INDR 450/550 Additional Exercise, March 21, 2023

The following data gives the sheep population of England and Wales from 1867 to 1939.

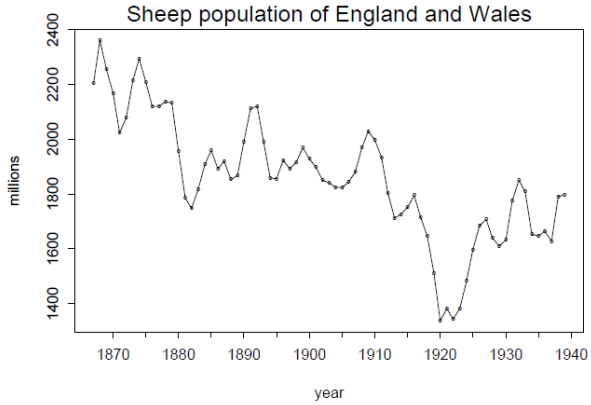

Figure 7-31: *Sheep population (in millions) of England and Wales from 1867–1939. (Source: Kendall, 1976.)*

The data shows trend, so differencing is necessary to obtain a stationary series. And the following is the ACF and PACF plots after differencing:

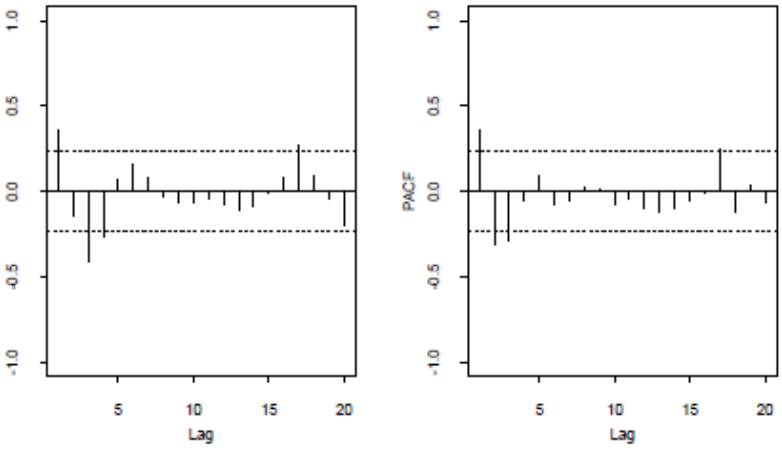

Figure 7-32: *ACF and PACF of differences for sheep data.*

Assume you decide the fit the following model:

$$Y_t = Y_{t-1} + \phi_1(Y_{t-1} - Y_{t-2}) + \phi_2(Y_{t-2} - Y_{t-3}) + \phi_3(Y_{t-3} - Y_{t-4}) + e_t$$

1. Explain why this model is appropriate looking at the ACF and PACF plots.

Answer: A reasonable fit based on the ACF and the PACF would be an ARIMA(3,1,0) model. The data series in Figure 7.31 shows that there is a trend and data is not stationary. Therefore, we need to take first differences before thinking about the AR and MA terms. PACF of the first differences has a spike at the first 3 lags. Furthermore, ACF has a sine-wave pattern. Therefore, AR(3) is appropriate for the differenced data.

2. Assume that the estimated paramaters are for the above model are as follows ϕ_1=0.42, ϕ_2=-0.2, ϕ_3=-0.3. and that the last five years of data are:

Year	1935	1936	1937	1938	1939
Millions of sheep	1648	1665	1627	1791	1797

Give the forecasts for years 1940,41,42.

Answer: The model is written as

$$Y_t - Y_{t-1} = 0.42(Y_{t-1} - Y_{t-2}) - 0.20(Y_{t-2} - Y_{t-3}) - 0.30(Y_{t-3} - Y_{t-4}) + \epsilon_t$$

$$Y_t = 1.42Y_{t-1} - 0.62Y_{t-2} - 0.10Y_{t-3} + 0.30Y_{t-4} + \epsilon_t$$

$$\hat{Y}_{1940} = 1.42Y_{1939} - 0.62Y_{1938} - 0.10Y_{1937} + 0.30Y_{1936} + \epsilon_{1940}$$

$$\hat{Y}_{1940} = (1.42)(1797) - (0.62)(1791) - (0.10)(1627) + (0.30)(1665) + 0 = 1778.1$$

$$\hat{Y}_{1941} = 1.42Y_{1940} - 0.62Y_{1939} - 0.10Y_{1938} + 0.30Y_{1937} + \epsilon_{1941}$$

$$\hat{Y}_{1941} = (1.42)(1778.1) - (0.62)(1797) - (0.10)(1791) + (0.30)(1627) + 0 = 1719.8$$

$$\hat{Y}_{1942} = 1.42Y_{1941} - 0.62Y_{1940} - 0.10Y_{1939} + 0.30Y_{1938} + \epsilon_{1942}$$

$$\hat{Y}_{1942} = (1.42)(1719.8) - (0.62)(1778.1\ ) - (0.10)(1797) + (0.30)(1791) + 0 = 1697.3$$