INDR 450/550

Spring 2022

Lecture 24: Mini Case
(Prescriptive Analytics)

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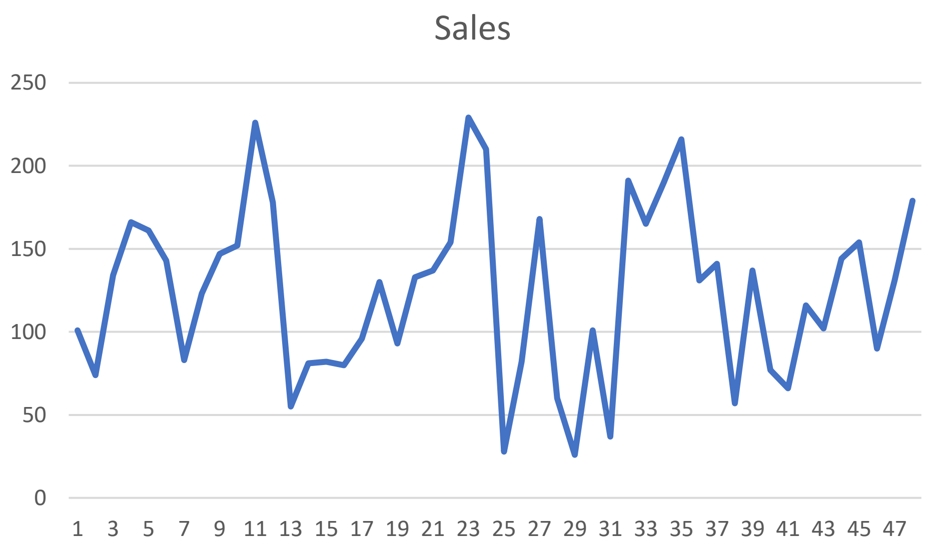
‘Mini’ Case: prescriptive analytics

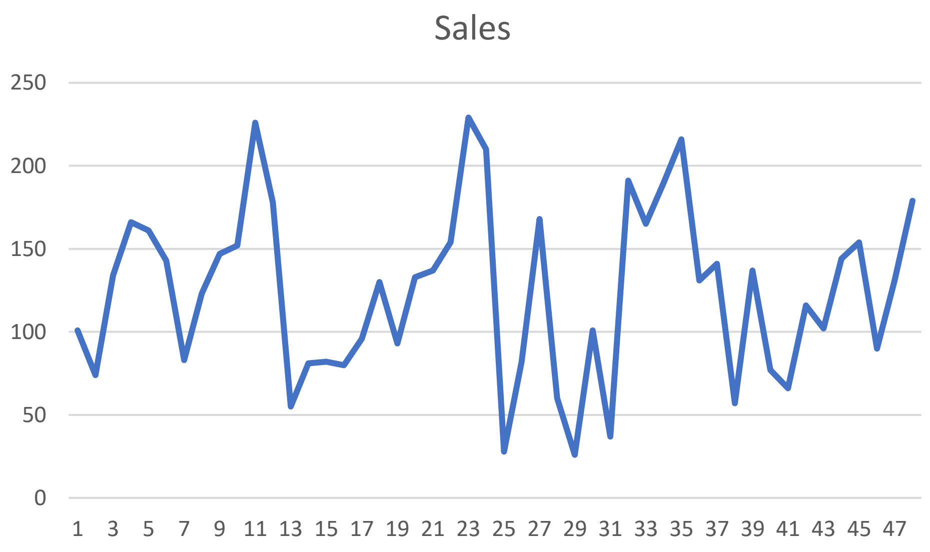
- Here’s a short prescriptive analytics case study.
- Implementation by Bijan Bibak (many thanks)
- We’ll consider the monthly sales of Minis in Turkey



- Data from 2016 to beginning of 2021.
- A few complications: no data recorded for 2018 and 2019, we’ll skip those two years.

‘Mini’ Case: prescriptive analytics

- We’ll consider the monthly inventory ordering problem: set the target inventory each month, assuming c_u=10, c_o = 2.



‘Mini’ Case: prescriptive analytics

- There’s no major visible trend or seasonality (except for the month of December).
- Let us assume that the sample corresponds to i.i.d. Observations.
- We can then use Sample Average Approximation.
- We’ll solve the corresponding stochastic optimization problem.
- As usual, we randomly separate the data to a training set and a test set.

‘Mini’ Case: prescriptive analytics

We solve the below LP using Gurobi Solver.

$$\min_{Q} R(Q) = \frac{1}{n}\sum_{i=1}^{n} c_u\,(d_i - Q)^+ + c_o(Q - d_i)^+$$

$$\equiv \min_{Q} \frac{1}{n}\sum_{i=1}^{n} c_u\, {z_i}^+ + c_o {z_i}^-$$

$$\begin{array}{lcll} s.t. & & & \\ \quad {z_i}^+ & \geq & d_i - Q & i = 1,2,\ldots,n \\ \quad {z_i}^- & \geq & Q - d_i & i = 1,2,\ldots,n \\ {z_i}^+, {z_i}^- & \geq & 0 & i = 1,2,\ldots,n \end{array}$$

‘Mini’ Case: prescriptive analytics

- Results of Sample Average Approximation:

```
Empirical Risk Minimization (train) ------------
Objective function is: 152.667
Optimal Order is: 165.0
Empirical Risk Minimization (test) ------------
Average cost is: 194.727
```

‘Mini’ Case: prescriptive analytics

- Next, we try a joint estimation and optimization formulation using the following predictors:
- t: to capture the effect of trend (if any)
- Dec: a binary variable for the month of December
- Lag 1 ,2, 3: The change in sales at lags 1, 2 and 3.

	A	B	C	D	E	F	G
1	Month	Sales	t	Dec	Lag1	Lag2	Lag3
2	1/1/2016	101	1	0			
3	2/1/2016	74	2	0			
4	3/1/2016	134	3	0	-27		
5	4/1/2016	166	4	0	60	-27	
6	5/1/2016	161	5	0	32	60	-27
7	6/1/2016	143	6	0	-5	32	60
8	7/1/2016	83	7	0	-18	-5	32
9	8/1/2016	123	8	0	-60	-18	-5
10	9/1/2016	147	9	0	40	-60	-18
11	10/1/2016	152	10	0	24	40	-60
12	11/1/2016	226	11	0	5	24	40
13	12/1/2016	178	12	1	74	5	24
14	1/1/2017	55	13	0	-48	74	5
15	2/1/2017	81	14	0	-123	-48	74

‘Mini’ Case: prescriptive analytics

- We then solve the following optimization problem:

$$\min_{Q(\cdot)} R(Q(\cdot), S_n) = \frac{1}{n}\sum_{i=1}^{n} c_u\,(d_i - Q(\mathrm{x}_i))^+ + c_o(Q(\mathrm{x}_i) - d_i)^+$$

$$\equiv \min_{Q=(q^1,q^2,\ldots q^p)} \frac{1}{n}\sum_{i=1}^{n} c_u\,{z_i}^+ + c_o {z_i}^-$$

$s.t.$

$$\begin{aligned}
{z_i}^+ &\geq d_i - \left(q^0 + \sum_{j=1}^{p} q^j x_i^j\right) && i = 1,2,,\ldots,n \\
{z_i}^- &\geq q^0 + \sum_{j=1}^{p} q^j x_i^j - d_i && i = 1,2,,\ldots,n \\
{z_i}^+, {z_i}^- &\geq 0 && i = 1,2,,\ldots,n
\end{aligned}$$

$$Q(\mathrm{x}) = Q((x^1, x^2, \ldots, x^p)) = q^0 + \sum_{j=1}^{p} q^j\, x^j$$

‘Mini’ Case: prescriptive analytics

- Here are the results when we use all five predictors:

```
Feature Based Newsvendor (train) ------------ Objective function is:
108.27052529494726
Average Order Quantity is: 160.84910722851487
Feature Based Newsvendor (test) ------------
Average cost is: 344.65686264558195
Average Order Quantity is: 131.0496792694867
```

$$Q(\mathrm{x}) = Q((x^1, x^2, \ldots, x^p)) = q^0 + \sum_{j=1}^{p} q^j x^j$$

q^0: 195.319, q^1: -1.562, q^2: -14.894, q^3: 0.245 q^4: 0.574, q^5: 0.535

It appears that there is significant overfitting here!

‘Mini’ Case: prescriptive analytics

- Here are the results when we use only the dummy for December

```
Feature Based Newsvendor (train) ------------
Objective function is: 149.273
Average Order is: 162.030
Feature Based Newsvendor (test) ------------
Average cost is: 173.273
Average Order is: 164.09
```

$$Q(\mathrm{x}) = Q(x^1) = q^0 + q^1\, x^1$$

q^0: 161, q^1: 17,

Very simple ordering rule: order 178 in December and 161 in all other months!