INDR 422/522 - Spring 2023 Fikri Karaesmen

HOMEWORK 1, Due Date: March 31, 2022

- Please work in groups of two or (individually) and submit one file for each group with all names.
- The data is on total monthly average price index of Kenyan tea in commodity markets as reported by the IMF commodity price data portal (data found in the blackboard page). The data starts from January 2013 and ends in December 2022 (included).
- Please perform all computations in python. You are expected to implement your own forecasts (don't use packages/functions from libraries). Please submit (opload on blackboard) your commented (with explanations) python notebooks.
- In addition to the python notebook, **submit a short typed summary report** that includes the results (error tables, prediction intervals etc.) of all exercises. Also add a general assessment of the methods (which method is the best, which should be avoided etc.). **The report is part of the overall grade.**

Exercises

1. Forecasting Kenyan Tea Prices (monthly average price index of Kenyan tea in commodity markets as reported by the IMF commodity price data portal)
   (a) Plot the data and visually assess whether there is significant trend and seasonality.
   (b) To obtain a benchmark for errors, implement the naive one-month ahead forecast i) $\hat{y}_t = y_{t-1}$. Report the Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Root Mean Squared Error (RMSE) of the naive forecast for years 2014 through 2022. These error measures constitute a simple benchmark for all other approaches (i.e. hopefully you will obtain lower errors by more sophisticated methods). There is plenty of data in this case and we can start forecasting from 2013 but different methods require different initializations in the first year so it's

best to use 2013 as the warm-up period and compare the errors from 2014 to the end of 2021 and we use 2022 for some basic testing.

(c) Use a 5-period moving average to forecast the one month-ahead price. Report the MAE, MAPE and RMSE of the forecast for years 2014 through 2021. Report 95 percent prediction intervals (using the RMSE estimated in years 2014 to 2021) for the one-month ahead forecasts for year 2022. How do your prediction intervals perform for 2022?

(d) Use exponential smoothing to forecast the one-month ahead price. Perform an exhaustive search for the best smoothing constant α (that leads to the minimum MSE). Report the MAE, MAPE and RMSE of the forecast for years 2014 through 2021. Report 95 percent prediction intervals (using the RMSE estimated in years 2014 through 2021) for the one-month ahead forecasts for 2022. How do these compare with the benchmark and the MA-5 forecast?

(e) The data seems to have trend in some parts. Implement a naive forecast that includes trend: $\hat{y}_t = y_{t-1} + (y_{t-2} - y_{t-3})$ in order to forecast the one-month ahead price. Report the MAE, MAPE and RMSE of the forecast for years 2014 through 2021. Report 95 percent prediction intervals (using the RMSE estimated in years 2014 through 2021) for the one-month ahead forecasts for 2022. How do these compare with the previous forecasts?

(f) Implement an exponentially smoothed version of the previous forecast (you can take $\alpha = 0.5$ and $\beta = 0.1$): $\hat{y} = \alpha y_{t-1} + (1 - \alpha)\hat{y}_{t-1} + \beta z_{t-1} + (1-\beta)\hat{z}_{t-1}$ where $z_t = y_t - y_{t-1}$ and $\hat{z}_t = \hat{y}_t - \hat{y}_{t-1}$ for one-month ahead prices. Report the MAE, MAPE and RMSE of the forecast for years 2014 through 2021. Report 95 percent prediction intervals (using the RMSE estimated in years 2014 through 2021) for the one-month ahead forecasts for 2022 . How do these compare with the previous forecasts?

(g) Find the values of α and β that minimize the RMSE for **six-month ahead forecasts** for years 2014 through 2021. For the optimal values of the parameters, predict the mean Kenyan tea prices for the next six months from June 2022 to December 2022. Note that $\hat{y}_{t+k} = \alpha y_{t-1} + (1-\alpha)\hat{y}_{t-1} + (k+1)(\beta z_{t-1} + (1-\beta)\hat{z}_{t-1})$. Report the MAE, MAPE and RMSE of the six-month ahead forecast for years 2014 through 2021.

(h) Your report for the exercise should include a table similar to the one below.

Method	**Spec.**	**RMSE**	**MAPE**
Benchmark	-		
MA-5	-		
ES	-		
Trend	-		
Smoothed Trend			
6-month ahead			

Note that the model specification for exponential smoothing is: $\alpha^* = ...$, $\beta^* = ...$.

2. Generate 500 realizations from an AR-1 process $Y_t = c + \phi_1 Y_{t-1} + \epsilon_t$ where $c = 50$, $\phi_1 = 0.6$ and ϵ_t are normally distributed with mean zero and standard deviation 20.

   (a) Plot the generated observations.

   (b) Plot the Auto-Correlation Function (start from observation 100 to eliminate the effects of initialization).

   (c) Implement a naive forecast on the data $\hat{y}_t = y_{t-1}$. Compute the RMSE starting from period 100.

3. Investigating the Auto-correlation structure of the Kenyan Tea Price Index Data.

   (a) Plot the ACF and PACF of the tea price index time series. State if there are interesting observations.

   (b) Difference the data: $u_t = y_t - y_{t-1}$. Plot the ACF and PACF of the differences u_t. State if there are interesting observations.

   (c) Based on the previous AC analysis, are you able to propose a model for the original data series?