INDR 422/522 - Spring 2023 Fikri Karaesmen

CLASS EXERCISE, March 9, 2023

1. Let $Y_t = \mu + \epsilon_t$ where ϵ_t are i.i.d random variables with mean zero and variance σ^2. Consider the forecast: $\hat{y}_{t+1} = (y_t + y_{t-1})/2$.

   (a) $\hat{Y}_{t+1}$ and Y_{t+1} are independent
   Solution: $\hat{Y}_{t+1}$ depends Y_t and Y_{t-1} but by definition of the process (ϵ_t are independent) does not depend on Y_{t+1}. Therefore, the are independent.

   (b) $\hat{Y}_{t+1}$ and Y_t are independent
   Solution: $\hat{Y}_{t+1}$ depends on Y_t. They are dependent and positively correlated.

   (c) $\hat{Y}_{t+1}$ and $\hat{Y}_{t+2}$ are independent
   Solution: $\hat{Y}_{t+1}$ and $\hat{Y}_{t+2}$ both depend on Y_t. Therefore, they are dependent and positively correlated.

   (d) Denote the error by $E(t) = Y(t) - \hat{Y}_t$. What is the variance of the error.
   Solution:

$$Var(E(t)) = Var(Y(t) - \hat{Y}_t) = \sigma^2 + (-1)^2\sigma^2/2 = 3\sigma^2/2.$$

2. Consider the following demand process: $Y_t = mu_1 + \epsilon_t$ if t is odd and $Y_t = mu_2 + \epsilon_t$ if t is even, where ϵ_t are i.i.d random variables with mean zero and variance σ^2 and $\mu_1 > mu_2$.

   (a) Consider $\hat{y}_{t+1} = y_t$. Is this unbiased?
   Solution: If t is even then $E[\hat{Y}_{t+1}] = \mu_2$ but $E[Y_{t+1}] = \mu_1$ since $t+1$ is odd. Therefore, this forecast is not unbiased.

   (b) Consider $\hat{y}_{t+1} = (y_t + y_{t-1} + y_{t-2})/3$. Is this unbiased?
   Solution: If t is even then $E[\hat{Y}_{t+1}] = (2\mu_2 + \mu_1)/3$ but $E[Y_{t+1}] = \mu_1$ since $t+1$ is odd. Therefore, this forecast is not unbiased.

   (c) Consider $\hat{y}_{t+1} = y_{t-1}$. Is this unbiased?
   Solution: If t is even then $E[\hat{Y}_{t+1}] = \mu_1$ and $E[Y_{t+1}] = \mu_1$ since $t+1$ is odd. A symmetric argument holds when t is odd. Therefore, this forecast is unbiased.