INDR 422/522 - Spring 2023 Fikri Karaesmen

CLASS EXERCISE, March 28, 2023

1. We have a stationary time series data. We fit an ARIMA(1,1,0) model. The intercept c is estimated as 100 and the AR(1) coefficient as -0.4. Assume that y_{98} =120, y_{99} =110, what is your forecast for period 100?

   (a) What is your forecast for period 100?
   Solution: We saw in the previous exercise that the open form for ARIMA(1,1,0) is:

   $$Y_t = c + Y_{t-1} + \phi_1(Y_{t-1} - Y_{t-2}) + \epsilon_t = c + (1 + \phi_1)Y_{t-1} - \phi_1 Y_{t-2} + \epsilon_t$$

   therefore, we have:

   $$\hat{y}_{100} = 100 + 110 - 0.4(110 - 120) = 214$$

   (b) What is your forecast for period 101?
   Solution:

   $$\hat{y}_{101} = 100 + \hat{y}_{100} - 0.4(\hat{y}_{100} - 110) = 100 + 214 - 0.4(214 - 110) = 272.4$$

2. Consider the same stationary time series data set and we experiment with different models. Which statements are true?

   (a) ARIMA(0,0,0) estimates the mean of the series. *True*

   (b) We would expect a lower MSE with an ARIMA(1,0,0) than with an ARIMA(0,0,0). *True* ARIMA(1,0,0) includes ARIMA(0,0,0) as a special case.

   (c) We would expect lower AIC with an ARIMA(1,0,0) then with an ARIMA(0,0,0). *False* This is not always true. ARIMA(0,0,0) only estimates the intercept but ARIMA(1,0,0) estimates the intercept and the AR coefficient. Therefore ARIMA(1,0,0) would be penalized for using one more parameter.

   (d) We would expect a lower MSE with an ARIMA(1,0,0) than with an ARIMA(0,0,1). *False*: The two models are not special cases of each other. There is not a clear comparison.

(e) We would expect a lower MSE with an ARIMA(1,0,1) than with an ARIMA(1,0,0) *True*

(f) We would expect a lower MSE with an ARIMA(1,1,0) than with an ARIMA(1,0,0). *False*