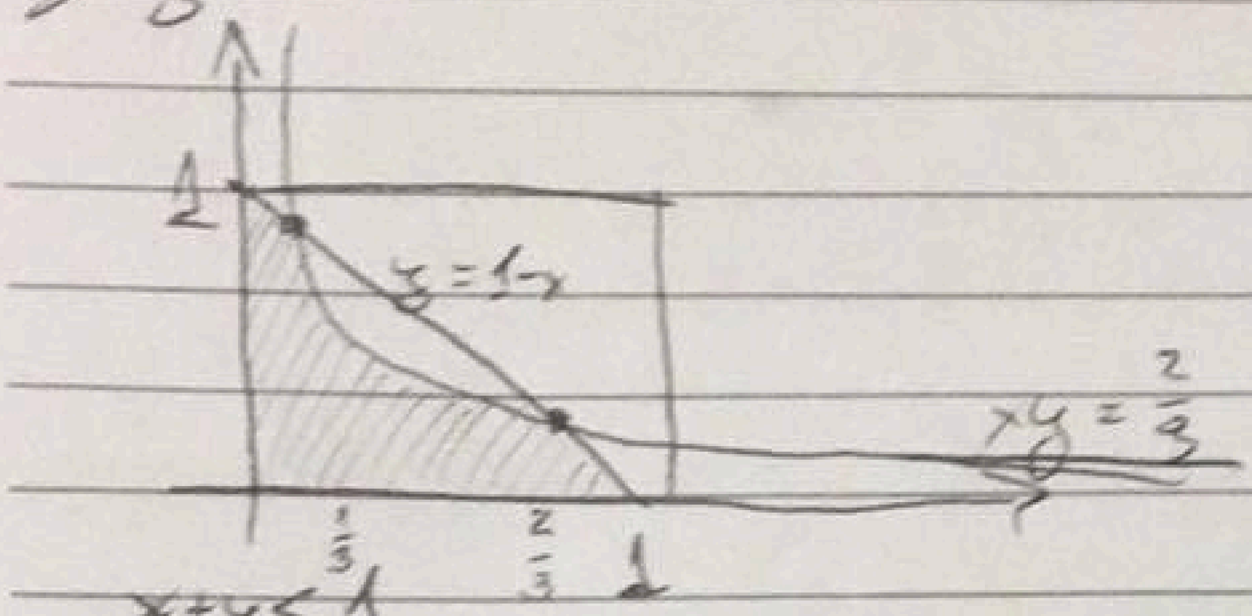
13.11.13 Статистика - Упражнения 7 ([illegible])

зад. 8



$$\begin{cases} x+y=1 \\ xy=\frac{2}{9} \end{cases}$$

$$t^2 - t + \frac{2}{9} = 0$$

$$t_{1,2} = \frac{1 \pm \sqrt{\frac{1}{9}}}{2} = \frac{1}{3}, \frac{2}{3}$$

$x+y<1$

$xy<\frac{2}{9}$

$$(x,y) = \left(\frac{2}{3}, \frac{1}{3}\right), \left(\frac{1}{3}, \frac{2}{3}\right)$$

$$\frac{1}{2} - \int_{\frac{1}{3}}^{\frac{2}{3}} \left(1 - x - \frac{2}{9x}\right) dx = \frac{1}{2} - \left(x\Big|_{\frac{1}{3}}^{\frac{2}{3}} - \frac{x^2}{2}\Big|_{\frac{1}{3}}^{\frac{2}{3}} - \frac{2}{9}\ln x\Big|_{\frac{1}{3}}^{\frac{2}{3}} \right)$$

$$= \frac{1}{2} - \left(\frac{2}{3} - \frac{1}{3} - \frac{\left(\frac{2}{3}\right)^2}{2} + \frac{\left(\frac{1}{3}\right)^2}{2} - \frac{2}{9}\left(\ln\frac{2}{3} - \ln\frac{1}{3}\right) \right)$$

Биномно разпределение

Случайна величина - величина, която приема случайно своите стойности
- функция

X	k_1	k_2	k_3	...
p	p_1	p_2	p_3	...

$$\sum_i p_i = 1$$

$\{X = k_i\}$ - хипотези

EX - очакване (expectation)

EX число

$EX = \sum_i p_i \cdot x_i$

Нека $X_1, X_2, X_3 \dots$ са еднакво разп. сл. величини с $EX_i = \mu$.

i. i.d.

$$\frac{\sum_{i=1}^{n} X_i}{n} \xrightarrow[n\to\infty]{} \mu$$ - интуитивно предположение

$X \sim B$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$k = 0, 1, 2, \dots n$

$EX = np$

1 $\binom{10}{5}\left(\frac{1}{2}\right)^5\left(1-\frac{1}{2}\right)^5$

$\binom{10}{4}\left(\frac{1}{2}\right)^4\left(1-\frac{1}{2}\right)^6 + \binom{10}{6}\left(\frac{1}{2}\right)^6\left(1-\frac{1}{2}\right)^4$

$$\begin{cases} |x-y| = 2 \\ x+y = 10 \end{cases}$$

② X - чиста печалба

x	-5	0	95
p	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

$$EX = -5\cdot\frac{25}{36} + \frac{10}{36}\cdot 0 + \frac{95}{36} = -\frac{30}{36} < 0$$

③ сума 6: $\frac{5}{36}$

$$\binom{5}{2}\left(\frac{5}{36}\right)^2\left(1-\frac{5}{36}\right)^3$$

$$X \sim Bi\left(5, \frac{5}{36}\right)$$

④

A	B
1	0
2	0
2	1
3	0
3	1
3	2

$$P(x) = \binom{3}{1}\left(\frac{1}{2}\right)^3 \cdot \binom{2}{0}\left(\frac{1}{2}\right)^2 + \binom{3}{2}\left(\frac{1}{2}\right)^3 \cdot \binom{2}{0}\left(\frac{1}{2}\right)^2$$

$+ \ldots$

x – спечелил е A
y – B хвърля един герб

$$P(y|x) = \frac{P(y\cap x)}{P(x)}$$

$$P(y\cap x) = \binom{3}{2}\left(\frac{1}{2}\right)^3 \cdot \binom{2}{1}\left(\frac{1}{2}\right)^2 + \binom{3}{3}\left(\frac{1}{2}\right)^3 \cdot \binom{2}{1}\left(\frac{1}{2}\right)^2$$

~~A~~

A	3	-2
p	$\frac{1}{2}$	$\frac{1}{2}$

B	3	-2
p	$\frac{1}{2}$	$\frac{1}{2}$

$$EA = 2\frac{1}{2} + 3\left(-\frac{1}{2}\right) = -\frac{1}{2}$$

r-ти

$$EB = \frac{1}{2}$$

⑤ бернулиеви опити

$$\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_}_{k+r-1 \text{ опита}} \quad \underset{k+r-\text{ти}}{y}$$

точно r-1 успеха

$$\binom{k+r-1}{r-1} p^{r-1}(1-p)^{k+r-1-(r-1)=k} \cdot p$$

→ степен

отр. биномно разпределение

величината, която казва на кой опит ще е r-тия успех

⑥

$$\underbrace{\_\ \_\ \_\ \_\ \_}\quad \underline{z}$$

$$2\binom{2n-k}{n}\left(\frac{1}{2}\right)^{2n-k} \cdot \frac{1}{2}$$