INDR 422/522 - Spring 2023 Fikri Karaesmen

CLASS EXERCISE, March 21, 2023

1. Consider the process $D_t = 100 - \epsilon_{t-1}/2 + \epsilon_t$ where ϵ_t are i.i.d with mean zero and variance σ^2. Which of the following is true?

   (a) 1-lag AC is negative : True. We have an MA(1) process: $D_t = \theta_0 + \theta_1\epsilon_{t-1} + \epsilon_t$. The AC at lag 1 has the same sign as the coefficient θ_1 (and $\theta_1 = -1/2$).

   (b) 1-lag AC is positive : False

   (c) The theoretical value of the one-lag AC is -1/2: False. The first lag AC for the MA(1) case is given by:

   $$\rho_1 = \frac{\theta_1}{1 + \theta_1^2} = -2/5$$

   (d) 2-lag AC is negative: This is false. For an MA(1) process, the second lag AC equals 0. In fact, the AC equals zero at all higher lags than one.

   (e) 2-lag AC is zero: True.

   (f) If D_{t-1} is below average, then D_t is more likely to be above average: True because of negative correlation at lag 1.

   (g) If D_{t-2} is below average, then D_t is more likely to be above average: False, D_{t-2} and D_t are independent.

2. Consider the data from the slides. Which of the following are true?

   (a) The data may come from an i.i.d process: False. The ACF and PACF show that there is clear auto-correlation at multiple lags.

   (b) The data may come from an AR(1) process: False. The ACF does not have a geometrically decreasing pattern and the PACF has spikes at multiple lags. In an AR(1) process, in the PACF we would see a single spike on the first lag only.

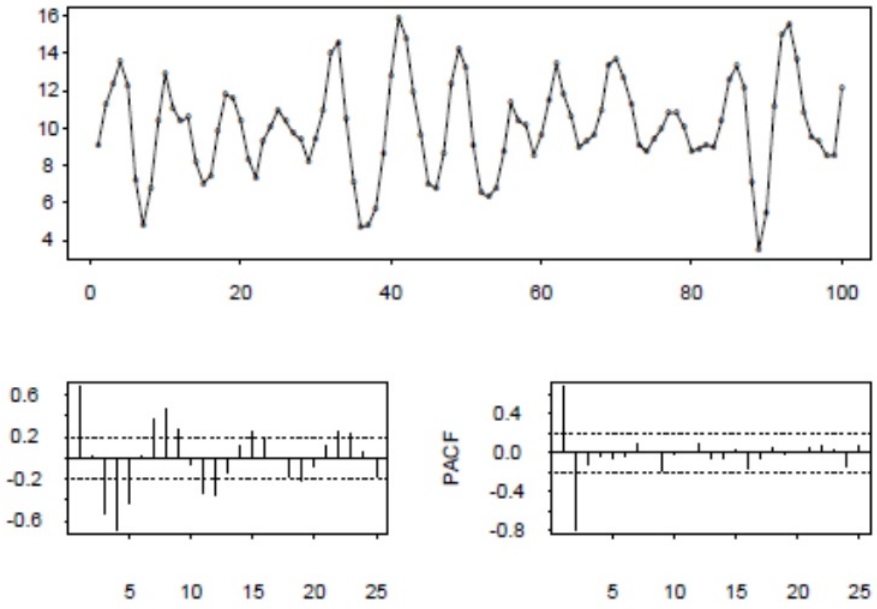
   (c) The data may come from an MA(1) process: False. With an MA(1) we would see a single spike at lag 1 on the ACF.

(d) The spikes on the PACF for the first two lags may correspond to AR terms on the first two lags: This may be true and the PACF should show single spikes corresponding to each AR term and this is the case here.

(e) The first spike on the PACF may correspond to a positive AR coefficient on the first term: True

(f) The first spike on the PACF may correspond to a positive AR coefficient on the first term: True

(g) The data may come from an AR(2) process with a positive coefficient at the first lag and a negative coefficient at the second lag:True. We can combine the above evidence to conclude that an AR(2) model would be a good fit.


16
14
12
10
8
6
4
0
20
40
60
80
100
0.6
0.2
-0.2
-0.6
5
10
15
20
25
PACF
0.4
0.0
-0.4
-0.8
5
10
15
20
25