INDR 422/522 - Spring 2023 Fikri Karaesmen

ADDITIONAL EXERCISE, March, 24, 2023

Use the backshift notation to write the explicit forms of the following processes:

1. ARIMA(1,2,1)
   Let's do this completely. From the lecture slides we can write using the backshift notation:

$$(1-\phi_1 B)(1-B)^2 y_t = c + (1+\theta_1 B)\epsilon_t$$

We can now expand the left hand side:

$$\begin{aligned}
(1-\phi_1 B)(1-B)^2 y_t &= (1-\phi_1 B)(1-2B+B^2)y_t \\
&= (1-2B+B^2-\phi_1 B+2\phi_1 B^2-\phi_1 B^3)y_t \\
&= (1-(2+\phi_1)B+(1+2\phi_1)B^2-\phi_1 B^3)y_t \\
&= y_t-(2+\phi_1)y_{t-1}+(1+2\phi_1)y_{t-2}-\phi_1 y_{t-3}
\end{aligned}$$

Now leaving y_t alone on the left-hand side:

$$y_t = c+(2+\phi_1)y_{t-1}-(1+2\phi_1)y_{t-2}+\phi_1 y_{t-3}+\theta_1\epsilon_{t-1}+\epsilon_t$$

So a forecast for y_t uses terms $y_{t-1}, y_{t-2}, y_{t-3}, \epsilon_{t-1}$. Note that the expression looks very busy but there are only two coefficients to estimate: c and ϕ_1.

2. ARIMA(2,1,2) We can express this as:

$$(1-\phi_1 B-\phi_2 B^2)(1-B)y_t = c+(1+\theta_1 B+\theta_2 B^2)\epsilon_t$$

3. SARIMA(1,1,1)(1,1,0,4)

$$(1-\phi_1 B)(1-\Phi_1 B^4)(1-B)(1-B^4)y_t = c+(1+\theta_1 B)\epsilon_t$$