tp_i = \frac{2Z_2}{Z_2 + Z_1} p_i \tag{1.4}$$

The intensity of the beam is given by:

$$I = \frac{pu_z}{2} \tag{1.5}$$

where u_z is the particle velocity given by:

$$u_z = \frac{p}{Z} \tag{1.6}$$

So we can easily derive:

$$I = \frac{p^2}{2Z} \tag{1.7}$$

Let I_i, I_r, I_t be the intensity of the incident beam, reflected beam and the transmitted wave each separately, we can easily derive:

$$I_i = \frac{p_i^2}{2Z_1} \tag{1.8}$$

$$I_r = \frac{p_r^2}{2Z_1} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} \frac{p_i^2}{2Z_1} \tag{1.9}$$

$$I_t = \frac{p_t^2}{2Z_2} = \frac{4Z_2^2}{(Z_2 + Z_1)^2} \frac{p_i^2}{2Z_2} \tag{1.10}$$

The total beam intensity at the boundary is:

$$I_{total} = I_r + I_t = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} \frac{p_i^2}{2Z_1} + \frac{4Z_2^2}{(Z_2 + Z_1)^2} \frac{p_i^2}{2Z_2} \tag{1.11}$$

$$= \frac{p_i^2}{2} \frac{Z_2(Z_1^2 + Z_2^2 - 2Z_1Z_2) + 4Z_2^2 Z_1}{Z_1 Z_2 (Z_1 + Z_2)^2} \tag{1.12}$$

$$= \frac{p_i^2}{2} \frac{Z_2 Z_1^2 + Z_2^3 - 2Z_1 Z_2^2 + 4Z_2^2 Z_1}{Z_1 Z_2 (Z_1 + Z_2)^2} \tag{1.13}$$

$$= \frac{p_i^2}{2} \frac{Z_1^2 + Z_2^2 + 2Z_1 Z_2}{Z_1(Z_1 + Z_2)^2} \tag{1.14}$$

$$= \frac{p_i^2}{2} \frac{(Z_1 + Z_2)^2}{Z_1(Z_1 + Z_2)^2} \tag{1.15}$$

$$= \frac{p_i^2}{2Z_1} \tag{1.16}$$

Thus, we can prove:

$$I_i = I_r + I_t \tag{1.17}$$

i.e. transmission and reflection at the boundary conserve total beam intensity.

2 Exercise 2

2.1 NFB

$$\lambda_{fat} = \frac{c_{fat}}{f} = \frac{1476}{3 \times 10^6} = 4.92 \times 10^{-4} m \tag{2.1}$$

$$NFB \approx \frac{r^2}{\lambda_{fat}} = \frac{1.5^2 \times 10^{-6}}{4.92 \times 10^{-4}} = 4.57 \times 10^{-3} m \tag{2.2}$$

2.2 On-axis Pressure and Intensity Levels Entering the Muscle Layer

Pressure at NFB:

$$P_{fat} = \sqrt{2Z_{fat} I_{NFB}} = \sqrt{2 \times 1.36 \times 100 \times 10^6} = 1.65 \times 10^4 Pa \tag{2.3}$$

Considering the attenuation in fat:

$$attenuation_{fat} = 0.5 * 3 * (1 - 0.457) = 0.81 dB \tag{2.4}$$

We can easily derive:

$$\frac{p_{fat}}{p'_{fat}} = 10^{\frac{0.81}{20}} \tag{2.5}$$

Pressure at boundary at the fat side:

$$p'_{fat} = 1.50 \times 10^4 Pa \tag{2.6}$$

Pressure at boundary at the muscle side:

$$p_{muscle} = t_{fat \to muscle} p'_{fat} = \frac{2Z_{muscle}}{Z_{muscle} + Z_{fat}} p_{fat} = 1.65 \times 10^4 Pa \tag{2.7}$$

Intensity level at boundary at the muscle side:

$$I_{muscle} = \frac{p^2_{muscle}}{2Z_{muscle}} = \frac{1.65^2 \times 10^8}{2 \times 1.66 \times 10^6} = 82 W/m^2 \tag{2.8}$$

2.3 Peak Particle Velocity

$$u_z = \frac{p_{muscle}}{Z_{muscle}} = \frac{1.65 \times 10^4 Pa}{1.66 \times 10^6} = 9.94 \times 10^{-3} m/s \tag{2.9}$$

2.4 Depth in the Muscle when the Sound Intensity Dropped to 1 mW/cm2

When the sound intensity drops to 1 mW/cm2, the pressure drops to:

$$P'_{muscle} = \sqrt{2Z_{muscle} I} = \sqrt{2 \times 1.66 \times 10^6 \times 10} = 5.76 \times 10^3 Pa \tag{2.10}$$

Suppose the sound intensity drop to 1 mW/cm2 at depth $z_0 + \Delta z$, where z_0 is the depth of the fat, we can derive the following equation:

$$\frac{p_{muscle}}{p'_{muscle}} = \frac{1.65 \times 10^4 Pa}{5.76 \times 10^3 Pa} = 10^{\frac{2 \times 3 \times (1 + \Delta z)}{20}} \tag{2.11}$$

Solving the equation and we get:

$$\Delta z = 1.52cm \tag{2.12}$$

So:

$$Z = Z_0 + \Delta z = 2.52cm \tag{2.13}$$

2.5 Lateral Resolution

$$\lambda_{fat} = \frac{c_{fat}}{f} = 0.49 \times 10^{-3} \tag{2.14}$$

$$\theta_{fat} = arcsin(\frac{0.61\lambda_{fat}}{r}) = 11.49^\circ \tag{2.15}$$

$$\lambda_{muscle} = \frac{c_{muscle}}{f} = 0.52 \times 10^{-3} \tag{2.16}$$

$$\theta_{muscle} = arcsin(\frac{0.61\lambda_{muscle}}{r}) = 12.33^\circ \tag{2.17}$$

So we can obtain the lateral resolution:

$$\Delta x = d + 2(x_{fat} - NFB)tan\theta_{fat} + 2\Delta z tan\theta_{muscle} = 1.16 \times 10^{-2}cm \tag{2.18}$$

2.6 Axial Resolution in Fat and Muscle

$$\Delta z_{fat} = \frac{p_d c_{fat}}{2} = 7.38 \times 10^{-4}m \tag{2.19}$$

$$\Delta z_{muscle} = \frac{p_d c_{muscle}}{2} = 7.84 \times 10^{-4}m \tag{2.20}$$