INDR 422/522

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Spring 2023

Regression for Time Series- 2

March 30, 2023

Reminders

- Third lab available, please take a look and work on the exercises
- Fourth lab will be available this Friday
- Participation taken. Please participate in polls.
- HW 1 (due-date March 31, 2023)
- So far, we used material from Chapters 2, 5, 8 and 9 of Athanasopoulos and Hyndman's book: Forecasting Principles and Practice
  - Link on blackboard:
  - New edition: https://otexts.com/fpp3/

Summary of ACF and PACF patterns for simple AR and MA models

Process	ACF	PACF
AR(1)	Exponential decay: on positive side if $\phi_1 > 0$ and alternating in sign starting on negative side if $\phi_1 < 0$.	Spike at lag 1, then cuts off to zero: spike positive if $\phi_1 > 0$, negative if $\phi_1 < 0$.
AR(p)	Exponential decay or damped sine-wave. The exact pattern depends on the signs and sizes of $\phi_1, \ldots, \phi_p$.	Spikes at lags 1 to p, then cuts off to zero.
MA(1)	Spike at lag 1 then cuts off to zero: spike positive if $\theta_1 < 0$, negative if $\theta_1 > 0$.	Exponential decay: on negative side if $\theta_1 > 0$ and alternating in sign starting on positive side if $\theta_1 < 0$.
MA(q)	Spikes at lags 1 to q, then cuts off to zero.	Exponential decay or damped sine-wave. The exact pattern depends on the signs and sizes of $\theta_1, \ldots, \theta_q$.

Table 7-2: *Expected patterns in the ACF and PACF for simple AR and MA models.*

Please note that the MA-terms are defined with a negative sign in this Reference. This is why the signs are reversed in the examples in Lab 3.

$$Y_t = c - \theta_1 \varepsilon_{t-1} - \theta_2\, \varepsilon_{t-2} + \varepsilon_t$$

Makridakis, Wheelwright and Hyndman (1997)

Class Exercise from last lecture

CLASS EXERCISE, March 28, 2023

1. We have a stationary time series data. We fit an ARIMA(1,1,0) model. The intercept c is estimated as 100 and the AR(1) coefficient as -0.4. Assume that y_{98} =120, y_{99} =110, what is your forecast for period 100?

   (a) What is your forecast for period 100?

   Solution: We saw in the previous exercise that the open form for ARIMA(1,1,0) is:

   $$Y_t = c + Y_{t-1} + \phi_1(Y_{t-1} - Y_{t-2}) + \epsilon_t = c + (1 + \phi_1)Y_{t-1} - \phi_1 Y_{t-2} + \epsilon_t$$

   therefore, we have:

   $$\hat{y}_{100} = 100 + 110 - 0.4(110 - 120) = 214$$

   (b) What is your forecast for period 101?

   Solution:

   $$\hat{y}_{101} = 100+\hat{y}_{100}-0.4(\hat{y}_{100}-110) = 100+214-0.4(214-110) = 272.4$$

Class Exercise from last lecture

2. Consider the same stationary time series data set and we experiment with different models. Which statements are true?

   (a) ARIMA(0,0,0) estimates the mean of the series. *True*

   (b) We would expect a lower MSE with an ARIMA(1,0,0) than with an ARIMA(0,0,0). *True* ARIMA(1,0,0) includes ARIMA(0,0,0) as a special case.

   (c) We would expect lower AIC with an ARIMA(1,0,0) then with an ARIMA(0,0,0). *False* This is not always true. ARIMA(0,0,0) only estimates the intercept but ARIMA(1,0,0) estimates the intercept and the AR coefficient. Therefore ARIMA(1,0,0) would be penalized for using one more parameter.

   (d) We would expect a lower MSE with an ARIMA(1,0,0) than with an ARIMA(0,0,1). *False*: The two models are not special cases of each other. There is not a clear comparison.

   (e) We would expect a lower MSE with an ARIMA(1,0,1) than with an ARIMA(1,0,0) *True*

   (f) We would expect a lower MSE with an ARIMA(1,1,0) than with an ARIMA(1,0,0). *False*

Operational Example

- Assume we forecast the number of calls to arrive at a call center
- Typically forecasts are for half-hour intervals
- ARIMA models are good fits
  - Typically for a given interval (9-9.30), we take as input the observed number of calls at the same interval for the previous days
- We use our favorite software to estimate the ARIMA coefficients

Operational Example: here's an estimation

ARIMA(1,0,0): $D_t = c + \phi_1 D_{t-1} + \varepsilon_t$

```
7]: # Fit the model
modar = sm.tsa.statespace.SARIMAX(y_ar[100:499], trend='c', order=(1,0,0))
res = modar.fit(disp=False)
print(res.summary())
```

```
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                  399
Model:               SARIMAX(1, 0, 0)   Log Likelihood               -1458.647
Date:                Tue, 01 Mar 2022   AIC                           2923.295
Time:                        10:09:29   BIC                           2935.261
Sample:                             0   HQIC                          2928.034
                                - 399
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept    293.9478     33.949      8.658      0.000     227.408     360.487
ar.L1          0.6470      0.041     15.866      0.000       0.567       0.727
sigma2        87.7254      6.350     13.815      0.000      75.279     100.172
===================================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):                 3.89
Prob(Q):                              0.95   Prob(JB):                         0.14
Heteroskedasticity (H):               1.44   Skew:                            -0.24
Prob(H) (two-sided):                  0.04   Kurtosis:                         3.01
===================================================================================
```

Operational Example:

- Consider a call center that receives random call arrivals. The number of total calls that arrive between 9 and 9.30 is estimated from the data according to the process:

$$D_t = 100 + 0.6D_{t-1} + \epsilon_t$$

- Assume that $d_{t-1} = 80$ and ϵ_t is normal with mean zero and variance $\hat{\sigma}^2 = 900$.
- How many calls are expected at time interval t?
- Each call takes an average of 3 minutes to answer. How many agents should be scheduled to handle average demand in time interval t?
- The call center aims to answer at least 95% of all incoming calls, what is the minimum number of agents needed to answer 95% of calls in time interval t. ($z_{0.95} = 1.64$.)

Operational Example:

- How many calls are expected at time interval t?

$$\hat{d}_t = 100 + 0.6(80) = 148$$

- Each call takes an average of 3 minutes to answer. How many agents should be scheduled to handle average demand in time interval t?
- Each agent answers 10 calls in half and hour. To respond to 148 calls, 15 agents are needed.
- The call center aims to answer at least 95% of all incoming calls, what is the minimum number of agents needed to answer 95% of calls in time interval t. ($z_{0.95} = 1.64$.)

$$s^* = \hat{d}_t + z_{0.95}\hat{\sigma} = 148 + 1.64(30) = 197.2$$

We need at least 20 agents for this case.

Summary last lecture

- Regression for time series
- First, we'll look at how to model the calendar (i.e. time related information that is always available to us)
  - There are many!

Regression for Time Series

- Consider the following linear model:

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \ldots + \beta_n x_{nt} + \epsilon_t$$

- y_t is the forecast and x_{kt} are the predictors.
- We are therefore looking for a linear relationship between the predictors and the response (the forecast).
- Note that in the setting of forecasting, this is somewhat different than designing a controlled experiment where we can control the levels of the predictors. The predictors that are available to us cannot be controlled in general.

Regression for Time Series

- The ordinary least squares regression finds the parameters $\beta_0, \beta_1, \ldots, \beta_n$ to minimize:

$$\sum_{t=1}^{T} \epsilon_t^2 = \sum_{t=1}^{T} (y_t - (\beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \ldots + \beta_n x_{nt}))^2$$

- The above is an unconstrained convex optimization problem. In addition, the derivative with respect to each parameter β_k of the objective function is a linear function.
- Finding the minimizer then boils down to solving $n+1$ linear equations in $n+1$ unknowns.

Regression for Time Series

- Finding the minimizer then boils down to solving $n+1$ linear equations in $n+1$ unknowns.
- We can therefore easily find the coefficients $\hat{\beta}_0, \hat{\beta}_1, ..., \hat{\beta}_n$ that minimize the total square error.
- To have a prediction, we can then use:

$$\hat{y}_t = \hat{\beta}_0 + \hat{\beta}_1 x_{1t} + ... + \hat{\beta}_n x_{nt}$$

Regression for Time Series: Goodness of Fit

- We measure the goodness of fit by the coefficient of determination R^2:

$$R^2 = \frac{\sum_t (y_t - \bar{y})^2 - \sum_t (y_t - \hat{y}_t)^2}{\sum_t (y_t - \bar{y})^2} = \frac{\sum_t (\hat{y}_t - \bar{y})^2}{\sum_t (y_t - \bar{y})^2}$$

- Note that $R^2 = Corr(Y, \hat{Y})^2$ the square of the correlation between the predictions and the data. The least squares optimization leads to the parameters that maximizes the correlation.
- We'll see that while R^2 is an important measure, we cannot rely on it completely without additional checks.

Regression for Time Series: Basic Predictors

- Here are some basic predictors that can capture the patterns in the data:
- To capture simple linear trend, we can use:

$$y_t = \beta_0 + \beta_1 t + \epsilon_t$$

- We'll see that non-linear trends can also be handled, for instance:

$$y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \epsilon_t$$

Q&A

- Consider the AR(1) model: $Y_t = \text{c} + \phi_1 \, Y_{t-1} + \epsilon_t$
- And the linear regression model:

$$y_t = \beta_0 + \beta_1 \, y_{t-1} + \epsilon_t$$

- They look very similar but
  - Note that the AR(1) model is a stochastic process, we can compute the probability distribution of Y_t conditional on past observations.
  - The regression typically is not concerned with the evolution of the predictors. We don't have a complete model of how the system evolves and where it will evolve to in the future.

Q&A

- The AR(1) model: $Y_t = \mathrm{c} + \phi_1 \, Y_{t-1} + \epsilon_t$
- And the linear regression model:

$$y_t = \beta_0 + \beta_1 \, y_{t-1} + \epsilon_t$$

- However, in terms of fitting the parameters from data, they are similar:
  - AR model fits parameters using MLE.
  - OLS regression fits the parameters based on squared residual minimization.
  - The resulting fits should be similar.

Q&A

- But there is no such easy correspondence for MA terms. This is why the general estimation tool for ARIMA processes is MLE.
- ARIMA processes perform quite well despite the restrictive assumptions.
- Regression allows more flexibility in modeling.

Regression for Time Series: Basic Predictors: Google Share Price

- The Google Share Price Data has a strong trend.
- Let's try a simple trend based regression.

$$y_t = \beta_0 + \beta_1 t + \epsilon_t$$

	Price	Day
1	2064.879883	1
2	2070.860107	2
3	2095.169922	3
4	2031.359985	4
5	2036.859985	5

Regression for Time Series: Basic Predictors: Google Share Price

$$y_t = \beta_0 + \beta_1 t + \epsilon_t$$

$$\hat{\beta}_0 = 2197.62, \hat{\beta}_0 = 3.31$$

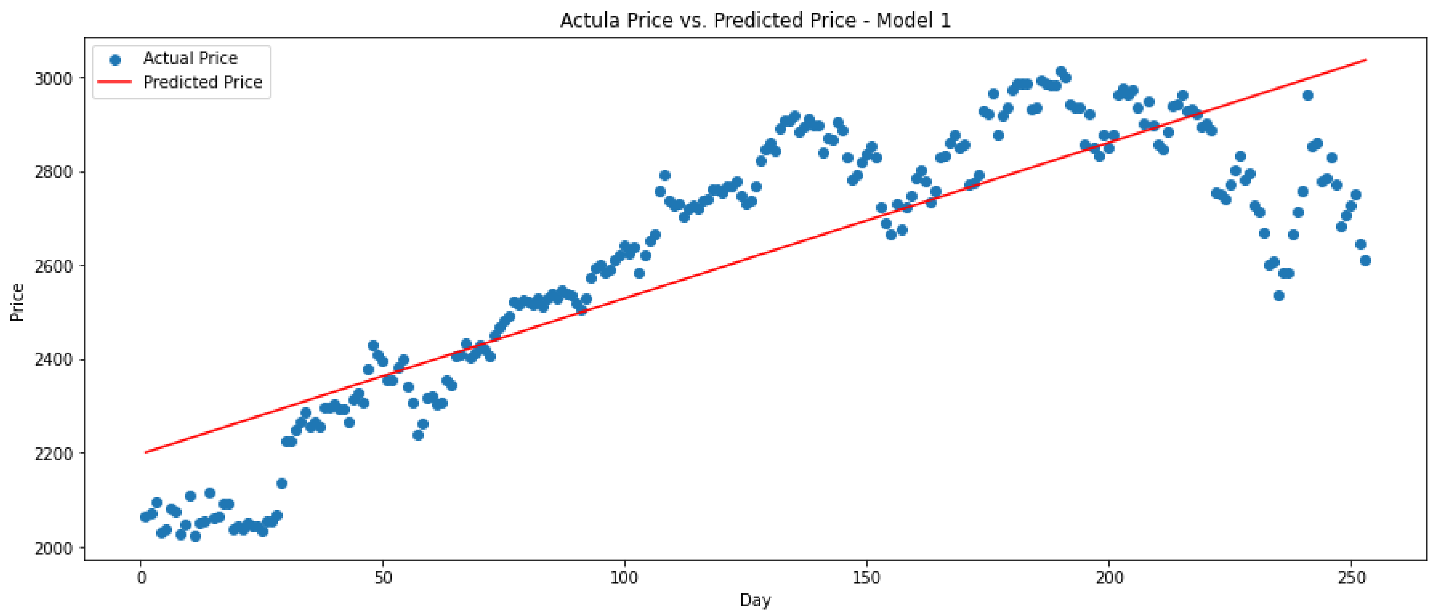
Out[3]:

OLS Regression Results

Dep. Variable:	Price	**R-squared:**	0.712
Model:	OLS	**Adj. R-squared:**	0.711
Method:	Least Squares	**F-statistic:**	619.5
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	9.84e-70
Time:	17:39:05	**Log-Likelihood:**	-1633.4
No. Observations:	253	**AIC:**	3271.
Df Residuals:	251	**BIC:**	3278.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	2197.6246	19.501	112.694	0.000	2159.218	2236.031
Day	3.3131	0.133	24.890	0.000	3.051	3.575

Omnibus:	13.705	**Durbin-Watson:**	0.068
Prob(Omnibus):	0.001	**Jarque-Bera (JB):**	15.010
Skew:	-0.590	**Prob(JB):**	0.000550
Kurtosis:	2.818	**Cond. No.**	294.



Regression for Time Series: Basic Predictors: Google Share Price

- Since the execution is very easy, we are tempted to try other predictors, let us try:

$$y_t = \beta_0 + \beta_1 t + \beta_2 \sqrt{t} + \beta_3 t^2 + \epsilon_t$$

Out[2]:

	Price	Day	Sqrtd	Sqrd
1	2064.879883	1	1.000000	1
2	2070.860107	2	1.414214	4
3	2095.169922	3	1.732051	9
4	2031.359985	4	2.000000	16
5	2036.859985	5	2.236068	25

In [12]:
```
lm2 = sm.OLS.from_formula('Price ~ Day + Sqrtd+ Sqrd', df)
result2 = lm2.fit()
```

Let us call this model Model 2.

Regression for Time Series: Basic Predictors: Google Share Price

13]:

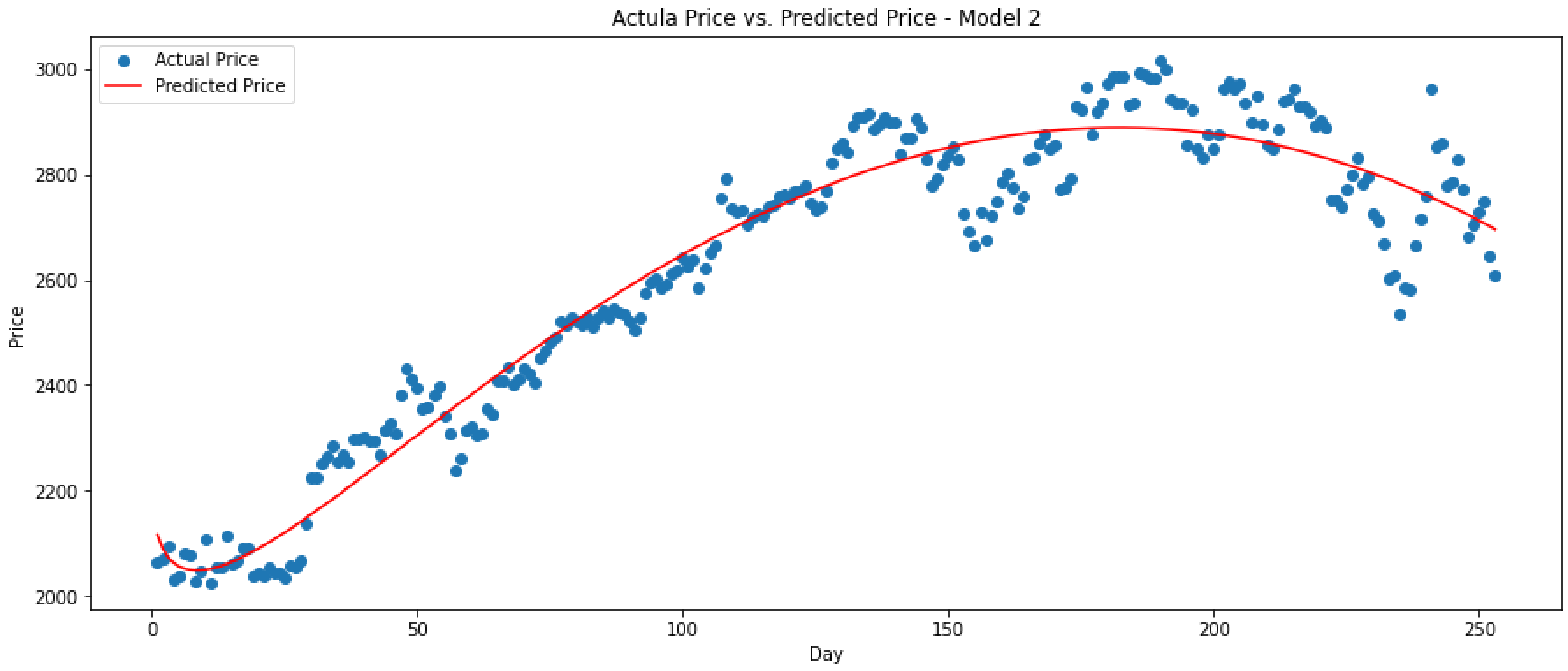
OLS Regression Results

Dep. Variable:	Price	**R-squared:**	0.937
Model:	OLS	**Adj. R-squared:**	0.936
Method:	Least Squares	**F-statistic:**	1236.
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	3.40e-149
Time:	12:17:45	**Log-Likelihood:**	-1440.8
No. Observations:	253	**AIC:**	2890.
Df Residuals:	249	**BIC:**	2904.
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	2207.3790	43.714	50.496	0.000	2121.282	2293.476
Day	19.8417	1.304	15.219	0.000	17.274	22.409
Sqrtd	-111.2141	14.968	-7.430	0.000	-140.695	-81.733
Sqrd	-0.0432	0.002	-18.802	0.000	-0.048	-0.039

Omnibus:	9.814	**Durbin-Watson:**	0.308
Prob(Omnibus):	0.007	**Jarque-Bera (JB):**	9.833
Skew:	-0.450	**Prob(JB):**	0.00733
Kurtosis:	3.352	**Cond. No.**	2.90e+05

$$y_t = \beta_0 + \beta_1 t + \beta_2 \sqrt{t} + \beta_3 t^2 + \epsilon_t$$



Regression for Time Series: Basic Predictors: Google Share Price

$$y_t = \beta_0 + \beta_1 t + \beta_2 \sqrt{t} + \beta_3 t^2 + \epsilon_t$$

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	2207.3790	43.714	50.496	0.000	2121.282	2293.476
Day	19.8417	1.304	15.219	0.000	17.274	22.409
Sqrtd	-111.2141	14.968	-7.430	0.000	-140.695	-81.733
Sqrd	-0.0432	0.002	-18.802	0.000	-0.048	-0.039

- Note that $\hat{\beta}_2$ and $\hat{\beta}_3$ are also statistically significant.

Regression for Time Series: Basic Predictors: Google Share Price

- We can become even more aggressive and try adding a log term to the model

$$y_t = \beta_0 + \beta_1 t + \beta_2 \sqrt{t} + \beta_3 t^2 + \beta_4 \log(t) + \epsilon_t$$

Out[19]:

OLS Regression Results

Dep. Variable:	Price	**R-squared:**	0.938
Model:	OLS	**Adj. R-squared:**	0.937
Method:	Least Squares	**F-statistic:**	931.6
Date:	Mon, 14 Mar 2022	**Prob (F-statistic):**	4.69e-148
Time:	17:18:57	**Log-Likelihood:**	-1439.8
No. Observations:	253	**AIC:**	2890.
Df Residuals:	248	**BIC:**	2907.
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	2225.6083	45.448	48.971	0.000	2136.096	2315.121
Day	24.3821	3.432	7.105	0.000	17.623	31.141
np.sqrt(Day)	-199.9354	63.822	-3.133	0.002	-325.637	-74.234
np.square(Day)	-0.0481	0.004	-11.574	0.000	-0.056	-0.040
np.log(Day)	100.0696	69.986	1.430	0.154	-37.773	237.912

Let us call this model Model 3.

The log term is not statistically significant but R^2 improved!

Regression for Time Series: Crucial Questions

- Crucial Question 1: Is there a relationship between the response and the predictor.
- We test the null hypothesis:

$$H_0 : \beta_1 = \beta_2 = ... = \beta_n = 0$$

versus the alternative hypothesis

$$H_1 : \text{at least one } \beta_k \text{ is non-zero}$$

- This is computed through the F-statistic. We typically check for the p-value of the F-statistic being less than 5%.

Regression for Time Series: Crucial Questions

- Crucial Question 1b: Is there a relationship between a particular response and the predictor.
- We test the null hypothesis:

$$H_0 : \beta_k = 0$$

versus the alternative hypothesis

$$H_1 : \beta_k \neq 0$$

- This is computed through the t-statistic. We typically check for the p-value of the t-statistic being less than 5%.

Regression for Time Series: Crucial Questions

- Crucial Question 2: Which variables are important?
- Assume that the F-test has a small p-value and the regression is significant. Which predictors are important?
- One answer would be to fit all models that include subsets of all potential predictors. For each model, we can then check adjusted R^2, AIC, BIC and other similar criteria. Unfortunately, when there are n potential predictors, the total number of subsets is 2^n. This is impractical when n is large.
- We'll spend some time later on this critical question.

Regression for Time Series: Crucial Questions - Example

- Recall that we fitted three different models to the Google share price data (all data points), Model 1 (one predictor), Model 2 (three predictors), Model 3 (four predictors). Model 3 has the highest R^2 followed by Model 2 and Model 1 last.
- It turns out that we can always improve R^2 and MSE by adding more terms.
  - This comes from the least squares optimization formulation. If we have more degrees of freedom, we can always improve the objective function.

Regression for Time Series: Crucial Questions - Example

- To avoid model overfitting, the appropriate approach for model selection is to separate the training and test samples
  - Let’s take the first 180 days as the training data
  - And the remaining 73 days as the test data
- We fit the models on the training data and check its error performance on the test data.

Regression for Time Series: Crucial Questions - Example

- Here are the results:

Model	Test Set RMSE
Model 1	415.68
Model 2	105.02
Model 3	116.42

- Model 3 has more predictors but performs worse than Model 2 on the test set. This is a sign of overfitting.
- Based on this particular train-test split, we'll prefer Model 2
  - But we can run more validation tests with different train-test splits.

Regression for Time Series: Crucial Questions

- Crucial Question 3: How strong is the model fit?
- We check R^2 and RMSE (or its corrected version the Residual Standard Error (RSE))

$$\mathrm{RSE} = \sqrt{\frac{\mathrm{RSS}}{T - p - 1}}$$

where

$$\mathrm{RSS} = \sum_{t=1}^{T} (y_t - \hat{y}_t)^2$$

- Plot the data and the predictions

Regression for Time Series: Crucial Questions

- Crucial Question 4: Predictions
- The regression gives the mean of the predicted response but using the RSE we can compute confidence intervals.
- A $(1-\alpha)\%$ confidence interval is expected to contain the true observation $(1-\alpha)\%$ of the time.

Regression for Time Series: Basic Predictors: Categorical Variables

Other useful modeling tricks for time series:

- We can use dummy variables to incorporate the effects of non-numerical (categorical predictors). For instance, assume that we are trying to predict daily demand but notice that weekends are considerably different than weekdays. We can then use a binary variable x_{wt} that takes the value of 1 if day t corresponds to a weekend day.

$$y_t = \beta_0 + \beta_1 x_{wt} + \epsilon_t$$

Note that our prediction would be $\hat{y}_t = \hat{\beta}_0$ for a weekday and $\hat{y}_t = \hat{\beta}_0 + \hat{\beta}_1$ for a weekend day.

Note that we only need one predictor for a binary variable (weekend vs. weekday).
If it's not a weekend then it must be a week day.

Regression for Time Series: Basic Predictors: Categorical Variables

- Dummies can be used to model multiple categories. We can distinguish six dummies to mark the different days of the week. We don't need the seventh one since it would be dependent on the other six. x_{1t} is equal to 1 if it's a Monday and is 0 otherwise, x_{2t} is equal to 1 if it's a Tuesday and is 0 otherwise,..., x_{6t} is equal to 1 if it's a Saturday and is 0 otherwise, The model is then:

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + ... + \beta_6 x_{6t} + \epsilon_t$$

  Note that our prediction for a Monday would be $\hat{y}_t = \hat{\beta}_0 + \hat{\beta}_1$. For a sunday it would be $\hat{y}_t = \hat{\beta}_0$.

Regression for Time Series: Basic Predictors: Categorical Variables

- We can use dummies to mark months of the year, quarters of the year, hours of the day etc.
- We can also use dummies to mark irregular (non-seasonal) exceptions (holidays, days of Ramadan, promotions, school holidays etc.)
- This is great but note that we may easily end up with a very large number of dummies!

Regression for Time Series: Basic Predictors: Australian Beer Production

- The Australian Beer Production Data is strongly seasonal. We can try to fit:

$$y_t = \beta_0 + \beta_1 t + \beta_2 x_{1t} + \beta_3 x_{2t} + \ldots + \beta_{12} x_{11,t} + \epsilon_t$$

- where $x_{1t}, \ldots, x_{11,t}$ are the monthly dummies.

```
In [16]: df = pd.read_csv('ausbeer_dummies.csv', index_col=0)
         df.head()
```

Out[16]:

Month	Production	t	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11
1	164	1	1	0	0	0	0	0	0	0	0	0	0
2	148	2	0	1	0	0	0	0	0	0	0	0	0
3	152	3	0	0	1	0	0	0	0	0	0	0	0
4	144	4	0	0	0	1	0	0	0	0	0	0	0
5	155	5	0	0	0	0	1	0	0	0	0	0	0

Regression for Time Series: Basic Predictors: Australian Beer Production

$$y_t = \beta_0 + \beta_1 t + \beta_2 x_{1t} + \beta_3 x_{2t} + \ldots + \beta_{12} x_{11,t} + \epsilon_t$$

Dep. Variable:	Production	**R-squared:**	0.836
Model:	OLS	**Adj. R-squared:**	0.791
Method:	Least Squares	**F-statistic:**	18.30
Date:	Tue, 08 Mar 2022	**Prob (F-statistic):**	3.97e-13
Time:	12:20:51	**Log-Likelihood:**	-194.93
No. Observations:	56	**AIC:**	415.9
Df Residuals:	43	**BIC:**	442.2
Df Model:	12		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	192.9750	5.015	38.477	0.000	182.861	203.089
t	-0.2158	0.075	-2.887	0.006	-0.367	-0.065
M1	-39.7792	6.030	-6.597	0.000	-51.940	-27.619
M2	-48.5633	6.026	-8.059	0.000	-60.715	-36.411
M3	-30.9475	6.023	-5.139	0.000	-43.093	-18.802
M4	-46.7317	6.020	-7.762	0.000	-58.873	-34.591
M5	-46.1158	6.019	-7.662	0.000	-58.254	-33.978
M6	-58.5000	6.018	-9.720	0.000	-70.637	-46.363
M7	-51.8842	6.019	-8.620	0.000	-64.022	-39.746
M8	-42.2683	6.020	-7.021	0.000	-54.409	-30.127
M9	-46.6475	6.348	-7.349	0.000	-59.449	-33.846
M10	-19.6817	6.346	-3.102	0.003	-32.479	-6.884
M11	-2.9658	6.344	-0.467	0.643	-15.760	9.829

Regression for Time Series: Basic Predictors: Australian Beer Production

$$y_t = \beta_0 + \beta_1 t + \beta_2 x_{1t} + \beta_3 x_{2t} + \ldots + \beta_{12} x_{11,t} + \epsilon_t$$

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	192.9750	5.015	38.477	0.000	182.861	203.089
t	-0.2158	0.075	-2.887	0.006	-0.367	-0.065
M1	-39.7792	6.030	-6.597	0.000	-51.940	-27.619
M2	-48.5633	6.026	-8.059	0.000	-60.715	-36.411
M3	-30.9475	6.023	-5.139	0.000	-43.093	-18.802
M4	-46.7317	6.020	-7.762	0.000	-58.873	-34.591
M5	-46.1158	6.019	-7.662	0.000	-58.254	-33.978
M6	-58.5000	6.018	-9.720	0.000	-70.637	-46.363
M7	-51.8842	6.019	-8.620	0.000	-64.022	-39.746
M8	-42.2683	6.020	-7.021	0.000	-54.409	-30.127
M9	-46.6475	6.348	-7.349	0.000	-59.449	-33.846
M10	-19.6817	6.346	-3.102	0.003	-32.479	-6.884
M11	-2.9658	6.344	-0.467	0.643	-15.760	9.829

Our prediction for month 4 is: 192.98 -0.2158 (4) -46.73

Our prediction for month 11 is: 192.98 -0.2158 (11) – 2.97

Our prediction for month 12 is: 192.98 -0.2158 (12)

Our prediction for month 26 is: 192.98 -0.2158 (2) -48.56

Month 12 is clearly the peak month for sales, all other months have negative seasonality factors wrt to month 12.

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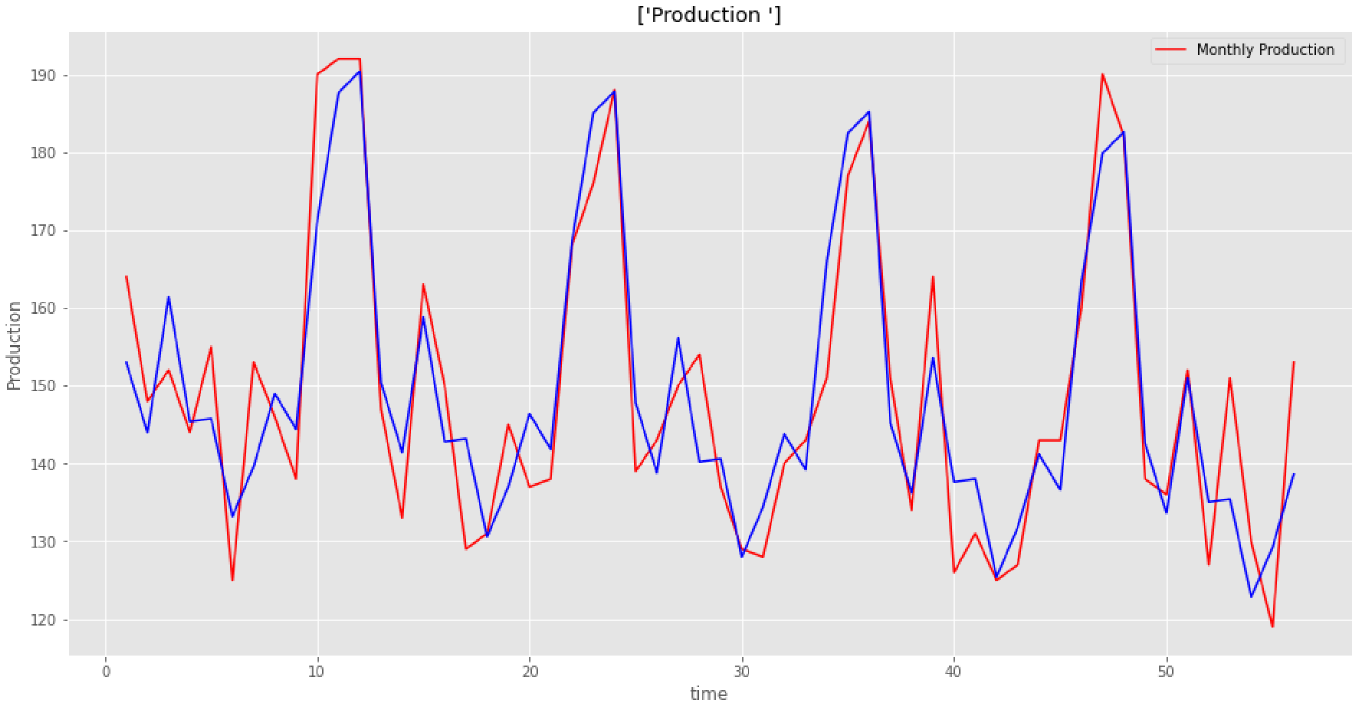

In-sample predictions in blue, and the observed production in red.

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In [9]: error_beer = prod - result_beer.fittedvalues
```

```
In [10]: mse_beer = np.mean(np.square(error_beer ))
         rmse_beer = np.sqrt(mse_beer)
         mae_beer = np.mean(np.abs(error_beer ))
         mape_beer = np.mean(np.abs(error_beer )/prod)
         print('MSE Beer = ', mse_beer)
         print('RMSE Beer = ', rmse_beer)
         print('MAE Beer = ', mae_beer)
         print('MAPE Beer = ', mape_beer)

MSE Beer =  61.805178571428556
RMSE Beer =  7.861626967201417
MAE Beer =  6.441249999999998
MAPE Beer =  0.04378739163608506
```

Regression for Time Series: Basic Predictors: Categorical Variables

- We can use dummies to mark months of the year, quarters of the year, hours of the day etc.
- We can also use dummies to mark irregular (non-seasonal) exceptions (holidays, days of Ramadan, promotions, school holidays etc.)
- This is great but note that we may easily end up with a very large number of dummies!

Regression for Time Series: General Case

- Calendar related indicators capture the patterns (trend, seasonality, bayram, ramadan, etc)
- But on top of that regression allows us to integrate other predictors that might explain the variability of the response
- For instance, for share price data, we can include the weekly change in the composite index, trading volume, etc.
- Anything reasonable can be a predictor to be tested
  - Some evidence that the market is more likely to go up on sunny days
  - Maybe add the weather forecast