INDR 422/522 - Spring 2023 Fikri Karaesmen

CLASS EXERCISE, March 23, 2023

1. Take the time series Y_t and let $W_t = Y_t - Y_{t-1}$ Consider an ARIMA(1,1,0) model for Y_t. This is equal to:

   Solution: ARIMA(1,1,0) uses 1 order of differencing (middle term) and an 1 AR term after differencing (first term). We therefore have:

   $$W_t = c + \phi_1 W_{t-1} + \epsilon_t$$

   Reverting the transformation, we have:

   $$Y_t = W_t + Y_{t-1} = c + \phi_1 W_{t-1} + \epsilon_t$$

   We can simplify further to:

   $$Y_t = c + Y_{t-1} + \phi_1(Y_{t-1} - Y_{t-2}) + \epsilon_t = c + (1 + \phi_1)Y_{t-1} - \phi_1 Y_{t-2} + \epsilon_t$$

2. An ARIMA (0,2,0) model for Y_t corresponds to:

   Solution: ARIMA(0,2,0) uses 2 orders of differencing (middle term) but no AR and MA terms. We have $W_t = Y_t - Y_{t-1}$ and $U_t = W_t - W_{t-1}$ and

   $$U_t = c + \epsilon_t$$

   We can then revert the transformations: $W_t = U_t + W_{t-1}$ and $Y_t = W_t + Y_{t-1}$

   $$Y_t = Y_{t-1} + U_t + W_{t-1} = Y_{t-1} + c + \epsilon_t + (Y_{t-1} - Y_{t-2})$$

   Simplifying, we obtain:

   $$Y_t = c + 2Y_{t-1} - Y_{t-2} + \epsilon_t$$

3. The backshift notation representation for ARIMA(1,3,0) is:

   Solution: The representation is:

   $$(1 - \phi_1 B)(1 - B)^3 y_t = c + \epsilon_t$$

4. In backshift notation, $Y_t = c + 2Y_{t-1} + Y_{t-2} + \epsilon_t$ corresponds to:

   Solution:

$$Y_t = c + 2BY_t + B^2 Y_t + \epsilon_t$$

   or

$$Y_t - 2BY_t + B^2 Y_t = (1 - B)^2 Y_t = c\epsilon_t$$